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Crude Oil Hedging Strategies Using
Dynamic Multivariate GARCH[*]

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Abstract

The paper examines the performance of four multivariate volatility models, namely CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK, for the crude oil spot and futures returns of two major benchmark international crude oil markets, Brent and WTI, to calculate optimal portfolio weights and optimal hedge ratios, and to suggest a crude oil hedge strategy. The empirical results show that the optimal portfolio weights of all multivariate volatility models for Brent suggest holding futures in larger proportions than spot. For WTI, however, DCC, BEKK and diagonal BEKK suggest holding crude oil futures to spot, but CCC and VARMA-GARCH suggest holding crude oil spot to futures. In addition, the calculated optimal hedge ratios (OHRs) from each multivariate conditional volatility model give the time-varying hedge ratios, and recommend to short in crude oil futures with a high proportion of one dollar long in crude oil spot. Finally, the hedging effectiveness indicates that diagonal BEKK (BEKK) is the best (worst) model for OHR calculation in terms of reducing the variance of the portfolio.

1. Introduction

As the structure of world industries changed in the 1970s, the expansion of the oil market has continually grown to have now become the world's biggest commodity market. This market has developed from a primarily physical product activity into a sophisticated financial market. Over the last decade, crude oil markets have matured greatly, and their range and depth could allow a wide range of participants, such as crude oil producers, crude oil physical traders, and refining and oil companies, to hedge oil price risk. Risk in the crude oil commodity market is likely to occur due to unexpected jumps in global oil demand, a decrease in the capacity of crude oil production and refinery capacity, petroleum reserve policy, OPEC spare capacity and policy, major regional and global economic crises risk (including sovereign debt risk, counter-party risk, liquidity risk, and solvency risk), and geopolitical risks.

A futures contract is an agreement between two parties to buy and sell a given amount of a commodity at an agreed upon certain date in the future, at an agreed upon price, and at a given location. Furthermore, a futures contract is the instrument primarily designed to minimize one's exposure to unwanted risk. Futures traders are traditionally placed in one of two groups, namely hedgers and speculators. Hedgers typically include producers and consumers of a commodity, or the owners of an asset, who have an interest in the underlying asset, and are attempting to offset exposure to price fluctuations in some opposite position in another market. Unlike hedgers, speculators do not intend to minimize risk but rather to make a profit from the inherently risky nature of the commodity market by predicting market movements. Hedgers want to minimize risk, regardless of what they are investing in, while speculators want to increase their risk and thereby maximize profits.

Conceptually, hedging through trading futures contracts is a procedure used to restrain or reduce the risk of unfavourable price changes because cash and futures prices for the same commodity tend to move together. Therefore, changes in the value of a cash position are offset by changes in the value of an opposite futures position. In addition, futures contracts are favoured as a hedging tool because of their liquidity, speed and lower transaction costs.

Among the industries and firms that are more likely to use a hedging strategy is the oil and gas industry. Firms will hedge only if they expect that an unfavourable event will arise. Knill et al. (2006) suggested that if an oil and gas company uses futures contracts to hedge risk, they hedge only the downside risk. When an industry perspective is good (bad), it will scale down (up) on the futures usage, thereby pushing futures prices higher (lower). Hedging by the crude oil producers normally involves selling the commodity futures because producers or refiners use futures contracts to lock the futures selling prices or a price floor. Thus, they tend to take short positions in futures. At the same time, energy traders, investors or fuel oil users focusing to lock in a futures purchase price or price ceiling tend to long positions in futures. Daniel (2001) shows that hedging strategies can substantially reduce oil price volatility without significantly reducing returns, and with the added benefit of greater predictability and certainty.

Theoretically, issues in hedging involve the determination of the optimal hedge ratio (OHR). One of the most widely-used hedging strategies is based on the minimization of the variance of the portfolio, the so-called minimum variance hedge ratio (see Chen et al. (2003) for a review of the futures hedge ratio, and Lien and Tse (2002) for some recent developments in futures hedging). With the minimum-variance criterion, risk management requires determination of the OHR (the optimal amount of futures bought or sold expressed as a proportion of the cash position). In order to estimate such a ratio, early research simply used the slope of the classical linear regression model of cash on the futures price, which assumed a time-invariant hedge ratio (see, for example, Ederington (1979), Figlewski (1985), and Myers and Thomson (1989)).

However, it is now widely agreed that financial asset returns volatility, covariances and correlations are time-varying with persistent dynamics, and rely on techniques such as conditional volatility (CV) and stochastic volatility (SV) models. Baillie and Myers (1991) claim that, if the joint distribution of cash prices and futures prices changes over time, estimating a constant hedge ratio may not be appropriate. In this paper, alternative multivariate conditional volatility models are used to investigate the time-varying optimal hedge ratio and optimal portfolio weights, and the performance of these hedge ratios is compared in terms of risk reduction.

The widely used ARCH and GARCH models appear to be ideal for estimating time-varying OHRs, and a number of applications have concluded that such ratios seem to display considerable variability over time (see, for example, Cecchetti et al. (1988), Baillie and Myers (1991), Myers (1991), and Kroner and Sultan (1993)). Typically, the hedging model is constructed for a decision maker who allocates wealth between a risk-free asset and two risky assets, namely the physical commodity and the corresponding futures. OHR is defined as $\text{OHR}_t = \text{cov}(s_t, f_t | F_{t-1}) / \text{var}(f_t | F_{t-1})$, where s_t and f_t are spot price and futures price, respectively, and F_{t-1} is the information set. Therefore, OHR_t can be calculated given the knowledge of the time-dependent covariance matrix for cash and futures prices, which can be estimated using multivariate GARCH models.

In the literature, research has been conducted on the volatility of crude spot, forward and futures returns. Lanza et al. (2006) applied the constant conditional correlation (CCC) model of Bollerslev (1990) and the dynamic conditional correlation (DCC) model of Engle (2002) for West Texas Intermediate (WTI) oil forward and futures returns. Manera et al. (2006) used CCC, the vector autoregressive moving average (VARMA-GARCH) model of Ling and McAleer (2003), the VARMA- Asymmetric GARCH model of McAleer et al. (2009), and DCC to spot and forward return in the Tapis market. Chang et al. (2009a and 2009b) estimated multivariate conditional volatility and examined volatility spillovers for the returns on spot, forward and futures returns for Brent, WTI, Dubai and Tapis to aid risk diversification in crude oil markets.

For estimated time-varying hedge ratios using multivariate conditional volatility models, Haigh and Holt (2002) modelled the time-varying hedge ratio among crude oil (WTI), heating oil and unleaded gasoline futures contracts of crack spread in decreasing price volatility for an energy trader with the BEKK model of Engle and Kroner (1995) and linear diagonal VEC model of Bollerslev et al 1988, and accounted for volatility spillovers. Alizadeh et al. (2004) examined appropriate futures contracts, and investigated the effectiveness of hedging marine bunker price fluctuations in Rotterdam, Singapore and Houston using different crude oil and petroleum futures contracts traded on the New York Mercantile Exchange (NYMEX) and the International Petroleum Exchange (IPE) in London, using the VECM and BEKK models. Jalali-Naini and Kazemi-Manesh (2006) examined

hedge ratios using weekly spot prices of WTI and futures prices of crude oil contracts one month to four months on NYMEX. The results from the BEKK model showed that the OHRs are time varying for all contracts, and higher duration contracts had higher perceived risk, a higher OHR mean, and standard deviations.

Recently, Chang et al. (2010) estimated OHR and optimal portfolio weights of the crude oil portfolio using only the VARMA-GARCH model. However, they did not focus on the optimal portfolio weights and optimal hedging strategy based on a wide range of multivariate conditional volatility models, and did not compare their results in terms of risk reduction or hedge strategies. As WTI and Brent are major benchmarks in the world of international trading and the reference crudes for the USA and North Sea, respectively, the empirical results of this paper show different optimal portfolio weights, optimal hedging strategy and their explanation to aid in risk management in crude oil markets.

The purpose of the paper is three-fold. First, we estimate alternative multivariate conditional volatility models, namely CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK for the returns on spot and futures prices for Brent and WTI markets. Second, we calculate the optimal portfolio weights and OHRs from the conditional covariance matrices for effective optimal portfolio design and hedging strategies. Finally, we investigate and compare the performance of the OHRs from the estimated multivariate conditional volatility models by applying the hedging effectiveness index.

The structure of the remainder of the paper is as follows. Section 2 discusses the multivariate GARCH models to be estimated, and the derivation of the OHR and hedging effective index. Section 3 describes the data, descriptive statistics, unit root test and cointegration test statistics. Section 4 analyses the empirical estimates from empirical modelling. Some concluding remarks are given in Section 5.

2. Econometric Models

2.1 Multivariate Conditional Volatility Models

This section presents the CCC model of Bollerslev (1990), VARMA-GARCH model of Ling and McAleer (2003), DCC model of Engle (2002), BEKK model of Engle and Kroner (1995) and Diagonal BEKK. The first two models assume constant conditional correlations, while the last two models accommodate dynamic conditional correlations.

Consider the CCC multivariate GARCH model of Bollerslev (1990):

$$y_t = E\left(y_t \middle| F_{t-1}\right) + \varepsilon_t \quad, \quad \varepsilon_t = D_t \eta_t \tag{1}$$

$$\mathrm{var}\left(\varepsilon_t \mid F_{t-1}\right) = D_t \Gamma D_t$$

where $y_t = \left(y_{1t}, ..., y_{mt}\right)'$, $\eta_t = \left(\eta_{1t}, ..., \eta_{mt}\right)'$ is a sequence of independently and identically distributed (i.i.d.) random vectors, F_t is the past information available at time t, $D_t = \mathrm{diag}\left(h_1^{1/2}, ..., h_m^{1/2}\right)$, m is the number of returns, and $t = 1, ..., n$, (see, for example, McAleer (2005) and Bauwens et al. (2006)). As $\Gamma = E\left(\eta_t \eta_t' \middle| F_{t-1}\right) = E\left(\eta_t \eta_t'\right)$, where $\Gamma = \left\{\rho_{ij}\right\}$ for $i, j = 1, ..., m$, the constant conditional correlation matrix of the unconditional shocks, η_t, is equivalent to the constant conditional covariance matrix of the conditional shocks, ε_t, from (1), $\varepsilon_t \varepsilon_t' = D_t \eta_t \eta_t' D_t$, $D_t = \left(\mathrm{diag}\, Q_t\right)^{1/2}$, and $E\left(\varepsilon_t \varepsilon_t' \middle| F_{t-1}\right) = Q_t = D_t \Gamma D_t$, where Q_t is the conditional covariance matrix. The conditional covariance matrix is positive definite if and only if all the n conditional variance are positive and Γ is positive definite.

The CCC model of Bollerslev (1990) assumes that the conditional variance for each return, h_{it}, $i = 1, .., m$, follows a univariate GARCH process, that is

$$h_{it} = \omega_i + \sum_{j=1}^{r} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{s} \beta_{ij} h_{i,t-j} \quad, \tag{2}$$

where α_{ij} represents the ARCH effect, or short run persistence of shocks to return i, β_{ij} represents the GARCH effect, and $\sum_{j=1}^{r} \alpha_{ij} + \sum_{j=1}^{s} \beta_{ij}$ denotes the long run persistence.

In order to accommodate interdependencies of volatility across different assets and/or markets, Ling and McAleer (2003) proposed a vector autoregressive moving average (VARMA) specification of the conditional mean, and the following specification for the conditional variance:

$$Y_t = E\left(Y_t \middle| F_{t-1}\right) + \varepsilon_t \tag{3}$$

$$\Phi\left(L\right)\left(Y_t - \mu\right) = \Psi\left(L\right)\varepsilon_t \tag{4}$$

$$\varepsilon_t = D_t \eta_t \tag{5}$$

$$H_t = W_t + \sum_{l=1}^{r} A_l \vec{\varepsilon}_{t-l} + \sum_{l=1}^{s} B_l H_{i,t-l} \tag{6}$$

where W_t, A_l and B_l are $m \times m$ matrices, with typical elements α_{ij} and β_{ij}, respectively. $H_t = \left(h_{1t},...,h_{mt}\right)'$, $\vec{\varepsilon} = \left(\varepsilon_{1t}^2,...\varepsilon_{mt}^2\right)'$, $\Phi\left(L\right) = I_m - \Phi_1 L - ... - \Phi_p L^p$ and $\Psi\left(L\right) = I_m - \Psi_1 L - ...$ $-\Psi_q L^q$ are polynomials in L, the lag operator. It is clear that when A_l and B_l are diagonal matrices, (6) reduces to (2). Theoretically, GARCH(1,1) captures infinite ARCH process (Bollerslev (1986)). However, on a practical level, a multivariate GARCH model with a greater number of lags can be problematic.

The VARMA-GARCH model assumes that negative and positive shocks of equal magnitude have identical impacts on the conditional variance. McAleer et al. (2009) extended the VARMA-GARCH to accommodate the asymmetric impacts of the unconditional shocks on the conditional variance, and proposed the VARMA-AGARCH specification of the conditional variance as follows:

$$H_t = W + \sum_{i=1}^{r} A_i \vec{\varepsilon}_{t-i} + \sum_{i=1}^{r} C_i I_{t-i} \vec{\varepsilon}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j} \quad , \tag{7}$$

where C_i are $m \times m$ matrices for $i = 1,..,r$ with typical element γ_{ij}, and $I_t = \mathrm{diag}\left(I_{1t},...,I_{mt}\right)$, is an indicator function, given as

$$I(\eta_{it}) = \begin{cases} 0, & \varepsilon_{it} > 0 \\ 1, & \varepsilon_{it} \leq 0 \end{cases} \tag{8}.$$

If $m = 1$, (7) collapses to the asymmetric GARCH (or GJR) model of Glosten et al. (1992). Moreover, VARMA-AGARCH reduces to VARMA-GARCH when $C_i = 0$ for all i. If $C_i = 0$ and A_i and B_j are diagonal matrices for all i and j, then VARMA-AGARCH reduces to the CCC model. The structural and statistical properties of the model, including necessary and sufficient conditions for stationarity and ergodicity of VARMA-GARCH and VARMA-AGARCH, are explained in detail in Ling and McAleer (2003) and McAleer et al. (2009), respectively. The parameters of model (1)-(7) are obtained by maximum likelihood estimation (MLE) using a joint normal density. When η_t does not follow a joint multivariate normal distribution, the appropriate estimator is QMLE.

The assumption that the conditional correlations are constant may seem unrealistic in many empirical results, particularly in previous studies about crude oil returns (see, for example, Lanza et al. (2006), Manera et al. (2006), and Chang et al. (2009a, 2009b, 2010)). In order to make the conditional correlation matrix time dependent, Engle (2002) proposed a dynamic conditional correlation (DCC) model, which is defined as

$$y_t \mid F_{t-1} \square (0, Q_t) \quad , \quad t = 1, 2, ..., n \tag{9}$$

$$Q_t = D_t \Gamma_t D_t, \tag{10}$$

where $D_t = \text{diag}\left(h_1^{1/2}, ..., h_m^{1/2}\right)$ is a diagonal matrix of conditional variances, and F_t is the information set available at time t. The conditional variance, h_{it}, can be defined as a univariate GARCH model, as follows:

$$h_{it} = \omega_i + \sum_{k=1}^{p} \alpha_{ik} \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_{il} h_{i,t-l} . \tag{11}$$

If η_t is a vector of i.i.d. random variables, with zero mean and unit variance, Q_t in (12) is the conditional covariance matrix (after standardization, $\eta_{it} = y_{it} / \sqrt{h_{it}}$). The η_{it} are used to estimate the dynamic conditional correlations, as follows:

$$\Gamma_t = \left\{ (diag(Q_t)^{-1/2} \right\} Q_t \left\{ (diag(Q_t)^{-1/2} \right\} \tag{12}$$

where the $k \times k$ symmetric positive definite matrix Q_t is given by

$$Q_t = (1 - \theta_1 - \theta_2)\overline{Q} + \theta_1 \eta_{t-1} \eta'_{t-1} + \theta_2 Q_{t-1} , \tag{13}$$

in which θ_1 and θ_2 are scalar parameters to capture the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, and θ_1 and θ_2 are non-negative scalar parameters satisfying $\theta_1 + \theta_2 < 1$, which implies that $Q_t > 0$. When $\theta_1 = \theta_2 = 0$, Q_t in (13) is equivalent to CCC. As Q_t is a conditional on the vector of standardized residuals, (13) is a conditional covariance matrix, and $\overline{Q}$ is the $k \times k$ unconditional variance matrix of η_t. DCC is not linear, but may be estimated simply using a two-step method based on the likelihood function, the first step being a series of univariate GARCH estimates and the second step being the correlation estimates (see Caporin and McAleer (2009) for further details and caveats).

An alternative dynamic conditional model is BEKK, which has the attractive property that the conditional covariance matrices are positive definite. However, BEKK suffers from the so-called "curse of dimentionality" (see McAleer et al. (2009) for a comparison of the number of parameters in various multivariate conditional volatility models). The BEKK model for multivariate GARCH(1,1) is given as:

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\varepsilon_{t-1}\varepsilon'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} , \tag{14}$$

where the individual element for the matrices $\mathbf{C}$, $\mathbf{A}$ and $\mathbf{B}$ matrices are given as

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}$$

with $\sum_{j=1}^{q} \sum_{k=1}^{K} (A_{kj} \otimes A_{kj}) + \sum_{j=1}^{q} \sum_{k=1}^{K} (B_{kj} \otimes B_{kj})$, where $\otimes$ denotes the Kronecker product of two matrices, are less than one in the modulus for covariance stationary (Silvennoinen and Teräsvirta (2008)). In this diagonal representation, the conditional variances are functions of their own lagged values and own lagged returns shocks, while the conditional covariances are functions of the lagged covariances and lagged cross-products of the corresponding returns shocks. Moreover, this formulation guarantees $\mathbf{H}_t$ to be positive definite almost surely for all t. The BEKK(1,1) model gives $N(5N+1)/2$ parameters. For further details and a comparison between BEKK and DCC, see Caporin and McAleer (2008, 2009). In order to reduce the number of estimated parameters, by setting $\mathbf{B} = \mathbf{AD}$ where $\mathbf{D}$ is diagonal matrix, (14) becomes

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A} + \mathbf{D}E[\mathbf{A}'\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A}]\mathbf{D} \tag{15}$$

with $a_{ii}^2 + b_{ii}^2 < 1$, $i = 1, 2$ for stationary. The parameters of the covariance equation ($h_{ij,t}, i \neq j$) are products of the corresponding parameters of the two variance equations ($h_{ij,t}$).

2.2 Optimal Hedge Ratios and Optimal Portfolio Weights

Market participants in futures markets choose a hedging strategy that reflects their attitudes toward risk and their individual goals. Consider the case of an oil company, which usually wants to protect exposure to crude oil price fluctuations. The return on the oil company's portfolio of spot and futures position can be denoted as:

$$R_{H,t} = R_{S,t} - \gamma_t R_{F,t} , \tag{15}$$

where $R_{H,t}$ is the return on holding the portfolio between $t-1$ and t, $R_{S,t}$ and $R_{F,t}$ are the returns on holding spot and futures positions between t and $t-1$, and γ is the hedge ratio,

that is, the number of futures contracts that the hedger must sell for each unit of spot commodity on which price risk is borne.

According to Johnson (1960), the variance of the returns of the hedged portfolio, conditional on the information set available at time $t-1$, is given by

$$\text{var}\left(R_{H,t} \mid \Omega_{t-1}\right) = \text{var}\left(R_{S,t} \mid \Omega_{t-1}\right) - 2\gamma_t \, \text{cov}\left(R_{S,t}, R_{F,t} \mid \Omega_{t-1}\right) + \gamma_t^2 \, \text{var}\left(R_{F,t} \mid \Omega_{t-1}\right), \quad (16)$$

where $\text{var}\left(R_{S,t} \mid \Omega_{t-1}\right)$, $\text{var}\left(R_{F,t} \mid \Omega_{t-1}\right)$ and $\text{cov}\left(R_{S,t}, R_{F,t} \mid \Omega_{t-1}\right)$ are the conditional variance and covariance of the spot and futures returns, respectively. The OHRs are defined as the value of γ_t which minimizes the conditional variance (risk) of the hedged portfolio returns, that is, $\min_{\gamma_t}\left[\text{var}\left(R_{H,t} \mid \Omega_{t-1}\right)\right]$. Taking the partial derivative of (16) with respect to γ_t, setting it equal to zero and solving for γ_t, yields the OHR$_t$ conditional on the information available at $t-1$ (see, for example, Baillie and Myers (1991)):

$$\gamma_t^* \mid \Omega_{t-1} = \frac{\text{cov}\left(R_{S,t}, R_{F,t} \mid \Omega_{t-1}\right)}{\text{var}\left(R_{F,t} \mid \Omega_{t-1}\right)} \quad (17)$$

where returns are defined as the logarithmic differences of spot and futures prices.

From the multivariate conditional volatility model, the conditional covariance matrix is obtained, such that the OHR is given as:

$$\gamma_t^* \mid \Omega_{t-1} = \frac{h_{SF,t}}{h_{F,t}}, \quad (18)$$

where $h_{SF,t}$ is the conditional covariance between spot and futures returns, and $h_{F,t}$ is the conditional variance of futures returns.

In order to compare the performance of OHRs obtained from different multivariate conditional volatility models, Ku et al. (2007) suggest that a more accurate model of

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conditional volatility should also be superior in terms of hedging effectiveness, as measured by the variance reduction for any hedged portfolio compared with the unhedged portfolio. Thus, a hedging effective index (HE) is given as:

$$HE = \left[\frac{var_{unhedged} - var_{hedged}}{var_{unhedged}} \right],$$ (19)

where the variances of the hedge portfolio are obtained from the variance of the rate of return, $R_{H,t}$, and the variance of the unhedged portfolio is the variance of spot returns (see, for example, Ripple and Moosa (2007)). A higher HE indicates a higher hedging effectiveness and larger risk reduction, such that a hedging method with a higher HE is regarded as a superior hedging strategy.

Alternatively, in order to construct an optimal portfolio design that minimizes risk without lowering expected returns, and applying the methods of Kroner and Ng (1998) and Hammoudeh et al. (2009), the optimal portfolio weight of crude oil spot/futures holding is given by:

$$w_{SF,t} = \frac{h_{F,t} - h_{SF,t}}{h_{S,t} - 2h_{SF,t} + h_{F,t}}$$ (20)

and

$$w_{SF,t} = \begin{cases} 0, & \text{if } w_{SF,t} < 0 \\ w_{SF,t}, & \text{if } 0 < w_{SF,t} < 0 \\ 1, & \text{if } w_{SF,t} > 0 \end{cases}$$ (21)

where $w_{SF,t}$ $(1 - w_{SF,t})$ is the weight of the spot (futures) in a one dollar portfolio of crude oil spot/futures at time t.

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3. Data

Daily synchronous closing prices of spot and nearby futures contract (that is, the contract for which the maturity is closest to the current date) of crude oil prices from two major crude oil markets, namely Brent and WTI, are used in the empirical analysis. The 3,132 price observations from 4 November 1997 to 4 November 2009 are obtained from the DataStream database. The returns of crude oil prices i of market j at time t in a continuous compound basis are calculated as $r_{ij,t} = \log\left(P_{ij,t} / P_{ij,t-1}\right)$, where $P_{ij,t}$ and $P_{ij,t-1}$ are the closing prices of crude oil price i in market j for days t and $t-1$, respectively.

[Insert Table 1 and 2 here]

Table 1 presents the descriptive statistics for the prices and returns series of crude oil prices. The ADF and PP unit root tests for spot and futures prices in Table 2 are not statistically significant, so they contain a unit root, and hence are $I(1)$. The market efficiency hypothesis requires that the current futures prices and the future spot price are cointegrated, meaning that futures prices are unbiased predictors of spot prices at maturity (Dwyer and Wallace (1992), Chowdhury (1991), Crowder and Hamed (1993) and Moosa (1996)). Consequently, the agent can buy or sell a contract in the futures market for a commodity and undertakes to receive or deliver the commodity at a certain time in the futures, based on a price determined today (Chow and McAleer (2000)).

The Johansen (1988, 1991, 1995) test for cointegation between spot and futures prices is presented in Table 3. The trace (λ_{trace}) and maximal (λ_{max}) eigenvalue test statistics are used, based on minimizing AIC. Under the null hypothesis of no cointegrating vectors, $r = 0$, both tests are statistically significant, while the alternative hypothesis of at least one cointegrating vector of λ_{trace} and one cointegrating vector of λ_{max} are statistically insignificant. These results indicate that spot and futures prices are cointegrated with one cointegrating vector.

[Insert Table 3 here]

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The average returns of spot and futures in Brent and WTI are similar and very low, but the corresponding variance of returns is much higher. These crude oil returns series have high kurtosis, which indicates the presence of fat tails. The negative skewness statistics signify the series has a longer left tail (extreme losses) than right tail (extreme gains). The Jarque-Bera Lagrange multiplier statistics of crude oil returns in each market are statistically significant, thereby implying that the distribution of these returns is not normal. Based on the coefficient of variation, the historical volatility among all crude oil returns are not especially different.

Figure 1 presents the plot of synchronous crude oil price prices. All prices move in the same pattern, suggesting they are contemporaneously highly correlated. The calculated contemporaneous correlations between crude oil spot and futures returns for Brent and WTI markets are both 0.99. Figure 2 shows the plot of crude oil returns. These indicate volatility clustering, or periods of high volatility followed by periods of relative tranquility. Figure 3 displays the volatilities of crude oil returns, where volatilities are calculated as the square of the estimated residuals from an ARMA(1,1) process. These plots are similar in all four returns, with volatility clustering and an apparent outlier.

[Insert Figures 1-3 here]

Standard econometric practice in the analysis of financial time series data begins with an examination of unit roots. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to test for all crude oil returns in each market under the null hypothesis of a unit root against the alternative hypothesis of stationarity. The results from unit root tests are presented in Table 2. The tests yield large negative values in all cases for levels, such that the individual returns series reject the null hypothesis at the 1% significance level, so that all returns series are stationary.

4. Empirical Results

An important task is to model the conditional mean and conditional variances of the returns series. Therefore, univariate ARMA-GARCH models are estimated, with the appropriate univariate conditional volatility model given as ARMA(1,1)-GARCH(1,1). These results are

available upon request. All multivariate conditional volatility models in this paper are estimated using the RATS 6.2 econometric software package.

Table 4 presents the estimates for the CCC model, with $p = q = r = s = 1$. The two entries corresponding to each of the parameters are the estimate and the Bollerslev-Wooldridge (1992) robust t-ratios. The ARCH and GARCH estimates of the conditional variance between crude oil spot and futures returns in Brent and WTI are statistically significant. The ARCH (α) estimates are generally small (less than 0.1), and the GARCH (β) estimates are generally high and close to one. Therefore, the long run persistence, is generally close to one, indicating a near long memory process, signifying that a shock in the volatility series impacts on futures volatility over a long horizon. In addition, as $\alpha + \beta < 1$, all markets satisfy the second moment and log-moment condition, which is a sufficient condition for the QMLE to be consistent and asymptotically normal (see McAleer et al. (2007)). The CCC estimates between the volatility of spot and futures returns of Brent and WTI are high, with the highest being 0.923 between the standardized shocks to volatility in the crude oil spot and futures returns of the WTI market.

[Insert Table 4 here]

Table 5 reports the estimates of the conditional mean and variance for VARMA(1,1)-GARCH(1,1) models. The ARCH (α) and GARCH (β) estimates, which refer to the own past shocks and volatility effects, respectively, are statistically significant in all markets. The degree of short run persistence, α, varies across those returns. In the case of the Brent market, the shock dependency in the short run of futures returns (0.100) is higher than that of spot returns (0.069). In the WTI market, spot returns (0.211) are higher than futures returns (0.066). However, the degree of long run persistence, $\alpha + \beta$, of futures returns in both markets is higher than for spot returns. This indicates that convergence to the long run equilibrium after shocks to futures returns is faster than for spot returns. Moreover, volatility spillover effects between volatility of spot and futures returns are found in both markets, especially the interdependency of spot and futures returns in the Brent market, 0.712 and 0.212. This means that the conditional variances of spot and futures returns of the Brent market are affected by the previous long run shocks from each other, while the conditional

variance of spot returns is only affected by the previous long run shocks from futures returns, 0.654 in the case of the WTI market.

[Insert Table 5 here]

The DCC estimates of the conditional correlations between the volatilities of spot and futures returns based on estimating the univariate GARCH(1,1) model for each market are given in Table 6. Based on the Bollerslev and Wooldridge (1992) robust t-ratios, the estimates of the DCC parameters, $\hat{\theta}_1$ and $\hat{\theta}_2$, are statistically significant in all cases. This indicates that the assumption of constant conditional correlation for all shocks to returns is not supported empirically. The short run persistence of shocks on the dynamic conditional correlations is greatest for WTI at 0.139, while the largest long run persistence of shocks to the conditional correlations is 0.986 (= 0.070 + 0.916) for Brent. The time-varying conditional correlations between spot and futures returns are given in Figure 4. It is clear that there is significant variation in the conditional correlations over time, especially in the spot and futures returns of Brent.

[Insert Table 6 here]
[Insert Figure 4 here]

The estimates for BEKK and 11 parameters are given in Table 7. The elements of the 2×2 parameter matrices, **A** and **B**, are statistically significant. Therefore, the conditional variances depend only on their own lags and lagged shocks, while the conditional covariances are a function of the lagged covariances and lagged cross-products of the shocks. In addition, in both markets the estimates of a_{12}, a_{21}, b_{12} and b_{21} are statistically significant, such that there are cross-effects between the variability of spot and futures returns. Table 8 presents the estimates for diagonal BEKK and 7 parameters, all of which are statistically significant. The estimated coefficients of the conditional variances and covariances in both markets are such that the sum of the ARCH and GARCH effects are close to one.

[Insert Tables 7 and 8 here]

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Table 9 gives the optimal portfolio weights, OHRs and hedge effectiveness. The average value of $w_{SF,t}$, calculated from (20) and (21), based on the Brent and WTI markets, are reported in the first and second columns. In the case of the Brent market, the optimal portfolio weights from each model are not particularly different, suggesting that the portfolio constructions give similar results. For example, the largest average value of $w_{SF,t}$ of the portfolio comprising crude oil spot and futures from the CCC model is 0.383, meaning that investors should have more crude oil futures than spot in their portfolio in order to minimize risk without lowering expected returns. In addition, the optimal holding of spot in one dollar of crude oil spot/futures portfolio is 38.3 cents, and 61.7 cents for futures.

[Insert Table 9 here]

In the case of the WTI market, optimal portfolio weights from constant conditional correlation models, namely CCC and VARMA-GARCH, are different and smaller than those from the dynamic conditional correlation models, namely DCC and BEKK. For example, the largest $w_{SF,t}$ is 0.571 from the BEKK model, while the smallest $w_{SF,t}$ is 0.350 from the CCC model, thereby signifying that the dynamic conditional correlation models suggest holding crude oil spot (57.1 cents for spot) more than futures (42.9 cents for futures), whereas the constant conditional correlation models suggest holding crude oil futures (65 cents for futures) than spot (35 cents for futures) of a one dollar spot/futures portfolio.

Figure 5 presents the calculated time-varying OHRs from each multivariate conditional volatility model. There are clearly time-varying hedge ratios. The third and fourth columns in Table 8 report the average OHR values. As the hedge ratios are identified by the second moments of the spot and futures returns, we conclude that the different multivariate conditional volatility models provide the different of OHR. The average OHR values of the Brent market obtained from several different multivariate conditional volatility models are high and have similar patterns to those of the WTI market. In addition, the constant conditional correlations of both markets recommend to short futures as compared with the dynamic conditional correlations.

Each multivariate conditional volatility model provides an average OHR value of the WTI market that is more than the Brent market, such that shorting a short position in a WTI portfolio requires more futures contracts than shorting the same position in a Brent portfolio. For example, the largest average OHR values are 0.846 and 0.956 from VARMA-GARCH of Brent and WTI, suggesting that, in order to minimize risk for short hedgers, one dollar long (buy) in the crude oil spot is shorted (sold) by about 84.6 and 95.6 cents of futures, respectively.

These results can be explained as follows. First, WTI crude oil is of a much higher quality than Brent, with API gravity of 30.6 degrees and containing only 0.24 percent of sulfur, so it can refine a large portion of gasoline. Although Brent is a light crude oil, it is not quite as light as WTI because its API gravity is 38.3 and it contains 0.37 percent of sulfur. Therefore, WTI is more expensive that Brent. Second, as the oil volume and open interest of WTI is greater than for Brent, in terms of the volume of crude or the number of market participants, WTI has higher liquidity than Brent. Therefore, WTI is generally used as a benchmark in oil pricing. Third, as traders profit from wider price swings, increasing volatility makes it more expensive for producers and consumers to use futures as a hedge. Table 1 shows that the standard deviation of the crude oil price of Brent is higher than for WTI, and the standard deviation and conditional volatility of crude oil returns of Brent are also higher than for WTI.

[Insert Figure 5 here]

As risk is given by the variance of changes in the value of the hedge portfolio, the hedging effectiveness in columns five and six in Table 7 shows that all four multivariate conditional volatility models effectively reduce the variances of the portfolio, and perform better in the WTI market than the Brent market (the HE indices are around 80% for WTI and 56% for Brent). Of the multivariate GARCH models, the largest HE value of the Brent market and WTI market is obtained from diagonal BEKK, such that diagonal BEKK is the best model for OHR calculation in terms of the variance of portfolio reduction. In contrast, the lowest HE value in both markets is obtained from BEKK model. Therefore, the BEKK model is the worst model in terms of the variance of portfolio reduction.

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5. Conclusion

This paper estimated four multivariate volatility models, namely CCC, VARMA-GARCH, DCC, BEKK and diagonal BEKK, for the crude oil spot and futures returns of two major benchmark international crude oil markets, namely Brent and WTI. The estimated conditional covariance matrices from these models were used to calculate the optimal portfolio weights and optimal hedge ratios, and to indicate crude oil hedge strategies. Moreover, in order to compare the ability of variance portfolio reduction due to different multivariate volatility models, the hedging effective index was also estimated.

The empirical results for daily data from 4 November 1997 to 4 November 2009 showed that, for the Brent market, the optimal portfolio weights of all multivariate volatility models suggested holding futures in larger proportion than spot. On the contrary, for the WTI market, BEKK, recommended holding spot in larger proportion than futures, but the CCC, VARMA-GARCH and DCC suggested holding futures in larger proportion than spot. The calculated OHRs from each multivariate conditional volatility model presented the time-varying hedge ratios, and recommended short hedger to short in crude oil futures, with a high proportion of one dollar long in crude oil spot. The hedging effectiveness indicated that diagonal BEKK (BEKK) was the best (worst) model for OHR calculation in terms of the variance of portfolio reduction.

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Table 1. Descriptive Statistics

Panel a: Crude Oil Prices

Prices	Mean	Max	Min	SD	Skewness	Kurtosis	Jarque-Bera
BRSP	43.103	144.07	9.220	26.837	1.163	4.100	863.70
BRFU	43.103	144.07	9.220	26.837	1.163	4.100	863.70
WTISP	44.675	145.66	10.730	26.814	1.192	4.231	939.78
WTIFU	44.696	145.29	10.720	26.827	1.189	4.220	932.83

Panel b: Crude Oil Returns

Returns	Mean	Max	Min	SD	CV	Skewness	Kurtosis	Jarque-Bera
BRSP	0.0004	0.152	-0.170	0.025	0.016	-0.047	6.113	1265.547
BRFU	0.0004	0.129	-0.144	0.024	0.017	-0.142	5.576	876.642
WTISP	0.0004	0.213	-0.172	0.027	0.015	-0.002	7.932	3174.982
WTIFU	0.0004	0.164	-0.165	0.025	0.016	-0.120	7.164	2270.166

Table 2. Unit Root Tests

Panel a: Crude Oil Prices

Prices	ADF test (*t*-statistic)			Phillips-Perron test		
	None	**Constant**	**Constant and Trend**	**None**	**Constant**	**Constant and Trend**
BRSP	0.185	-1.074	-2.372	0.177	-1.079	-2.402
BRFU	0.325	-0.937	-2.187	0.262	-0.990	-2.298
WTISP	0.148	-1.132	-2.431	0.159	-1.119	-2.444
WTIFU	0.224	-1.054	-2.324	0.185	-1.096	-2.400

Panel b: Crude Oil Returns

Returns	ADF test (*t*-statistic)			Phillips-Perron test		
	None	**Constant**	**Constant and Trend**	**None**	**Constant**	**Constant and Trend**
BRSP	-55.266	-55.275	-55.267	-55.276	-55.280	-55.271
BRFU	-59.269	-59.281	-59.273	-59.239	-59.252	-59.244
WTISP	-56.678	-56.684	-56.676	-56.881	-56.906	-56.897
WTIFU	-42.218	-42.231	-42.224	-57.169	-57.191	-57.183

Note: Entries in bold are significant at the 1% level.

Table 3. Cointegration Test Using the Johansen Approach

Market	lag	λ_{trace} $k=0$	λ_{trace} $k \leq 1$	λ_{max} $k=0$	λ_{max} $k=1$
Brent	1	**126.51**	0.005	**126.51**	0.005
		(12.321)	(4.130)	(11.225)	(4.130)
WTI	1	**438.88**	0.016	**438.87**	0.016
		(12.321)	(4.130)	(11.225)	(4.130)

Notes: (1) Entries in bold indicate that the null hypothesis is rejected at the 5% level.

(2) The cointegrating vector is normalized with respect to S_t

Table 4. CCC Estimates

Panel a: BRSP_BRFU

Returns	C	AR	MA	ϖ	α	β	$\alpha + \beta$	Constant conditional correlation	Log-likelihood	AIC
BRSP	**1.878e-03**	**-0.841**	**0.859**	**6.871e-06**	**0.039**	**0.951**	0.990	**0.794**	16291.932	-10.399
	(2.648)	**(-8.338)**	**(9.045)**	**(5.636)**	**(13.72)**	**(256.6)**		**(159.65)**		
BRFU	**1.343e-03**	**-0.383**	**0.309**	**6.299**	**0.035**	**0.953**	0.988			
	(2,930)	**(-27.87)**	**(21.08)**	**(5.691)**	**(9.693)**	**(204.2)**				

Panel b: WTISP_WTIFU

Returns	C	AR	MA	ϖ	α	β	$\alpha + \beta$	Constant conditional correlation	Log-likelihood	AIC
WTISP	**1.086e-03**	-0.093	0.029	**2.069e-05**	**0.083**	**0.888**	0.971	**0.923**	17421.123	-11.1198
	(3.560)	(-0.768)	(0.240)	**(15.58)**	**(23.90)**	**(244.9)**		**(550.9)**		
WTIFU	**1.209e-03**	**-0.177**	**0.100**	**1.978e-05**	**0.083**	**0.888**	0.971			
	(4.005)	**(-18.21)**	**(8.240)**	**(11.55)**	**(20.818)**	**(163.0)**				

Table 5. VARMA-GARCH Estimates

Panel a: BRSP_BRFU

Returns	C	AR	MA	ϖ	α_{BRSP}	α_{BRFU}	β_{BRSP}	β_{BRFU}	$\alpha+\beta$	Constant conditional correlation	Log-likelihood	AIC
BRSP	**1.492e-03**	**-0.855**	**0.872**	3.644e-06	**0.069**	**-0.037**	**0.412**	**0.712**	0.481	**0.803**	16348.450	-10.432
	(2.066)	**(-9.824)**	**(10.673)**	(0.433)	**(4.590)**	**(-2.424)**	**(3.327)**	**(4.585)**		**(158.566)**		
BRFU	**1.183e-03**	**-0.384**	**0.308**	**7.749e-06**	**-0.064**	**0.100**	**0.212**	**0.762**	0.862			
	(2.634)	**(-25.856)**	**(21.408)**	**(2.481)**	**(-4.946)**	**(6.867)**	**(2.364)**	**(9.794)**				

Panel b: WTISP_WTIFU

Returns	C	AR	MA	ϖ	α_{WTISP}	α_{WTIFU}	β_{WTISP}	β_{WTIFU}	$\alpha+\beta$	Constant conditional correlation	Log-likelihood	AIC
WTISP	**1.011e-03**	-0.060	1.303e-03	**1.641e-05**	**0.211**	**-0.138**	**0.305**	**0.654**	0.516	**0.928**	17525.095	-11.184
	(3.414)	(-0.392)	(0.009)	**(3.382)**	**(21.250)**	**(-20.464)**	**(10.436)**	**(19.323)**		**(583.246)**		
WTIFU	**1.143e-03**	**-0.179**	**0.105**	**1.048e-05**	1.679	**0.066**	0.033	**0.887**	0.953			
	(3.914)	**(-18.196)**	**(9.717)**	**(8.172)**	(0.274)	**(11.045)**	(1.387)	**(40.623)**				

Notes: (1) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust *t*- ratios.
(2) Entries in bold are significant at the 5% level.

Table 6. DCC Estimates

Panel a: BRSP_BRFU

	C	AR	MA	ϖ	α	β	$\alpha + \beta$	θ_1	θ_2	Log-likelihood	AIC
BRSP	**1.821e-03**	**-0.762**	**0.776**	**7.742e-06**	**0.053**	**0.935**	0.988	**0.070**	**0.916**	16424.565	-10.483
	(2.671)	(-3.584)	(3.765)	(5.033)	(13.851)	(189.947)		(18.766)	(183.616)		
BRFU	**1.244e-03**	**-0.346**	**0.299**	**6.012e-06**	**0.043**	**0.946**	0.989				
	(2.789)	(-21.576)	(18.131)	(5.156)	(11.195)	(195.999)					

Panel b: WTISP_WTIFU

	C	AR	MA	ϖ	α	β	$\alpha + \beta$	θ_1	θ_2	Log-likelihood	AIC
WTISP	0.001	**-0.259**	**0.252**	**3.34E-05**	**0.151**	**0.774**	0.925	**0.139**	**0.458**	17618.890	-11.246
	(1.580)	(-5.655)	(5.160)	(2.118)	(3.142)	(10.295)		(1.981)	(0.174)		
WTIFU	0.0003	**0.626**	**-0.658**	**3.98E-05**	**0.151**	**0.789**	0.940				
	(1.796)	(6.871)	(-8.085)	(2.129)	(3.460)	(12.108)					

Notes: (1) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust *t*- ratios.
(2) Entries in bold are significant at the 5% level.

Table 7. BEKK Estimates

Panel a: BRSP_BRFU

Returns	C	AR	MA	C		A		B		Log-likelihood	AIC
BRSP	**0.002**	**-0.715**	**0.724**	-0.001		**-0.320**	**0.153**	**-0.151**	**-0.878**	16427.720	-10.483
	(2.527)	(-2.585)	(2.481)	(-1.286)		(-7.800)	(5.613)	(-6.583)	(-27.652)		
BRFU	**0.001**	**-0.331**	**0.285**	**0.005**	-0.0001	**0.182**	**-0.357**	**-0.897**	-0.043		
	(2.386)	(-19.748)	(12.605)	(6.616)	(-0.063)	(5.438)	(-13.812)	(-81.273)	(-0.967)		

Panel b: WTISP_WTIFU

Returns	C	AR	MA	C		A		B		Log-likelihood	AIC
WTISP	0.0004	**0.310**	**-0.387**	**0.002**		**-0.911**	**-0.018**	**0.494**	**-0.079**	18021.590	-11.501
	(1.289)	(2.370)	(-3.186)	(5.808)		(-5.941)	(-2.394)	(9.593)	(-62.908)		
WTIFU	0.0007	**-0.212**	**0.157**	**-0.003**	1.00E-06	**0.938**	**0.192**	**0.500**	**1.053**		
	(1.575)	(-6.440)	(4.043)	(-7.358)	(0.001)	(6.758)	(6.130)	(10.064)	(213.713)		

Notes: (1) $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}$ are the coefficient matrices from equation (14).

(2) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust t-ratios.

(3) Entries in bold are significant at the 5% level.

Table 8. Diagonal BEKK Estimates

Panel a Diagonal BEKK

BRSP_BRFU	C		A		B	
Coeff.	**6.55E-06**	**8.43E-06**	**0.218**		**0.971**	
	(6.739)	**(8.034)**	**(29.836)**		**(546.5)**	
		1.40E-05		**0.243**		**0.960**
		(7.836)		**(30.926)**		**(375.6)**
Log-likelihood	16341.97					
AIC	-10.431					
WTISP_WTIFU	C		A		B	
Coeff.	**7.54E-05**	**5.57E-05**	**0.377**		**0.870**	
	(19.449)	**(19.709)**	**(77.944)**		**(302.90)**	
		3.80E-05		**0.282**		**0.931**
		20.588.		**(78.404)**		**(768.65)**
Log-likelihood	17750.13					
AIC	-11.331					

Notes: (1) $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ & a_{22} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} b_{11} & \\ & b_{22} \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ & c_{22} \end{bmatrix}$ are the coefficient matrices from equation (14).

(2) The two entries for each parameter are their respective parameter estimates and Bollerslev and Wooldridge (1992) robust t- ratios.

(3) Entries in bold are significant at the 5% level.

Substituted coefficient Brent

GARCH1 = 6.549e-06+0.047*RESID1(-1)^2+0.943*GARCH1(-1)

GARCH2 = 1.397e-05+0.059*RESID2(-1)^2+0.921*GARCH2(-1)

COV1_2 = 8.432e-06 + 0.053*RESID1(-1)*RESID2(-1) + 0.932*COV1_2(-1)

Substituted coefficient WTI

GARCH1 = 7.544e-05+0.142*RESID1(-1)^2+0.757*GARCH1(-1)

GARCH2 = 3.797-05+0.080*RESID2(-1)^2+0.867*GARCH2(-1

COV1_2 = 5.571e-05 + 0.106*RESID1(-1)*RESID2(-1) + 0.810*COV1_2(-1)

Table 9. Alternative Hedging Strategies

Model	Optimal Portfolio Weights		Average OHR		Variance of Portfolios		Hedge Effectiveness (%)	
	Brent	WTI	Brent	WTI	Brent	WTI	Brent	WTI
CCC	0.383	0.350	0.840	0.955	2.682e-04	1.349e-04	56.724	80.857
VARMA-GARCH	0.377	0.351	0.846	0.956	2.706e-04	1.373e-04	56.346	80.513
DCC	0.366	0.478	0.824	0.923	2.663e-04	1.342e-04	57.045	80.942
BEKK	0.355	0.571	0.827	0.922	2.710e-04	1.417e-04	56.294	79.886
Diagonal BEKK	0.351	0.501	0.843	0.941	2.655 e-04	1.340 e-04	57.167	80.983
Unhedged Portfolio					6.199e-04	7.046e-04		

Note: The portfolio weights given are for the spot oil, and thus 1-spot weights for futures in the portfolio are warranted.
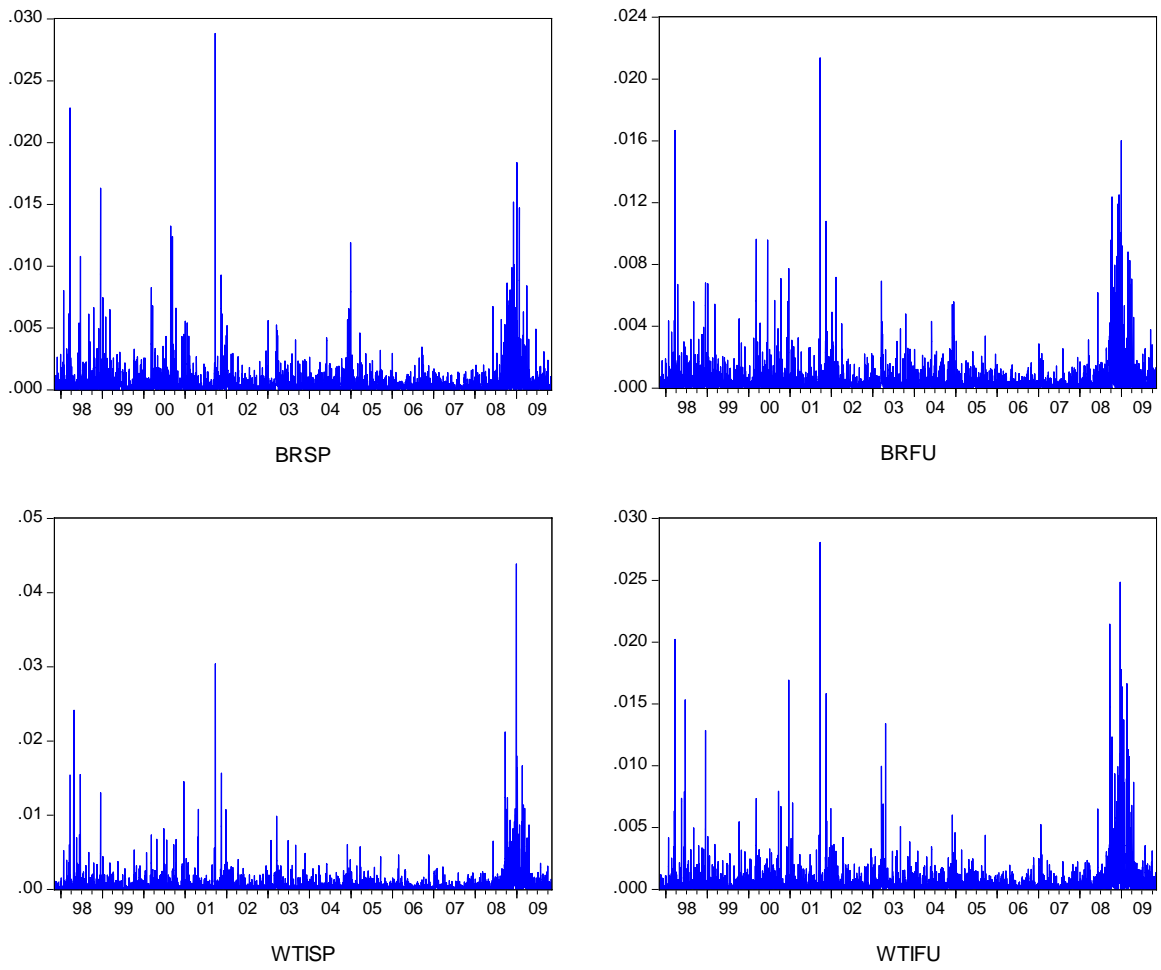
Figure 1. Crude Oil Spot and Futures Prices for Brent and WTI

BRSP

BRFU

WTIFU

WTISP
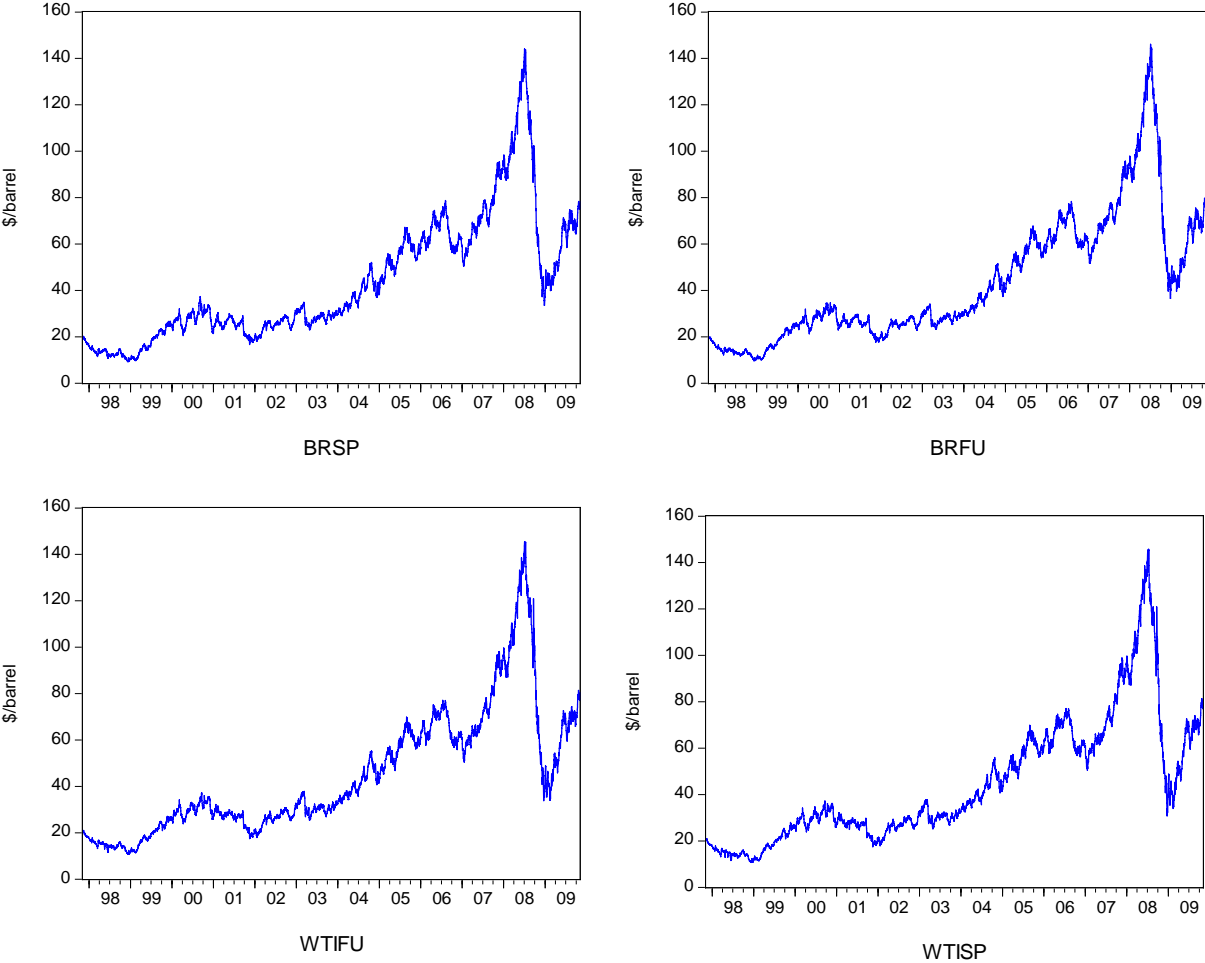
Figure 2. Logarithm of Daily Crude Oil Spot and Futures Returns for Brent and WTI

WTISP

WTIFU
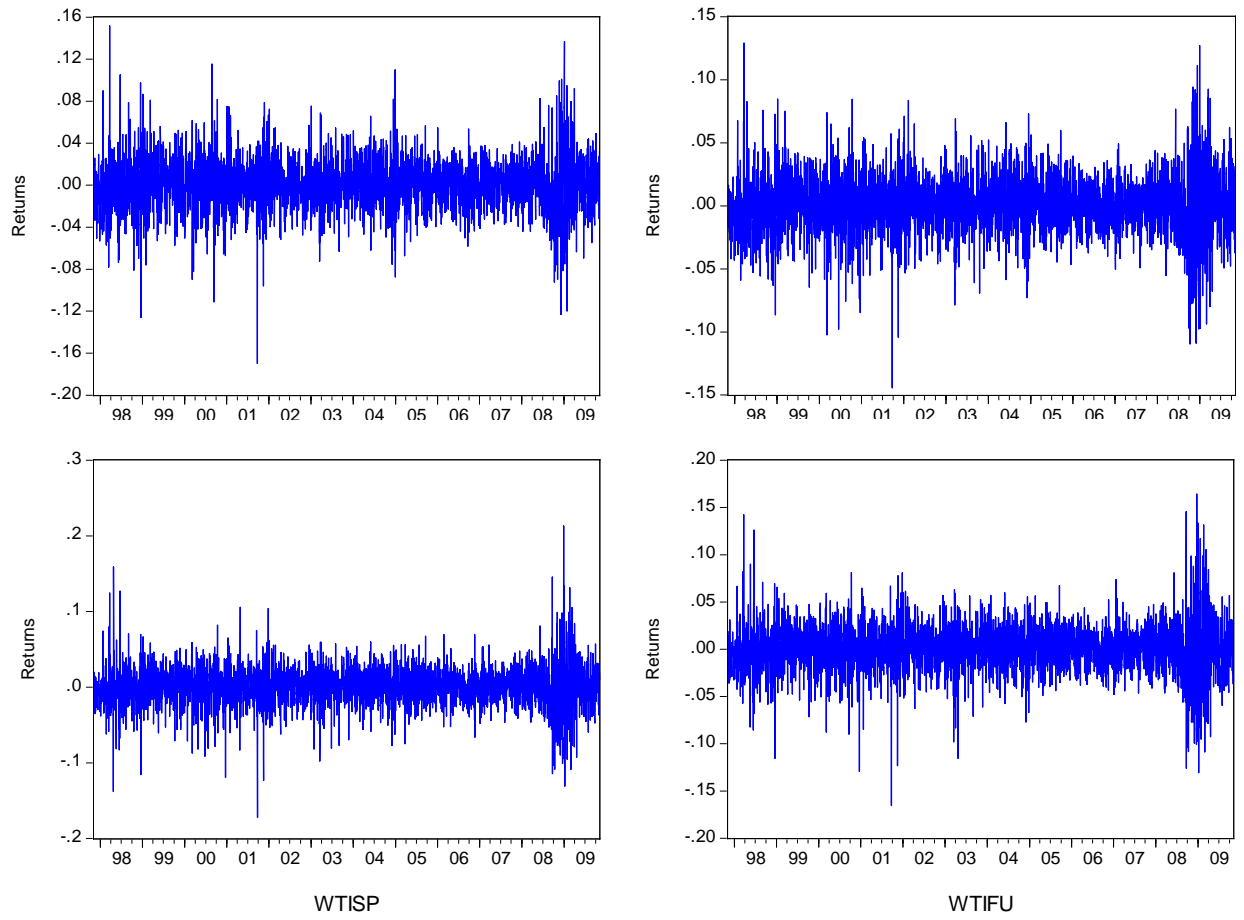
Figure 3. Estimated Conditional Volatilities of Returns for Brent and WTI

BRSP

BRFU

WTISP

WTIFU
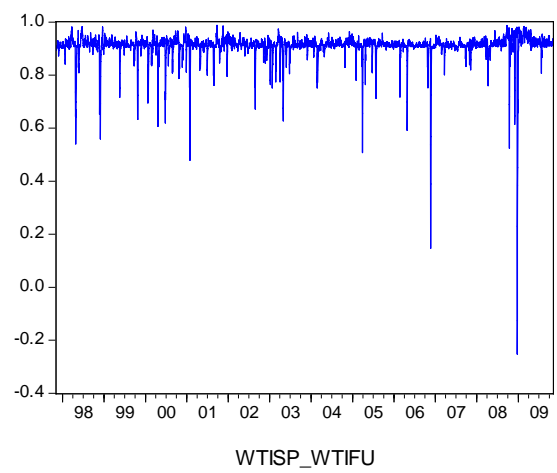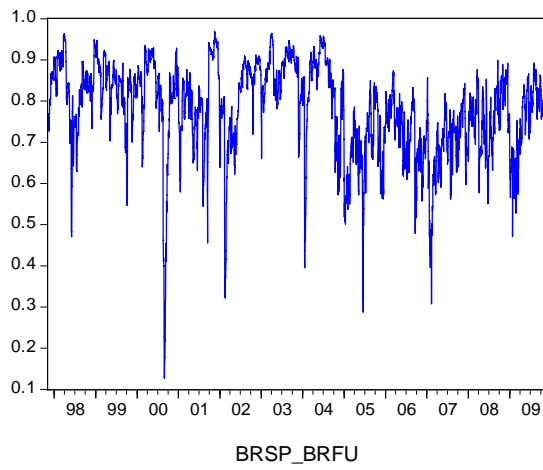
Figure 4. DCC Models Conditional Correlations

BRSP_BRFU

WTISP_WTIFU

Figure 5. Optimal Hedge Ratios

CCC:BRSP_BRFU

CCC:WTISP_WTIFU

DCC:BRSP_BRFU

DCC:WTISP_WTIFU

VARMA-GARCH:BRSP_BRFU

VARMA-GARCH:WTISP_WTIFU

BEKK:BRSP_BRFU

BEKK:WTISP_WTIFU

diagonal BEKK: BRSP_BRFU

diagonal BEKK:WTISP_WTIFU

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