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“Imperfect Interbank Markets and the Lender of Last Resort”

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Imperfect Interbank Markets and the Lender of Last Resort *

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Abstract

This paper presents a monetary model in which interbank markets bear limited commitment to contracts. Limited commitment reduces the proportion of assets that can be used as collateral, and thus banks with high liquidity demands face borrowing constraints in interbank markets. These constraints can be relieved by the central bank (a lender of last resort) through the provision of liquidity loans. I show that the constrained-efficient allocation can be decentralized by controlling only the money growth rate if commitment to interbank contracts is not limited. Otherwise, a proper combination of central bank loans and monetary policy is needed to bring the market equilibrium into a state of constrained efficiency.

Key words: Overlapping generations, money, interbank markets, limited commitment, the lender of last resort.

JEL Classification: E42, E51, G21

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1 Introduction

Interbank markets are one of the most important systems in a modern economy because they allow liquidity to be readily transferred from banks with a surplus to those with a deficit. Therefore, they are the focus of monetary policy and have significant effects on the whole economy. On occasion, however, the markets malfunction, as they did during the crisis that started in the summer of 2007. At these times, central banks needs to make the large-scale interventions to prevent the situation from futher deteriorating. The importance of a lender of last resort (LLR), dating back to Bagehot (1873), is stressed by many economists, but there is much less consensus on the nature of its role. For example, Fischer (1999, p. 86) states that "While there is considerable agreement on the need for a domestic lender oflast resort, some disagreements persist about what the lender of last resort should do."[1]

The purpose of this paper is to provide a monetary model for understanding the role of an LLR in an economy with an imperfect interbank market. In the model of this paper, interbank markets provide insurance for banks against the risk of sudden liquidity demands, but this insurance may be damaged by the limited commitment problem. Limited commitment denotes the inability of individual banks to fully commit to debt repayment. If this problem is significant, the banks' assets can be used as collateral, and banks that experience high liquidity shocks may be subject to borrowing constraints of the form studied by Kiyotaki and Moore (1997). In a crisis, limited commitment is attributed to the incapacity of troubled banks to borrow money from healthy banks.

My analysis is based on the works of Champ et al. (1996) and Smith (2002). I employ an overlapping generations model in which spatial separation and limited communication generate a transactions role for fiat money. At the end of each period, a fraction of agents is relocated to a different location. The only asset that they can use is fiat money. This allows money to be held even when dominated in the rate of return. Limited communication implies that relocated agents cannot transact using privately issued liabilities in the new location. However, agents who are

[1] See also Goodfriend and King (1988), Bordo (1990), and Kaufman (1991).

not relocated are not constrained by the rule of limited communication: they can pay for consumption goods with checks or other credit instruments when they are old. The other asset is a storage technology. The stochastic relocations act like shocks to agents' liquidity preferences, and create a role for banks to provide insurance against these shocks, as in Diamond and Dybvig (1983).

The model also assumes that a location is divided into a number of ex ante identical regions, each of which contain a number of depositors and a representative bank that behaves competitively. Different regions receive different liquidity shocks; this gives rise to regional heterogeneity, which motivates interbank trades. The basic role of interbank markets is to allow reallocations of liquidity from banks with an excess to banks with a deficit. As noted above, however, the markets may be imperfect because of the limited commitment problem.

The main results of the paper are as follows: (i) the market equilibrium can achieve constrained efficiency in perfect interbank markets when the central bank implements zero-inflation policy; (ii) when interbank markets malfunction because of limited commitment, banks cannot diversify their liquidity risks; consequently, the market equilibrium cannot achieve constrained efficiency, even if the central bank implements optimal monetary policy; (iii) if the central bank prints money and lends freely to the banking system at the same interest rate as interbank markets do, all banks that face borrowing constraints can meet liquidity demands by obtaining the central bank loans; consequently, the market equilibrium will achieve constrained efficiency under the implementation of zero-inflation policy.

Several other papers have studied the imperfections of interbank markets and the role of the central bank intervention in mitigating these imperfections. Aghion et al. (1999) and Allen and Gale (2000) analyze the spread of banking failure through interbank markets. Diamond and Rajan (2005) investigate optimal liquidity provision by a central bank when interbank markets are subject to aggregate liquidity shocks and contagious failure. Acharya et al. (2008) study the imperfections of interbank markets in times of crisis due to moral hazard, asymmetric information, and monopoly power, and show that central bank lending can ameliorate the inefficiency. Allen and Gale (2009) consider incomplete interbank markets that result in limited hedging opportunities for banks, and they show that a central bank can implement the constrained-efficient allocation by using open-market operations. Freixas et al. (2010) examine two different types of liquidity shocks to the banking system, and show that the central bank can implement the constrained-efficient allocation by setting interest rates that depend on the pattern of the shocks.

The main difference between these studies and mine is that my model explicitly assesses the role of money . In practice, the central bank has two important function: to control the money supply, and, as the LLR, to lend money to a banking systems. However, most of existing literature on the LLR does not consider monetary policy. In contrast, the model described here allows us to study not only optimal monetary policy and optimal LLR policy but also the interaction between these policies.

This paper bears a close theoretical similitude to the works of Antinolfi et al. (2001) and Antinolfi and Keister (2006). This is because it studies the role of LLR policy by combining the overlapping generations model with random relocation. Antinolfi et al. (2001) study the relationship between various LLR policies and inflationary equilibria in a pure-exchange economy. They show that an LLR policy in which the central bank lends money freely at a zero nominal interest rate generates Pareto optimal steady-state equilibrium, but also generates non-optimal inflationary equilibria, and suggest several LLR regimes that do not generate non-optimal equilibria. Antinolfi and Keister (2006) study an LLR policy and a monetary policy in a similar environment, and show that the policy combination achieves a state of the market equilibrium that closely approximates the first-best allocation of resources. Their LLR policy plays a key role in mitigating communication friction, which generates a transaction role for money. In contrast, this paper focuses on the inefficiency of interbank markets and a corrective LLR policy. That is, it focuses on using the LLR to reduce, not the communication friction, but friction caused by limited commitment within interbank markets. Thus, this paper considers how the constrained-efficient allocation, as chosen by a planner facing communication friction but not limited commitment, can be decentralized through monetary policy and LLR policy.

The rest of the paper is organized as follows. Section 2 presents the model of the study, from which Section 3 derives the constrained-efficient allocation. Section 4 reviews the behavior of banks in economies with perfect and imperfect interbank markets. Section 5 discusses equilibria in both types of economies, and Section 6 introduces the role of the LLR. Section 7 considers an LLR-created equilibrium, and how LLR policy leads market equilibrium to the constrained-efficient allocation. Finally, Section 8 concludes the investigation. All omitted proofs and some difficult derivations are contained in Appendices A and B, respectively.

2 The Model

I consider an economy consisting of an infinite sequence of overlapping generations that live for two periods. Periods are represented by $t = 0, 1, 2, \dots$. The world is divided into two spatially separated locations, and each location consists of a number of regions of unit mass. Each region is populated by a continuum of agents of unit mass. The two locations are completely symmetrical in terms of economic activity.

All young agents are ex ante identical. They are endowed with w units of goods when young, and none when old. In addition, there is a storage technology that convers one unit of goods stored at period t to $R > 1$ units of consumption at period $t + 1$.

All agents care only about second-period consumption. Let c_t denote the second-period consumption of a representative agent born at t. Agents have the same lifetime utility, $u(c) = \ln(c)$.[2]

As in Townsend (1987), I introduce a transaction role for money by emphasizing that the two locations are spatially separated and that communication between them is limited. Limited communication prevents privately issued liabilities from being verifiable in the other location.

[2]As in Champ et al. (1996) and others, this assumption of logarithmic utility allows us to solve the banks' problem analytically.

However, money is universally recognizable and impossible to noncounterfeitable, and is therefore accepted in both locations. In addition, during each period, agents can trade and communicate only with other agents in the same location.

After deposits have been allocated between investments and cash balances, a fraction π of young agents in each region is relocated to the other location. These agents are called "movers". The value of π is different across regions. Relocation plays the role of a "liquidity preference shock" in the Diamond and Dybvig (1983) model, and it is natural to assume that banks arise to insure agents against these shocks. The relocation probability π is a random variable, and because there is a continuum of young agents, it represents the fraction of all movers in each region. It also represents the aggregate liquidity in a region, and higher realizations of π correspond to higher demand for money. This is publicly observable, independent across regions, and identically distributed over time. Let F represent the distribution function, which is assumed to be smooth and strictly increasing on $[0, 1]$, and f the associated density function. The distribution F is common knowledge. Thus, the number of movers from each region within a location is $E(\pi) \equiv \int_0^1 \pi f(\pi) d\pi$.

To illustrate the role of interbank markets, I consider an economy where an intermediary is allowed to operate in only one region. In the past, legal restrictions of this form were common in the United States (US) and Japan. Even today, many banks in both countries operate only within a small region because of their size. After the realization of the liquidity shock, interbank markets open, and bank-to-bank transactions occur. Banks with high liquidity demands decide to borrow money through the markets, while banks with low liquidity demands decide to lend remaining cash reserves. It is assumed that a limited commitment problem exists in these financial markets. Banks that have borrowed money have the option of default, in which case the external enforcement agency can seize only a fraction of their assets. Thus, the assumption of limited commitment creates a role for collateral and the possibility of credit constraints.

Let M_t denote the per capita money supply at period t. The money supply grows at the exogenously selected gross rate σ, chosen once and for all in the initial period. Monetary injections are accomplished via lump-sum transfers to young agents. Let τ_t denote the real value of the transfers received by young agents at period t. In addition, let p_t denote the price levels at period t, and let $m_t = M_t/p_t$ denote real balances at period t. Given all this, the government budget constraint requires that

$$\tau_t = \frac{\sigma - 1}{\sigma} m_t. \tag{1}$$

The initial money supply, M_0, is given.

I assume that $\sigma R \geq 1$. In a steady-state equilibrium, σR is the market's nominal interest rate, and money is dominated in rate of return by storage technology. Note that the money growth rate satisfies $\sigma R = 1$, and thus implies the Friedman rule.

3 The Constrained-Efficient Allocation

First, I consider the constrained planning problem of a planner under the limited communication constraint[3]. This constraint not only bars the planner from transferring goods between locations, but prevents them from give goods stored in one location to movers from the other location. The constrained-efficient allocation maximizes the steady-state expected utility of a representative generation subject to both the limited communication constraint and the feasibility constraint. Let c_t^m and c_t^n denote the consumption allocated by the constrained planner to movers and non-movers born on period t, respectively. Also, denote s_t to be the amount that is stored by the planner at period t. Hence, the planner's problem can be written as

$$\max_{c_t^m, c_t^n, s_t} \int_0^1 \left\{ \pi \ln c_t^m + (1-\pi) \ln c_t^n \right\} f(\pi) d\pi$$

$$\text{s.t.} \quad E(\pi) c_t^m = w - s_{t+1} \tag{2}$$

$$[1 - E(\pi)] c_t^n = s_t R \tag{3}$$

$$s_t \leq w \quad \forall t. \tag{4}$$

[3]For details, see Haslag and Martin (2007) and Bhattacharya and Singh (2008)

Equation (2) states that all movers' consumption must be paid from a part of the current endowment collected by the planner. Equation (3) states that the goods provided to non-movers must be stored. Equation (4) states that storage investments cannot exceed the endowments collected by the planner. Given all this, the constrained-efficient allocation in a steady state $\{c^m, c^n, s\}$ is characterized by

$$c^m = w, \tag{5}$$

$$c^n = Rw, \tag{6}$$

$$s = [1 - E(\pi)]w. \tag{7}$$

Since the planner can transport goods across regions within a location, and since agents are risk-averse, it is optimal for the planner to equalize the consumption levels across regions. That is, the levels of consumption chosen by the planner should not be contingent on π.

4 A Banking Economy

As in Diamond and Dybvig (1983), the savings of all young agents will be intermediated. Banks take deposits from young agents in their regions, and choose how much to invest in storage i_t and money balances m_t. The rate of return on real balances between t and $t+1$ is p_t/p_{t+1}. Banks promise a return of $d_t^m(\pi)$ to each mover, and a return of $d_t(\pi)$ to each non-mover per unit on their deposits. These returns depend on the value of π. It is assumed that firms can freely enter the banking sector, and that banks are competitive in the sense that they accept as given the real return on assets. Thus, banks in each region are Nash competitors on the deposit side. That is, banks announce deposit return schedules $(d_t^m(\pi),\, d_t(\pi))$, taking the announced return schedules of other banks as given.

Let $\alpha_t(\pi)$ denote the fraction of cash reserves that the bank pays out at period t, and let $b_t(\pi)$ be the real balances that a bank borrows from or lends to interbank markets at the end of period t. If $b_t(\pi)$ is positive, a bank borrows cash from banks in the other regions; if it is negative, it lends them cash through an interbank market. If the interbank market is perfect, the bank can use it to borrow or lend cash freely at the market rate. Let ϕ_t denote the gross nominal interest rate of an interbank market at period t.

After banks create their portfolios and learn about the liquidity shocks of their region, interbank markets open, and they decide whether to borrow or to lend cash at ϕ_t. If, at the end of period t, a bank in a region experiencing high liquidity shock demands cash amounting to $b_t(\pi)$, it can borrow $p_t b_t(\pi)$ yen from other banks in regions experiencing a low liquidity shock, through an interbank market. During the following period, the borrowing bank must pay back $\phi_t b_t(\pi) p_t$ yen to the lending banks. Let $r_t^b \equiv \phi_t p_t / p_{t+1}$ denote the gross real interest rate in an interbank market. I use this rate as a substitute for ϕ_t in the following discussions.

The bank faces the following constraints on its choices i_t, m_t, $d_t^m(\pi)$, and $d_t(\pi)$. First, the bank's balance sheet requires that

$$i_t + m_t \leq w + \tau_t. \tag{8}$$

Second, payments to movers at period t, $\pi d_t^m(\pi)(w + \tau_t)$, cannot exceed the value of the bank's holdings at period $t+1$, and the borrowing of cash reserves. Therefore, it follows that

$$\pi d_t^m(\pi)(w + \tau_t) \leq \alpha_t(\pi) m_t \frac{p_t}{p_{t+1}} + b_t(\pi) \frac{p_t}{p_{t+1}}. \tag{9}$$

Finally, real payments to non-movers cannot exceed the value of the bank's remaining reserves plus the income from its investments minus the repayments of the interbank loan, so that

$$(1 - \pi) d_t(\pi)(w + \tau_t) \leq [1 - \alpha_t(\pi)] m_t \frac{p_t}{p_{t+1}} + R i_t - r_t^b b_t(\pi). \tag{10}$$

Of course, $0 \leq \alpha_t \leq 1$, $i_t \geq 0$, and $m_t \geq 0$ must hold.

In addition to the above constraints, banks also face borrowing constraints in interbank markets. From the assumption of limited commitment, an external enforcement agency can seize only a fraction $\theta \in (0, 1]$ of the investments of banks that choose to default on interbank loans.[4] To

[4]When $\theta = 0$, interbank markets collapse, and banks in each region fall into financial autarky. This situation is the same as that noted by Antinolfi et al. (2001), Smith (2002), and Antinolfi and Keister (2006). In this paper, I focus on the imperfections within interbank markets and the role of central bank loans in amending these imperfections, and then I rule out the case of $\theta = 0$.

prevent borrowers from defaulting strategically, i_t and $b_t(\pi)$ must satisfy a debt incentive constraint given by

$$Ri_t - r_t^b b_t(\pi) \geq (1-\theta)Ri_t.$$

The left-hand side of the constraint is what borrowers get if they decide to pay back their creditors, and the right-hand side is what they get if they default on their loan. The debt incentive constraint reduces to

$$r_t^b b_t(\pi) \leq \theta Ri_t. \tag{11}$$

Equation (11) implies that only a fraction θ of investment returns can be used as collateral for repayment in the interbank markets.[5]

Because banks behave as Nash competitors and have free entry, they will maximize the expected utility of a representative depositor in their region

$$\int_0^1 \Big\{\pi \ln[d_t^m(\pi)(w+\tau_t)] + (1-\pi)\ln[d_t(\pi)(w+\tau_t)]\Big\} f(\pi)d\pi, \tag{12}$$

subject to the constraints (8), (9), (10), (11), and the non-negativity constraints.

Let $\gamma_t \equiv m_t/(w+\tau_t)$ denote a bank's reserve-deposit ratio at period t, and let $\delta_t(\pi) \equiv b_t(\pi)/(w+\tau_t)$ denote the real value of a bank's borrowing or lending from the interbank markets per unit of deposits at period t.

Notice that θ represents the degree of imperfections in interbank markets. If θ is sufficiently large, an external enforcement agency can seize large enough quantities of assets owned by defaulting banks to give banks an ex ante incentive not to default strategically. As a result, borrowing constraints will be relaxed, and market perfections restored. It is easy to confirm that interbank markets are perfect only when $\theta = 1$ holds. I study the optimization problems of banks in two different interbank regimes in the remainder of this section.

4.1 The Case of $\theta = 1$: Perfect Interbank Markets

First, I analyze a regime featuring well-functioning interbank markets, that is, a regime in which $\theta = 1$. Within such a system, banks can

[5] As is typical in these models, defaults will not occur in equilibrium.

borrow or lend money freely at the interbank rate r_t^b. In this case, a debt incentive constraint never binds, and can be ignored in an examination of the optimization problem for banks. Both the fraction of bank reserves paid out to movers, α_t, and the real value of interbank loans, δ_t, are chosen after the realization of π, while γ_t is chosen before the realization of π. Hence, the optimal values of α_t and δ_t can be obtained via the functions of γ_t and π. That is, α_t and δ_t can be chosen in order to solve

$$\max_{\alpha_t \in [0,1], \delta_t} \pi \ln \left[\frac{\alpha_t \gamma_t}{\pi} \frac{p_t}{p_{t+1}} + \frac{\delta_t}{\pi} \frac{p_t}{p_{t+1}} \right] + (1-\pi) \ln \left[\frac{(1-\alpha_t)\gamma_t}{1-\pi} \frac{p_t}{p_{t+1}} + \frac{1-\gamma_t}{1-\pi} R - \frac{r_t^b \delta_t}{1-\pi} \right].$$

The solutions to these problems are

$$\alpha_t(\pi) = 1 \qquad \forall \pi \in [0,1], \tag{13}$$

$$\delta_t(\pi) = -(1-\pi)\gamma_t + \frac{\pi R}{r_t^b}(1-\gamma_t) \qquad \forall \pi \in [0,1]. \tag{14}$$

Equation (13) says that it is optimal for banks not to pay cash reserves to non-movers regardless of the value of π. When demand for liquidity is fairly low, that is, when equation (14) is negative, banks are able to meet the demand by using their own cash reserves. Under such circumstances, it is optimal for them to lend their remaining reserves to banks that are experiencing high liquidity shocks. Conversely, when demand for liquidity is high enough, that is, when equation (14) is positive, banks pays out all their reserves to movers, and borrow cash from banks that are experiencing low liquidity shocks.

I now proceed to solve the optimal value of γ_t. Substituting the optimal values of α_t and δ_t into the banks' objective function, I obtain the following problem:

$$\max_{\gamma_t \in [0,1]} \ln \left[\gamma_t + (1-\gamma_t)\frac{R}{r_t^b} \right] + \int_0^1 \left\{ \pi \ln \left(\frac{p_t}{p_{t+1}} \right) + (1-\pi) \ln r_t^b \right\} f(\pi) d\pi$$

The optimal choice for γ_t must be given by

$$\gamma_t(\pi) = \begin{cases} 0 & \text{if} \quad R > r_t^b \\ \in [0,1] & \text{if} \quad R = r_t^b \\ 1 & \text{if} \quad R < r_t^b. \end{cases} \tag{15}$$

If $R > r_t^b$, banks have an incentive to invest all deposits in storage because interbank lending is not beneficial and interbank borrowing is not expensive. However, if $R < r_t^b$, then it is optimal for banks to hold all deposits in cash because interbank lending is beneficial and interbank borrowing is expensive. Therefore, banks are neutral toward the choice between holding cash reserves and investing in storage only when $R = r_t^b$.

4.2 The Case of $0 < \theta < 1$: Imperfect Interbank Markets

I next examine a regime in which interbank markets are imperfect, that is, a regime in which $0 < \theta < 1$, because the limited commitment problem is severe. In this case, banks have to consider the possibility that a borrowing constraint may bind if they face high liquidity demands. Again, given the timing of the banks' decision, I can solve the optimal values of α_t and δ_t given γ_t and π. That is, I can choose α_t and δ_t to solve

$$\max_{\alpha_t \in [0,1], \delta_t} \pi \ln \left[\frac{\alpha_t \gamma_t}{\pi} \frac{p_t}{p_{t+1}} + \frac{\delta_t}{\pi} \frac{p_t}{p_{t+1}} \right] + (1-\pi) \ln \left[\frac{(1-\alpha_t)\gamma_t}{1-\pi} \frac{p_t}{p_{t+1}} + \frac{1-\gamma_t}{1-\pi} R - \frac{r_t^b \delta_t}{1-\pi} \right]$$

subject to

$$r_t^b \delta_t \leq \theta R (1-\gamma_t).$$

The solutions to these problems are

$$\alpha_t(\pi) = 1 \qquad \forall \pi \in [0,1], \tag{16}$$

$$\delta_t(\pi) = \begin{cases} -(1-\pi)\gamma_t + \frac{\pi R}{r_t^b}(1-\gamma_t) & \text{if} \quad 0 \leq \pi \leq \pi^* \\ \frac{\theta R}{r_t^b}(1-\gamma_t) & \text{if} \quad \pi^* < \pi \leq 1, \end{cases} \tag{17}$$

where

$$\pi^* \equiv \frac{r_t^b \gamma_t + \theta R (1-\gamma_t)}{r_t^b \gamma_t + R(1-\gamma_t)}. \tag{18}$$

As in the previous, equation (16) says that it is optimal for banks not to pay cash reserves to non-movers, regardless of the value of π. Equation

(17) states that, for realizations of the liquidity shock below the critical value π^*, banks can lend or borrow money freely in interbank markets; however, for realization of the liquidity shock is greater than π^*, banks cannot borrow the amount of money desired, and will face a "liquidity crisis." In the crisis, the banks cannot fully meet liquidity demands, and movers must receive a lower return.

This result follows from the trade-off between two forces. First, the return on cash balances is lower than the return on storage technology. Therefore, banks prefer to minimize cash reserves. At the same time, they strive to provide insurance by equalizing the returns between movers and non-movers for all realizations of π. To do so, they must hold sufficient cash reserves. At the margin, the welfare gains from equalizing the returns to movers and non-movers exactly offset the cost implied by the return dominance of storage investments over cash reserves.

I now proceed to solve the optimal value of γ_t. Substituting the optimal values of α_t and δ_t into the banks' objective function, I obtain the following problem:

$$\max_{\gamma_t \in [0,1]} \int_0^{\pi^*} \left\{ \pi \ln \frac{p_t}{p_{t+1}} \left[\gamma_t + (1-\gamma_t)\frac{R}{r_t^b} \right] + (1-\pi)\ln[\gamma_t r_t^b + (1-\gamma_t)R] \right\} f(\pi) d\pi$$
$$+ \int_{\pi^*}^1 \left\{ \pi \ln \frac{1}{\pi}\frac{p_t}{p_{t+1}} \left[\gamma_t + (1-\gamma_t)\frac{\theta R}{r_t^b} \right] + (1-\pi)\ln \frac{(1-\theta)(1-\gamma_t)R}{1-\pi} \right\} f(\pi) d\pi$$

The optimal value of γ_t is given by[6]

$$\gamma_t = \begin{cases} 0 & \text{if} \quad r_t^b < \frac{\theta R}{1-\int_\theta^1 F(\pi)d\pi} \\ 1 - \frac{r_t^b}{r_t^b - \theta R}\int_{\pi^*}^1 F(\pi)d\pi & \text{if} \quad \frac{\theta R}{1-\int_\theta^1 F(\pi)d\pi} \le r_t^b \le R \\ 1 & \text{if} \quad r_t^b > R. \end{cases} \tag{19}$$

Note that γ_t implicitly defined by (19) is increasing in r_t^b. Intuition is simple. An increase in r_t^b makes borrowing interbank loans costly, and it gives an incentive for banks to hold more cash reserves. Having solved the optimization problem for banks, I now turn to an analysis of general equilibrium.

[6] I provide the derivation in Appendix B.

5 Equilibrium

An equilibrium consists of sequences for prices $\{p_t, r_t^b\}$ and for the decision rules of banks $\{\gamma_t, \alpha_t, \delta_t\}$, such that (i) given $\{p_t, r_t^b\}$, the decision rules solve the banks' problems in each period; (ii) the market where money is traded for goods at the beginning of each period clears; (iii) the government budget constraint in equation (1) holds in each period; and (iv) the interbank markets clear in each period, that is,

$$\int_0^1 \delta_t(\pi) f(\pi) d\pi = 0. \tag{20}$$

From this clearing condition and the optimal reserve-deposit ratio given in equation (15) or equation (19), the values of the pair (γ_t^*, r_t^{b*}) can be determined. It is easy to check that, under the logarithmic utility, these values do not depend on the inflation rate p_{t+1}/p_t, and the steady state is the only equilibrium of the economy. Thus, the time subscript can be dropped from in the following discussions.

The money market clears if $m = \gamma^*(w + \tau)$. Substituting the government budget constraint in equation (1) into this equation yields the following:

$$w + \tau = \frac{w}{1 - \frac{\sigma - 1}{\sigma}\gamma^*}.$$

Then, I can obtain the steady-state equilibrium values of real balances and investments,

$$m = \frac{w\gamma^*}{1 - \frac{\sigma - 1}{\sigma}\gamma^*},$$

$$i = \frac{w[1 - \gamma^*]}{1 - \frac{\sigma - 1}{\sigma}\gamma^*}.$$

5.1 Equilibrium with Perfect Interbank Markets

First, I derive an equilibrium with perfect interbank markets. Substituting the optimal value $\delta(\pi)$ into the interbank market clearing condition in equation (20), I obtain

$$\gamma = \frac{E(\pi)R}{E(\pi)R + [1 - E(\pi)]r^b}, \tag{21}$$

which is decreasing in r^b. An decrease in γ increases the number of borrowers and decreases the number of lenders in interbank markets. Then, it pushes up the interbank rates. A steady-state equilibrium is characterized by the pair (γ^*, r^{b*}) satisfying equations (15) and (21). Figure 1 illustrates the steady-state equilibrium with well-functioning interbank markets. The horizontal axis shows r_t^b, and the vertical axis shows γ_t. Equation (15) crosses equation (21) exactly once, and a unique steady-state equilibrium exists at that point. In this equilibrium, the optimal reserve-deposit ratio is given by $\gamma^* = E(\pi)$, and the real interest rate in an interbank market is given by $r^{b*} = R$. From these results, I obtain $\delta(\pi) = \pi - E(\pi)$. This solution means that banks in each region will hold in money a share of deposits equal to the share of movers in regions as a whole, which is represented by $E(\pi)$. For liquidity shocks below the value $E(\pi)$, banks will borrow cash from other banks at a rate of $r^b = R$, and pay out all their reserves, plus the liquidity they obtain from loans, to movers in their respective regions. When liquidity shocks are greater than $E(\pi)$, banks will pay out only a fraction of their reserves to movers, and lend the remainder to other banks through interbank markets.

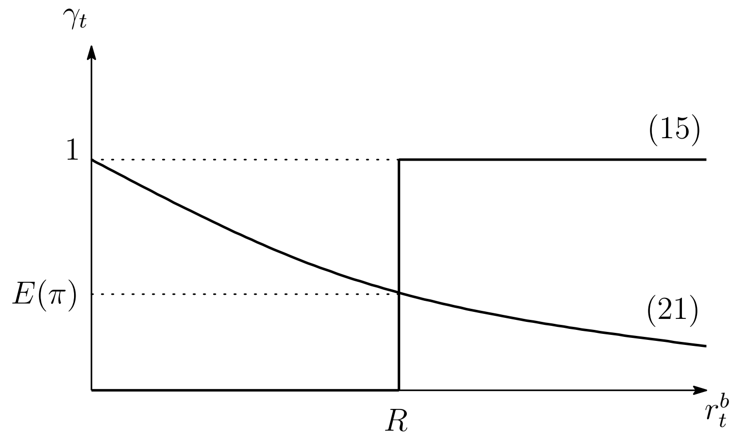

Figure 1: Equilibrium with perfect interbank markets

Therefore, expressions for m, i, and $b(\pi)$ are as follows:

$$\begin{aligned} m &= \frac{wE(\pi)}{1 - \frac{\sigma-1}{\sigma}E(\pi)}, \\ i &= \frac{w[1 - E(\pi)]}{1 - \frac{\sigma-1}{\sigma}E(\pi)}, \\ b(\pi) &= \frac{w[\pi - E(\pi)]}{1 - \frac{\sigma-1}{\sigma}E(\pi)}. \end{aligned}$$

From the above results, I obtain the value of equilibrium consumption:

$$c^m = \frac{w}{\sigma - (\sigma - 1)E(\pi)}, \tag{22}$$

$$c^n = \frac{wR}{1 - \frac{\sigma-1}{\sigma}E(\pi)}. \tag{23}$$

Note that the consumption levels of movers and non-movers do not depend on π. This implies that consumption levels are equalized between regions, and that agents receive complete insurance against liquidity shocks. In addition, note that $c^n = \sigma R c^m$. In the steady-state equilibrium, the wedge between the return paid to movers and the return paid to non-movers depends on the money growth rate. The higher the inflation rate, the larger the gap between c^m and c^n, and the farther agents are from being completely insured.

Now, consider a situation where a central bank implements zero-inflation policy, which means that $\sigma = 1$. Consumption levels for movers and non-movers are then given by the following equations:

$$\begin{aligned} c^m &= w, \\ c^n &= wR. \end{aligned}$$

This allocation is identical to that dispensed in the constrained planning problem. The Friedman rule, $\sigma = 1/R$, equalizes consumption between movers and non-movers. In addition, it states that agents' deposits decrease, and that banks invest less in storage technology because the seigniorage collected by the government is rebated to young agents. Zero-inflation policy is a fair trade-off between productive efficiency and risk sharing, and, in this case, constitutes optimal monetary policy.

This result can be summarized in the following proposition.

Proposition 1 *When interbank markets are perfect (i.e., when $\theta = 1$), the market equilibrium under zero-inflation policy (in which $\sigma = 1$) achieves constrained efficiency.*

Goodfriend and King (1988) argue that, in well-functioning financial markets, a solvent institution cannot be illiquid, and conclude that the central bank should focus solely on implementing open-market operations, and should refrain from helping specific banks. The results obtained in this subsection coincide with their views. In an economy with perfect interbank markets, the constrained-efficient allocation can be decentralized by controlling only the money growth rate.

5.2 Equilibrium with Imperfect Interbank Markets

Next, I derive an equilibrium with imperfect interbank markets. Substituting the optimal value $\delta(\pi)$ into the interbank market clearing condition expressed in equation (20), I obtain

$$\frac{\theta R(1-\gamma)}{r^b\gamma + R(1-\gamma)} = \int_0^{\pi^*} F(\pi)d\pi, \tag{24}$$

which implicitly defines the relationship between γ and r^b as in the previous. The value of γ defined by (24) is decreasing in r^b. A steady-state equilibrium with imperfect interbank markets is characterized by the pair (γ^{**}, r^{b**}) satisfying equations (19) and (24). Figure 2 illustrates a steady-state equilibrium with imperfect interbank markets. Equation (19) crosses equation (24) exactly once, and a unique steady-state equilibrium exists at that point.

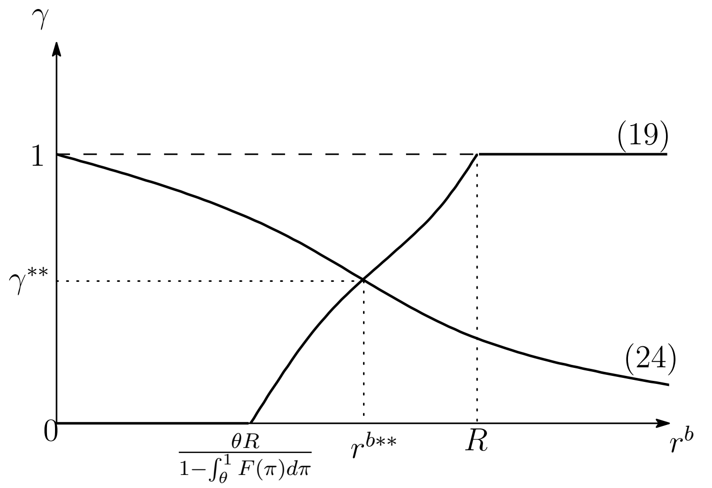

Figure 2: Equilibrium with Imperfect Interbank Markets.

Let us compare the reserve-deposit ratio of an economy with a perfect interbank markets to that of an economy with an imperfect interbank market. I have the following result.

Lemma 1 *For any $\theta \in (0,1)$, $\gamma^{**} < E(\pi) = \gamma^*$ holds.*

Lemma 1 states that banks that may face a liquidity constraint in interbank markets hold lower levels of cash reserves than banks that do not. This is because $r_t^b < R$ in an economy with an imperfect interbank market. Banks that receive relatively high liquidity shocks and borrow from interbank markets have an incentive to reduce their cash reserves because interbank borrowing is less costly. On the other hand, banks that receive relatively small shocks and lend to interbank markets also have an incentive to reduce their cash reserves because storage investments produce more profit than interbank lending does. Consequently, it is ex ante optimal for all banks to reduce cash reserves and invest in storage.

I now examine how the steady-state values γ^{**} and r^{b**} depend on parameters. Specifically, I am interested in the effect of a change in the limited enforcement parameter θ on γ^{**} and r^{b**}. Note that an increase in θ has two contrasting effects on γ^{**}. Figure 3 illustrates this change.

Before shocks are realized, storage investments provide more collateral capacity and are more attractive to banks because they are easier to borrow against. This collateral effect decreases the value of γ^{**}. On the other hand, an increase in θ also increases the amount of money demanded by borrowers because borrowing constraints have been relaxed. However, at the same time, the amount of money supplied by lenders remains unchanged. This drives up the price of interbank loans until the demand for loans equals their supply. This general equilibrium price effect increases γ^{**} because an increase in r^{b**} makes interbank loans costly, and it gives an incentive to hold larger cash reserves.

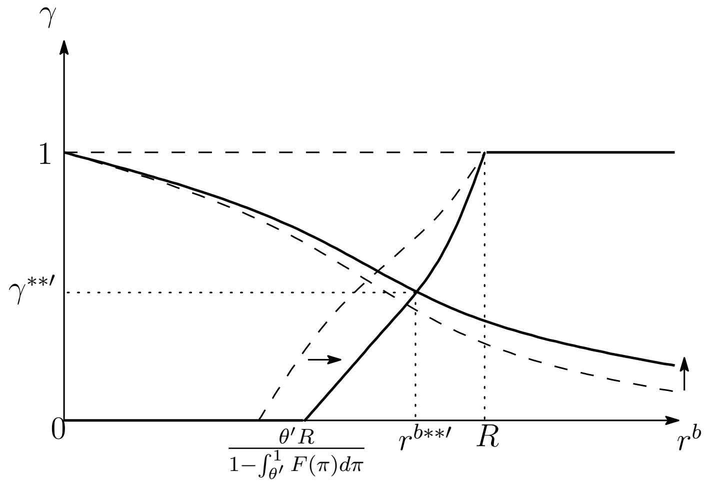

Figure 3: An increase in θ

With the steady-state values γ^{**} and r^{b**}, I can obtain equilibrium consumption as follows:

$$c^m(\pi) = \begin{cases} \frac{w[\gamma^{**}+\frac{R}{r^{b**}}(1-\gamma^{**})]}{\sigma-(\sigma-1)\gamma^{**}} & \text{if} \;\; 0 \leq \pi < \pi^* \\ \frac{w[\gamma^{**}+\frac{\theta R}{r^{b**}}(1-\gamma^{**})]}{\pi[\sigma-(\sigma-1)\gamma^{**}]} & \text{if} \;\; \pi^* \leq \pi \leq 1, \end{cases} \tag{25}$$

$$c^n(\pi) = \begin{cases} \frac{w[\gamma^{**}r^{b**}+R(1-\gamma^{**})]}{1-\frac{\sigma-1}{\sigma}\gamma^{**}} & \text{if} \;\; 0 \leq \pi < \pi^* \\ \frac{w(1-\theta)(1-\gamma^{**})R}{(1-\pi)[1-\frac{\sigma-1}{\sigma}\gamma^{**}]} & \text{if} \;\; \pi^* \leq \pi \leq 1. \end{cases} \tag{26}$$

The values of c^m and c^n depend on the realization of π; therefore agents do not hedge the risk of liquidity shock. The limited enforcement of contracts in interbank markets prevents banks from diversifying liquidity risk, thus exposing agents to the risk.

In this situation, I am interested in optimal monetary policy, which is the value at which σ would maximize the steady-state expected utility of a representative depositor.[7] Specifically, the problem of the central bank is as follows:

$$\max_{\sigma \geq \frac{1}{R}} \int_0^1 \Big\{ \pi \ln c^m(\pi) + (1-\pi) \ln c^n(\pi) \Big\} f(\pi) d\pi \tag{27}$$

Here, $c^m(\pi)$ and $c^n(\pi)$ are given by equations (25) and (26), respectively. The optimality of zero-inflation policy (i.e., a policy where $\sigma = 1$) is a fairly well-known result in the overlapping generations model with random relocation.[8] As previously shown, this result holds in an economy with a perfect interbank market. However, the following proposition states that zero inflation is not optimal in an economy with an imperfect interbank market.

Proposition 2 *When interbank markets are imperfect (i.e., when* $0 < \theta < 1$*), zero inflation is not optimal, and deflation dominates zero inflation.*

In the problem, the government chooses a value of σ to balance a trade-off between productivity efficiency and risk-sharing. Low money demands results in smaller government seigniorage and smaller transfers to young agents. In an economy with an imperfect interbank market, the marginal benefit from an increase in inflation is relatively small, and the government must prioritize risk-sharing against idiosyncratic shocks over transfers to young agents.

However, the equilibrium consumption levels still depend on the value of π, even if the central bank implements the optimal monetary policy. The inefficiency produced by limited commitment to interbank contracts cannot be corrected only by controlling the money growth rate. This result can be summarized in the following proposition.

[7] I ignore the initial old generations in welfare calculations, as Smith (2002) and others have done.

[8] See Bhattacharya, Haslag, and Russell (2005), and Haslag and Martin (2007).

Proposition 3 *When interbank markets are imperfect (i.e., when* $0 < \theta < 1$*), the market equilibrium cannot achieve constrained efficiency for any* $\sigma \geq 1/R$*.*

6 The Lender of Last Resort

In this section, I analyze a regime in which interbank markets are imperfect, but the central bank can make one-period loans of money at a fixed rate as a LLR. This is in line with the famous lesson from Bagehot (1873) that a LLR should "lend freely, at a penalty rate" during crises. After π is realized, banks determine the real amount $e_t \geq 0$ that it would like to borrow at the end of period t, and the central bank prints $p_t e_t$ yens for that banks. During the next period, the bank must repay $\phi^c e_t p_t$ yen to the central bank where ϕ^c is the gross nominal interest rate on the liquidity loan at period t. I assume that the central bank destroys $p_t e_t$ of these yen and uses the remaining $(\phi^c - 1)p_t e_t$ to purchase goods so that the stock of base money remains unaffected. Let $r^c \equiv \phi^c p_t / p_{t+1}$ denote the gross real interest rate of the central bank loans, [9] and substitute this real rate for ϕ^c in the following discussions. Further, assume that the real interest rate of the central bank loans is higher than that of a loan obtainable from the interbank market. In other words, suppose that $r^c \geq r_t^b$. This means that the central bank lends money to private banks at a "penalty" rate. [10] Otherwise, there are no banks which borrow the interbank loans, and interbank markets collapse. For simplicity's sake, I assume that agents derive no utility from the revenue that the central bank earns from these interest payments.

[9] I assume that r^c is time-invariant and controllable by the central banks. In fact, the assumption is that the central bank can sets $\phi^c = \tilde{r}^c p_{t+1}/p_t$ and controls $\tilde{r}^c$.

[10] It is implicitly assumed that the central bank is able to overcome the problem of limited contract commitment, or that there are no upper limits on the central bank loans. A simple interpretation is as follows. The central bank has enough wealth to keep track of defaulting borrowers and to collect debts in their entirety. Consequently, banks cannot evade debt payments, and private banks will not face borrowing constraints on loans from the central bank. It would be interesting to considering credit constraints on the central bank loans, but I present a simpler case here.

Defining $\eta_t \equiv e_t/(w + \tau_t)$ to be real borrowing from the central bank per unit of deposits. Since banks cannot lend money to the central bank, η_t must be non-negative. Again, given the timing of banks' decision, the optimal values of α_t, δ_t, and η_t can be obtained given γ_t and π

$$\max_{\alpha_t, \delta_t, \eta_t} \pi \ln \left[\frac{\alpha_t \gamma_t}{\pi} \frac{p_t}{p_{t+1}} + \frac{\delta_t + \eta_t}{\pi} \frac{p_t}{p_{t+1}} \right] + (1 - \pi) \ln \left[\frac{(1 - \alpha_t)\gamma_t}{1 - \pi} \frac{p_t}{p_{t+1}} + \frac{1 - \gamma_t}{1 - \pi} R - \frac{r_t^b \delta_t}{1 - \pi} - \frac{r^c \eta_t}{1 - \pi} \right]$$

subject to

$$r_t^b \delta_t \leq \theta R(1 - \gamma_t),$$
$$\eta_t \geq 0.$$

The solutions to these problems are

$$\alpha_t(\pi) = 1 \qquad \forall \pi \in [0, 1], \tag{28}$$

$$\delta_t(\pi) = \begin{cases} -(1 - \pi)\gamma_t + \frac{\pi R}{r_t^b}(1 - \gamma_t) & \text{if} \quad 0 \leq \pi \leq \pi^* \\ \frac{\theta R}{r_t^b}(1 - \gamma_t) & \text{if} \quad \pi^* < \pi \leq 1, \end{cases} \tag{29}$$

$$\eta_t(\pi) = \begin{cases} 0 & \text{if} \quad 0 \leq \pi \leq \pi^{**} \\ -(1 - \pi)[\gamma_t + \frac{\theta R}{r_t^b}(1 - \gamma_t)] + (1 - \theta)\frac{\pi R}{r^c}(1 - \gamma_t) & \text{if} \quad \pi^{**} < \pi \leq 1, \end{cases} \tag{30}$$

where π^* continues to be given by equation (18), and

$$\pi^{**} \equiv \frac{r^c[r_t^b \gamma_t + \theta R(1 - \gamma_t)]}{r^c[r_t^b \gamma_t + \theta R(1 - \gamma_t)] + (1 - \gamma_t)(1 - \theta) R r_t^b}. \tag{31}$$

Note that $\pi^* \leq \pi^{**} < 1$ if and only if $r_t^b \leq r^c$.

For realizations of the liquidity shock below the first critical value π^*, banks can lend or borrow money freely via interbank markets, and meet the liquidity demands in their respective regions. When a liquidity shock $\pi \in [\pi^*, \pi^{**}]$ is realized, borrowing constraints are binding in interbank markets, but the banks do not resort to the central bank loans because such loans are costly for the banks. Finally, when a liquidity shock is greater than π^{**}, borrowing constraints are binding, and the banks obtain the liquidity loans from the central bank.

I now proceed to solve the optimal value of γ_t. Substituting the optimal values of α_t, δ_t, and η_t into the banks' objective function, I obtain the following problem:

$$
\begin{aligned}
\max_{\gamma_t \in [0,1]} & \int_0^{\pi^*} \left\{ \pi \ln \frac{p_t}{p_{t+1}} \left[\gamma_t + (1-\gamma_t) \frac{R}{r_t^b} \right] + (1-\pi) \ln[\gamma_t r_t^b + (1-\gamma_t) R] \right\} f(\pi) d\pi \\
& + \int_{\pi^*}^{\pi^{**}} \left\{ \pi \ln \frac{1}{\pi} \frac{p_t}{p_{t+1}} \left[\gamma_t + (1-\gamma_t) \frac{\theta R}{r_t^b} \right] + (1-\pi) \ln \frac{(1-\theta)(1-\gamma_t) R}{1-\pi} \right\} f(\pi) d\pi \\
& \quad + \int_{\pi^{**}}^{1} \left\{ \pi \ln \frac{p_t}{p_{t+1}} \left[\gamma_t + (1-\gamma_t) R \left\{ \frac{\theta}{r_t^b} + \frac{1-\theta}{r^c} \right\} \right] \right. \\
& + (1-\pi) \ln \left[(1-\theta)(1-\gamma_t) R + r^c \left[\gamma_t + (1-\gamma_t) \frac{\theta R}{r_t^b} \right] \right] \left. \vphantom{\frac{\theta}{r_t^b}} \right\} f(\pi) d\pi
\end{aligned}
$$

The optimal value of reserve-deposit ratio must be given by[11]

$$
\gamma_t = \begin{cases} 0 & \text{if} \quad r_t^b < \frac{\theta R}{\lambda - \int_\theta^\lambda F(\pi) d\pi} \\ 1 - \frac{1}{\pi^{**}} \frac{\lambda r_t^b}{\lambda r_t^b - \theta R} \int_{\pi^*}^{\pi^{**}} F(\pi) d\pi & \text{if} \quad \frac{\theta R}{\lambda - \int_\theta^\lambda F(\pi) d\pi} \leq r_t^b \leq R \\ 1 & \text{if} \quad r_t^b > R \end{cases} \tag{32}
$$

where

$$
\lambda_t \equiv \frac{\theta r^c}{\theta r^c + (1-\theta) r_t^b}.
$$

Equation (32) implicitly defines the solution to the banks' problem in an economy with an LLR. I now turn to an analysis of general equilibrium in the context of such an economy.

7 Equilibrium with the Lender of Last Resort

As in the previous, a steady-state equilibrium is characterized by the pair ($\hat{\gamma}^{**}$, $\hat{r}^{b**}$) satisfying equations (24) and (32). Note that equation

[11]The intermediate steps are provided in Appendix B.

(24) continues to stipulate that interbank markets must clear. Figure 4 illustrates the steady-state equilibrium with an imperfect interbank market and an LLR. At one point, the locus defined by equation (32) crosses the locus defined by equation (24). Here exists a unique steady-state equilibrium. In addition, the locus defined by equation (32) lies to the right of the locus defined by equation (19). This implies that the introduction of the LLR shifts the locus defined by equation (19) to the right. The ability to borrow from the central bank encourages banks to reduce cash reserves and to increase investments because it makes a liquidity crisis a less costly event. As a result, the interest rate on interbank loans increases because the proportion of banks lending money to the markets decreases.

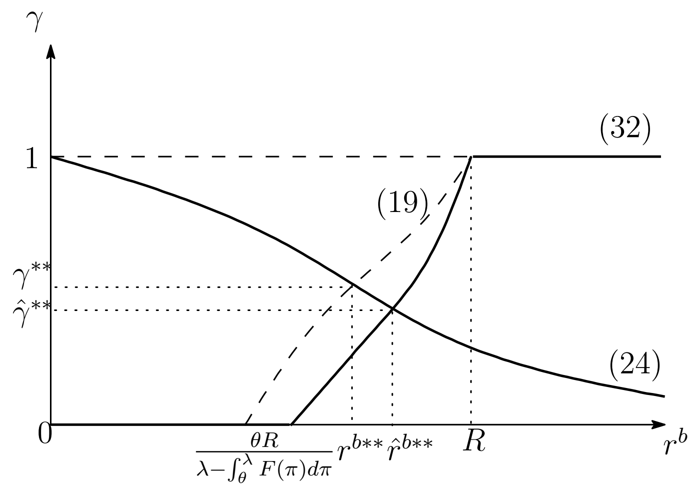

Figure 4: Equilibrium with the LLR

I now examine the dependence of the steady-state values $\hat{\gamma}^{**}$ and $\hat{r}^{b**}$ upon certain parameters. Specifically, I am interesting in the effect of a change in the real interest rate r^c on the value of $\hat{\gamma}^{**}$ and $\hat{r}^{b**}$. An increase in r^c shifts the locus defined by equation (32) to the left. Consequently, the optimal reserve-deposit ratio $\hat{\gamma}^{**}$ increases, and the interest rate $\hat{r}^{b**}$ decreases. This change is illustrated in Figure 5. Intuitively, when borrowing money from the central bank becomes more costly, banks increase cash reserves in order to avoid borrowing from the LLR. Consequently, the interest rate on interbank loans falls because a rise in cash reserves increases the proportion of interbank lenders to borrowers.

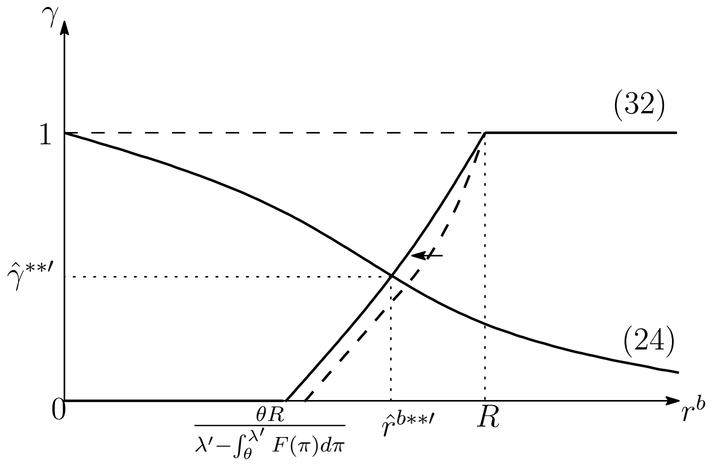

Figure 5: An increase in r^c

Next, I consider what happens if the central bank lowers the rate of the central bank loan toward the interbank loan rate. This signifies that the degree of "penalty" on the central bank loan decreases, and increases the number of banks borrowing money from the central bank. In the limit $r^c \to r^b$, I obtain the following results from equations (18), (31), and (32),

$$
\begin{aligned}
\hat{\gamma}^{**} &\longrightarrow \tilde{\gamma}^{**}, \\
\pi^{*} &\longrightarrow \tilde{\gamma}^{**} + \theta(1 - \tilde{\gamma}^{**}), \\
\pi^{**} &\longrightarrow \tilde{\gamma}^{**} + \theta(1 - \tilde{\gamma}^{**}),
\end{aligned}
$$

where $\tilde{\gamma}^{**}$ satisfies

$$\theta(1-\gamma) = \int_0^{\gamma+\theta(1-\gamma)} F(\pi) d\pi. \tag{33}$$

Note that $\pi^* = \pi^{**}$ when $r^b = r^c$. In this limited case, all banks that face borrowing constraints in interbank markets resort to the central bank loans, so every bank is able to meet its liquidity demands. Banks that receive a liquidity shock below the critical value $\tilde{\gamma}^{**}+\theta(1-\tilde{\gamma}^{**})$ use their own cash reserves, the interbank loans, or both. Conversely, banks that receive a liquidity shock above the critical value $\tilde{\gamma}^{**}+\theta(1-\tilde{\gamma}^{**})$ use their own cash reserves, the interbank loans, and the central bank loans. When the interest rate on the central bank loans is equal to the market rate (i.e., when $r^b = r^c$), the optimal reserve-deposit ratio γ is identical to the solution of banks problem with perfect interbank markets. In other words, the shape of equation (32) is identical to that of equation (15) in the $(r^b,\, \gamma)$ plane. Thus, the pair $(\tilde{\gamma}^{**}, R)$ is characterized as an equilibrium in which $r^b = r^c$. Figure 6 illustrates the resulting steady-state equilibrium.

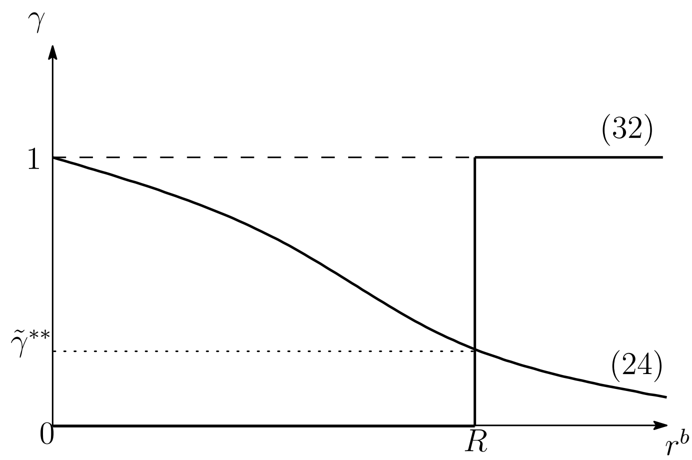

Figure 6: Equilibrium with the LLR in the limit $r^c \to r^b$.

Let us compare the reserve-deposit ratio chosen in this economy, $\tilde{\gamma}^{**}$, with the one chosen in an economy with a perfect interbank market, $\gamma^* = E(\pi)$. I have the following result.

Lemma 2 *For any θ and π, $\tilde{\gamma}^{**} < \gamma^*$.*

This lemma states that the optimal reserves for banks in imperfect interbank markets, with the LLR that charges the market rates are less than the optimal reserves for banks in perfect interbank markets. In other words, investment levels with the LLR are higher than investment levels in a perfect interbank markets. This result is a bit of surprising. The extra goods produced by storage technology goes to the central bank as lending revenue. The market equilibrium in an economy with the LLR is not exactly the same as equilibrium with perfect interbank markets.

In the steady-state equilibrium where $r^c = r^b$, the following equations are obtained from equations (29) and (30),

$$\delta(\pi) = \begin{cases} \pi - \tilde{\gamma}^{**} & \text{if} \quad 0 \leq \pi \leq \tilde{\gamma}^{**} + \theta(1-\tilde{\gamma}^{**}) \\ \theta(1-\tilde{\gamma}^{**}) & \text{if} \quad \tilde{\gamma}^{**} + \theta(1-\tilde{\gamma}^{**}) < \pi \leq 1, \end{cases} \tag{34}$$

$$\eta(\pi) = \begin{cases} 0 & \text{if} \quad 0 \leq \pi \leq \tilde{\gamma}^{**} + \theta(1-\tilde{\gamma}^{**}) \\ \pi - \tilde{\gamma}^{**} - \theta(1-\tilde{\gamma}^{**}) & \text{if} \quad \tilde{\gamma}^{**} + \theta(1-\tilde{\gamma}^{**}) < \pi \leq 1. \end{cases} \tag{35}$$

From these, it follows that

$$\gamma + \delta(\pi) + \eta(\pi) = \pi \qquad \forall \pi \in [0,1].$$

From these equations, I obtain the value of equilibrium consumption for movers and non-movers when $r^c = r^b$,

$$c^m = \frac{w}{\sigma - (\sigma - 1)\tilde{\gamma}^{**}}, \tag{36}$$

$$c^n = \frac{wR}{1 - \frac{\sigma-1}{\sigma}\tilde{\gamma}^{**}}. \tag{37}$$

Note that these are independent of the realization of π. This implies that the LLR that lends money at a non-penalty rate helps banks to diversify their liquidity risks, and allows depositors to receive complete insurance.

As in the previous, I consider a situation where the central bank implements zero-inflation policy, which means that $\sigma = 1$. Consumption levels are then given by

$$c^m = w,$$
$$c^n = wR.$$

This allocation is identical to that produced in the constrained planning problem, and can be summarized in the following proposition.

Proposition 4 *When interbank markets are imperfect (i.e., when $0 < \theta < 1$), and the LLR lends money freely to the banking system at the interbank rate, the market equilibrium under zero-inflation policy achieves constrained efficiency.*

Proposition 4 states that, if the limited commitment problem is significant, both monetary policy and LLR policy are needed to achieve the constrained-efficient allocation. Monetary policy balances the trade-off between productive efficiency and risk-sharing, while LLR policy corrects the inefficiencies of the interbank markets that are caused by the limited commitment problem.

8 Conclusions

This paper provides a monetary model that allows us to analyze the role of an LLR in an economy with an interbank market. I have shown that there is no need for an LLR in a fully functioning interbank market. In this case, the only role of the central bank is to choose the optimal money growth rate. This result concurs with the argument by Goodfriend and King (1988). In contrast, an LLR is required in a malfunctioning interbank market. I have also shown that if the central bank lends money to banks facing collateral constraints at the same interest rate as interbank markets do, banks can diversify liquidity risks and the market equilibrium can achieve efficiency. In such an economy, both the discount window lending to particular banks and the money supply must be controlled in order to achieve the constrained-efficient allocation.

I have hitherto ignored the implications of insolvency. In my model, all banks facing credit constraints are illiquid but solvent because they do not default as a result of having undertaken risky projects. In practice, however, insolvent banks demand loans from the LLR, and there are some difficulties in distinguishing solvent banks from insolvent ones. [12] Thus, the LLR policy can create moral hazard because banks will respond to their policies by taking greater risks, and the public will lose its

[12] See Goodhart (1985, 1987) and Solow (1982).

incentive to monitor these problems. This is a very important problem for which the model described here is unable to provide a satisfactory answer. Risky assets and the moral hazard problem of the LLR should be considered in the context of this model. The optimal LLR policy may be to lend money at the penalty rate, which is higher than the interbank rate, in the presence of solvency shocks. I leave this important question for future research.

A Appendix

Proof of Lemma 1 The equilibrium pair (γ^{**}, r^{b**}) satisfies equations (19) and (24). Note that

$$\begin{aligned}\int_0^{\pi^*} F(\pi)d\pi &= \int_0^1 F(\pi)d\pi - \int_{\pi^*}^1 F(\pi)d\pi \\ &= 1 - E(\pi) - \int_{\pi^*}^1 F(\pi)d\pi.\end{aligned}$$

Rearranging (24) yields

$$\int_{\pi^*}^1 F(\pi)d\pi = 1 - E(\pi) - \frac{\theta R(1-\gamma^{**})}{r^{b**}\gamma^{**} + R(1-\gamma^{**})}.$$

Substituting this into (19) and rearranging terms yields

$$\left(1 - \frac{\theta R}{r^{b**}}\right)(1-\gamma^{**}) = 1 - E(\pi) - \frac{\theta R(1-\gamma^{**})}{r^{b**}\gamma^{**} + R(1-\gamma^{**})}.$$

With algebraic manipulations, I obtain

$$E(\pi) - \gamma^{**} = \frac{\theta R(1-\gamma^{**})^2(R - r^{b**})}{r^{b**}[r^{b**}\gamma^{**} + R(1-\gamma^{**})]}.$$

Since $R > r^{b**}$, $E(\pi) > \gamma^{**}$ holds. ■

Proof of Proposition 2 Substituting (25) and (26) into the government's objective function in equation (27) and dropping the terms not depending on σ yields the problem

$$\max_{\sigma \geq \frac{1}{R}} \quad \int_0^{\pi^*} \left\{ \pi \ln \left[\frac{1}{\sigma(1 - \frac{\sigma-1}{\sigma}\gamma^{**})} \right] + (1-\pi) \ln \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma^{**}} \right\}$$
$$+ \int_{\pi^*}^{1} \left\{ \pi \ln \left[\frac{1}{\sigma(1 - \frac{\sigma-1}{\sigma}\gamma^{**})} \right] + (1-\pi) \ln \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma^{**}} \right\},$$

which can be reduced to

$$\max_{\sigma \geq \frac{1}{R}} \quad \ln \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma^{**}} + E(\pi) \ln \frac{1}{\sigma}.$$

The solution to this problem is

$$\sigma^* = \max \left\{ \frac{\gamma^{**}[1 - E(\pi)]}{E(\pi)(1-\gamma^{**})}, \frac{1}{R} \right\}. \tag{38}$$

Since $\gamma^{**} < E(\pi)$ holds for any $\theta \in (0,1)$ from Lemma 1, it is easy to check that the deflationary policy is optimal, which means that $\sigma^* < 1$. ■

Proof of Lemma 2 In order to examine the effect of a change in θ on $\tilde{\gamma}^{**}$, differentiating both side of equation (33) with respect to θ yields

$$1 - \tilde{\gamma}^{**} - \theta \frac{d\tilde{\gamma}^{**}}{d\theta} = F[\tilde{\gamma}^{**} + \theta(1 - \tilde{\gamma}^{**})] \left(1 - \tilde{\gamma}^{**} + (1-\theta) \frac{d\tilde{\gamma}^{**}}{d\theta} \right),$$

which is reduced to

$$\frac{d\tilde{\gamma}^{**}}{d\theta} = \frac{(1-\tilde{\gamma}^{**})(1 - F[\tilde{\gamma}^{**} + \theta(1-\tilde{\gamma}^{**})])}{\theta + (1-\theta)F[\tilde{\gamma}^{**} + \theta(1-\tilde{\gamma}^{**})]} > 0.$$

In addition, from (33), I have

$$\lim_{\theta \to 1} \tilde{\gamma}^{**} = E(\pi) = \gamma^*.$$

By continuity and monotonicity, I obtain $\tilde{\gamma}^{**} < \gamma^*$. ■

B Appendix

B.1 Derivation of (19)

Let $H(\gamma_t, p_t/p_{t+1})$ denote the objective function, Because this function is strictly concave and the constraint set is compact, there is a unique solution to the problem for any p_t/p_{t+1}. The first derivative of the objective function can be written as

$$H_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = \frac{r_t^b - R}{r_t^b\gamma_t + (1-\gamma_t)R}F(\pi^*) + \frac{r_t^b - \theta R}{r_t^b\gamma_t + (1-\gamma_t)\theta R}\int_{\pi^*}^1 \pi f(\pi)d\pi - \int_{\pi^*}^1 \frac{1-\pi}{1-\gamma_t}f(\pi)d\pi \quad (39)$$

Using L'Hôpital's rule, I can show that

$$\lim_{\gamma_t \to 1} H_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = 1 - \frac{R}{r_t^b}. \quad (40)$$

If $r_t^b > R$ holds, then this limit is positive, and the solution must be $\gamma_t = 1$.

On the other hand, it is easy to show that

$$\lim_{\gamma_t \to 0} H_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = \frac{r_t^b}{\theta R}\left[1 - \int_\theta^1 F(\pi)d\pi\right] - 1. \quad (41)$$

If $r_t^b < \theta R/(1 - \int_\theta^1 F(\pi)d\pi)$ holds, then this limit is negative, and the solution must be $\gamma_t = 0$. For values of $r_t^b \in [\theta R/(1 - \int_\theta^1 F(\pi)d\pi), R]$, the solution to the problem is defined by the first-order condition,

$$H_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = 0$$

which yields

$$\gamma_t = 1 - \frac{r_t^b}{r_t^b - \theta R}\int_{\pi^*}^1 F(\pi)d\pi. \quad (42)$$

This implicitly defines the optimal reserve-deposit ratio of banks.

B.2 Derivation of (32)

Let $J(\gamma_t, p_t/p_{t+1})$ denote the objective function, Because this function is strictly concave and the constraint set is compact, there is a unique solution to the problem for any p_t/p_{t+1}. The first derivative of the objective function can be written as

$$J_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = \frac{r^c(r_t^b - \theta R) - (1-\theta)Rr^b}{r^c[r_t^b\gamma_t + (1-\gamma_t)\theta R] + (1-\theta)(1-\gamma_t)r_t^b R} - \frac{r_t^b}{(1-\gamma_t)[r_t^b\gamma_t + (1-\gamma_t)\theta R]}\int_{\pi^*}^{\pi^{**}} F(\pi)d\pi. \quad (43)$$

Using L'Hôpital's rule, I can show that

$$\lim_{\gamma_t \to 1} J_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = 1 - \frac{R}{r_t^b}. \quad (44)$$

If $r_t^b > R$ holds, then this limit is positive, and the solution must be $\gamma_t = 1$.

On the other hand, it is easy to show that

$$\lim_{\gamma_t \to 0} J_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = \frac{r_t^b\lambda}{\theta R} - 1 - \frac{r^b}{\theta R}\int_\theta^\lambda F(\pi)d\pi. \quad (45)$$

If $r_t^b < \theta R/(\lambda - \int_\theta^\lambda F(\pi)d\pi)$ holds, then this limit is negative, and the solution must be $\gamma_t = 0$. For values of $r_t^b \in [\theta R/(\lambda - \int_\theta^\lambda F(\pi)d\pi), R]$, the solution to the problem is defined by the first-order condition,

$$J_1\left(\gamma_t, \frac{p_t}{p_{t+1}}\right) = 0$$

which yields

$$\gamma_t = 1 - \frac{1}{\pi^{**}}\frac{\lambda r_t^b}{\lambda r_t^b - \theta R}\int_{\pi^*}^{\pi^{**}} F(\pi)d\pi. \quad (46)$$

This implicitly defines the optimal reserve-deposit ratio of banks.

References

[1] Acharya, V., D. Gromb, and T. Yorulmazer (2008)"Imperfect competition in the inter-bank market for liquidity as a rationale for central banking," Working Paper, London Business School.

[2] Allen, F. and D. Gale (2000) "Financial contagion," *Journal of Political Economy* 108: 1-33.

[3] Allen, F., E. Carletti, and D. Gale (2009) "Interbank market liquidity and central bank intervention," *Journal of Monetary Economics* 56: 639-652.

[4] Antinolfi, G., E. Huybens, and T. Keister (2001) "Monetary stability and liquidity crises: the role of the lender of last resort," *Journal of Economic Theory* 99: 187-219.

[5] Antinolfi, G. and T. Keister (2006) "Discount window policy, banking crises, and indeterminacy of equilibrium," *Macroeconomic Dynamics* 10:1-19.

[6] Bagehot, W. (1873) *Lombard Street: A Description of the Money Market.* William Clowers and Sons, London.

[7] Bhattacharya, J., J. Haslag, and S. Russell (2005) "The role of money in two alternative models: when, and why is the Friedman rule optimal," *Journal of Monetary Economics* 52: 1401-1433.

[8] Bhattacharya, J., and R. Singh (2008) "Usefulness of the constrained planning problem in a model of money,"*Macroeconomic Dynamics* 12:503-525.

[9] Bordo, M.D. (1990) "The lender of last resort: Alternative views and historical experience," *Federal Reserve Bank of Richmond Economic Review* 18-29

[10] Champ, B., B. Smith, and S. Williamson (1996) "Currency elasticity and banking panics: theory and evidence," *Canadian Journal of Economics* 29: 828-864.

[11] Diamond, D. and P. Dybvig (1983) "Bank runs, deposit insurance and liquidity," *Journal of Political Economy* 91: 401-419.

[12] Diamond, D. and R. Rajan (2005) "Liquidity shortages and banking crises," *Journal of Finance* 60: 615-647.

[13] Fischer, S. (1999) "On the need for an international lender of last resort," *Journal of Economic Perspectives* 13: 85-104.

[14] Freixas, X., A. Martin, and D. Skeie (2010)"Bank liquidity, interbank markets, and monetary policy," European Banking Center Discussion Paper No. 2010-08S.

[15] Goodfriend, Marvin, and Robert G. King (1988) "Financial deregulation, monetary policy, and central banking," *Federal Reserve Bank of Richmond Economic Review* 74: 3-22.

[16] Goodhart, C. A. E. (1985) *The Evolution of Central Banks.* London: London School of Economics and Political Science.

[17] Goodhart, C. A. E. (1987) "Why do banks need a central bank?," *Oxford Economic Papers* 39: 75-89.

[18] Haslag J., and A. Martin (2007) "Optimality of the Friedman rule in an overlapping generations model with spatial separation," *Journal of Money, Credit, and Banking* 39: 1741-1758.

[19] Kaufman, G.G. (1991) "Lender of last resort: A contemporary perspective," *Journal of Financial Services Research* 5: 95-110.

[20] Kiyotaki, N, and J. Moore (1997) "Credit cycles," *Journal of Political Economy* 105: 211-248.

[21] Smith, B.D. (2002) "Monetary policy, banking crises, and the Friedman rule," *American Economic Review* 92: 128-134.

[22] Solow, R. M. (1982) "On the Lender of Last Resort." In C. P. Kindleberger and J. P. Laffargue (eds), *Financial Crises: Theory, History and Policy.* Cambridge: Cambridge University Press.

[23] Townsend, R. (1987) "Economic organization with limited communication," *American Economic Review* 77: 954-971.