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Synopsis

Hitherto the vibrations of earth dams have been discussed as to shearing ones. But according to the results of numerical calculations by the finite difference method on the fundamental dam section as a two-dimensional body, the vibration should be considered to be a two-dimensional one with vertical as well as horizontal displacements. The theoretical two-dimensional vibration is compared with the shearing one in steady and transitional states.

Introduction

Earth dams, embankments or levees, with fundamental triangle sections, have the bases much longer than the heights. When the earth dam is disturbed in the horizontal direction at the base, the vibrational distortions due to bending are very small, so that the problems of earth dams have been treated as to the shearing vibration. But as the results of field experiments, model tests and some theoretical considerations, on which some papers were already published by the authors (1957), the vibration should be considered to be a two-dimensional one, taking vertical as well as horizontal displacements into account. In addition, the test of model dams, which were made of mixed cement, wheat flour and machine oil and fixed on the shaking table, showed the fact that the first cracks were developed perpendicular to the dam slope near the bases of dams and that the second cracks followed the first and so on, until the dam body was broken down. The cracks in early stages would be caused by the tensile stresses near the surface of the dam.

In this paper the earth dam is supposed to be an elastic body on the rigid ground and to be applied by horizontal disturbance at its base, and some numerical calculations are made by the finite difference method on the fundamental dam section as a two-dimensional body. The displacements and the principal stresses of points in the dam body are derived from these calculations. From these results, the horizontal displacements and the shearing stresses of the fundamental dam section as a two-dimensional body are calculated and they are compared with those of the shearing vibration in steady and transitional states.

Differential Equations of Motion and Finite Difference Equations for the Symmetrical Wedge-Shaped Dam

1. Differential equations

The assumed section of the dam which is symmetrical and uniform in the longitudinal direction is treated. We apply the finite difference method to analyse the vibrations of dams with the slopes of 1 : 1 and 1 : 2. Taking coordinates shown in Fig. 1, u and v are the components of the elastic displacements in the x and y directions respectively, so that the equations of motion of the dam are expressed by the following ones,

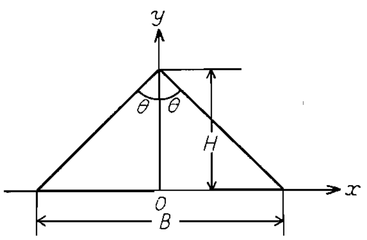

Fig. 1. Coordinate system.

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda+2\mu) \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + (\lambda+\mu) \frac{\partial^2 v}{\partial x \partial y} \quad \cdots\cdots\cdots\cdots\cdots\cdots(1)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda+2\mu) \frac{\partial^2 v}{\partial y^2} + \mu \frac{\partial^2 v}{\partial x^2} + (\lambda+\mu) \frac{\partial^2 u}{\partial x \partial y} \quad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where λ and μ are Lame's constants and ρ is the density of the dam. The stress-components are expressed as follows,

$$\left.\begin{aligned} \sigma_x &= \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu \frac{\partial u}{\partial x} \\ \sigma_y &= \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu \frac{\partial v}{\partial y} \\ \tau_{xy} &= \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

The solution of Eqs. (1) and (2) must be satisfied by the following conditions :

(1) The normal stress and the shearing stress should vanish on the free surface of the dam at all time, and therefore, in the case of the slope of 1 : 1, boundary conditions are expressed by the following equations :

$$\left.\begin{aligned}(\lambda+\mu)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)&=\pm\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\\ \frac{\partial u}{\partial x}&=\frac{\partial v}{\partial y}\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

In the case of the slope of 1 : 2, it follows :

$$\left.\begin{aligned}5\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)&=\pm2\mu\left(\frac{\partial u}{\partial x}+2\frac{\partial u'}{\partial y}+4\frac{\partial v}{\partial y}+2\frac{\partial v}{\partial x}\right)\\ 4\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)&=\pm3\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\end{aligned}\right\}\quad\cdots\cdots\cdots(5)$$

(2) The transversal wave, which is expressed $u=A\sin pt$, should act at the base of the dam.

2. Finite difference equations

The first step in the solution of the differential equation is the replacement of the differential equations by a set of finite difference approximations. The dam is divided into constant spacings h and k in the x and y directions respectively. Any point in the dam denotes a typical node 0 at any time of t, and u and v are the displacements in the x and y directions. Eight surrounding nodes 1, 2, 3,......7, 8 suffixes will be used to indicate values of the variables at correspondingly numbered nodes. And the displacements of node 0 at the time of $t-\tau$ and $t+\tau$ are u_I, v_I and u_{II}, v_{II} as shown in Fig. 2. Each partial differential terms of Eqs. (1) and (2) are expressed approximately by Taylor's series. From Eqs. (1) and (2) the components of the displacement after one time interval τ will be expressed by the following finite difference equations in two cases respectively. In the case of the interval for the finite difference $h=k$, it follows :

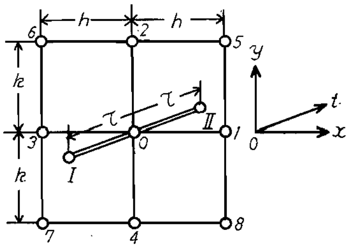

Fig. 2. Point pattern.

$$u_{II}=2\left\{1-\left(\frac{\lambda+3\mu}{\rho}\right)\left(\frac{\tau}{h}\right)^2\right\}u_0-u_I+\left(\frac{\lambda+2\mu}{\rho}\right)\left(\frac{\tau}{h}\right)^2(u_1+u_3)$$
$$+\left(\frac{\mu}{\rho}\right)\left(\frac{\tau}{h}\right)^2(u_2+u_4)+\left(\frac{\lambda+\mu}{4\rho}\right)\left(\frac{\tau}{h}\right)^2(v_5-v_6+v_7-v_8)\quad\cdots\cdots(6)$$

$$v_{II}=2\left\{1-\left(\frac{\lambda+3\mu}{\rho}\right)\left(\frac{\tau}{h}\right)^2\right\}v_0-v_I+\left(\frac{\lambda+2\mu}{\rho}\right)\left(\frac{\tau}{h}\right)^2(v_2+v_4)$$
$$+\left(\frac{\mu}{\rho}\right)\left(\frac{\tau}{h}\right)^2(v_1+v_3)+\left(\frac{\lambda+\mu}{4\rho}\right)\left(\frac{\tau}{h}\right)^2(u_5-u_6+u_7-u_8)\cdots\cdots(7)$$

In the case of the interval for the finite difference $h=2k$, it follows:

$$u_{II}=2\left\{1-\left(\frac{\lambda+6\mu}{4\rho}\right)\left(\frac{\tau}{k}\right)^2\right\}u_0-u_I+\left(\frac{\lambda+2\mu}{4\rho}\right)\left(\frac{\tau}{k}\right)^2(u_1+u_3)$$
$$+\left(\frac{\mu}{\rho}\right)\left(\frac{\tau}{k}\right)^2(u_2+u_4)+\left(\frac{\lambda+\mu}{8\rho}\right)\left(\frac{\tau}{k}\right)^2(v_5-v_6+v_7-v_8)\cdots\cdots(8)$$
$$v_{II}=2\left\{1-\left(\frac{4\lambda+9\mu}{4\rho}\right)\left(\frac{\tau}{k}\right)^2\right\}v_0-v_I+\left(\frac{\lambda+2\mu}{\rho}\right)\left(\frac{\tau}{k}\right)^2(v_2+v_4)$$
$$+\left(\frac{\mu}{4\rho}\right)\left(\frac{\tau}{k}\right)^2(v_1+v_3)+\left(\frac{\lambda+\mu}{8\rho}\right)\left(\frac{\tau}{k}\right)^2(u_5-u_6+u_7-u_8)\cdots\cdots(9)$$

Similarly, boundary conditions on the free surface of the dam are expressed by the following equations from Eqs. (4) and (5) in two cases respectively.

$$\left.\begin{aligned}u_2-u_4+v_1-v_3&=\pm2\{(\lambda+\mu)/\mu\}(u_1-u_3)\\&=\pm2\{(\lambda+\mu)/\mu\}(v_2-v_4)\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

or

$$\left.\begin{aligned}u_5+u_6-u_7-u_8+v_5-v_6-v_7+v_8&=\pm2\{(\lambda+\mu)/\mu\}(u_5-u_6-u_7+u_8)\\&=\pm2\{(\lambda+\mu)/\mu\}(v_5+v_6-v_7-v_8)\end{aligned}\right\}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots(11)$$

In the case of $h=2k$, it follows:

$$\left.\begin{aligned}u_1-u_3+2(v_2-v_4)&=\pm\frac{2\mu}{5\lambda}\{u_1-u_3+8(v_2-v_4)+4(u_2-u_4)+2(v_1-v_3)\}\\u_1-u_3-2(v_2-v_4)&=\pm\frac{3}{4}\{2(u_2-u_4)+(v_1-v_3)\}\end{aligned}\right\}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots(12)$$

or

$$\left.\begin{aligned}&u_5-u_6-u_7+u_8+2(v_5+v_6-v_7-v_8)\\&\quad=\pm\frac{2\mu}{5\lambda}\{5u_5+3u_6-5u_7-3u_8+2(5v_5+v_6-5v_7-3v_8)\}\\&u_5-u_6-u_7+u_8-2(v_5+v_6-v_7-v_8)\\&\quad=\pm\frac{3}{4}\{2(u_5+u_6-u_7-u_8)+(v_5-v_6-v_7+v_8)\}\end{aligned}\right\}\cdots\cdots\cdots(13)$$

We can perform the numerical calculations on the vibrations of the dam by using above Eqs. (6), (7), (10), (11), or (8), (9), (12), (13) step by step at every short time interval.

Numerical Calculations

We assume that the maximum amplitude of the transversal wave is 1 and that longitudinal and transversal wave velocities are $V_p = 200$ m/s and $V_s = 100$ m/s respectively. Therefore, Poisson's ratio of the dam is 1/3. Numerical calculations are made in the following two cases.

(1) We suppose that the height of the dam is $H = 10$ m, the width of its bottom $B = 20$ m, the interval for the finite difference method $h = k = 2$ m

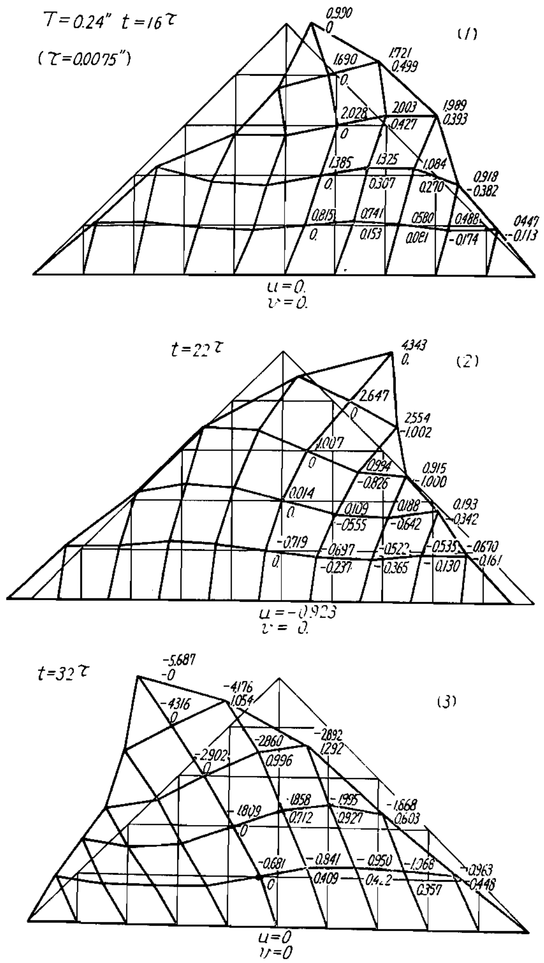

Fig. 3. Displacements due to the two-dimensional vibration by the external wave period 0.24 sec.

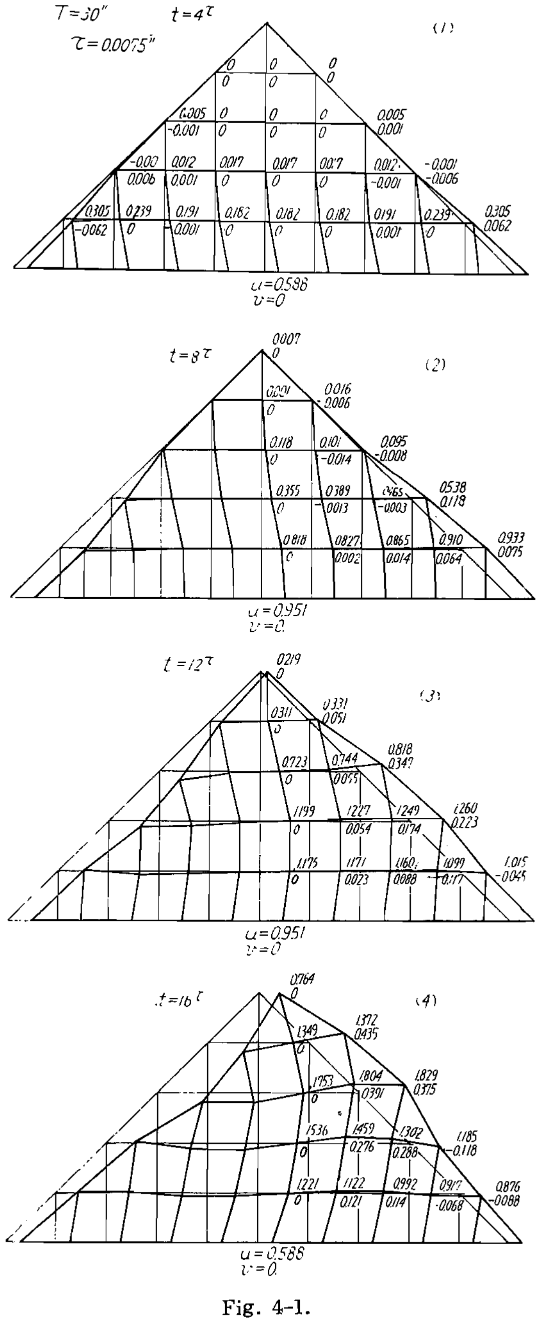
T=30″
t=4τ
τ=0.0075″
(1)
u=0.586
v=0
t=8τ
(2)
u=0.951
v=0
t=12τ
(3)
u=0.951
v=0
t=16τ
(4)
u=0.588
v=0

Fig. 4-1.

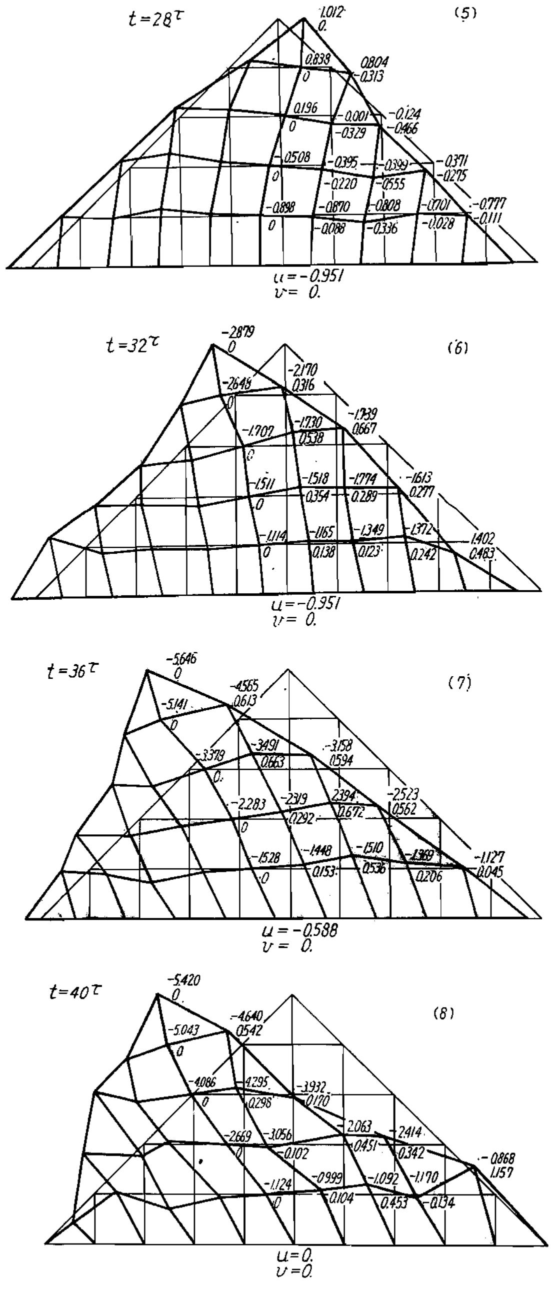

Fig. 4-2.

Fig. 4. Displacements due to the two-dimensional vibration by the external wave period 0.30 sec.

and the periods of the incidental wave acting at the base $T=0.24''$, $0.30''$ and $0.42''$ Dividing one period of $T=0.24''$, $0.30''$ and $0.42''$ into thirty, forty and sixty parts respectively, corresponding one time intervals are $0.0075''$, $0.0075''$ and $0.0070''$ Some results of calculations are shown in Figs. 3, 4 and 5.

(2) We suppose that the height of the dam is $H=10$ m, the width of its bottom $B=40$ m, the interval for the wave acting at the base $T=0.24''$ Dividing one period into thirty-two parts, one time interval is $0.0075''$.

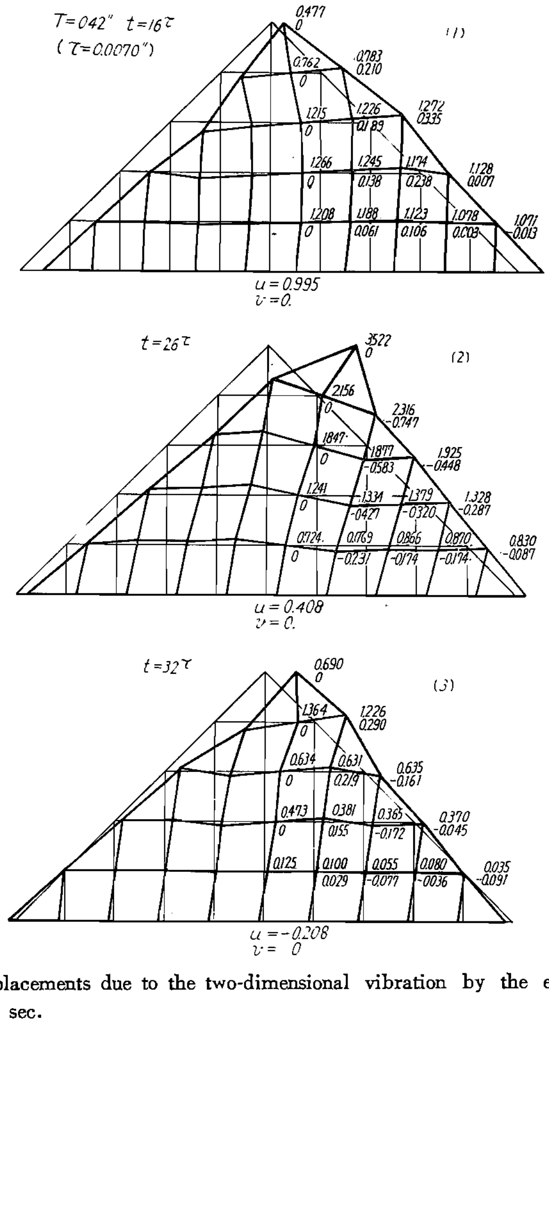

Fig. 5. Displacements due to the two-dimensional vibration by the external wave period 0.42 sec.

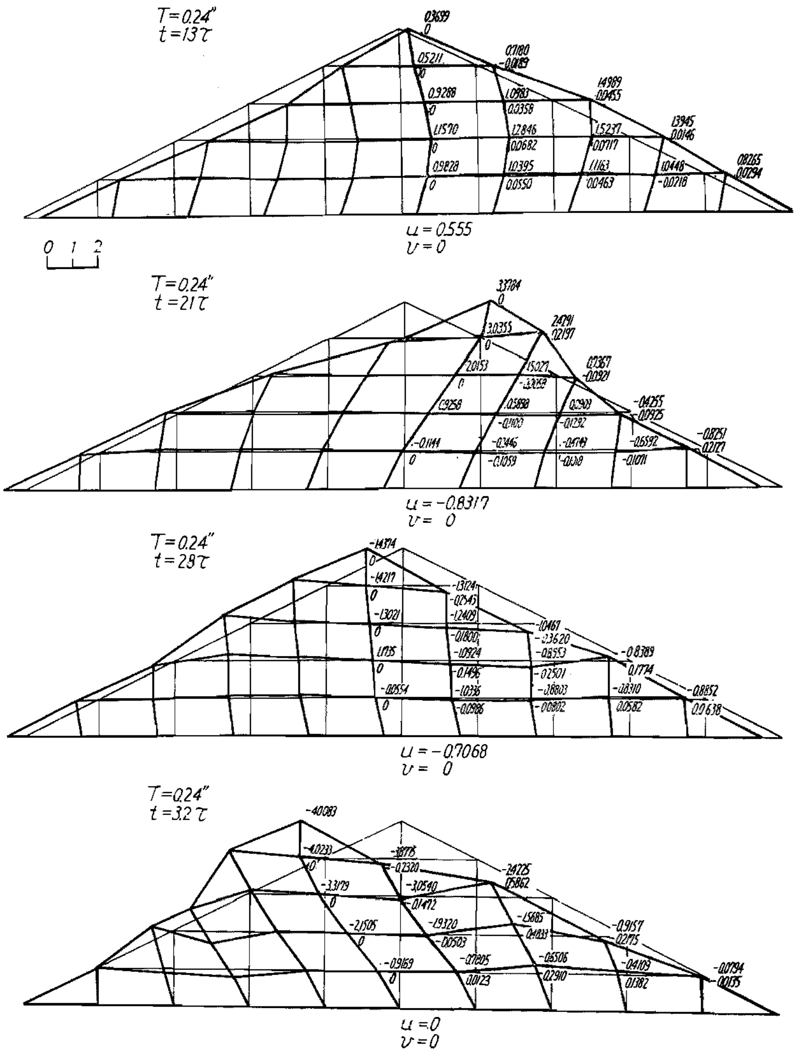

Fig. 6. Displacements due to the two-dimensional vibration by the external wave period 0.24 sec.

Some results of calculations are shown in Fig. 6.

Considerations on Displacements

Several examples of numerical calculations confirm the following considerations.

(1) It is clear that the incidental wave propagates in the dam with the passage of time and that the dam deforms gradually.

(2) All displacements at any points of the dam are almost horizontal until the incidental wave arrives at the top of the dam. But, with the elapse of time, the vertical displacement gradually increases after the wave had arrived at the top of the dam.

(3) In the center line of the symmetrical dam, the displacements are always horizontal. In the center of the dam, therefore, the vibration resembles the shearing one.

(4) As the wave propagates in the dam from the base to the top, the dam begins to expand and contract on the right and left sides of the lower parts of the dam respectively. But after a while, the dam begins to contract and expand on the right and left sides of the lower parts of the dam. Such a condition proceeds upward until the slopes of the right and left sides are entirely contracted and expanded. Next, at the top, the right side of the dam is expanded and the left side is contracted. And in one period, the right and left sides are entirely expanded and contracted respectively. The vibrational characteristics of the gentle slope dam have quite the same tendency as the steep slope dam.

(5) The inner parts as well as the surface of the dam are expanded and contracted.

(6) The vibration which accompanies the expansion and contraction of the dam body is entirely different from the shearing one as explained above.

Calculations of Principal Stresses

The displacements of any points of the dam at any time have been derived from the above-mentioned calculations, so the principal stress and its direction may be easily calculated. Maximum and minimum stresses may be expressed approximately by the following relationships:

In the case of the interval for the finite difference $h=k$, it follows;

$$\sigma_{\substack{max \\ min}}=\frac{\lambda+\mu}{2h}(u_1-u_3+v_2-v_4)\pm\frac{\mu}{2h}\sqrt{(u_1-u_3-v_2+v_4)^2+(u_2-u_4+v_1-v_3)} \qquad \cdots\cdots(14)$$

In the case of the interval for the finite difference $h=2k$, it follows;

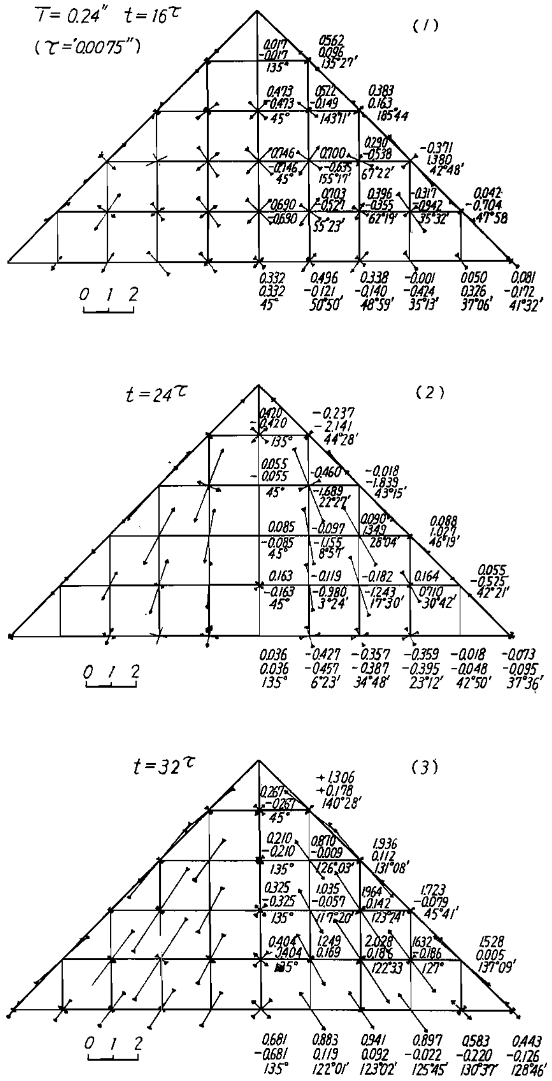

Fig. 7. Principal stress distributions due to the two-dimensional vibration by the external wave period 0.24 sec.

$T=0.30''$ ($\tau=0.0075''$)

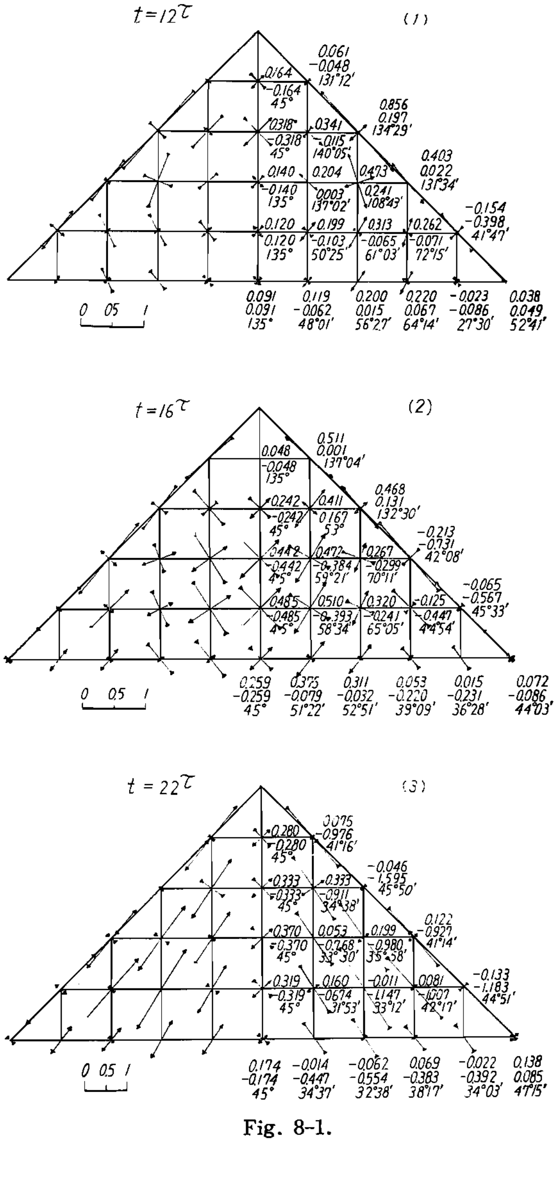

Fig. 8-1.

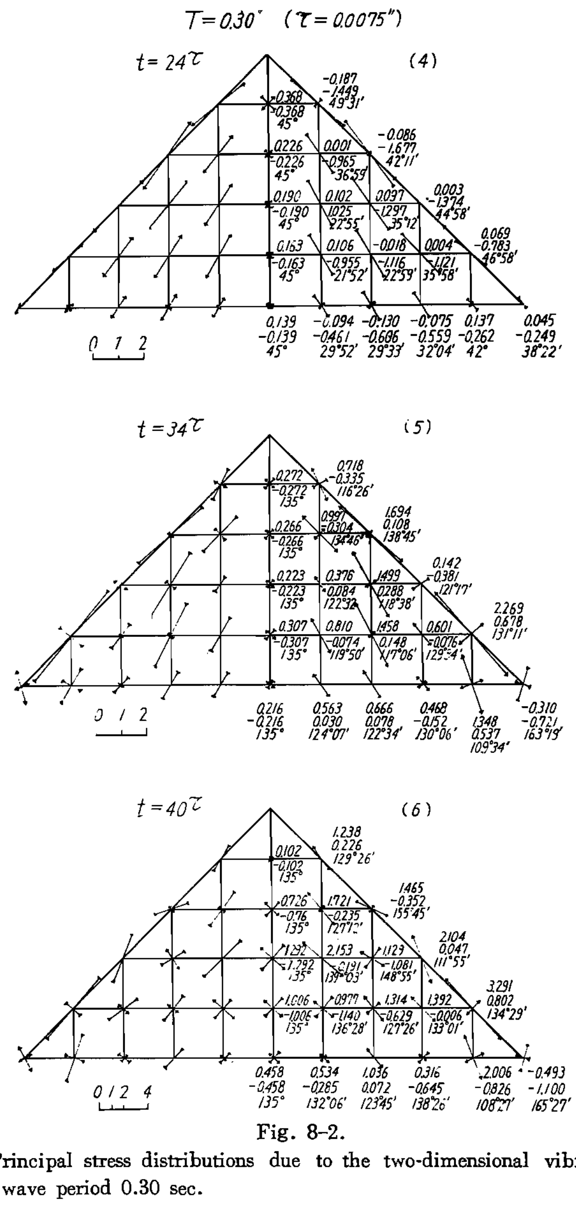

Fig. 8-2.

Fig. 8. Principal stress distributions due to the two-dimensional vibration by the external wave period 0.30 sec.

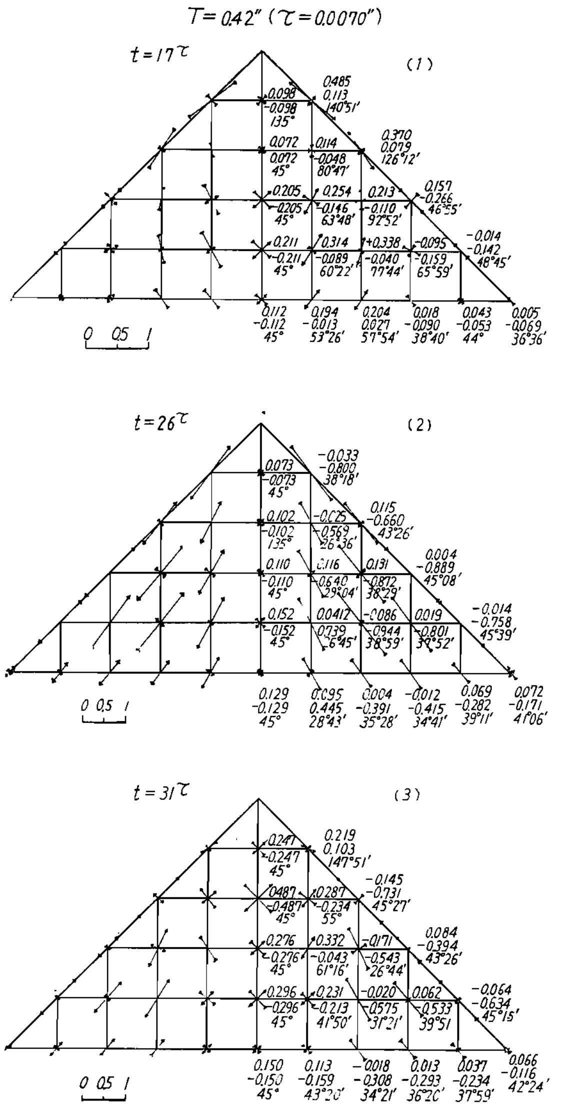

Fig. 9. Principal stress distributions due to the two-dimensional vibration by the external wave period 0.42 sec.

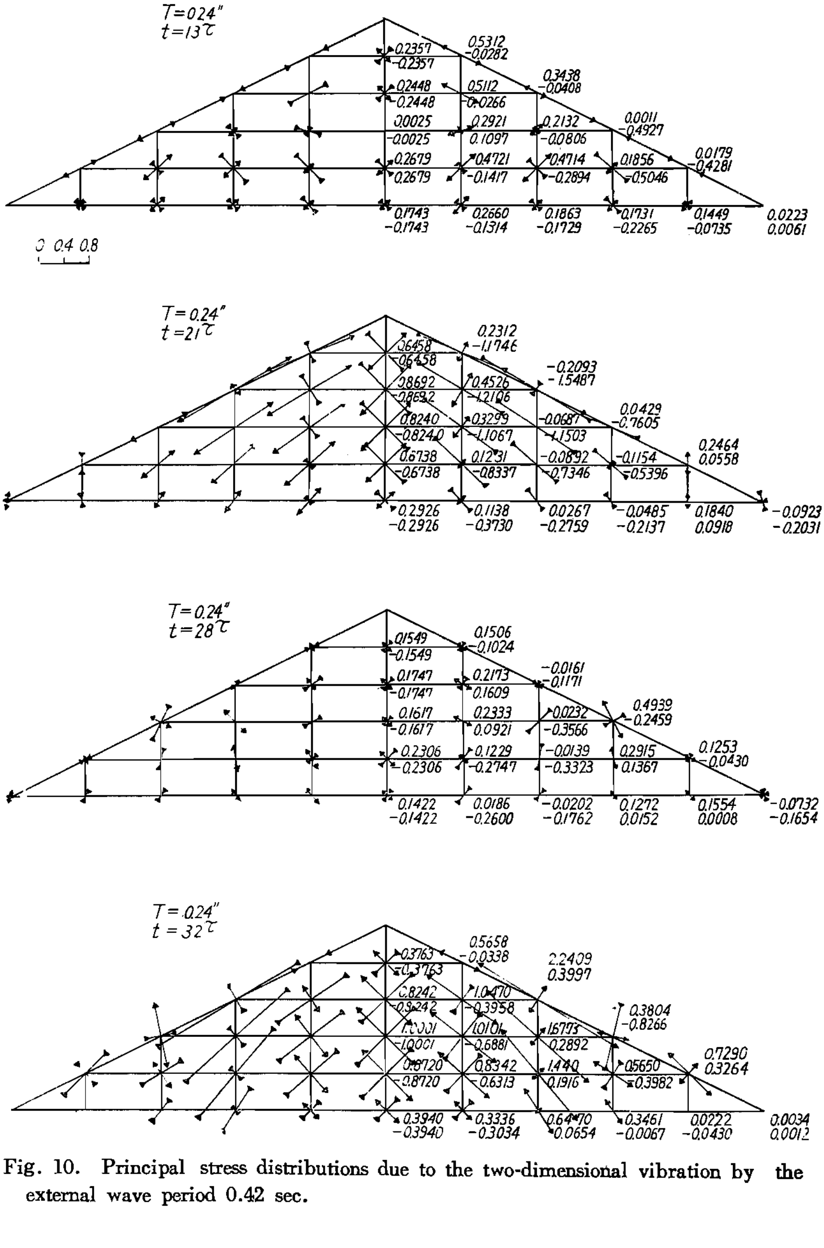

Fig. 10. Principal stress distributions due to the two-dimensional vibration by the external wave period 0.42 sec.

$$\sigma_{\substack{max\\min}}=\frac{\lambda+\mu}{4k}\{u_1-u_3+2(v_2-v_4)\}$$

$$\pm\frac{\mu}{4k}\sqrt{\{u_1-u_3-2(v_2-v_4)\}^2+\{2(u_2-u_4)+v_1-v_3\}^2}\cdots\cdots(15)$$

The angle of inclination of the principal stress may be expressed similarly. In the case of $h=k$, it follows ;

$$\tan 2\theta=\frac{u_2-u_4+v_1-v_3}{u_1-u_3-v_2+v_4}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(16)$$

In the case of $h=2k$, it follows ;

$$\tan 2\theta=\frac{2(u_2-u_4)+v_1-v_3}{u_1-u_3-2(v_2-v_4)}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(17)$$

Considerations on Principal Stresses

Some results of the calculations are shown in Figs. 7, 8, 9 and 10. The states of the expansion and contraction during the vibration are clearly shown by those figures. Several examples of the numerical calculations have led to the following considerations.

(1) On the surface of the dam, maximum principal stresses grow along the dam slopes. In the inner parts of the dam, the directions of the principal stresses are parallel to the surface slope, though the angles of the principal stresses are somewhat varied.

(2) As the wave propagates in the dam from the base, at first the tension grows on one side of the lower surface. With passage of time, the position of the maximum tensile stress moves upward and downward.

(3) By the fact that the tension and compression grow at the surface and the inner parts of the dam the vibration of the dam should be considered to be a two-dimensional one with vertical as well as horizontal displacements, so that the treatment of the dam by the shearing vibration would be imperfect.

(4) It is an obvious fact that the earth structures are easily cracked by tension. Earth dams when subjected to the seismic disturbance may have the cracks perpendicular to the slope surface by tension. As aforesaid, the results of the breaking tests of the model dams indicated the fact that the cracks grow in the slope at the right angle to the surface. However, the above-mentioned results are obtained from the calculations on the

assumption of the elastic body. The response of such structures to strong gound motions would give the non-elastic behavior, so the analytical treatments of such problems would be extremely difficult.

Considerations on Shearing Vibrations

The problems of the vibration of earth dams have been treated as those of the shearing vibration, which has been mainly treated in steady state. The shearing vibration merely takes into consideration the horizontal deformation due to shear, so it is different from the above-mentioned two-dimensional vibration. However, we will try to compare the horizontal displacement and shearing stress in the dam as the two-dimensional elastic body with those of the shearing vibration in steady and transitional states.

1. The differential equation of motion and finite difference equation

Referring to coordinate shown in Fig. 11, u is the component of the elastic displacement in the x direction and ρ is the density of the dam body. Assuming that the dam section is symmetrical and that the width of the dam in the x direction changes in proportion to dam height, equation of motion may be expressed as follows:

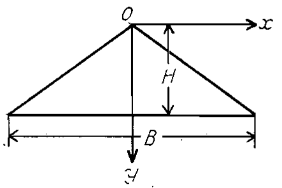

Fig. 11. Coordinate system.

$$\frac{\partial^2 u}{\partial t^2}=\frac{\mu}{\rho}\cdot\frac{1}{y}\cdot\frac{\partial}{\partial y}\left(y\frac{\partial u}{\partial y}\right) \qquad \cdots\cdots\cdots\cdots\cdots\cdots(18)$$

The boundary conditions for the Eq. (18) are given by $\frac{\partial u}{\partial y}=0$ at $y=0$ and $u=A\sin pt$ at $y=H$.

(A) In the case of steady state, the solution which satisfies Eq. (18) is obtained as follows:

$$u=A\cdot\frac{J_0\left(\frac{p}{C}y\right)}{J_0\left(\frac{p}{C}H\right)}\cdot\sin pt \qquad \cdots\cdots\cdots\cdots\cdots\cdots(19)$$

where $C=(u/\rho)^{1/2}$ = transversal wave velocity.

(B) In the case of transitional state, the solution of Eq. (18) is made by the calculus for finite differences taking account of time. The dam body is divided into the constant spacing h in the y direction. The finite difference equation direvied from the Eq. (18) by considering the point pattern illustrated in Fig. 12 is

$$u_{II}=\frac{\tau^2 C^2}{h^2}\left\{\frac{1}{2i}(u_2-u_1)+(u_1+u_2-2u_0)\right\}+2u_0-u_I \quad \cdots\cdots(20)$$

where i is the number of division from the top of the dam.

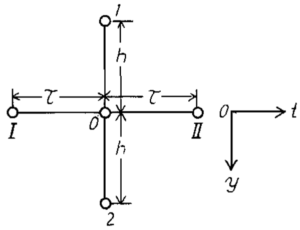

Fig. 12. Point pattern.

At the top of the dam the difference equation is

$$u_{II}=\frac{4\tau^2 C^2}{h^2}(u_2-u_0)+2u_0-u_I \quad \cdots\cdots\cdots\cdots\cdots\cdots(21)$$

We can perform the numerical calculations on the vibration of earth dams by the above equations step by step.

2. Numerical calculations

It is supposed that the height of the dam is $H=10$ m, the velocity of the transversal wave of the dam is $V_s=100$ m/s, maximum amplitude of the transversal wave is 1 and the fundamental period of the dam in elastic range is 0.261″. In transitional state, the interval for finite difference method is taken to be $h=H/10=1$ m. One period of incidental wave at $T=0.24''$, 0.30″ and 0.42″ is divided into thirty, forty and sixy parts respectively. The calculations are performed about one period.

3. Comparison between the two-dimensional vibration and the shearing one of earth dams

We compare the horizontal displacement and shearing stress of the dam as the two-dimensional elastic body with those of the shearing vibration in steady and transitional states.

(A) Comparison of vibrations taking the elapse of time into consideration

One example of the displacement and shearing stress distributions of the two-dimensional and shearing vibrations in transitional state is shown in Fig. 13. From this figure, we have obtained some considerations as follows.

(a) On the two-dimensional vibration, the displacement and stress distributions at the surface of the dam are fairly different from those in the center of the dam.

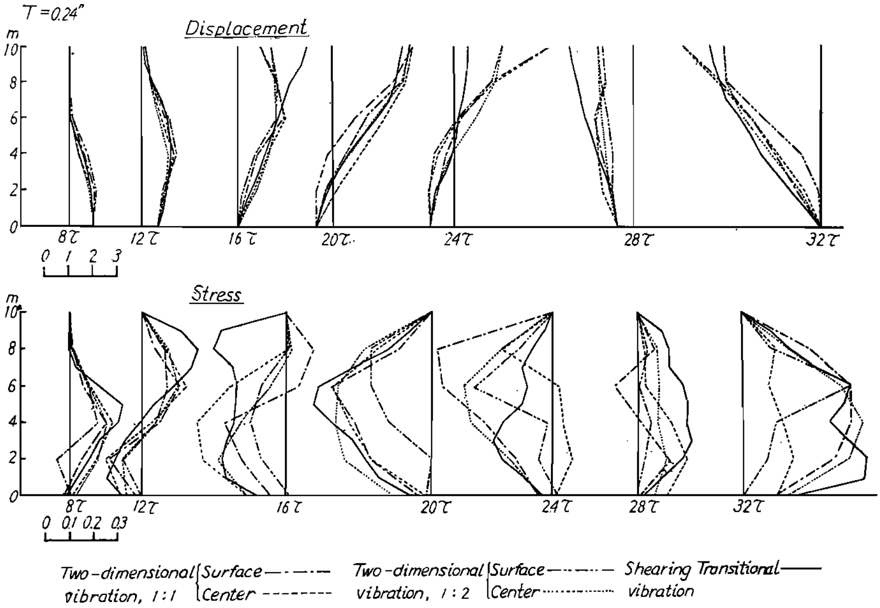

Fig. 13. Displacements and shearing stress distributions in the horizontal direction.

(b) When the displacement becomes large, the displacement distributions show generally to be regular in the case of the two-dimensional vibration of the dam with the steep slopes and they show the shape to be curved inside at the top of the dam if the slopes are gentle. The distributions of the shearing vibration are analogous to those of the two-dimensional vibration of the dam with the gentle slopes.

(B) Comparison of vibrations irrespective of time

In order that three vibrations may be generally compared with each other, the distributions of the horizontal displacement when the displacement at the top of the dam is maximum and those of the maximum stress which shows the maximum value irrespective of time, are considered about during one period.

(a) Distributions of horizontal displacement : The distributions of the horizontal displacement are shown in Fig. 14. The modes of vibrations are compared as follows ; (1) On the two-dimensional vibration, the mode of the slope surface of the dam is like that of the center of the dam. (2) In the case of $T=0.30''$, the modes of the three vibrations make no great difference. (3) In the case of $T=0.24''$, the displacements make great

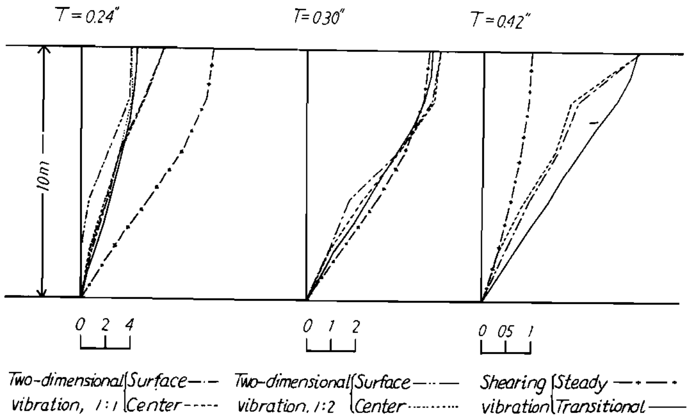

Fig. 14. Displacements in the horizontal direction, caused by the maximum displacement at the top of the dam.

difference between the shearing vibration in steady state and the other two. (4) In the case of $T=0.42''$, the displacements are generally small and those of the shearing vibration in steady state are much smaller than the other two.

(b) Distributions maximum shearing stresses: The distributions of the maximum shearing stresses are shown in Fig. 15. From this figure, they are compared as follows; (1) In the case of the incidental wave period $T=0.24''$, on the two-dimensional vibration, the stress at the surface of the dam is less than that at the center of the dam. The stress of the sharing vibration in steady state is much greater than that in transitional state and the maximum stress appears roughly at the same position. The stress at the bottom in transitional state is less than that in steady state. (2) In the case of $T=0.30''$, the stress of the shearing vibration in steady state is nearly the same as

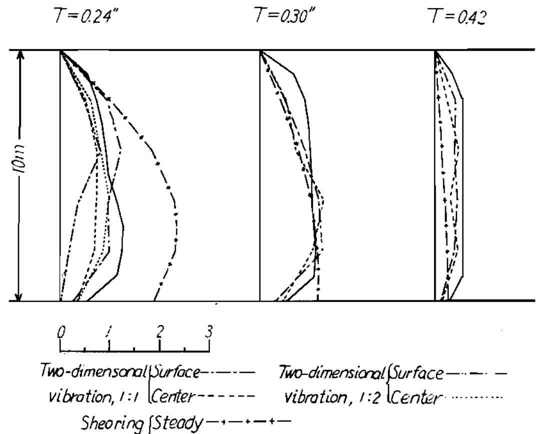

Fig. 15. Maximum shearing stress distributions.

that in transitional state, but at the upper part of the dam, the stress in transitional state is larger than that in steady state. The stress distribution of this vibration in transitional state is fairly uniform in the direction of the height. On the two-dimensional vibration, distribution of it is like the case of the shearing vibration in steady state and the stress at the bottom of the dam is small. (3) In the case of $T=0.42''$, the stress of the shearing vibration in transitional state is larger than that in steady state. The stress distribution of it in transitional state is uniform in the direction of the height. The stress distribution of the two-dimensional vibration is like the shearing vibration in transitional state.

4. Considerations

From the above-mentioned results, we are aware that the shearing vibration in transitional state is more similar to the two-dimensional vibration than to the shearing vibration in steady state, as the two-dimensional one is also treated in transitional state. And yet, these vibrations are fairly different, though we have not taken the soil damping into account in these calculations. But setting the case that the vibration is in steady state, when the incidental wave period is confirmed to the fundamental period of the dam, the stress and the displacement of the dam are maximum. In this calculation, the fundamental period of the dam is $0.26''$ for the shearing vibration, which is nearer to the incidental wave period $0.24''$ As many natural periods grow at the two-dimensional vibration, it is difficult to find out the natural period in this numerical calculation. However, in the case of the incidental external wave period $T=0.24''$, there is the greatest difference between the two-dimensional vibration and the shearing one. We have compared the two vibrations in above-mentioned calculations, but it is clearly shown from the results of the numerical calculation of the two-dimensional vibration that the dam body expands and contracts during the vibration. So the treatment of the shearing vibration would be imperfact.

Summary

We have considered the vibrational behaviors of earth dams, using numerical calculations by the finite difference method on the fundamental dam section treated as the two-dimensional body. Hitherto, the vibration

of earth structures which have the fundamental triangle section has been discussed as to the shearing vibration, so we have compared the two-dimensional vibration in transitional state with the shearing vibrations in steady and transitional states. From the numerical calculations developed above, we have made the following basic considerations.

(1) Since tension and compression are caused at the surface and inner parts of the dam body, the vibration of the dam body should be considered as the two-dimensional one with vertical as well as horizontal displacements.

(2) Comparing the distributions of the horizontal displacements and stresses of these vibrations, the two-dimensional vibration differs materially from the shearing vibration.

From the above-mentioned considerations, it would be imperfect to treat the vibration of earth dams as the shearing one. However, in this calculation, we assume the earth dam to be an elastic body on the absolutely rigid base and we have not taken the soil damping into consideration. Earth-dams, embankments or levees are constructed on the comparatively soft ground, namely alluvium. The complicated characteristics of soils, which are affected by density, cohesion, angle of internal friction, pore pressure and deformation of soil particles, should be taken into consideration on the vibration of earth structures. So we are going to have complicated problems to be solved as to the vibrational behaviors of earth dams.

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