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"Production Structure,
Household Time Allocation, and Fertility"

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Production Structure, Household Time Allocation, and Fertility

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Abstract

This paper develops an overlapping generations model that incorporates two-sector (market and non-market) production, sexual difference, and fertility choice. Our model could explain the joint evolution of production structure, household time allocation, and fertility broadly observed in the 19th and 20th centuries in the Western world as part of a single process of economic development: (i) production has shifted out of households and into the market, (ii) males first increased their labor supply to the market, and then females increased it; married-female participation in wage work outside the home dramatically increased in the latter half of the 20th century, and (iii) there has been the secular decline in fertility over the last 200 years, but there was the temporary rise in the middle of the 20th century (inverted N-shaped fertility dynamics). We also provide the quantitative analysis and examine how well our model replicates the patterns observed in U.S. data.

Keywords Fertility · Overlapping generations model · Structural change · Gender gap

JEL Classification J13 · J16 · O11 · O41

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1 Introduction

It is generally agreed that fertility is strongly correlated with economic growth. In actual, several patterns of fertility transition in the course of economic development are remarkably similar across countries. This paper focuses on a pattern broadly observed in the 19th and 20th centuries in the Western world: there has been the secular decline in fertility, but there was the temporary rise in the middle of the twentieth century (inverted N-shaped fertility dynamics). Over the same period of time, production structure, family time use, and sexual division of labor have also changed dramatically in those countries: production has shifted out of households and into the market; males first increased their labor supply to the market, and then females increased it. We explain such a joint evolution of production structure, household time allocation, and fertility based on a general equilibrium model incorporating two-sector production, sexual difference, and fertility choice.

We extend the model of Galor and Weil (1996) (henceforth GW) by incorporating non-market production, which is characterized by a diminishing-returns production technology for each household. GW develop a growth model, in which males and females have equal endowments of mental input, but males have more physical strength than females. Since physical capital complements mental-intensive tasks more than physical-intensive tasks, physical capital accumulation associated with economic growth reduces the gender wage gap. The rise in women's relative wages induces the increase in married-female participation in wage work and the decrease in time spent for child rearing, generating fertility declines. The model of GW is groundbreaking in that the relative wages of women and men endogenously evolve in the process of economic development. Considered as a theory explaining the fertility transition historically observed, however, it neglects an important fact that rises in married-female participation in wage work are the "latecomers" in the fertility transition. Figure 1 depicts the experiences of the United States, Great Britain, and Sweden: fertility steadily declined while married-female labor-force participation (married-FLP) increased very little in the 19th century; it was in the middle of the 20th century that married-FLP began to rise sharply.[1] Similar trends were

[1]Due to limited availability of data, we present the data on married-FLP in place of data on the proportion of wage workers for married women in Figure 1. Although employees, employers, own-account workers, and unpaid family workers are classified as in the labor force, it is paid employees that we wish to focus on here. Thus we should discount a little of what the plotted data show. For the 19th century, even data on married-FLP are not available, but Mammen and Paxson (2000) provide an evidence to support the view that married-female participation in wage work increased very little in the 19th century. Their estimates indicate that the ratio of women who participate in the labor force exhibits a U-shaped pattern with respect to per capita GDP, whereas the ratio of female workers who receive wage is positively correlated with per capita GDP. We could infer from their estimation that the ratio of married-female employees was low through the course of the 19th century.

broadly observed in the Western world. Our extended version of the GW model could explain this pattern of the dynamics of fertility and married-FLP.

In our model, market or firm production and non-market or household production coexist. In the market sector, firms produce goods employing labor and capital as inputs. On the other hand, the non-market production takes place at home individually; each household faces a diminishing-returns production technology using labor as the only input. The non-market production in our model is a reduced-form representation of composite activities workable at the household level. For instance, it contains family business, cottage industry, household chores, and agricultural task. The dimishing-returns technology captures the idea that each household engages in these activities in descending order of productivity. We model the structural change from non-market production to market one based on the supply-side explanation. It is assumed that both the production processes produce the same good; alternatively, it can also be interpreted that the good produced in market sector and the good produced in non-market sector are perfectly substitutable and the relative price is fixed.[2] The sector experiencing the higher growth in productivity pulls more workers.[3] Many researchers have developed macroeconomic models incorporating two-sector production, e.g., Benhabib et al. (1991), Greenwood and Hercowitz (1991), Hansen and Prescott (2002), Parente et al. (2000), and Restuccia (2004).[4] There are also several papers that combine two-sector production in macroeconomic models with endogenous fertility, e.g., Bar and Leukhina (2009), Doepke (2004), Greenwood and Seshadri (2002), Kimura and Yasui (2007), Lord and Rangazas (2006), and Moe (1998).[5] The model developed in this paper is differentiated from these models by taking a difference between men and women into consideration. We demonstrate that incorporating a sexual difference into the model with structural change can generate the historical sequence of changes in fertility and married-female participation in wage work.

The main mechanism of our model is summarized as follows. By the difference in the degree of

[2]Given the perfect substitutability, it seems to be unreasonable that the non-market production includes household chores. In reality, however, labor resources have shifted out of households and into the market since the 19th century, as well as goods used to be regarded as home-produced ones have become more readily available outside the home. This indicates that our assumption would be a decent approximation in the long-run model.

[3]This is the mechanism featured by many researchers since Lewis (1954). Matsuyama (2008) provides a concise review of the literature on structural change.

[4]Hansen and Prescott (2002) and Restuccia (2004) assume that the goods produced by two sectors are perfectly substitutable, whereas Benhabib et al. (1991), Greenwood and Hercowitz (1991), and Parente et al. (2000) assume that they are not perfect substitutes. The difference partly comes from differences in what they are focusing on: Hansen and Prescott (2002) and Restuccia (2004) focus on the long-run structural change, whereas Benhabib et al. (1991), Greenwood and Hercowitz (1991), and Parente et al. (2000) on short-run fluctuations.

[5]Our formulation of non-market technology is similar to the family-production technology of Lord and Rangazas (2006) in that each household owns a diminishing-returns production technology.

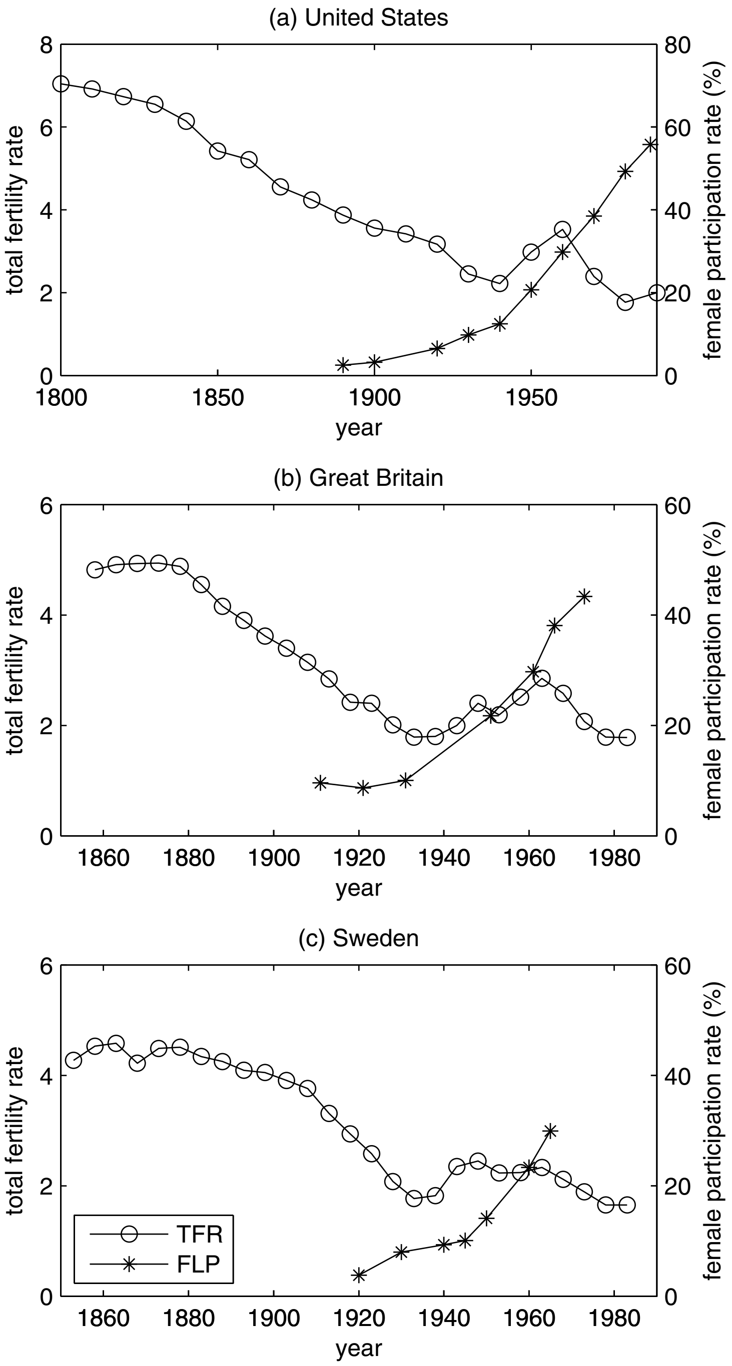

Figure 1: Total fertility rate and married-female labor-force participation. (a) United States, (b) Great Britain, (c) Sweden. *Notes:* Panel (a): TFR and FLP of U.S. are those for white women. Panel (b): TFR of Great Britain is that for England and Wales. *Sources:* Panel (a): Haines (2000, p.156, Table 4.3) and Goldin (1990, p. 17, Table 2.1). Panel (b): Chesnais (1992, pp. 549-50, Table A2.5) and Matthews et al. (1982, pp. 564, Table C.3). Panel (c): Chesnais, op. cit. and Silenstam (1970, p. 105, Tabell A:15).

capital accumulation, the economy is classified into four stages of development: (I) since there is little capital and thus the return to market work is sufficiently low, all agents engage in non-market work, (II) males increase their labor supply to the market as capital accumulates, whereas females do not, (III) gender-based specialization is observed: males specialize in market work while females specialize in non-market work and child rearing, and (IV) females also increase their labor supply to the market as capital accumulates.[6] The time lag between increases in males' labor supply to the market and increases in females' one is due to the gender wage gap. In each stage, households face a trade-off between consumption and children: for increasing the number of children, utility-maximizing households decrease the time spent on a marginally-lowest-return activity, which is non-market work in the stages (I) and (III) and is market one in the stages (II) and (IV). In the stages (II) and (IV), the rise of wages induced by capital accumulation has a dominant substitution effect on fertility, generating fertility declines. In the stage (III), it has a dominant income effect on fertility, generating fertility rises. In the course of economic development, (I)-(IV), fertility exhibits inverted N-shaped dynamics; only in the stage (IV), rises of married-female participation in wage work and declines of fertility are simultaneously observed.

The distinction between market work and non-market work in our model is made on the basis of whether it is performed for wage outside the home, not on whether produced goods are used solely by family members or traded. In the era of early industrialization, home and workplace were physically unified more often than at present. In such a form of production, married women were often drawn into the family business: for instance, when the store and house were physically unified, wives were frequently employed in retail trades without pay. Although comprehensive data on married-FLP of U.S. prior to 1890 are not available, several researchers have investigated the picture of female labor at that time. Goldin (1990, p.11) says "women on farms and in cities were active participants when the home and workplace were unified, and their participation likely declined as the marketplace widened and the specialization of tasks was enlarged. ... the married women's labor force participation rate is vaguely U-shaped over our history." According to the criterion adopted by our model, both husbands and wives working in the workplace at home are not considered to be supplying labor to the market. Our view that married-female participation in wage work was very low through the course of the 19th century does not contradict the familiar view, such as Goldin (1990), on married-female labor of the

[6]The stage (IV) is almost the same as the model developed by GW except that a part of women's time is used for production at home. In our model, the economy must accumulate sufficient amount of capital to reach the stage (IV): our model aims at explaining the longer-term phenomenon.

19th century.

Soares and Falcão (2008) share a common motivation for explaining the evolutions of FLP and fertility: increases in married-FLP appear only in the latter stages of fertility reduction. In their model, reductions in child mortality reduce fertility but do not raise FLP, whereas gains in adult longevity reduce fertility and raise FLP. A time lag between reductions in child mortality and gains in adult longevity leads to the pattern of evolutions of FLP and fertility consistent with historical data: rises in FLP appear only later on. Our mechanism is quite different from theirs. In our model, a time lag between increases in males' labor supply to the market and increases in females' one endogenously arises in the process of economic development. The changes in sexual division of labor along the growth path generate the inverted N-shaped fertility dynamics: only the latter phase of fertility reduction is accompanied by rises in FLP. An advantage over Soares and Falcão (2008) is that our model can explain declines in fertility "net of child mortality" during the period of low FLP.[7] The model of Soares and Falcão (2008) predicts that reductions in child mortality negatively affect "gross" fertility but have no effect on fertility net of child mortality: the number of surviving children is insensitive to variations of child mortality. However, historical data of most countries indicate that "net" fertility also decreased during the period characterized by declines of "gross" fertility: see Table 1 in Section 3 for U.S. and Chesnais (1992, p. 122, Table 4.7) for European countries.

We provide not only the qualitative analysis but also the quantitative one. There are now many papers that quantitatively examine the extent of the model's fitness for actual fertility transitions (e.g., Bar and Leukhina, 2009; Doepke et al., 2007; Greenwood and Seshadri, 2002; Greenwood et al., 2005; Lagerlöf, 2006; Lord and Rangazas, 2006; Moe, 1998). With regard to the motivation for explaining the non-monotonic dynamics of fertility in the 19th and 20th centuries, our paper is most related to Greenwood et al. (2005). They attribute the temporary rise of fertility in the course of secular declines to the technological progress in household sector: the introduction of electricity and the development of associated household products such as appliances and frozen foods reduced the time needed for child rearing, leading to the temporary rise of fertility in the middle of the 20th century, the so-called "baby boom." The exogenous rises of the productivity in household sector generate the baby boom in their paper. Our paper is also related to Doepke et al. (2007) in that the interaction between fertility and FLP is examined as a cause of the baby boom. They attribute the baby boom to the demand shock

[7] Although we do not model child mortality explicitly, we simulate the model based on infant-mortality-adjusted fertility rate in the numerical analysis (Table 1).

for female labor caused by World War II. Women of the war generation, whose husbands served in the armed forces, participated in the labor market and accumulated the work experience during the war. After the war, younger women who were still in school during the war faced increased competition with men who returned from the war and older women who had accumulated the work experience during the war. Women of the post-war generation were crowded out of the labor market and chose to have more children, leading to the baby boom. The exogenous one-time decline in the availability of male labor generates the baby boom in their paper.[8]

We think that both Doepke et al. (2007) and Greenwood et al. (2005) provide plausible explanations for the baby boom, but our view for it in this paper is rather different from theirs. We regard it as a phenomenon structurally embodied in the course of economic development rather than a boom caused by exogenous shocks: the time lag between increases in males' labor supply to the market and increases in females' one resulting from economic growth leads to the non-monotonic fertility dynamics. We are partly motivated by the fact that neutral countries where the impact of the war was relatively small also had the temporary rise of fertility in the last century, even if its scale was smaller than that in countries involved in the war; among the three countries in Figure 1, Sweden remained neutral in the war.[9] The fact that the baby boom started earlier in richer countries also supports our view that it is a by-product of economic development.[10] Butz and Ward (1979, p.318) say "the "baby boom" of the 1950's can be explained as a response to rising male income, whereas the baby bust of the 1960's was due primarily to increases in female wages and income." based on their empirical results. This paper shows that the phenomenon in their statement is a natural by-product of economic growth using a general-equilibrium overlapping-generations framework.

The remainder of this paper is organized as follows. Section 2 presents the model. In Section 3, we conduct the numerical analysis, which shows that our model does well at reproducing the main qualitative features of U.S. data. Finally, Section 4 concludes this paper.

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[8]There are some alternative explanations for the baby boom. For instance, most well-known explanations are the Easterlin's "relative income" hypothesis (Easterlin, 1961) and the "catch-up fertility" hypothesis. The Easterlin's hypothesis is the idea that the prosperity of the post-war years derived the gap between actual and expected material well-being, inducing couples to increase the demand for children. One of the problems of this hypothesis is that the timing is not right: the cohorts of women contributing the most to the baby boom were too young to be directly affected by the Great Depression and World War II. The catch-up fertility hypothesis is the idea that couples who could not have babies during the war had babies after the war. This hypothesis is also problematic: a pure catch-up hypothesis should have had no influence on lifetime fertility, but lifetime fertility actually rose. See Greenwood et al. (2005) for details of this discussion.

[9]Doepke et al. (2007) provide a concise comparison between the size of baby boom in the Allied countries and that in neutral countries. While they explain the baby boom as a by-product of World War II, they also recognize that smaller post-war baby booms in neutral countries must be due to other mechanisms.

[10]According to Greenwood et al. (2005), among 16 OECD countries which experienced the baby boom, the relationship between the start of the baby boom and country's income is significantly negative at the 95-percent confidence level.

2 Model

We consider an overlapping generations model in which each agent lives for three periods: childhood, adulthood, and old age. In childhood, an agent does not make any decisions and consumes a fixed quantity of time from his or her parents. In adulthood, each agent raises children, supplies labor to the market, and/or engages in non-market production. To keep things simple, suppose that the agent consumes nothing in adulthood. In old age, each agent only consumes savings from the previous period.

The economy is populated by two kinds of agents: men and women. Men and women differ in terms of the ability to earn wages in the labor market, which is the only difference between them.[11] The mechanism generating the gender wage gap is identical to that of GW: men and women have equal endowments of mental input, but males have more physical strength than females, and thus the gender wage gap reflecting the difference of physical strength exists.[12] A man and a woman form a family and jointly decide the allocation of their time; they are assumed to have joint consumption and joint utility. Our basic unit of analysis is the couple and they are assumed to be together since birth so that we need not consider issues related to the formation of families.

2.1 Production

There is a single final good, the numeraire, which can either be consumed or invested. The final goods can be produced in two sectors: non-market sector, where the only input is labor, and market sector, which is relatively capital-intensive.

2.1.1 Market Production

There are three factors in market production: physical capital, K_t, physical labor, L_t^p, and mental labor, L_t^m. We assume that physical capital and mental labor are more complementary than physical capital

[11]There is no difference between men and women in the abilities to do non-market work and raise children. The assumption that there is the only sexual difference in the return to market work is employed for simplification, not crucial for the main result of this paper. What matters is that men have a comparative advantage in market work.

[12]Many researchers have invented various mechanisms generating gender wage gaps for analyzing issues related to female labor. Doepke et al. (2007) and Greenwood et al. (2005) assume the differential productivity in labor market. These papers, including GW and us, have a common feature that gender wage gaps are attributed to innate differences. Albanesi and Olivetti (2009) and Lagerlöf (2003) derive wage gaps between husband and wife without ex-ante biological differences. In Albanesi and Olivetti (2009), an agent devoting more time to home work faces lower wage rate. In Lagerlöf (2003), there is a continuum of Nash equilibria: as men have more human capital, women have less, and vice versa. Without ex-ante gender differences, however, why women devote more time to home work than men in the real world is not explainable in terms of those models.

and physical labor. For simplicity, it is assumed that physical labor is a perfect substitute for other factors of production. The aggregate production function incorporating such an idea is

$$Y_t = A\left[K_t^{\alpha}\left(L_t^m\right)^{1-\alpha} + bL_t^p\right],$$

where $A > 0$, $b > 0$, and $\alpha \in (0,1)$.

Assuming perfectly competitive factor markets, the return to a unit of physical labor at time t, w_t^p, the return to mental labor at time t, w_t^m, and the return to physical capital at time t, r_t, are respectively,

$$w_t^p = Ab, \tag{1}$$

$$w_t^m = A(1-\alpha)K_t^{\alpha}\left(L_t^m\right)^{-\alpha}, \tag{2}$$

$$r_t + \delta = A\alpha K_t^{\alpha-1}\left(L_t^m\right)^{1-\alpha}, \tag{3}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital.

If all the time available were devoted to market work, men could supply one unit of physical labor and one unit of mental labor and earn $w_t^p + w_t^m$, while women could only supply one unit of mental labor and earn w_t^m. As GW, for the sake of brevity, we assume that women have no endowment of physical input for market production.

2.1.2 Non-market Production

Instead of supplying labor to the market and receiving the wage, each household can produce final goods by its own technology. The technology of non-market production is given by $f(\cdot)$. If a couple devote h_t of their time to non-market production at time t, the output of non-market production by the couple at time t is $f(h_t)$. It is assumed that males and females have equal endowments of input to non-market production. The total time input to non-market production of the couple is simply the sum of the time inputs of the husband and the wife: $h_t = h_t^H + h_t^W$, where h_t^H and h_t^W denote the husband's time spent on non-market work and the wife's one respectively.

We use the term "non-market production" to refer to the composite of productive activities excluding market work and child rearing, such as family business, cottage industry, household chores, and agricultural task. It is assumed that $f'(h_t) > 0$, $f''(h_t) < 0$, and $\lim_{h_t \to 0} f'(h_t) = \infty$. The concavity of $f(\cdot)$ captures the idea that there are various productive activities in household sector and the couple engages in them in descending order of productivity. The condition $\lim_{h_t \to 0} f'(h_t) = \infty$ is not necessary for our results, but assumed for simplification of analysis.[13]

2.2 Couples' Decision Problem

Couples receive utility from the number of children that they have and from consumption in old age. The utility function is

$$u_t = \gamma \ln n_t + (1-\gamma) \ln c_{t+1}, \tag{4}$$

where n_t and c_{t+1} respectively represent the number of "pairs" of children and consumption, and $\gamma \in (0,1)$ denotes the relative weight given to children. It is noted that the basic unit in this model is a couple and thus n_t essentially represents the number of pairs.

Each person is endowed with a unit of time that can be devoted to market work, non-market work, and child rearing. The time constraint of person i is given by

$$l_t^i + h_t^i + q_t^i = 1,\ i \in \{H, W\},$$

where l_t^i, h_t^i, and q_t^i denote the time spent on market work, non-market work, and child rearing, respectively. The person indexed by the superscript H (resp. W) is the husband (resp. wife). Raising a pair of children takes fraction $z \in (0,1)$ of the time endowment of one person. The time constraint for raising children can be written as

$$q_t^H + q_t^W = zn_t.$$

Since the couple consume goods only in their old age, all of their earnings are saved for future consumption. The couple's savings, s_t, are

$$s_t = \left(w_t^m + w_t^p\right) l_t^H + w_t^m l_t^W + f\left(h_t^H + h_t^W\right), \tag{5}$$

where $\left(w_t^m + w_t^p\right) l_t^H$ is the wage earnings of the husband, $w_t^m l_t^W$ is the wage earnings of the wife, and $f\left(h_t^H + h_t^W\right)$ is the output by their non-market work.

In the old age, the couple consumes the value of their savings with accrued interest:

$$c_{t+1} = (1 + r_{t+1}) s_t. \tag{6}$$

[13] With this condition, we need not condiser the case where the couple spend no time on non-market work.

The decisions that the household makes are how many children to have and how to earn income. We characterize the household's maximization in two-step process: (i) specifying the couple's time allocation that maximizes their income and plotting the attainable value of savings (s_t) for any possible amount of time for child rearing ($zn_t \in [0,2]$), we can obtain the set of feasible actions for the household in the (s_t, zn_t) space, and then (ii) we search for the utility-maximizing point from the set.

2.2.1 Couple's Budget Set

Here, we derive the couple's budget set, that is, the set of their feasible actions in the (s_t, zn_t) space. For that purpose, we specify the couple's time allocation maximizing their income, where the couple give more priority to a higher-return activity so that they can earn as much income as possible in a given time available for working. Let $\hat{h}_1$ and $\hat{h}_2$ denote the time spent on non-market work such that the marginal product of non-market production is equal to the men's wage rate $w_t^m + w_t^p$ and the women's wage rate w_t^m, respectively:

$$f'(\hat{h}_1) = w_t^m + w_t^p \ \text{ and } \ f'(\hat{h}_2) = w_t^m. \tag{7}$$

It follows from the concavity of $f(\cdot)$ that $\hat{h}_1 < \hat{h}_2$. When $h_t < \hat{h}_1$, the marginal product of non-market production is larger than the return to market work of husband; when $h_t > \hat{h}_2$, the marginal product of non-market production is smaller than the return to market work of wife.

Let us now shape the budget set using (7). Since both $\hat{h}_1$ and $\hat{h}_2$ are the functions of w_t^m, the form of budget set depends on the value of w_t^m.[14] It is noted that the couple's time allocation maximizing income must satisfy the conditions that $l_t^H \in [0,1]$, $l_t^W \in [0,1)$, $l_t^H \geq l_t^W$ with strict inequality if $l_t^H > 0$, and $l_t^W = 0$ if $l_t^H < 1$.[15] There are, depending on the value of w_t^m, four types of the form of budget set (Figure 2):

(a) The case of $w_t^m + w_t^p \leq f'(2)$, that is, $\hat{h}_1 \geq 2$: even if all the couple's time, 2, is devoted to non-market production, the marginal product of non-market production is still larger than the return to market work of husband. Both the husband and the wife must not spend any time on market work if they are to maximize their consumption level.

[14] Since $\hat{h}_1$ also depends on w_t^p, the value of w_t^p naturally affects the household's behavior. As is apparent from (1), however, w_t^p is excusively determined by parameters. Thus, we focus on the relationship between w_t^m and the household's behavior.

[15] By the assumption of $\lim_{h_t \to 0} f'(h_t) = \infty$, the possibility that the wife spends no time on non-market work, that is, the possibility of $l_t^W = 1$ is excluded.

(b) The case of $f'(2) < w_t^m + w_t^p \leq f'(1)$, that is, $1 \leq \hat{h}_1 < 2$: the marginal product of non-market production is smaller than the return to market work of husband when all the couple's time, 2, is devoted to non-market production; even if all the one person's time, 1, is devoted to non-market production, however, the marginal product of non-market production is larger than the return to market work of husband. The husband might spend some of his time on market work, but must not spend all of his time on market work.

(c) The case of $w_t^m \leq f'(1) < w_t^m + w_t^p$, that is, $\hat{h}_1 < 1 \leq \hat{h}_2$: when all the one person's time, 1, is devoted to non-market production, the marginal product of non-market production is smaller than the return to market work of husband, but larger than the return to market work of wife. The husband might spend all of his time on market work, but the wife must not spend any time on market work.

(d) The case of $f'(1) < w_t^m$, that is, $\hat{h}_2 < 1$: when all the one person's time, 1, is devoted to non-market production, the marginal product of non-market production is smaller than the return to market work of wife. The wife might spend some of her time on market work.

The wage rate of mental labor, w_t^m, determines what type of shape the couple's budget set exhibits. As w_t^m rises, the shape of the couple's budget set evolves as (a) $\rightarrow$ (b) $\rightarrow$ (c) $\rightarrow$ (d).

2.2.2 Couple's Optimization

The couple chooses the utility-maximizing point from the budget set. It should be noted that a unique maximum exists because the budget set is convex and the utility function is strictly quasiconcave. The couple's utility-maximizing behavior is characterized by the first-order conditions, which are classified into the following four cases. In what follows, variables with superscript "$*$" represent ones chosen optimally by the couple.

Case 1: $zn_t^* \geq 2 - \hat{h}_1$

If $zn_t \geq 2 - \hat{h}_1$, the couple spend all their time engaging in non-market production except for child rearing. The points A, B, D, and G in Figure 2 correspond to this case. The couple's time allocation is

$$\left(l_t^H, l_t^W, h_t^H, h_t^W, q_t^H, q_t^W\right) = \left(0, 0, h_t^H, h_t^W, q_t^H, q_t^W\right), \tag{8}$$

$$\text{where } h_t^H + h_t^W = 2 - zn_t,\ q_t^H + q_t^W = zn_t,\ h_t^H + q_t^H = 1,\ h_t^W + q_t^W = 1. \tag{9}$$

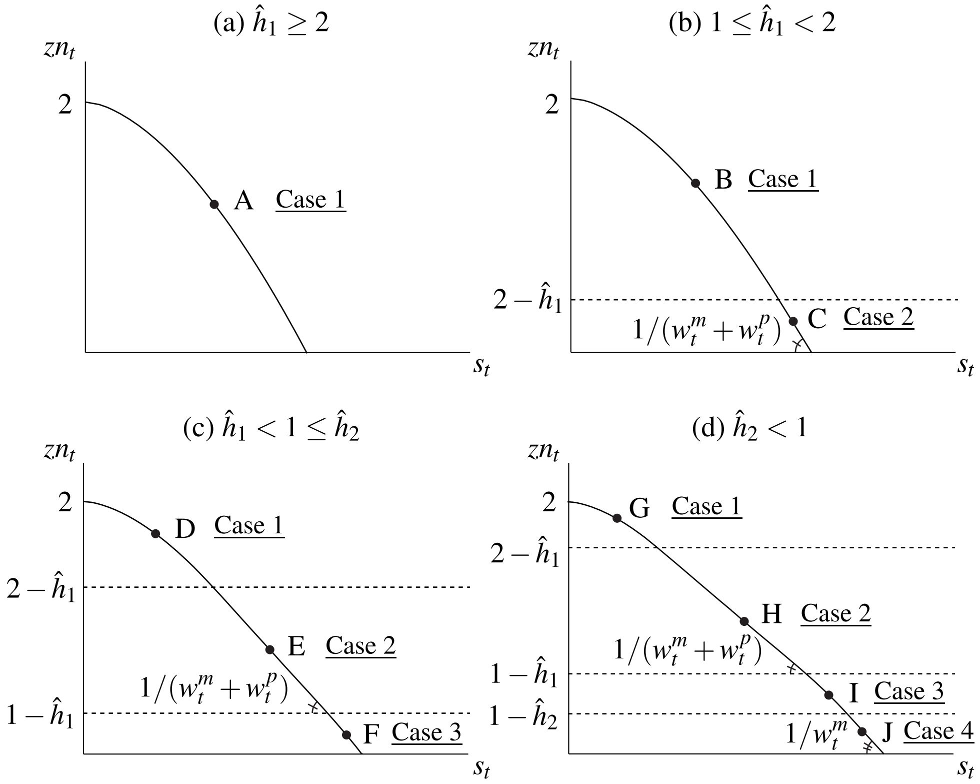

Figure 2: The couple's budget set. (a) $\hat{h}_1 \geq 2$, (b) $1 \leq \hat{h}_1 < 2$, (c) $\hat{h}_1 < 1 \leq \hat{h}_2$, (d) $\hat{h}_2 < 1$. *Notes:* Case 1: $l_t^H = l_t^W = 0$. Case 2: $0 < l_t^H < 1$, $l_t^W = 0$. Case 3: $l_t^H = 1$, $l_t^W = 0$. Case 4: $l_t^H = 1$, $0 < l_t^W < 1$.

It is not uniquely determined how the couple share child rearing and non-market work: there are numerous time allocations satisfying (9). Their savings are

$$s_t = f(2 - zn_t). \tag{10}$$

Maximizing (4) with respect to n_t subject to (6) and (10), we obtain

$$\frac{\gamma}{n_t^*} = \frac{(1-\gamma) f'(2 - zn_t^*) z}{f(2 - zn_t^*)}. \tag{11}$$

When the household chooses a point such as A, B, D, and G in Figure 2, this first-order condition must hold. It follows from (11) that the number of children is independent of the return to mental labor in the market. In this case, the household does not supply any labor to the market, thus naturally, the fertility is not affected by the market wage rate.

Case 2: $1-\hat{h}_1 \leq zn_t^* < 2-\hat{h}_1$

If $1-\hat{h}_1 \leq zn_t < 2-\hat{h}_1$, the wife spends no time on market work and the husband spends some time on market work. The household's time allocation between non-market work and market work is determined so as to equalize the marginal product of non-market production to the men's wage rate. The points C, E, and H in Figure 2 correspond to this case. The couple's time allocation is

$$\left(l_t^H, l_t^W, h_t^H, h_t^W, q_t^H, q_t^W\right) = (2 - zn_t - \hat{h}_1, 0, h_t^H, h_t^W, q_t^H, q_t^W), \tag{12}$$

$$\text{where } h^H + h^W = \hat{h}_1,\ q_t^H + q_t^W = zn_t,\ h_t^H + q_t^H = 1,\ h_t^W + q_t^W = 1. \tag{13}$$

It is not uniquely determined how the couple share child rearing and non-market work: there are numerous time allocations satisfying (13). Their savings are

$$s_t = (2 - zn_t - \hat{h}_1)\left(w_t^m + w_t^p\right) + f(\hat{h}_1). \tag{14}$$

Maximizing (4) with respect to n_t subject to (6) and (14), we obtain

$$n_t^* = \frac{\gamma}{z} \frac{(2-\hat{h}_1)\left(w_t^m + w_t^p\right) + f(\hat{h}_1)}{w_t^m + w_t^p}. \tag{15}$$

When the household chooses a point such as C, E, and H in Figure 2, this first-order condition must hold. It follows that

$$\frac{\partial n_t^*}{\partial w_t^m} = -\frac{\gamma}{z} \frac{f(\hat{h}_1)}{\left(w_t^m + w_t^p\right)^2} < 0.$$

The number of children is decreasing in the wage rate of mental labor. A rise in the wage rate expands the budget set and makes the slope of the frontier of budget set in the interval $zn_t \in (1-\hat{h}_1, 2-\hat{h}_1)$ flatter, and thus has a positive income effect and a negative substitution effect on the demand for children. Since the substitution effect on fertility dominates the income effect, fertility falls. It is noted that the presence of non-market production has an important role in deriving the negative relationship between w_t^m and n_t. If non-market production does not exist, the rise in w_t^m rotates the frontier of budget set upward around a point on $s_t = 0$. Such change of budget set does not change fertility under the logarithmic utility because the substitution effect on fertility and the income effect cancel out. In this model, however, the household devotes a part of working time to non-market work; the larger the amount of time devoted to non-market work, the smaller the effect of the rise in market wage rate on the total income. The presence of non-market production operates to mitigate the income effect of the rise in wage rate, making the substitution effect dominant.

Differentiating the quantity of husband's labor supply to market l_t^H with respect to the wage rate of mental labor w_t^m, we obtain

$$\frac{\partial l_t^{H*}}{\partial w_t^m} = -z\frac{\partial n_t^*}{\partial w_t^m} - \frac{d\hat{h}_1}{dw_t^m} > 0.$$

As the wage rate of mental labor rises, the quantity of husband's labor supply to market increases. In this case, if the couple is to devote more time to child rearing, the husband must reduce the time spent on market work: the trade-off arises between child rearing and the husband's market work, as is apparent from Figure 2. It is noted that this does not necessarily mean that the husband cares for children. In this case, as we have mentioned before, it is indeterminate how the couple share child rearing and household production. Irrespective of which cares for children, the trade-off exists between the number of children and the activity whose marginal product is smallest at the optimum, which is the husband's market work in this case. The rise in the husband's wage rate increases the value of his market work, reducing fertility and increasing his labor supply to the market, through the dominant substitution effect.

Case 3: $1-\hat{h}_2 \leq zn_t^* < 1-\hat{h}_1$

If $1-\hat{h}_2 \leq zn_t < 1-\hat{h}_1$, the husband specializes in market work and the wife spends all her time in non-market work and child rearing. The points F and I in Figure 2 correspond to this case. The couple's time allocation is uniquely determined:

$$\left(l_t^H, l_t^W, h_t^H, h_t^W, q_t^H, q_t^W\right) = (1, 0, 0, 1-zn_t, 0, zn_t). \tag{16}$$

Their savings are

$$s_t = w_t^m + w_t^p + f(1-zn_t). \tag{17}$$

Maximizing (4) with respect to n_t subject to (6) and (17), we obtain

$$\frac{\gamma}{n_t^*} = \frac{(1-\gamma) f'(1-zn_t^*) z}{w_t^m + w_t^p + f(1-zn_t^*)}. \tag{18}$$

When the household chooses a point such as F and I in Figure 2, this first-order condition must hold. By totally differentiating this condition, we obtain the following relationship:

$$\frac{dn_t^*}{dw_t^m} = \frac{\gamma}{z\left[f'\left(1-zn_t^*\right)-(1-\gamma)f''\left(1-z^*n_t\right)zn_t^*\right]} > 0.$$

As the wage rate of mental labor rises, the number of children increases. Suppose that (s_t^*, zn_t^*) is an optimum. Consider the impact of the rise in wage rate of mental labor. On the one hand, the associated rise of husband's wage rate expands the budget set. On the other hand, the slope of the frontier of budget set evaluated at zn_t^* does not change. As a result, the fertility rises through the income effect. In this case, the trade-off arises between child rearing and the wife's non-market work. The rise in the husband's wage rate makes the household wealthier, but does not directly change the marginal product of non-market production. As a result, it raises fertility and reduces wife's time spent on non-market work.

Case 4: $zn_t^* < 1-\hat{h}_2$

If $zn_t < 1-\hat{h}_2$, the husband specializes in market work and the wife engages in market work, non-market work, and child rearing. The wife's time allocation between non-market work and market work is determined so as to equalize the marginal product of non-market production to the women's wage rate. The point J in Figure 2 corresponds to this case. The couple's time allocation is uniquely determined:

$$\left(l_t^H, l_t^W, h_t^H, h_t^W, q_t^H, q_t^W\right) = \left(1, 1-\hat{h}_2-zn_t, 0, \hat{h}_2, 0, zn_t\right). \tag{19}$$

Their saving are

$$s_t = w_t^m(2-\hat{h}_2-zn_t)+w_t^p+f(\hat{h}_2). \tag{20}$$

Maximizing (4) with respect to n_t subject to (6) and (20), we obtain

$$n_t^* = \frac{\gamma}{z}\frac{w_t^m(2-\hat{h}_2)+w_t^p+f(\hat{h}_2)}{w_t^m}. \tag{21}$$

When the household chooses a point such as J in Figure 2, this first-order condition must hold. It follows that

$$\frac{\partial n_t^*}{\partial w_t^m} = -\frac{\gamma}{z}\frac{w_t^p+f(\hat{h}_2)}{(w_t^m)^2} < 0.$$

This comparative static result can be explained by the analogous argument in Case 2. The trade-off arises between child rearing and the wife's market work. A rise in the wage rate of mental labor raises both the husband's wage rate and the wife's wage rate. By the presence of non-market production operating to mitigate the income effect, the substitution effect is dominant and fertility falls.

The effects of the rise of wage rate on the wife's time spent on non-market work and that on market work are, respectively,

$$\frac{\partial h_t^{W*}}{\partial w_t^m} = \frac{d\hat{h}_2}{dw_t^m} < 0 \text{ and } \frac{\partial l_t^{W*}}{\partial w_t^m} = -\frac{d\hat{h}_2}{dw_t^m} - z\frac{\partial n_t^*}{\partial w_t^m} > 0.$$

As the wage rate of mental labor rises, the wife decreases her time spent on non-market work and increases her time spent on market work.

The question which we must consider next is which case is chosen. We derive the condition for each case to be chosen from investigating the slope of indifference curve and that of budget constraint in the (s_t, zn_t) space. The slope of indifference curve in the (s_t, zn_t) space is

$$\frac{d(zn_t)}{ds_t} = -\frac{1-\gamma}{\gamma}\frac{zn_t}{s_t}.$$

First, compare Case 1 ($zn_t \geq 2-\hat{h}_1$) with Case 2 ($1-\hat{h}_1 \leq zn_t < 2-\hat{h}_1$). If the absolute value of the slope of indifference curve evaluated at $(f(\hat{h}_1), 2-\hat{h}_1)$ is smaller than that of budget constraint in the interval of $zn_t \in [1-\hat{h}_1, 2-\hat{h}_1]$, that is, $1/\left(w_t^m + w_t^p\right)$, then there exists a feasible point in $zn_t \geq 2-\hat{h}_1$ such that the household prefers it to any feasible point in $zn_t < 2-\hat{h}_1$. Therefore, the condition for the household to prefer Case 1 to Case 2 is

$$\left| \left. \frac{d(zn_t)}{ds_t} \right|_{(f(\hat{h}_1), 2-\hat{h}_1)} \right| \leq \frac{1}{w_t^m + w_t^p}$$

$$\leftrightarrow \quad \frac{1-\gamma}{\gamma}(2-\hat{h}_1) \leq \frac{f(\hat{h}_1)}{w_t^m + w_t^p}. \tag{22}$$

Next, compare Case 2 ($1-\hat{h}_1 \leq zn_t < 2-\hat{h}_1$) with Case 3 ($1-\hat{h}_2 \leq zn_t < 1-\hat{h}_1$). If the absolute value of the slope of indifference curve evaluated at $(f(\hat{h}_1) + w_t^m + w_t^p, 1-\hat{h}_1)$ is smaller than that of budget constraint in the interval of $zn_t \in [1-\hat{h}_1, 2-\hat{h}_1]$, that is, $1/\left(w_t^m + w_t^p\right)$, there exists a feasible point in $zn_t \geq 1-\hat{h}_1$ such that the household prefers it to any feasible point in $zn_t < 1-\hat{h}_1$. Therefore, the condition for the household to prefer Case 2 to Case 3 is

$$\left| \left. \frac{d(zn_t)}{ds_t} \right|_{(f(\hat{h}_1)+w_t^m+w_t^p, 1-\hat{h}_1)} \right| \leq \frac{1}{w_t^m + w_t^p}$$

$$\leftrightarrow \quad \frac{1-\gamma}{\gamma}(1-\hat{h}_1) \leq \frac{f(\hat{h}_1)+w_t^m+w_t^p}{w_t^m+w_t^p}. \tag{23}$$

Last, compare Case 3 ($1-\hat{h}_2 \leq zn_t < 1-\hat{h}_1$) with Case 4 ($zn_t < 1-\hat{h}_2$). If the absolute value of the slope of indifference curve evaluated at $(f(\hat{h}_2)+w_t^m+w_t^p, 1-\hat{h}_2)$ is smaller than that of budget constraint in the interval of $zn_t \leq 1-\hat{h}_2$, that is, $1/w_t^m$, there exists a feasible point in $zn_t \geq 1-\hat{h}_2$ such that the household prefers it to any feasible point in $zn_t < 1-\hat{h}_2$. Therefore, the condition for the household to prefer Case 3 to Case 4 is

$$\left| \left. \frac{d(zn_t)}{ds_t} \right|_{(f(\hat{h}_2)+w_t^m+w_t^p, 1-\hat{h}_2)} \right| \leq \frac{1}{w_t^m}$$

$$\leftrightarrow \quad \frac{1-\gamma}{\gamma}(1-\hat{h}_2) \leq \frac{f(\hat{h}_2)+w_t^m+w_t^p}{w_t^m}. \tag{24}$$

Denote the mental wage rates which equalize the LHS to the RHS in (22), (23), and (24) by $\tilde{w}_1^m$, $\tilde{w}_2^m$, and $\tilde{w}_3^m$ respectively. Since the LHS's of (22), (23), and (24) are increasing in w_t^m and the RHS's of (22), (23), and (24) are decreasing in w_t^m, the thresholds $\tilde{w}_1^m$, $\tilde{w}_2^m$, and $\tilde{w}_3^m$ are each determined uniquely. The LHS of (22) is larger than the LHS of (23) for any w_t^m, and the LHS of (23) is larger than the LHS of (24) for any w_t^m. The RHS of (22) is smaller than the RHS of (23) for any w_t^m, and the RHS of (23) is smaller than the RHS of (24). Thus, we obtain the following relationship:

$$\tilde{w}_1^m < \tilde{w}_2^m < \tilde{w}_3^m.$$

It follows from the above description that (i) when $w_t^m \leq \tilde{w}_1^m$, the behavior of household is characterized by (8)-(11) (Case 1), (ii) when $\tilde{w}_1^m < w_t^m \leq \tilde{w}_2^m$, it is characterized by (12)-(15) (Case 2), (iii) when $\tilde{w}_2^m < w_t^m \leq \tilde{w}_3^m$, it is characterized by (16)-(18) (Case 3), and (iv) when $\tilde{w}_3^m < w_t^m$, it is characterized by (19)-(21) (Case 4).

The number of children, n_t, the husband's amount of time devoted to market work, l_t^H, and the wife's one, l_t^W, can be expressed as functions of the wage rate of mental labor w_t^m. Figure 3 depicts the relationships among them: as the mental wage rate rises, the husband first increases his labor

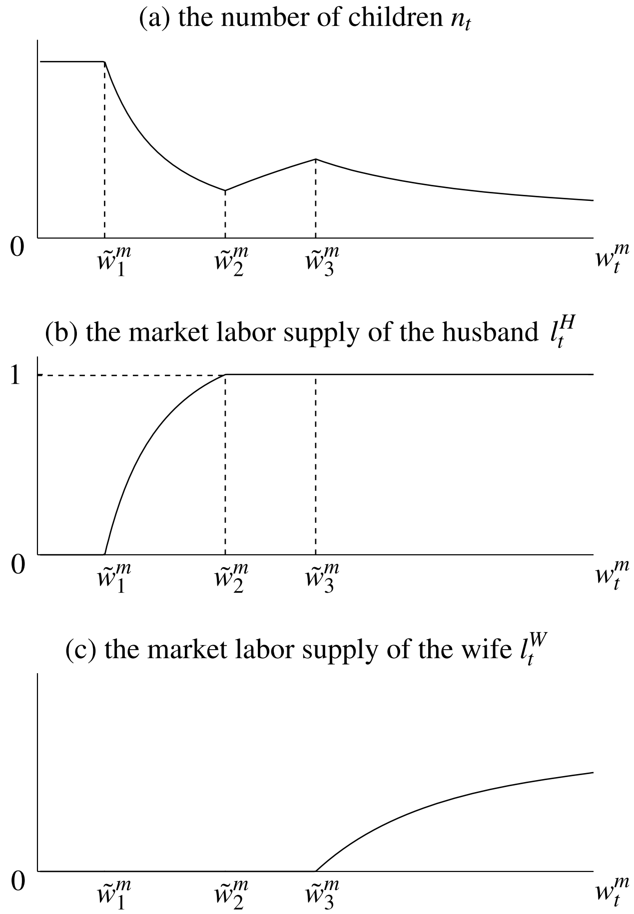

Figure 3: (a) The number of children, (b) the market labor supply of the husband, and (c) the market labor supply of the wife.

supply to the market, and then the wife increases hers; the number of children is non-monotonic with respect to the mental wage rate (inverted N-shaped dynamics). Note that it is not uniquely determined whether the husband's wage positively affects the number of children or negatively affects it. When $\tilde{w}_1^m < w_t^m \leq \tilde{w}_2^m$, that is, when the labor-supply behavior of husband is elastic with respect to the wage rate, the rise of the wage rate decreases the number of children through the dominant substitution effect. When $\tilde{w}_2^m < w_t^m \leq \tilde{w}_3^m$, that is, when the quantity of labor supply of husband is sufficiently large such that his labor-supply behavior is inelastic with respect to the wage rate, the rise of the wage rate increases the number of children through the dominant income effect.

2.3 Temporary Competitive Equilibrium

We have investigated the behavior of each household taking the wage rate as given. In fact, however, the wage rate is an endogenous variable that is determined in the model. In what follows, we conduct the equilibrium analysis. In this subsection, we characterize a temporary competitive equilibrium where the capital stock, K_t, and the number of couples, N_t, are given, that is, the capital stock per couple, $k_t \equiv K_t/N_t$, is given.

In equilibrium, the quantity of labor supply to market is the same across all households because all agents face a single return to mental labor and all households supply labor so as to equalize the market wage to the marginal product of non-market production. Using the results of the preceding subsection, we can express the household's supply of mental labor to market l_t, which is identical to the sum of the husband's time spent on market work and the wife's one $l_t^H + l_t^W$, as a function of the wage rate of mental labor:

$$l_t\left(w_t^m\right) = \begin{cases} 0 & \text{if} \quad w_t^m \le \tilde{w}_1^m, \\ 2 - zn_t\left(w_t^m\right) - \hat{h}_1 & \text{if} \quad \tilde{w}_1^m < w_t^m \le \tilde{w}_2^m, \\ 1 & \text{if} \quad \tilde{w}_2^m < w_t^m \le \tilde{w}_3^m, \\ 2 - zn_t\left(w_t^m\right) - \hat{h}_2 & \text{if} \quad \tilde{w}_3^m < w_t^m. \end{cases} \tag{25}$$

The derivative of $l_t\left(w_t^m\right)$ is

$$l_t'\left(w_t^m\right) \begin{cases} = 0 & \text{if} \quad w_t^m < \tilde{w}_1^m, \\ > 0 & \text{if} \quad \tilde{w}_1^m < w_t^m < \tilde{w}_2^m, \\ = 0 & \text{if} \quad \tilde{w}_2^m < w_t^m < \tilde{w}_3^m, \\ > 0 & \text{if} \quad \tilde{w}_3^m < w_t^m. \end{cases} \tag{26}$$

Although each household determines the quantity of labor supply taking the wage rate as given, in equilibrium, it must be consistent with the competitive wage rate endogenously determined. The aggregate supply of mental labor is written as $L_t^m\left(w_t^m\right) = N_t l_t\left(w_t^m\right)$. It follows from this equation and (2) that the equilibrium condition is given by

$$w_t^m = A\left(1-\alpha\right)\left[\frac{k_t}{l_t\left(w_t^m\right)}\right]^{\alpha} \equiv \Psi\left(w_t^m; k_t\right). \tag{27}$$

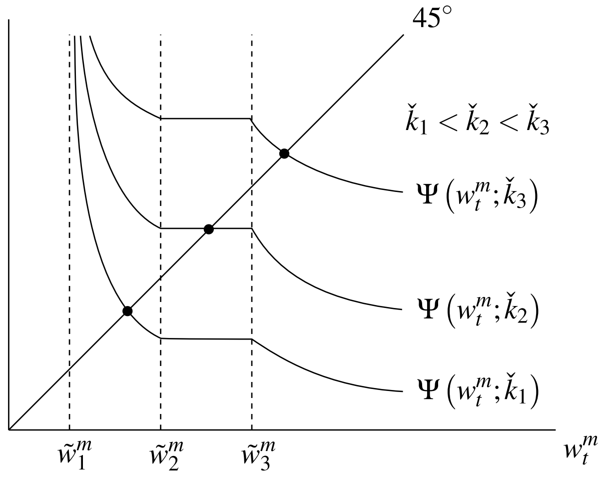

Figure 4: Temporary competitive equilibrium.

The equilibrium wage rate of mental labor is given by a fixed point of $\Psi(\cdot)$. It follows from (25) and (26) that $\Psi(\cdot)$ is not increasing in w_t^m and $\lim_{w_t^m \to \tilde{w}_1^m} \Psi = \infty$. Therefore, the unique equilibrium wage rate exists in $(\tilde{w}_1^m, \infty)$ if $k_t > 0$. The increase in k_t shifts $\Psi(\cdot)$ upward, thus the equilibrium wage rate of mental labor rises as the capital stock per couple increases (Figure 4).

2.4 Intertemporal Competitive Equilibrium

In the preceding subsection, we derived the equilibrium taking the per-couple capital as given. In fact, given an initial stock of per-couple capital k_0, the sequence of per-couple capital stock $\{k_t\}_{t=0}^{\infty}$ is endogenously determined in the model. In this subsection, we characterize the intertemporal competitive equilibrium where k_t endogenously evolves. It follows from (25), (26), and (27) that, given k_t, w_t^m is uniquely determined. Thus, we focus on the sequence of wage rate of mental labor $\{w_t^m\}_{t=0}^{\infty}$ rather than the sequence of per-couple capital stock $\{k_t\}_{t=0}^{\infty}$.

Using the results of couple's decision problem, we can express the couple's savings at time t as a function of the wage rate of mental labor, $s_t\left(w_t^m\right)$. The stock of capital at time $t+1$, K_{t+1}, is determined by the aggregate supply of savings at time t: $K_{t+1} = N_t s_t\left(w_t^m\right)$. The number of households at time $t+1$ is $N_{t+1} = N_t n_t\left(w_t^m\right)$. It follows that the per-couple capital stock at time $t+1$ is $k_{t+1} = s_t\left(w_t^m\right)/n_t\left(w_t^m\right)$. Using this equation and (27), we obtain the difference equation which determines the transition of the

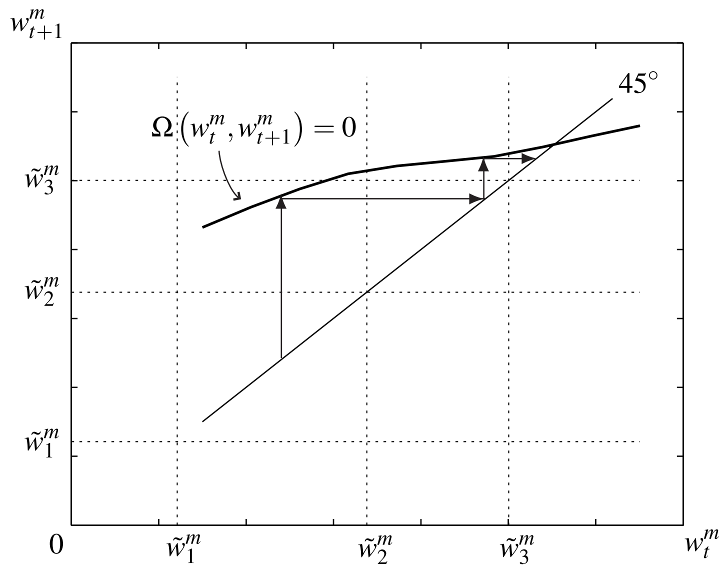

Figure 5: A numerical example of the dynamics of the wage rate of mental labor.

wage rate of mental labor:

$$w_{t+1}^m = A(1-\alpha)\left[\frac{s_t\left(w_t^m\right)}{l_{t+1}\left(w_{t+1}^m\right) n_t\left(w_t^m\right)}\right]^{\alpha}. \tag{28}$$

If the initial level of per-couple capital stock, k_0, is historically given, the dynamic equilibrium sequence $\{w_t^m\}_{t=0}^{\infty}$ is determined by the households' fertility and saving decisions, (10), (11), (14), (15), (17), (18), (20), and (21), the labor supply function, (25), and the wage-dynamics equation, (28).[16] Figure 5 demonstrates a numerical example of the dynamics of the wage rate of mental labor, where $\Omega\left(w_t^m, w_{t+1}^m\right) = 0$ is the difference equation (28): the economy starts from Case 2, goes to Case 3 in the next period, and converges to the steady state in Case 4. Along the growth path, fertility and labor supply to market move from point to point in Figure 3.

3 Numerical Analysis

To see how well the model presented above replicates the fertility transition observed in U.S. data, we solve the model numerically in this section. Thus far we have not taken infant mortality into consideration, but it is not negligible for simulating the model over long periods; in fact, infant mortality

[16]By the implicit function theorem, it is verified that $dw_{t+1}^m/dw_t^m > 0$ for any $w_t^m > \tilde{w}_1^m$. Therefore, the time paths for w_t^m do not exhibit oscillatory behavior. Although we do not observe the multiplicity under a large variety of parameter sets, we cannot exclude the possibility of multiple steady states. An acceleration in the rate of growth associated with sectoral shift leads to the possibility.

Table 1: Total fertility rate, infant mortality rate, and total fertility rate net of infant mortality for white, 1850-1990. *Source:* Haines (2000, p.156, Table 4.3).

	[1] total fertility rate	[2] infant mortality rate	[3] net fertility rate
1850	5.42	216.8	4.24
1870	4.55	175.5	3.75
1890	3.87	150.7	3.29
1910	3.42	96.5	3.09
1930	2.45	60.1	2.30
1950	2.98	26.8	2.90
1970	2.39	17.8	2.35
1990	2.00	7.6	1.98

is much lower today than 200 years ago. Although we do not explicitly model infant mortality, we assume that n_t represents the number of pairs of "surviving" children following the convention of literature. Therefore, we use the total fertility rate net of infant mortality as the target of simulation (Table 1,[17] column [3]).

3.1 Calibration

Take the length of a period in the model to be 20 years so that an individual lives for 20 years as a child, for 20 years as an adult, and for 20 years as an old. There will be 8 model periods between 1850 and 1990.

To be able to simulate the model, we give a parametric form to the non-market production function:

$$f(h) = \eta h^{\xi}, \tag{29}$$

where $\eta > 0$ and $\xi \in (0,1)$.

Time paths for TFP similar to those found in the United States between 1850 and 1990 are given to the model. Let $\{A_{1850}, A_{1870}, A_{1890}, A_{1910}, A_{1930}, A_{1950}, A_{1970}, A_{1990}\} = \{1.00, 1.15, 1.33, 1.62, 2.18, 3.59, 4.93, 5.72\}$. The initial level of TFP is normalized to unity. The estimates of the growth rate of TFP for the 1850-1890 periods are taken from Gallman (2000, p.15, Table 1.4), those for the 1890-1950 periods are taken from Carter et al. (2006, Series Cg270 and Cg278), those for the 1950-1990

[17] The infant mortality rate is the number of infant deaths per 1,000 live births per annum. The total fertility rate and the infant mortality rate are taken from Haines (2000). We construct the net fertility rate using them.

periods are taken from Bureau of Labor Statistics.[18]

Next, we choose values for the parameters governing tastes and technology. The parameter values are chosen on the basis of one of the following two criteria: (i) the parameter values themselves should be reasonable, and (ii) the values of the endogenous variable that follow from those parameter values should be reasonable. The values of α (physical capital share), δ (depreciation rate of physical capital), and z (time cost of children) are chosen on the basis of the former criterion: we choose them using a priori information. On the other hand, the values of γ (weight of children in utility function), ξ (curvature of non-market production function), η (efficiency of non-market production), and b (marginal productivity of physical labor) are chosen on the basis of the latter criterion.

We set $\alpha = 0.30$ because it is well known that capital share of income is roughly 30 percent. Strictly speaking, α is not equivalent to the capital share of income in our production technology. However, we choose $\alpha = 0.30$ as the baseline because the result is extremely insensitive to variations of α (see Subsection 3.3). The depreciation rate of physical capital, δ, is set to be 1.0 for simplification because the variables on which this paper focuses, such as time allocation and fertility, are not affected by the choice of δ at all.

The parameter z represents the time cost for having a pair of children. Although it is not easy to determine which value is realistic, Haveman and Wolfe (1995) and Knowles (1999) provide a guideline evidence: they indicate that the opportunity cost of a child is equivalent to about 20% of the parents' time endowment using U.S. data in 1992.[19] Following previous studies, e.g., de la Croix and Doepke (2003, 2004), Doepke (2004), and Lagerlöf (2006), we choose the value of z based on this empirical result. It is noted that zn_t is the time needed for rearing n_t-"pairs" of children. Thus, the value of z must be doubled: we choose $z = 0.40$. Although we assume that the value of z is constant over time in the baseline case, there is no evidence to support this assumption. There are many factors affecting the time cost, such as the prevalence of appliances and frozen foods, the introduction of child labor law, changes in child mortality, and quantity-quality trade-off. It seems to be reasonable to regard that the value of z varies over time. We examine the effect of varying z in Subsection 3.3.

The parameter γ, which is the weight of children in the utility function, directly influences the

[18] Source: ftp://ftp.bls.gov/pub/special.requests/opt/mp/prod3.mfptablehis.zip.

[19] There is a range of values in their estimation. We assume that the opportunity cost of a child is equal to 20% of the time endowment in the baseline case, while we conduct the sensitivity analysis with regard to the opportunity cost in Subsection 3.3.

number of children in each household. Using (7), (21), and (29), we obtain

$$\lim_{w_t^m \to \infty} n_t = \lim_{w_t^m \to \infty} \frac{\gamma}{z}\left[2+\frac{Ab}{w_t^m}+\frac{1-\xi}{\xi}\left(\frac{\eta\xi}{w_t^m}\right)^{\frac{1}{1-\xi}}\right] = \frac{2\gamma}{z}.$$

It follows that, given a value of z, the lower bound on the number of children is determined exclusively by the value of γ. We set it so that the lower bound is 0.8. This leads to $\gamma = 0.16$ because we have set $z = 0.40$ above.

The parameter $\xi \in (0,1)$ describes the curvature of the non-market production function. Using (11) and (29), we can compute the number of children in the case without physical capital, which is the upper bound on the number of children:

$$n_t\left(w_t^m \le \tilde{w}_1^m\right) = \frac{2\gamma}{z\left[\xi\left(1-\gamma\right)+\gamma\right]}.$$

The industrial revolution of the United States is generally considered to have started in the beginning of the 19th century. The total fertility rate was 7.04 in 1800. Since the comprehensive data on infant mortality prior to 1850 are not available, we need to guess the infant mortality rate in 1800 for computing the net fertility rate in 1800. Assuming that infant mortality linearly decreased between 1800 and 1990, we obtain the estimated value, 276.2, using the data of Haines (2000). Thus, we suppose that the net total fertility rate in 1800 was 5.10. We choose $\xi = 0.183$ so that the fertility rate in the economy without physical capital, that is, the economy with $w_t^m \le \tilde{w}_1^m$, is consistent with the figure: $n_t\left(w_t^m \le \tilde{w}_1^m\right) = 2.55$.

According to Ruggles (2001), the proportion of wage and salary workers for white men in 1850, which corresponds to the initial period in our simulation, was about 45%. Suppose that the economy starts from a steady state belonging to Case 2. We specify the relationship between the marginal productivity of physical labor, b, and the efficiency of non-market production, η, as $b(\eta)$ such that $l_t^H = 0.45$ in the steady-state equilibrium from which the simulation starts off. It follows from (14), (15), (25), (28), and $w_t^m = w_{t+1}^m$ that

$$b(\eta) = \frac{\eta\xi}{A_{1850}}\left[\frac{\gamma(1-\xi)+\xi}{\xi\left(2-2\gamma-0.45\right)}\right]^{1-\xi} - (1-\alpha)\left\{\frac{z(1-\gamma)}{0.45\gamma}\left[\eta\xi\left[\frac{\gamma(1-\xi)+\xi}{\xi\left(2-2\gamma-0.45\right)}\right]^{1-\xi}\right]\right\}^{\alpha}.$$

Table 2: The baseline parameter values

parameter	interpretation	value
α	Physical capital share	0.3
b	Marginal productivity of physical labor	0.860
δ	Depreciation rate of physical capital	1
z	Time cost of children	0.4
γ	Weight of children in utility function	0.16
η	Efficiency of non-market production	9.506
ξ	Curvature of non-market production function	0.183

Denote the value of η associated with $b = 0$ by η_0:

$$\eta_0 = \frac{1}{\xi}\left[A_{1850}\left(1-\alpha\right)\left(\frac{z\left(1-\gamma\right)}{0.45\gamma}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}}\left[\frac{\gamma(1-\xi)+\xi}{\xi\left(2-2\gamma-0.45\right)}\right]^{\xi-1}.$$

We pin down the value of η so as to fit the gender wage gap predicted by our model into the actual data: the ratio of women's median annual earnings to the men's was 0.537 in 1950, 0.587 in 1970 and 0.694 in 1990.[20] We minimize the sum of squared differences between actual and predicted female/male wage ratios in the last three model periods:[21]

$$\min_{\eta}\left(\frac{w^m_{1950}}{w^m_{1950}+A_{1950}b}-0.537\right)^2+\left(\frac{w^m_{1970}}{w^m_{1970}+A_{1970}b}-0.587\right)^2+\left(\frac{w^m_{1990}}{w^m_{1990}+A_{1990}b}-0.694\right)^2,$$
$$\text{s.t.}\quad \eta>\eta_0.$$

It follows from this minimization that the value of η and the attendant value of b are determined: $\eta = 9.506$ and $b = 0.860$ (the predicted female/male wage ratios are 0.603 in 1950, 0.603 in 1970, and 0.621 in 1990).

Table 2 lists the parameter values obtained from the procedures above.

[20] The figure for 1950 is calculated using data on manufacturing workers (Goldin, 1990, Table 3.1). The figures for 1970 and 1990 are calculated using data on full-time white workers (U.S. Census Bureau, Historical Income Tables, Table P-40 [http://www.census.gov/hhes/www/income/histinc/p40.html]).

[21] In the model economy, females supply labor to the market only in the last three periods. Altough we can compute the potential wages for females in the first five periods, females actually do not receive them in those periods. Thus, we estimate η focusing on the last three periods.

3.2 Transitional Dynamics

Imagine starting the economy off in 1850. Suppose that the economy is initially in a steady state belonging to Case 2. Figure 6 presents the simulations against the actual data.

The plotted data on male labor-market participation represent the evolution of the proportion of wage and salary workers for white men. This measure fits well with the purpose of our analysis. On the other hand, the women's equivalent is not available; we use the married-female labor-force participation in place of the proportion of wage and salary workers for married women; we should discount a little of what the actual data show.

The model captures the qualitative patterns of fertility and labor-force participation in the 19th and 20th centuries: (i) production has shifted out of households and into the market, (ii) males first increased their labor supply to the market, and then females increased it; married-female labor-market participation dramatically rose in the latter half of the 20th century, and (iii) there has been the secular decline in fertility, but there was the temporary rise after World War II (inverted N-shaped fertility dynamics).

We find that the model does well at explaining the main trend in fertility transition, but fails to replicate the significant decline prior to the baby boom: there is considerable discrepancy between the steepness of fertility decline from 1910 to 1930 in the data and that in the model. This discrepancy might be partly explainable by the absence of effects of World War II in our model. Suppose that the availability of time for male reaching adulthood in the fifth period, which corresponds to 1930 in the real world, exogenously declines, as in the simulation of Doepke et al. (2007). Our model would then generate the reduction in fertility in response to such an exogenous shock because it decreases the resources available for households and lowers both consumption level and fertility. Our model is more likely to complement rather than contradict the models focusing on effects of the war, such as Doepke et al. (2007). If the war had a considerable influence upon fertility choice, the fact that our model without any effect of the war cannot replicate the significant decline in fertility at that time indicates the relevance of our mechanism, rather than the defect. Historical data show that many developed countries experienced the inverted N-shaped fertility dynamics in the last two centuries, regardless of whether they joined the war, and the amplitude is larger in countries entering the war than in those not (Doepke et al., 2007, Figure 11-12; Galor, 2005, p.202, Figure 23). Such data provide a justification for our theory.

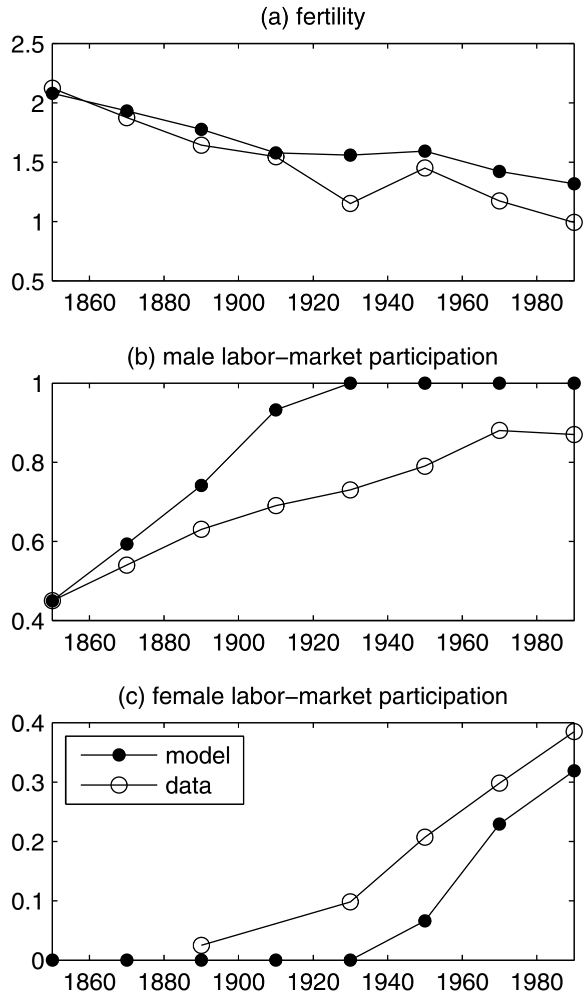

Figure 6: Simulations and actual data. (a) Fertility, (b) Male labor-market participation rate, (c) Female labor-market participation rate. *Sources:* Haines (2000), Goldin (1990), and Ruggles (2001).

3.3 Sensitivity Analysis

We conduct the sensitivity analysis here. All the tables and figures are in Appendix.

3.3.1 Sensitivity to α, γ, and ξ

In the baseline case, we set $\alpha = 0.3$ tentatively: despite the discrepancy between this parameter and capital income share, we chose the value from a priori information about capital income share. We find that the quantitative results obtained above are extremely robust with respect to changes in α (Table 3, column [1] and [2], and Figure 7).

The parameter γ represents the weight on children in the utility function and determines the lower bound of fertility for a given value of z. Given the baseline choice of $z = 0.4$, we change γ from 0.16

to 0.12 and 0.20 so that the lower bound of fertility changes from 0.8 to 0.6 and 1.0 respectively. We then recalibrate the values for ξ, η, and b in the same way as in the baseline case (Table 3, column [3] and [4]). As is easily expected, the higher value of γ results in higher fertility rates on any stage of development. However, the main trends are preserved (Figure 8).

The parameter ξ represents the curvature of the non-market production function and determines the upper bound of fertility for given values of z and γ. Given the baseline choice of $z = 0.4$ and $\gamma = 0.16$, we change ξ from 0.183 to 0.215 and 0.156 so that the upper bound of fertility changes from 2.55 to 2.35 and 2.75 respectively. We then recalibrate the values for η and b in the same way as in the baseline case (Table 3, column [5] and [6]). The value of ξ crucially affects the relative price of children to consumption. The lower value of ξ means that the non-market production exhibits strong diminishing-returns and household cannot increase consumption level so much even if it decreases time spent for child rearing: the relative price of children is low. The lower value of ξ results in higher fertility rates on any stage of development, but the main trends are insensitive to changes of ξ (Figure 9).

3.3.2 Sensitivity to z

The parameter z represents the time cost for having a pair of children. Since there is no information on the time series of z, we set $z = 0.4$ throughout all periods in the baseline case following the approach often used by previous studies. This approach would be rather problematic, especially when simulating the model over long periods, because there is no reason to assume that z is constant over time. There are many factors affecting the time cost: for instance, the prevalence of appliances and frozen foods and the higher availability of child-care services reduce parental time needed for child rearing; on the other hand, the introduction of child labor law and parents' motivation to spend more time educating offspring might imply the increase in it. To estimate the sequence of z over time in consideration of those factors is beyond the scope of this paper. We conduct three experiments here: (i) we change the value of z while retaining the assumption that z is constant over time, (ii) we consider the changes of z accompanied with changes of child mortality, and (iii) we consider the changes of z accompanied with changes of child mortality and school enrollment rate.

First, consider the changes of z while retaining the constant assumption. We change z from 0.4 to 0.3 and 0.5. Given the baseline choice that the lower bound of fertility is 0.8 and the upper bound is 2.55, the values for γ and ξ change with the change of z. We then recalibrate the values for η and b in the same way as in the baseline case (Table 3, column [7] and [8]). Figure 10 depicts the result. Since we recalibrate the variables other than z so that the upper bound and the lower bound of fertility do not change, the fertility rates does not change so much compared to the baseline case. However, the value of z considerably affects FLP: the lower z is, the steeper the rise of FLP. Furthermore, the lower value of z operates to accelerate the economic development: when $z = 0.3$, the baby boom occurs in the fifth period. Figure 11 presents the experiment where we change z from 0.4 to 0.35 and 0.45 without recalibration, that is, the variables other than z are the same as in the baseline case. As is easily expected, the higher value of z results in lower fertility rates on any stage of development. However, the main trends are preserved.

Next, consider the changes of z accompanied with changes of child mortality. The basic idea is that the time cost of raising a surviving child declines as more newborns survive to adulthood because parental time is also used for children not surviving to adulthood. Suppose that the time cost for having a pair of children is 0.4 when infant mortality rate is zero. Dividing $z = 0.4$ by the surviving rate ($1-$ (infant mortality rate)) of each period, we get $\{z_{1850}, z_{1870}, ..., z_{1970}, z_{1990}\} = \{0.511, 0.485, 0.471, 0.443, 0.426, 0.411, 0.407, 0.403\}$. Using these values, we recalibrate other variables in the same way as in the baseline case (Table 4, column [2]).[22] Although the steepness of fertility declines becomes small compared to the baseline case, the main trends are preserved (Figure 12).We also provide the result of the experiment where the time cost for having a pair of children is 0.3 and 0.5 when infant mortality rate is 0 (column [1] and [3] in Table 4, and Figure 12).

Lastly, we further take the changes of parents' motivation to educate children into consideration. The time cost for having a child only accrues as long as the child is a dependent family member. We assume that all children from ages 0 to 4 are dependent on their parents whereas children from ages 5 to 19 are dependent only if they attend school. Furthermore, assume that parents with children from ages 0 to 4 must bear the infant mortality risk. The time cost function in period t incorporating such ideas is $COST_t = 0.25 \cdot 1/(1 - IMR_t) + 0.75 \cdot SER_t$, where IMR_t is the infant mortality rate and SER_t is the school enrollment rate. Using this function and the historical data for infant mortality rate and school enrollment rate, we calculate $\{z_t\}$ so that z_{1990} is equal to 0.4: $z_t = 0.4 \cdot COST_t / COST_{1990}$.[23] We get $\{z_{1850}, z_{1870}, ..., z_{1970}, z_{1990}\} = \{0.313, 0.301, 0.308, 0.312, 0.338, 0.360, 0.396, 0.400\}$. This sequence

[22] The upper bound of fertility is computed based on z_{1800}, and the lower bound of fertility is computed based on $z = 0.4$.

[23] The school enrollment rate of 5-19 year-olds white for the 1850-1990 periods are taken from Goldin (1999), $\{SER_{1850}, SER_{1870}, ..., SER_{1970}, SER_{1990}\} = \{0.562, 0.544, 0.579, 0.613, 0.712, 0.793, 0.908, 0.925\}$. We also need SER_{1800} in order to equate the model's upper bound on the number of children to the actual fertility rate in 1800. Since the data for 1800 is not available, we substitute SER_{1850} for it.

of z_t exhibits an upward trend, which contrasts markedly with that derived in the previous experiment adjusting only changes of child mortality. Calibrated parameters are listed in Table 4, column [5], and Figure 13 depicts the result. The simulation manifests improvement in the steepness of fertility declines and FLP rises. However, the temporary rise of fertility does not occur because the downward pressure on fertility caused by the rise of time cost is greater than the positive income effect of the rise of male's wage.

3.3.3 Sensitivity to b and η

In the baseline, we specified the relationship between b and η so that the predicted proportion of wage and salary workers for men in the first period was equal to the actual one: $l^H_{1850} = 0.45$. Given the baseline choice of $z = 0.4$, $\gamma = 0.16$, and $\xi = 0.183$, we change the relationship between b and η so that l^H_{1850} changes from 0.45 to 0.35 and 0.55. We then minimize the sum of squared differences between actual and predicted female/male wage ratios in the last three model periods as in the baseline case (Table 4, column [7] and [8]). Figure 14 depicts the result. The higher l^H_{1850} implies the higher relative productivity of market work, inducing the structural transformation: both MLP and FLP rise more dramatically. When $l^H_{1850} = 0.55$, the baby boom occurs in the fifth period. Figure 15 and Figure 16 present the experiments where we change b and η without recalibration respectively, that is, the variables other than b and η are the same as in the baseline case.

4 Conclusion

This paper presents a general equilibrium model that incorporates both the difference between men and women and the allocation of time across market work, non-market work, and child rearing. In spite of its simple form, the model exhibits fairly rich dynamics related to production structure, family time use, sexual division of labor, and fertility. In particular, our model can replicate the inverted N-shaped fertility dynamics and the sequence of fertility and FLP changes broadly observed in the 19th and 20th centuries in the Western world. The point we wish to emphasize is that such results are derived as inherent features of the model, not by exogenous shocks.

The determinants of fertility are likely to be more complex than those embodied in our model: for instance, legal and social changes, gains in life expectancy, technological progress lowering the cost of having children, quantity-quality trade-off, and contraceptive availability are also important factors affecting fertility. Inevitably, there are still some features that cannot be explained in terms of our model. A prominent example is the fact that married-female labor-market participation had started to rise at the onset of baby boom. One possible solution is introducing heterogeneity among women: women with more market-oriented ability start to increase labor supply to the market before women with less one do. Alternatively, we can introduce the technological progress lowering the cost of having children as in Greenwood et al. (2005). This paper is more likely to complement rather than contradict the studies embodying the factors mentioned above, such as Greenwood et al. (2005). Although it is natural that one model cannot explain all facts, the general equilibrium model combining non-market production and differences between men and women should be a useful framework for explaining the fertility dynamics in line with economic growth.

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Appendix

This appendix presents the results of sensitivity analyses in Section 3.3.

Table 3: Recalibrated parameters.

	change in α		change in γ		change in ξ		change in z	
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	$\alpha = 0.2$	0.4	$\gamma = 0.12$	0.20	$\xi = 0.215$	0.156	$z = 0.3$	0.5
α	0.2	0.4	0.3	0.3	0.3	0.3	0.3	0.3
b	0.778	0.954	0.974	0.783	0.854	0.866	0.844	0.894
z	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.5
γ	0.16	0.16	0.12	0.2	0.16	0.16	0.12	0.2
η	8.46	10.73	15.14	6.55	8.67	10.33	9.723	9.243
ξ	0.183	0.183	0.131	0.241	0.215	0.156	0.220	0.142

Table 4: Recalibrated parameters.

	decreasing z_t			increasing z_t			change in l^H_{1850}	
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	$z_{IMR=0} = 0.3$	0.4	0.5	$z_{1990} = 0.3$	0.4	0.5	$l^H_{1850} = 0.35$	0.55
α	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
b	0.882	0.904	0.937	0.799	0.814	0.832	0.891	0.838
z							0.4	0.4
γ	0.12	0.16	0.2	0.12	0.16	0.2	0.16	0.16
η	14.878	14.310	13.351	6.635	6.542	6.405	10.92	8.37
ξ	0.122	0.080	0.034	0.329	0.297	0.262	0.183	0.183

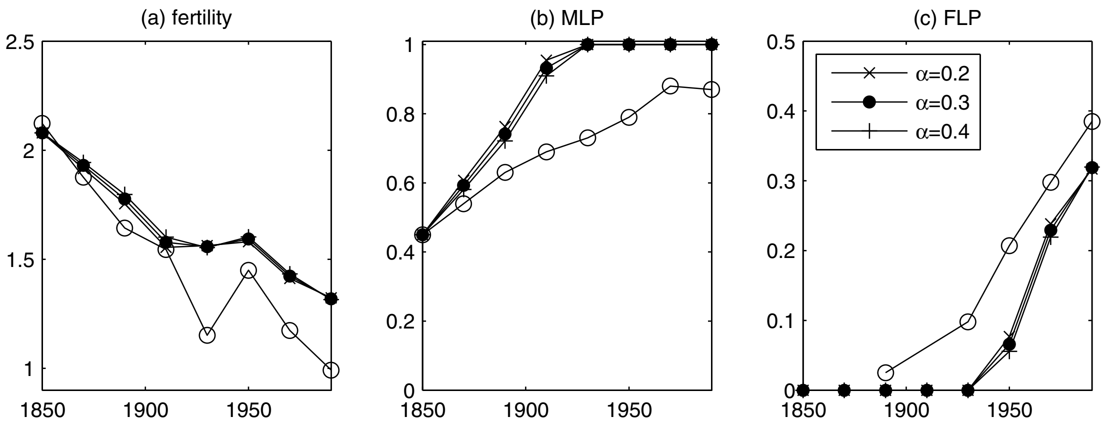

Figure 7: The effect of change in α. *Note:* Open circles "○" represent actual data (see Figure 6 for the sources). We use the same mark in Figure 8 - Figure 16.

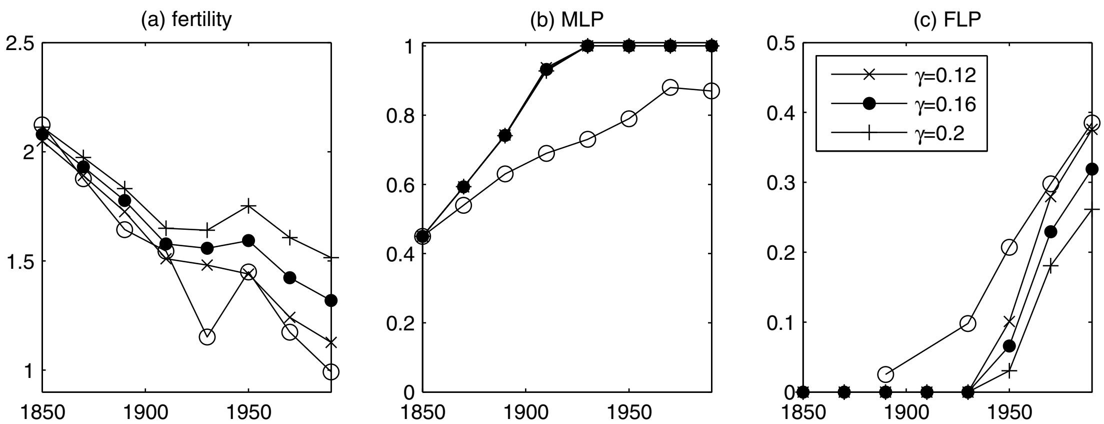

Figure 8: The effect of changes in γ.

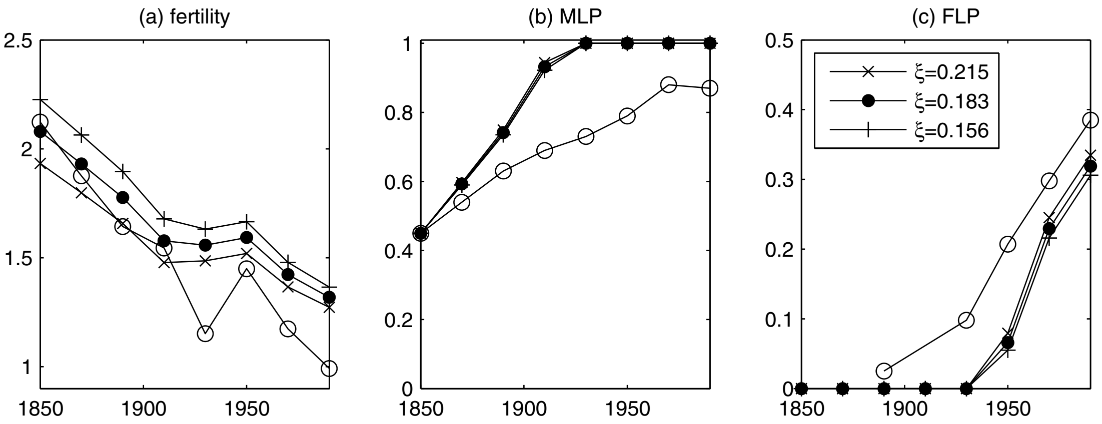

Figure 9: The effect of change in ξ.

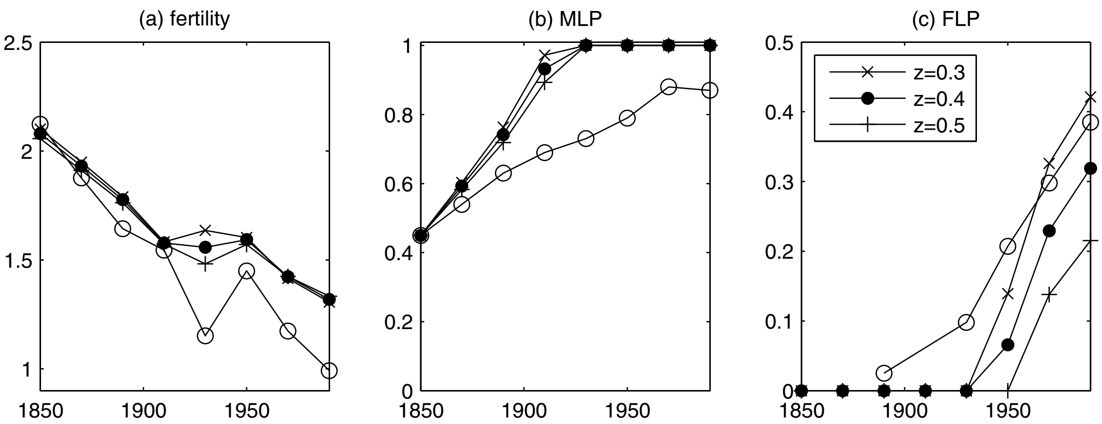

Figure 10: The effect of change in z.

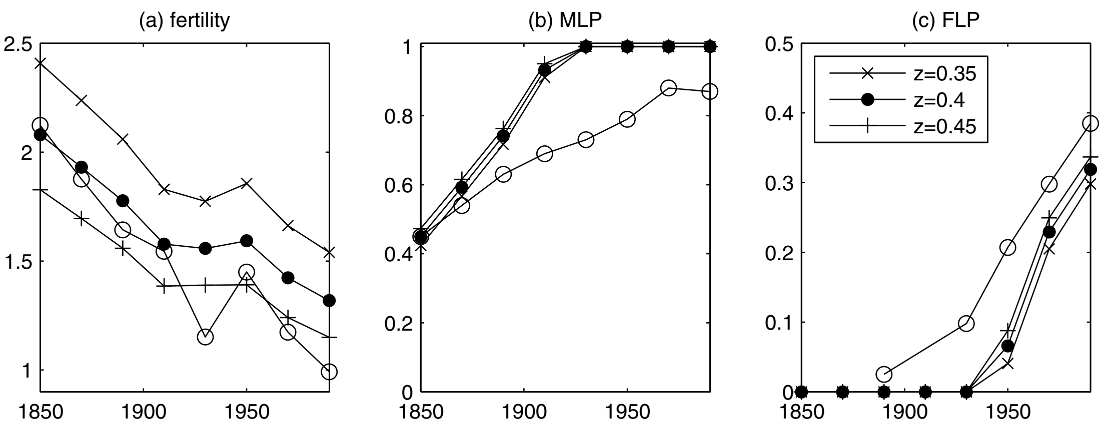

Figure 11: The effect of exogenous change in z.

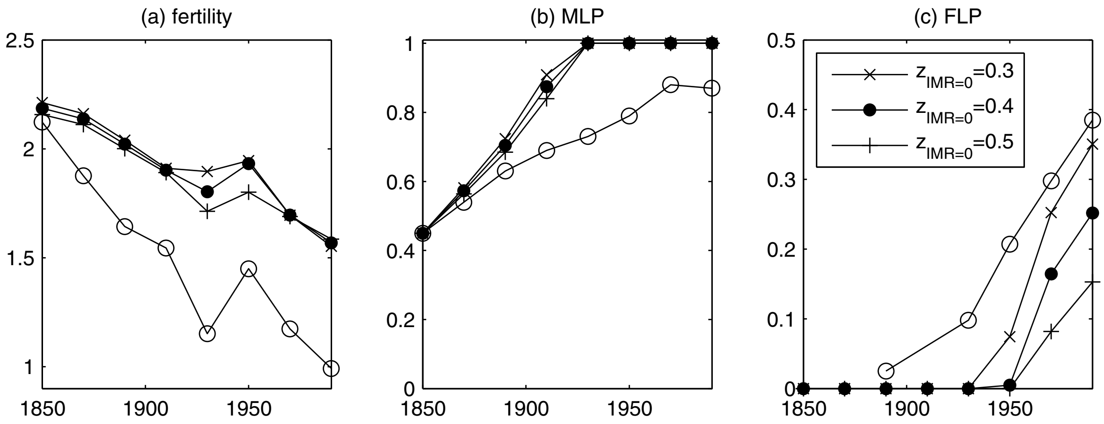

Figure 12: The effect of decreasing z_t.

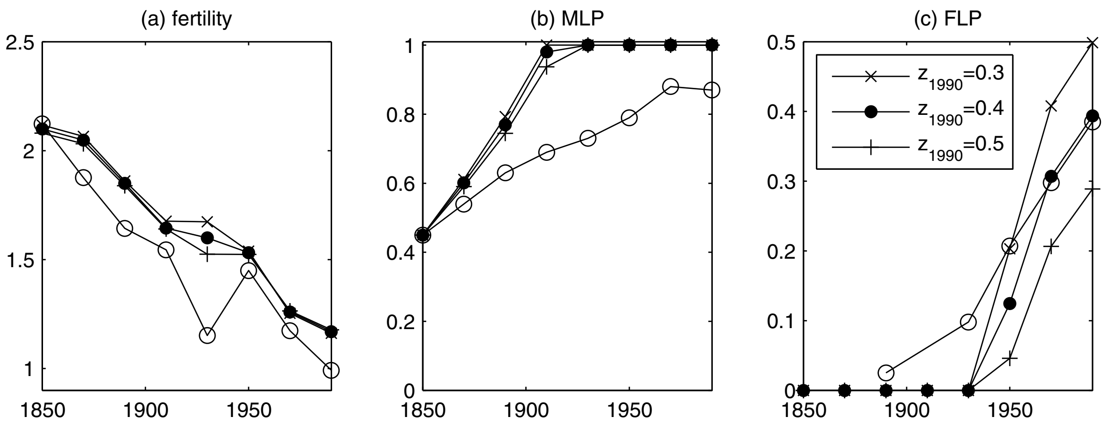

Figure 13: The effect of increasing z_t.

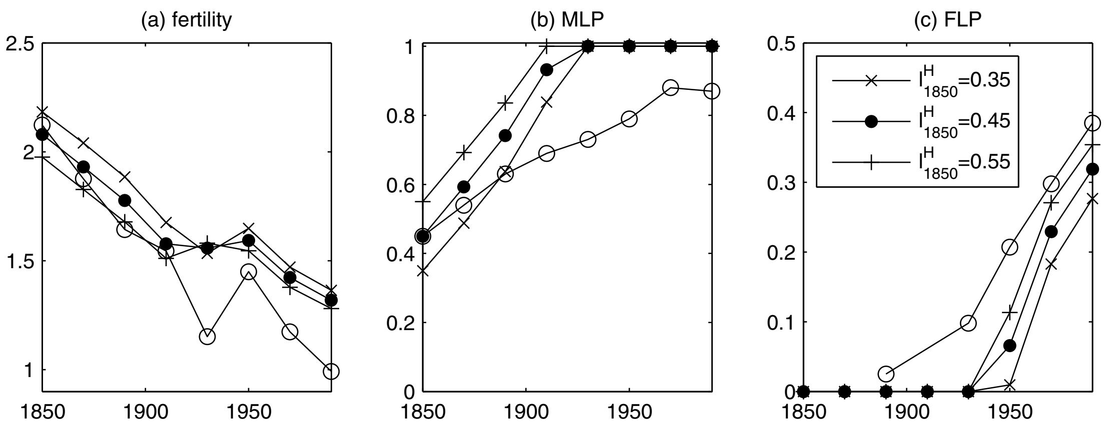

Figure 14: The effect of change in l^H_{1850}.

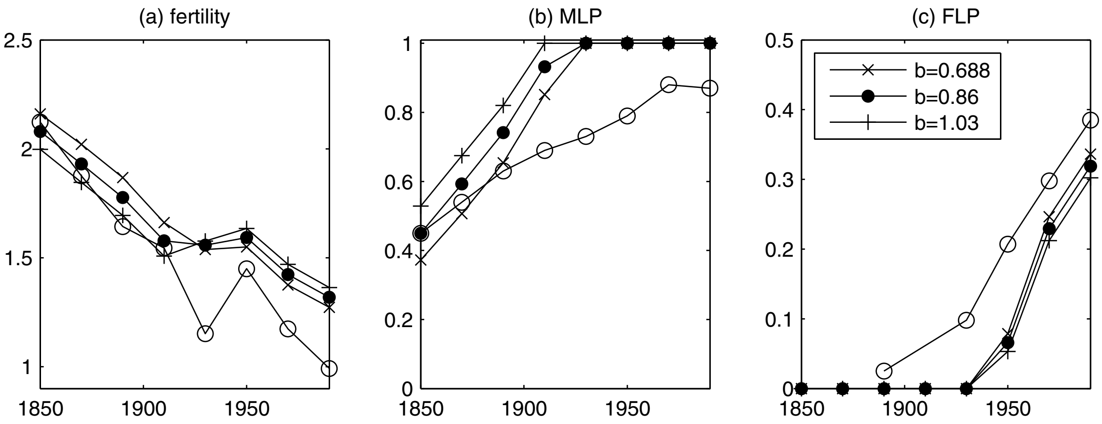

Figure 15: The effect of exogenous change in b.

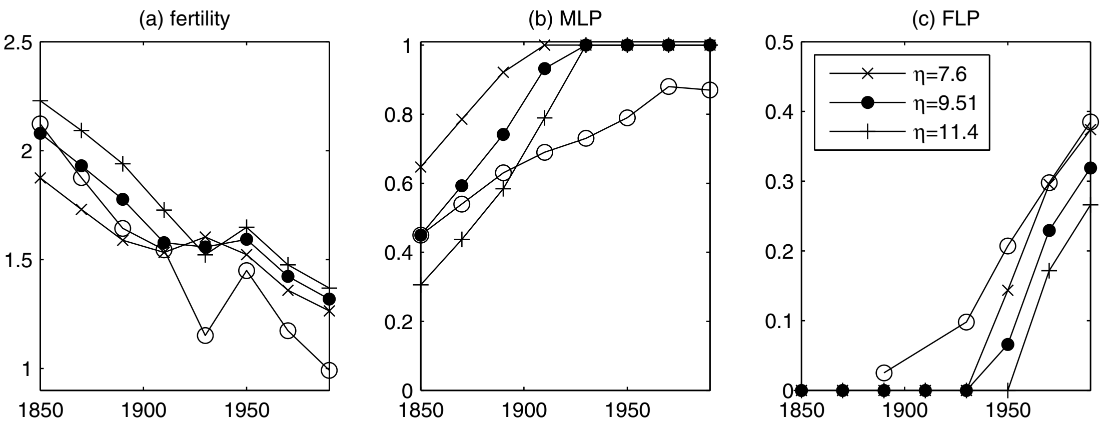

Figure 16: The effect of exogenous change in η.

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