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ON THE STRENGTH DISTRIBUTION OF THE EARTH'S CRUST AND THE UPPER MANTLE, AND THE DISTRIBUTION OF THE GREAT EARTHQUAKES WITH DEPTH

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Abstract

It seems that the strength of the earth's crust and upper mantle has some relation with the magnitude of the earthquakes. In this paper, we try to estimate the strength distribution of the earth's crust and upper mantle from the experimental results carried out under high pressure and temperature.

We compare the above result with the distribution of the great earthquakes with depth, which occurred in and near Japan in 1926-1956. It seems that the both are fairly resemble each other in tendency.

Introduction

The geographycal distribution of the earthquakes has been studied in detail and well known as the seismic zone. On the other hand, noticing the distribution with depth, it will be seen that the distribution of the hypocentres of the great earthquakes with depth has the considerably conspicious feature, even of the shallow focus earthquakes within the depth of 100 k.m..

The abundant problems pertaining to the mechanism of the earthquakes will not be able to be proved till we get the sufficient knowledges for the time and space distribution of the forces in the crust and upper mantle, and for the constitution of there. Nevertheless, it will be the reasonable inference that the magnitude of the earthquakes will depend on the mechanical strength of the hypocentre, and it will not be the futile attempt to estimate the strength of the crust and upper mantle.

Because the large amount of energy should be stored in the region where the strength is large, and the less do in the region of the small strength.

In this paper, we try to estimate the distribution of the strength of the crust and upper mantle with depth by the experimental results on the strength of rocks under high pressure and temperature. Then we consider the distribution of the great earthquakes with depth, which occurred in and near Japan, with references to the above estimated results.

Pressure, Temperature and Constituents of the Crust and Upper Mantle

The strength of rocks is strongly affected by confining pressure and temperature, and these effects are different for rocks of different types. Then we must know the pressure and temperature distributions and the constituent rocks of the crust and upper mantle to evaluate the strength there.

Of the pressure distribution, we have had the sufficient knowledge for our purposes[1]. This is shown in Fig. 1.

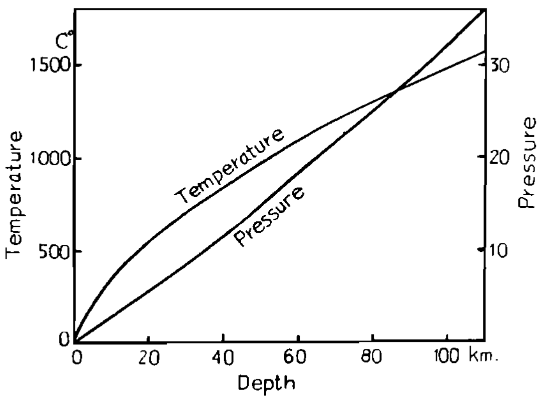

Fig. 1. Pressure and temperature distribution in the earth's crust and the upper mantle.

Of the temperature distribution, on the other hand, the various estimations have been presented by many authors[1] and we have had no definitive distribution. These distribution curves, however, are approximately coincident in tendeny and magnitude one another. So we will take the average of these curves as the temperature distribution of the crust and upper mantle. This curve is shown in Fig. 1, which will not give so large error for the temperature within the superfiical layers of the earth.

Next we consider the constituent rocks of the crust and upper mantle. The boundary between the crust and the upper mantle shows very remarkable discontinuity and has been well known as Mohorovicic discontinuity. From the seismological and geological evidences, it has been clarified that the crust is composed of relatively acidic rocks such as granite and that the upper mantle is done of basic rocks such as dunite or so. In the following discussions, we assume the simplest model of the crust and upper mantle, that is, the crust is composed of granite and the main constituent rocks of the upper mantle are dunite or similar rocks.

Variation of the Strength of Rocks with Confining Pressure

In the previous paper[2], we studied the variation of strength with various confining pressures for Kitashirakawa granite. This and subsequently obtained results are shown in Fig. 2. The empirical formula of the strength versus pressure for this rock is given

$$P_p^* = P_0^*(k \cdot p + 1)^{1/2},$$

where P_0^*, P_p^* are the strength at an atmospheric pressure and confining pressure of p k.b., respectively, and k is the characteristic constant for the rock.

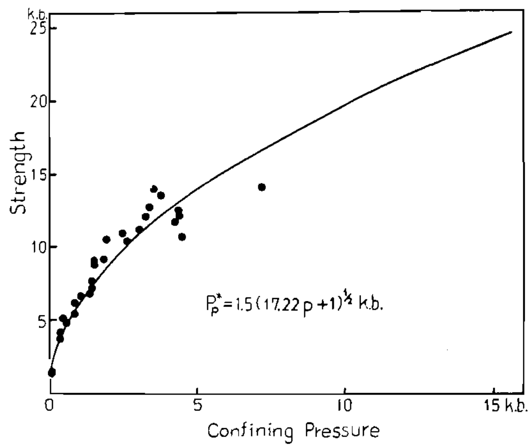

Fig. 2. Variation of strength of granite with confining pressure at room temperature.

Here we assume that the above formula holds in all types of silicate rocks, though we can not assert strongly this assumption for the accumulation of the experimental results are not sufficient to determine the plausible formula.

As k and P_0^* for granite, we use the experimentally derived values for Kitashirakawa granite, that is, $k = 17.22$ (k.b.)$^{-1}$ and $P_0^* = 1.5$ k.b.. The average value of P_0^*, listed in "Handbook of Physical Constants[3]", is 1.48 k.b., so that the above value seems to be appropriate.

Next, we must find the values of k and P_0^* for dunitic rocks. There has been no experimentally derived value of k for this sort of rocks, so we will determine this value from the values of P_0^* and P_p^* (strength under certain confining pressure). In "Handbook of Physical Constants", average value of P_0^* for gabbro, diabase, etc. is listed. This is 1.8 k.b.. The strength of gabbro, basalt and diabase, which we obtained, was about 2.0 k.b.. Then we put $P_0^*=2.0$ k.b. for dunitic rocks as the approximate value. D. T. Griggs et al.[4] has observed the stress strain relations for dunite under 5.0 k.b. confining pressure. In their report, the strength of dunite at room temperature is shown as about 21 k.b.. From the values of P_0^* and P_5^*, we can obtain the value of $k=22.0$ (k.b.)$^{-1}$ for dunitic rocks.

In Table 1, the strength of granitic and dunitic rocks under pressures corresponding to the various depths from the earth's surface, which is calculated by the above mentioned way, is listed.

Table 1. Strength of granite and dunitic rocks under pressures corresponding to the depths from the earth's surface.

Depth k.m.	Pressure k.b.	Strength Granite k.b.	Strength Dunitic rocks k.b.
0	0	1.5	2.0
5	1.3	7.3	10.9
10	2.7	10.3	15.5
20	5.5	14.7	22.1
30	8.5	18.2	27.4
40	11.5	21.2	31.9
50	15.0	24.2	36.4
60	18.5	26.8	40.4
80	25.5	31.5	47.4
100	32.0	35.3	53.1

Reduction of the Strength caused by the Elevated Temperature

It has been currently accepted that the strength of rocks is considerably reduced by the elevated temperature, but it has not been made clear quantitatively how much amount of reduction is brought about.

Recently, D. T. Griggs et al., as above mentioned, has experimentally investigated the stress-strain relations for various sorts of rocks at the temperatures up to 800°C and pressures of 5.0 k.b..

We will quote their results and picture again the temperature-strength relations for granite and dunite. This is shown in Fig. 3. As seen in this figure, it seems that the linear relation exists between the strength and temperature for granite, and exponentially decreasing relation for dunite. Here we assume that the following formula can be adopted for granite as the empirical one and that this formula is held all over the pressure range, that is,

$$ {}_{T}P_{p}^{*} = {}_{0}P_{p}^{*}(1 - T/1100), $$

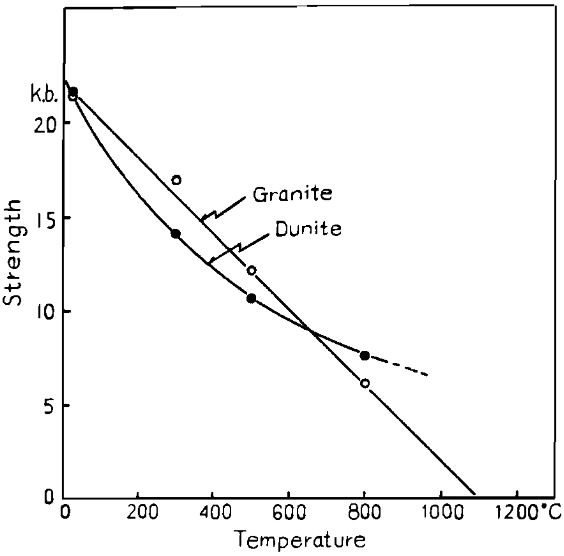

Fig. 3. Variation of strength of granite and dunite with temperature, at 5.0 k.b confining pressure (after D. T. Griggs et al.).

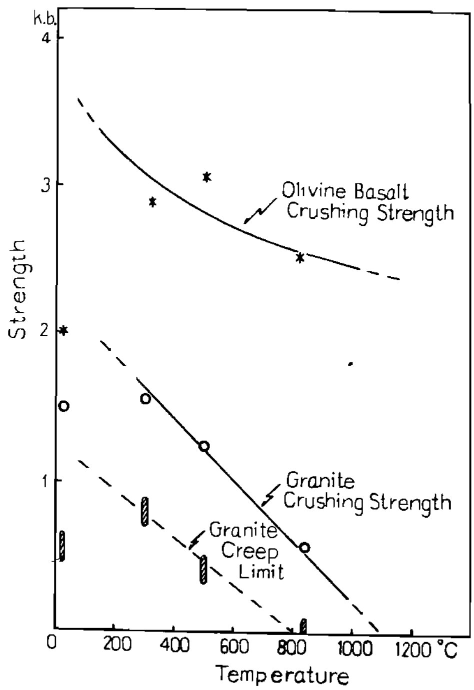

Fig. 4. Variation of strength of granite and olivine basalt with temperature, at an atmospheric pressure. The upper and lower solid curves show the crushing strength of olivine basalt and granite respectively, and the dotted curve shows the estimated creep limit of granite.

where ${}_TP_p^*$, ${}_0P_p^*$ are the strength at temperatures of T°C and 0°C under confining pressure p k.b., respectively.

The temperature-strength relation for dunite is not given in such a simple form as for granite, then we evaluate the strength reduction graphycally from the curve in Fig. 3. That is, the strength at T°C, p k.b. is given as the product of the strength at 0°C, p k.b., which was estimated in the former section, by the ratio of strength at T°C and 0°C under 5 k.b.. This is given formally as follows,

$${}_TP_p^* = {}_0P_p^* \left({}_TP_5^* / {}_0P_5^*\right).$$

In addition to the above argument, we refer to the temperature de-

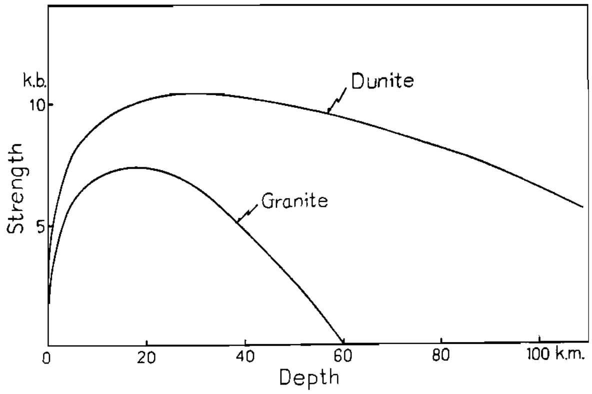

Fig. 5. Strength distribution in the earth's crust and the upper mantle.

Table 2. Strength of the earth's crust and the upper mantle taking account of the temperature and pressure.

Depth k.m.	Pressure k.b.	Temperature °C	Strength Granite k.b.	Strength Dunitic rocks k.b.
0	0	0	1.5	2.0
5	1.3	200	5.95	8.0
10	2.7	350	7.05	9.15
20	5.5	550	7.35	10.15
30	8.5	700	6.6	10.6
40	11.5	850	4.8	10.15
50	15.0	980	2.6	9.9
60	18.5	1100	0.0	9.55
80	25.5	1300		8.2
100	32.0	1500		6.3

pendence of strength of granite and olivine basalt in our experiments at an atmospheric pressure[6]. This is shown in Fig. 4. This results will suggest that the above treatment is not so unreasonable.

Thus we can estimate the strength distribution within the earth's crust and upper mantle taking account of the temperature and pressure of there. This is shown in Table 2 and Fig. 5.

Creep Limit of the Earth's Crust

If the earthquakes are caused by forces generated suddenly[6], the crushing (or yield) strength obtained in the foregoing sections should give the actual one in the crust and upper mantle. On the other hand, if the earthquakes are caused by forces accumulated gradually, the above obtained strength does not give the real strength. The stress is released constantly by the flow or the creep in such a rather static process, and rupture must be occurred by further lower stress than the crushing strength. Then we must estimate the creep limit in the crust and upper mantle.

For this problem, we scarcely have any experimental data to be available. Nevertheless, we will try to estimate the rather qualititative distribution of the creep limit within the earth's crust, using the very rough-and-ready presumptions. Creep limit can be obtained only by the very long period tests, but such experiments are considerably difficult to be carried out sufficiently, especially at high pressure and temperature. Thus we assume that the creep limit corresponds to the point at which the curve of the rate of creep versus stress comes in contact asymptotically with the stress axis.

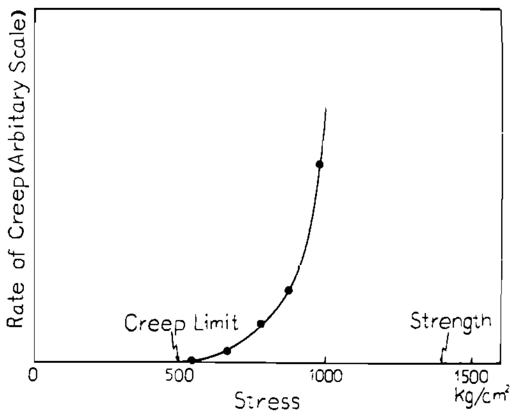

Fig. 6. Relation between the rate of creep and stress, and the estimated creep limit for Kitashirakawa granite, at an atmospheric pressure and room temperature.

In Fig. 6, the curve of the rate of creep versus stress for Kitashirakawa granite at an atmospheric pressure and room temperature is shown. Fig. 7 shows the variation of creep limit with confining pressures for Kitashirakawa granite obtained by the above method. The dotted line in Fig. 4 shows the relation between creep limit and temperature at an atmospheric pressure for this granite.

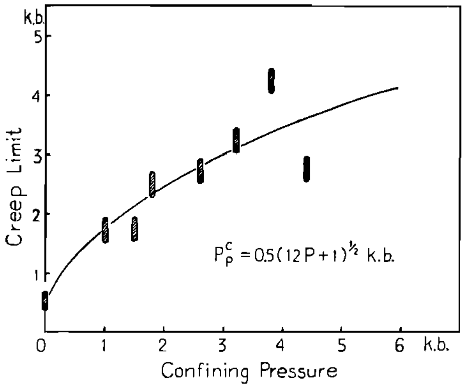

Fig. 7. Variation of the estimated creep limit with confining pressure for Kitashrakawa granite, at room temperature.

It will be seen from the figures that the empirical formulas of the variation of creep limit with confining pressure and temperature have the same form as the one of crushing (or yield) strength. That is,

$$P_p{}^c = P_0{}^c (k\cdot p+1)^{1/2},$$

and

$$_TP_p{}^c = {}_0P_p{}^c (1-T/800),$$

where the index c instead of * appeared in crushing (or yield) strength denotes the creep limit.

Using the results shown in Fig. 7 and Fig. 4, we can roughly estimate the creep limit of the earth's crust. The result is shown in Fig. 8. Of course, the quantitatively plausible result must be recomputed after the accurate tests of creep under high pressure and temperature.

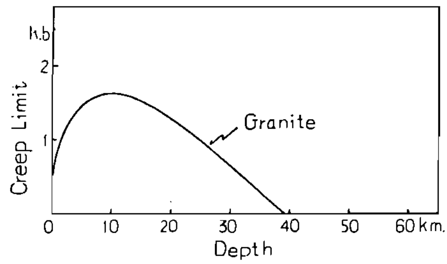

Fig. 8. Distribution of the creep limit with depth in the crust.

Distribution of the Great Earthquakes with Depth

Let us consider the distribution of hypocentres of the great earthquakes with depth. We pick up the shallow focus earthquakes, the magnitudes of which are larger than 6.3, which occurred in and near

Japan from 1926 to 1956. The data are listed in "The Seismological Bulletin of the Japan Meteorological Agency, Supplementary Volume No. 1[7]".

We consider the seimic zone in Japan dividing into two parts by Fuji Volcanic Belt. In North-eastern Japan, the earthquakes have occurred mainly under the Pacific Ocean along the coast and these hypocentres are rather deep in general. On the other hand, in South-western Japan, the earthquakes have occurred mainly under the land and the depth of hypocentres is shallow. This means, that is to say, that the earthquakes have occurred in the upper mantle in North-eastern Japan and in the

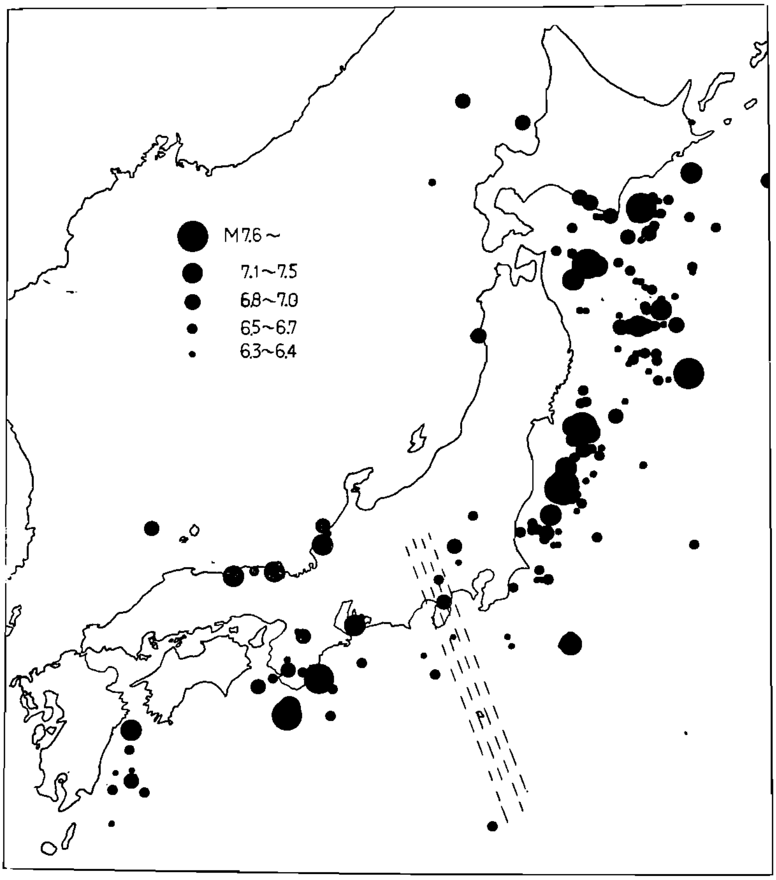

Fig. 9. Geographycal distribution of the great earthquakes which occurred in and near Japan in 1926-1956.

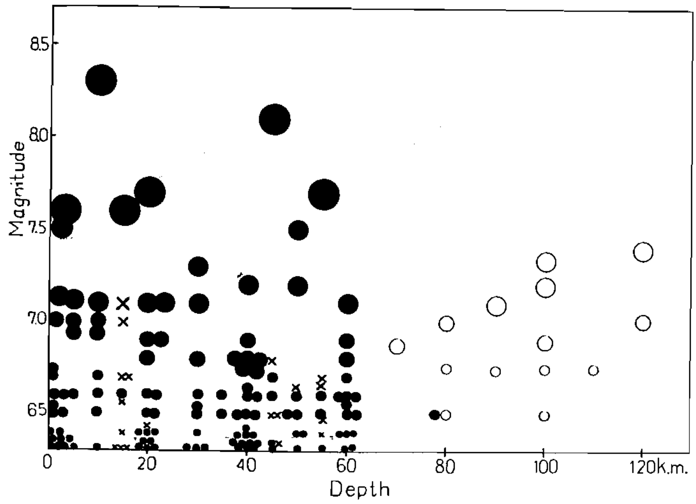

Fig. 10(a).

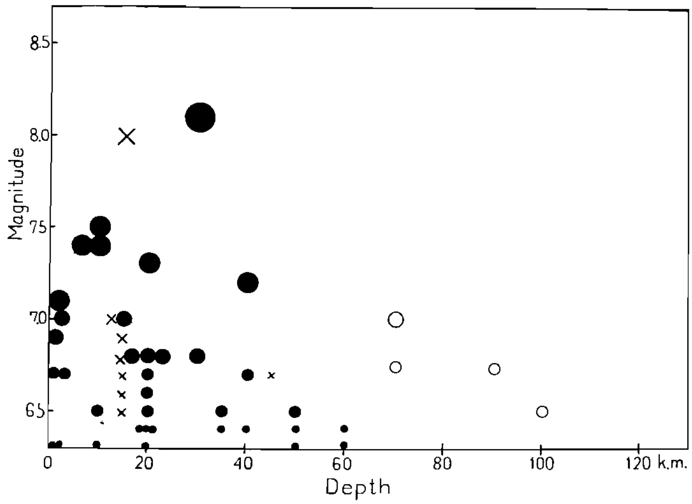

Fig. 10(b).

Fig. 10. Distribution of the great earthquakes with depth, which occurred in and near Japth, in 1926-1956. (a) In North-eastern Japan, and (b) in South-western Japan. The full circles and empty circles express th magnitudes determined by the Japan Meteor. Agency and by B. Gutenberg respectively, and the X marks express the earthquakes, the depths of which are not determined definitely.

crust in South-western Japan. Fig. 9 shows the geographycal distributions of epicentres in and near Japan. Fig. 10(a), (b) show the distributions of hypocentres in North-eastern and South-western Japan respectively.

Comparing Fig. 10 with Fig. 5, it seems that the tendencies of both distributions with depth are fairly alike to each other. Fig. 10(a) corresponds to the upper curve in Fig. 5, strength of dunitic rocks, and Fig. 10(b) to the lower curve, strength of granite.

If the energy of earthquake is chosen as the vertical coordinate, the curves in Fig. 10 show the rather sharp maximum. This will mean that the concept of the "earthquake volume[8]" is very available, because the earthquake which occurs in the region with great strength should have the much larger volume than the one which occurs in the region with less strength.

Acknowledgement

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References

1) B. Gutenberg, "Internal constitution of the Earth.", (1949).
2) S. Matsushima, "On the deformation and fracture of granite under high confining pressure.", Disaster Prevention Research Inst. Bull. Kyoto Univ. No. **36** (1960).
3) F. Birch, "Handbook of Physical Constants.", Geol. Soc. Amer. Sp. No. **36** (1942).
4) D. T. Griggs et. al., "Deformation of rocks at 500° to 800°C.", Geol. Soc. Amer. Mem. **79**, 39-104 (1960).
5) S. Matsushima, unpublished.
6) T. Matsuzawa, "Feldtheorie der Erdbeben." Bull. Earthquake Res. Inst. Tokyo Univ. **31**, 179-201 (1953).
7) Japan Meteorological Agency, "Catalogue of major earthquakes which occurred in and near Japan.", Seism. Bull. Jap. Meteor. Agency Supple. Vol. No. **1** (1958).
8) C. Tsuboi, "On seismic activities in and near Japan.", Contributions in Geophysics 87-112 (1958).

Shibata, 1958.

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