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ON IMBEDDINGS OF SOME SEPARABLE EXTENSIONS IN GALOIS EXTENSIONS

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Throughout B will mean a ring with identity element 1, and all ring extensions of B will be assumed to have the (common) identity element 1. As a sequel of imbedding theorems in field theory, we have the following problem: *If a ring extension A/B is f. g. projective and separable then can A/B be imbedded in a Galois extension N/B ?*

First, we consider a commutative ring A which is a ring extension of B. By [1], M. Auslander and O. Goldman proved that A/B is f. g. free and separable then A/B can be imbedded in a Galois extension. Moreover, in [8], O. E. Villamayor proved that this result is also true for any projective separable extension with projective rank. However, more generally, there holds that *any projective separable extension can be imbedded in a Galois extension* ([6]).

Now, let $B[X]$ be a polynomial ring over a commutative ring B. Then, if a monic $f \in B[X]$ is separable then the discriminant $\delta(f)$ is inversible in B, and conversely; when this is the case, the factor ring $B[X]/(f)$ (which is a ring extension of B) can be imbedded in a Galois extension N/B such that $N = B[x_1, \ldots, x_n]$, $B[x_1] = B[X]/(f)$, and $f = (X - x_1) \cdots (X - x_n)$, which will be called *a splitting ring of f* ([5]). In case B is irreducible, there exists an (irreducible) polynomial closure

C(B) so that any separable polynomial in $B[X]$ has a splitting ring in *C(B)*, and *C(B)* is generated by the subrings of *C(B)* which are splitting rings of separable polynomials in $B[X]$ ([2], [4]).

Next, we consider a non commutative ring B and a skew polynomial ring $B[X;\rho]$ of ρ-automorphisms type. Then, for a polynomial f in $B[X;\rho]$ of degree *2*, f is Galois if and only if f is $\tilde{\rho}$-separable ([3], [7], [8]). Moreover, *if* $f \in B[X;\rho]$ *is* $\tilde{\rho}$*-separable then* $B[X;\rho]/(f)$ *can be imbedded in a Galois extension which is a splitting ring of* f*. Further, in case* B *is simple, we can discuss (simple) splitting rings of separable polynomials in* $B[X;\rho]$ *and (simple) polynomial closures* $C(B;\rho)$ *in various ways.* Those results will be seen later in a paper (to appear).

References

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