A STUDY ON THE DYNAMIC PROCESS OF THE LONG-TERM RUNOFF

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ERRATA

page	line	Correct	Error
5	1	.. the important tasks ..	.. the important task ..
7	24	The process on the ..	These process on the ..
12	19	.. time invariant in an ..	.. time invariant in a ..
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31	21	.. these equations ..	.. these equation ..
39	7	component.	components.
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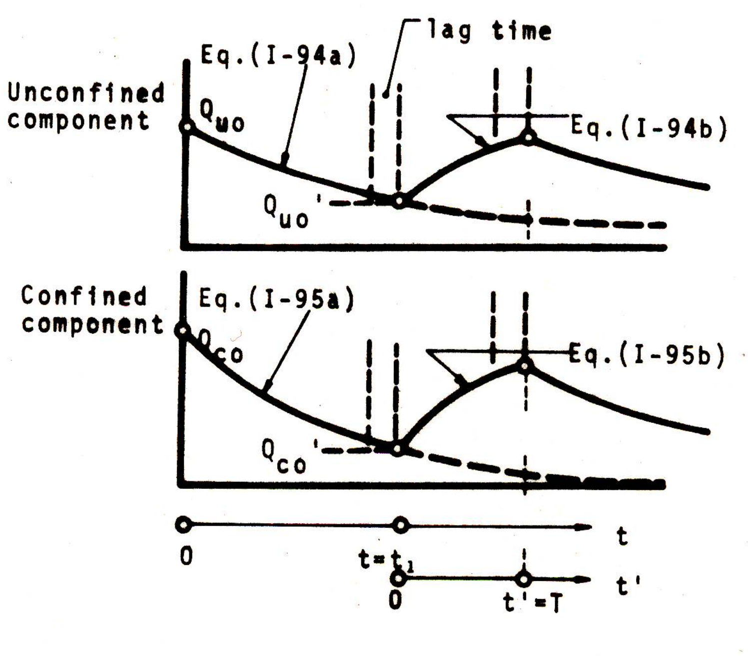
92	7	$\gamma \frac{\partial H_g}{\partial t} = \sum \frac{\partial}{\partial x_i} \{ \cdots$	$\gamma \frac{\partial H_g}{\partial t} = \frac{\partial}{\partial x_i} \{ \cdots$
102	16	.. closely related to ..	.. closely related with ..
111	22	$\frac{\Lambda_{gj}}{\Gamma_{gj}} \simeq \frac{2\beta_j}{L_j^2} (\cdots$	$\frac{\Lambda_{gj}}{\Gamma_{gj}} \simeq \frac{2\beta_j}{L_j} (\cdots$
137	Table I-9	Unit : mm	--
181		Correct the arrow heads for Eq.(I-94b) and Eq.(I-95b) as follows;	



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INTRODUCTORY STATEMENT

The acute water shortage has not been brought to the attention of scientists till recent years and hydrologists have devoted their efforts to solve the flood disaster problems. Hydrology, therefore, has been developed by the research works on the direct runoff and its related phenomena. As well known, however, the problems on the long-term runoff have recently attracted the special interest of the hydrologists with respect to the water resources problem[1)*]. Research works on the long-term runoff are expanded into the various fields concerned[2)3)4)5)6)].

As a matter of course, the direct runoff is also involved in the long-term runoff as one of its components. But it is limited in short periods in the runoff phenomena in an extended period. Phenomenological consideration leads that the main component of the long-term runoff is so-called ground water runoff which supplies the runoff discharge unceasingly in a prolonged period. Regarding both the components, it should be also emphasized that the fields of water movement of the ground water runoff completely differ from those of the direct runoff. That is, movement of the ground water runoff takes place in aquifers and is governed by Darcy's law, on the other hand, the direct runoff occurs over the ground surface or in very shallow layer under ground. Therefore, there are great differences between the mechanisms which govern the process of ground water runoff and that of the direct runoff. Numerous researches have been made on the direct runoff from various angles, and behaviour of it has been considerably revealed, whereas the mechanism and kinematics of the ground water runoff have remained still

* The numbers in parentheses refer to the list of references.

unsolved. For the purpose of investigation of the long-term runoff, therefore, the ground water runoff should be studied in the phenomenological meaning.

In the present paper, the ground water runoff is only treated as the major runoff component of the long-term runoff. Although the research on ground water runoff is not a relatively new field of study, it has remained at the estimation of empirical laws or conceptual treatment and has been left behind from the studies on flood. Moreover, no investigation has been carried out on the dynamical process of ground water runoff. The reason is not only the scant social needs for the long-term runoff but the complexity and slowness of the phenomena which conceal dominant factors out of our sights. This situation will be understood in finding the fact that the complex and slow phenomena such as effects of soil moisture and evapo-transpiration on water loss are left yet unsolved even in the flood hydrology.

With this situation, the problems on the ground water runoff should be approached from the kinematical or hydrodynamical view point. Realizing the problems, the author has tackled the problems on the ground water runoff, focusing the points at its dynamical process and the kinematical characters. The purpose of the present paper is to investigate the kinematical behaviour of the ground water runoff as a major component in the long-term runoff and to clarify the physical significances of the several quantities which characterize the ground water runoff. To carry out the investigation, two kinds of approaches are adopted.

In the first approach, the theoretical discussions have been made based on hydraulic considerations by means of the hydrodynamic models as a lumped system. The method[7)8)9)] has given us an useful tool to clarify the physical significances of ground water runoff to some extent.

Moreover, it has pointed out the component which plays an important role in the runoff process in a prolonged period. The application of this method has been attempted[10)] for the simulation technique of stream discharge due to ground water runoff in an extended period. Although the major points of the research have been carried out successfully, several problems remain to be solved.

In order to clarify some of the problems unsolved in the first approach and also to contribute to the construction of new method for future hydrology, the author has treated the ground water runoff from a different angle[11)]. In this second approach, the author has attempted to formulate a general law which governs the behaviour of the basinwide water.

Part I concerns with the variation of ground water runoff in comparatively simple basins by means of the first approach. The investigation is made on the basic conception that the behaviour of water which appears in the stream as ground water runoff are characterized by the mechanism of flow from the aquifer. Accordingly, some components of ground water which differ with each other in kinematical mechanism must affect the characteristics of ground water runoff in a different way. The runoff components coming from the unconfined and confined aquifers are treated by means of the theoretical and phenomenological considerations. From the first chapter to the third, discussions will be made especially on ground water runoff phenomena from the hydrodynamic and phenomenological point of view, and the fourth chapter is devoted to the engineering problem as to how we can simulate the daily discharge in an extended period.

As an introduction to the first approach, some conceptions on ground water runoff on which many investigations have been made are conclusively stated and discussed in the first chapter. The main task in the first chapter is to describe and demonstrate the fundamental conception of the first approach in the present study. Differences in the mechanisms between unconfined and confined components will be discussed, thereupon, runoff models for these components will also be formulated.

The second chapter concerns with the recession characteristics of ground water runoff during no rainfall period. It is demonstrated that besides the recession equation being the fractional function with respect to time for the unconfined component, the recession of confined runoff component is expressed by the exponential function. Furthermore, the roles of these components in the runoff process in a prolonged period will be demonstrated and the physical significances of several parameters will be also clarified in terms of hydraulic, hydrological and geological factors. The research is successfully verified in Sections 2-4, 2-5 and 2-6 by a considerable number of field data in the several watersheds in Japan. The results obtained in the second chapter have established the basic foundations of the first approach for the better understandings of ground water runoff and have given available information to stimulate the investigation on the rising state due to rainfall which will be treated in the third chapter.

In the third chapter, the rising state of each component due to rainfall is treated. The theoretical considerations are made with respect to the flow through the models under the condition of the constant recharge due to rainfall. The recharge regime to the ground water table is discussed with respect to several assumptions with which the theoretical treatment is developed, especially regarding the infiltration process through the zone of aeration under ground. Thereafter, the problems as to how the ground water runoff will increase resulting from the rainfall, will be solved with help of the theoretical equations and field data. Furthermore, one of the important tasks in this chapter is also to answer to questions what are the significant factors in the rising state of both of runoff components. The roles of both components in the variation of ground water runoff due to rainfall will be also presented.

The first half of each of the second and the third chapter treats the theoretical discussions on the mechanism of the variation of discharge and the considerations about the significances of the several parameters. Verification is made in the latter half with help of the actual data as well as discussions on the runoff process. Through these chapters it may be concluded that the unconfined component supplies the stream discharge unceasingly during a prolonged period although the amount of discharge is relatively small, while the discharge of the confined components reaches at a considerable amount at the early stage of recession but decreases very fast and ceases in a few days. The existences of these components in the actual watershed are also examined and verified.

The fourth chapter is primarily intended to utilize the analytical method derived in the previous chapters to the engineering problems. That is, the establishment of the simulation technique of the long-term runoff is aimed. The comparison of the estimated and actual hydrographs is described in Section 4-3 as well as the simulation technique. The applicability of the method to the simulation technique will be also demonstrated by successful results obtained at the four basins in Japan.

Throughout the first part, the method explains well the behaviour of ground water runoff in small mountainous region and certain basins with very large catchment area, however, we can not clarify several characteristics definitely in the basins of middle size because of the scattering of several parameters resulting from the temporal non-homogeneity of the distributions of hydrologic quantities and the interactions among several water components such as ground water, stream water and others.

While the first part of the present paper deals with the ground water runoff by the hydrodynamic model as a lumped system, the second part, which concerns with the second approach, treats the distributed model. The second approach is originally introduced in order to solve the problems which have remained unsolved by the first approach. The second approach, therefore, has been aimed to find a law or a quantity which governs the behaviour of all water components as a whole. The variational expression is mathematically formulated for the behaviour of all water within a basin involving the interactions of them. On the basis of the variational principle so derived, the variation of the recession characteristics in the runoff process is theoretically investigated.

In the first chapter, the fundamental concepts of the relationship between the runoff phenomena and the behaviour of whole water components are described as well as of the kinematical system in the runoff process. The entire basin in which the runoff takes place is then considered as links of several kinematical systems.

As a general law on which the behaviour of all water components depends, the variational formulation is made in the second chapter based on the concepts of the local equilibrium. The most important result obtained in this chapter is that the behaviour of all water components within a kinematical system occurs in such a manner that the integral of the newly defined local potential all over the region of water movement and any time interval takes the mathematical stationary value. Furthermore, the physical significances of the local potential and the variational principle are also discussed.

The third chapter concerns with one of the application of the variational principle established in the previous chapters to the hydrologic problems. That is to say, the variation of the characteristics of the ground water runoff in the runoff process is discussed, stating again, the averaging process of the recession characteristics of water coming from various regions within a basin is treated in connection with the behaviour of the ground water flow and the stream flow as a whole. In this part, the unconfined ground water is only treated as an dominant component in a long-term runoff, and then the recession factor introduced in Part I is used as the measure of the recession characteristics. By the theoretical consideration, it is concluded that the recession factor for a kinematical system as a whole is expressed as the weighted mean of those of the individual small regions. In other words, the recession factor varies from upstreams to downstreams according to the averaging process mentioned above.

The weights in the averaging process depend mainly on the water distributions in a basin. Therefore, the recession factor does not change temporally in the small specified basins due to the homogeneity of the hydrologic quantities, but it varies due to the temporal non-homogeneity in the basins of middle size. In the very large basins, however, since the roles of the runoff from the individual small region are relatively slight in the runoff phenomena at the entire basin, the statistical averaging process dominates and rather definite characteristics is visually observed. The process on the recession characteristics is discussed in details by the theoretical considerations and also verified by the field observations to some extent.

All of the subjects in the present paper were promoted by the author with cooperation of students in the hydraulic Laboratory, Department of Civil Engineering and the Disaster Prevention Research

Institute, Kyoto University under the supervision of Dr. Tojiro Ishihara, Professor of River Engineering, and numerous suggestions given by Dr. Yasuo Ishihara, Professor of Hydrology. The research has been continuing also in the Department of Civil Engineering, Nagoya University, since April, 1967. The original purpose to establish the analytical methodology to clarify the dynamic process of the ground water runoff in details is not fully accomplished. However, the mechanism of the ground water runoff has been essentially revealed by the theoretical verifications and the examinations by field data and the present paper will contribute to the better understanding of the hydrodynamical process of the long-term runoff.

Part I

THE HYDRODYNAMIC BEHAVIOUR OF GROUND WATER RUNOFF THROUGH THE IDEALIZED MODEL

Chapter 1 GENERAL SCOPE OF PROBLEMS

The first part of the present study aims to investigate the mechanical process of ground water runoff and its behaviour in a watershed by constructing hydrodynamic models. With respect to each of kinematical characteristics of flow through the unconfined and confined aquifers, the runoff models are formulated. The equations for the variation of ground water runoff are derived on the basis of the theoretical treatment of the models. Furthermore, the roles of these components in the runoff process will be demonstrated, and physical significances of several parameters are also clarified as well as the reason why the unique state appears in the recession limb of hydrograph in a specified basin.

Before discussing the behaviour of ground water runoff on the kinematical point of view in details, the author wishes to review the previous fundamental conceptions on the ground water runoff and to state clearly the problems to be solved, though the detail discussion will not be stated here since there exist many books[12] and references[13] which deal with the bird's eye view of those problems.

1-1 Previous Conceptions on Ground Water Runoff

1-1-1 Recession Characteristics

Beginning of the research works on ground water runoff goes way back to the thirtieth. In this era referred to as "the inchoate stage of modern hydrology", many famous researches were made, especially Horton's researches[14][15][16] which were the guiding star of hydrology and the unit graph method proposed by L. K. Sherman[17].

In this spirited period, Barnes[18)] and Horton had found empirically that the special shape in the long tail of the recession segment of hydrograph in each watershed appears. Then, they made many efforts to formulate the mathematical expression for the recession segment in order to understand precisely the flood runoff. Although they were interested in ground water runoff itself[19)], the problems were discussed with respect to the effect of basin states on the direct runoff, namely, roles of the infiltration in the runoff process.

Since Barnes gave the exponential expression for the recession curve[18)], many investigators have considered that the recession state represents implicitly the characteristics of ground water runoff. Although numerous number of researches[20)21)22)] have been published on the recession equations, the fundamental concepts of them are reduced to a few conceptions, as the exponential recession[18)], the normal recession curve[22)] and the recession equation in a statistical meaning[23)]. Among these conceptions, as well known, the exponential recession equation has been widely accepted.

In the natural world, we often encounter with exponential recession phenomena such as radiation and electric condenser in which the intensity of outflow is proportional to the storage. These phenomena have given us suggestions on the behaviour of ground water runoff within a watershed. This is the reason why many investigators, as will be shown later, have formulated runoff models of ground water runoff regarding the storage of water in a watershed.

The exponential recession equation, however, can not express accurately the actual recession curve during an extended period and aparts from the observed long tail of recession limb in hydrograph. For the purpose of supplement of the deficiency, various modifications have been made--for this example, we may take the series type expression[21)] and others. Regarding all of recession equations which have been proposed, it may be pointed out that they are no better than the empirical expression. Even though the recession state has been expressed mathematically or graphically, few phenomenological considerations have been made. That is, the parameters in the expressions have been only evaluated by the great number of data, but the investigators could not answer the questions as to what the parameter means and why the unique recession state appears in each basin. Although research works on recession characteristics have given us the fundamental conceptions for the models on ground water runoff, the physical significances of the recession state, namely the physics and mechanics of ground water runoff remain unsolved yet.

1-1-2 Runoff Models of Ground Water Runoff

Reviewing the research works in hydrology, it is naturally understood that the recent studies on ground water runoff are often based on the concepts and methodology developed in the flood hydrology. However, as we have seen, the ground water runoff cannot be treated by the model for the direct runoff itself, since it's mechanism is distinguished from that of the direct runoff. In this meaning, it should be emphasized that the recession characteristics is what has led the basic concept on which the investigators have formulated the runoff models for the ground water runoff.

Recently many investigators have focused their attentions to complete the simulation technique of discharge in an extended period. Therefore, many of the models for the ground water runoff have been formulated in connection with runoff models for the whole system. As will be discussed later, since the roles of the stream may be actually ignored in the process of ground water runoff, the ground water runoff discharge has been expressed as a summation of runoff discharges from each model which corresponds to the various aquifers respectively. In this meaning, various kinds of models have been treated, for example the unit graph method[24], tank or reservoir models[25][26][27][28], Stanford University Models[29][30][31] and others[32][33].

As a matter of course, the model will lose physical significances, if the basic relationship is given without any fixed principle. Therefore, the models for ground water runoff have been often formulated on the basis of the essential assumption of the exponential recession. Furthermore, it should be noted that the linear assumption used for the ground water runoff[24][25] is originally derived by the exponential recession. That is, almost all models proposed for the ground water runoff treat the decreasing process of water storage within a basin or superposition of the processes. In other words, the linear kernel and the impulse response function consist of only the recession limb and the rising has been often assumed to occur instantaneously to reach a certain discharge.

Regarding the linear assumption, however, it should be remembered that the most important reason why the method is used successfully for the flood runoff is that the system is of time-invariant[34][35][36] in an approximate meaning. For the ground water runoff, however, the phenomenological consideration implies the time-variant system and the exponential recession aparts from the long tail of the hydrograph. Moreover, the relationship between the runoff discharge and the corresponding rainfall is complicated and very vague. Hence lengthy and careful consideration should be given to these problems when we adopt the linear assumption to the ground water runoff.

Recently, the techniques to treat ground water runoff as problems on the time series of hydrologic quantities from the statistical view point[37] have arisen. Some investigators have devoted their efforts to separate the various runoff components with various periods[38][39][40]. And others[41][42][43] have tried to find the linear kernel most suitable in statistical meaning, to correlate the time series of rainfall and daily discharge in a long period. These approaches may give us fruitful tools to analyze the very complex phenomena of ground water runoff. But, the terms "most suitable in statistical meaning or statistics" might lead vague physical meaning of the several factors and results, if the methods are applied without sufficient discussions.

As stated above, many approaches have been discussed for ground water runoff, however, the fundamental concept of the models for the ground water runoff is reduced to storage effect within the basin based on the exponential recession. In addition, it should be emphasized that a considerable number of models have been treated conceptually but none has been developed with respect to the dynamical process of ground water runoff.

1-2 Statement of Problems

In order to understand ground water runoff as kinematical phenomena and to clarify the physical significances of several characteristics, it will be hoped that the research is carried out on the kinematical point of view.

Although the term "ground water runoff" has been defined conceptually by many investigators[44][45][46], we often discuss the variant components as ground water runoff, even based on the exponential recession. In an extended period or drought period, the stream discharge is mainly sustained by water coming from aquifers with exception of special cases in basins with large lakes and/or ponds. Therefore, the stream discharge may be characterized by ground water runoff alone after ceasing of direct runoff. In the present paper, all stream discharge after ceasing of direct runoff is treated as ground water runoff.

1-2-1 Runoff Process

Let us consider that portion of precipitation which appears again to ground surface as ground water runoff. From the time the rain water reaches at ground surface until it flows out from the outlet of the river basin, the rain water passes through various processes which characterize the flow kinematically. After phenomenological consideration the processes may be classified into the following categories;

a. infiltration process through the intermediate zone and/or recharge flow process through the orifices, cracks and chinks under ground,

b. infiltration process in the saturated water aquifers,

c. overland flow and stream flow.

As a rule, infiltration processes have close relationships with soil moisture at that time. Through the intermediate zone, the water will infiltrate downwards under the gravitational potential when it exceeds the capirally potential. Regarding the runoff phenomena, the infiltration process plays a definite role as a boundary phenomenon which divides the rain water into the direct runoff component and the ground water runoff. The limitation of the discharge which is one of the most important characteristics as well as the slowness and the storage effect described below, is caused by this infiltration process, since it controls the recharge rate to the ground water table.

In the saturated zone, the water behaves according to Darcy's law during a prolonged period. The slowness and storage effect of ground water runoff phenomena are due to the effect of the great friction for water movement in this stage. In other words, the flow mechanism in the saturated zone determines the variation of runoff phenomena especially in a prolonged period.

Water movement in over-land-flow process or stream is governed by the law for open channel flow, such as Manning's, Chézy's law. However, it is of very short period for water to flow along overland and streams comparing with that passed through the saturated zone. Therefore, even though the characteristics of ground water runoff is temporally changed in this process, the effect of this process may be by far smaller for the runoff phenomena in an extended period than that of the saturated zone.

As the result, among these processes, the most important ones which govern the characteristics of ground water runoff, are infiltration or recharge one and the process in the saturated aquifer. The water particles which appear at the ground surface as ground water runoff, spend almost all the time under kinematical transformation in the infiltration process and process in the saturated zone. In other words, the distinguished characteristics of the ground water runoff in a prolonged period may be established in these two processes.

Generally, since the infiltration and recharge occur simultaneously with the direct runoff, it is rather difficult to analyze the several features of rising limb of hydrograph. Moreover, as forementioned, the infiltration process should be considered as the boundary phenomenon with respect to ground water runoff. On the other hand, the recession state is understood to be so ideal that it enables discussion on several behaviour of ground water runoff, because only the process in saturated zone is active with no other external forces during drought period. Thus the recession characteristics have been also considered as a measure of the basin characteristics for runoff phenomena. This is the reason why many investigators have directed their attentions to the recession features with respect to the ground water runoff.

Consequently, it may be concluded that water when it flows out to the ground surface, takes its dominant kinematical characters from the zone of saturation. Even though our problem is reduced to this point, it is really impossible to find exact solutions on the flow mechanisms in each aquifer which is complicated in vertical and horizontal distributions within the watershed. In a macroscopic meaning, some components with different mechanisms in the ground water aquifer must contribute and affect to the runoff characteristics in different ways.

When we inspect the flowing regimes from aquifers to the ground surface, different kinds of regimes are observed such as seepage from the unconfined aquifer and leakage from the confined one. Although both of them are flows through porous media, the dominant term in the fundamental equation is different for these components with each other. That is, the diffusion term will be significant in the flow equation for the unconfined component and the pressure term for the confined one. In other words, the flow in the unconfined aquifer is kinematically somewhat similar to that of diffusion process, while the pressure plays an important role in the flow regime in the confined one. Stating again, the seepage occurs when the pressure near the outlet of the aquifer is almost equal to atmospheric one, whereas leakage is due to excess pressure. The schematic representations of these aquifers are given in Fig. I-1.

If we discuss ground water runoff from kinematical view points, different characters between those components may give us a clue for the better understanding of the kinematical characteristics of ground water runoff. In the actual basin, there must exist various and complicated combinations and distributions of aquifers, and they supply the runoff discharge in forms of seepage and/or leakage. The factors affecting the runoff regime in the basin are also of wide divergences, so that it is practically impossible to measure them in details. Therefore, it seems rather worthwhile to hold the runoff phenomena by means of rather simple models.

1-2-2 Runoff Models

For the practical treatment, we have to formulate the mathematical expressions for the ground water runoff. Generally, two kinds of approaches are available for the runoff analyses. The first one is the lumped hydrological system model which expresses the phenomena in watershed grossly. The second one is the distributed model[47)48)] by which the investigators may discuss various behaviour within the basin. It goes without saying that the distributed model is essentially superior to the lumped system model, if sufficient information is available. Since it is very difficult, however, attempts by the distributed model are now still in progress even in the flood hydrology[47)48)].

At the first step of the approach, the author has adopted the lumped models. In the procedure of formulation of the models, attention has been focused how to simplify the models in such way that they represent the major kinematical mechanism of flows through the unconfined and confined aquifers.

In the confined aquifer, the water movement is due to the pressure gradient. Moreover, although the domain of the ground water flow in the confined ground water might be changeable in space and time, the effect of the temporal change of the domain may be reflected to the variation of the pressure according to the equation of continuity and Darcy's law. In other words, in order to define the model, all we have to do is to express the effect of pressure reasonably instead of considering the change of the domain. Giving the kinematical priority to this characteristics, the confined aquifer model of a definite region is connected with the water tank which expresses synthetically the effect of pressure and the change of the domain or the variations of the water storage in the aquifer.

In the flow through the unconfined aquifer, what distinguishes the flow in the unconfined aquifer from that in the confined one, is the process similar to the diffusion one which is originally caused from the existence of free surface of ground water. From this point of view, we may adopt very simple runoff model which points out the diffusion process, instead of the microscopic presentation of the shape of the aquifer and hydraulic quantities.

To define the runoff models, considerations should be made of the basin states as well as the kinematics of the ground water flow. The actual distributions of the aquifer are of course closely related to the vertical and horizontal distributions of the geological and geographical quantities in the regions. It seems, however, that the unconfined aquifers distribute homogeneously in macroscopic meaning all over the regions.

On the other hand, although the greater quantity of confined ground water is distributed in the volcanic rock region, rather small quantity behaves in connection with the runoff discharge, because the confined water is located discontinuously and deeply below the ground surface. That is, there must exist many confined aquifers with various properties. However, if we treat them grossly or pick up the major one aquifer, we may synthesize the kinematical characters into a dynamical model.

The divides of the region in which ground water movement takes place, must variate occasionally according to the changes of hydraulic, hydrologic, geological and meteorological factors. For these problems, Macdonald and Kenyon reported[49)] that the catchment area of ground water flow variates in the range of 0.7-1.33 times of that of the surface flow. Their report concerns with the field researches at the calcareous watershed in England. Moreover, it appears in general that the gradient of ground water surface is almost equal to that of the ground surface in the alluvial and diluvium regions. Consequently, change of the catchment area for the ground water is not taken into considerations in the present paper. For the brief treatment, besides the aquifers assumed homogeneous and isotropic, the horizontal aquifers are treated in order that the attention is focused to the fundamental characteristics of the mechanism of the ground water runoff.

On the basis of the considerations stated above and the kinematical characteristics of the unconfined and confined aquifers, the runoff models as shown in Figs. I-2 and I-3 are formulated for both components in the present paper. Moreover, both of the models are assumed to be of unit width and the effect of basin area will be introduced later. The symbols denote as follows;

- q_u : runoff discharge per unit width of the unconfined model,
- q_c : runoff discharge per unit width of the confined model,
- H_u : depth of water in the unconfined aquifer,
- H_c : water depth in the tank of the confined model,
- k_u : permeability coefficient of the unconfined aquifer,
- k_c : permeability coefficient of the confined aquifer,
- γ_u : porosity of the unconfined aquifer,
- γ_c : porosity of the confined aquifer,
- L_u : length of stratum of the unconfined aquifer,
- L_c : length of stratum of the confined aquifer,
- f, F : cross sectional area per unit width of confined stratum and of a tank at the upstream end, respectively,
- r_e : recharge intensity to the unit area of the unconfined ground water table,
- R_c : recharge intensity to the confined component,
- x : distance along the strata downwards positive.

Chapter 2 RECESSION CHARACTERISTICS OF GROUND WATER RUNOFF

2-1 Fundamental Equation of Recession

2-1-1 Confined Component

For the recession state, we have the following equation of continuity for the water movement in the model in Fig. I-3,

$$f \cdot v_c = - F \frac{dH_c}{dt} , \tag{I-1}$$

in which v_c is the filter velocity through the aquifer.

Employing Bernoulli's equation for the flow between the points A and B in Fig. I-3, we may write equation of motion as follows:

$$H_c + \frac{1}{2g} \left(\frac{dH_c}{dt} \right)^2 = \frac{1}{2g} v_c^2 + h_B . \tag{I-2}$$

Moreover, we gain for the flow pipe from the point B to C,

$$\frac{v_c^2}{2g} + h_l = \frac{v_c^2}{2g} + h_B , \tag{I-3}$$

in which h_l indicates the head loss in the confined layer and h_B the water head at the point B. Equation (I-2) is reduced to

$$h_B = H_c + \frac{1}{2g} \left\{ \left(\frac{dH_c}{dt} \right)^2 - v_c^2 \right\}, \tag{I-4}$$

and from Eq.(I-3), we have

$$h_B = h_l . \tag{I-5}$$

On the other hand, we may estimate the head loss h_l in the layer by means of Darcy's law as follows;

$$h_l = \frac{L_c}{k_c} v_c . \tag{I-6}$$

Hence, Eqs.(I-1) ∿ (I-4) are reduced to

$$H_c = h_B + \frac{1}{2g}\left\{v_c^2 + \left(\frac{dH_c}{dt}\right)^2\right\}$$

$$= \frac{L_c}{k_c} v_c + \frac{1}{2g}\left(1 - \frac{f^2}{F^2}\right) v_c^2 . \tag{I-7}$$

Differentiating this equation with respect to t, we obtain

$$\frac{dH_c}{dt} = -\frac{f}{F} v_c = \frac{L_c}{k_c}\frac{dv_c}{dt} + \frac{v_c}{g}\left(1 - \frac{f^2}{F^2}\right)\frac{d(fv_c)}{dt} ,$$

or

$$-\frac{fv_c}{F} = \frac{L_c}{k_c f}\frac{d(fv_c)}{dt} + \frac{fv_c}{f^2 g}\left(1 - \frac{f^2}{F^2}\right)\frac{d(fv_c)}{dt} . \tag{I-8}$$

This equation may be further rewritten into the following form incorporating with the relationship $f \cdot v_c = q_c$.

$$1 = -\frac{L_c F}{k_c f}\frac{1}{q_c}\frac{dq_c}{dt} - \frac{F}{f^2 g}\left(1 - \frac{f^2}{F^2}\right)\frac{dq_c}{dt} . \tag{I-9}$$

Thereafter, by integrating this equation over any time interval, the solution is resulted in

$$-\frac{L_c F}{k_c f} \log q_c - \frac{F}{f^2 g}\left(1 - \frac{f^2}{F^2}\right) q_c = t + C . \tag{I-10}$$

For the initial condition

$$q_c = q_{co} \quad \text{at} \quad t = 0 , \tag{I-11}$$

at last, we obtain the solution for the recession state of the confined component as follows.

$$- \frac{L_c F}{k_c f} \log q_c - \frac{F}{f^2 g}\left(1 - \frac{f^2}{F^2}\right) q_c$$

$$= t - \frac{L_c F}{k_c f} \log q_{co} - \frac{F}{f^2 g}\left(1 - \frac{f^2}{F^2}\right) q_{co} \quad . \qquad \text{(I-12)}$$

2-1-2 Unconfined Component

2-1-2-1 Fundamental Equation

For the unconfined component, to prescribe the water head is the first task for the formulation of the fundamental equation because of the existence of the free surface. For the brief treatment, we assume that the slope of the free surface is satisfactorily gentle and Dupuit-Forchheimer's assumption is valid.

Remembering recharge to the ground water table is zero in the recession period, Darcy's law and the equation of continuity for the model in Fig. I-2 are written as

$$v_u = - \frac{\partial}{\partial x}\left\{ k_u \cdot H_u(x,t) \right\} \quad , \qquad \text{(I-13)}$$

and

$$\frac{\partial(\gamma_u H_u)}{\partial t} = - \frac{\partial}{\partial x}\left\{ H_u(x,t) \cdot v_u \right\} \quad , \qquad \text{(I-14)}$$

respectively.

In cooperation of these equations, the fundamental equation becomes as follows;

$$\frac{\partial(\gamma_u H_u)}{\partial t} = \frac{\partial}{\partial x}\left\{ H_u \cdot \frac{\partial}{\partial x}(k_u H_u) \right\} . \qquad \text{(I-15)}$$

We further assume that the permeability k_u and porosity γ_u are constant and the slope of the free surface is very gentle, hence the term $k_u\left(\frac{\partial H_u}{\partial x}\right)^2$ in the equation may be ignored as higher infinitesimal comparing with the others. The fundamental equation thus can be reduced to the simple differential equation;

$$\frac{\partial H_u}{\partial t} = \beta H_u \cdot \frac{\partial^2 H_u}{\partial x^2} \quad , \qquad \text{(I-16)}$$

in which

$$\beta = \frac{k_u}{\gamma_u} \quad . \qquad \text{(I-17)}$$

2-1-2-2 Boundary and Initial Conditions

To find the appropriate solution, the boundary and initial conditions should be prescribed for the differential equation. However, there are difficulties in defining them for an actual river runoff. In an actual basin, the ground water and channel water interact* with each other. In other words, the boundary conditions for each of them cannot be prescribed mathematically unless the behaviour of the other is defined.

In order to circumvent the difficulty, although it is the only way to observe the actual states of flow in the watershed, we will again face another difficulty because of the small number of actual data.

On the other hand, the macroscopic consideration on the significant phenomena in the runoff process in low flow stage, leads that the ground water runoff may dominantly affect the behaviour of stream water but reversely is slightly controlled by stream water, especially in upstream mountainous regions. Remembering the purpose of this research, the author has focused his attention to the gentle variation of ground water runoff and has not taken the effect of stream water into consideration. Thus, in this part, the following boundary and initial conditions are adopted;

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* The detail discussion will be made in Part II in the present paper.

$$H_u(0,0) = H_{uo} , \tag{I-18a}$$

$$H_u(L_u,0) = h_{uo} , \tag{I-18b}$$

$$\left.\frac{\partial H_u}{\partial x}\right|_{x=0} = 0 . \tag{I-18c}$$

Validity of these conditions should be left to later discussions and examination on the analyzed data which are obtained in actual basins through this method.

2-1-2-3 Solution of Equation

Now our problem has been reduced to solve Eq.(I-16) under the conditions (I-18). If $H_u(x,t)$ is variable separable

$$H_u(x,t) = T(t)\cdot X(x),$$

the original equation (I-16) may be written as

$$\frac{1}{T^2}\frac{dT}{dt} = \frac{d^2X}{dx^2} = -\lambda < 0 ,$$

in which λ is an Eigen value for the conditions (I-18). Integrating the above equation, we obtain

$$H_u = \frac{1}{\lambda t + C_1}\left\{-\frac{\lambda}{2\beta}x^2 + C_2 x + C_3\right\} , \tag{I-19}$$

in which C_1, C_2 and C_3 are integral constants. Then with respect to the conditions (I-18a,b) the solution becomes,

$$H_u(x,t) = \frac{1}{\lambda t + 1}\left\{-\frac{\lambda}{2\beta}x^2 + \frac{h_{uo} - H_{uo}}{L_u}x + \frac{\lambda}{2\beta}L_u x + H_{uo}\right\} , \tag{I-20}$$

in which λ is defined by the boundary condition (I-18c) like

$$\lambda = \frac{2\beta}{L_u^{\,2}}(H_{uo} - h_{uo}) . \tag{I-21}$$

Arranging the equation, finally we obtain the solution for the behaviour of the unconfined component during recession state as follows;

$$H_u(x,t) = \frac{1}{\frac{2\beta}{L_u^2}(H_{uo} - h_{uo})t + 1}\left\{-\frac{H_{uo} - h_{uo}}{L_u^2}x^2 + H_{uo}\right\}. \qquad \text{(I-22)}$$

The variation of discharge q_u coming from the unconfined aquifer may be estimated by use of the above solution per unit width of the model, that is,

$$q_u(t) = k_u H_u \left.\frac{\partial H_u}{\partial x}\right|_{x=L_u} = \frac{1}{\left\{\frac{2\beta}{L_u^2}(H_{uo} - h_{uo})t + 1\right\}^2}\cdot k_u H_u \cdot \left.\frac{\partial H_u}{\partial x}\right|_{x=L_u}$$

i.e. (I-23)

$$q_u(t) = \frac{q_{uo}}{(at+1)^2}\ , \qquad \text{(I-24)}$$

in which

$$a = \frac{2\beta}{L_u^2}(H_{uo} - h_{uo}) \qquad \text{(I-25)}$$

and q_{uo} indicates the initial discharge out of the unit width of the model and is given by the following expression.

$$q_{uo} = k_u H_u \left.\frac{\partial H_u}{\partial x}\right|_{\substack{x=L_u \\ t=0}}. \qquad \text{(I-26)}$$

2-2 Recession Equations and the Roles of Components in Runoff Process

2-2-1 Recession Equations

As we have seen, the recession equations of runoff discharge are given by Eqs.(I-24) and (I-10) with respect to the unconfined and confined components, respectively.

$$q_u(t) = \frac{q_{uo}}{(\alpha t+1)^2} \quad , \tag{I-24}$$

$$-\frac{L_c F}{k_c f} \log q_c - \frac{F}{f^2 g}\left(1 - \frac{f^2}{F^2}\right) q_c = t + C \ . \tag{I-10}$$

Eq.(I-24) shows the variation of discharge from the unconfined component explicitly in terms of time t, on the contrary, Eq.(I-10) includes the time t implicitly. For the convenience of treatment, let us rewrite Eq.(I-10) as an explicit function of time t.

If we use km-day units for the length and time, the permeability coefficient is of the order of about $10^{-6} \sim 10^{-7}$, the acceleration of gravity g of 10^8. Since $\frac{FL_c}{f}$ and $\frac{F^2}{f}$ are nearly of the same order with each other, the second term on the left hand side of Eq.(I-10) may be ignored by comparing with the first term. The recession equation of the confined component may be thus rewritten approximately into the exponential form. Figure I-4 shows a numerical example of the solution Eq.(I-12) plotted on the semi-logarithmic paper. From the figure, Eq. (I-12) i.e. Eq.(I-10) apparently asymptotes towards the dotted straight line

$$-\frac{FL_c}{fk_c} \log q_c = t + C \ . \tag{I-27}$$

The value at the intersection of the dotted line with q-axis $(t=0)$ is nearly equal to the initial discharge of the confined component. Consequently, the variation of the runoff discharge of the confined component becomes approximately

$$q_c(t) = q_{co} \cdot exp(-\alpha t) \quad , \tag{I-28}$$

in which q_{co} denotes the initial discharge of the confined component and

$$\alpha = \frac{fk_c}{FL_c} \ . \tag{I-29}$$

Equations (I-24) and (I-28) express the recession curves of the both components per unit width of the models. In order to use these equations in the actual analyses, the factors which represent the basin area should be taken into consideration and the equation should be rewritten for the actual discharge. Write the actual river discharge due to the ground water runoff Q and the confined and unconfined components in actual basins Q_c and Q_u, respectively. Indicate the initial discharges of both components by the subscript 0. Further introducing new factors B_u and B_c which represent the width of the outlets, we may write the discharge as follows;

$$Q_u = B_u \cdot q_u \,, \qquad Q_c = B_c \cdot q_c \,. \tag{I-30}$$

Therefore, the recession curves of the two components are written as;

$$Q_u = \frac{Q_{uo}}{(at+1)^2} \,, \tag{I-31}$$

and

$$Q_c = Q_{co} \cdot exp(-\alpha t) \,, \tag{I-32}$$

in which constants a and α are given by Eqs.(I-25) and (I-29).

On the actual runoff, there still exists another question as to how the two components are combined. For the simplicity, in the present paper, the author prescribes the actual discharge as the summation of both components, that is,

$$Q(t) = Q_u(t) + Q_c(t) = \frac{Q_{uo}}{(at+1)^2} + Q_{co} \cdot exp(-\alpha t) \,, \tag{I-33}$$

where constants a and α represent synthetically the recession state of the runoff discharge. More detailed discussions on these factors and other relationships derived theoretically will be made later.

2-2-2 Roles of Components in Runoff Process

The ratio of the confined component to the unconfined one tends to zero when the time t tends to infinity.

$$\lim_{t\to\infty} \frac{Q_c(t)}{Q_u(t)} = \lim_{t\to\infty} \frac{Q_{co}}{Q_{uo}} \cdot exp(-\alpha t)\cdot(\alpha t+1)^2 = 0. \qquad \text{(I-34)}$$

This expression means that the confined component decreases faster than the unconfined one with increasing time. So that, if the no-rainfall days continue for an excessive period of certain extents, the river discharge is supplied by only the unconfined component because the confined one practically vanishes in a few days. That is, it should be emphasized that the unconfined component plays a dominant role in the runoff process during the prolonged period. Furthermore it will be shown in the next chapter that the unconfined component will supply a relatively small quantity of river discharge even in the early stage of the recession. On the contrary, although the confined component decreases fast and vanishes only in a few days, later discussion will clarify that it supplies considerable amount of discharge in the early stage of the recession.

2-3 Recession Characteristics of Ground Water Runoff

2-3-1 Confined Component

The recession equation of the runoff discharge from the confined aquifer is given by Eq.(I-32),

$$Q_c(t) = Q_{co}\cdot exp(-\alpha t) , \qquad \text{(I-32)}$$

in which

$$\alpha = \frac{fk_c}{FL_c} . \qquad \text{(I-29)}$$

It should be noted here that the equation derived with respect to the flow under pressure gradient results the exponential recession curve. That means, a component with the exponential recession exists within a basin.

As seen from Eq.(I-32), the recession state is synthetically governed by the initial discharge Q_{co} and the recession factor α. Since detail discussion about the initial discharge is left to the next chapter concerning variation of discharge due to rainfall, it is only stated here that the initial discharge may reach at considerable amount in the early stage of the recession.

It is the recession factor α which defines the recession rate of the confined component. That is, the factor indicates synthetically the recession state in a whole basin. The factor, as seen from Eq.(I-29), consists of the geological and geographical quantities F, f and L_c and the hydraulic quantity k_c within a basin. Moreover, it should be noted that the recession factor is proportional to permeability and to reciprocal of the length of aquifer but independent on porosity of the aquifer. In general, we may consider all of these quantities as constants in a basin.

Conclusively, we may state that the recession factor α is an invariant in a specified basin for any occasion of recession regardless of the initial state of the basin. Moreover, it should be emphasized that we may write the recession factor in terms of geological, topological and hydraulic quantities in a basin.

In a basin for which the recession factor α is an unique constant, the discharge should decrease along the same curve, so-called normal recession curve, regardless of any initial state.

The storage of this component may be derived as proportional to the runoff discharge at any time by simple calculation.

2-3-2 Unconfined Component

The recession curve of the unconfined component is given by Eqs. (I-31) and (I-25).

$$Q_u(t) = \frac{Q_{uo}}{(at + 1)^2} , \qquad \text{(I-31)}$$

$$a = \frac{2\beta}{L_u^2} (H_{uo} - h_{uo}) . \qquad \text{(I-25)}$$

These equations show that the runoff component which is kinematically characterized by diffusion process, decreases according to the fractional function of t during no rainfall period.

2-3-2-1 Recession Factor K and Normal Recession Curve

The recession features of the unconfined component are governed by the initial discharge Q_{uo} and recession factor a given by Eq.(I-25). Discussion on the initial discharge Q_{uo} itself will be made later in Chapter 3 because it is closely connected with increment of discharge due to the previous rainfall. So, the recession factor a is only treated in this article.

Since what form the recession factor a are not only the geological and hydraulic quantities such as the length of the unconfined aquifer L_u, the porosity γ_u and the permeability coefficient k_u, but also the initial state H_{uo} and h_{uo}, the recession factor a varies with the initial state of the water within a basin and is not an unique constant even for a specified basin.

On the other hand, the initial discharge Q_{uo} must be related to the initial water depth in the aquifer H_{uo} and h_{uo}. Therefore, the recession factor a may be expressed as a function of the initial discharge through the quantities H_{uo} and h_{uo} as well as the geological quantities. If the discussion is restricted to the recession factor a in a certain basin, as the geological quantities and the permeability seem to be constant, the recession factor a should be a function of the initial discharge Q_{uo} alone.

To derive the relationship between the recession factor a and the initial discharge Q_{uo}, we assume that there exists a normal recession curve also for the unconfined component as well as the confined one. That is to say, regarding the schematic representation of recession curve shown in Fig. I-5, if the discharge of which initial value is at point A will decrease along the curve ABCD, the other recession which starts from the point B is assumed to decrease along the curve BCD, and further any recession is also assumed to follow the same curve.

Write the discharge at time $t=0$, $t=t_1$ and $t=t_1+t_2$ as Q_{uo}, Q_{u1} and Q_{u2}, respectively, then we have

$$Q_{u1} = \frac{Q_{uo}}{(at_1 + 1)^2} ,$$

$$Q_{u2} = \frac{Q_{uo}}{\{a(t_1+t_2) + 1\}^2} .$$

For the initial discharge Q_{u1}, the discharge Q_{u2} at $t=t_1+t_2$ may be written by means of the above assumption,

$$Q_{u2} = \frac{Q_{u1}}{(a't_2 + 1)^2} ,$$

in which a' is the recession factor for the time axis of its origin at $t=t_1$.

The condition that these equations should be satisfied for arbitrary time t, yields the relationship

$$\frac{a}{a'} = \frac{\sqrt{Q_{uo}}}{\sqrt{Q_{u1}}} .$$

In other words, if there exists a normal recession curve regardless of any initial discharge, the relationship between the initial discharge and the recession factor a is obtained as

$$a = K \cdot \sqrt{Q_{uo}} \quad , \tag{I-35}$$

in which K is an unique coefficient.

The new factor K, which represents the recession rate instead of the factor a, should be constant for a basin with a normal recession curve but not for the basin for which the recession rate varies occasionally. It may be thus concluded that the use of the new factor K as a measure of recession characteristics is advantageous over the recession factor a.

2-3-2-2 Significance of Recession Factor K

Let us consider the properties of the recession factor K. The solution for the unconfined component Eq.(I-22) leads the following relationship with help of Eq.(I-26),

$$h_{uo} = \sqrt{\frac{2L_u}{k_u}} \cdot \frac{1}{\sqrt{m - 1}} \cdot \sqrt{q_{uo}} \quad , \tag{I-36}$$

in which $m=H_{uo}/h_{uo}$. With cooperation of Eqs.(I-24), (I-25) and (I-36), the recession factor a is then rewritten as follows:

$$a = \frac{2\sqrt{2k_u}}{\gamma_u L_u \sqrt{L_u} \sqrt{B_u}} \sqrt{m - 1} \sqrt{Q_{uo}} \quad . \tag{I-37}$$

Therefore, the recession factor K is described as;

$$K = \frac{2\sqrt{2k_u}}{\gamma_u L_u \sqrt{L_u} \sqrt{B_u}} \sqrt{m - 1} \ . \tag{I-38}$$

This is an expression of the recession factor K in terms of the geological and hydraulic factors. As seen from the expression, K

consists of the geological and topological factors γ_u, L_u, B_u, the hydraulic properties k_u and the initial distributions of water within a basin $m=H_{uo}/h_{uo}$. In other words, K value expresses the basin characteristics in the macroscopic meaning, involving the contributions of the individual factors to the recession state.

Since the geological and topological factors and the macroscopic permeability coefficient for an entire basin are apparently invariant in a basin, the factor m, which is closely connected with the initial distributions of water within a basin, should be constant for a basin with the unique recession factor K. That is, if the water distributions in the basin do not change in time or do only slightly, the unique recession factor K will appear for any occasions of the recession.

2-4 Procedure of Hydrograph Analyses

2-4-1 Watersheds and Hydrologic Data

The forementioned method has been applied and examined for several actual river basins in Japan. Some of them are located in the Kansai District and the others in central Japan. Although the detail descriptions of these basins are omitted here, the catchment areas are summarized in Table I-1 and the brief features are shown in Fig. I-6.

Daily discharges and daily rainfalls are available as the fundamental hydrologic data for each watershed, while others such as the distributions of the geophysical or geological features and others are also used, when required.

For the recession analyses, to clarify the pure characteristics as well as we can, it is hoped that the prolonged recession limb is treated for the hydrograph analyses. Therefore, in this chapter, the author has analyzed several recession limbs so chosen from the sequences of the daily discharges during several years up to ten years. The recession limbs analyzed here are listed in Table I-2, in which the day at which the peak discharge occurs and the recession state follows, is also shown together with the peak discharge.

2-4-2 Separation of Components

The characteristics of the unconfined component will appear in the long tail of the hydrograph, since the stream discharge in this situation consists of the unconfined component alone. This fact enables us to separate the unconfined component and the confined one from the actual hydrograph.

The recession equation (I-31) for the unconfined component is rewritten as follows:

$$\sqrt{\frac{1}{Q_u}} = a\sqrt{\frac{1}{Q_{uo}}}\,t + \sqrt{\frac{1}{Q_{uo}}} \quad . \tag{I-39}$$

Since the quantities Q_{uo} and a are constant for a recession state, this expression means a straight line on the $(t, \sqrt{\frac{1}{Q_u}})$-plane as shown in Fig. I-7. Therefore, the unconfined component on each day should be plotted along a straight line in the recession state. In other words, the long tail of the actual hydrograph should be plotted straight on $(t, \sqrt{\frac{1}{Q_u}})$-plane. In the practical procedure, with use of the intersection of this straight line with $\sqrt{\frac{1}{Q_u}}$ axis $(t=0)$ and the gradient which corresponds to the K-value in Eq.(I-35), we may estimate the initial discharge Q_{uo} and a for each recession state.

The differences between the straight line and the plots of daily discharges are the confined component. We may also define the recession factor α and the initial discharge of the confined component Q_{co}, plotting this component on the semi-logarithmic paper. Schematic explanation of these procedures are given in Figs. I-8 and I-9.

For a practical treatment, the initial time should be defined for each recession. In view of the complexity of the behaviour of ground water runoff, it is doubtful that the initial day of the recession state would be defined simply. However, in macroscopic meaning, based on the previous knowledge[50)51)52)] on the hydrograph analysis, the lag time from ceasing of rainfall until occurrence of the peak of the ground water runoff is defined as follows empirically for each watershed concerned.

River	Watershed	Lag time
Yura River	Arakura	1 day
	Kado	2 days
Yoshino River	Terao	1 day
Kako River	Kunikane	2 days
Nagara River	Takasu	1 day
	Tsurugi	1 day
	Sugihara	1 day
	Kamita	2 days
Kizu River	Inooka	2 days
	Kamo	2 days

2-5 Actual Evaluation of Confined Component

2-5-1 Recession Factor α

The α-values obtained are summarized in Table I-3. The values are also plotted versus the corresponding initial discharges Q_{co} in Fig. I-10. The straight line drawn in the figure indicates the mean value α_o for the basin. The recession factor for Takasu gauging station with very small catchment area, has never been defined, because the confined component decreases so fast that we cannot hold clear characteristics by such small number of data.

In the figure, we can see the concentrative distribution of α values around the mean value. In other words, the α values for these basins seem to be almost constant regardless the initial state. This fact proves the presentation aforesaid of the factor α. Moreover, it is satisfactory to consider the physical property of the recession factor α which is an unique constant for the basin, as to represent the basin characteristics in terms of the geographical and hydraulic properties in the basin as a lumped system.

If we carefully observe the plot in Fig. I-10, for relatively large basins as the Kado gauging station, the values scatter considerably and we can not define the unique α value for the basin. The causes of the scattering in the large basin, which are unsolved yet in details, may be understood as the temporal and spatial nonhomogeneity in the distributions of various factors within the basins, such as the variation of spatial rainfall distributions and other governing factors. Nevertheless, as will be shown by the normal recession curve in Fig. I-11, it appears that the scattering of the recession factors for each case found for Kado gauging station has not a large effect on the normal recession curve in the macroscopic meaning.

2-5-2 Normal Recession Curve

If we move the sequences of the recession discharge for this component along time axis and trace the various recession segments in such position as to overlap each others, then we may draw the normal recession curve. In the figure I-11, discharge are so plotted for each recession state together with the recession curves corresponding to the mean recession factor α_o.

Apparently, at the basin for which the recession factor α is invariant, as seen from Fig. I-11, the discharge decreases along the normal recession curve given by Eq.(I-32). Especially for the gauging stations Terao, Tsurugi and Kamita, very slight scatterings of the plots are found. Although we could not define the unique recession factor α for Kado and Kunikane gauging stations, great discrepancy between the plotting values and the normal recession curve is not observed in Fig. I-11, especially for the initial stage of the recession.

Consequently, we may recognize that there actually exists the confined runoff component which decreases along the normal recession curve in small basins and also in the basins with rather large catchment area in spite of the special exceptions. It is also noted from Fig. I-11 that the confined component supplies a considerable amount of runoff discharge in the early stage of the recession but it decreases very fast and vanishes only in a few days.

2-5-3 Relationship between Recession Factor α and Catchment Area

Actually many phenomena take place and great number of factors are active within a basin with various distributions in time and space. Strictly speaking, it is impossible to discuss the recession characteristics in connection with all of these factors. However, since the recession factors α are closely related to the geological, geographical and hydraulic factors within the basin, the differences of these factors will result in differences of recession states.

We consider the basin area as one of the most important quantities which represent the topological or geographical properties of the basin. On the confined component, in general, the recession factor α becomes smaller with the increasing basin area as seen from Table I-3 and Fig. I-10.

Of those which constitute the recession factor α, what has direct contact with the size of the basin is the length of the confined aquifer L_c, since others are rather conceptual ones regarding the geological and hydraulic properties. In this part, remembering that our purpose is to make clear the characteristics of the basin as a lumped system, we temporarily assume the following relations between the length L_c of the confined aquifer and the catchment area A, as

$$L_c \propto \sqrt{A} \quad . \tag{I-40}$$

Then, Eq.(I-29) leads,

$$\alpha \propto \frac{1}{\sqrt{A}} \quad . \tag{I-41}$$

That is, inverse proportionality between the recession factor and the square root of the basin area may be expected.

Figure I-12 shows the α_o-value versus square root of basin area. In the figure, the plots may be classified into two groups, the first group for the River Nagara and the second for the other basins all of which are located in the Kansai District. For all the plots, we cannot hold the clear relationship as Eq.(I-41), but for the each group we can see the relationship somehow similar to Eq.(I-41). It is very interesting that all watersheds in the Kansai District belong to one group and the River Nagara basins in central Japan to the other.

Up to date, the exponential index of the recession state has been often estimated in numerous basins. And it has been also stated often that the exponential index, in general, becomes smaller with the increasing of basin area. In this meaning, the relationship (I-41) which is not of course sufficient is interesting to explain the tendency.

Of course, the other quantities such as f, F and k_c in Eq.(I-29) may also affect the recession factor considerably. The fact mentioned above might suggest the great discrepancy between sets of these quantities in both groups.

2-6 Actual Evaluation of Unconfined Component

2-6-1 Recession Factor K

The recession factors K obtained through the recession analyses are summarized in Table I-4 together with the recession factors a and their corresponding initial discharges Q_{uo}. In the table, K_o indicates the mean value of K for each basin.

Figure I-13 illustrates the obtained recession factor K for each basin versus the corresponding initial discharge of the unconfined component . Though the plots scatter in the gauging stations Kunikane and Kado with rather large catchment area, we can see generally the concentrative distribution of K-values around the mean value for each basin. Especially the recession factors for the River Nagara, i.e. Takasu, Tsurugi, Kamita and Sugihara gauging stations, seem to be constant with imperceptible scattering. At Arakura station, we may also observe the constant K-value except two points which will be treated later.

The fact observed above proves that the recession characteristics of the unconfined component may be actually expressed by the recession factor K for these basins. It will be also understood that the factor, as seen from Eq.(I-38), expresses synthetically the basin characteristics in terms of the geological and hydrological quantities within the watershed.

The basins with the unique recession factor K in the above discussion are mountainous regions with rather small basin area. Furthermore, the normal recession curve, as discussed later, suggests that the K-value is also an invariant at the very large basins such as Kamo and Inooka gauging stations in Kizu River Basin.

On the other hand, for the gauging stations with basins of middle size such as Kado in the River Yura and Kunikane in the River Kako, considerable scatterings are observed in the recession factor K in Fig. I-13. The causes of scattering are considered due to the change or variations of rainfall distributions in space and those of the hydrologic quantities for each recession state. On the scattering of K-value, discussion will be also made in details in Part II.

2-6-2 Normal Recession Curve

In the case that the recession factor K is an unique constant for a basin, the unconfined component should decrease along the normal recession curve in disregard of any initial discharge as well as the confined component. The normal recession curve becomes a straight line with the unique gradient K in $(t, \sqrt{\frac{1}{Q_u}})$-plane as we have seen in Subsection 2-4-2. Figure I-14 illustrates some examples of the normal recession curve. In the figure, the original daily discharges observed are plotted and the straight line is evaluated by the mean value of recession factor K_o. From the figure, it is obvious that the plots asymptote towards the straight line and the points for individual recession state are plotted almost straightly after the lapse of several days from the initial time of recession. This fact proves that the stream discharge consists of the unconfined component alone in the prolonged period of recession. On the other hand, since the difference between the plots and the straight line in the early stage of recession corresponds to the confined component, it is also understood how fast the confined component decreases comparing with the unconfined one.

The most important point, however, is that the plots for any recession state at the gauging stations with definite K-value in Fig. I-14 also asymptote towards only one-straight line no matter how much quantity of discharge runs off from the unconfined aquifer. In other words, the straight line is considered as the normal recession curve for the unconfined component. In the figure, it is also noted that the definite normal recession curves are observed for Kamo and Inooka gauging stations in the River Kizu in spite of the very large catchment area.

Regarding the basins of middle size such as Kado and Kunikane, the recession curves for each occasion do not asymptote to the same line. But the existence of the unconfined component is clearly recognized in Fig. I-14 for each recession, even though the recession state and the factor K are changeable due to the circumstances in these basins. For the intuitive convenience, some examples of the analyzed recession curve of the unconfined component are also illustrated in (t, Q_u)-plane in Fig. I-15.

2-6-3 Variation of Recession Factor K

To discuss the recession factor K, we consider the relationship between the factor K and the catchment area as the major geographical factor of the basin.

The K-value is expressed by Eq.(I-38) in terms of the geological and hydraulic factors and initial condition, that is,

$$K = \frac{2\sqrt{2k_u}}{\gamma_u L_u \sqrt{L_u}\sqrt{B_u}} \sqrt{m - 1} \quad \text{(I-38)}$$

Of those which form the recession factor K, the quantities directly correlated with basin area are L_u and B_u. Although it might be an unrealistic assumption to prescribe the definite relations between the factors B_u, L_u and the catchment area A, we temporarily assume

$$L_u \propto \sqrt{A} \quad \text{and} \quad B_u \propto \sqrt{A} \,. \quad \text{(I-42)}$$

Then, Eq.(I-38) leads the following relations

$$K \propto A^{-1} \,. \quad \text{(I-43)}$$

The mean recession factor K_o obtained for each basin is plotted in Fig. I-16 versus the basin area. As seen from the figure, we can not derive the definite relationship among all the plots. However, the plots show similar tendency as given by Eq.(I-43) for certain ranges of the catchment area, say, for the group of the very small basins of which catchment area are smaller than about 100 km^2, the second group of the basins with 150 ∿ 700 km^2 and the third group for the rather large basins.

The reason why such tendency appears for these groups individually, is vague in the present knowledge. But the fact might show the difference among the properties of the watersheds as lumped systems in these three groups. According to the phenomenological consideration of the usual watershed in Japan, very small basin consists of small valley and steep mountainous regions but the basins over about 150 km^2 may cover the various elements such as valleys, tributaries, cup-shaped regions and others. Furthermore, the large basins involve the plain region in a considerable fraction to the whole area. In other words, there are great differences in basin characteristics among these groups.

In addition, a large basin involves several number of small sub-basins with tributaries. Therefore, it seems that the kinematical characteristics of runoff may depend on a law within a certain range of catchment area but the law may be converted, if the basin area exceeds the range.

As a matter of course, it is rather insufficient to hold the properties of the recession factor K in terms of the catchment area alone. However, as far as the author had analyzed, we might understand and explain the tendency of the plots for each group with help of Eq. (I-43) as discussed above. Moreover the difference among the groups may be understood due to the transitions of the hydrologic properties of the basin as a lumped system.

Regarding the effect of the basin area on the recession factor K, it should be repronounced that we can define the unique recession factor K in small basins and certain basins with very large area but the factor scatter considerably in the basins of middle size.

The phenomenological consideration implies that the temporal non-homogeneity of several factors in the basin causes the scattering of the K-value in the basin of middle size. For example, in the gentle recession states K'-1 and K'-2 as seen from Figs. I-13, I-14 and I-15 the rainfall concentrates in the upstream region of the basin, while it distribute homogeneously all over the basin in the other cases. That is, it may appear that the homogeneity of hydrologic quantities leads the definite recession factor in the small basin and the statistical cancellation results in the definite value in the very large basins. On these problems, the detail discussion will be made theoretically in Part II.

2-7 Conclusion

In this chapter, as the first step of the approach, the recession characteristics, which represent the elemental basin characteristics, have been discussed by means of the hydrodynamic models. The results obtained through the research in this chapter, may be summarized as follows;

i) Behaviour of ground water runoff is characterized by mechanical characteristics of the runoff components coming from the unconfined and the confined aquifers.

ii) According to the theoretical discussion, it has been made clear that the unconfined component plays a dominant role in the runoff process in a prolonged period and supplies the stream discharge unceasingly. On the contrary, the confined component decreases very fast and ceases in a few days, although it supplies much quantity of discharge in the early stage of recession.

iii) The recession states of the runoff discharge for both components are theoretically derived and examined by field data. That is, the recession curve for the unconfined component is expressed by the fractional function with respect to time t, on the other hand, the exponential function gives the recession state of the confined one.

iv) Physical significances of several parameters which represent the basin characteristics, have been clarified in terms of the geophysical and topological factors within the basin.

v) Several parameters α and K for the confined and the unconfined components are recognized as invariants in each basin, especially in small mountainous regions. In very large basins, we have also defined the invariant constant for the K-value. In basins of middle size, however, the recession factor K scatters to some extent.

vi) As the results described above, the clear normal recession curves are recognized for both of the components in large basins as well as small ones with special exceptions.

vii) Moreover, effects of several factors on the recession factors are also discussed by semi-theoretical procedure with help of the empirical considerations.

viii) From the results described here, it may be concluded that the analytical method focusing to the hydrodynamical process is available and useful for the better understanding of mechanism of ground water runoff.

Chapter 3 VARIATION OF GROUND WATER RUNOFF DUE TO RAINFALL

3-1 Basic Remarks

In Chapter 2, we have clarified that the recession state of ground water runoff is characterized by those of the unconfined and confined components. The author has tackled[8)9)] the next problems as to how the ground water runoff variates or rises due to the rainfall. This problem, however, is actually very difficult since many factors are involved in this process rather than the recession state. As we have discussed in Section 1-2, the infiltration through the zone of aeration takes place and furthermore the direct runoff occurs simultaneously within the watershed.

Infiltration plays an important role as the boundary phenomena in the hydrologic cycle. That is to say, it divides the rain-water into two runoff components, say, the direct runoff (surface runoff and inter-flow runoff) and the indirect runoff (ground water runoff). The research works on the infiltration have been also promoted in the flood hydrology[15)50)] as well as other hydrological activities. Therefore, many researches[51)] have aimed to treat the problems as to how much fraction of rainfall becomes "the loss" for the direct runoff. On the other hand, if we consider the phenomena with respect to the recharge rate to the ground water table, it appears that the infiltration process through the zone of aeration is very significant. Generally speaking, this process is of the vertical infiltration through the unsaturated porous media under coexistence of the three phases. Many investigations[51)52)53)54)55)] have been also made on the unsaturated infiltration in various research fields. The author has also carried out the experimental research works[56)57)] especially on the roles of void air in the phenomena and clarified the special characteristics on the behaviour of infiltrating water, void air and their interactions. However, there are still many problems to be solved, and an extention of these researches to the explanations of basinwide problems is beyond our present knowledge. The only way to circumvent this difficulty is to consider as a whole the bulk properties of the infiltration with partial help of the research works from many points of view.

The second problem we will encounter with respect to the rising state of ground water runoff due to rainfall is that the direct runoff takes place simultaneously within the basin. In other words, the existence of the direct runoff conceals almost all characteristics and factors of the ground water runoff by the dominant quantities of the direct runoff. Therefore, it is impossible to investigate the variation of ground water runoff due to rainfall, unless we treat it on the foundation of the bulk characteristics interpolated and extrapolated by the essential characteristics of the mechanism of the ground water runoff.

Reviewing the discussions made in Chapter 2 suggests that the variation of ground water runoff also reflects the difference of mechanisms of the unconfined and confined components. That is, the mechanisms of these components may form the foundation for the investigation of the variation of ground water runoff too. If we study the problems in this chapter on that ground, the validity will remain on the essential properties though several complex phenomena which are treated temporarily here, may be solved by further examination in the future.

3-2 Fundamental Equation

3-2-1 Confined Component

Rising of runoff discharge occurs due to recharge of rain-water to the ground water table. Let us write R_c for the constant recharge intensity to the water tank of the confined runoff model in Fig. I-3, then we have the following equations for the water movement through the confined aquifer.

$$v_c = \frac{k_c \cdot h_l}{L_c} , \tag{I-44}$$

$$H_c + \frac{1}{2g}\left(\frac{dH_c}{dt}\right)^2 = h_l + \frac{v_c{}^2}{2g} , \tag{I-45}$$

$$F \frac{dH_c}{dt} = R_c - f \cdot v_c . \tag{I-46}$$

In Eq.(I-45), we may ignore the term $\left(\frac{dH_c}{dt}\right)^2$ in comparison with the others as well as in Section 2-1. From these equations, simple calculation leads the following equation with respect to the filter velocity v_c, that is,

$$F\left(\frac{L_c}{k_c} + \frac{v_c}{g}\right)\frac{dv_c}{dt} = R_c - f \cdot v_c . \tag{I-47}$$

Then, we obtain the solution for the discharge $q_c = f \cdot v_c$ through the confined aquifer as follows;

$$-\frac{F}{f}\left(\frac{L_c}{k_c} + \frac{R_c}{fg}\right) log\ (R_c - q_c) - F\frac{q_c - R_c}{f^2 g} = t + C_1 : \quad f \cdot v_{co}{}' < R_c \tag{I-48a}$$

$$-\frac{F}{f}\left(\frac{L_c}{k_c} + \frac{R_c}{fg}\right) log\ (q_c - R_c) - F\frac{q_c - R_c}{f^2 g} = t + C_2 : \quad f \cdot v_{co}{}' > R_c , \tag{I-48b}$$

in which C_1 and C_2 are the integral constants defined by the initial condition, and $v_{co}{}'$ is the initial velocity of water through the aquifer. It should be noted here that the initial time in this chapter should be chosen at the time when the recharge to the ground water table begins.

In the above solution, the second term on the left hand side may be ignored comparing with the first term by the order estimation. Therefore, the solution (I-48a,b) may be rewritten into the following simple form

$$-\frac{F}{f}\left(\frac{L_c}{k_c}+\frac{R_c}{fg}\right)log_e\left|\frac{R_c - q_c}{R_c - q_{co}'}\right| = t \quad , \tag{I-49}$$

that is,

$$q_c(t) = R_c\left[1 - exp(-\delta t)\right] + q_{co}' \; exp(-\delta t) \quad , \tag{I-50}$$

in which

$$\delta = \frac{f}{F}\cdot\frac{1}{\dfrac{L_c}{k_c}+\dfrac{R_c}{fg}} \quad , \tag{I-51}$$

and q_{co}' is the initial discharge of this component, and a prime is applied in order to distinguish this quantity from the initial discharge q_{co} for the recession state in the precedent chapter.

It is obvious from Eq.(I-50) that the runoff discharge q_c during the constant recharge consists of the effects of the initial state, say, the recession states from the previous period and the increment of discharge due to the recharge. Consequently the increment of the discharge due to the recharge $\Delta q_c(t)$ per unit width of the model is given by

$$\Delta q_c(t) = R_c\left[1 - exp(-\delta t)\right] \quad . \tag{I-52}$$

3-2-2 Unconfined Component

For this component, we write $r_e(t)$ for the recharge intensity per unit area of ground water table. Assume that the water table is almost horizontal, namely the gradient of water surface with respect to x is very small, then we may ignore the higher infinitesimal $\left(\frac{\partial H_u}{\partial t}\right)^2$. Then with cooperation of Darcy's law the equation of continuity, that is, the fundamental equation for the model shown in Fig. I-2 becomes approximately

$$\frac{\partial H_u}{\partial t} = \beta \cdot H_u \frac{\partial^2 H_u}{\partial x^2} + \eta \cdot r_e(t) \quad , \tag{I-53}$$

where

$$\eta = \frac{1}{\gamma_u} \quad , \qquad \beta = \frac{k_u}{\gamma_u} \quad . \tag{I-54}$$

In order to find the proper solution of this equation, the boundary and initial conditions should be prescribed. Granted that the water stage in the stream may vary rapidly and is not constant, violent variation of the water stage in the stream is limited to a short period. Therefore, it is assumed that the water stage in the stream is, in macroscopic meaning, equal to the mean water stage h_{uo}' during the rising state. For the brief treatment in this chapter, we use the following boundary and initial conditions.

$$H_u(x, 0) = f(x) \ , \tag{I-55a}$$

$$\left.\frac{\partial H_u}{\partial x}\right|_{x=0} = 0 \ , \tag{I-55b}$$

$$H_u(L_u, t) = h_{uo}' \ . \tag{I-55c}$$

Furthermore, in order to circumvent the mathematical difficulty of the nonlinearity, we write

$$H_u(L_u, t) = H_{uo}' + h(x, t) \quad , \tag{I-56}$$

in which

$$H_{uo}' \equiv H_u(0,\ 0) \quad . \tag{I-57}$$

If we assume

$$H_{uo}' \gg h(x,\ t) \quad , \tag{I-58}$$

then the fundamental equation (I-53) is linearized and becomes

$$\frac{\partial h}{\partial t} = \beta \cdot H_{uo}' \frac{\partial^2 h}{\partial x^2} + \eta \cdot r_e(t) \quad , \tag{I-59}$$

and the condition (I-55) is rewritten as,

$$h(x,\ 0) = f(x) - H_{uo}' = F(x) \ , \tag{I-60a}$$

$$h(L_u,\ t) = h_{uo}' - H_{uo}' \ , \tag{I-60b}$$

$$\left.\frac{\partial h}{\partial t}\right|_{x=0} = 0 \quad . \tag{I-60c}$$

Eq.(I-59) may be reduced to the following equations on the newly defined quantities $h_1(x,\ t)$ and $h_2(x,\ t)$. That is, separate the function $h(x,\ t)$ into two parts,

$$h(x,\ t) = h_1(x,\ t) + h_2(x,\ t) \ , \tag{I-61}$$

then our problem is to find the solutions $h_1(x,\ t)$ and $h_2(x,\ t)$ which satisfy the following systems of the differential equation and conditions, respectively.

$$\left.\begin{aligned} &\frac{\partial h_1}{\partial t} = \kappa^2 \frac{\partial^2 h_1}{\partial x^2} \ , \\ &h_1(x,\ 0) = F(x) \ , \\ &\left.\frac{\partial h_1}{\partial x}\right|_{x=0} = 0 \ , \\ &h_1(L_u,\ t) = h_{uo}' - H_{uo}' \ . \end{aligned}\right\} \text{ for } \ h_1(x,\ t), \tag{I-62}$$

$$\left.\begin{aligned} &\frac{\partial h_2}{\partial t} = \kappa^2 \frac{\partial^2 h_2}{\partial x^2} + \eta \cdot r_e(t) \ , \\ &h_2(x,\ 0) = 0 \ , \\ &\left.\frac{\partial h_2}{\partial x}\right|_{x=0} = 0 \ , \\ &h_2(L_u,\ 0) = 0 \ . \end{aligned}\right\} \text{for} \quad h_2(x,\ t), \tag{I-63}$$

in which

$$\kappa^2 = \beta \cdot H_{uo}' \ , \qquad \beta = \frac{k_u}{\gamma_u} \ . \tag{I-64}$$

Since these equations are linear, the solutions are easily found and they become

$$\begin{aligned} h_1(x,\ t) = {} & h_{uo}' - H_{uo}' \\ & + \frac{2}{L_u}\sum_{s=0}^{\infty}\left[exp\left(-\frac{\kappa^2(2s+1)^2\pi^2}{4L_u^2}\,t\right) cos\left(\frac{2(s+1)\pi}{2L_u}\,x\right)\right. \\ & \times \left.\left\{ \int_0^{L_u} F(\lambda) cos\,\frac{(2s+1)\pi\lambda}{2L_u}\,d\lambda - (-1)^s\,\frac{2L_u(h_{uo}' - H_{uo}')}{(2s+1)\pi}\right\}\right] \ , \end{aligned} \tag{I-65}$$

$$\begin{aligned} h_2(x,\ t) = {} & \frac{2}{L_u}\sum_{s=0}^{\infty}\left[cos\,\frac{(2s+1)\pi x}{2L_u} \int_0^{L_u} exp\left(-\frac{\kappa^2(2s+1)^2\pi^2}{4L_u^2}\,t\right)\right. \\ & \times \left\{ C + \int \eta\cdot r_e(t)\ exp\left(\int \frac{\kappa^2(2s+1)^2\pi^2}{4L_u^2}\,dt\right) dt\right\} \\ & \times \left. cos\left(\frac{(2s+1)\pi\lambda}{2L_u}\right) d\lambda \right] \ . \end{aligned} \tag{I-66}$$

Summation of H_{uo}', h_1 and h_2, thus, becomes the approximate solution of Eq.(I-53). Then, we may obtain the runoff discharge through the unconfined aquifer $q_u(t)$ as a function of t by the following expression.

$$q_u(t) = k_u H_u(x,\ t) \left.\frac{\partial H_u(x,\ t)}{\partial x}\right|_{x=L_u} = k_u h_{uo}' \left.\frac{\partial H_u(x,\ t)}{\partial x}\right|_{x=L_u} . \tag{I-67}$$

If we assume here that r_e is constant, then the runoff discharge $q_u(t)$ is given by:

$$\begin{aligned} q_u(t) = {} & \frac{2k_u}{L_u} h_{uo}' \sum_{s=0}^{\infty} \left[exp\left(\frac{-\kappa^2 (2s+1)^2 \pi^2}{4L_u^{\ 2}} t \right) \frac{(2s+1)\pi}{2L_u} (-1)^s \right. \\ & \left. \times \left\{ \int_0^{L_u} f(\lambda) cos \frac{(2s+1)\pi\lambda}{2L_u} d\lambda - (-1)^s \frac{2L_u (h_{uo}' - H_{uo}')}{(2s+1)\pi} \right\} \right] \\ & + \frac{2k_u}{L_u} h_{uo}' \sum_{s=0}^{\infty} \left[\frac{4L_u^{\ 2}}{\kappa^2 (2s+1)^2 \pi^2} \cdot \frac{r_e}{\gamma_u} \right. \\ & \left. \times \left\{ 1 - exp\left(- \frac{\kappa^2 (2s+1)^2 \pi^2}{4L_u^{\ 2}} t \right) \right\} \right] . \end{aligned} \tag{I-68}$$

As seen from the system of equations (I-62) and (I-63), the behaviour of ground water in the unconfined component is expressed as superposition of the effects of the recharge of the rain-water and the initial states. That is, since the solution $h_1(x,\ t)$ describes the variation of water table in the model during no rainfall period, say, the effect of initial state, it corresponds to the variation in the recession state. On the other hand, $h_2(x,\ t)$ expresses the effect of the recharge on the variation of ground water table. Furthermore, the first term on the right hand side of Eq.(I-68) is derived from the solution $h_1(x,\ t)$ and the second term from $h_2(x,\ t)$. In other words, the first and second terms of Eq.(I-68) give the contributions of the recession of the initial state and the constant recharge of the rain-water to the variation of runoff discharge, respectively. From this, we get the expression for the increment of runoff discharge due to the constant recharge as follows,

$$\Delta q_u(t) = \frac{2k_u}{L_u} h_{uo}' \sum_{s=0}^{\infty} \frac{4L_u^2}{\beta H_{uo}'(2s+1)^2\pi^2} \frac{r_e}{\gamma_u} \times \left\{1 - exp\left(- \frac{\beta H_{uo}'(2s+1)^2\pi^2}{4L_u^2} t\right)\right\} \quad \text{(I-69)}$$

3-3 Fundamental Expressions of Variation of Runoff Discharge due to Rainfall

3-3-1 Confined Component

As has been derived, the variation of runoff discharge of the confined component is expressed by Eq.(I-50). This equation corresponds to the variation of discharge due to rainfall per unit width of the model in Fig. I-3. Introducing the factor B_c representing the width of the outlet of the aquifer, we have an expression for the actual confined component like

$$Q_c(t) = R_c B_c \left[1 - exp(-\delta t)\right] + Q_{co}' \, exp(-\delta t) , \quad \text{(I-70)}$$

in which Q_{co}' indicates the actual discharge at the initial time when the recharge begins.

The variation rate of discharge is synthetically expressed by the exponential index δ given by Eq.(I-51).

$$\delta = \frac{f}{F} \cdot \frac{1}{\frac{L_c}{k_c} + \frac{R_c}{fg}} \quad \text{(I-51)}$$

Of those which are involved in the factor δ, the quantities F, f, L_c and k_c may be considered constant in a specified river basin, so that, the variation rate δ should be a function of R_c only. The effect of R_c on the factor δ may be ignored by the order estimation. That is, since k_c is of the order $10^{-6} \sim 10^{-7}$(km/day), g of 10^8(km/day^2) and R_c smaller than unity (km^3/day/km), the term R_c/fg is the higher order infinitesimal comparing with L_c/k_c. If the term R_c/fg is thus ignored, the value δ becomes

$$\delta \approx \frac{fk_c}{FL_c} \quad . \tag{I-71}$$

This quantity corresponds to the value in the case of $R_c=0$, namely, the recession state without any recharge. Furthermore it may be remembered that this value is then equal to the recession factor α given by Eq. (I-29) in the second chapter, i.e.,

$$\delta \approx \alpha \ . \tag{I-72}$$

The original equation (I-70) may thus be rewritten approximately as follows:

$$Q_c(t) = R_c B_c \left[1 - exp(-\alpha t)\right] + Q_{co}' \ exp(-\alpha t) \ . \tag{I-73}$$

Moreover, the increment of discharge $\Delta Q_c(t)$ due to rainfall is given by the first term of Eq.(I-73), that is

$$\Delta Q_c(t) = R_c B_c \left[1 - exp(-\alpha t)\right] \quad . \tag{I-74}$$

In addition, Eq.(I-73) shows that, if the confined aquifer is supplied by the water with constant recharge during extended period, the discharge will continue to increase or decrease unceasingly and reach a steady state, because the following relations hold at any time as far as recharge continues:

$$Q > Q_{co}' \qquad \text{for} \qquad R_c B_c > Q_{co}' \ ,$$

$$Q < Q_{co}' \qquad \text{for} \qquad R_c B_c < Q_{co}' \ . \tag{I-75}$$

The governing parameters in the variation of discharge, as easily seen from Eq.(I-74), are $R_c B_c$ and the exponential index α. That is to say, the value $R_c B_c$ governs the increment of discharge in the amount and the exponential index α defines the variation rate. As to the value α, with respect to the recession characteristics we have already discussed in details in the last chapter.

On the other hand, the value $R_c B_c$ obviously depends on the recharge regimes. Although detail discussion can not be made on the recharge regime to the confined component because of the complexity, it will be clarified later through semi-empirical procedure that the recharge intensity R_c is governed by the rainfall intensity, and the proportionality is observed between them. This proportionality, as contrasted with the case of the unconfined component, results in the considerable amount of discharge from the rainfall.

3-3-2 Unconfined Component

3-3-2-1 Approximate Expression

For this component, the increment of discharge is given by Eq.(I-69). The equation (I-69) is rewritten as

$$\Delta q_u(t) = \frac{8}{\pi}\frac{h_{uo}'}{H_{uo}'} r_e L_u \sum_{s=0}^{\infty} \frac{1}{(2s+1)^2}\left\{1 - exp\left(- \frac{\beta_u H_{uo}'(2s+1)^2\pi^2}{4L_u^2} t\right)\right\}. \tag{I-76}$$

In this series, the first term is dominant and the other terms are 1/10 or much less in order of the first term. So the increment of discharge may be approximately given by

$$\Delta q_u(t) = \frac{8}{\pi}\frac{h_{uo}'}{H_{uo}'} r_e L_u \left\{1 - exp\left(- \frac{k_u H_{uo}'\pi^2}{4\gamma_u L_u^2} t\right)\right\}. \tag{I-77}$$

The equation obtained above does not satisfy the equation of continuity, because $\Delta q_u(t)$ tends to $\frac{8}{\pi}\frac{h_{uo}'}{H_{uo}'} r_e L_u$ when t tends to infinity. The causes of the error are apparently due to several assumptions and the simplification in the mathematical procedure. Therefore, let us correct the coefficient so that the equation of continuity holds, then we obtain

$$\Delta q_u(t) = r_e L_u \left\{1 - exp\left(- \frac{k_u}{\gamma_u} \frac{\pi^2}{4L_u{}^2} H_{uo}'\ t\right)\right\}. \qquad (I-78)$$

Furthermore, the equation is transformed into that for the actual discharge, introducing the factor B_u as well as in Chapter 2, that is,

$$\Delta Q_u(t) = r_e B_u L_u \left[1 - exp(-\varepsilon t)\right] , \qquad (I-79)$$

in which

$$\varepsilon = \frac{k_u}{\gamma_u} \frac{\pi^2 H_{uo}'}{4L_u{}^2} . \qquad (I-80)$$

This is the approximate expression for the increment of actual runoff discharge from the unconfined component due to the constant recharge intensity r_e.

The equation (I-79) shows that the recharge intensity r_e, the duration time of rainfall and the exponential index ε are most important factors in the rising state of discharge. Besides the value $r_e L_u B_u$ gives the upper bound of the increment, the exponential index ε corresponds to the variation rate. In addition, the duration time defines what quantity of increment the actual discharge will rise and reach in each rainfall occasion.

3-3-2-2 Upper Limit of Increment of Discharge

As stated in the last article, the upper limit of the increment of discharge depends on the recharge intensity r_e. In the basinwide meaning, since the ground water seems to distribute homogeneously all over the basin, the recharge to it mainly depends on the vertical infiltration of rainwater. Therefore, the recharge intensity may be considered to be limited in its variation range by the infiltration capacity of the basin.

If the recharge intensity is truly an invariant constant in a basin, as assumed in the earlier section and also proved in Sub-section 3-7-1, the factor $r_e L_u B_u$ should be also an invariant. Then, due to rainfall, the runoff discharge of this component increases until it reaches at a definite value, say, the value $r_e L_u B_u$, but the increment of discharge since then may remain at the value no matter how long the rainfall excess the value r_e continues. This fact, which will be discussed later, forms a remarkable contrast to the confined component in the amount of the increment of discharge due to rainfall.

3-3-2-3 Effect of Initial State

Of those which govern the rising state, the factor closely related to the basin state is the exponential index ε. The exponential index ε which expresses the variation rate of the rising state is given by Eq. (I-80), that is,

$$\varepsilon = \frac{k_u}{\gamma_u} \frac{\pi^2 H_{uo}'}{4L_u^2} \quad . \tag{I-80}$$

It is obviously seen that the variation of this index in each occasion is due to the initial water depth H_{uo}' which expresses synthetically the initial state of basinwide water.

Before the rainfall, the runoff should be at recession state, so that, the initial water depth H_{uo}' for the rising state is also considered as the water depth at the precedent recession state. Thereupon, with use of several expressions for the recession state, we get theoretically the relationship between this initial water level H_{uo}' and the unconfined discharge at the same moment Q_{uo}' as follows;

$$H_{uo}' \propto \sqrt{Q_{uo}'} \quad . \tag{I-81}$$

Besides being the discharge at the preceding recession state, Q_{uo}' is the initial discharge of the unconfined component for the following rising state. Then Eqs.(I-80) and (I-81) can be combined to give

$$\varepsilon \approx n'\sqrt{Q_{uo}'} \quad , \tag{I-82}$$

in which n' is a constant.

In accordance with Eq.(I-82), the variation rate is governed by the initial discharge, say, the initial state of the basin. Therefore, the variation of discharge of this component in a basin is actually governed by the initial state of the basin and the rainfall duration because the recharge intensity may be considered as an invariant in a basin, as will be stated later.

3-4 Estimation of Increments of Discharge and Storage

For the purpose of evaluation of increments of discharge and storage, since the direct runoff takes place simultaneously within the basin during the rising state of the ground water runoff, we have to resort to the recession characteristics of both the components as the essential foundation. Figure I-17 shows the schematic representation of the variation of discharge hydrograph due to rainfall. From rainfall, the ground water runoff will rise, for instance, along the broken line O'C in the figure, and the precedent recession curve I may be replaced by the new recession curve II. Therefore, we can see the macroscopic features of the increment of runoff discharge and storage in terms of the quantities with respect to the recession characteristics of curve I and curve II.

3-4-1 Increment of Runoff Discharge

Let us write the recession characteristics for curve II α, Q_{co}, K, Q_{uo} for the time origin at $t=t_c$, and the initial discharges for both components at the point O' in Fig. I-17 Q_{co}' and Q_{uo}', If there is no rainfall, the discharge must decrease along the curve ABCD. Denote the confined and unconfined runoff discharge in the precedent recession at $t=t_c$, say, at the point C' as Q_{cC}' and Q_{uC}', respectively, then the increments of the discharge ΔQ_c and ΔQ_u for these components at time $t=t_c$, may be easily calculated as follows:

for the confined component,

$$\Delta Q_c = Q_{co} - Q_{cC}' , \tag{I-83}$$

for the unconfined component,

$$\Delta Q_u = Q_{uo} - Q_{uC}' . \tag{I-84}$$

These expressions, of course, give the increments of discharge at the moment $t=t_c$ only. There is no way to define the variation of the increments of discharge corresponding the broken line O'C in Fig. I-17 which is actually concealed by the direct runoff, unless we interporate or extrapolate it by the increments obtained by the above method with various durations of recharge.

3-4-2 Increment of Ground Water Storage

For each of the components, the total amount of water stored in the basin at any moment which will flow out to the stream as the ground water runoff in future, is approximately given by the integral of the hydrograph from the moment $t=t$ to $t=\infty$. This total amount may be considered as the ground water storage concerned with the ground water runoff. Therefore, the increment of the storage ΔS which corresponds to the hatched area in Fig. I-17, is approximately given by

$$\Delta S_u = \int_0^\infty \frac{Q_{uo}}{(K\sqrt{Q_{uo}}\ t + 1)^2}\, dt - \int_0^\infty \frac{Q_{uC}'}{(K\sqrt{Q_{uC}'}\ t + 1)^2}\, dt$$

$$= \frac{\sqrt{Q_{uo}}}{K} - \frac{\sqrt{Q_{uC}'}}{K}, \qquad \text{(I-85)}$$

for the unconfined component, and

$$\Delta S_c = \int_0^\infty Q_{co}\, exp(-\alpha t) dt - \int_0^\infty Q_{cC}'\, exp(-\alpha t) dt = \frac{Q_{co}}{\alpha} - \frac{Q_{cC}'}{\alpha}, \qquad \text{(I-86)}$$

for the confined component.

In order to evaluate the increments of discharge and storage by the above equations, it should be noted that the recession characteristics must be clarified definitely, because a small error in the recession analysis may cause a considerable error in the estimation of the increments. Therefore, in this chapter, we will treat only the data for certain basins of which recession characteristics have been clearly defined. The evaluated values of the increments of discharge and storage are shown in Table I-5 together with the other quantities.

3-4-3 Lag Time and Duration of Recharge

In the practical procedure, the lag time from the rainfall to the variation of discharge or the duration of recharge should be defined. As will be discussed later, the lag time from the rainfall to the recharge may be understood as to be shorter than one day. So that, in this paper, the lag time for the variation of discharge is assumed empirically[50)58)] as follows.

Gauging station	Lag time
Arakura	
Takasu	1 day
Tsurugi	

Kamita 2 days

Furthermore, the duration of recharge is also assumed to be equal to that of the rainfall, as will be discussed in Section 3-8.

3-5 Recharge Regime to both Components

Regarding to the ground water runoff, as seen from Eqs.(I-74) and (I-79), the most important problem to be solved with respect to the recharge regime may be reduced to the problems as summarized below;

a) the intensity of the recharge to the ground water table,
b) the duration of the recharge,
c) the time lag expected for the water movement from the ground surface to the ground water table, and
d) the ability of the soil or layer to sustain the water, that is, the quantity of water exhausted in the zone of aeration without reaching the ground water table.

The intensity of the recharge will govern the increment of discharge quantitatively and the duration will define how long the rising state continues and then how much quantity of the discharge the runoff will reach. There is no needs to say how important the lag time is for the better understanding of the runoff phenomena, furthermore, the ability of the soil to sustain the water in the zone of aeration is of great importance regarding the loss for the ground water runoff and also the direct runoff. With respect to these problems, discussions will be phenomenologically made in Sections 3-6, 3-7 and 3-8 on the basis of the field data, especially about the infiltration process.

In this section, the author wishes to summarize the recharge regime to the aquifer and also to contrast the effects of the recharge regimes on the variation of both of the components.

In the basinwide meaning, since the unconfined ground water seems to distribute homogeneously all over the basin, the recharge to it mainly depends on the infiltration of rainwater through the zone of aeration. On the other hand, a confined aquifer may be considered to have the catchment area for itself, and a certain fraction of rainfall reaching the area supplies the confined ground water. However, the distribution of the confined aquifer is closely related to the local features of the geological factors, therefore, we cannot define clearly the recharge regime to the confined aquifer.

Some examples of the increments of discharge evaluated in the precedent section are plotted in Fig. I-18. In the figure the increments of the confined component ΔQ_c are plotted against those of the unconfined component ΔQ_u in each occasion. What is evident from the figure is that there is a certain upper limit for ΔQ_u but on the contrary ΔQ_c may reach at a considerably large amount. Although there must be actually a certain upper limit for the increment of the confined component too, we cannot find it from the values obtained here.

Generally, the increments of discharge of both components are directly related to the recharge intensity. Therefore, the fact observed in Fig. I-18 suggests the difference between the recharge regime to the both components. The existence of the upper limit for the unconfined component implies that the infiltration process governs the recharge to the unconfined component, on the contrary, the large increment found for the confined component suggests the considerable role of the water flow through chinks, cracks and orifices below the ground in the recharge process to this aquifer.

3-6 Evaluation of Variation of Confined Component

As derived in Section 3-3, the variation of confined component is expressed by Eq.(I-74), that is,

$$\Delta Q_c(t) = R_c B_c \left[1 - exp(-\alpha t)\right] \quad . \tag{I-74}$$

In other words, to clarify the behaviour of this runoff component is reduced to the evaluation of the quantity $R_c B_c$, because the exponential index α has been already clarified.

3-6-1 Relationship between Increment of Discharge and Rainfall

The effect of rainfall on the recharge intensity and the variation of discharge is by itself of a compensating nature. Provided the constant recharge continues during a certain period, the increment of discharge will increase according to Eq.(I-74). The factor $R_c B_c$ which has been originally introduced for the recharge rate, implies the upper bound of the increment of discharge due to the constant recharge R_c. Since the quantity R_c seems to variate or change in each occasion of rainfall and the value B_c is rather a conceptual one, there is no way to define the value $R_c B_c$, unless we estimate it with help of Eq.(I-74) and the increment of discharge analyzed by the field data. Thereupon, the value $R_c B_c$ is estimated by the reverse calculation of Eq.(I-74) by using the recession factor α obtained in the second chapter, the duration of rainfall T and the increment of discharge ΔQ_c for individual occasion of rainfall. The strongest correlation was then observed between $R_c B_c$ so obtained and the mean daily rainfall. Fig. I-19 shows the value $R_c B_c$ versus the mean daily rainfall R_m during the rainfall period.

The figure suggests that the recharge rate seems to variate with the rainfall intensity. The relationship is temporarily held as the proportionality in the present paper. Furthermore, in spite of the semi-empirical procedure, the recharge intensity may be considered nearly constant during the rainfall period as far as the bulk characteristics are treated, that is,

$$R_cB_c = D\cdot R_m \,. \tag{I-87}$$

For the convenience of the following discussion, the coefficient D which combines the value R_cB_c and the mean daily rainfall are estimated as shown in eight column in Table I-5 though is of rough estimation. The straight line in Fig. I-19 corresponds to the relation (I-87) with the estimated value D.

The recharge regime of which intensity is proportional to the rainfall intensity is considered in general as the flow somehow similar to the one through the closed channel, namely, the recharge through the chinks, cracks and orifices below the ground. Accordingly, this fact mentioned above appears to be the proof that the considerable fraction of the recharge to the confined component is due to the flow through the chinks and cracks. This is one of the reason why the confined component reaches at a considerable amount of discharge due to the rainfall and also in the early stage of recession. It is very interesting that the fact forms a marked contrast to the case for the unconfined component.

3-6-2 Regional Distribution of Confined Component

The investigation in this chapter is carried by the lumped system model, accordingly it is actually impossible to confirm the regional distributions of the confined runoff component in a region. However, to define the value D for a certain basin is equivalent to express how much fraction of rainfall runs off as the confined component. The value D for each basin is temporally estimated as follows.

	D	Ratio of confined component to effective rainfall
Arakura	0.61	0.331
Tsurugi	0.50	0.193
Takasu	0.10	0.132
Kamita	0.67	0.081

These values suggest that the ratio of the confined component to the rainfall is greater in the small mountainous region than in large basins. An evaluation of the value D itself for the discussion is not intended in the present study as it requires a larger amount of data to establish the relationship of the spatial distributions of the confined aquifer within a basin. However, it is believed that the confined ground water seems to be considerably distributed in rather mountainous regions. Furthermore, this fact does not contradict with the intuitive consideration of the actual states of the confined aquifers.

3-6-3 Variation of the Confined Component

In cooperation with the relationship Eq.(I-87), Eq.(I-74) becomes

$$\Delta Q_c(t) = D \cdot R_m \left[1 - exp(-\alpha t) \right] . \qquad \text{(I-88)}$$

As seen from Eq.(I-88), the increment of discharge of the confined component in the actual watershed is mainly defined by the rainfall intensity and the duration time of the rainfall. Therefore, if the parameters α and D are defined in a certain basin and sufficient rainfall information is given, we may estimate the runoff discharge in the rising state for the parameter R_m. The family of curves in Fig. I-20 illustrates the estimated values $\Delta Q_c(t)$ by Eq.(I-88) for the parameter R_m. The dotted plots indicate the increments of discharge obtained from the observed data for each occasion of rainfall, and the number described for each plot is the mean daily rainfall.

From the figure, we may conclude that the estimated curve explains the actual increment of the confined discharge considerably well all at once, in spite of a few exceptions. The fact proves synthetically several characteristics of the variation of the confined component due to the rainfall. Consequently, the characteristics of this component are summarized as:

a) The variation of the discharge may be given by the exponential function of type Eq.(I-74).

b) The exponential index α is common with the recession factor. That is, through this factor, the basin characteristics such as geological, geographical and hydraulic quantities affect the hydrodynamic behaviour of the confined component due to rainfall.

c) The increment of discharge is considerably affected by the external factors to the phenomena within the basin. That is, the quantitative characteristics of the increment are governed by the rainfall condition. Besides mean rainfall intensity defining the upper limit of the increment, the duration of the rainfall governs what quantity the discharge will rise and reach at.

Consequently, these results obtained in this section confirm that the rising state of the confined component may be characterized by the kinematics of the hydrodynamic runoff model in Fig. I-3 considerably well as well as the recession characteristics.

3-7 Evaluation of Variation of Unconfined Component

In this case, the variation of the runoff discharge is expressed by Eq.(I-79), that is,

$$\Delta Q_u(t) = r_e B_u L_u \left[1 - exp(-\varepsilon t)\right] . \qquad \text{(I-79)}$$

In this section, we will evaluate the governing factors in turns with help of the results analyzed at the actual watersheds.

3-7-1 Recharge Intensity and Increment of Discharge

Let us consider at first the recharge intensity r_e to ground water table. Figure I-21 shows the relationship between the increment of storage of the unconfined ground water estimated in Sub-section 3-4-2 and the duration of rainfall. In the figure, we may find the proportional relationship between the two quantities. In other words, the recharge to the unconfined aquifer occurs actually at almost constant rate in the duration of the rainfall. It wil be also demonstrated below that the recharge rate is almost equal to the final infiltration capacity. These natures strongly suggest that the recharge to the unconfined aquifer is mainly caused by the vertical infiltration of rain-water. Thus, the suggestion made in preceding sections is verified as well as the assumption of constant r_e in the mathematical procedure.

The recharge intensity r_e is constant, then the upper limit of the increment of discharge should be a definite invariant for a specified basin and may correspond to the upper limit observed for the unconfined component in Fig. I-18. In the present paper, the values $r_e L_u B_u$ for several river basins are estimated from Fig. I-18 as shown in eleventh column in Table I-5.

The value $r_e L_u B_u$ so obtained for each basin is rewritten into the recharge intensity as follows:

Takasu	:	0.1883 mm/hr	(3.4 m^3/s)
Tsurugi	:	0.2098	(13.0)
Kamita	:	0.1187	(23.5)
Arakura	:	0.2264	(10.0)
Kamo	:	0.1236	(50.0)
Inooka	:	0.0462	(20.0)

On the other hand, the final infiltration capacity has been estimated by Ishihara and others[50)58)] from various points of view, say, by the runoff analyses and the field observations. The value estimated by them is

$$i_c = 0.23 \sim 0.27 \text{ mm/hr}$$

at Ohno watershed, which is located just downstream reaches of Arakura in the River Yura, with the catchment area 350 km^2. As a matter of course, the final infiltration capacity at each basin, differs from each other, but this value will give us an available measurement for the order estimation of it with respect to the usual basin.

The comparison of this value with the estimated ones suggests that the recharge to the unconfined component takes place with almost the same intensity as the final infiltration capacity in the watershed, in macroscopic meaning, in spite of an exception for the basin of Inooka gauging station which is located just at the outlet of the cup-shaped region.

In review of the above discussion, we draw the conclusion that the recharge to the unconfined component is caused by the infiltration phenomena and the recharge intensity is almost equal to the final infiltration capacity. This process governs the increment of discharge, and then the maximum increment of discharge is unique constant for a specified basin but not affected by the rainfall intensity.

3-7-2 Variation Rate ε and Initial State

The variation rate of runoff discharge due to rainfall is given by the parameter ε. The parameter is, as obtained in Eq.(I-82), correlated with the initial discharge Q_{uo}' as follows,

$$\varepsilon \simeq n'\sqrt{Q_{uo}'} \quad . \qquad \text{(I-82)}$$

The values ε estimated for the several cases in the watersheds concerned are plotted against the square root of the initial discharge for each occasion in Fig. I-22. For the estimation of the index ε, Eq. (I-79) was used together with the observed increment of discharge, the duration time T for individual occasions and the value $r_e L_u B_u$ defined in the precedent article.

In Fig. I-22, the plots scatter to a considerable extent but we may guess the relationship similar to Eq.(I-82). Thereupon, the coefficient n' combining the value ε with the initial discharge is temporarily defined as follows.

Takasu	:	0.10	$m^{-3/2}sec^{1/2}day^{-1}$
Tsurugi	:	0.05	
Kamita	:	0.04	
Arakura	:	0.08	

The straight line drawn in Fig. I-22 indicates Eq.(I-82) with the value n' so defined.

From Fig. I-22 the rising rate of the unconfined component is understood as to be affected by the initial state and increases with the initial discharge. This conclusion is not only interesting but also of practical importance. That is, the actual increment of the unconfined component is defined quantitatively by the initial state as well as the duration of rainfall disregard of any rainfall intensity.

3-7-3 Variation of Discharge of Unconfined Component

The equation (I-79) which gives the increment of the unconfined component may be rewritten with use of Eq.(I-82) as follows:

$$\Delta Q_u(t) = r_e L_u B_u \left[1 - exp(-n'\sqrt{Q_{uo}'}\ t)\right] . \qquad \text{(I-89)}$$

This equation suggests that the most significant factors which govern the increment of this component are the duration of rainfall, the initial state of the basinwide water and the basin characteristics such as $r_e L_u B_u$ and n'. Therefore, if these factors are defined and the rainfall information is given, we may estimate the rising state of the unconfined component by Eq.(I-89).

The rising state so evaluated is shown as the family of curves in

Fig. I-23 with the parameter Q_{uo}', namely, the initial state. The plots indicate the observed discharge and the number denotes the initial discharge for each of the rising states. In the figure the family of curves may be understood as to coincide fairy well with the observed values. In other words, the rising state of the unconfined component may be expressed sufficiently by Eq.(I-79).

Consequently, we can summarize the several characteristics of the rising state of the unconfined component as below:

a) The variation of the discharge may be given by the exponential function of type Eq.(I-79).

b) The coefficient $r_e L_u B_u$ which represents the upper limit of the increment of discharge is an invariant in a certain basin. The value $r_e L_u B_u$ per unit area corresponds to the final infiltration capacity.

c) The variation rate in the exponent is proportional to the square root of the initial discharge of this component, that is, it is governed by the initial basin state.

d) The rainfall condition affects the variation of this component through the duration time alone.

3-8 Recharge Regimes

At the end of this chapter, the author wishes to discuss about the recharge regime to the aquifers, since the problems are of great importance in the analyses of the ground water runoff with respect to the water loss as well as the recharge resources.

Generally speaking, regimes of the recharge to the ground water table may be classified into two categories as follows;

a) the recharge by the infiltration or seepage of rainwater through the zone of aeration, and

b) the recharge due to the water flow through the cracks, chinks and orifices in the ground, which is somewhat similar to those of open-channel and/or closed-channel flows.

Of two recharge regimes mentioned above, the recharge through the cracks, chinks and orifices involves very difficult problems to be handled generally and its contributions to the ground water in the basinwide sense are beyond our present knowledge, since the local distributions of the chinks, cracks and orifices, which govern the recharge regime to very great extent, cannot be expressed in the general form. For the purpose of the discussion of this regime, therefore, it appears worthwhile that we deal the problems empirically with help of the several characteristics on the other hydrologic quantities affected by the recharge regime.

On the other hand, a considerable number of the publications report on the infiltration process from various points of view. The subsurface occurrence of ground water may be divided into zones of saturation and aeration. Generally, it is the zone of aeration through which the vertical infiltration takes place.

The zone of aeration may be further divided into the soil water zone just below the ground surface, the intermediate zone and capillary zone. The mechanism of the infiltration through each of these zones is not presented here, but we will summarize only several characteristics closely related to the discussions in this paper.

As well known, the moisture content in the soil water zone may vary in wide divergences from very dried state during no-rainfall to the almost saturated state during period of excessive rainfall. Therefore, this zone is sometimes considered as "the zone of major hydrologic activity" especially as regards to the direct runoff process[59)60)61)].

In the intermediate zone, the range of variation of the moisture content is very narrow, for instance, about 5-10%[61]. The moisture content in this zone remains almost constant in the direction of depth as well as in time[61)56)]. Moreover, the hydraulic gradient for the water movement through the zone is also almost constant in time and space[56]. That is to say, the characteristics of this zone for the infiltration are kept almost invariant in time and space.

Some parts of rain-water reaching the ground surface infiltrate through the soil water zone, the intermediate zone and capillary zone in turns, supplying the soil moisture in each zone. However, if we remember the facts discussed earlier in this article, it will be easily understood that almost all of water which supply the moisture content will be exhausted in the soil water zone, but less in the intermediate zone. In other words, the intermediate zone plays a role only to transport the water flowing down from the soil water zone to the ground water table[56)57)]. Furthermore, the phenomena may be considered to be kept at almost the same state from the head of the intermediate zone to the bottom in space as well as in time.

Therefore, the final infiltration capacity in the Horton's expression of the infiltration equation may correspond to the infiltration rate through the intermediate zone and the recharge intensity to the ground water table. In other words, it appears that the recharge rate due to infiltration is almost constant regardless of the rainfall intensity, although in the actual infiltration phenomena the recharge will gradually begin and reach at the final infiltration rate.

The lag time expected for the infiltration intensity to reach near the final one is generally not long, for example, a few hours estimated by Dreibelbis's field observations[59)60)] in Coshocton Watersheds, Ohio, U.S.A.. The time until the infiltration capacity to reach almost at the final one in the Yura River Basin is shown in Table I-6[50)58)]. These values in the table have been evaluated from the infiltration equation defined by Ishihara and others[50)58)]. The lag time from the rainfall to the response of the runoff discharge has been thus defined empirically as presented in Sub-section 3-4-3.

It does not always follow that the duration time of recharge equals to that of rainfall or precipitation. But there must be a close relationship between them, and it seems that the intensity of infiltration will become very small after a reasonable lag time from the ceasing of rainfall. For the purpose of the practical analyses, the recharge in the type of infiltration is assumed to begin and cease with the lag time stated in Sub-section 3-4-3 from the beginning and the ceasing of rainfall on referring to the values in Table I-6. That is, the duration of recharge is assumed to equal that of rainfall.

Regarding the recharge regime through chinks, cracks and orifices under ground, although there are many points which are uncertain in the present stage, the lag time has been assumed as stated earlier, since the lag time of water flow through the orifices may be considered as probably shorter than that of the infiltration recharge.

3-9 Conclusion

Several characteristics on the variation of the runoff discharge coming from the unconfined and confined aquifers are discussed on the basis of the hydrodynamic treatment of the runoff models. The results obtained may be summarized as follows:

i) The rising states of the ground water runoff may be also characterized by the mechanisms of both the runoff components.

ii) Theoretical treatment leads the exponential functions with respect to time t, that is, Eqs.(I-88) and (I-89) for the variation of the components.

iii) For the confined component, the most significant factor which governs the increment of discharge quantitatively, is the daily rainfall intensity. The rising rate which is mathematically involved as the exponential index in Eq.(I-88) is an invariant in a certain basin which represents the basin characteristics as a whole and equals to the recession factor α. And, the runoff discharge may rise considerably to a large amount due to the great intensity of rainfall.

iv) On the contrary, for the unconfined component, the recharge to the ground water table takes place in a form of the vertical infiltration, therefore, its rate almost equals to the final infiltration capacity of the basin. Then, there is an upper limit for the increment of runoff discharge regardless of the rainfall intensity. Further, the rising rate is variable with the initial state of the basin, namely, the initial discharge for the rising. That is, the rising state of the runoff discharge is mainly governed by the initial state.

As summarized above, in spite of the great difference between the mechanisms of the two components, the discussions in this chapter lead the similar mathematical expressions for both the components. Although this depends on the several mathematical assumptions for the unconfined component, however, it should be emphasized here that the behaviour of both the components forms a remarkable contrast with each other due to the difference of the mechanisms.

Chapter 4 SIMULATION TECHNIQUE OF GROUND WATER RUNOFF

4-1 Basic Remarks

The first half of Part I has been devoted to the scientific discussions on the ground water runoff process.

On the other hand, in referring to the water resources, there are many problems of wide divergences to be solved and counterplans to be considered. With respect to these problems, the primary problem will arise in connection with the characteristics on the daily discharge in a prolonged period. These characteristics will give the fundamental information to the water policy as to what we can do for the redistributions of water in space and time, how much quantity we may yield and utilize and how to treat the problems. Therefore, the simulation techniques of the daily discharges and its variation state are of great importance as the engineering problems for the masterplans or hydraulic and hydrologic designs for the water resources problems.

With this situation, provided the several kinematical characteristics of the ground water runoff are clarified to some extent, the method may be utilized for the simulation of the daily discharge due to long-term runoff as well as the other problems. Realizing the facts, the author has commenced a study[10)] to establish the simulation technique.

The purpose of this chapter is primarily intended to utilize the hydrodynamic runoff models discussed in the first half of this part, that is to say, the ground water runoff components are only treated. The main reason why the direct runoff is not taken into consideration, as stated in the introductory statement in the present paper, should be understood as due to the kinematical difference between the runoff processes of it and of ground water runoff. To establish the complete simulation of the daily discharge in connection with whole runoff components, the direct runoff component should be estimated by the other method[12)50)] treated in flood hydrology and added to the estimated values in this chapter.

In the simulation procedure of runoff discharge from the rainfall information, we will face the problems as to what fraction of rainfall becomes the loss for ground water runoff. This problem is of course closely related to the water loss for the direct runoff component,however, the water loss for the ground water runoff is apparently different from that for the direct one. The reason is that some part of the water loss for the direct runoff will appear to the ground surface as the ground water runoff. In other words, the difference depends on how long it takes for the water which flows into the ground to appear again on the ground surface. Therefore, the discussion on the water loss for the ground water runoff will be made in the beginning section of this chapter.

Another important problem arises with regard to the simulation for the runoff discharge in a prolonged period, that is, the water balance. The water balance will be examined based on the simulation results. The simulation which is concerned as the main task in this chapter is made by the computer by cooperating the several equations derived in the previous chapters.

4-2 Water Loss and its Evaluation

The water loss within the runoff process is mainly caused by the infiltration and the evapo-transpiration. If we consider that the evapotranspiration occurs from the soil water under ground, we may hold the evapo-transpiration phenomena by discussing the change of the soil water, say, the soil moisture in the zone of aeration. We consider the variation of the soil moisture on the basis of the infiltration equation:

$$i = (i_o - i_c)\cdot exp(-\nu t') + i_c \, , \tag{I-90}$$

in which the symbols denote i: the infiltration capacity at arbitrary time t', i_o: the initial infiltration capacity at $t'=0$, i_c: the final infiltration capacity and the exponential index ν: the coefficient representing the variation rate of the infiltration capacity.

As we have seen in Chapter 3, the state of the soil moisture are homogeneous in space in the intermediate zone after the infiltration capacity has become almost equal to the final capacity $i=i_c$, in addition no temporal change occurs in the soil moisture. Therefore, the water which infiltrates into the ground at the intensity i_c is transmitted through the intermediate zone and reaches the ground water table at the same intensity i_c. This means that the water amount exhausted to supply the want of soil moisture in the zone of aeration, is approximately given by the integral of the first term in Eq.(I-90) with respect to t'.

According to the description mentioned above, the water amount exhausted for the replenishment of the want of soil moisture varies with the initial infiltration capacity i_o. The variation of the initial infiltration capacity is expressed by the infiltration recovery curve. The initial infiltration intensity will reach at its maximum value i_{omax}, if no rainfall state continues in a long period. The integral of the first term in Eq.(I-90) with i_{omax} , then, is to correspond to the maximum amount of water M_o which can be sustained in the zone of aeration.

After the cease of rainfall, the water amount M sustained in the zone of aeration at any time decreases due to the evapo-transpiration from this zone, and then the initial infiltration capacity at any moment increases along the infiltration recovery curve. Conclusively, we may estimate the water loss for the whole runoff components in terms of such quantities as the maximum amount of water M_o, the water amount M sustained in the zone at that time and the rainfall intensity.

The soil moisture $M(t'')$ at time t'' after the cease of rainfall is given by

$$M = M_o' \cdot exp(-\mu t''), \qquad (I-91)$$

in which M_o' is the soil moisture at $t''=0$ and μ the exponential index in the infiltration recovery curve. If we have rainfall, water amount

$$M_o - M$$

is available for the replenishment of the want of soil moisture. Consequently, for the daily rainfall R the effective rainfall may be evaluated by the following equation,

$$R_e = \begin{cases} R - (M_o - M) & : \text{for } R > M_o - M, \qquad (I-92a) \\ 0 & : \text{for } R < M_o - M, \qquad (I-92b) \end{cases}$$

and then the soil moisture increases and reaches at the following value for the each case in Eq.(I-92), respectively.

$$M_o \quad : \quad \text{for} \quad R > M_o - M, \qquad (I-93a)$$

$$M + R : \quad \text{for} \quad R < M_o - M, \qquad (I-93b)$$

The procedure of the estimation of water loss is illustrated in Fig. I-24 schematically.

For the practical calculation the value M_o and μ should be defined for each of the basins. Regarding the infiltration equation and the recovery curve in Yura River Basin, Ishihara and others[50)] have made the detail discussions in their research project for the runoff process, and they estimated the several parameters in the equations. Therefore, in the present study the values of the parameters are used for Yura River Basin in order to evaluate water loss. Unfortunately, no-research has been made for the other basins concerned in this study, because of the insufficient number of data. Then the maximum water amount M_o which is sustained in the zone of aeration has been evaluated on the basis of water balance.

On the other hand, for the basins in Nagara River the exponential index μ in the infiltration recovery is temporarily assumed to equal to the value at Yura River Basin[50]. This assumption has been also examined by the annual water balance in Nagara River Basins.

Consequently, we adopt the values of these parameters as follow:

	M_o *(mm)*	μ *(1/day)*
Arakura	20.0	0.184
Takasu	25.0	0.184
Tsurugi	25.0	0.184
Kamita	25.0	0.184

4-3 Simulation of Ground Water Runoff

Provided the several parameters with respect to the unconfined and confined runoff components are defined and the water loss is reasonably estimated, we may estimate the ground water runoff in an extended period with use of the equations for the variation of the ground water runoff from the rainfall information as the input data. Therefore, in this chapter the simulation will be made only at Yura River Basin and Nagara River Basins for which we have defined rather clear characteristics in the previous chapters.

For the simulation, the runoff components coming from the unconfined and confined aquifers are individually calculated by the rainfall information and the ground water runoff on each day is defined as their summation. The equations used for the simulation are summarized as follows:

For the unconfined component:

$$Q_u(t) = \frac{Q_{uo}}{(K\sqrt{Q_{uo}}t + 1)^2} \quad ; \qquad 0 \le t \le t_1 \qquad \text{(I-94a)}$$

$$Q_u(t') = r_e B_u L_u \left[1 - exp(-n'\sqrt{Q_{uo}{}'}\, t')\right] + \frac{Q_{uo}{}'}{(K\sqrt{Q_{uo}{}'}t + 1)^2} \quad ;$$

$$0 \le t' \le T \ , \qquad \text{(I-94b)}$$

For the confined component:

$$Q_c(t) = Q_{co}\cdot exp(-\alpha t) \quad ; \qquad 0 \le t \le t_1 \qquad \text{(I-95a)}$$

$$Q_c(t') = DR_m\left[1 - exp(-\alpha t')\right] + Q_{co}{}'\cdot exp(-\alpha t') \quad ;$$

$$0 \le t' \le T \qquad \text{(I-95b)}$$

The time origins for t and t' in these equations are explained schematically in Fig. I-25 together with the time t_1. Moreover, T denotes the duration of the rainfall and t_g the lag time. Each recession state and its following rising state are calculated by the equations above mentioned for the initial discharge for the recession. The procedure is then repeated for every cycle from the recession to the rising in turns. The mean daily effective rainfall is adopted for the rainfall intensity R_m in Eq.(I-95b) as the input data as well as the duration T of rainfall and no-rainfall period t_1. In addition, for the purpose of comparisons, some examples are simulated in such way that the discharge of each day is calculated in turns from the values of one day before as the initial state. For this case, the value R_m is given by the effective rainfall of each day. To distinguish the two kinds of computations, we name the former case as Case 1 and the latter as Case 2.

Further, since no considerations are taken with respect to the runoff due to the melting snow, the winter season is excluded from the simulation. The values of the parameters used for the simulation in this chapter are summarized in Table I-7 and the initial conditions which have been defined by the recession analyses for each case are also tabulated in Table I-8.

Figure I-26 illustrates some examples of estimated discharge of ground water runoff together with the observed value. As a matter of course, the estimated values differ from the observed ones in the duration of the direct runoff which is not treated here. A careful comparison of the estimated value with the observed one in the figure, reveals that although the differences among the hydrograph are small, they are systematic. That is, the simulated values are rather larger than those in the actual phenomena in Summer season. The reason may be understood that the mean values of the several parameters are used for the whole period, accordingly, the effective rainfall may be over-estimated in Summer.

With exceptions for the duration of direct runoff and Summer season, it may be concluded that the shapes and of course the discharge ordinates of the simulated hydrograph of the ground water runoff agree considerably well with the observed discharge in whole periods, say, in five to six months. Furthermore we may find a rather good correlation between them especially in the long tails of recession states of the hydrographs. For Case 2 which has been treated for the comparison with Case 1, we may also see the similar tendencies and good agreement in the calculated values. In addition Case 2 gives an almost equivalent estimation in the ordinate with Case 1. In other words, it suggests that we may use the average values during the rainfall as the parameters for the computations in every one day steps. But for Case 2, it should be examined if the relationship Eq.(I-82) holds during the rising state. The equation (I-82) is originally defined for the recession state, but the initial state for the estimation in Case 2 is not at the recession state for every steps of calculation except for the first day in the rising state. But in the present study, we could not see if the similarity between Cases 1 and 2 suggests the generality of Eq.(I-82) or not.

For the improvement of this simulation technique, there are many problems to be discussed and to be solved, since many factors such as depression storage, interception and others are not taken into considerations and the mechanism of water loss and the lag time are also treated temporarily. Recently, Hosoi has made the simulation of daily discharge for some watersheds in central Japan[62]. He used the author's method for the ground water runoff simulation and the conceptual model proposed by Tsuchiya and others[63] for the direct runoff component, and he has obtained very accurate results illustrated in Fig. I-27. From the figure, and the discussions mentioned above, in spite of several restrictions, it seems that the method presented here may be available and applicable for the simulation of the daily discharge if it is used together with simulation technique for the direct runoff and the reasonable estimation of other factors.

4-4 Water Balance

The water balance in the whole period of each simulation is summarized in Table I-9. As seen from the table, about one third of total rainfall is lost and the remaining two thirds are effective rainfall. This fact agrees quite well with the previous estimations of annual water balance in numerous watersheds in Japan[64]. That is, although several problems remain to be solved as will be discussed below, the simulation technique seems to give the reasonable water balance.

For each duration of rainfall, total rainfall, effective rainfall and ground water runoff are shown in Fig. I-28. As a matter of course, according to the initial states of basin, effective rainfall varies occasionally in wide divergencies even for the case of the same total rainfall. Besides for small rainfall the ratio of water loss to total one scattering, for heavy rainfall, relatively constant water loss may be observed in a macroscopic meaning. This will be easily understood considering that the maximum amount of water sustained in the zone of aeration is assumed as constant in the simulation. The differences between water losses of various cases of heavy rainfall are mainly due to those of initial states and durations of rainfall because we assumed that evapotranspiration occurs every day whether it rains or not.

The ratio of ground water runoff to effective rainfall also varies widely. In Fig. I-28, it should be noted that in case of small rainfall, ground water runoff sometimes exceeds effective rainfall and that of direct runoff is estimated as negative. This contradiction may be caused due to the deficiency of the assumptions. That is, the recharge intensity is assumed here as constant in the duration of recharge or rainfall, however, the state should be primarily restricted in such case that there exists sufficient water supply on the ground surface.

If the several problems mentioned above are solved in the future, the simulation technique treated in this chapter will give much better estimations in discharge amount and water balance. This modification is being undertaken by the author.

Chapter 5 SUMMARY AND CONCLUSION

The investigation in Part I has dealt with the dynamic process of the ground water runoff by means of the idealized runoff models, and special attention is focused to the mechanism of water flow through the aquifers, since the ground water runoff may be understood to be characterized by the kinematics of water coming from the aquifers. So that, both of the unconfined and confined components are treated as the fundamental components.

For the idealized condition, the recession states of ground water runoff are treated as the first step of the approach. Then, the discussion is extended with regard to the variation of ground water runoff due to rainfall. Further, the proposed method is utilized as the engineering or technological problems regarding the simulation technique of the stream discharge coming from the ground water runoff.

The analytical solutions for the recession state of the components as well as for the variations due to rainfall are derived based on runoff models. In addition, the theoretical treatment on the several characteristics which enable to clarify the physical significances of runoff process is made. The equations and characteristics introduced theoretically are thus examined and discussed for the field data in several actual watersheds with help of the semi-empirical procedures. The main results obtained through the research in Part I are summarized as follows:

1) The ground water runoff is characterized kinematically by the unconfined and confined components. The confined component variates quantitatively in wide divergences but ceases in a few days after the ceasing of rainfall. On the contrary, the unconfined compo-

nent supplies not so much discharge even in the duration of rainfall but decreases very slowly in no-rainfall period. That is to say, the unconfined component supplies unceasingly the stream discharge, and then, it becomes obvious that this component plays an important role in the runoff process in a prolonged period.

2) Regarding the recession characteristics, the significances of the recession parameters are clarified in terms of the geological and topological factors within the basin. And then, the reasons why we can see the rather unique recession segment in the long tail of the hydrograph are also discussed.

3) The results obtained by the runoff analyses show that the method can explain very well the ground water runoff in the small mountainous basins and also some basins with very large catchment area, that is, there are clear normal recession curves for both of the components and the recession factors are clearly defined, for the homogeneity of the several hydrologic parameters in the small basin and the statistical cancelations among them in the very large basin. In other words, the small basin with catchment area 50∿200 km^2 and also some basins with very large area for the bulk characteristics may be treated as the unit basin for this approach. On the other hand, in the considerable large basin with catchment area over 300∿500 km^2 the considerable scattering of the several parameters appear for the various states of distributions of the hydrologic quantities. Although the reasons why we can hold the runoff characteristics in small and very large basins but not in rather large basins can not be observed by the empirical considerations, the author dares to present them here because of the clear relationship with the discussions which will be made in Part II.

4) Both runoff components variate in different ways due to the rainfall. The analytical solutions give the same type of equation,

namely the exponential function with respect to time, for the rising state of both. However, the increment of discharge coming from the unconfined aquifer is quantitatively limited by the recharge intensity through the infiltration process, on the other hand, for the confined component it is closely related to the rainfall intensity, hence the confined runoff component rises to a considerable amount of discharge due to the rainfall. Furthermore, the variation rate for the confined component is an unique constant and equals to the recession factor, but for the unconfined one it is governed by the initial discharge which represents the initial state of water within the basin. Moreover, it is pointed out that the duration time of rainfall is also one of the important factors effecting the variation of ground water discharge.

5) Applicability of the method are also demonstrated for the establishment of simulation technique of the ground water discharge from the rainfall information in a prolonged period.

As often presented, the emphasis should be laid throughout the present research on the conceptions based on the flow mechanisms of water near the outlets of the aquifers.

Part II

AN ANALYSIS OF THE BEHAVIOUR OF BASIN WATER BY MEANS OF THE VARIATIONAL METHOD

Chapter 1 GENERAL SCOPE OF PROBLEMS ON THE BEHAVIOUR OF BASINWIDE WATER

Generally speaking, runoff phenomena occur according to the variation of state of whole water within a watershed. That is, runoff shows only one phase of basinwide water behaviour. However, if the relationship between the causes and the results may be relatively conspicuous and the factors which play dominant roles in the phenomena are easily found, as in the case of flood due to the excess rainfall, we may pick up a certain phenomenon alone and discuss its characteristics with respect to such dominant factors. In pursuing such analysis, external actions and other factors are treated as the initial and boundary conditions and parameters.

On the contrary, in the case that many factors and elements interact very complicatedly and so the dominant factors cannot be picked up, it becomes difficult to hold some of the parameters in definite way and then problems are very difficult to be understood or to be solved. In the runoff process, the problems on the ground water runoff and the long-term runoff are very difficult examples to be treated in this meaning. The fact the research in Part I has explained considerably well the behaviour of the ground water runoff suggests that the models used may express the main factors in the ground water runoff process.

For large basins, however, it is rather difficult to clarify the several characteristics because of the scatterings and variations of several parameters in various cases. The variation is due to the temporal change of mechanical transformations in the runoff process; for example the interactions among stream water, ground water and hydrological quantities with different distributions in time and space.

In other words, the different way of interactions and different distributions of water in each case has large effects on ground water runoff when it appears in our sights. Furthermore, in the plain region and cup-shaped region these phenomena become of rather great importance in the runoff process. Therefore, it may be hoped that the problems on ground water runoff in the rather large basin are discussed in connection with the behaviour of all water components in basins.

Runoff process, in general, is of uni-directional system, that means several phenomena in downstream reaches are affected by those in upstream ones. That is, several physical quantities are transmitted from upstream to downstream in turns. Through that process, the runoff water varies its kinematical characteristics. However, phenomenological consideration in details indicates that in a watershed there are situated such regions that each of them acts as a kinematical system. In the region, besides giving considerable effects on water state in downstreams, the phenomena in upstreams are also controlled by the state in downstreams. In other words, the phenomena take place within a region according to a mechanism as a whole. For this example, we may take the plain regions and cup-shaped regions. Water coming from the upstream region behaves in this region as a kinematical system together with several water components. Thereafter it flows out from the outlet. Therefore, field of runoff in actual basins may be understood as the links of several kinematical systems.

It is generally pronounced that natural phenomena in a kinematical system occur in pursuit of the most stable and equilibrium state through the averaging and uniformalizing processes. The process towards the equilibrium state itself is understood as to be dependent upon a certain kinematical balance. Therefore, it may be expected that the behaviour of whole water components also follows a balance in a system as a whole in the same meaning. As the results and one phase of the behaviour, we observe the runoff phenomena and the variations of stream discharge.

There are various kinds of components in the basin water; stream water, ground water, soil moisture and others. Regarding the respect mentioned above, it may be considered that these components also behave following to their own balances under such situation that the basin water, as a whole, behaves in chase of the stable and equilibrium state.

The movement of each component is expressed by the fundamental equation. It is, however, still unknown what quantity and law govern the balance and process of behaviour of total basin water in a system. Such quantity and law, if could be found, may contribute to the understanding of the runoff phenomena in connection with all components in basin water.

In many cases of water resources problems, we plan and construct several hydraulic structures at the boundaries of these kinematical systems, for example dams at narrow valleys which separate downstream from upstream region, and the pipe line and watercourses from the head of alluvial zones. Due to the artificial water controls, we will face changes of the state of basin water in the system. That is, it is hoped that the characteristics of each system and the behaviour of water within the system are clarified as well as those for the whole basin. In these problems, as well known, we should handle several water components in a system together.

Realizing the point, the author has commenced a study of the problem whether we can formulate any law which governs the basin water behaviour all at once. As the first step of the approach, the author has focused his attention to ground water, stream water, and their interactions in a runoff process by means of mathematical model. The purpose of the present paper is to introduce a variational formulation for the basin water behaviour, especially with respect to the process of ground water runoff. The method was then applied for the investigation on the recession characteristics. That is, discussions are made with respect to the relationship between the recession characteristics at the outlet of the basin and those in the individual small basins. In other words, the problems as to how the characteristics of runoff water are averaged in the runoff process and what characteristics will appear at the outlet of the basin due to the interactions of several water components, are discussed. Further, it may be pronounced that the proposed method in the present paper will give us an useful tool for many engineering problems.

Throughout the second part, it should be noted that the term "variation" means sometimes the mathematical variation and sometimes the change of the recession characteristics in the runoff process, while it denotes the variation of ground water runoff discharge itself in time in the first part.

Chapter 2 VARIATIONAL FORMULATION FOR BASIN WATER BEHAVIOUR

2-1 Variational Principle for Energy Dissipative System

There are many variational expressions for kinematical systems, including Lagrangian and Hamiltonian[65]. In general, the methods treat the energy conservative system, but not the energy dissipative systems. Prigogine and his colleagues carried out studies on the problems whether there are any functions which play such roles in the dissipative system as Lagrangian function in a conservative system[66]. Thereafter, the variational principles have been formulated for the linear system at first and extended for the nonlinear system[67][68]. The variational principle so introduced is now applied as a new tool to the macroscopic and statistical treatment of various phenomena. The fundamental assumption is so called "principle of local equilibrium"[68][69], that is, although the system as a whole will not be equilibrium, there exists at every point a state of local equilibrium with local entropy defined by the classical Gibbs formula.

Recently, the method has been applied in various fields of hydrodynamics by W.H.Reid[70], P.Glansdorff and D.F.Hays[71], R.S.Schechter and D.M.Himmelblau[72] and others. The author considered the method proposed by Prigogine may give us an useful tool to understand the hydrological problems described in the last chapter.

In this chapter, the variational formulation is attempted for the basin water behaviour in a kinematical system. The expression partially differs from that after Prigogine because of the particularity of runoff system. The method discussed here is of course not perfect as the numerous problems are yet unsolved, but it will become an initiator for better understanding of runoff phenomena.

2-2 Variational Formulation for Ground Water Behaviour

Let us consider the movement of ground water which occurs in the ground water region G as shown in Fig. II-1. We assume that the aquifer is homogeneous and isotropic for the brief treatment. The fundamental equation of the motion is then written as well known:

$$\gamma \frac{\partial H_g}{\partial t} = \sum_i \frac{\partial}{\partial x_i} \left\{ k \cdot H_g \frac{\partial H_g}{\partial x_i} - f_i H_g \right\} + r \quad . \qquad \text{(II-1)}$$

The symbols in the figure and the equation denote:

G : ground water region,

γ : porosity,

k : permeability coefficient,

H_g : water depth of ground water,

r : recharge intensity per unit area of the ground water region,

x_i : the rectangular coordinate downward positive, i=1,2,

θ_i : inclinations of the impermeable bed, $\sin\theta_i = -\frac{\partial z}{\partial x_i}$,

f_i : $= k \sin\theta_i = -k \frac{\partial z}{\partial x_i}$,

z : elevation of the impermeable bed from the reference horizon.

The water depth H_g considered here may be expressed as of the macroscopic water depth or mean water depth $H_g{}^*$ plus the small arbitrary variations δH_g around the macroscopic water depth. Both of them are functions of space coordinates as well as time, that is,

$$H_g(x_i,t) = H_g{}^*(x_i,t) + \delta H_g(x_i,t) \ , \qquad \text{(II-2)}$$

in which x_i denotes x_1 and x_2, for example $H_g(x_i,t) \equiv H_g(x_1,x_2,t)$.

Moreover, we assume

$$| H_g{}^*(x_i, t) | \quad >> \quad | \delta H_g(x_i, t) | \ . \tag{II-3}$$

Provided the gradient of ground water surface and bed are very small, Dupuit-Forchheimer's assumption leads that the potential energy per unit area of ground water region is proportional to

$$H_g(x_i, t) + z \ . \tag{II-4}$$

Thereafter, the variation of potential energy due to the small variation of water depth δH_g, is given by

$$\delta(H_g + z) \ ,$$

in which z does not change its value due to the variation of water depth, that is,

$$\delta z = 0 \ . \tag{II-5}$$

Multiplying $-\delta(H_g+z)$ to both hand sides of Eq.(II-1) and replacing H_g by $H_g{}^* + \delta H_g$, we obtain the following relationship after simple calculations :

$$- \gamma \cdot \delta(H_g+z) \cdot \frac{\partial \delta H_g}{\partial t}$$

$$= \gamma \frac{\partial H_g{}^*}{\partial t} \delta(H_g+z) + \frac{1}{2} \sum_i k H_g{}^* \delta \left(\frac{\partial(H_g+z)}{\partial x_i} \right)^2 - r \ \delta(H_g+z)$$

$$- \sum_i \frac{\partial}{\partial x_i} \left\{ \left(k H_g \frac{\partial H_g}{\partial x_i} - f_i H_g \right) \delta(H_g+z) \right\} \ .$$

In the deduction of this expression, the higher order infinitesimals are ignored. Integrating this equation over the region G in Fig.II-1 and any time interval, we obtain :

$$- \frac{1}{2} \int_G \int_t \gamma \frac{\partial}{\partial t} \left(\delta(H_g+z) \right)^2 dx_i dt$$

$$= -\frac{1}{2}\int_G \gamma\ (\delta(H_g+z))^2\ dx_i$$

$$= \delta \int_G\int_t [\ \gamma\ \frac{\partial H_g{}^*}{\partial t}(H_g+z) + \frac{1}{2}\sum_i kH_g{}^* \left(\frac{\partial(H_g+z)}{\partial x_i}\right)^2 - r\ (H_g+z)\ ]\ dx_i dt$$

$$+ \int_C\int_t [\ \Big\{\left(kH_g{}^*\frac{\partial H_g}{\partial x_i} - f_1 H_g{}^*\right)\frac{dx_2}{ds}$$

$$- \left(kH_g{}^*\frac{\partial H_g{}^*}{\partial x_2} - f_2 H_g{}^*\right)\frac{dx_1}{ds}\Big\}\ \delta(H_g+z)\ ]\ \ dsdt \leqq 0\ . \qquad \text{(II-6)}$$

In this equation, $\int_G dx_i$ means double integral over the ground water region G, ie. $\int_G dx_1\ dx_2$, and $\int_C ds$ line integral along the boundary C of ground water region. The first term of the right hand side in the equation may also be written as;

$$\delta \int_G\int_t [\ \gamma\ \frac{\partial H_g{}^*}{\partial t}(H_g+z) + \sum_i \left(kH_g{}^*\frac{\partial H_g}{\partial x_i} - f_i H_g{}^*\right)\left(\frac{\partial H_g}{\partial x_i} + \frac{\partial z}{\partial x_i}\right)$$

$$- r\ (H_g+z)\ ]\ dx_i dt. \qquad \text{(II-7)}$$

The equality in Eq.(II-6) should be satisfied only in the case that the water depth $H_g(x_i,t)$ equals to the macroscopic one all over the region at any moment, that is,

$$H_g(x_i,t) \equiv H_g{}^*(x_i,t)\ . \qquad \text{(II-8)}$$

Moreover, the last term on the right hand side of Eq.(II-6) vanishes provided either boundary conditions are given on the boundary C or no flow flux crosses the boundary. In such case, since only the first term remains, we may write the equation of motion as,

$$\delta \int_G\int_t [\ \gamma\ \frac{\partial H_g{}^*}{\partial t}(H_g+z) + \frac{1}{2}\sum_i kH_g{}^* \left(\frac{\partial(H_g+z)}{\partial x_i}\right)^2 - r(H_g+z)\ ]\ dx_i dt = 0, \qquad \text{(II-9)}$$

or approximately

$$\delta \iint_{G}\int_{t} \left[\gamma \frac{\partial H_g^*}{\partial t} (H_g+z) + \sum_i \left(kH_g^* \frac{\partial H_g^*}{\partial x_i} - f_i H_g^* \right) \frac{\partial (H_g+z)}{\partial x_i} - r(H_g+z) \right] dx_i dt = 0 \,. \qquad \text{(II-9')}$$

In the equation, the variation should be taken with respect to only the quantity H_g, keeping the macroscopic water depth H_g^* fixed according to the principle of local equilibrium. As well known, these equations have as their Euler-Lagrangian equation the equation of motion for the macroscopic distributions. These presentations of the problem need the subsidiary condition Eq.(II-8).

As introduced above, although the system is dissipative with respect to the energy, we may rewrite the fundamental equation Eq.(II-1) for the movement of ground water into the variational form as given by Eq.(II-9) or approximately Eq.(II-9').

2-3 Variational Formulation for Stream Water Flow

For the stream water flow in the region S as shown in Fig. II-1, the one dimensional equation of continuity is

$$B_s \frac{\partial H_s}{\partial t} + \frac{\partial Q_s}{\partial s} = 0 \,. \qquad \text{(II-10)}$$

Manning's formula is

$$v = \frac{1}{n} H_s^{2/3} \left\{ - \frac{\partial H_s}{\partial s} + \sin \theta_s \right\}^{1/2} \,. \qquad \text{(II-11)}$$

In these equations, the symbols denote as follows;

H_s : water depth in the stream,

B_s : width of the stream,

Q_s : discharge,

v : mean velocity,

s : distance along the stream, downward positive,

θ_s : bed slope of the stream, $\sin\theta_s = -\frac{\partial z}{\partial s}$,

n : Manning's roughness coefficient.

We write the water depth $H_s(s,t)$ in the stream as the summation of macroscopic depth distribution $H_s{}^*(s,t)$ and the small arbitrary deviations $\delta H_s(s,t)$ around the macroscopic distribution, that is,

$$H_s(s,t) = H_s{}^*(s,t) + \delta H_s(s,t) \ . \tag{II-12}$$

The following inequality is also assumed,

$$|\ H_s(s,t)\ | >> |\ \delta H_s(s,t)\ | \ . \tag{II-13}$$

For the brief treatment, let us consider the case of stream with very gentle slope, that is,

$$\cos\theta_s \simeq 1 \ ,$$

then the variation of the potential energy due to the variation of water depth δH_s may be proportional to

$$\delta\ (\ H_s + z\ )\ ,$$

in which δz is equal to zero because the stream bed does not change due to the variation of water depth. Then the following expression is obtained in the same way as described in the last section.

$$\begin{aligned} -\frac{1}{2} B_s \int_S (\delta(H_s+z))^2 ds \\ = \delta\iint_{S\,t} B_s \frac{\partial H_s{}^*}{\partial t}(H_s+z) + \frac{2}{3}\frac{B_s}{n} H_s{}^{*5/3}\left(-\frac{\partial H_s}{\partial s} - \frac{\partial z}{\partial s}\right)^{3/2} dsdt \\ + \int_t [\frac{B_s}{n} H_s{}^{5/3}\left(-\frac{\partial H_s}{\partial s} - \frac{\partial z}{\partial s}\right)^{1/2}\delta(H_s+z)\ ]_{s=0}^{s=l}\ dt \leqq 0\ , \end{aligned} \tag{II-14}$$

in which the higher infinitesimals are ignored.

Thus it may be concluded that the equality in this equation holds in the case the water depth equals to the macroscopic water depth all over the resion S at any moment.

$$H_s(s,t) = H_s{}^*(s,t) . \qquad \text{(II-15)}$$

Moreover, the equation leads the following variational formulation provided either the boundary conditions are given or no flow flux crosses the boundary.

$$\delta \iint_{S\,t} B_s \frac{\partial H_s{}^*}{\partial t}(H_s+z) + \frac{2}{3}\frac{B_s}{n} H_s{}^{*5/3}\left(-\frac{\partial H_s}{\partial s} - \frac{\partial z}{\partial s}\right)^{3/2} ds\,dt = 0 . \qquad \text{(II-16)}$$

This equation is also written approximately in the form of

$$\delta \iint_{S\,t} B_s \frac{\partial H_s{}^*}{\partial t}(H_s+z) + Q_s{}^*\left(-\frac{\partial H_s}{\partial s} - \frac{\partial z}{\partial s}\right) ds\,dt = 0 \qquad \text{(II-16')}$$

If we treat Chézy flow, we gain the same type of equation, as

$$\delta \iint_{S\,t} B_s \frac{\partial H_s{}^*}{\partial t}(H_s+z) + \frac{2}{3} B_s C' H_s{}^{*3/2}\left(-\frac{\partial H_s}{\partial s} - \frac{\partial z}{\partial s}\right)^{3/2} ds\,dt = 0. \qquad \text{(II-17)}$$

in which C' is Chézy's roughness coefficient. These equations have the equation of motion for the macroscopic behaviour of stream flow as their Euler-Lagrangian equation with the help of the subsidiary condition Eq. (II-15). In this case too, the variation should be taken with respect to only the quantity H_s, keeping the macroscopic $H_s{}^*$ fixed. In other words, the variational form introduced above is mathematically equivalent to the system of equations (II-10) and (II-11) for the stream water flow. Consequently, we establish the variational formulation for the movement of stream water.

2-4 Variational Formulation for Basin Water Behaviour

In this section, we will introduce the variational formulation for the behaviour of the basin water in a system as a whole, on the basis of the expressions obtained in the last two sections.

Analogized from the variational forms introduced in the previous sections, we consider the water movement in a system S and G containing the ground water, stream water and their interactions in the mathematical model as shown in Fig. II-1. We treat then the following variational problem;

$$\delta \left\{ \iint_{G} \int_{t} \boldsymbol{L}_g \ dx_i dt \ + \iint_{S} \int_{t} \boldsymbol{L}_s \ ds dt \right\} = 0 \ . \qquad \text{(II-18)}$$

in which,

$$\boldsymbol{L}_g = \begin{cases} \gamma \dfrac{\partial H_g^*}{\partial t}(H_g+z) + \dfrac{1}{2}\sum kH_g^* \left(\dfrac{\partial H_g}{\partial x_i} + \dfrac{\partial z}{\partial x_i}\right)^2 - r(H_g+z). & \text{(II-19)} \\ \text{or approximately,} & \\ \gamma \dfrac{\partial H_g^*}{\partial t}(H_g+z) + \sum_i \left(kH_g^* \dfrac{\partial H_g}{\partial x_i} - f_i H_g^*\right) \dfrac{\partial (H_g+z)}{\partial x_i} - r(H_g+z). & \text{(II-19')} \end{cases}$$

$$\boldsymbol{L}_s = \begin{cases} B_s \dfrac{\partial H_s^*}{\partial t}(H_s+z) + \dfrac{2}{3}\dfrac{B_s}{n} H_s^{*5/3} \left(- \dfrac{\partial H_s}{\partial s} - \dfrac{\partial z}{\partial s}\right)^{3/2} , & \text{(II-20)} \\ \text{or approximately,} & \\ B_s \dfrac{\partial H_s^*}{\partial t}(H_s+z) + Q_s^* \left(- \dfrac{\partial H_s}{\partial s} - \dfrac{\partial z}{\partial s}\right). & \text{(II-20')} \end{cases}$$

In Eq.(II-18), variation is taken with respect to only H_g and H_s , keeping H_g^* and H_s^* fixed. In this meaning, the quantities $\boldsymbol{L}_g$ and $\boldsymbol{L}_s$ are called "local potential". Now we have the following equations as the Euler-Lagrangian equation for the variational problem Eq.(II-18) and as the natural boundary condition along the boundary between ground water region and the stream, together with subsidiary conditions Eqs. (II-8) and (II-15).

Euler-Lagrangian equation;

$$\gamma \frac{\partial H_g{}^*}{\partial t} - \Sigma \frac{\partial}{\partial x_i} \left\{ kH_g{}^* \left(\frac{\partial H_g{}^*}{\partial x_i} + \frac{\partial z}{\partial x_i} \right) \right\} - r = 0. \tag{II-21}$$

Natural boundary condition along the stream;

$$B_s \frac{\partial H_s{}^*}{\partial t} + \frac{\partial Q_s{}^*}{\partial s}$$
$$+ \left(kH_g{}^* \frac{\partial H_g{}^*}{\partial x_1} - f_1 H_g{}^* \right) \frac{dx_2}{ds} - \left(kH_g{}^* \frac{\partial H_g{}^*}{\partial x_2} - f_2 H_g{}^* \right) \frac{dx_1}{ds} = 0. \tag{II-22}$$

Take $H_g{}^* = H_s{}^*$ at the boundary between ground water and stream, then Euler-Lagrangian equation leads to the fundamental equation of ground water movement and the natural boundary condition to the equation of motion of stream water containing the interaction with ground water. The discharge $Q_s{}^*$ in Eq.(II-22) should be expressed in terms of $H_s{}^*$ and its derivatives with respect to the space coordinates. Consequently, we may write the basin water behaviour as a whole in terms of a simple variational form Eq.(II-18). In other words, it may be understood that the water movement in stream and ground water region occurs as the result of the behaviour of whole basin water which follows the variational principle.

Moreover, since $\boldsymbol{L}_g$ and $\boldsymbol{L}_s$ in Eqs.(II-19), (II-19'), (II-20) and (II-20') do not contain $\frac{\partial H_g}{\partial t}$ and $\frac{\partial H_s}{\partial t}$ explicitly, time t is only a parameter in the variational calculation in Eq.(II-18). Therefore, we may write the variational formulation in the following form too,

$$\delta \left\{ \frac{\partial}{\partial t} \iint \boldsymbol{L}_g \, dx_i dt + \frac{\partial}{\partial t} \iint \boldsymbol{L}_s ds dt \right\} = 0, \tag{II-23}$$

or $$\delta \left\{ \int \boldsymbol{L}_g \, dx_i + \int \boldsymbol{L}_s ds \right\} = 0. \tag{II-23'}$$

It may be easily understood that the type of variation Eq.(II-18) holds for the evolution of the system during arbitrary time interval, in addition the latter form Eq.(II-23) holds for the whole system at any moment. That is, the behaviour of basin water seems to take place within a basin according to the same variational principle in any time interval as at any moment.

The variational form introduced here is not so advantageous over the original fundamental equation in the strict meaning, because we have to treat the same equation of motion as Euler-Lagrangian equation for the variational form. However, it will give us an initiator to understand and penetrate the general law which governs the behaviour of the basin water, further, it will be also available for the treatment of practical problems with helps of appropriate trial functions.

2-5 Physical Significances of Local Potential and Variational Principle

It is apparent that each term of the local potential shown in Eqs.(II-19), (II-19'), (II-20) and (II-20'), has dimension of flux of potential energy in terms of water head. In order to understand the physical significances of the local potentials, we will take them defined by the prime-system, that is, by Eqs.(II-19') and (II-20'). If we multiply H_g+z and H_s+z to Eqs.(II-21) and (II-22), which should be satisfied in the case $H_g = H_g{}^*$ and $H_s = H_s{}^*$, respectively, and thereafter subtract them from the local potentials Eqs.(II-19') and (II-20'), we have other expressions for $\boldsymbol{L}_g$ and $\boldsymbol{L}_s$ as follows:

$$\boldsymbol{L}_g = \sum_i - \frac{\partial}{\partial x_i}\left\{-(H_g+z)(kH_g{}^*\frac{\partial H_g{}^*}{\partial x_i} - f_i H_g{}^*)\right\} , \qquad \text{(II-24)}$$

and

$$\boldsymbol{L}_s = - \frac{\partial}{\partial s}\left(Q_s{}^*(H_s + z)\right)$$

$$+ \, (H_s+z)\left\{(kH_g{}^*\frac{\partial H_g{}^*}{\partial x_1} - f_1 H_g{}^*)\frac{dx_2}{ds} - (kH_g{}^*\frac{\partial H_g{}^*}{\partial x_2} - f_2 H_g{}^*)\frac{dx_1}{ds}\right\}. \tag{II-25}$$

These expressions show that the local potentials L_g and L_s are of the following physical significances:

In the case we change the water depth around the macroscopic ones, keeping the flow fluxes fixed, L_g corresponds to the summation of the inflow flux of potential energy per unit area due to the flow flux and that due to the work done by the pressure, and L_s corresponds to the summation of the inflow flux of potential energy per unit length of stream due to the flow flux and that due to the work done by the pressure.

These significances are rather loosely defined but it is apparent that the local potentials in the prime system correspond to the actual values of inflow fluxes of potential energy demonstrated above, when the water depth H_g and H_s are equal to the macroscopic one $H_g{}^*$ and $H_s{}^*$, respectively. That is, substitute H_g and H_s in the local potentials Eqs.(II-24) and (II-25) by $H_g{}^*$ and $H_s{}^*$, respectively, we get

$$L_g{}^* = \sum_i -\frac{\partial}{\partial x_i}\left\{-(H_g{}^*+z)(kH_g{}^*\frac{\partial H_g{}^*}{\partial x_i} - f_i H_g{}^*)\right\},$$

$$L_s{}^* = -\frac{\partial}{\partial s}\left(Q_s{}^*(H_s+z)\right)$$

$$+ \, (H_s{}^*+z)\left\{(kH_g{}^*\frac{\partial H_g{}^*}{\partial x_1} - f_1 H_g{}^*)\frac{dx_2}{ds} - (kH_g{}^*\frac{\partial H_g{}^*}{\partial x_2} - f_2 H_g{}^*)\frac{dx_1}{ds}\right\},$$

in which $L_g{}^*$ and $L_s{}^*$ denote the local potentials for $H_g = H_g{}^*$ and $H_s = H_s{}^*$ all over the region at any moment.

In the discussion here, it should be noted that we assume very gentle slopes of stream bed and ground water surface. Though there are certain restrictions, the physical significances of each term in local potential Eqs.(II-19) and (II-20) may also be summarized as Table II-1. Thus the local potential is closely related to the potential energy of water. For the generalization of the local potential, if the kinetic energy which is not considered here because of the mathematical difficulty is introduced, we may also obtain the similar expressions of the local potential but in a rather complex form.

As mentioned above, it may be concluded that the behaviour of whole basin water takes place within the basin in such a manner that the local potential closely related to the potential energy has stationary value for the variation of water depth. For the approximate local potential in the prime system, although the variational problem degenerates, the equation (II-18) itself remains theoretically correct and may be available for the brief treatment of the practical problems of certain kinds.

Although the physical significances of the local potentials and the variational principle are rather loosely defined, it is interesting that they are closely related to the potential energy of basin water.

2-6 Applicability of the Variational Technique to the Engineering Problems

As obviously understood from the procedure of the introduction, the variational formulation will be available for the behaviour of basin water as far as expressed satisfactorily by the fundamental equations Eqs.(II-1), (II-10) and (II-11). Furthermore, it should be emphasized that the interaction between the ground water and stream water is also treated in the kinematical system all at once.

For the examples for which the variational technique is useful, we may take the variation of the flood characteristics in the runoff process in the plain region, the variation of behaviour of the basin water due to the artificial discharge or depth controls and other water policies. Although these problems, as a matter of course, are very difficult in the rigorous meaning because of the complexity of phenomena, the variational technique will give us an useful tool to find the approximate solutions with help of the modern development of the computer technique if the appropriate trial functions are adopted. In the present paper, we consider the averaging process of behaviours of basin water in the runoff process with respect to the recession characteristics of the ground water runoff by means of the variational formulation.

Chapter 3 VARIATION OF RECESSION CHARACTERISTICS OF GROUND WATER RUNOFF IN RUNOFF PROCESS

3-1 Basic Remarks

As described in the opening chapter in this Part II, the actual river basin consists of links of several regions and each of them forms a kinematical system in the runoff process individually. Furthermore, the field of the runoff phenomena may be pronounced as an uni-directional links of these kinematical systems. Realizing these facts, Dooge[27)] and Nash[28)] have developed their runoff models as links of reservoirs. That is to say, in their studies the reservoirs are regarded as kinematical lumped systems.

It is obvious that the research in Part I in the present study treats the ground water runoff in a watershed as a lumped system. As we have seen, however, we can not make the recession characteristics clear by the lumped model if the interaction between the ground water and the stream water takes place and the distributions of water state in time and space change in a basin, such as Kado and Kunikane basins with rather large catchment area.

The author recognized that the runoff phenomena would variate its characteristics within the region which is considered as a system with various distributions of hydrologic quantities. In this chapter, the discussions are made theoretically on the problems of what kinds of transformations and averagings happen in the system with respect to the recession characteristics of ground water runoff. In other words, to relate the recession characteristics of lumped system in Part I to those of the individual small regions in the runoff process is the main task in this chapter. Further, it aims also to solve the various effects of distributions of hydrologic quantities and the interactions among various factors, which have been left unsolved in Part I. As the first step, since the unconfined component plays an important role in the ground water runoff, only the unconfined component is treated. For these purposes, as will be easily understood, the variational technique formulated in the previous chapter may be available.

3-2 Assumptions

The behaviour of water within a system are expressed by the variational principle Eqs.(II-18) or (II-23). If we use proper trial functions for H_g, H_s, H_g* and H_s*, and define the values of the parameters in the trial functions in such a way that the variational principle holds, we will obtain the appropriate expressions for the behaviour of basin water as a whole. Strictly speaking, the procedure should be repeated until the obtained values of the parameters in the trial functions H_g and H_s become equal to the assumed ones for H_g* and H_s*. For the brief treatment, however, we treat the method by several assumptions.

For the discussions in this chapter, we consider the region as shown in Fig. II-2 which acts as a kinematical system. The region is assumed to consist of n-ground water regions with different characters and of n-stream reaches. Further, besides the individual regions and reaches being prismatic and homogeneous, flows in the ground water regions and through the stream reaches are also assumed one-dimensional. The new symbols indicate;

l : the width of the ground water region or the length of the each stream reach,

G_j : j-th ground water region (j=1,2,·····,n,),

S_j : j-th stream reach,

H_g : water depth in the ground water region,

H_s : water depth in the stream reach,

and the subscript g and s denote the values for the unconfined component of ground water and for the stream water, respectively. The subscript j indicates the value for the j-th ground water region and the stream reach.

With respect to the ground water flow, the behaviour in each ground water region G_j is assumed to follow the behaviour of the unconfined flow obtained in Part I, if in the region the interactions between hydrological situations do not take place. In other words, if we treat the recession characteristics, we may define several quantities or relationships introduced in the second chapter in Part I for each region, that is,

$$H_{gj}{}^*(x,t) = \frac{1}{\lambda_j t + 1}\left\{-\frac{\lambda_j}{2\beta_j}x^2 + \frac{h_{uoj} - H_{uoj}}{L_j}x + \frac{\lambda_j}{2\beta_j}L_j x + H_{uoj}\right\}, \qquad \text{(II-26)}$$

$$\lambda_j = \frac{2\beta_j}{L_j}(H_{uoj} - h_{uoj}), \quad (= a_j), \qquad \text{(II-27)}$$

$$a_j = K_j\sqrt{Q_{uoj}}\,, \qquad \text{(II-28)}$$

for Eqs.(I-20), (I-21) and (I-35), respectively. Based on the above assumption, Eq.(II-26) is used for the approximate trial functions $H_{gj}{}^*$ in the variational technique in Eq.(II-23) in each region G_j.

According to the results obtained through the research in Part I, each recession state is expressed considerably well by the equations on the unconfined component even for the rather large basins, although the values for each occasion scatter. In other words, it seems that the mean behaviour of the ground water runoff in the whole region G as a lumped system may also follow that of the unconfined one. Therefore, the same type of equation as Eq.(II-26) is also available as a trial function H_g for the behaviour of ground water runoff in a whole region. However, the parameters involved in the trial functions mentioned above may vary in the averaging process due to the interactions and compensation among the hydrologic quantities in individual regions G_j. In the macroscopic meaning for a lumped system, the behaviour of ground water runoff may be assumed to take place in each region G_j in the same way as that in the whole region G due to the interactions and the compensations. Thus we use the trial functions H_{gj} for H_g in each region as follows:

$$H_{gj}(x,t) = \frac{1}{\lambda t + 1}\left\{-\frac{\lambda}{2\beta_j}x^2 + \frac{h_{uoj} - H_{uoj}}{L_j}x + \frac{\lambda}{2\beta_j}L_j x + H_{uoj}\right\}, \quad \text{(II-29)}$$

in which value λ is common to all regions G_j ($j=1,2,\cdots\cdots,n$).

Regarding the channel water, we assume that the stream water flowing into the system is also characterized by Eq.(I-20) with recession coefficient λ_s, since it may be understood as the runoff discharge of ground water out of the upstream regions. The coefficient λ_s in the stream discharge is also assumed to keep its value constant, if no interaction would occur in the system. Moreover, assume the water depth distributed linearly within the stream reach S_j from the value H_{sj} at upstream end to that H_{sj+1} at downstream end, then we may use the following trial functions for $H_{sj}{}^*$ and H_{sj} in the reach S_j;

$$H_{sj}{}^*(s,t) = \frac{1}{\lambda_s t + 1}\left\{\frac{h_{uoj+1} - h_{uoj}}{l_j}s + h_{uoj}\right\}, \quad \text{(II-30)}$$

$$H_{sj}(s,t) = \frac{1}{\lambda t + 1}\left\{\frac{h_{uoj+1} - h_{uoj}}{l_j}s + h_{uoj}\right\}, \quad \text{(II-31)}$$

in which

$$H_{sj}(t) = H_{gj}(L_j, t) \quad \text{(II-32)}$$

$$H_{sj}(0) = h_{uoj} . \qquad \text{(II-33)}$$

In addition, we further assume that the relationship between the stream discharge and the water depth holds approximately in the reach S_j as follows;

$$Q_{sj} = C_j H_{sj}^{\;2} . \qquad \text{(II-34)}$$

3-3 Theoretical Treatment

In this article, we derive theoretically the expressions on variations of recession characteristics in the system by means of variational principle in the type of Eq.(II-23'). That is, the equation is written for the behaviour of ground water runoff in the system as,

$$\delta \left\{ \sum_j \int \boldsymbol{L}_{gj} \; dxdy + \sum_j \int \boldsymbol{L}_{sj} \; ds \right\} = 0 . \qquad \text{(II-35)}$$

To calculate the integrals in the blacket, we use the expressions of prime system for local potentials Eqs.(II-19') and (II-20') in terms of the trial functions Eqs.(II-26), (II-29), (II-30) and (II-31).

Since the trial functions for ground water flow Eqs.(II-26) and (II-29), as we have seen in the section 2-1 in Part I, are obtained as the solution of the approximate equation (I-16) instead of Eq.(I-15), we may assume that these trial functions are approximately the solution for the original equation (I-15) for the brief treatment. Then the following relationship should hold for the trial functions,

$$\gamma \frac{\partial H_g^{\,*}}{\partial t} = k \cdot H_g^{\,*} \cdot \frac{\partial^2 H_g^{\,*}}{\partial x^2} + k \cdot \left(\frac{\partial H_g^{\,*}}{\partial x} \right)^2 . \qquad \text{(II-36)}$$

If we multiply γH_g to both hand sides of this equation and subtract $\boldsymbol{L}_g$, we obtain approximately another expression of local potential as follows:

$$\boldsymbol{L}_{gj} = \gamma_j \frac{\partial}{\partial x} \left\{ \beta_j H_{gj}^{\,*} \frac{\partial H_{gj}^{\,*}}{\partial x} H_{gj} \right\} , \qquad \text{(II-37)}$$

for the trial functions H_{gj}^* and H_{gj}. Thereafter, we obtain the functional of the variational form by the simple calculation as follows:

$$\sum_j \iint \mathcal{L}_{gj}\, dxdy$$

$$= \sum_j \frac{\gamma_j \beta_j l_j}{(\lambda_j t+1)^2(\lambda t+1)} \left\{ \frac{(h_{uoj}^2 - H_{uoj}^2)(h_{uoj} - H_{uoj})}{L_j} - \frac{\lambda_j}{2\beta_j}(h_{uoj}^2 + H_{uoj}^2)L_j \right\}. \qquad \text{(II-38)}$$

On the other hand, the integral at the stream region becomes

$$\sum_j \iint \mathcal{L}_{sj}\, ds$$

$$= \sum_j \frac{1}{(\lambda_s t+1)^2(\lambda t+1)} \left\{ -\frac{D_j \lambda_s l_j}{3}(h_{uoj+1}^2 + h_{uoj}h_{uoj+1} + h_{uoj}^2) - \frac{C_j}{3}(h_{uoj+1}^3 - h_{uoj}^3) \right\}. \qquad \text{(II-39)}$$

Therefore, the value λ which satisfies Eq.(II-35), is defined by the following equation

$$\frac{\partial}{\partial \lambda}\left\{ \sum_j \iint \mathcal{L}_{gj}\, dxdy + \sum_j \iint \mathcal{L}_{sj}\, ds \right\} = 0. \qquad \text{(II-40)}$$

To find the behaviour of system as a whole, in average meaning, we write the recession factors λ_j as follows;

$$\lambda_j = \lambda + \delta\lambda_j, \qquad \text{(II-41)}$$

and assume

$$|\lambda| \gg |\delta\lambda_j|. \qquad \text{(II-42)}$$

In other words, we assume briefly that the behaviour of water in the individual small regions are nearly equal to that of the system as a whole. Although this might be an unrealistic assumption, the author has adopted it as a first step of the research. Then, the above equation (II-40) by which we define the unknown parameter λ, is reduced to

$$\sum_j \left\{\frac{1}{(\lambda+\delta\lambda_j)t+1}\right\}^2 (\lambda_s t+1)^2 \prod_{p=1}^{n}\left\{(\lambda+\delta\lambda_p)t+1\right\}^2 \left\{\Lambda_{gj}-\Gamma_{gj}(\lambda+\delta\lambda_j)\right\}$$

$$+\prod_{p=1}^{n}\left\{(\lambda+\delta\lambda_p)t+1\right\}^2 \sum_j\left\{\Lambda_{sj}-\Gamma_{sj}\lambda_s\right\} = 0, \qquad \text{(II-43)}$$

in which

$$\Lambda_{gj} = \frac{\gamma_j\beta_j l_j(h_{uoj}^2 - H_{uoj}^2)(h_{uoj} - H_{uoj})}{L_j}, \qquad \text{(II-44a)}$$

$$\Gamma_{gj} = \frac{\gamma_j l_j L_j}{2}\cdot(h_{uoj}^2 + H_{uoj}^2), \qquad \text{(II-44b)}$$

$$\Lambda_{sj} = -\frac{C_j}{3}(h_{uoj+1}^3 - h_{uoj}^3), \qquad \text{(II-44c)}$$

$$\Gamma_{sj} = \frac{B_s l_j}{3}(h_{uoj+1}^3 + h_{uoj}h_{uoj+1} + h_{uoj}^2). \qquad \text{(II-44d)}$$

In the rigorous meaning, the above equation should be satisfied for arbitrary time t. Since the above equation is arithmetic equation of the order 2n, the coefficients of the terms of every power of t should be equal to zero. However, we can not define generally such value λ as to hold all the conditions, because of the assumptions and the several approximations. Hence, we treat here the approximate solution. Since the coefficient of the term of t^p is also of p-th power with respect to λ and the value of λ is generally small and of the order $\frac{1}{10} \sim \frac{1}{100}$, the coefficient of the term t^p may be ignored if p is large. By the simple calculations, we finally get the following solution λ^* as the first approximation of λ with respect to the coefficients of the small power of t, that is,

$$\lambda^* = \frac{\Sigma \Lambda_{gj} + \Sigma(\Lambda_{sj} - \Gamma_{sj}\lambda_s)}{\Sigma \Gamma_{gj}} \qquad \text{(II-45)}$$

in which Λ_{gj}, Γ_{gj}, Λ_{sj} and Γ_{sj} are given by Eq.(II-44).

Although this equation is rather an approximate one, the value obtained corresponds to the recession characteristics at the downstream end of the system or also the one averaged all over the region which consists of n-ground water regions and stream reaches with different characteristics. As easily known from the comparison of Eqs.(II-26) and (II-29) with (I-21) and (I-25), λ^* correspond to the recession factor a^* of the ground water runoff for the system as a lumped system, in addition λ_j to the recession factor a_j for the individual regions G_j. In this meaning, we read a^* in stead of λ^* on the left hand side of Eq.(II-45). That is,

$$a^* = \frac{\Sigma \Lambda_{gj} + \Sigma(\Lambda_{sj} - \Gamma_{sj}a_s)}{\Sigma \Gamma_{gj}} \qquad \text{(II-46)}$$

in which a_s denotes the recession factor of the stream discharge as the ground water runoff which flows into the region from the upstream.

3-4 Variation of Recession Characteristics in the Runoff Process

3-4-1 Fundamental Relationship

The equation (II-46) is the fraction of series. If we divide each term in the first series in the numerator, by the corresponding term in the denominator, the remainder becomes nearly equal to the recession coefficient a_j for each ground water zone G_j, that is,

$$\frac{\Lambda_{gj}}{\Gamma_{gj}} \simeq \frac{2\beta_j}{L_j^2}(H_{uoj} - h_{uoj}) = a_j \, . \qquad \text{(II-47)}$$

Write m_j for H_{uoj}/h_{uoj}, then value m_j is constant for the basin with constant K_j in Eq.(II-28), as seen from Eq.(II-47), and we gain approximately the following relationship with help of Eq.(II-44), (II-46), (I-36) and (I-37)

$$\Gamma_{gj} = \left(\frac{1}{\beta_j}\right)^2 L_j^2 (m_j+1) Q_{uoj} \quad , \tag{II-48}$$

where Q_{uoj} denotes the initial discharge out of the j-th ground water zone G_j. Therefore, Eq.(II-28), (II-47) and (II-48) lead to the expression

$$\Lambda_{gj} = \left(\frac{1}{\beta_j}\right)^2 L_j^2 (m_j+1) K_j Q_{uoj}^{3/2} \quad . \tag{II-49}$$

On the other hand, if the channel is prismatic and uniform, that is, D_j and C_j are constant through the stream reaches, we may write

$$\Lambda_{sj} = \frac{1}{3\sqrt{C}} (Q_{uos}^{3/2} - Q_{uo}^{3/2}) \tag{II-50}$$

and

$$\Gamma_{sj} = \frac{B_s l}{C} Q_{uo} \quad , \tag{II-51}$$

in which Q_{uo} is the initial discharge at the outlet of the system, and Q_{uos} is that at the upstream of this region. Therefore, we have the following expression for a^*.

$$a^* = \frac{\sum p_j q_j^{3/2} K_j + \frac{1}{3\sqrt{C}}(q_s^{3/2} - 1) - \frac{B_s l}{C} q_s^{3/2} K_s}{\sum p_j q_j} \tag{II-52}$$

in which

$$p_j = \left(\frac{1}{\beta_j}\right)^2 L_j^2 (m_j+1) \tag{II-53a}$$

$$q_j = Q_{uoj} / Q_{uo} \quad , \tag{II-53b}$$

$$q_s = Q_{uos} / Q_{uo} \quad . \tag{II-53c}$$

In addition, if we define the recession factor K^* which is correlated with a^* according to Eqs.(II-28) or (I-35), for the recession characteristics of the system as a whole, the value K^* may be written in terms of the recession factors K_j in each region as;

$$K^* = \frac{\sum p_j q_j^{3/2} K_j + \frac{1}{3\sqrt{C}}(q_s^{3/2} - 1) - \frac{B_s l}{C} q_s^{3/2} K_s}{\sum p_j q_j} \quad . \qquad \text{(II-54)}$$

Gathering the water out of each ground water zone and these characteristics interacting, the stream water of which recession characteristics is K_s when it flows into the region, will flow out from the outlet of the region with the value K^*. Equation (II-54) indicates that the value K^* is a weighted mean with respect to the value K_j for the individual zone and the value K_s for the stream. In addition, the weights are defined by the geological and geographical factors as well as the initial distributions q_j and q_s within the region.

The quantity p_j in Eq.(II-54) consists of the geological, topological and hydraulic factors L_j, β_j and the initial state m_j of water in the individual region. As inferred from the discussions in Part I, the value m_j should be constant for a region with definite recession factor K_j in disregard of the initial state. Since the geological, topological and hydraulic quantities are invariant for a specified region, the quantity p_j should be also invariant. In addition, the factors such as C and B_s may be also considered as constants in the system. Therefore, what causes the variation of recession factor K^* for the whole basin for each occasion, if any, must be the factors $q(q_j$ and $q_s)$.

Though the above discussion is restricted to the case shown by Eq. (II-42), it may seem that the averaging process and interactions cause the variation of recession characteristics in the runoff process. In the actual watershed, it is apparent that besides being averaged in a system, the recession characteristics are averaged repeatedly through many systems in the runoff process.

3-4-2 Recession Characteristics in the Compound Watershed

3-4-2-1 Averaging Process of Recession Characteristics

As we have seen in the last article, the temporal changes of the factor q_j and q_s cause the variation of the recession factor $K*$. The factors q, which indicate the ratios of discharges from the small regions Q_{uoj} and of inflow through stream Q_{uos} to the total discharge Q_{uo} at the outlet, may change their values due to the rainfall distributions case by case. In Part I, the recession factor K is defined for each basin as lumped system. That is, the value estimated in Part I is considered to correspond to the value $K*$ which is averaged within the basin.

The discussion mentioned above and Eq.(II-54) lead that the phenomena in the basins for which we can observe the definite K value for any recession states may correspond to either of the following cases:

a) q_j and q_s are almost constant for every case,

b) the recession factor $K*$ is not substantially influenced by the q values, regardless the changes of q values.

In the small basins, the geological and geographical factors may be considered to distribute uniformly all over the region in the average sense. The small basin in general will be often covered entirely by a rainfall region, and the spatial distributions of rainfall are nearly the same state for any rainfall. The small basin may be thus considered as of the case a) described above. Regarding the above discussion, it should be remembered that we have observed and evaluated the definite K value for the actual small basin as a whole. For these examples, we may take the watersheds such as Arakura, Takasu and Tsurugi. In other words, this fact suggests that we may treat such small basin as an unit basin for the studies on ground water runoff.

On the contrary, in the very large basin the spatial and temporal distributions of rainfall will change their state in wide divergences. The runoff in such large basin, however, takes place in various ways in many regions with different properties. Since the roles of the individual basins are very slight in the whole basin, no matter how the q-values variate case by case, K^* value may be kept constant because of the dominant averaging processes in statistical meanings. This fact seems to be closely related to the Central Limit Theorem in Statistics. Although there remains the problem to be discussed whether the coefficients of K_j and K_s in the weighted mean Eq.(II-54) are independent or not, if the phenomena interact each others within very large regions, the distributions of K^* may asymptote to the normal distribution with variance of

$$\sum_j (K_j)^2(\sigma_j^2) + (K_s)^2(\sigma_s^2) ,$$

in which σ_j^2 and σ_s^2 indicate the variances of the weights referring to the j-th region and stream in the averaging process, respectively. Since the value q_j and q_s become very small for the large regions in which large number of the individual regions are included, in addition all of K_j, K_s, σ_j^2 and σ_s^2 may be considered smaller than unity, the variance of the recession factor K^* may be very small and we may observe the definite value K^* for the whole basin. This case which corresponds to the above case b), will explain well the tendency seen for the very large basins in Section 2-6-3.

In the basin of which catchment area is neither sufficiently large for the statistical treatment nor small as the unit basins, the runoff phenomena in the various individual regions will have appreciably large effects on the water behaviour in the whole basin. Therefore, it may be expected that the various states of rainfall distributions within the basin result in the variations of recession factor K for each recession state according to Eq.(II-54), as seen in Section 2-6-3.

With respect to the effect of the stream water on the averaging process, since the value q_s is smaller than unity and the other factors as to the stream characteristics such as C, B_s, l and K_s are positive, it will be understood that the stream plays a role to reduce the recession factor K in the runoff process. This is one of the reason why we observe the small value of the recession factor K in the large basin rather than in the upstream gauging station for the small basin.

As discussed above, the tendency of recession characteristics observed in the last chapter may be explained qualitatively by the averaging process expressed by Eq.(II-54).

3-4-2-2 Discussions

If we examine the averaging process and the recession characteristics in the complex regions based on the characteristics in the small individual regions, many hydrologic data should be necessary. Unfortunately, there are very few such detailed data for sufficient discussions. Therefore, we have to handle the actual problems by another way, that is, discussion on the value K for discharge at the gauging stations located along a stream. Here, we will check and examine the expression Eq.(II-54) for the Nagara River and the Yura River for the examples. In both of these river basins, several gauging stations are located along the main rivers. For the brief treatment, we choose two gauging stations, and consider the variation of the recession characteristics in the basin between these two stations as shown in Fig. II-3. Let us call the region between these stations as Basin 1. As Basin 1, the region between Tsurugi and Kamita gauging stations are treated for

Nagara River and region between Arakura and Kado gauging stations for Yura River. For the discussion, the available data obtained in Part I are treated for the recession factor K for each basin.

In order that the results of the preceding section may be useful, we consider the recession factor K at Tsurugi and Arakura to be K_s for the stream flow into Basin 1, and the recession factor K at Kamita and Kado to be K^* in Eq.(II-54), respectively. Moreover, we write the recession factor K in Basin 1 K_1.

Then, based on these values we may discuss the averaging process and interaction promesses in Basin 1. That is, since the values K^* and K_s are known quantities, the unknown factor K_1 and the weights for the averagings may be estimated by Eq.(II-54) and examined. In the practical computation, the second term in the numerator of Eq.(II-54) are ignored approximately for the brief treatment, and the values q_1 and q_s we calculated by the values in Table II-2 in which the main hydrologic data used are listed.

The distributions of K_1 value and the ratio of the weight in the averagings given by Eq.(II-54) are shown in Figs. II-4 and II-5. In the figures we can see the concentrative distributions around certain values with respect to both K_1 and the weight $\frac{1}{p}\cdot\frac{B_s l}{C}$. In other words, these values seem to be almost constant for the region, in spite of various values of q_1 and q_s for each recession occasion. This proves that the relationship Eq.(II-54) holds approximately for the recession characteristics in the actual river basin. Although there are still many problems to be solved, such as the validity of the assumption to adopt Basin 1 as a kinematical system, we may conclude from the fact mentioned above that the variation of recession characteristics in the rather large basin is caused by the temporal distributions of water state within the basin, in addition the averaging of recession states takes place within the basin according to the expression Eq.(II-54).

The discussion in this section has been made on the averaging process within a region as a kinematical system, such averaging process will take place repeatedly through the links of many kinematical systems in turns.

Detailed conclusions cannot be drawn due to the insufficient number of actual hydrologic data, however, the discussion may indicate the possibility that Eq.(II-54) will become a new tool to understand the variation or the averaging of recession characteristics in the runoff process. Moreover, the equation seems to become a clue to the statistical treatment of the ground water runoff.

Chapter 4 SUMMARY AND CONCLUSION

As the first step of studies of which the target is to clarify the runoff phenomena in connection with the behaviour of basin water within the watershed, the author has made the variational formulation for the movements of ground water, stream water and their interactions as a whole. The analyses indicate that the evolution of water behaviour takes place in a region as a kinematical system so that the integral of the local potential takes the stationary value. As the result following to such behaviour, the runoff phenomena and/or the interactions between water components appear in our sights. The physical significance of local potential is rather loosely discussed because of simplification and several assumptions. It should be noted, however, that the local potential is closely related to the potential energy of the water within the basin. Moreover, the result obtained through the analyses suggests the possibilities that we may understand synthetically the runoff phenomena in connection with the behaviour of water components as a whole. In addition, the variational form introduced seems to be available for many other engineering problems. The further generalization of the variational formulation and its physical significances are now in progress.

Regarding the recession characteristics of ground water runoff, the application of the method introduced has been attempted. The expression derived in this part seems to explain considerably well in what way the recession characteristics change through the runoff process which involves the various individual regions with different properties. The reason why we have observed the definite value of the recession factor K obviously in the small basin and in the very large basin, has been also explained qualitatively by the expression (II-54). Then the problems were reduced in the uniformity of hydrologic quantities for small basins and the repeated statistical process for the large basins. For the considerably large basin, the variation of the recession characteristics are explained well by the temporal change of the distributions of water within the basin.

The great difficulty encountered when we make the method useful for the practical problem is as to what region should be taken as a kinematical system for runoff phenomena. For the actual basin, there are many problems yet unsolved, and further investigations should be made for the detailed problems in future. However, the method proposed here contributes to the better understanding of runoff and also to relate the mechanism of runoff to the statistical treatments.

We have only noted one problem of the applications, however, many hydraulic and hydrological problems such as changes of behaviour of channel water and/or ground water due to artificial discharge controls may be satisfactorily analyzed with cooperation of the variational technique and the appropriate trial functions.

CONCLUSIVE STATEMENT

Recently the study of the long-term runoff process has developed rapidly in wide divergences with respect to the water resources problems and its achievements have become a centre of attraction in the field of hydrology. However, when the author started the research works, only few had been carried on the problems and the hydrologists were beginning to focus their attentions to the problems. At that time it was Professor Tojiro Ishihara who encouraged the author to tackle with the attractive research project, and Professor Yasuo Ishihara who gave the author the vivid suggestions.

As often stated, the research in the present paper has been made essentially focused to the kinematical characteristics of the ground water runoff. This point of view was apparently stimulated by the research works[24)35)36)50)52)] on flood runoff which had been promoted by many staffs and their colleagues during long years under the supervision of Professor Tojiro Ishihara at Kyoto University.

At the first step of the approach, the exponential recession of the stream discharge was examined, and the reason why the exponential recession equation aparts from the long tails of the actual hydrograph was also discussed. Through these procedure, the question as to what kinds of components which affect on the characteristics in different way exist in the ground water runoff, has been discussed. Then the phenomenological considerations and the field observations have led the attention to the different mechanisms between the flow through the unconfined and confined aquifers. The effect of these components to the behaviour of stream discharge due to the ground water runoff has been discussed by means of the simple models, at the first stage with respect to the recession characteristics and thereafter to the variation of the ground water runoff due to rainfall.

Several problems which had arisen in the procedure of the first approach for the actual watersheds required a more general treatment which enabled us to discuss the ground water runoff in connection with other water components within the basin and the interactions among them all at once. Then the author has devoted his effort to find the law or the quantity which govern the behaviour of basinwide water in a kinematical system. As a result of many considerations, the variational formulation has been made for the flow in the ground water aquifer and the stream water, in addition their interactions in a kinematical system.

The detail conclusions obtained throughout the present paper will be omitted here since they have been stated at the last chapter of each Part. As the conclusive statement, the brief review of the present paper is summarized as follows.

Part I concerns with the variation of ground water runoff by means of the idealized runoff models. This approach substantially treats the runoff models based on the mechanism of water at the outlet of aquifers to the ground surface. In this meaning, the approach may be stated as a newly developed analysis because many researches have treated the problems with respect to the models introduced from the regime of water storage within basins according to the exponential recession.

In the present paper, the flow from the unconfined and confined aquifers has been treated. Besides the kinematical characteristics of the former component being somehow similar to the diffusion process, the latter component is mainly governed by the pressure gradient. In the formulation of the runoff models, the attention has been focused to the characteristics for each of these components. The behaviour of the runoff discharge has been considered theoretically regarding the models so formulated.

It has been made clear that the unconfined component supplies a little amount of discharge but it supplies the stream discharge unceasingly in a prolonged period. In other words, the variation of the runoff discharge of this component is limited in a narrow band but very slow, so that, it may be demonstrated that the unconfined component plays an important roles in the runoff process in an extended period. The physical significances of the parameters included in the equations are also clarified with respect to the geological and topological factors within the watersheds.

Regarding the confined component, the runoff discharge reaches a considerable large amount due to the rainfall but ceases very fast in a few days. This component is of the exponential recession and the recession factor in the exponential index is an invariant for a certain watershed since it consists of the geological and topological factors of the basin.

As the results of the hydrograph analyses in the actual basins, the normal recession curves are clearly defined for both the components in small mountainous basins and certain basins with very large catchment area, but in the basins of middle size the recession state changes in each occasion due to the temporal change of the hydrologic quantities within the basins. In other words, the facts suggest that small mountainous region with catchment area about 100 $\sim$ 300 km^2 may be treated as the unit basin for the ground water runoff.

Part II is essentially based on the conceptions that the runoff phenomena appear as only one phase of the behaviour of the basinwide water. And the variational formulation has been accomplished as a new tool for the better understanding of the behaviour of whole basinwide water--as the first step in the present paper, the ground water and the stream water are treated as the water components. In the variational formulation, the concept of local potential closely related to the ponential energy of the water components is newly introduced. Thereafter, it is made clear that the behaviour of the water in a kinematical system takes place so that the integral of the local potential takes the stationary value in both in any time interval of the evolution and at any moment.

Considering the actual basin as links of several kinematical system, the variational technique has been approximately applied to solve the problems as to the variation of recession characteristics of the unconfined component in the runoff process and the averaging process of several hydrologic quantities in the various individual regions within the whole region as a kinematical system. Then the recession factor of the unconfined component for the lumped system is expressed by the weighted mean of those of the small individual regions. The weight of the averaging process consists of the geological and topological factors together with the distributions of water within the basin.

The reason why we can obtain the definite recession factor has been reduced to the homogeneity of the water distributions for small mountainous basins and the dominant averaging process in the statistical meaning for certain basins with very large catchment area. In addition, the variation of the recession characteristics in each occasion for basins of middle size is due to the change of the distributions of water in time and space. Therefore, the method will contribute to understand the runoff phenomena in the rather complex basins in connection with the several characteristics of water components in the small regions.

As a matter of course, the variational formulation in the present study is yet incomplete, however it will contribute to the better understanding of the hydrologic phenomena, in addition it will give us an useful tool to treat the various engineering problems. The generalization of the variational formulation and the physical meaning of the local potential is now in progress.

TABLES AND FIGURES

Table I-1 Catchment Areas of Watersheds

	River	Watershed, Gauging - station	Catchment area km^2
Kansai district	Yura River	Kado	585.0
		Arakura	159.0
	Yoshino R.	Terao	253.0
	Kako R.	Kunikane	1,674.0
	Kizu River	Kamo	1,456.0
		Inooka	1,559.0
Central Japan	Nagara River	Takasu	65.3
		Tsurugi	223.0
		Sugihara	102.0
		Shimotsuhara	118.2
		Horado	311.0
		Kamita	713.0

Table I-2 Recession Limbs in Hydrograph Analyses

River	Watershed, Gauging Station	Symbol	Peak Discharge Day	Peak Dis-charge m^3/s
Kako River	Kunikane 1674 km^2	Ku - 1	March 7, 1950	828.1
		Ku - 2	July 20, 1954	196.7
		Ku - 3	July 31, 1954	229.0
		Ku - 4	Sept. 29, 1954	187.3
		Ku - 5	July 24, 1956	184.3
		Ku - 6	Sept. 27, 1956	1722.0
		Ku - 7	Oct. 31, 1956	370.5
		Ku - 8	July 28, 1957	304.0
		Ku - 9	July 4, 1958	218.0
Yoshino River	Terao 253 km^2	Y - 1	July 6, 1949	68.1
		Y - 2	July 13, 1951	357.3
		Y - 3	Nov. 5, 1952	225.3
		Y - 4	July 24, 1955	128.5
		Y - 5	Aug. 27, 1955	1331.4
		Y - 6	July 23, 1958	145.6
Yura River	Arakura 159 km^2	A - 1	Aug. 3, 1950	88.6
		A - 2	Oct. 6, 1950	52.8
		A - 3	Sept. 15, 1952	42.3
		A - 4	June 8, 1953	90.7
		A - 5	July 5, 1953	172.1
		A - 6	Sept. 26, 1953	331.8
		A - 7	July 6, 1954	35.8
		A - 8	Sept. 29, 1954	19.5
		A - 9	June 19, 1955	15.5
		A -10	July 7, 1955	36.6
		A -11	July 24, 1956	92.3
		A -12	April 23, 1957	53.2
		A -13	Oct. 7, 1957	12.6
		A -14	Oct. 8, 1959	18.1
		A -15	Oct. 19, 1959	19.9
		A -16	Nov. 4, 1959	12.3

Table I-2 Recession Limbs in Hydrograph Analyses (Continued)

River	Watershed, Gauging Station	Symbol	Peak Discharge Day	Peak Dis- charge m^3/s
Yura River	Kado 585 km^2	K' - 1	July 30, 1949	229.0
		K' - 2	Oct. 6, 1949	69.2
		K' - 3	July 11, 1952	386.2
		K' - 4	July 30, 1954	66.6
		K' - 5	Sept. 28, 1954	53.1
		K' - 6	July 7, 1955	78.2
		K' - 7	July 24, 1955	13.2
		K' - 8	July 24, 1956	251.8
		K' - 9	April 24, 1957	102.9
		K' -10	Oct. 7, 1957	68.8
		K' -11	Oct. 8, 1959	61.2
		K' -12	Oct. 19, 1959	76.0
Nagara River	Takasu 65.3 km^2	T - 1	Sept. 17, 1965	41.6
		T - 2	July 21, 1965	25.7
		T - 3	Nov. 6, 1959	3.7
	Tsurugi 223 km^2	Ts - 1	Sept. 17, 1965	300.0
		Ts - 2	July 23, 1965	471.0
		Ts - 3	July 19, 1964	203.0
		Ts - 4	Aug. 31, 1963	46.8
		Ts - 5	Nov. 6, 1959	10.2
		Ts - 6	Oct. 7, 1954	108.0
		Ts - 7	Oct. 10, 1953	12.5
		Ts - 8	Nov. 5, 1961	8.3
	Kamita 713 km^2	Ka - 1	Sept. 17, 1965	613.0
		Ka - 2	July 23, 1965	959.0
		Ka - 3	July 19, 1964	390.0
		Ka - 4	Sept. 1, 1963	129.0
		Ka - 5	Nov. 4, 1959	163.0
		Ka - 6	Oct. 7, 1954	55.0
		Ka - 7	Oct. 10, 1953	53.2
		Ka - 8	Nov. 1, 1961	25.1
	Sugihara 102 km^2	S - 1	Sept. 18, 1965	58.2
		S - 2	July 23, 1965	50.2
		S - 3	Oct. 4, 1954	13.0
		S - 4	July 30, 1954	65.0

Table I-3 Recession Factor α and Initial Discharge of Confined Component

River	Watershed	Symbol	Q_{co} m^3/s	Recession Factor α 1/day	Recession Factor $α_o$ 1/day
Kako River	Kunikane 1674 km²	Ku - 1	53.3	-	0.443
		Ku - 2	6.9	-	
		Ku - 3	23.5	0.387	
		Ku - 4	16.6	0.363	
		Ku - 5	16.4	0.421	
		Ku - 6	26.8	0.431	
		Ku - 7	21.3	-	
		Ku - 8	27.4	0.504	
		Ku - 9	28.7	0.554	
Yoshino River	Terao 253 km²	Y - 1	23.9	0.569	0.634
		Y - 2	109.7	0.618	
		Y - 3	37.0	0.628	
		Y - 4	64.0	0.626	
		Y - 5	109.4	0.719	
		Y - 6	61.8	0.642	
Yura River	Arakura 159 km²	A - 1	24.0	0.774	0.649
		A - 2	17.1	0.462	
		A - 3	13.3	-	
		A - 4	40.0	0.770	
		A - 5	92.2	0.618	
		A - 6	18.4	0.685	
		A - 7	1.3	-	
		A - 8	-	-	
		A - 9	14.5	0.613	
		A -10	14.4	0.620	
		A -11	30.6	0.648	
		A -12	1.1	0.648	
		A -13	-	-	
		A -14	3.4	-	
		A -15	-	-	
		A -16	-	-	

Table I-3 Recession Factor α and Initial Discharge of Confined Component (Continued)

River	Watershed	Symbol	Q_{co} m^3/s	Recession Factor α 1/day	Recession Factor $α_o$ 1/day
Yura River	Kado 585 km^2	K' - 1	26.0	0.300	0.595
		K' - 2	6.1	0.366	
		K' - 3	1.4	-	
		K' - 4	10.9	0.790	
		K' - 5	-	-	
		K' - 6	22.0	0.524	
		K' - 7	1.5	0.742	
		K' - 8	16.1	0.847	
		K' - 9	3.1	0.595	
		K' -10	6.5	0.841	
		K' -11	10.9	0.652	
		K' -12	2.2	-	
Nagara River	Takasu		-	-	-
	Tsurugi 223 km^2	Ts - 1	1.2	-	0.458
		Ts - 2	14.5	0.535	
		Ts - 3	28.5	0.489	
		Ts - 4	-	-	
		Ts - 5	-	-	
		Ts - 6	0.3	-	
		Ts - 7	5.2	0.350	
		Ts - 8	-	-	
	Kamita 713 km^2	Ka - 1	19.8	0.355	0.348
		Ka - 2	16.5	0.356	
		Ka - 3	3.5	0.348	
		Ka - 4	17.5	0.357	
		Ka - 5	36.7	0.340	
		Ka - 6	44.0	0.369	
		Ka - 7	7.6	0.314	
		Ka - 8	-	-	
	Sugihara 102 km^2	S - 1	14.7	0.563	0.563
		S - 2	9.7	-	
		S - 3	6.0	-	
		S - 4	3.2	-	

Table I-4 Recession Factor K and Initial Discharge of Unconfined Component

River	Watershed	Symbol	Q_{uo}	Recession Factor	
			m^3/sec	K $m^{-\frac{1}{2}} sec^{\frac{1}{2}} day^{-1}$ $\times 10^{-2}$	K_o $m^{-\frac{1}{2}} sec^{\frac{1}{2}} day^{-1}$ $\times 10^{-2}$
Kako River	Kunikane 1674 km^2	Ku - 1	82.9	0.978	0.878
		Ku - 2	28.2	0.527	
		Ku - 3	19.5	0.544	
		Ku - 4	38.4	0.387	
		Ku - 5	15.9	1.530	
		Ku - 6	77.2	0.899	
		Ku - 7	47.2	0.888	
		Ku - 8	41.6	1.643	
		Ku - 9	22.3	0.508	
Yoshino River	Terao 253 km^2	Y - 1	18.6	0.441	0.880
		Y - 2	46.0	1.237	
		Y - 3	15.7	0.934	
		Y - 4	13.8	0.619	
		Y - 5	28.6	1.028	
		Y - 6	17.8	1.019	
Yura River	Arakura 159 km^2	A - 1	4.7	1.250	1.454
		A - 2	8.0	1.310	
		A - 3	9.4	-	
		A - 4	12.2	1.570	
		A - 5	22.2	1.670	
		A - 6	16.0	1.350	
		A - 7	18.2	1.550	
		A - 8	5.5	1.420	
		A - 9	4.9	1.230	
		A -10	7.7	1.720	
		A -11	7.7	1.150	
		A -12	11.5	1.730	
		A -13	6.8	(2.270)	
		A -14	8.6	(2.210)	
		A -15	6.1	1.410	
		A -16	3.3	1.540	
Kizu River	Kamo 1456 km^2		-	-	0.573
	Ino-oka 1559 km^2		-	-	0.355

Table I-4 Recession Factor K and Initial Discharge of Unconfined Component (Continued)

River	Watershed	Symbol	Q_{uo} m^3/sec	Recession Factor K $m^{-\frac{1}{2}}sec^{\frac{1}{2}}day^{-1}$ $\times10^{-2}$	Recession Factor K_o $m^{-\frac{1}{2}}sec^{\frac{1}{2}}day^{-1}$ $\times10^{-2}$
		K' - 1	23.2	0.353	
		K' - 2	27.0	0.520	
		K' - 3	66.0	1.674	
		K' - 4	16.4	1.700	
		K' - 5	16.5	1.060	
Yura River	Kado 585 km²	K' - 6	14.6	1.110	
		K' - 7	5.3	-	1.294
		K' - 8	29.3	1.240	
		K' - 9	29.0	1.320	
		K' -10	22.8	1.265	
		K' -11	25.5	1.005	
		K' -12	22.5	1.320	
	Takasu 65.3 km²	T - 1	4.6	1.320	
		T - 2	5.8	1.310	1.333
		T - 3	4.4	1.370	
		Ts - 1	15.0	0.820	
		Ts - 2	14.0	0.820	
		Ts - 3	14.0	0.850	
	Tsurugi 223 km²	Ts - 4	11.0	0.836	
		Ts - 5	7.5	0.750	0.800
		Ts - 6	12.3	0.720	
		Ts - 7	9.0	0.780	
Nagara River		Ts - 8	8.7	0.820	
		Ka - 1	32.0	0.380	
		Ka - 2	34.0	0.380	
		Ka - 3	43.0	0.337	
	Kamita 713 km²	Ka - 4	30.0	0.350	
		Ka - 5	40.0	0.331	0.354
		Ka - 6	35.0	0.350	
		Ka - 7	30.0	0.325	
		Ka - 8	13.8	0.380	
		S - 1	6.9	0.870	
	Sugihara 102 km²	S - 2	13.7	0.940	
		S - 3	8.5	1.090	0.998
		S - 4	6.2	1.090	

Table I-5 Results obtained for Rising States of Hydrograph due to Rainfall

River	Basin	Date	Rainfall: Mean daily rainfall R_m mm/d	Rainfall: Duration T d	Initial state Q_{uo}' m^3/s	Confined Component: Increment of discharge ΔQ_c m^3/s	Confined Component: R_cB_c m^3/s	Confined Component: D $\frac{m^3 \cdot day}{mm\ sec}$	Unconfined Component: Increment of discharge ΔQ_u m^3/s	Unconfined Component: Increment of storage ΔS_u $m^3 \times 10^4$	$r_eL_uB_u$ m^3/s
		Aug. 3, 1950	26.0	1	3.3	24.0	-		1.8	253	
		Oct. 6, 1950	30.2	4	2.6	17.1	18.5		5.8	405	
		Sept. 15, 1952	23.2	9	3.0	13.3	13.3		6.9	810	
		June 8, 1953	40.8	5	7.8	40.6	-		6.6	608	
Yura		July 5, 1953	-	-	-	92.2	-		-	-	
River	Arakura	Sept. 26, 1953	-	-	7.8	18.4	-	0.61	10.0	-	10.0
		July 6, 1954	14.3	5	15.5	1.3	1.4		7.1	531	
		June 19, 1955	-	-	4.6	4.5	-		1.1	-	
		July 7, 1955	28.3	4	3.7	14.4	15.6		4.9	607	
		July 24, 1956	73.0	2	4.4	30.6	42.1		3.8	152	
		April 26, 1961	21.0	2	3.7	1.7			0.9		
		May 5, 1961	30.0	2	3.2	1.6			0.8		
		Aug. 24, 1963	25.6	2	2.2	1.3			1.2		
		Aug. 31, 1963	33.8	2	2.7	3.1			0.9		
		Sept. 16, 1963	27.3	2	-	1.3			0.9		
	Takasu	Sept. 25, 1963	29.2	2	2.1	2.3	-	0.10	1.2		3.4
		Oct. 16, 1963	57.4	1	-	3.2			1.2		
		Oct. 27, 1963	11.4	2	-	1.2			0.5		
		July 9, 1964	137.3	2	-	9.7			3.2		
Nagara		Aug. 25, 1964	67.5	5	1.6	5.1			2.1		
River		Aug. 13, 1953	28.1	7	5.7	13.0			7.8		
		Sept. 19, 1953	103.5	2	10.9	35.3			3.9		
		Oct. 9, 1953	10.1	2	8.3	1.4			0.7		
		April 20, 1957	-	-	11.5	39.2			12.8		
		Oct. 7, 1957	55.5	1	29.3	7.2			2.9		
		May 1, 1958	52.0	1	11.7	12.8			2.2		
	Tsurugi	Sept. 3, 1959	6.8	1	7.2	0.9		0.50	1.1		13.0
		May 29, 1961	43.8	1	6.7	6.1			1.6		
		Oct. 19, 1961	24.5	3	8.8	3.2			3.7		
		Oct. 25, 1961	18.4	1	9.5	2.7			2.9		
		Aug. 11, 1963	51.7	2	4.1	18.3			2.9		
		Aug. 31, 1963	34.4	2	8.0	10.3			4.7		

Table I-5 Results obtained for Rising States of Hydrograph due to Rainfall (Continued)

River	Basin	Date	Rainfall: Mean daily rain-fall R_m mm/d	Rainfall: Duration T d	Ini-tial sta-te Q_{uo} m^3/s	Confined Component: Incr-ement of disc-harge ΔQ_c m^3/s	Confined Component: R_cB_c m^3/s	Confined Component: D $\frac{m^3 \cdot day}{mm \cdot sec}$	Unconfined Component: Incr-ement of disc-harge ΔQ_u m^3/s	Unconfined Component: Incr-ement of stor-age ΔS_u $m^3 \times 10^4$	$r_eL_uB_u$ m^3/s
Nagara River	Kamita	May 1, 1953	20.4	4	27.0	14.4		0.67	13.0		23.5
		Aug. 19, 1954	67.8	2	19.5	33.1			23.0		
		Aug. 14, 1956	33.4	2	14.0	12.0			5.3		
		July 27, 1960	19.6	3	11.8	6.5			2.5		
		Sept. 24, 1962	17.5	1	24.5	2.5			3.0		
		Oct. 27, 1963	19.9	2	19.0	3.5			10.2		
		Nov. 8, 1963	13.6	2	18.3	7.5			8.2		
		June 3, 1964	17.3	2	14.0	4.7			3.6		
		Aug. 25, 1964	-	2	14.3	68.0			23.0		
		Oct. 16, 1965	32.9	2	14.0	17.0			7.0		

Table I-6 Time expected for Variation of Infiltration Capacity

The antecedent no-rainfall days	The time expected for the infiltration intensity to reach nearly the final one at the Yura River Basin in Japan
4 days	0.89 days
2 ∿ 4	0.84
1 ∿ 2	0.78

Table I-7 Several Parameters used for Simulation

River			Yura River	Nagara River		
Gauging station			Arakura	Kamita	Tsurugi	Takasu
Unconfined component	Recession K		0.0145	0.0038	0.0032	0.0133
	Rising	$r_e B_u L_u$	10.0	25.0	15.0	3.5
		n	0.08	0.04	0.05	0.10
Confined component	Recession α		0.649	0.348	0.458	(0.628)
	Rising D		0.61	0.67	0.50	0.10
Rainfall loss	M_o		20.0	25.0		
	μ		0.184			
Lag time day			1	2	1	1

Unit, K: $m^{-2/3} sec^{1/2} day^{-1}$, $r_e B_u L_u$: m^3/sec, n: $m^{-2/3} sec^{1/2} day^{-1}$, α: 1/day, D: m^3day/mmsec
M_o: mm, μ: 1/day.

Table I-8 Initial Value for Simulation

Initial day	Case	Yura River		Nagara River					
		Arakura		Kamita		Tsurugi		Takasu	
		Q_{uo}	Q_{co}	Q_{uo}	Q_{co}	Q_{uo}	Q_{co}	Q_{uo}	Q_{co}
June 9, 1953	1	12.2	40.0	-	-	-	-	-	-
June 9, 1954	1	11.4	9.1	-	-	-	-	-	-
May 1, 1955	1	6.1	10.5	-	-	-	-	-	-
May 6, 1956	1	8.3	6.5	-	-	-	-	-	-
April 25, 1957	1	14.9	10.5	-	-	-	-	-	-
May 24, 1958	1	3.7	2.5	-	-	-	-	-	-
May 23, 1959	1	8.4	2.7	-	-	-	-	-	-
May 20, 1960	1	8.5	5.2	-	-	-	-	-	-
June 10, 1961	1	3.0	2.6	-	-	-	-	-	-
May 1, 1963	1	-	-	35.0	40.0	8.0	15.0	3.5	3.0
May 3, 1963	2	-	-	34.0	21.0	16.0	6.0	-	-
May 1, 1964	1	-	-	34.0	20.0	7.0	2.5	-	-
May 4, 1964	2	-	-	40.0	11.0	14.0	4.0	-	-
May 1, 1965	1	-	-	30.0	30.0	7.0	8.0	-	-
May 2, 1965	2	-	-	29.0	21.0	8.0	10.0	-	-

Unit : m^3/sec

Table I-9 Water Balance estimated for Whole Periods

Watershed	Year	Total Rainfall	Loss	Effective Rainfall		
				Total Effective Rainfall	Ground Water Runoff	Direct Runoff
Arakura	1953	1200.0	259.5	940.5	539.5	400.9
	1954	1161.0	363.0	798.0	465.2	332.8
	1955	1182.0	455.6	729.0	496.7	232.3
	1956	1379.0	455.3	864.5	612.2	252.3
	1957	1544.0	500.8	1035.0	636.0	399.0
	1958	1310.0	425.0	885.0	526.6	358.6
	1959	1518.0	389.3	1129.0	574.9	554.7
	1960	1311.0	416.9	894.1	524.8	354.7
	1961	1502.0	340.6	1161.4	629.1	532.7
Kamita	1963	2226.0	654.0	1571.4	346.6	1224.8
	1964	1587.0	543.2	1043.8	209.9	834.2
	1965	2132.0	534.6	1597.4	280.4	1317.5
Tsurugi	1963	1841.0	610.3	1250.4	562.2	688.2
	1964	1833.0	581.1	1251.9	473.2	779.6
	1965	2255.0	548.3	1706.7	591.2	1115.5

Unit: mm

Table II-1 Physical Significances of Each Term of Local Potential

	Physical Significances
$B_s \dfrac{\partial H_s^*}{\partial t} H_s^*$, $\gamma \dfrac{\partial H_g^*}{\partial t} H_g^*$,	Change of the potential energy stored in the region per unit time and unit area,
$- Q^* \dfrac{\partial H_s^*}{\partial s}$, $\sum_i \{ kH_g^* \dfrac{\partial H_g^*}{\partial x_i} - f_i H_g^* \} \dfrac{\partial H_g^*}{\partial x_i}$,	Potential energy carried into the region of unit area due to flow flux per unit time,
$H_g^*\{ kH_g^* \dfrac{\partial H_g^*}{\partial x_1} - f_1 H_g^*\}\dfrac{dx_2}{ds}$ $- H_g^*\{ kH_g^* \dfrac{\partial H_g^*}{\partial x_2} - f_2 H_g^*\}\dfrac{dx_1}{ds}$, $r \cdot H_g^*$,	Potential energy carried into the region of unit area per unit time,
$- H_s^* \dfrac{\partial Q^*}{\partial s}$, $\sum_i H_g^* \dfrac{\partial}{\partial x_i}\{kH_g^* \dfrac{\partial H_g^*}{\partial x_i} - f_i H_g^*\}$.	Potential energy carried into the region of unit area due to the works caused by the pressure per unit time

Table II-2 Several Quantities on Recession States Analyzed

	Initial day	Tsurugi		Kamita	
		Initial discharge	Recession factor K	Initial discharge	Recession factor K
Nagara River	Sept. 21, 1965	12.1 m³/s	820×10^{-5}	28.0 m³/s	380×10^{-5}
	July 27, 1965	14.0	820	34.0	380
	July 23, 1964	14.0	850	43.0	337
	Sept. 2, 1963	11.0	836	30.0	350
	Oct. 7, 1954	12.3	720	35.0	350
	Oct. 10, 1953	9.0	780	30.0	350

Unit, K : $m^{-3/2}sec^{1/2}day^{-1}$

	Initial day	Arakura		Kado	
		Initial discharge	Recession factor K	Initial discharge	Recession factor K
Yura River	Sept. 28, 1954	5.5 m³/s	1424×10^{-5}	16.5 m³/s	1060×10^{-5}
	July 7, 1955	7.7	1720	14.6	1100
	July 24, 1956	7.7	115C	29.3	1240
	April 23, 1957	11.5	1730	29.0	1080
	Oct. 5, 1957	6.8	2270	22.8	1265
	Oct. 8, 1959	8.6	2210	25.5	1005
	Oct. 19, 1959	6.1	1410	22.5	1320

Unit, K : $m^{-3/2}sec^{1/2}day^{-1}$

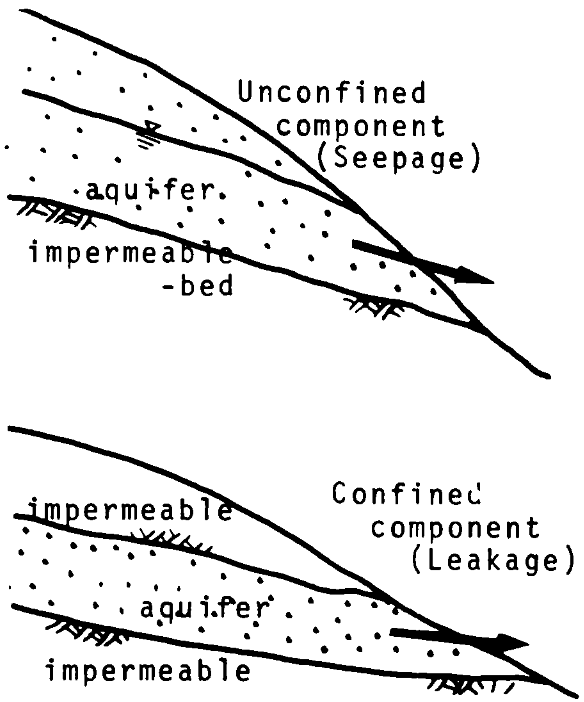


Fig. I-1 Schematic Representation of Unconfined and Confined Aquifers

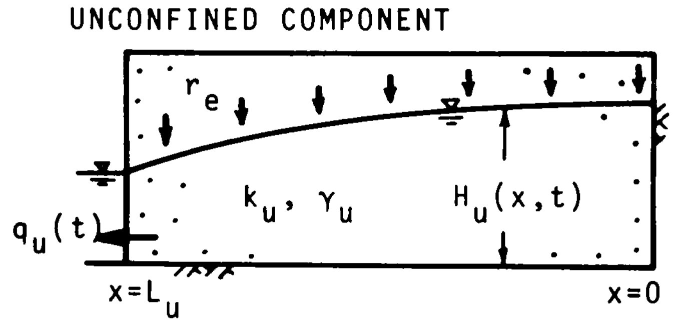


Fig. I-2 Runoff Model for Unconfined Component

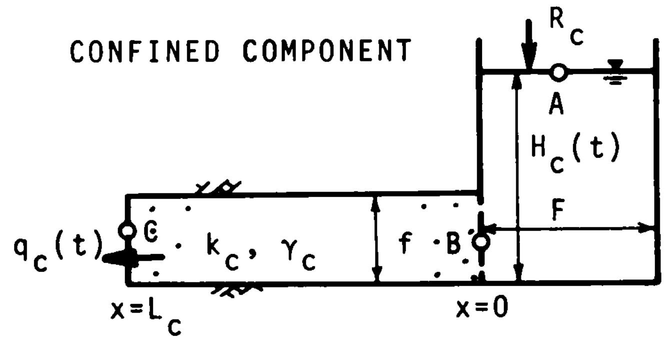


Fig. I-3 Runoff Model for Confined Component

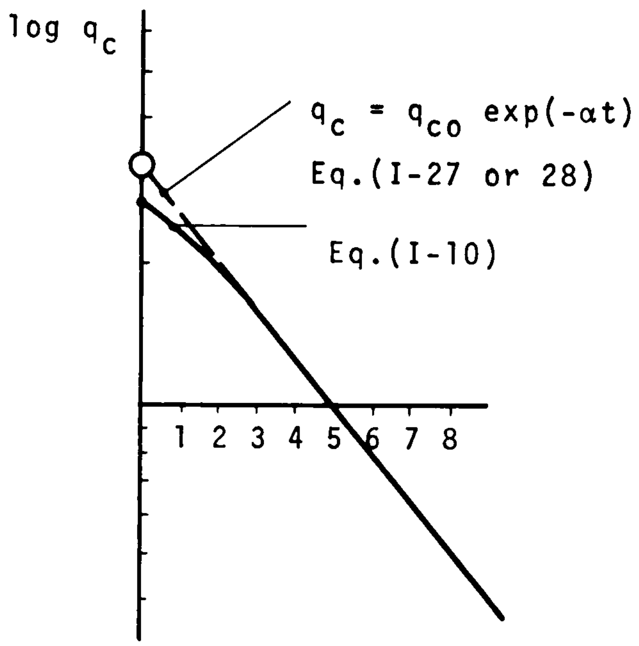


Fig. I-4 Graphical Explanation of Eq.(I-10) and Eq.(I-27)

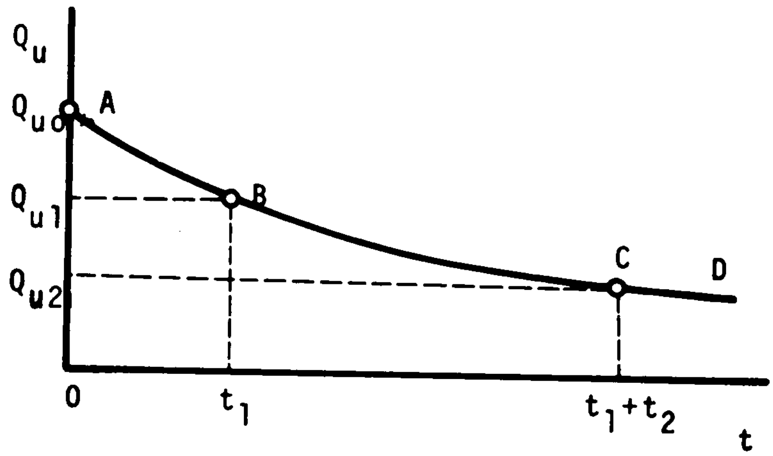


Fig. I-5 Graphical Explanation of Normal Recession Curve

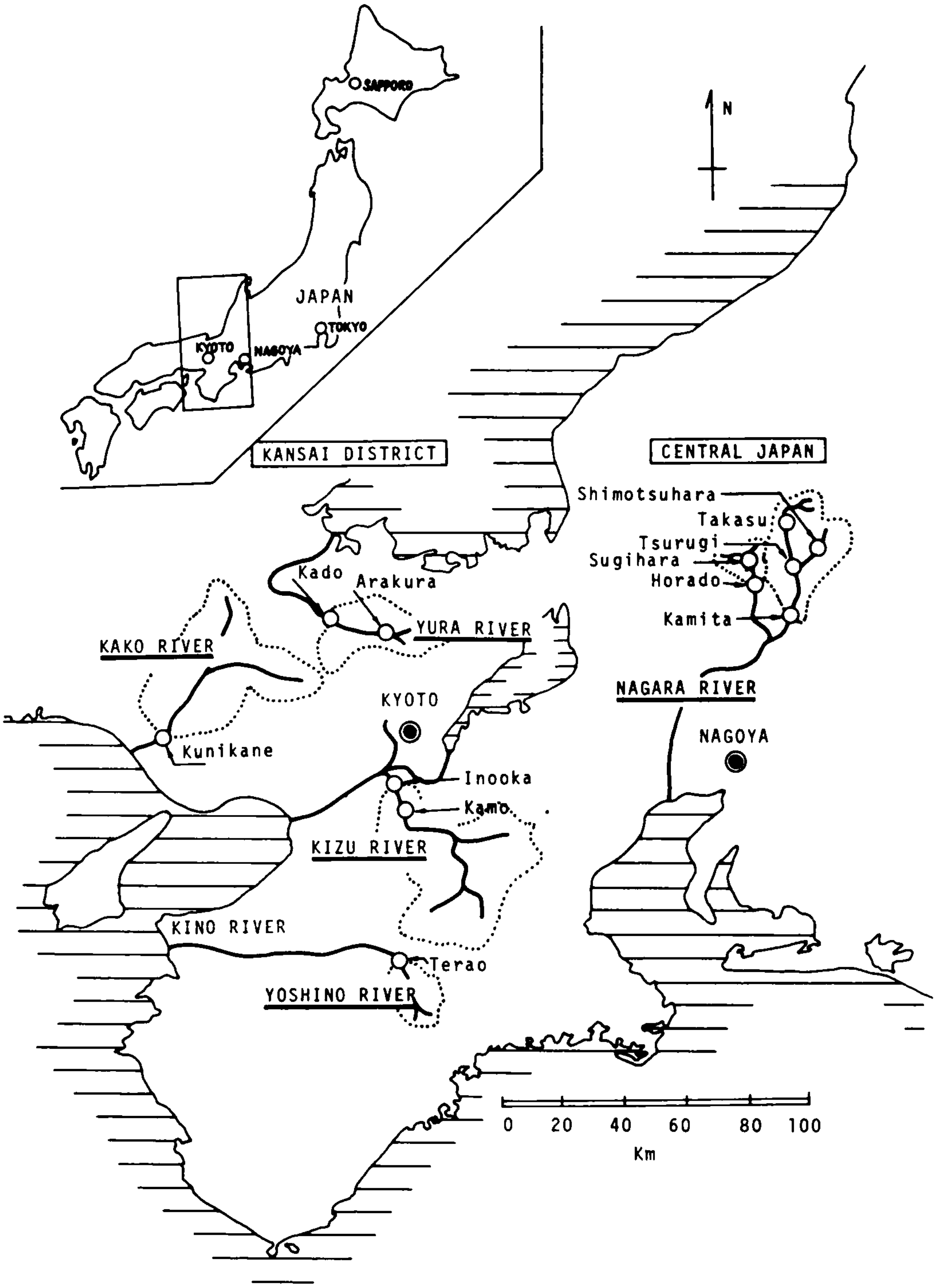


Fig. I-6 General Remarks of Watersheds

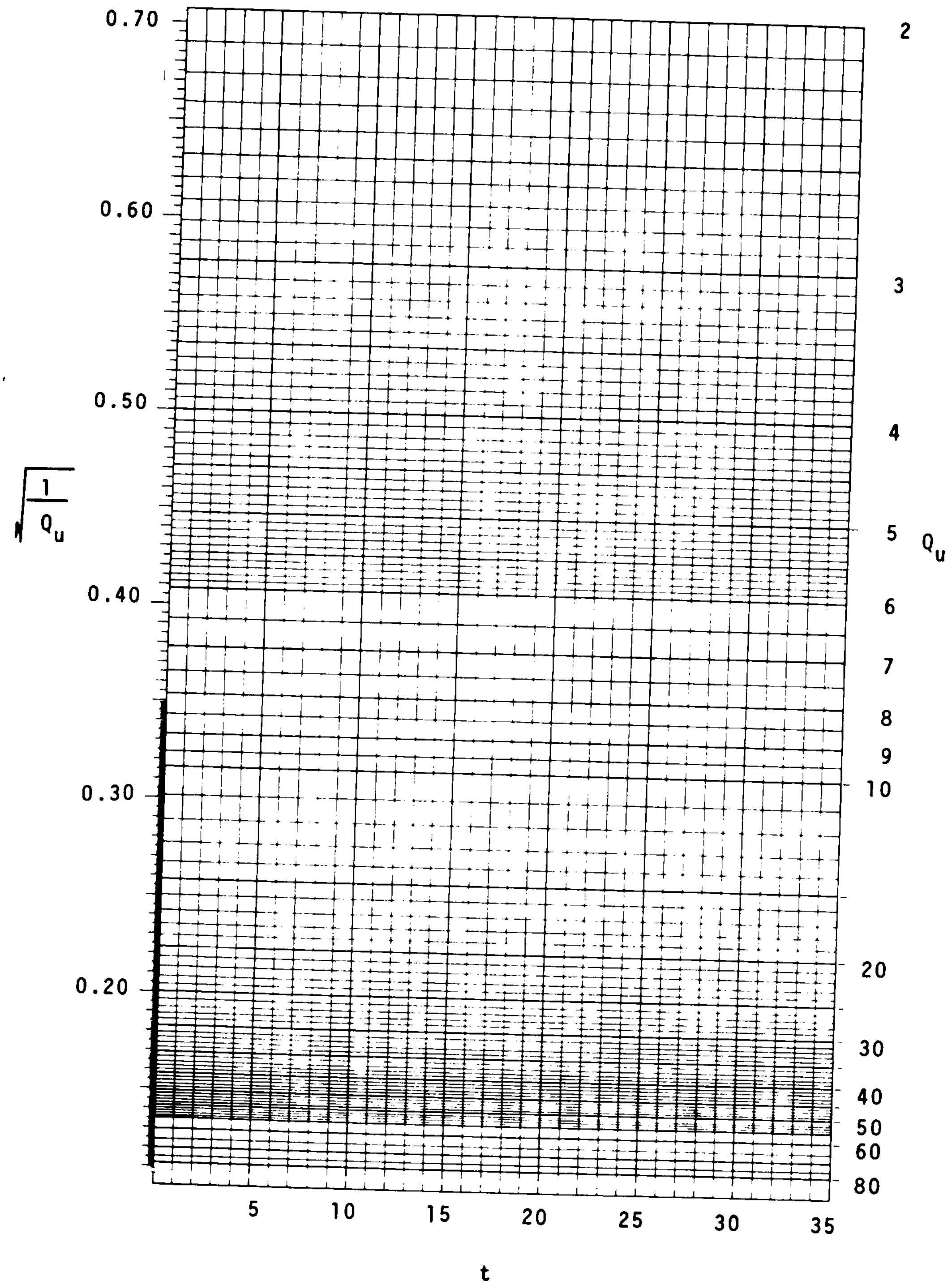


Fig. I-7 $(t, \sqrt{\frac{1}{Q_u}})$-plane

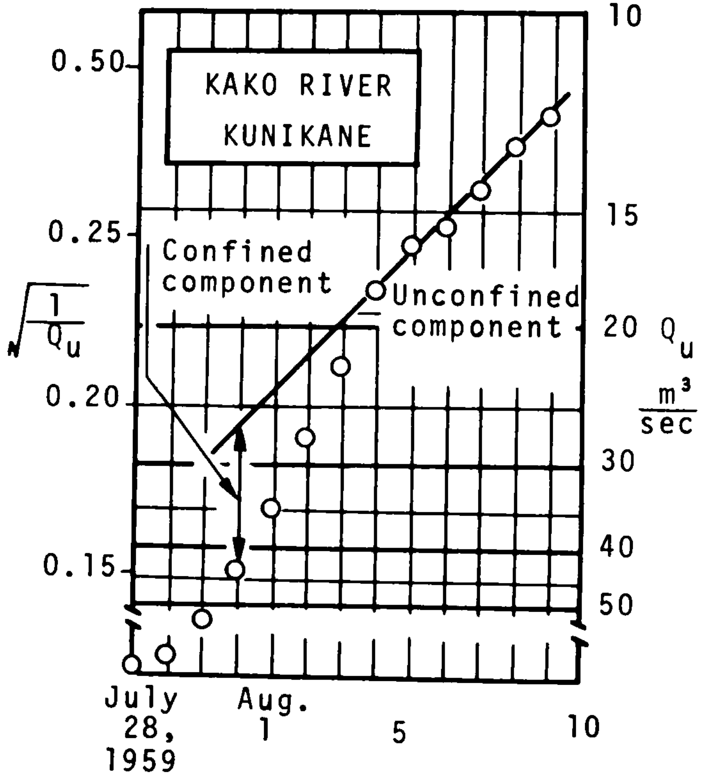


Fig. I-8 Schematic Explanation of Separation of Components

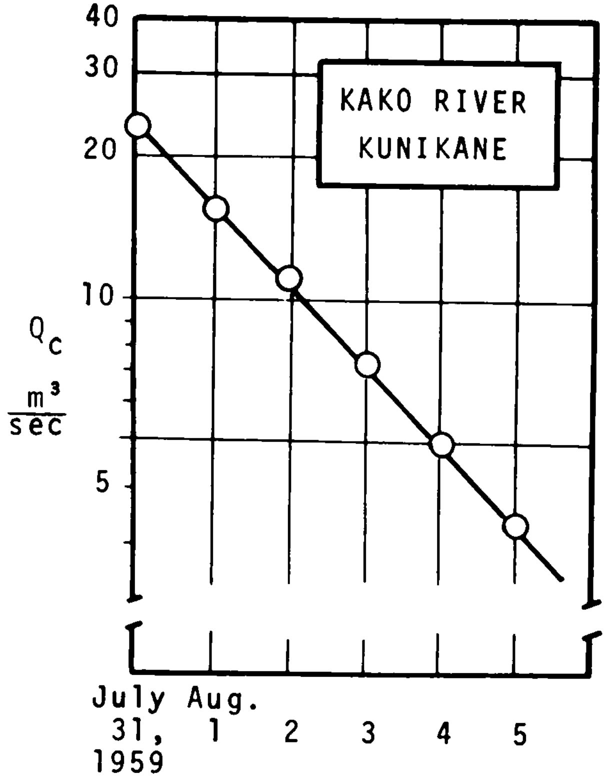
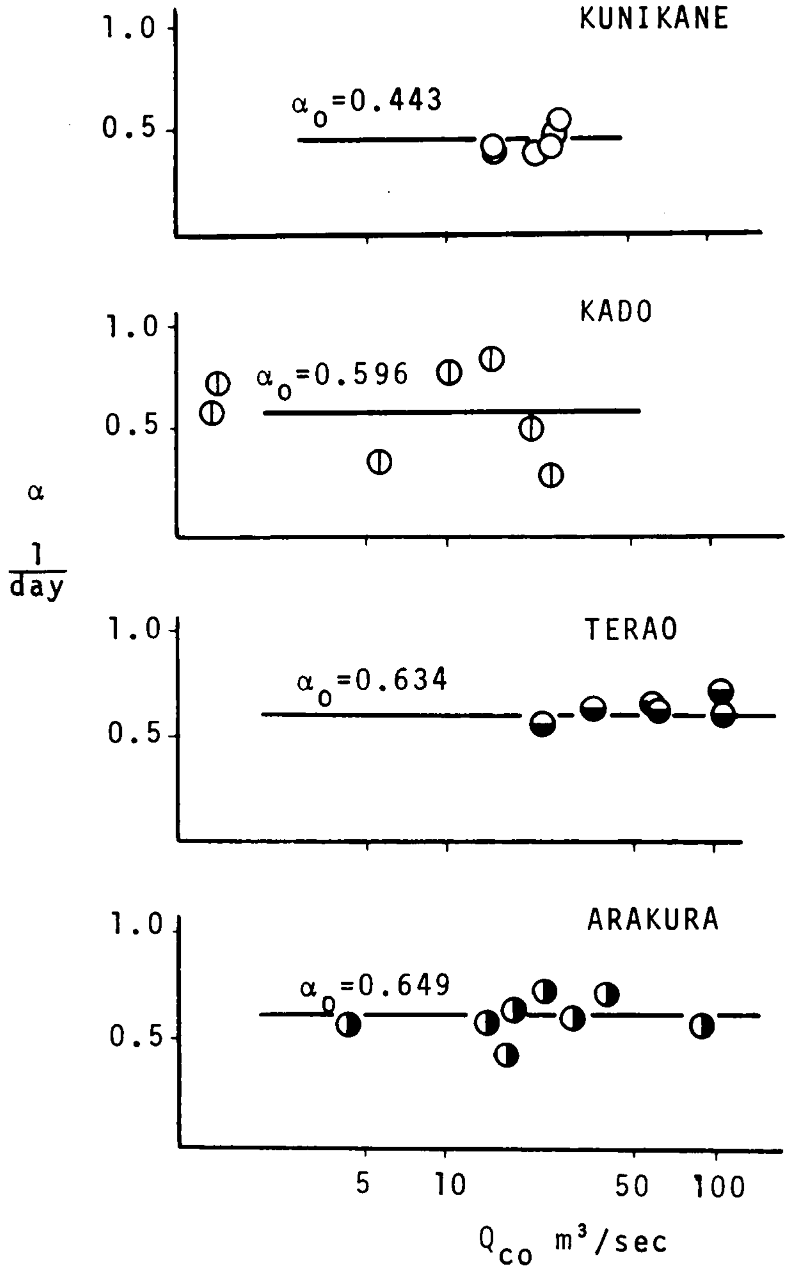


Fig. I-9 Schematic Explanation of Estimation of Confined Component



Fig,I-10 Relationship between Recession Factor α and Initial Discharge of Confined Component

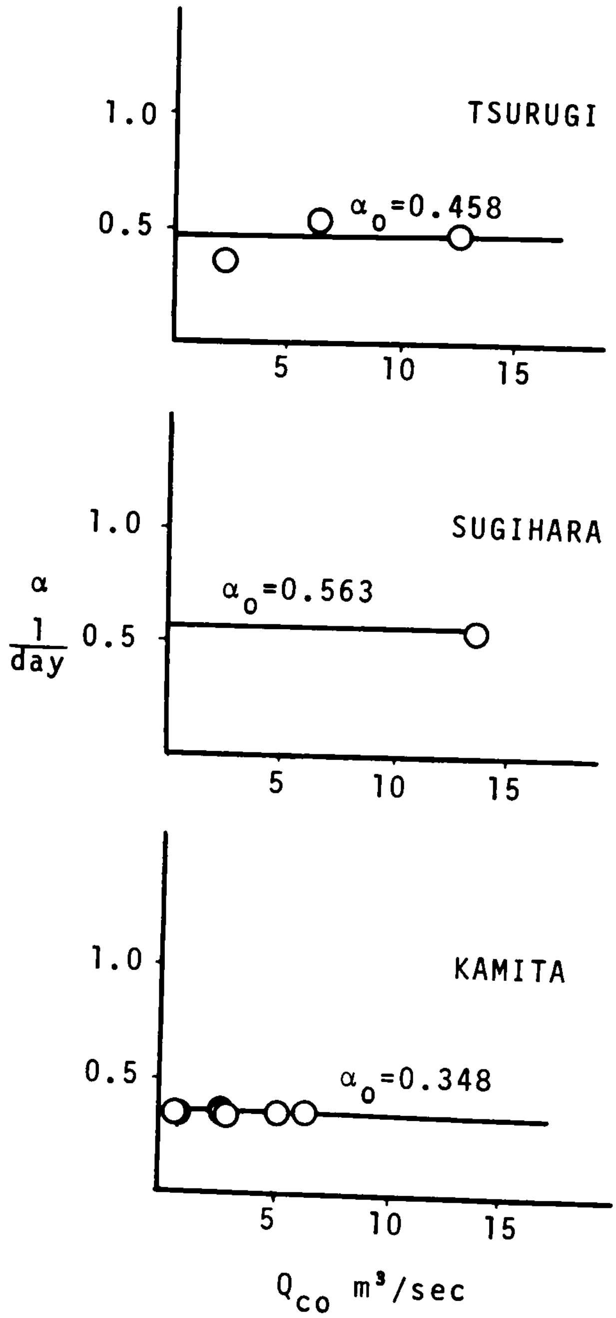


Fig. I-10 (Continued)

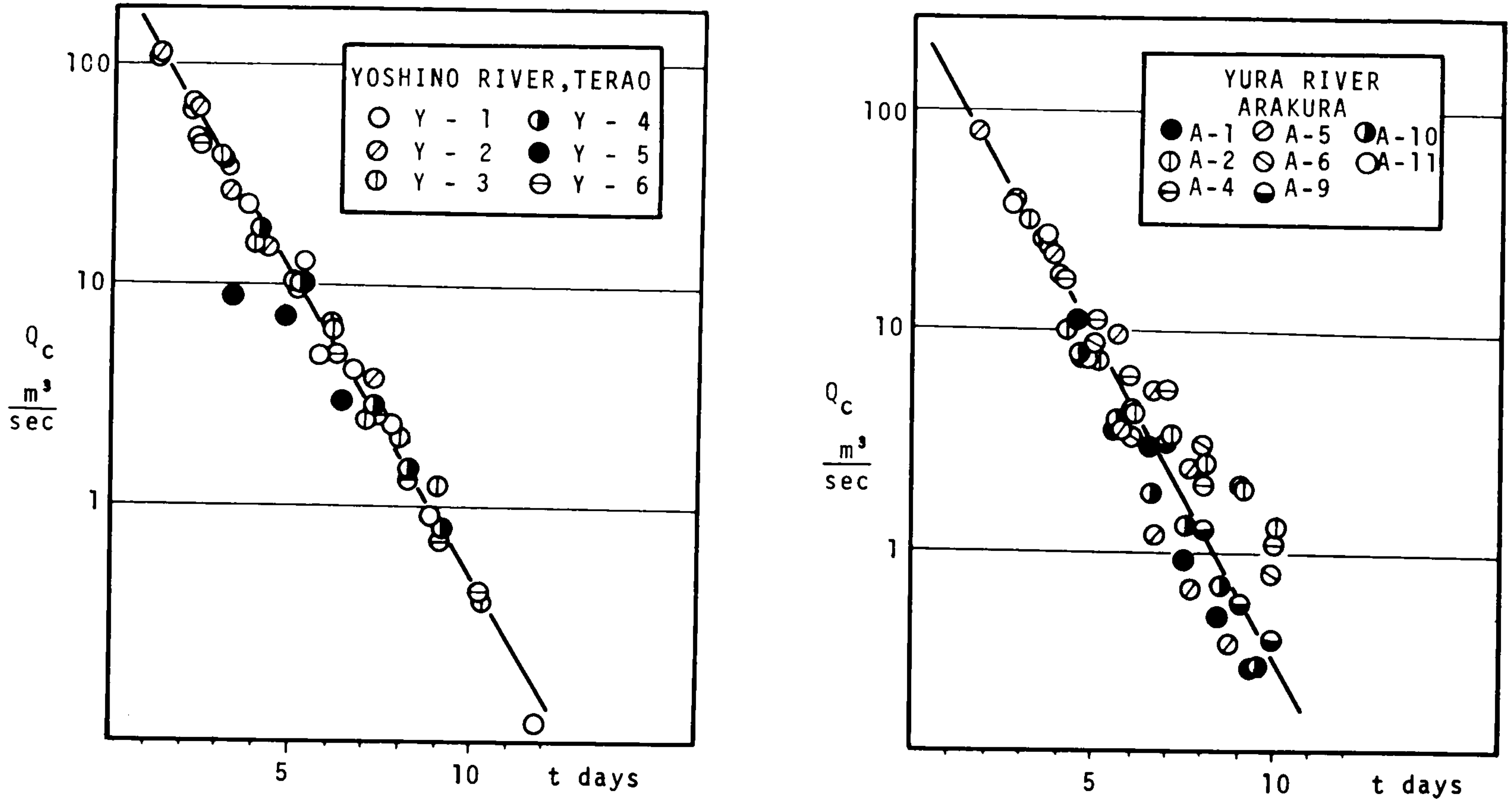


Fig. I-11 Normal Recession Curve of Confined Component

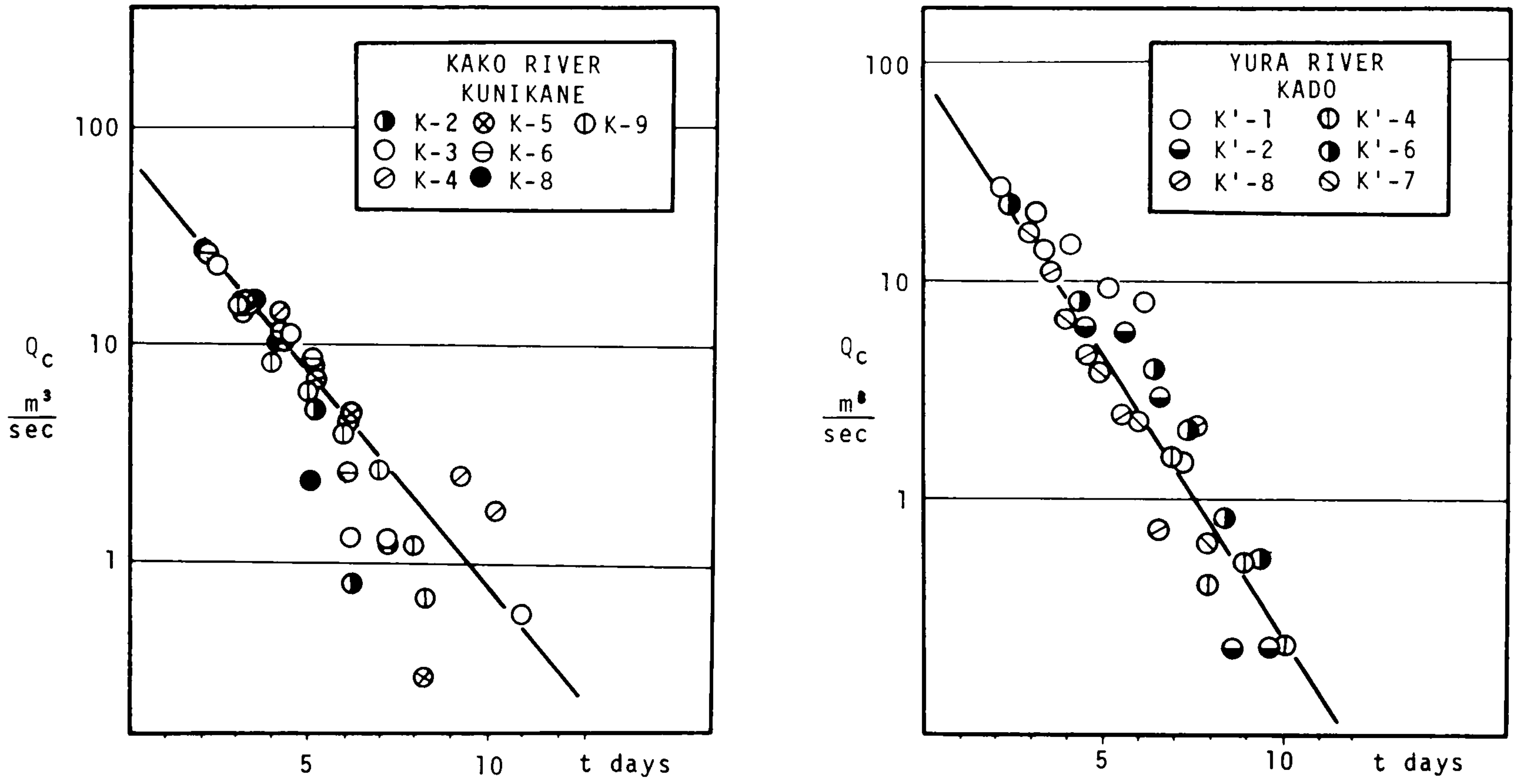
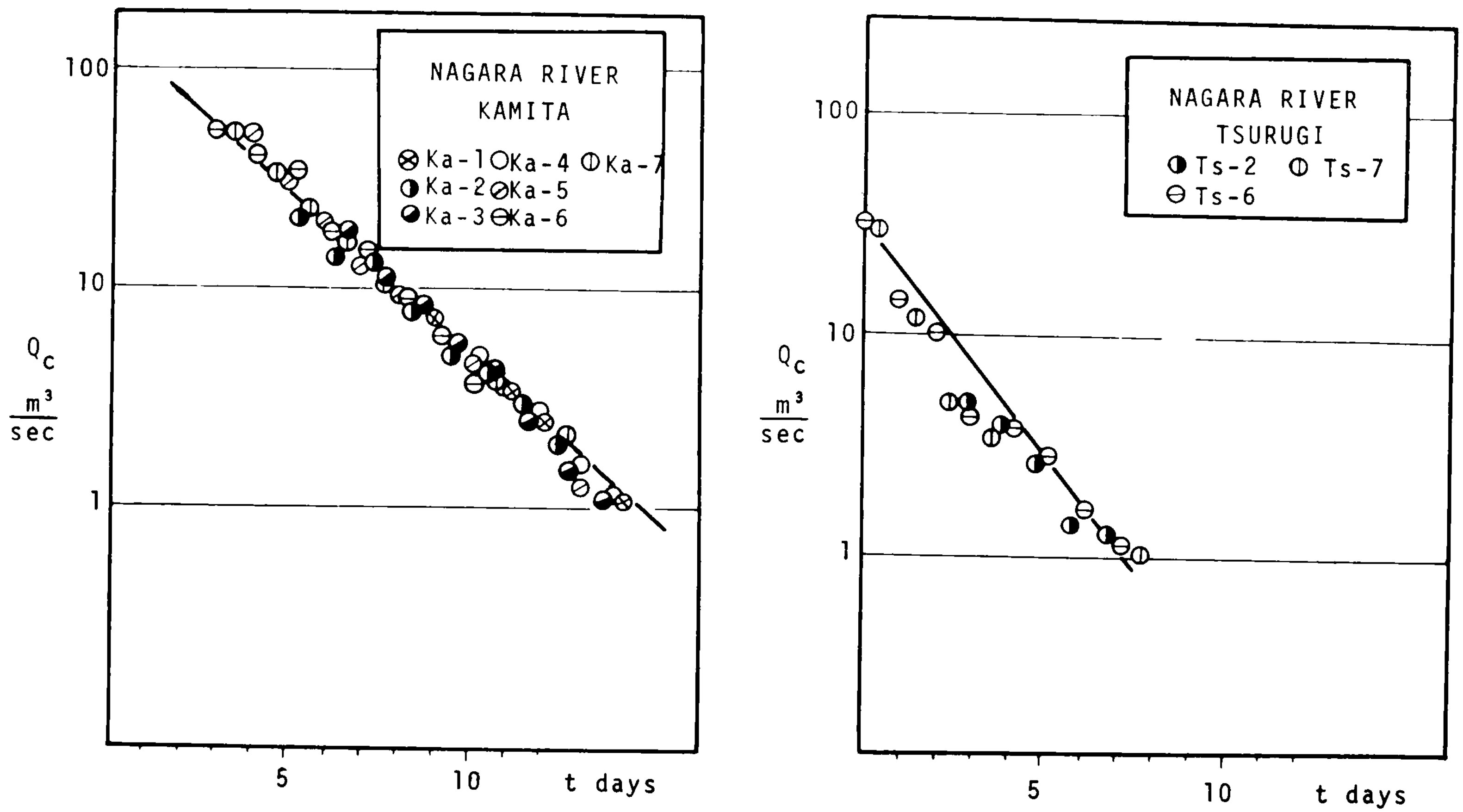


Fig. I-11 (Continued)



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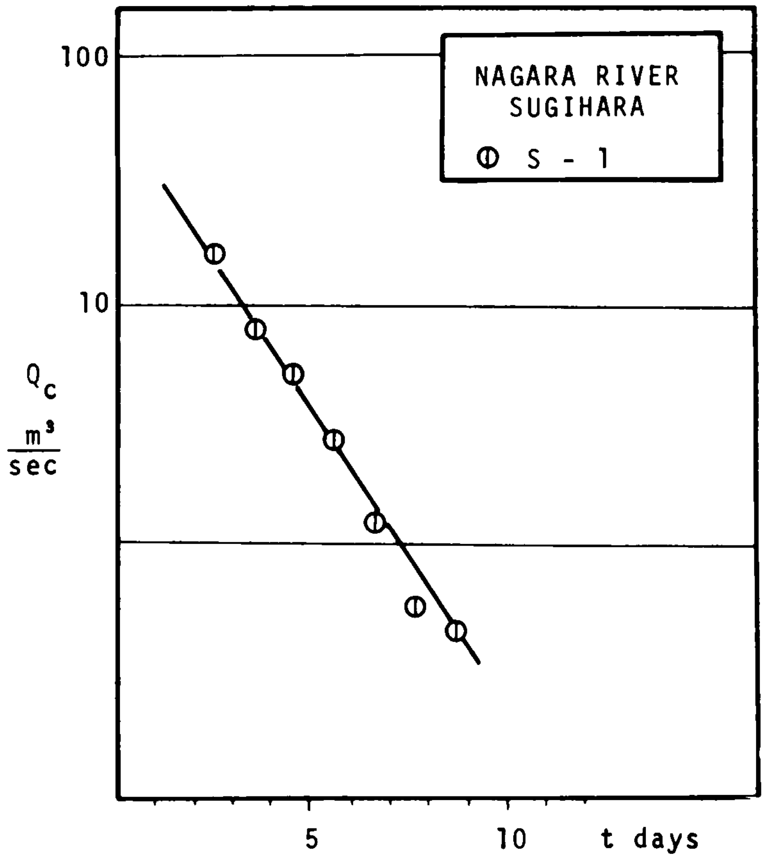

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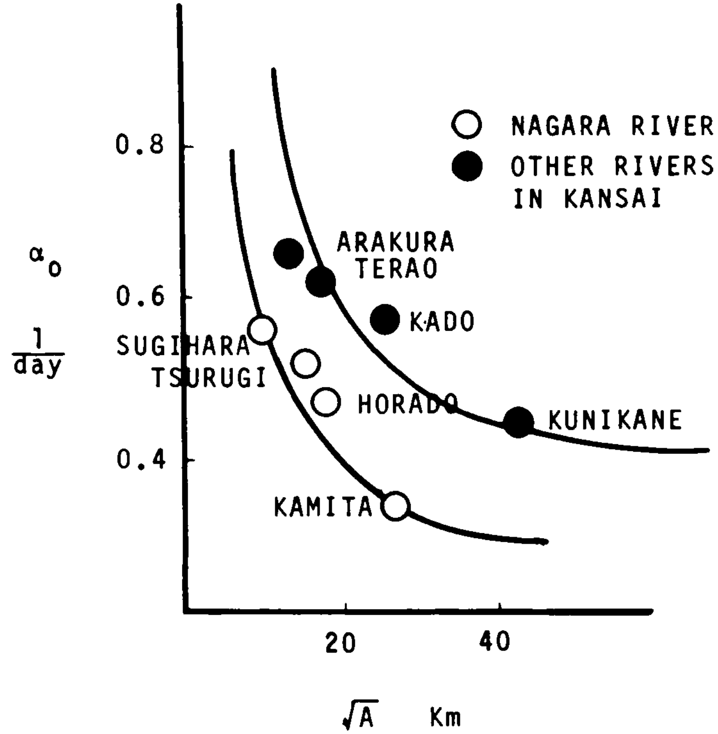
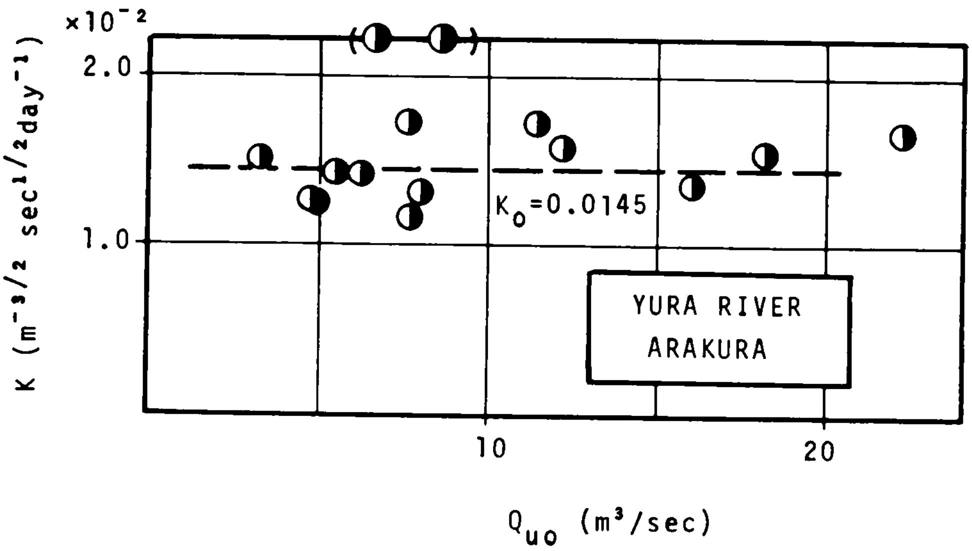


Fig. I-12 Relationship between Recession Factor α and Catchment Area



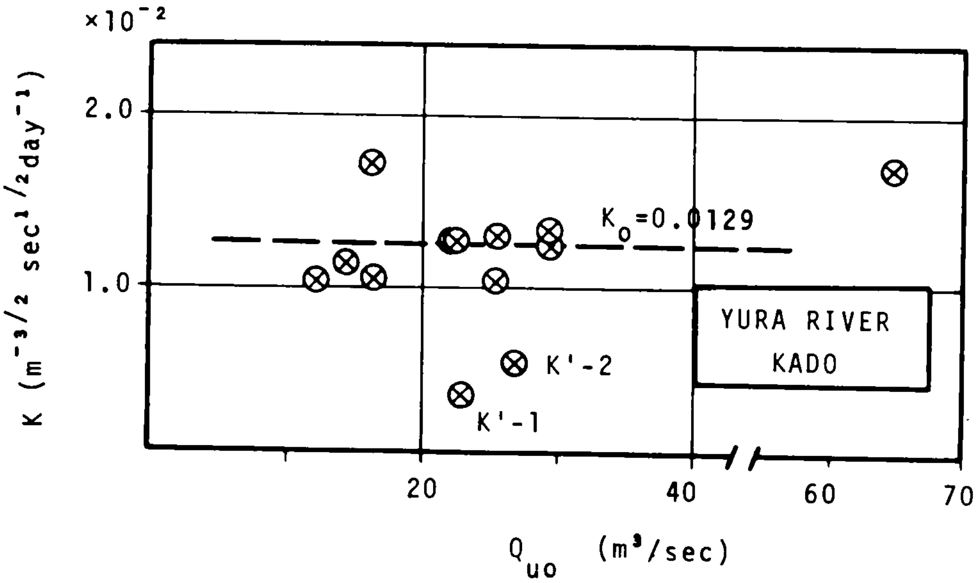
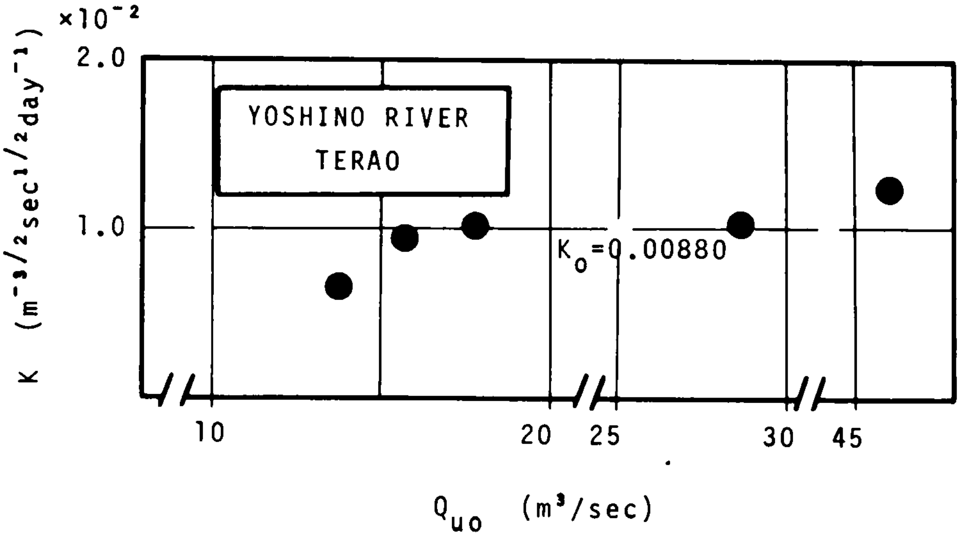


Fig. I-13 Recession Factor K versus Initial Discharge of Unconfined Component



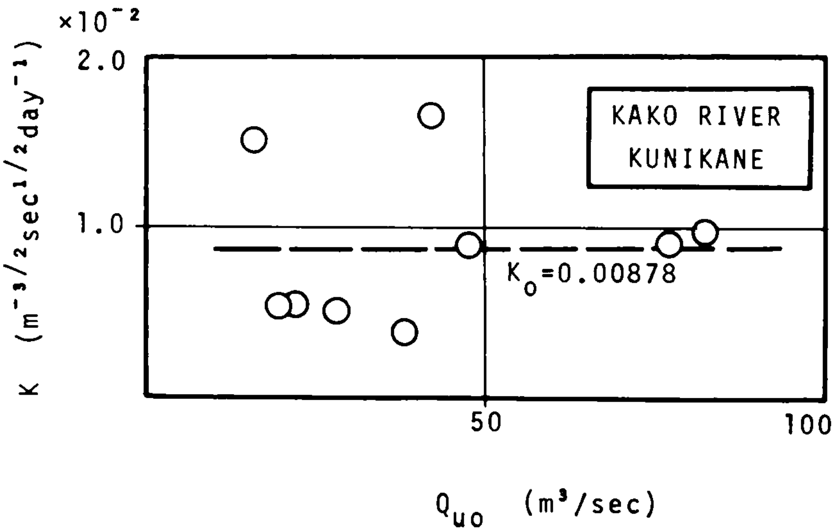
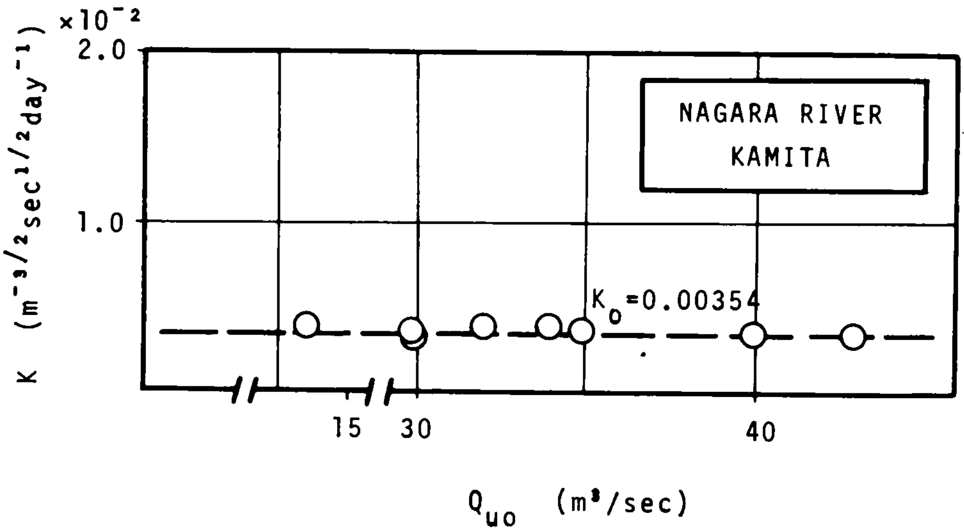
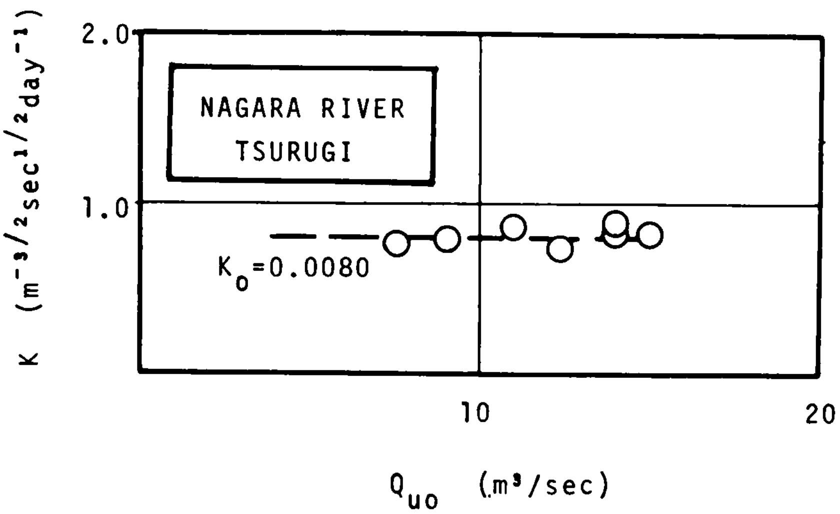


Fig. I-13 (Continued)





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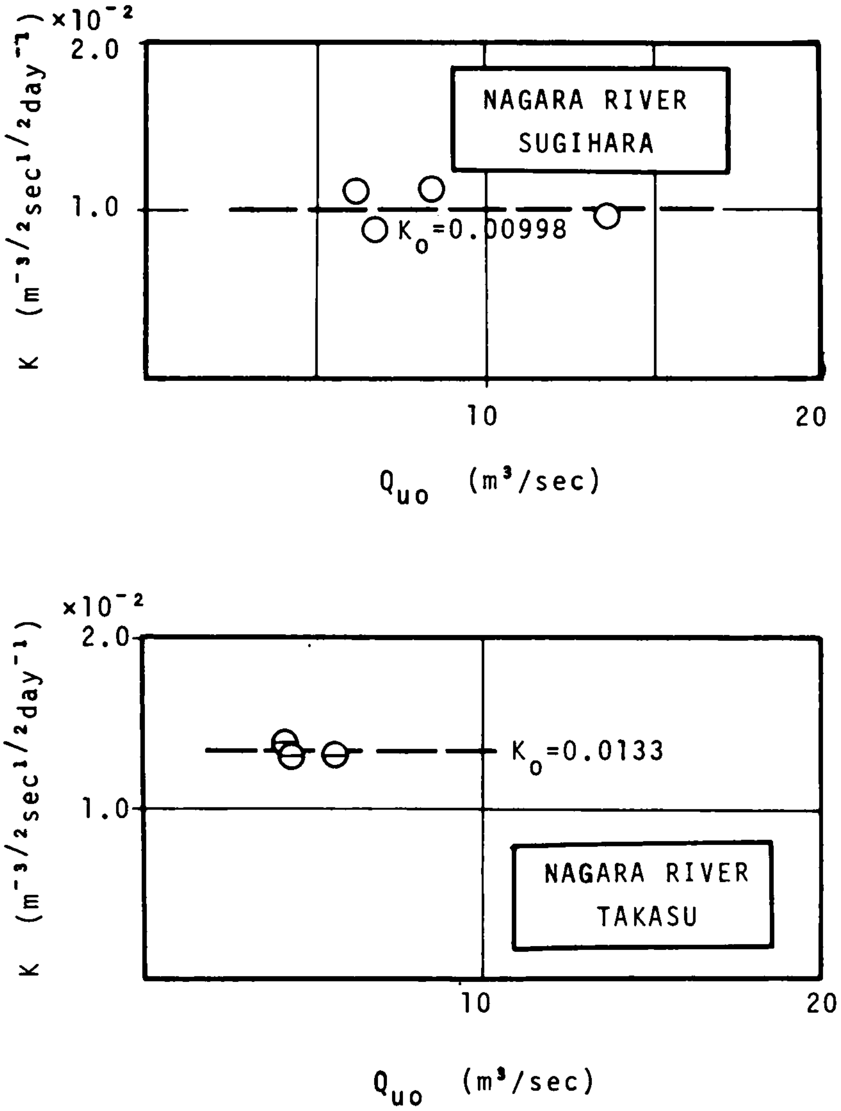

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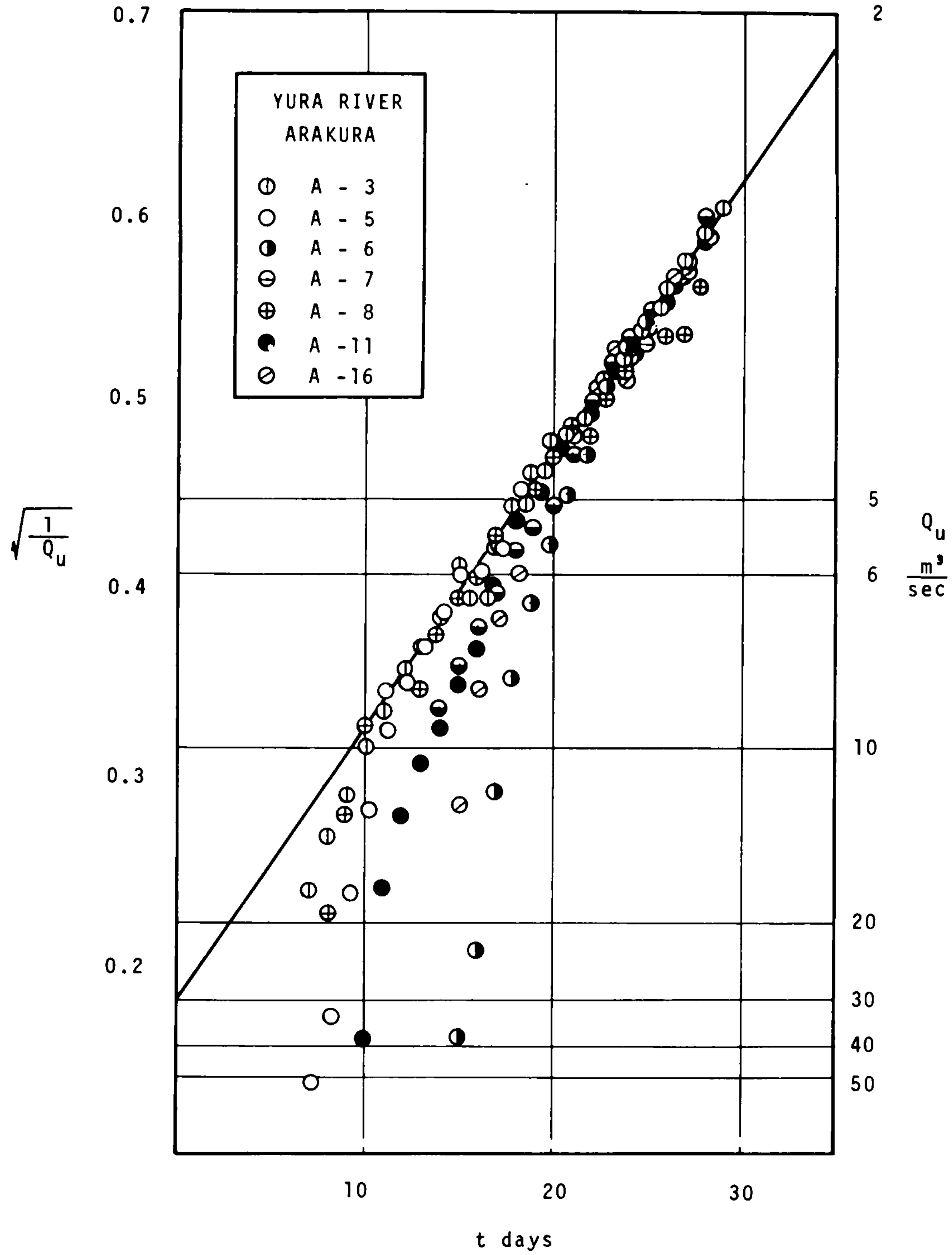
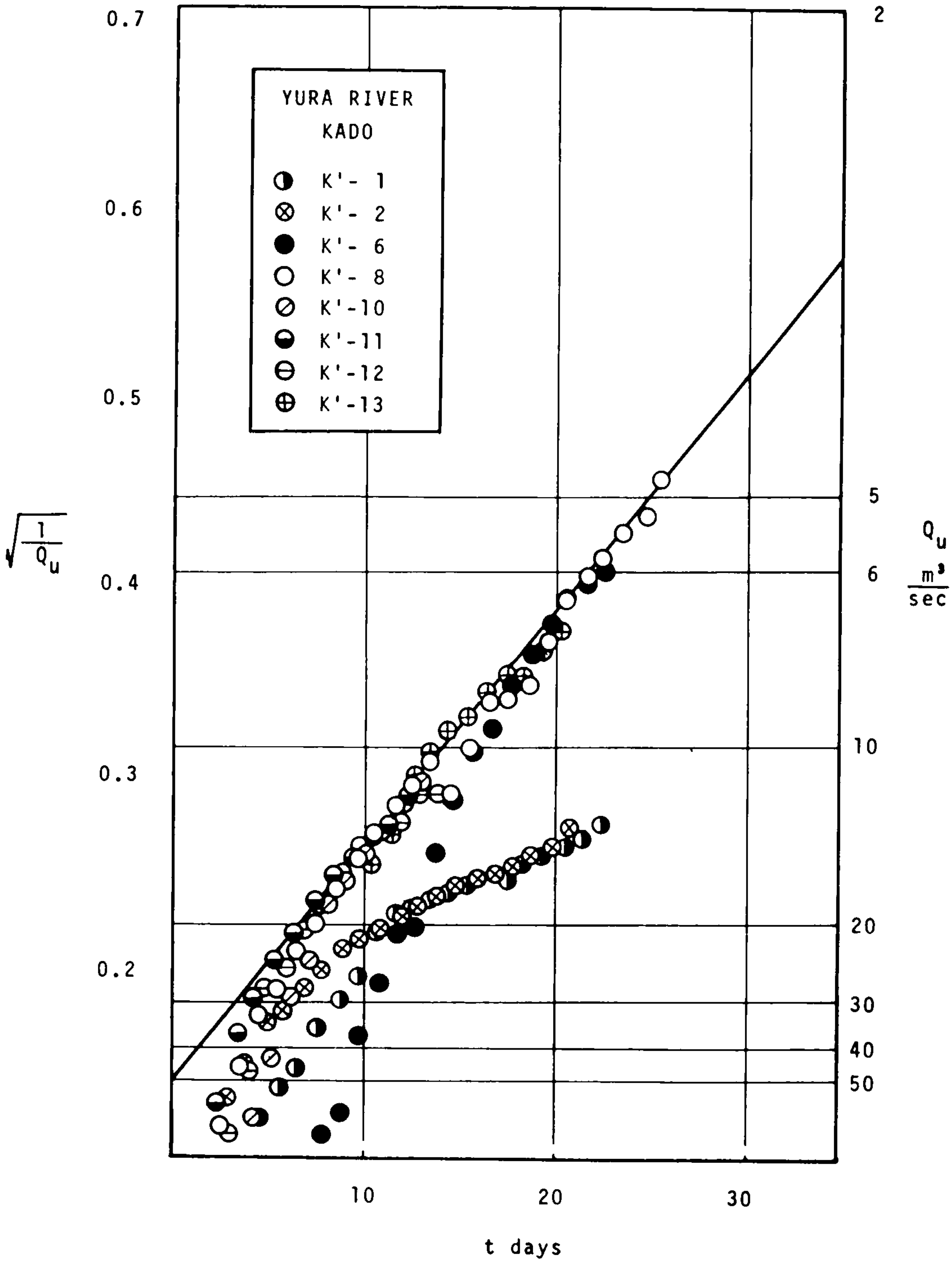


Fig. I-14 Recession Curve of Unconfined Component in $(t, \sqrt{\frac{1}{Q_u}})$-plane



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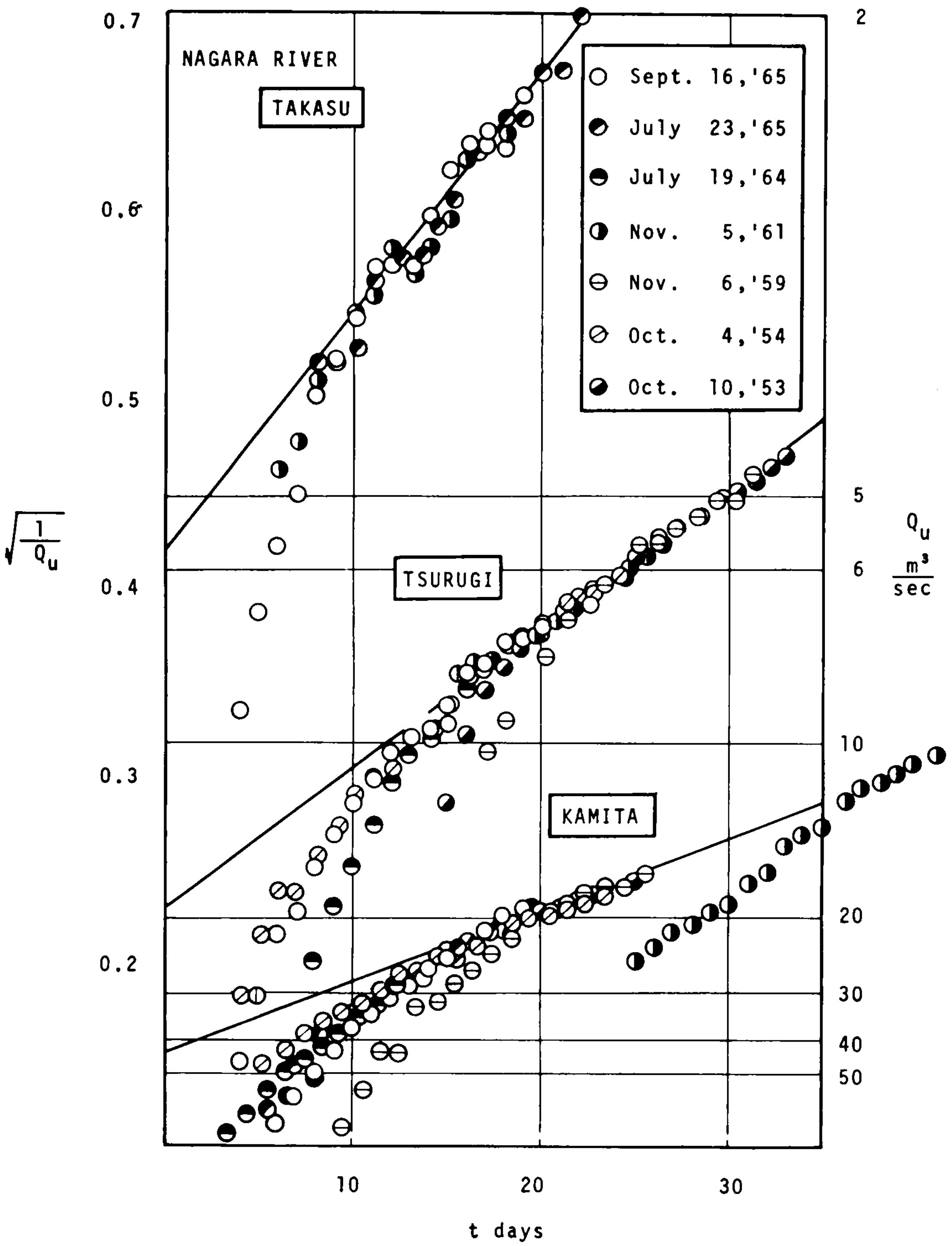

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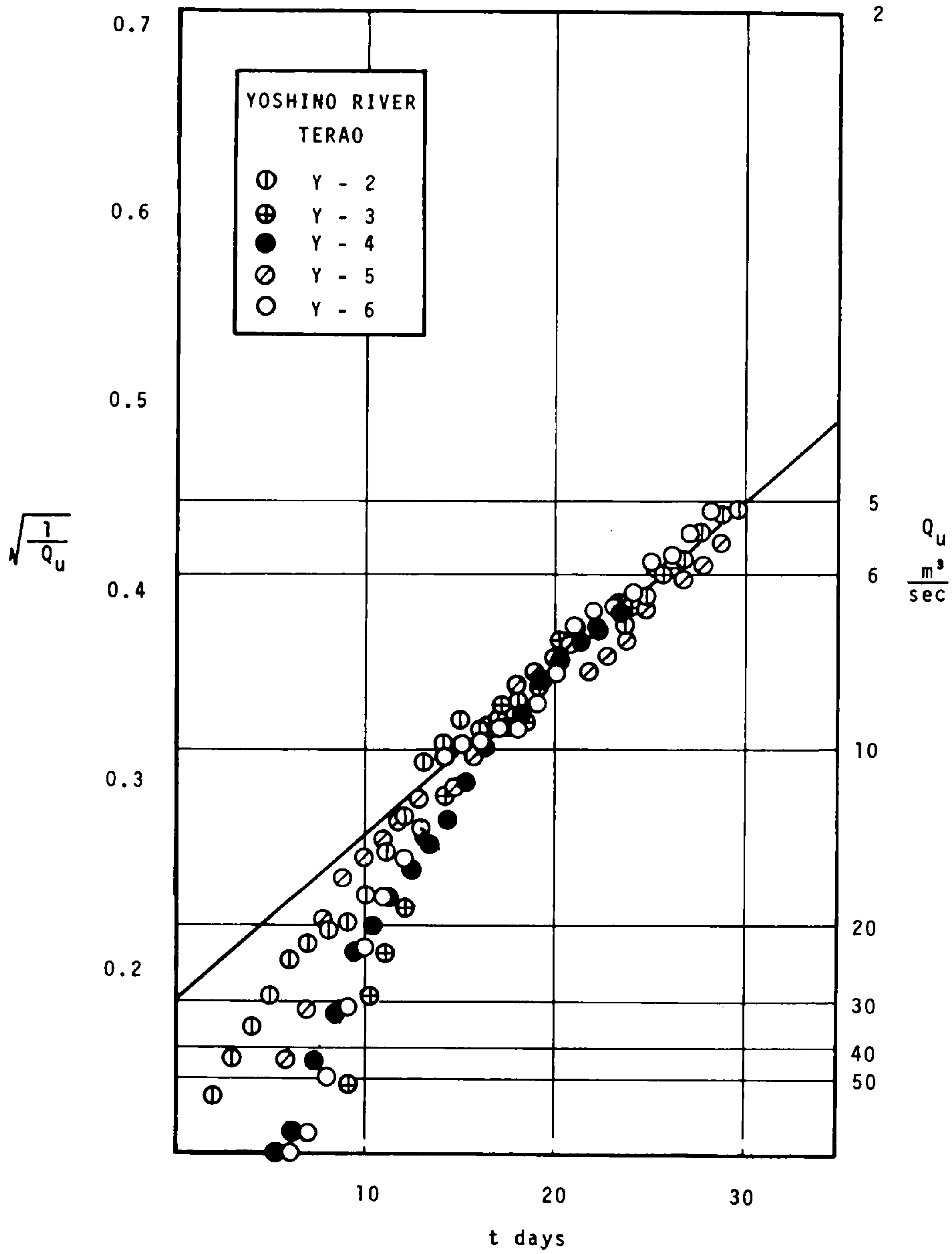

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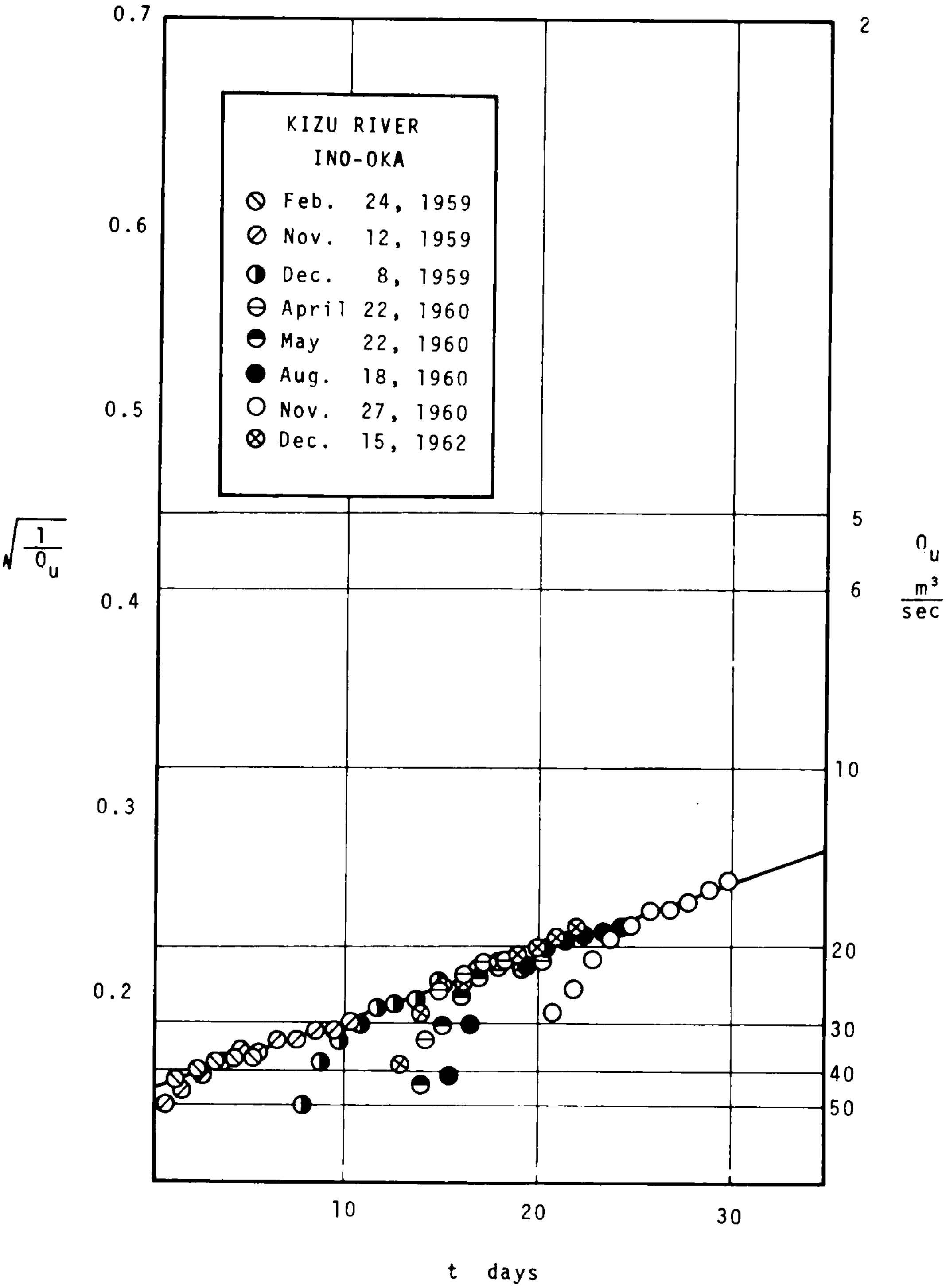

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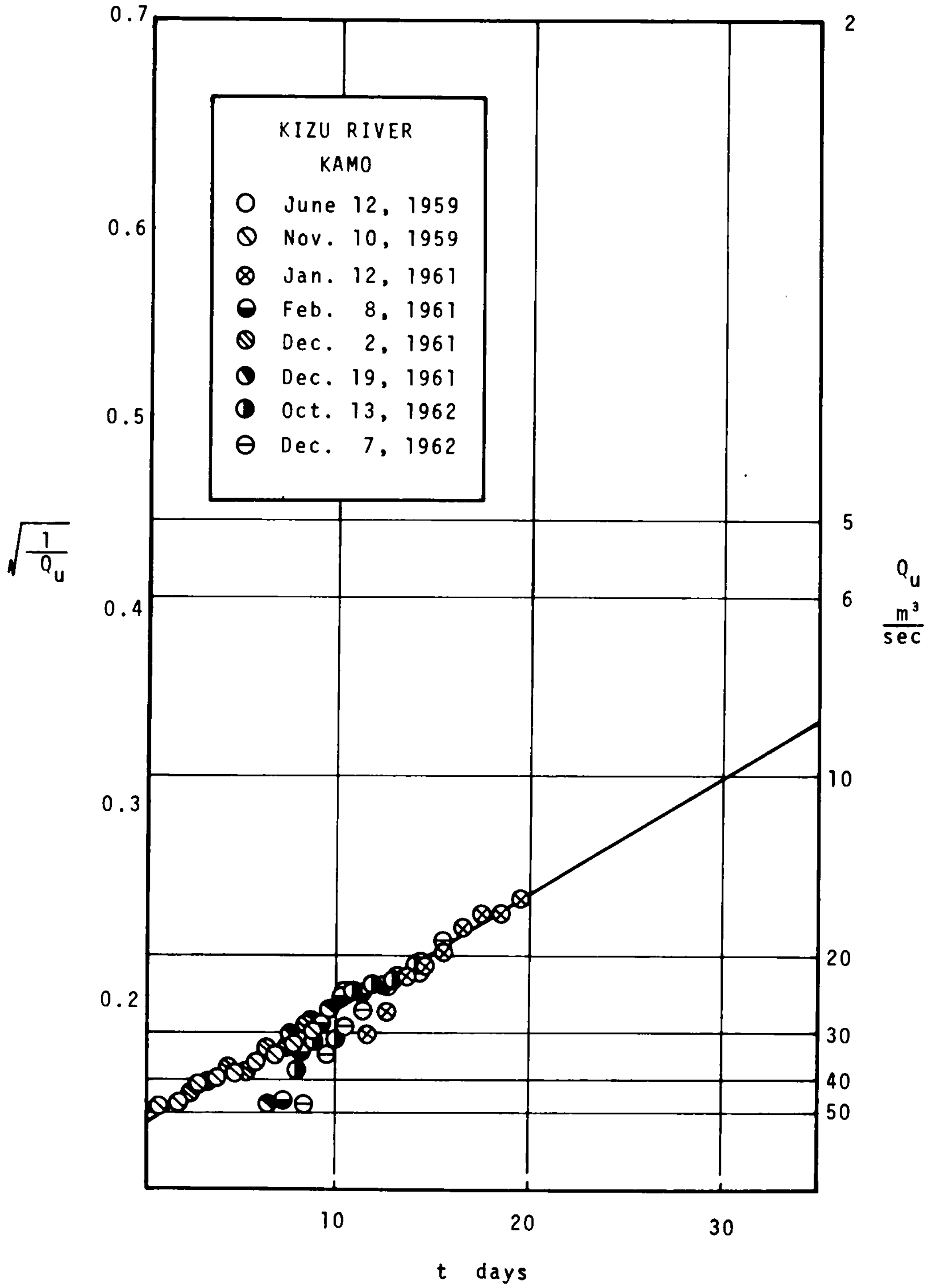

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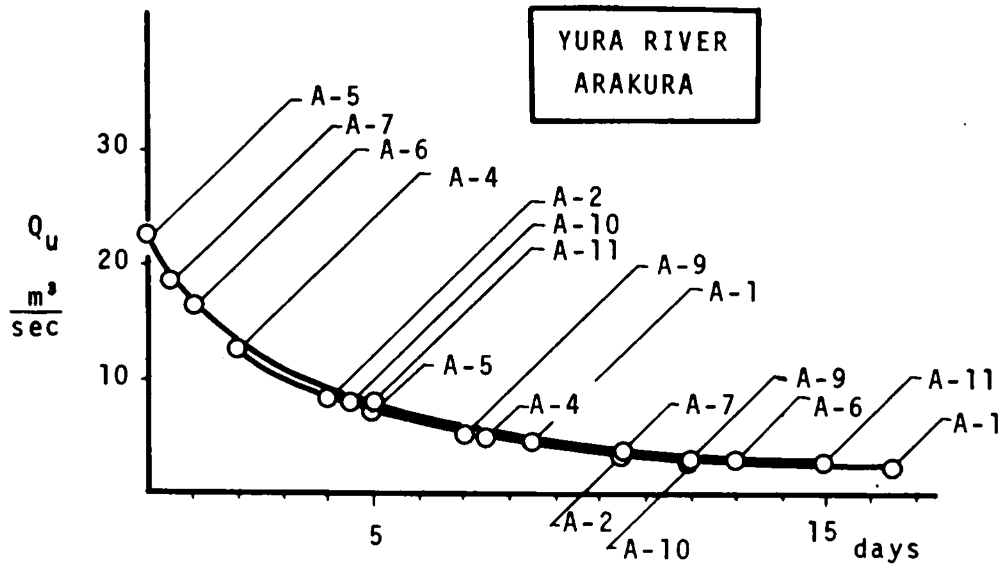


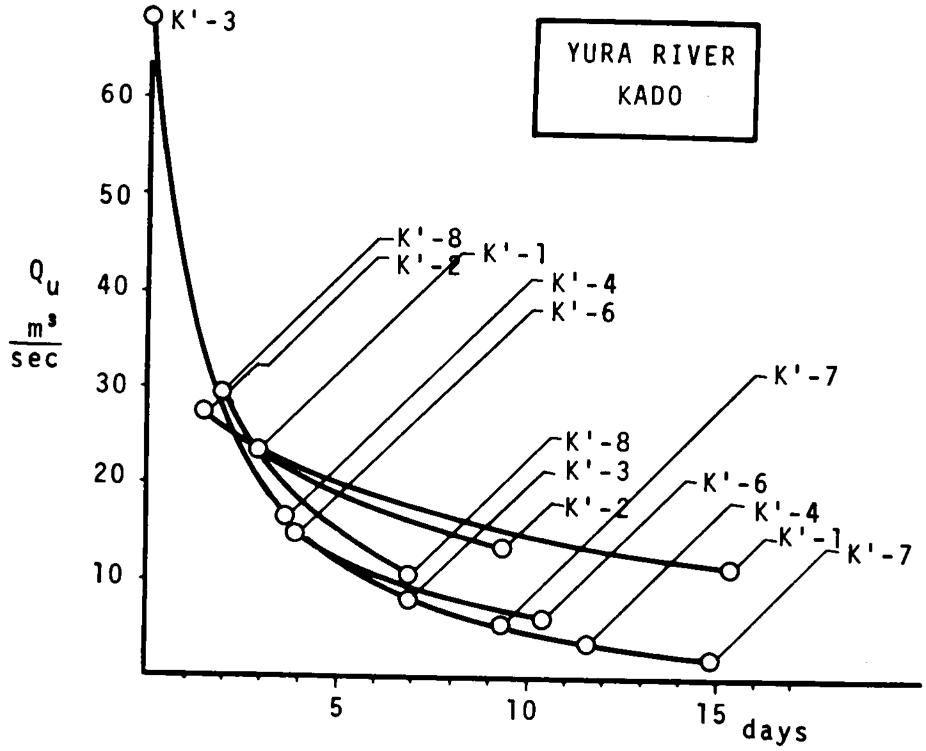
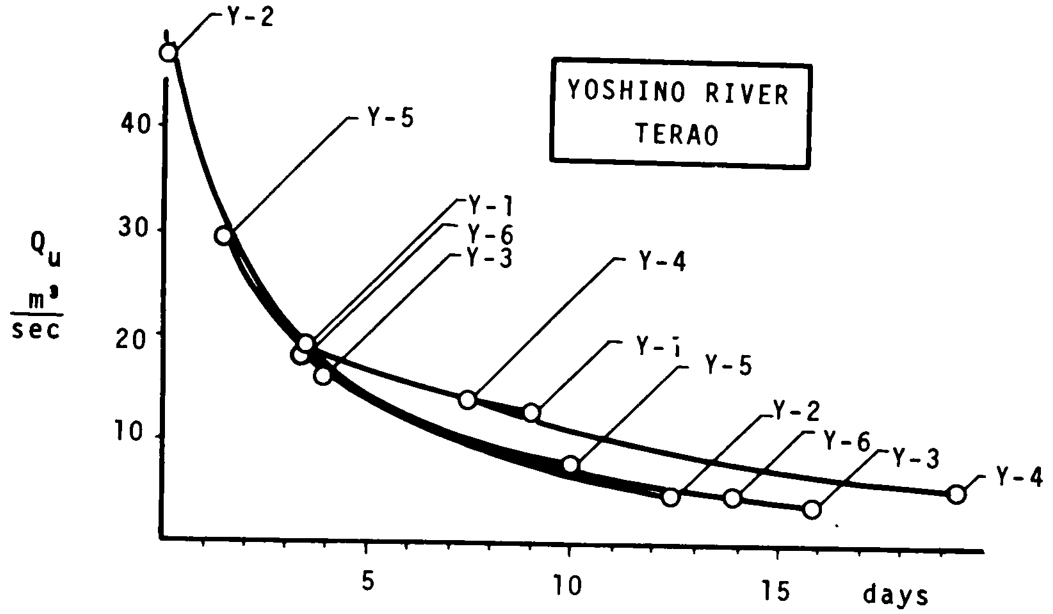


Fig. I-15 Recession Curve of Unconfined Component in (t, Q_u)-plane



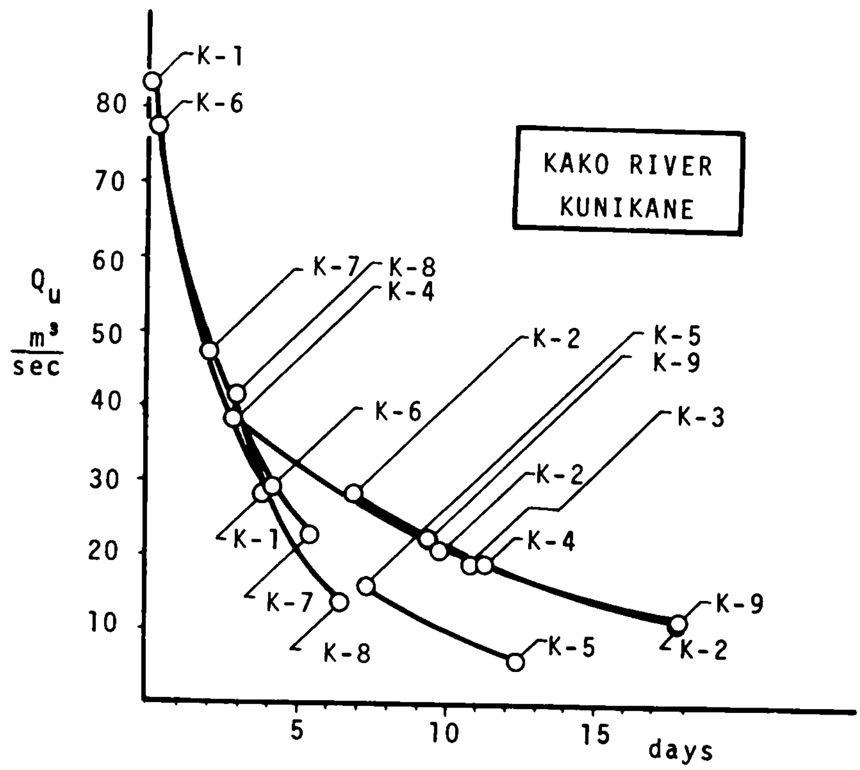


Fig. I-15 (Continued)

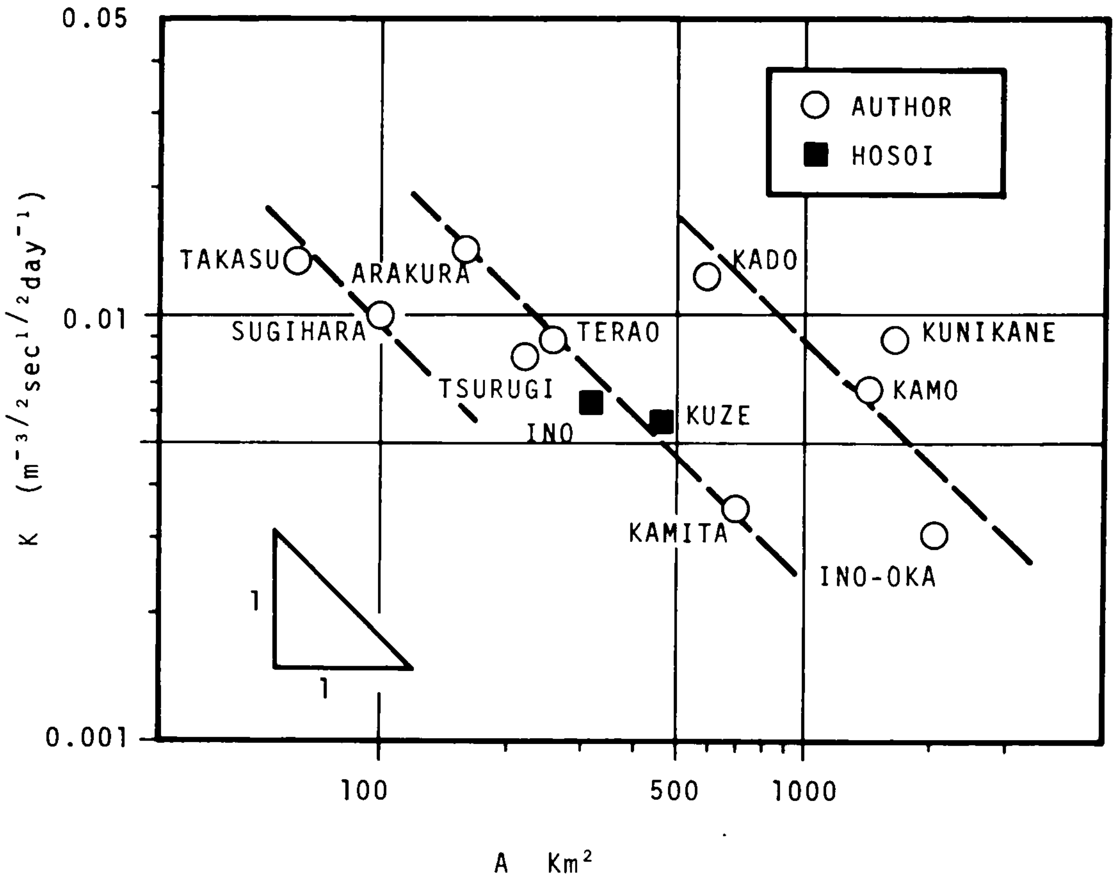


Fig. I-16 Relationship between Recession Factor K and Catchment Area
(Data analyzed by Hosoi are also plotted for comparison. He analyzed ground water runoff by means of the author's method)

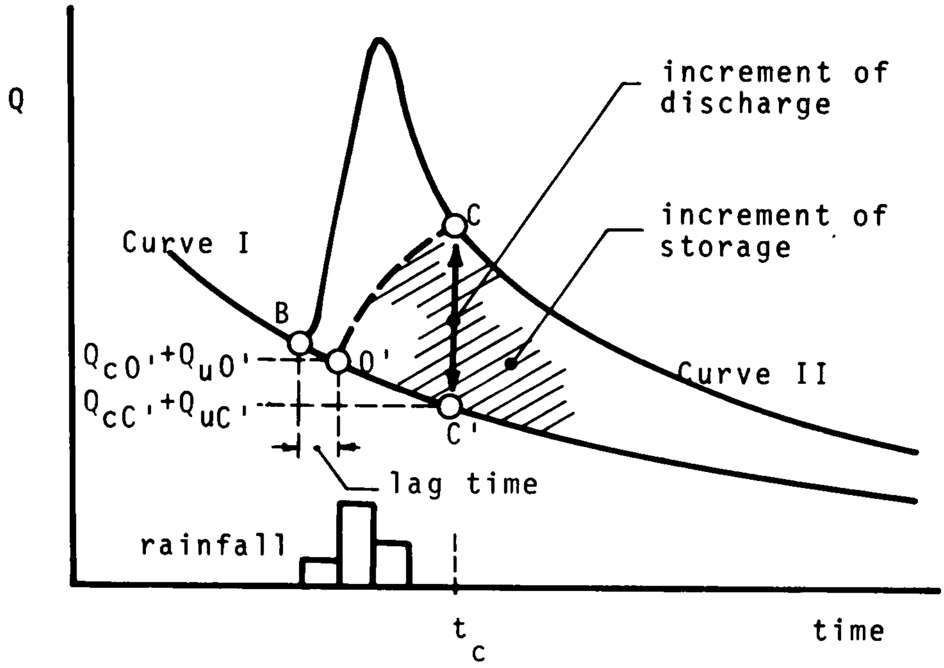


Fig. I-17 Schematic Representation of Rising State of Hydrograph resulting from Rainfall

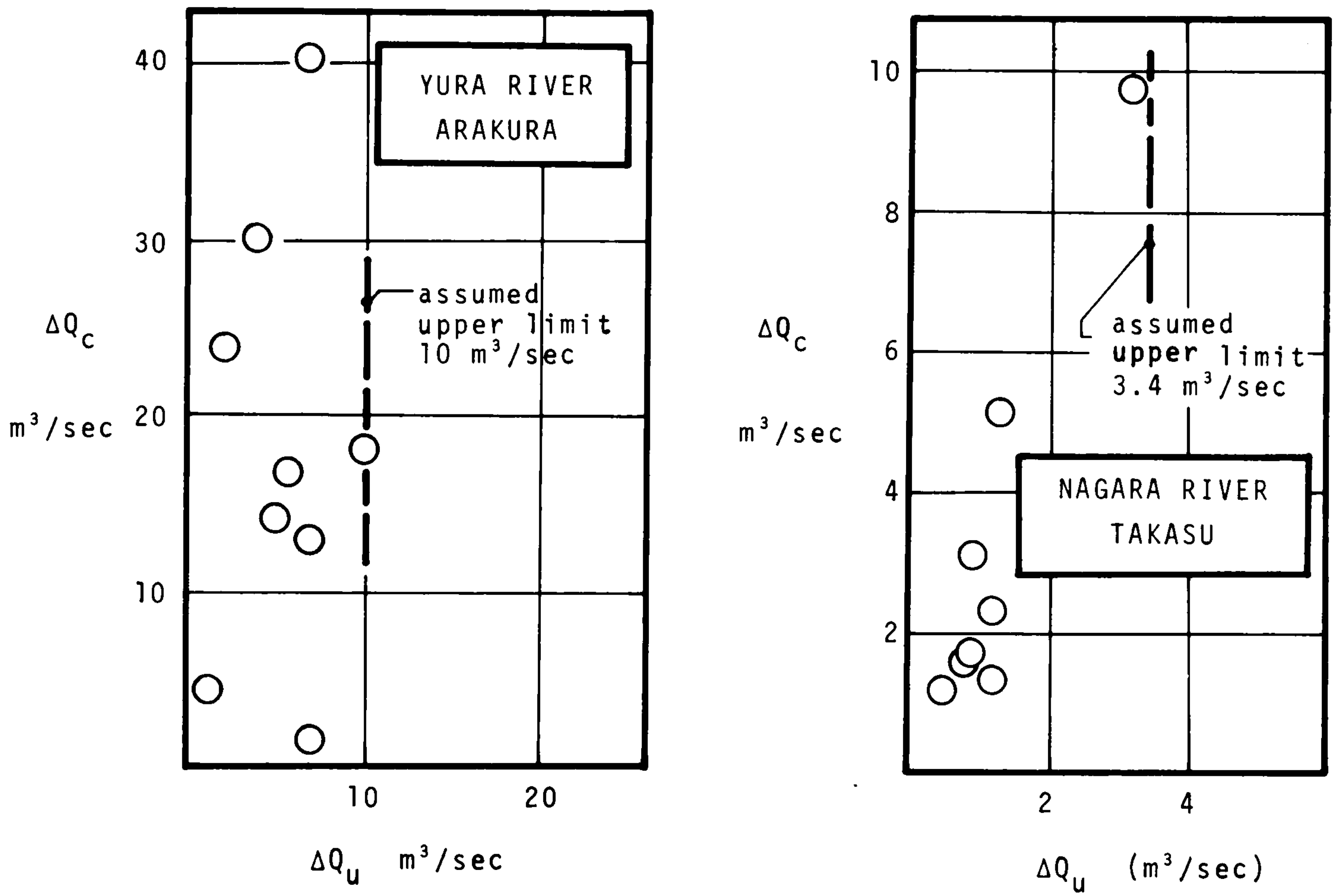


Fig. I-18 Relationship between Increments of Discharge due to Rainfall ΔQ_u and ΔQ_c

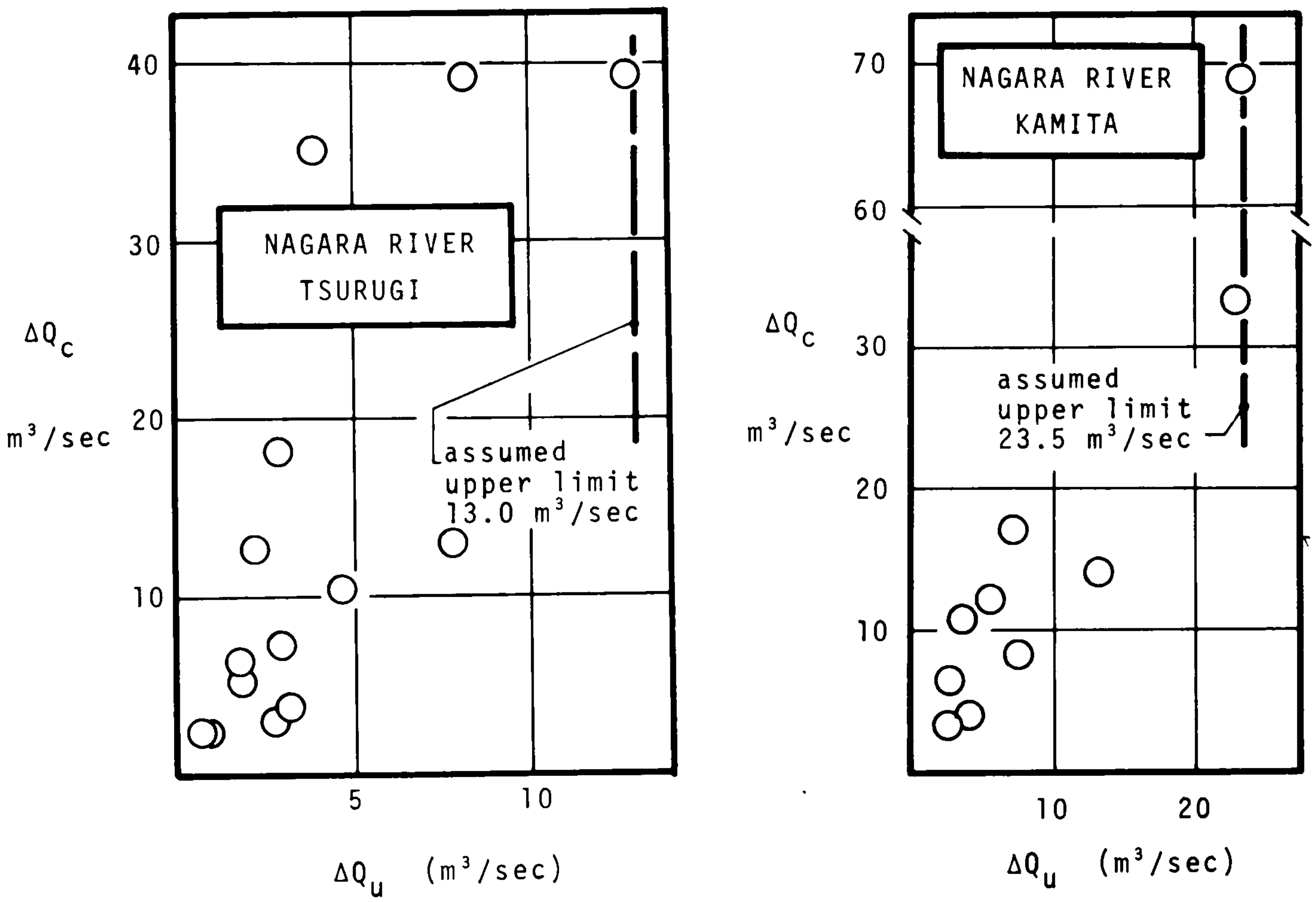


Fig. I-18 (Continued)

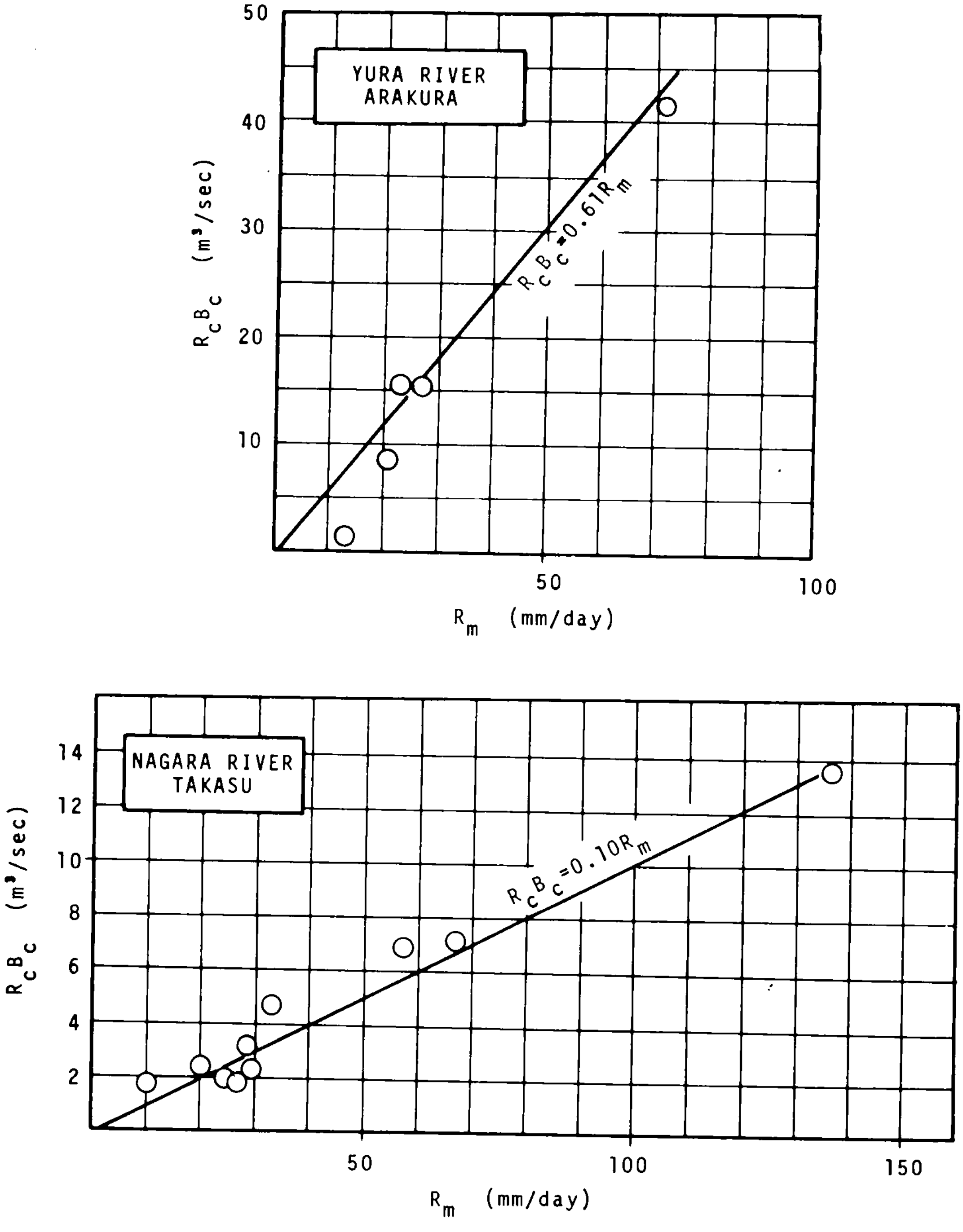
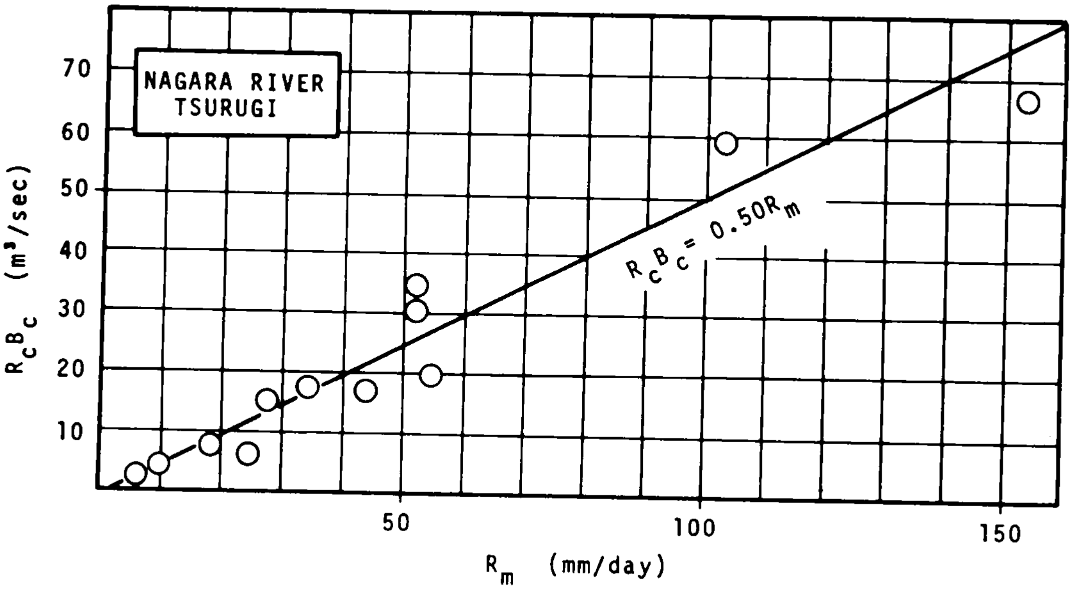


Fig. I-19 Relationship between Value R_cB_c and Mean Daily Rainfall R_m (Confined Component)



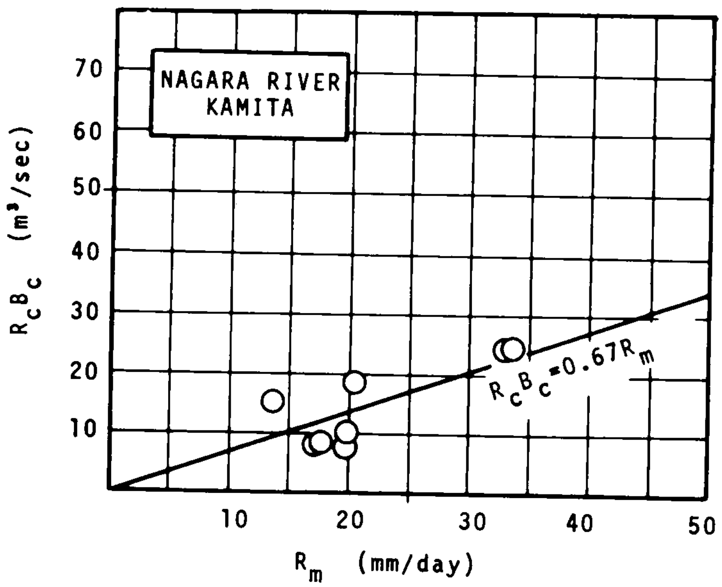


Fig. I-19 (Continued)

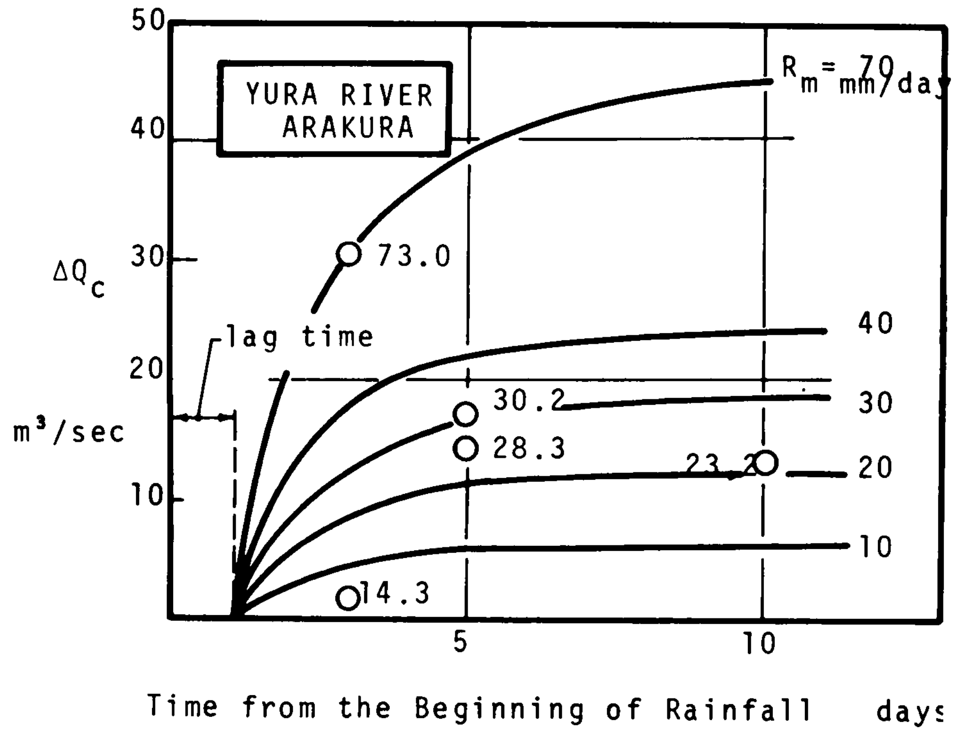
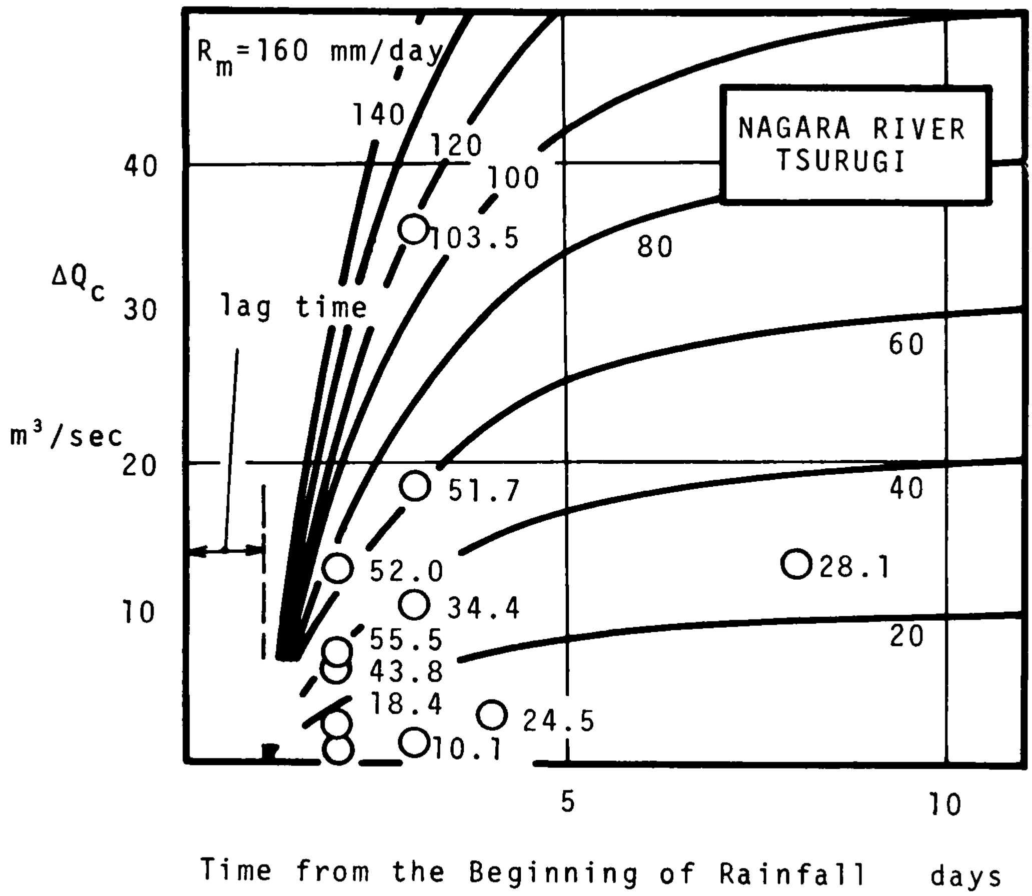


Fig. I-20 Variation of Confined Component due to Rainfall



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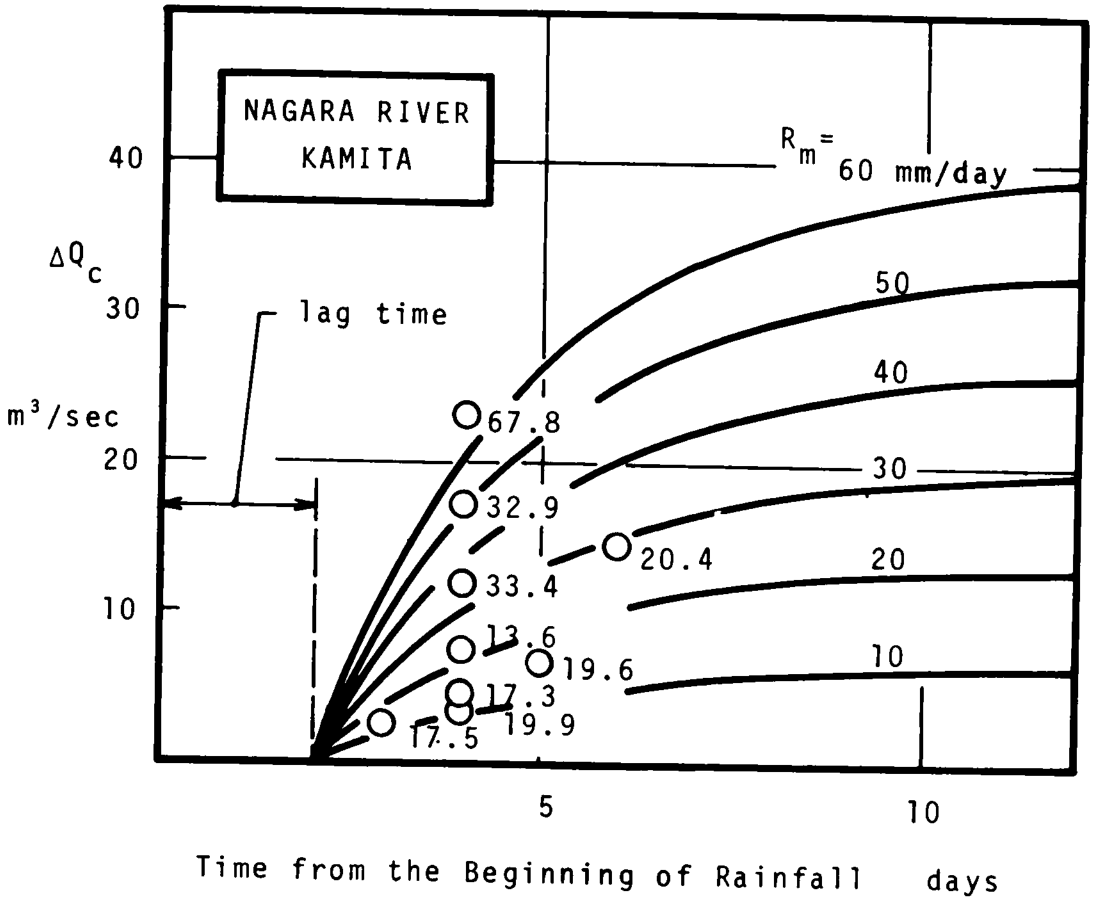

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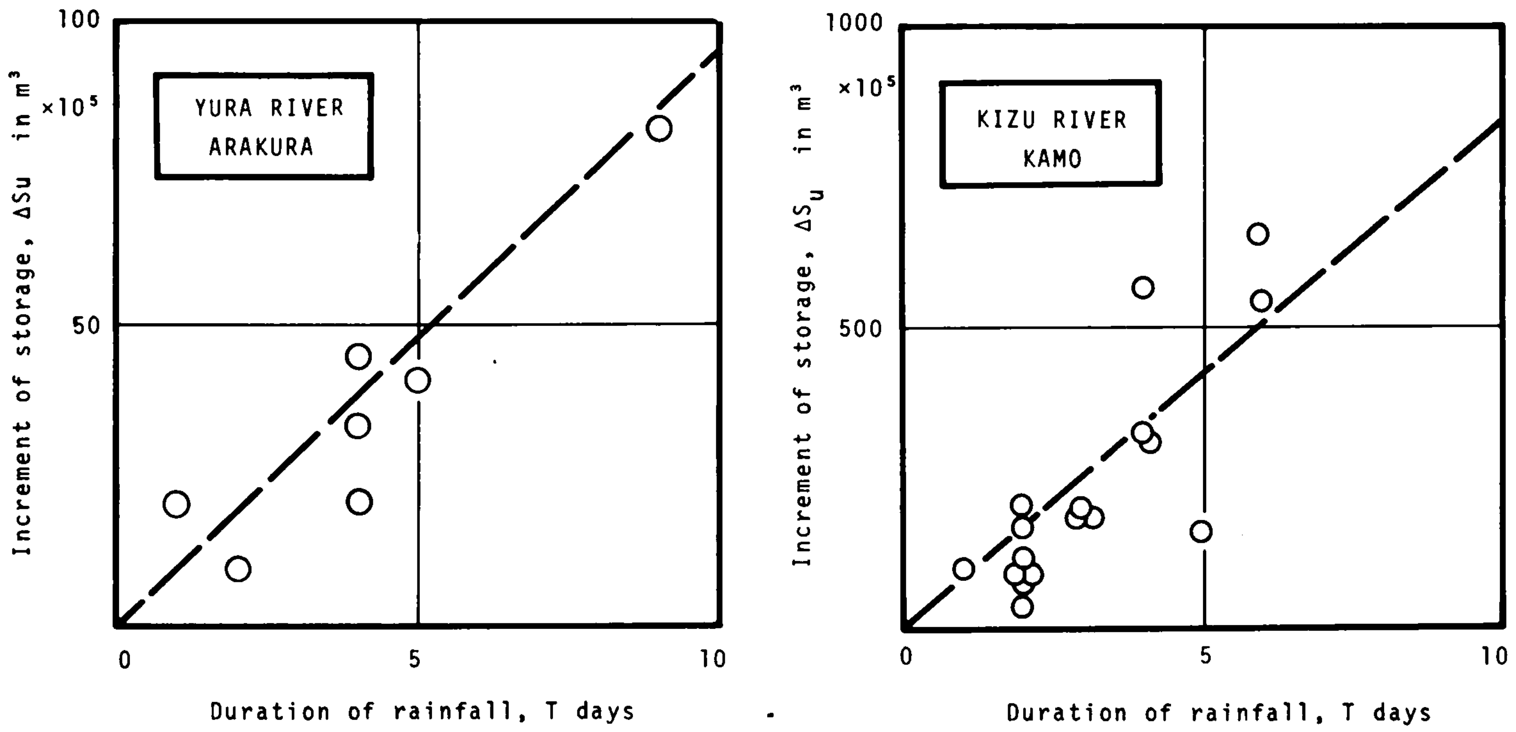


Fig. I-21 Relationship between Increment of Storage ΔS_u and Duration of Rainfall T

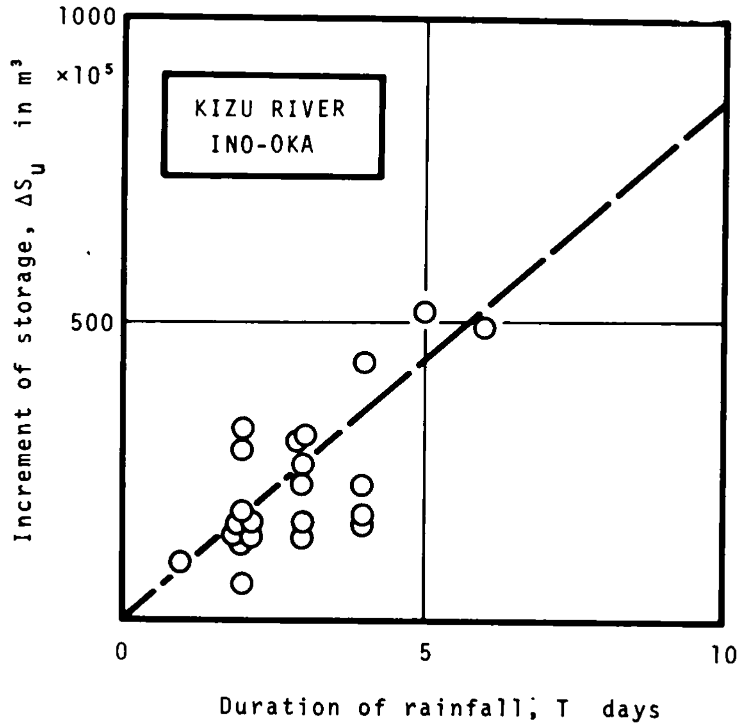


Fig. 21 (Continued)

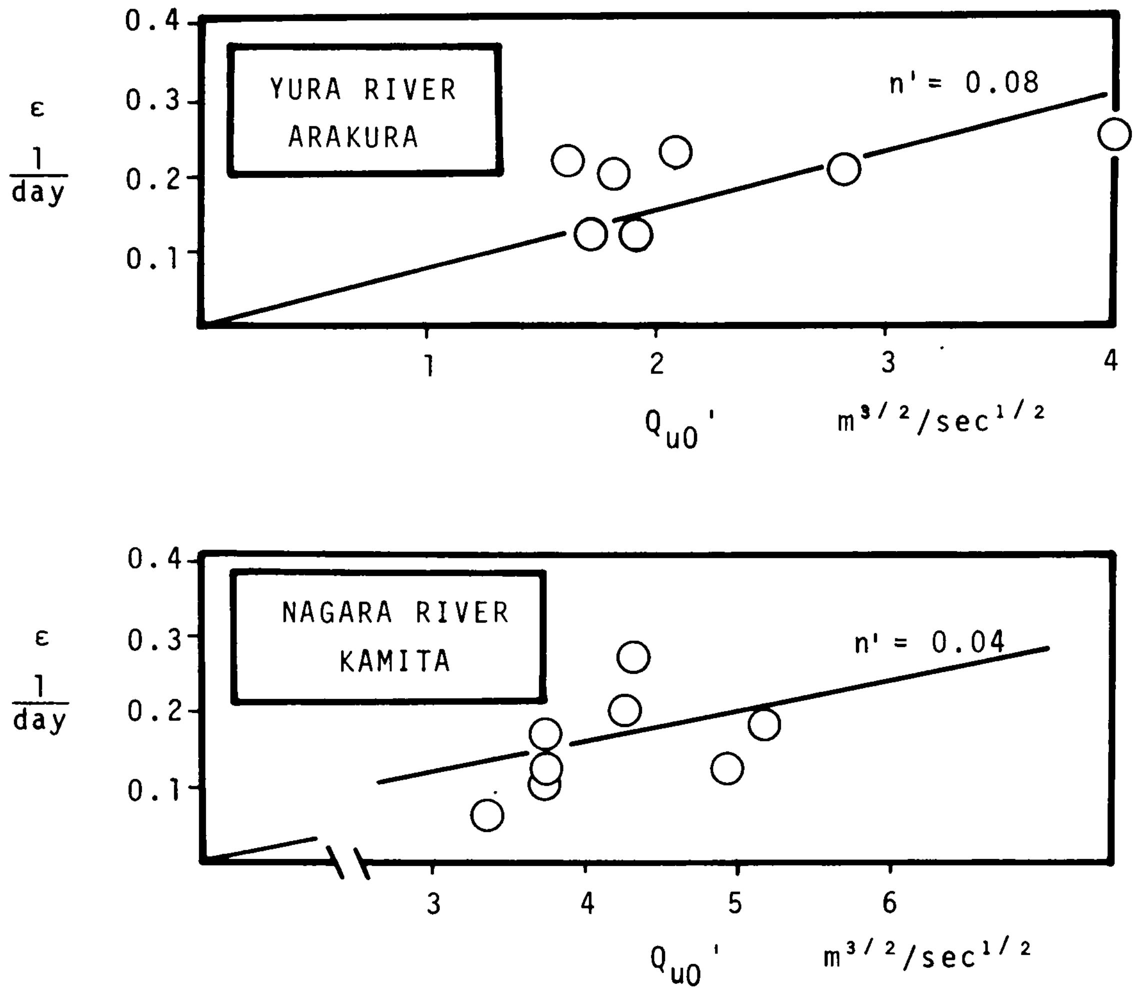
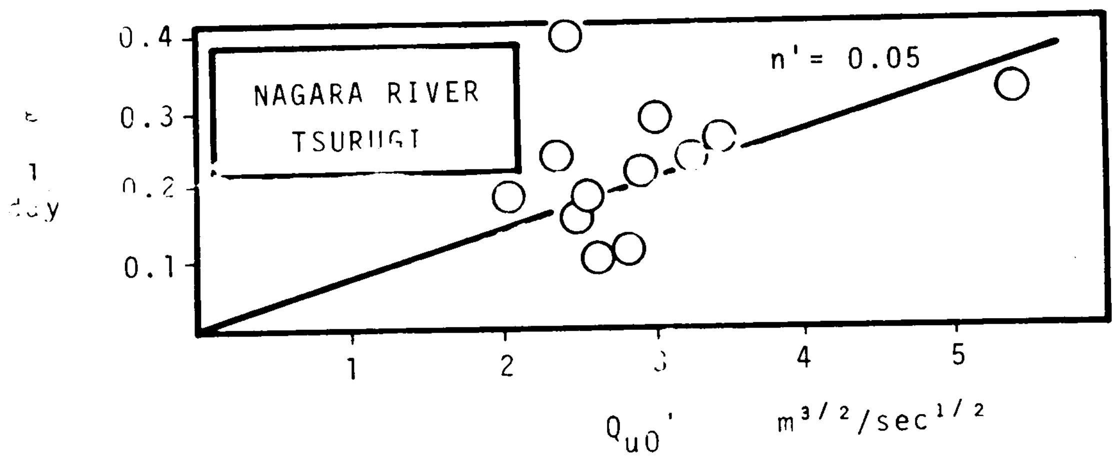


Fig. I-22 Relationship between Variation Rate ε and Initial Discharge for Rising State of Unconfined Component



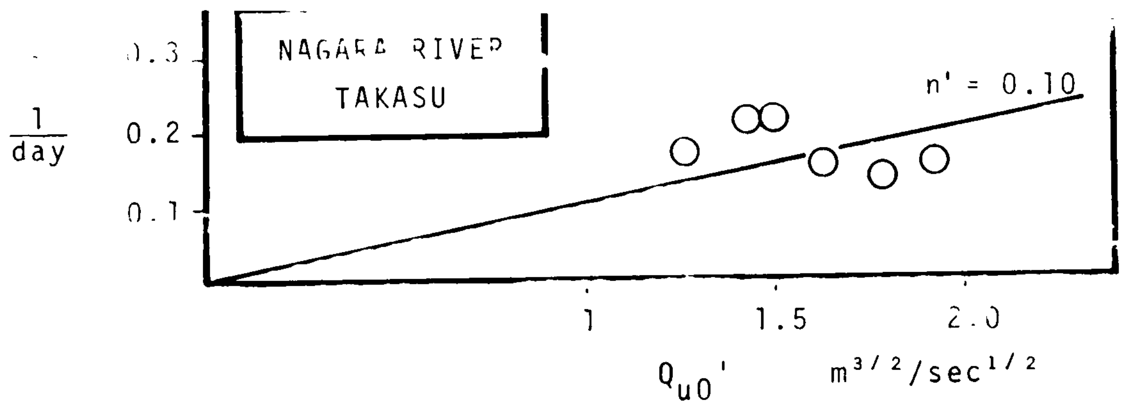


Fig. I-22 (Continued)

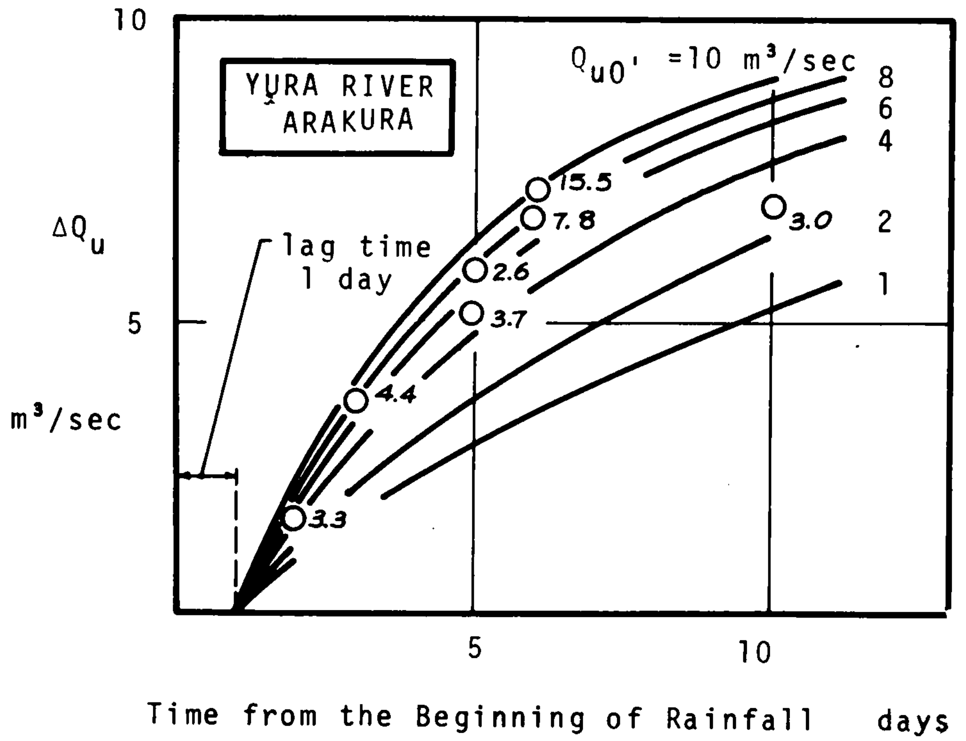
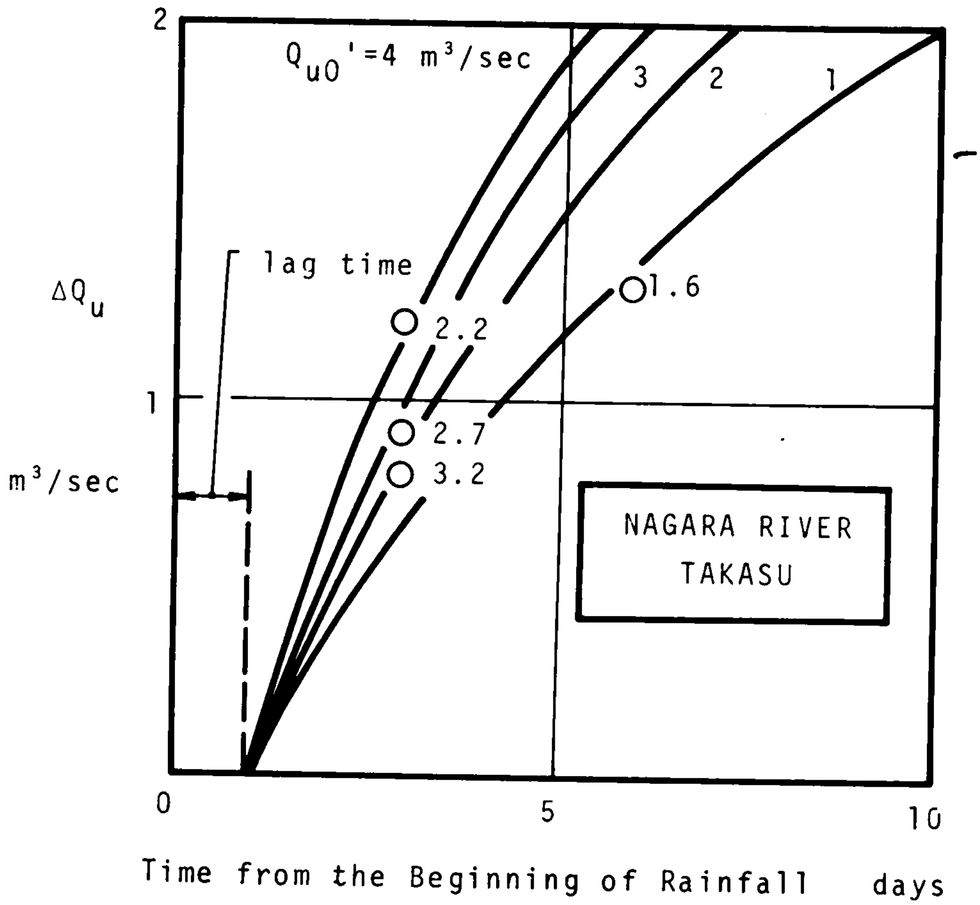


Fig. I-23 Variation of Unconfined Component due to Rainfall



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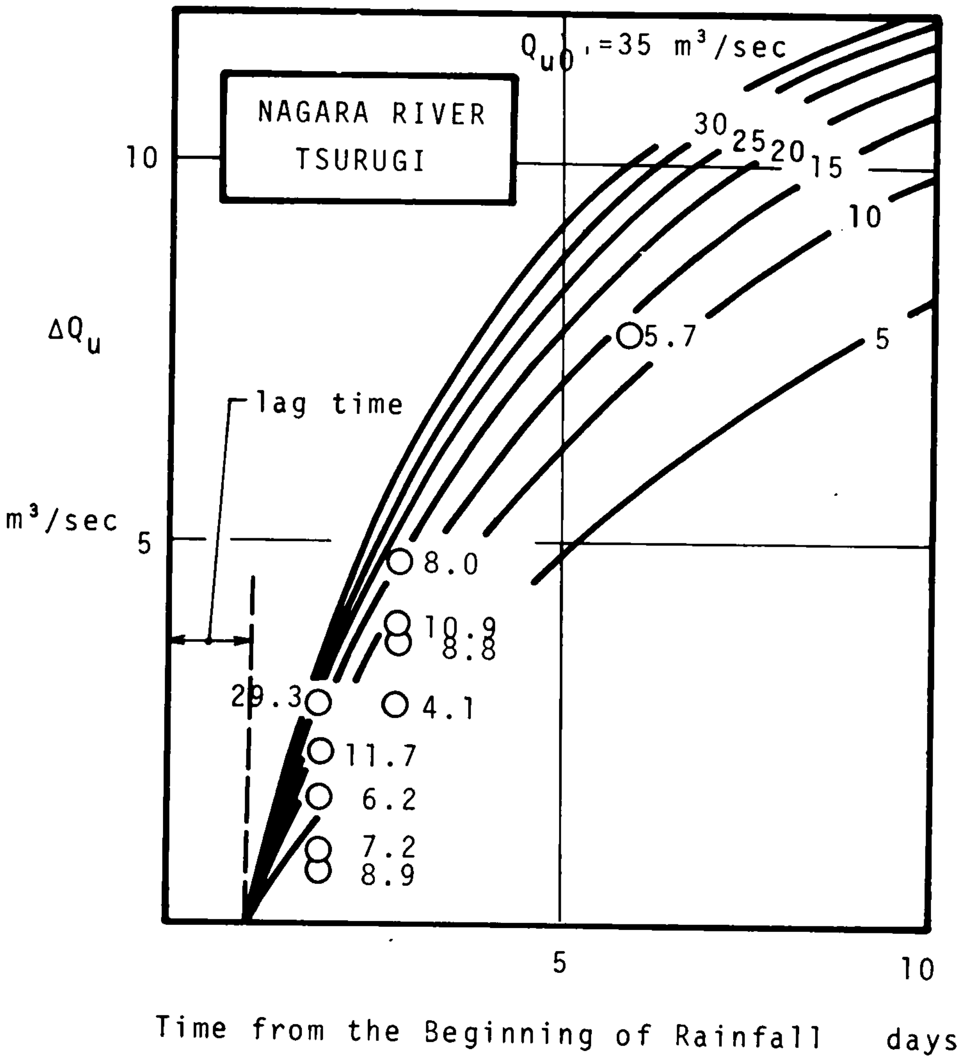

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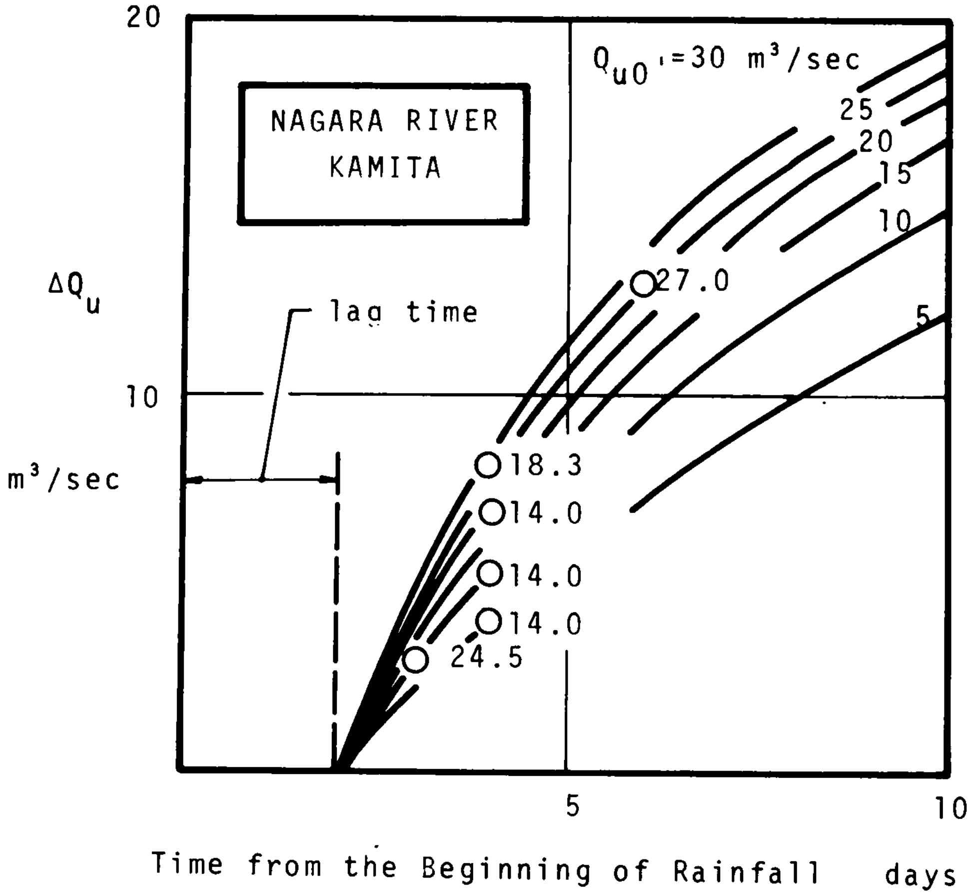

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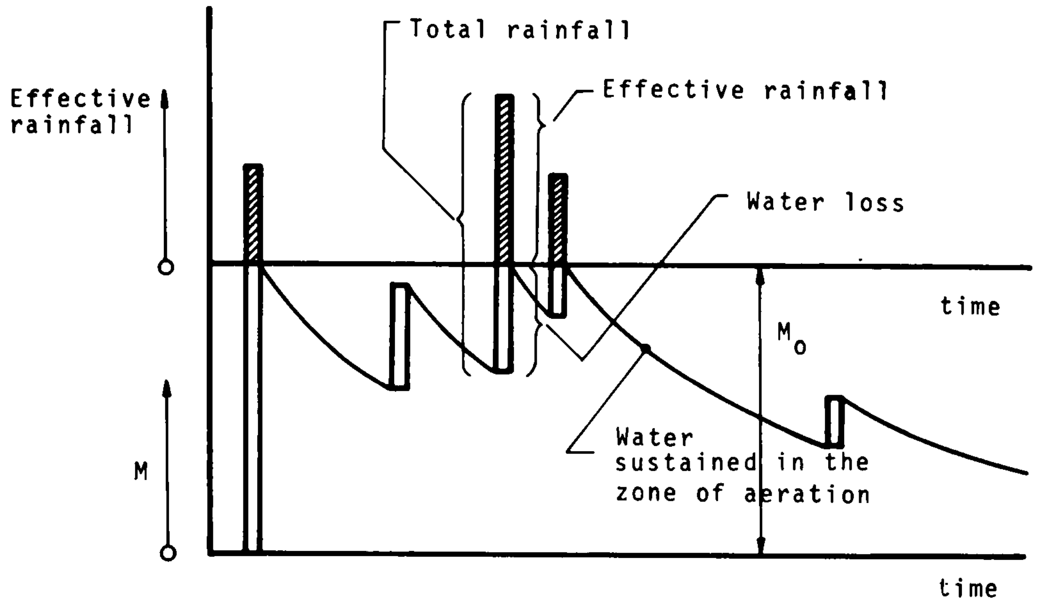


Fig. I-24 Schematic Explanation of Estimation of Water Loss and Effective Rainfall

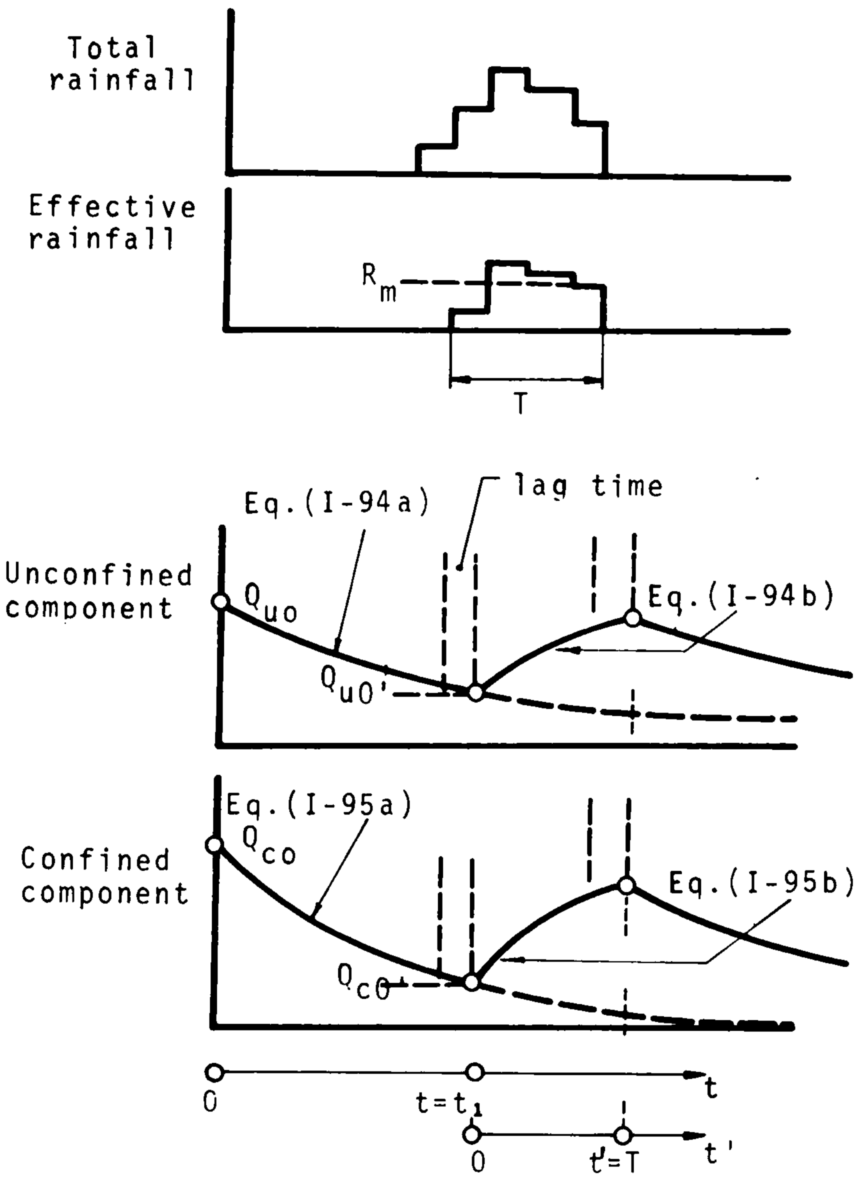


Fig. I-25 Schematic Explanation of Simulation Procedure

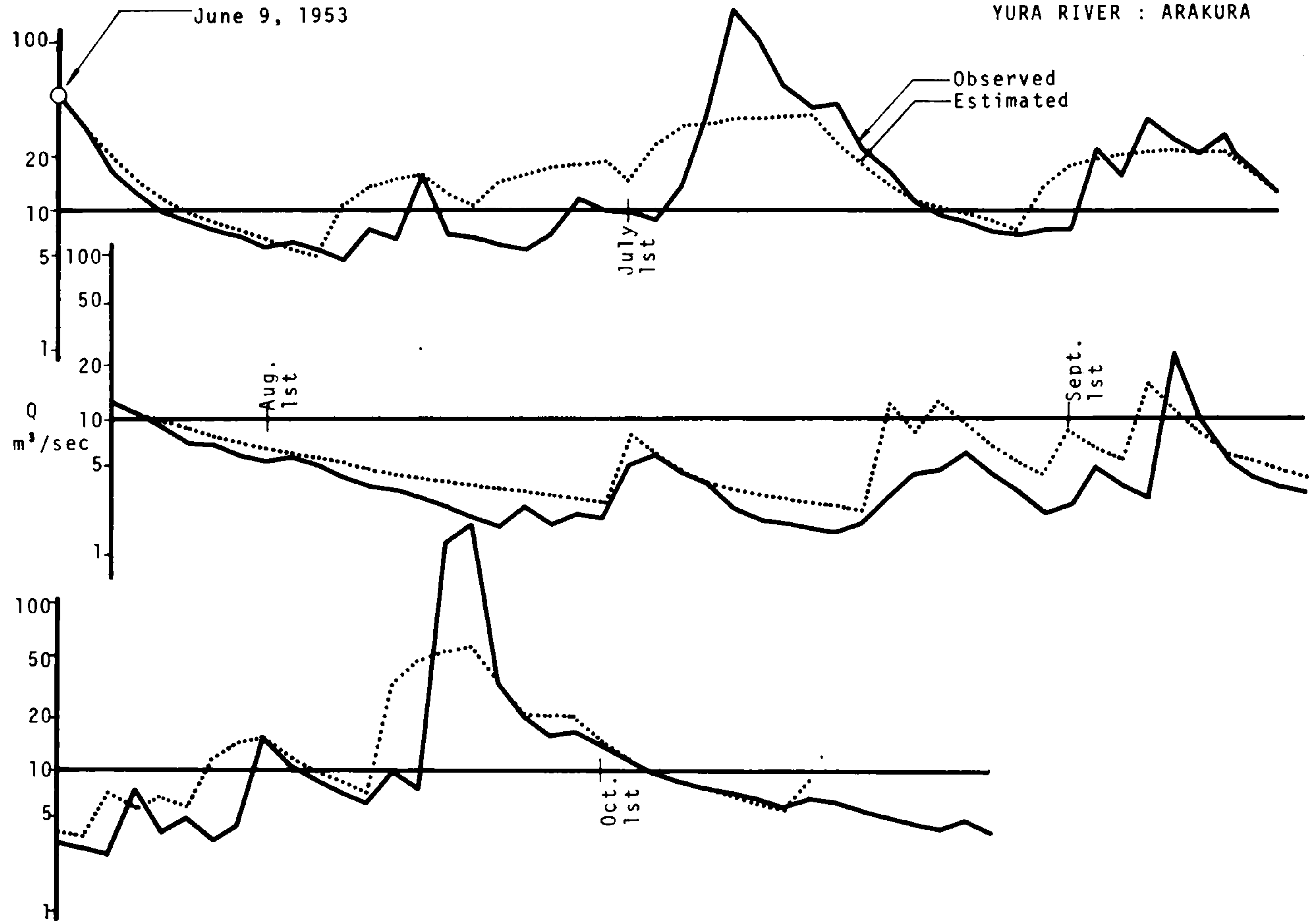
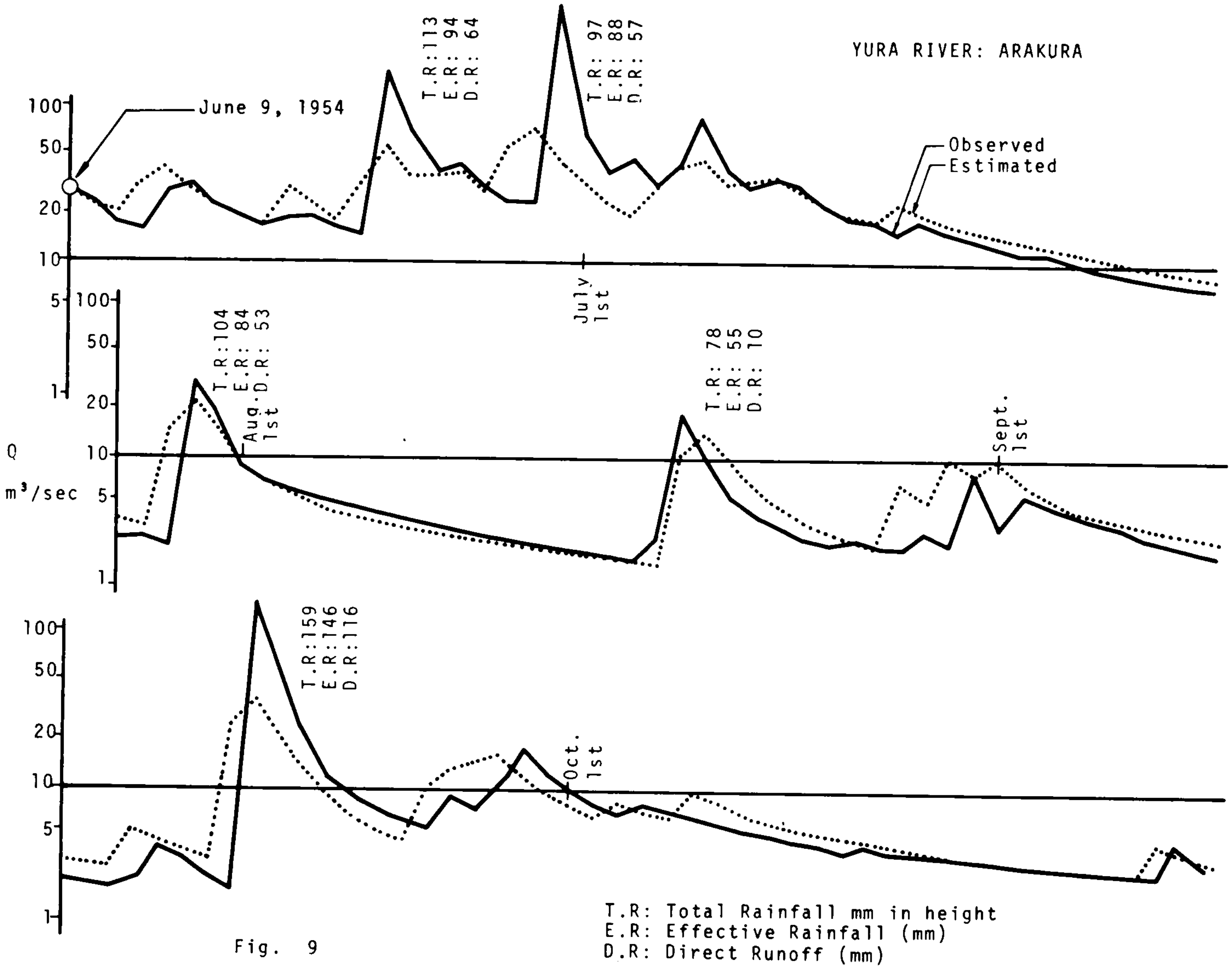


Fig. I-26 Comparison of Simulated Discharge with Observed Hydrograph,
(Ground water runoff is only simulated, but not direct runoff)



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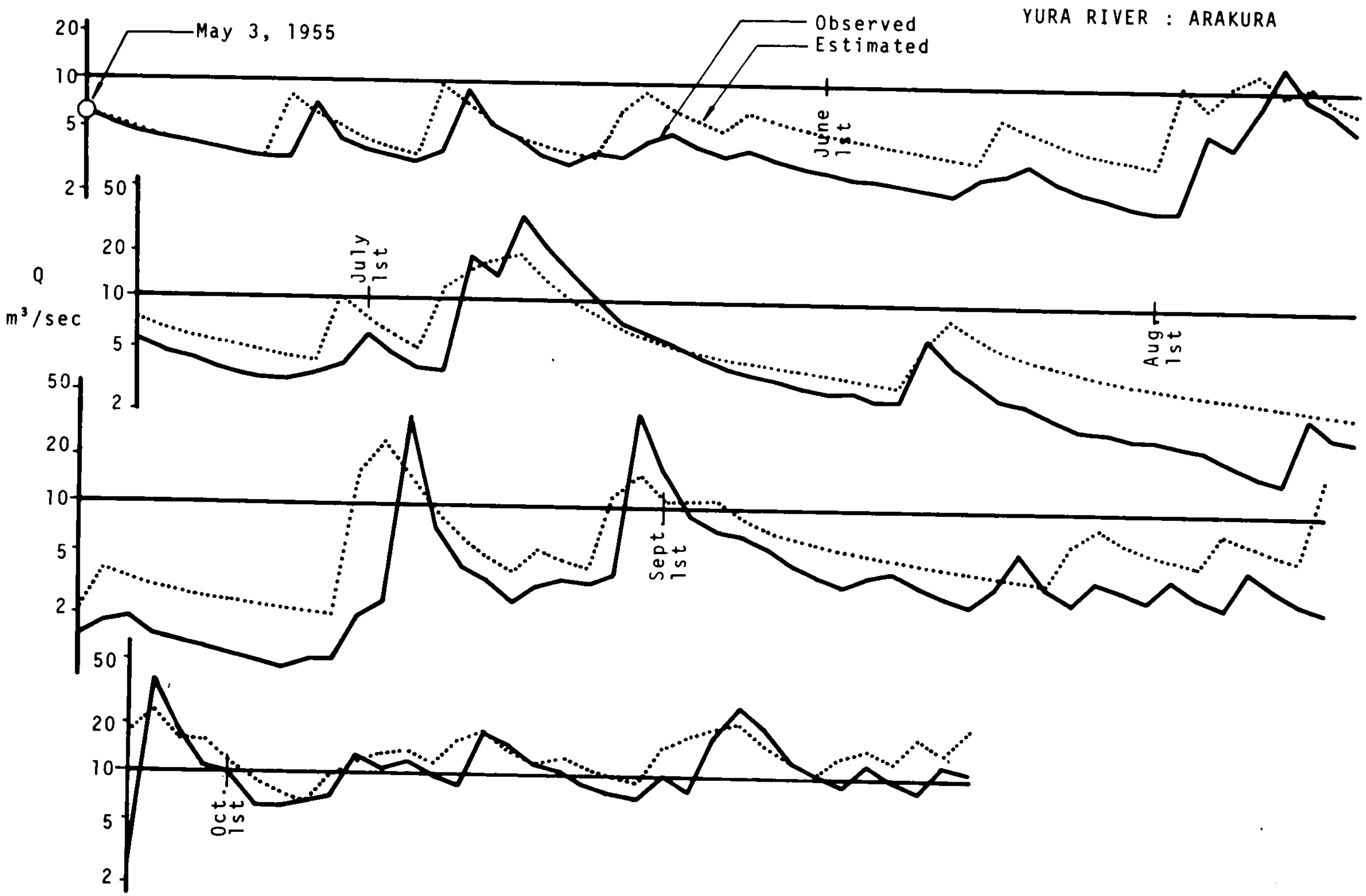

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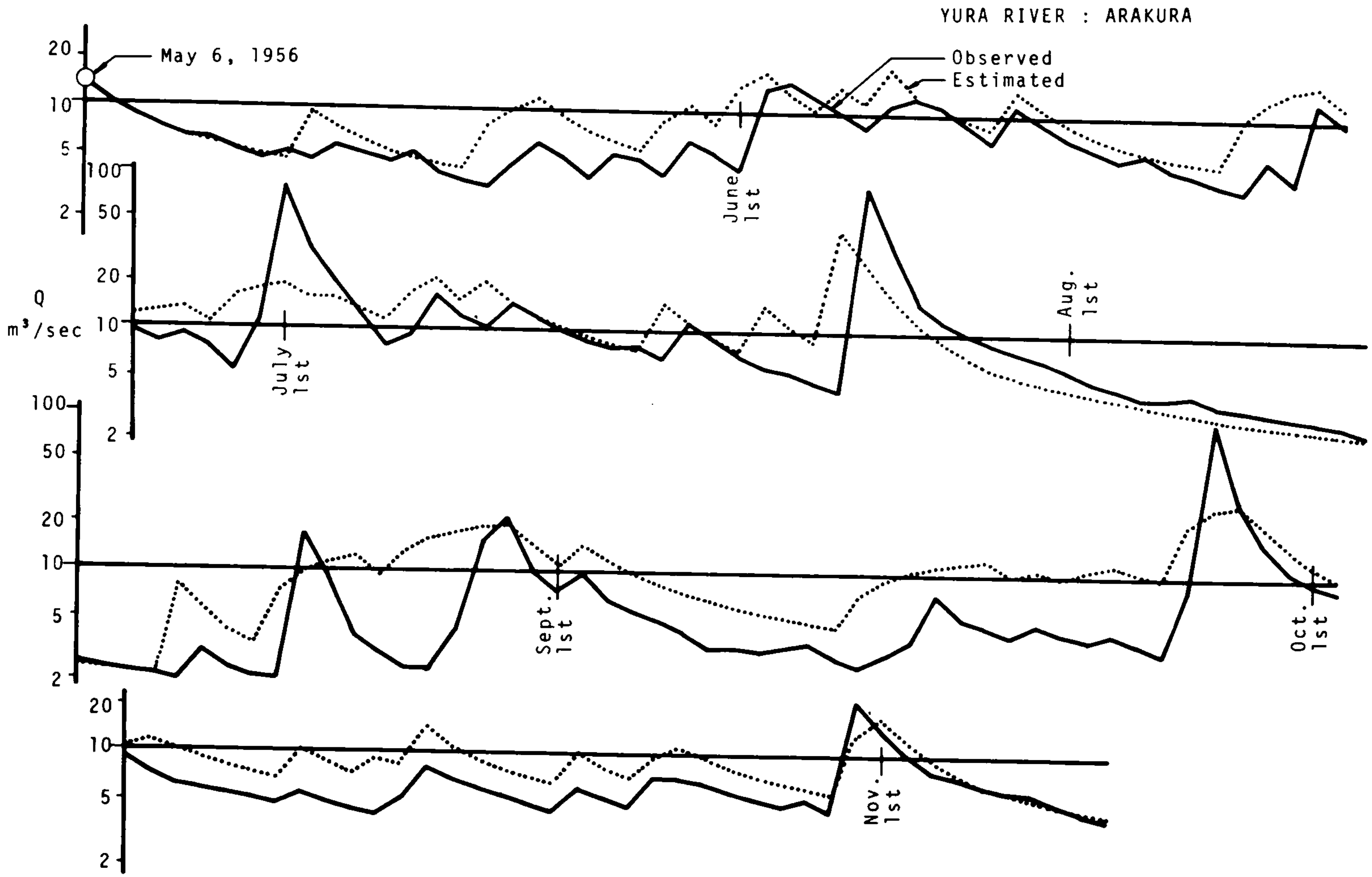
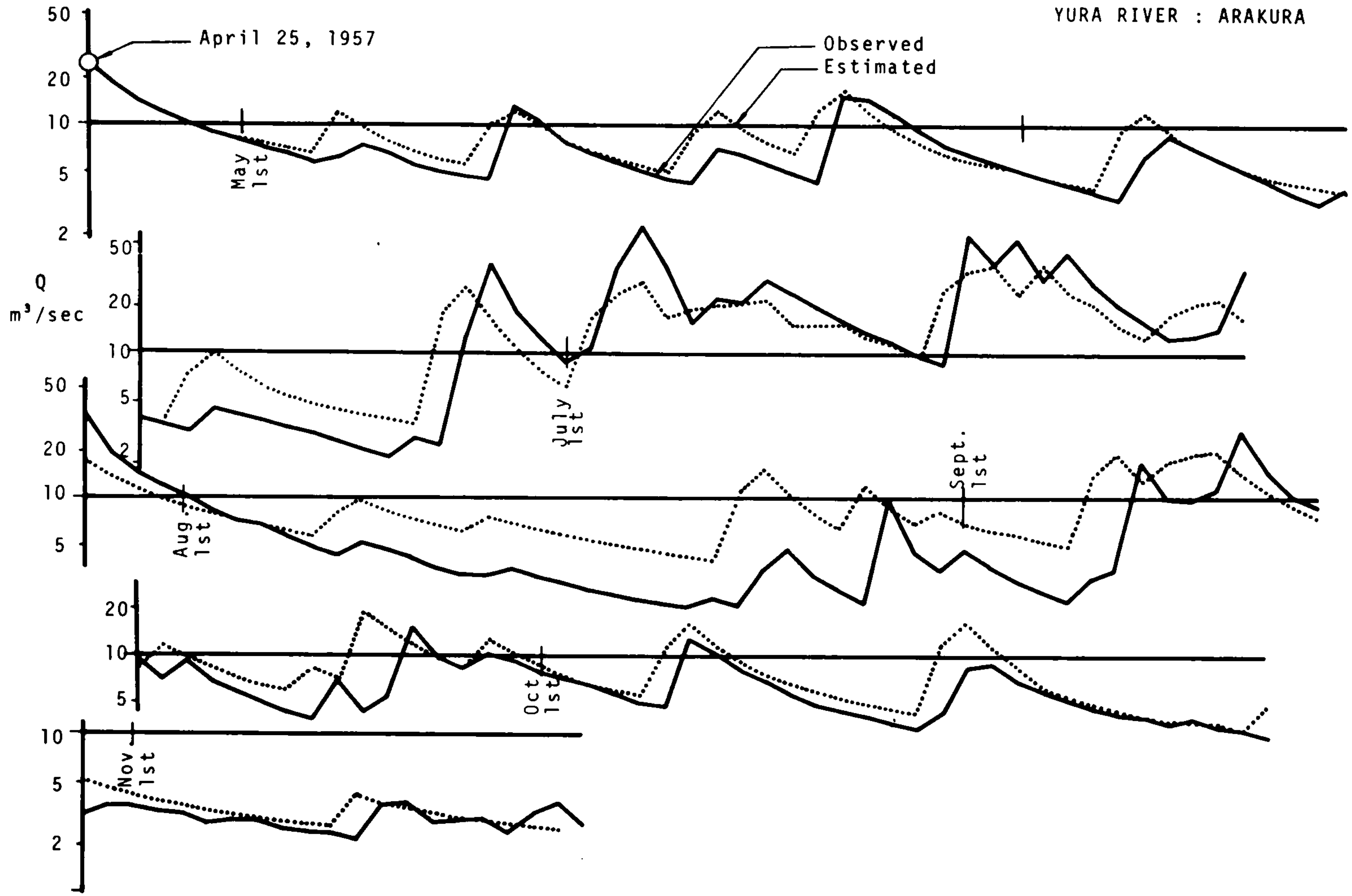


Fig. I-26 (Continued)



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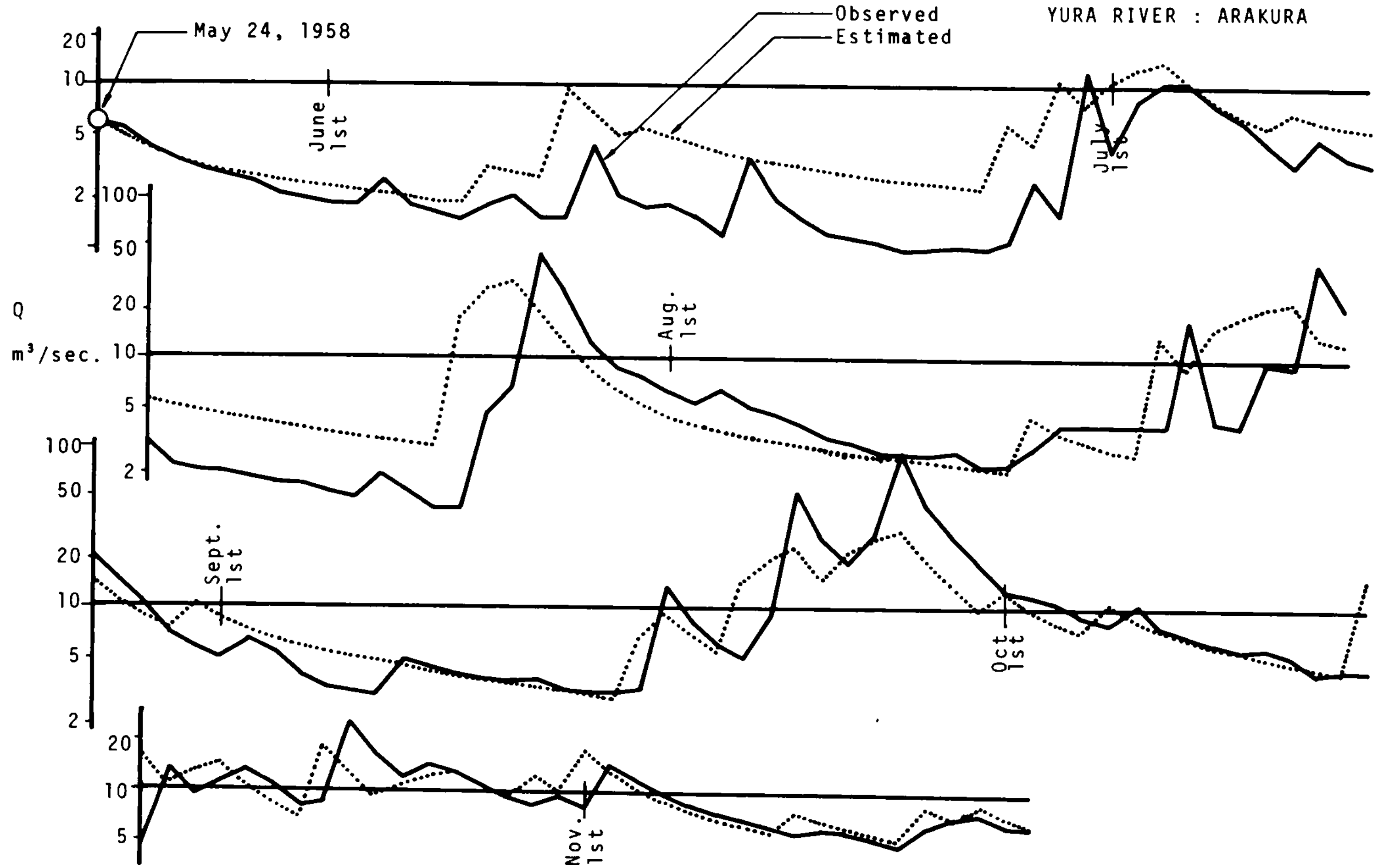

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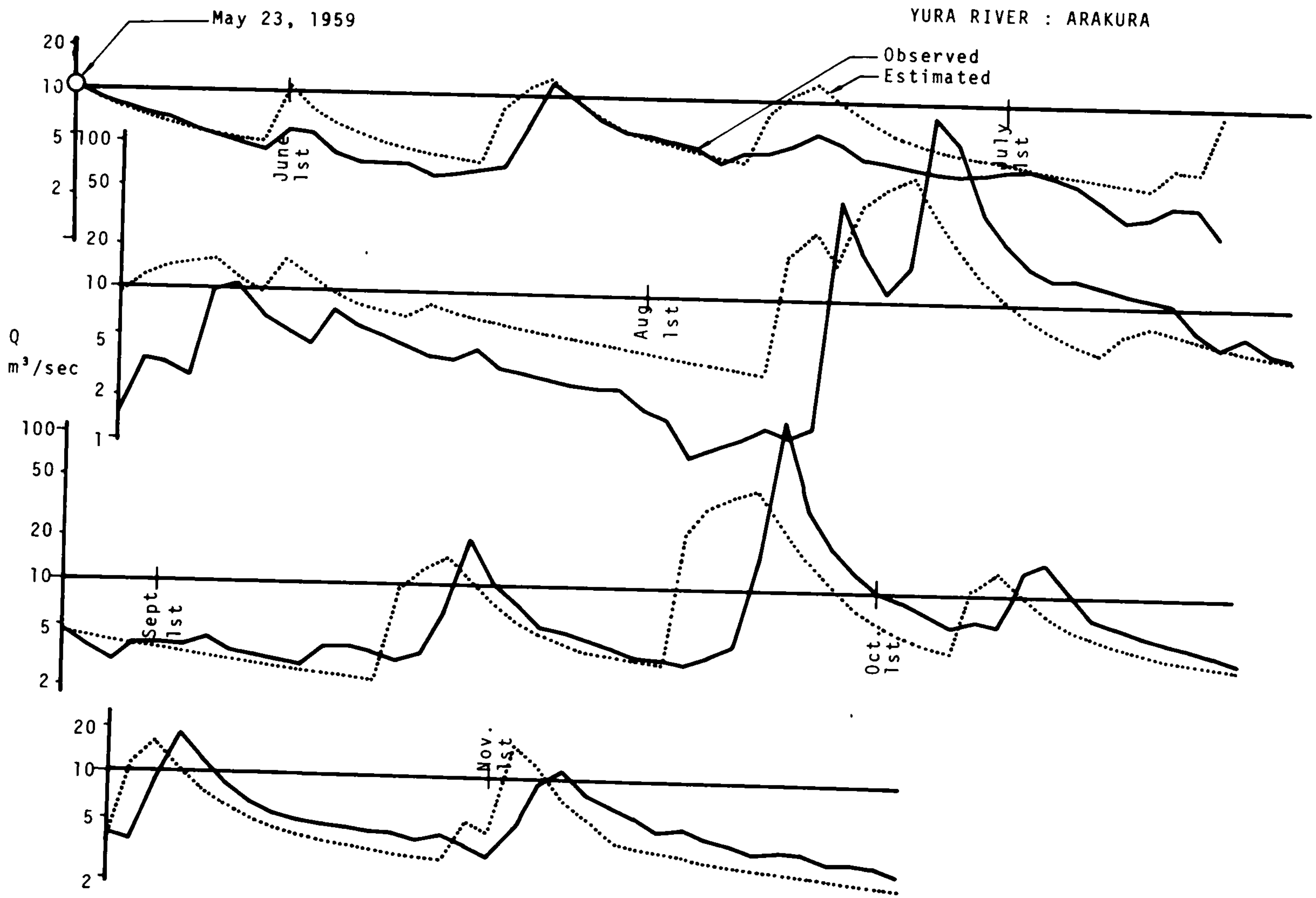
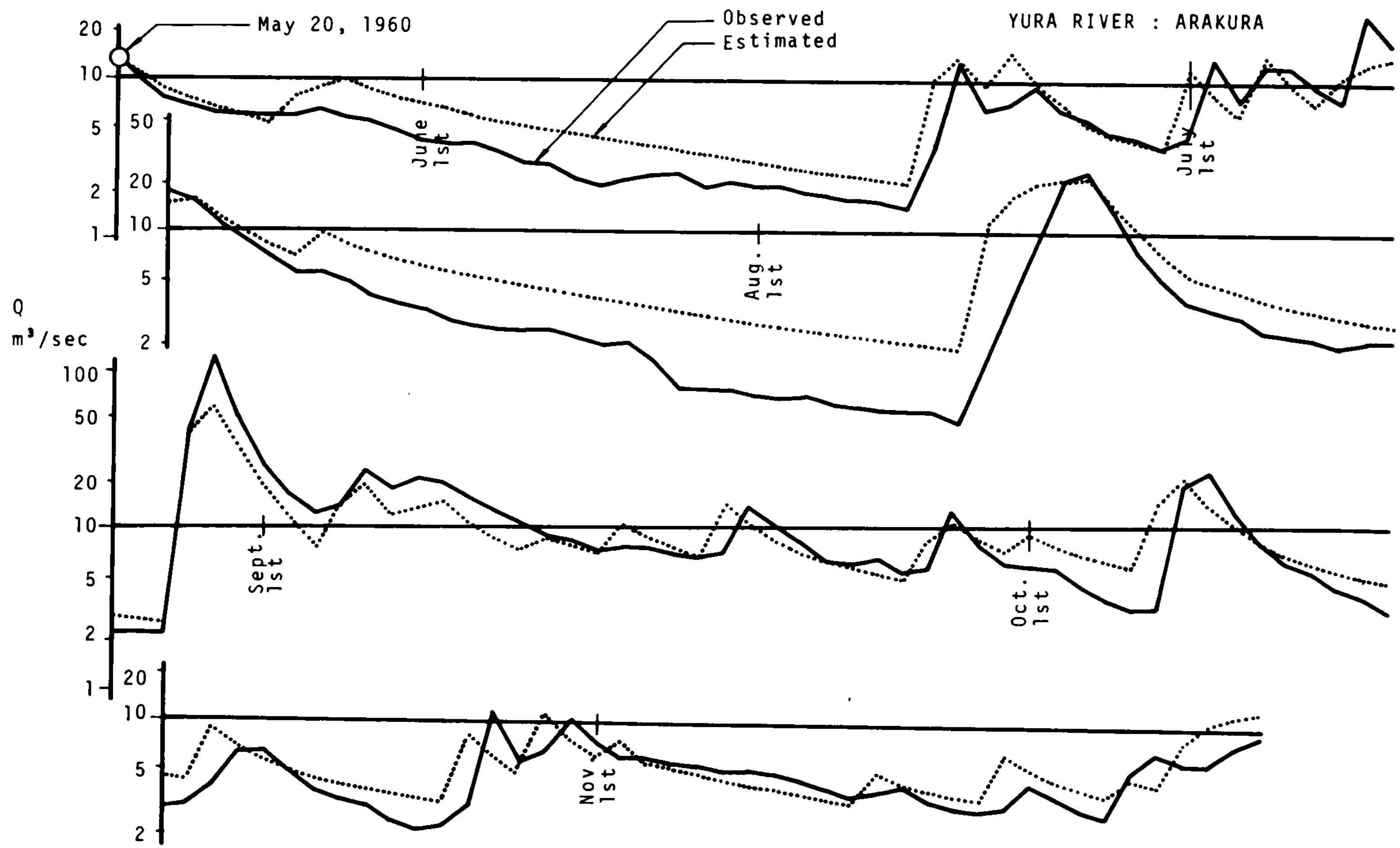


Fig. I-26 (Continued)



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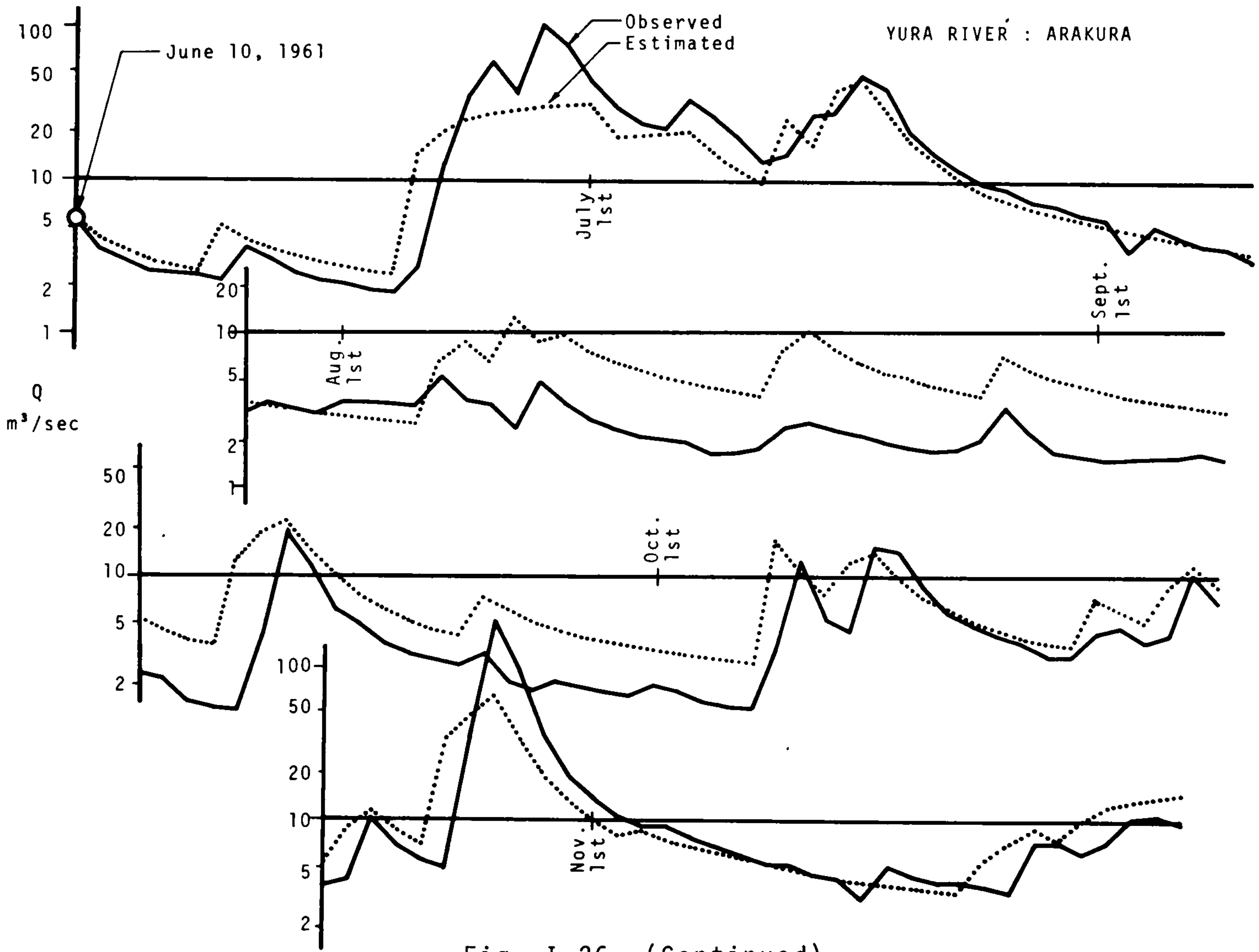
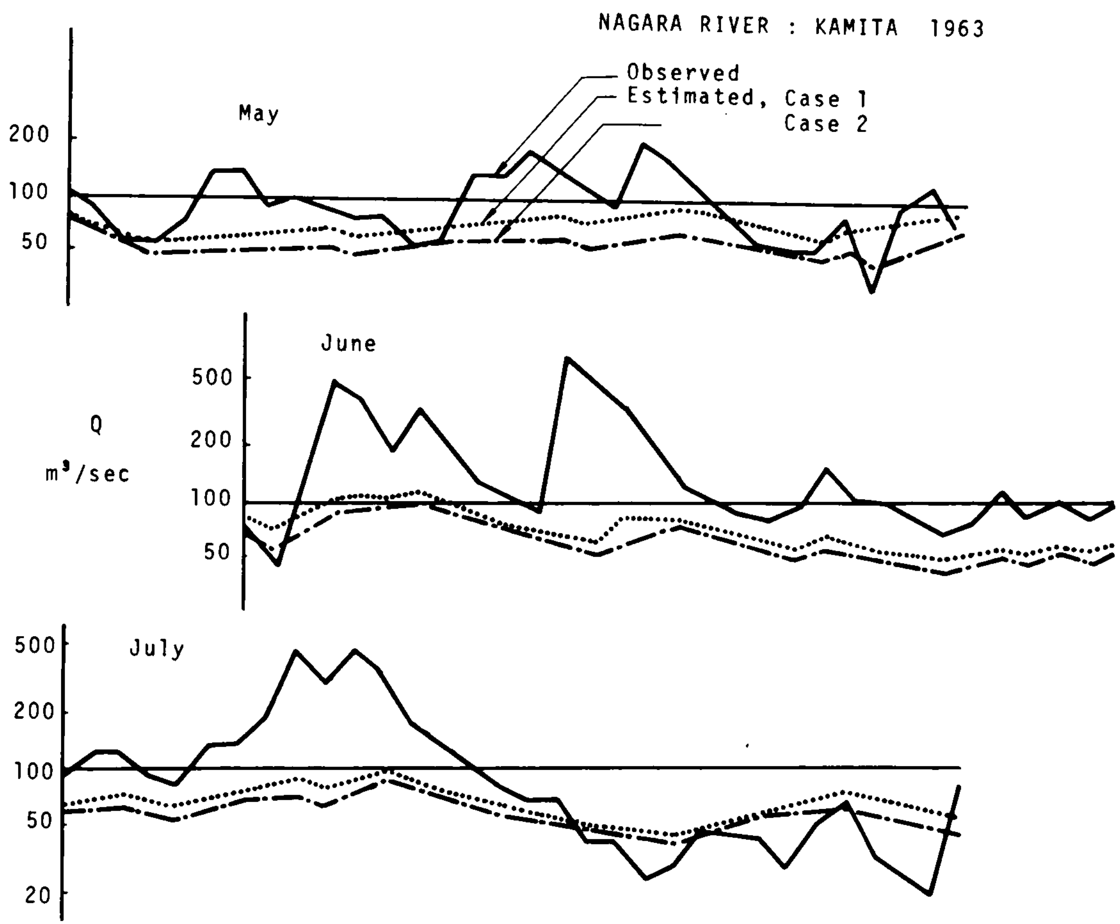


Fig. I-26 (Continued)



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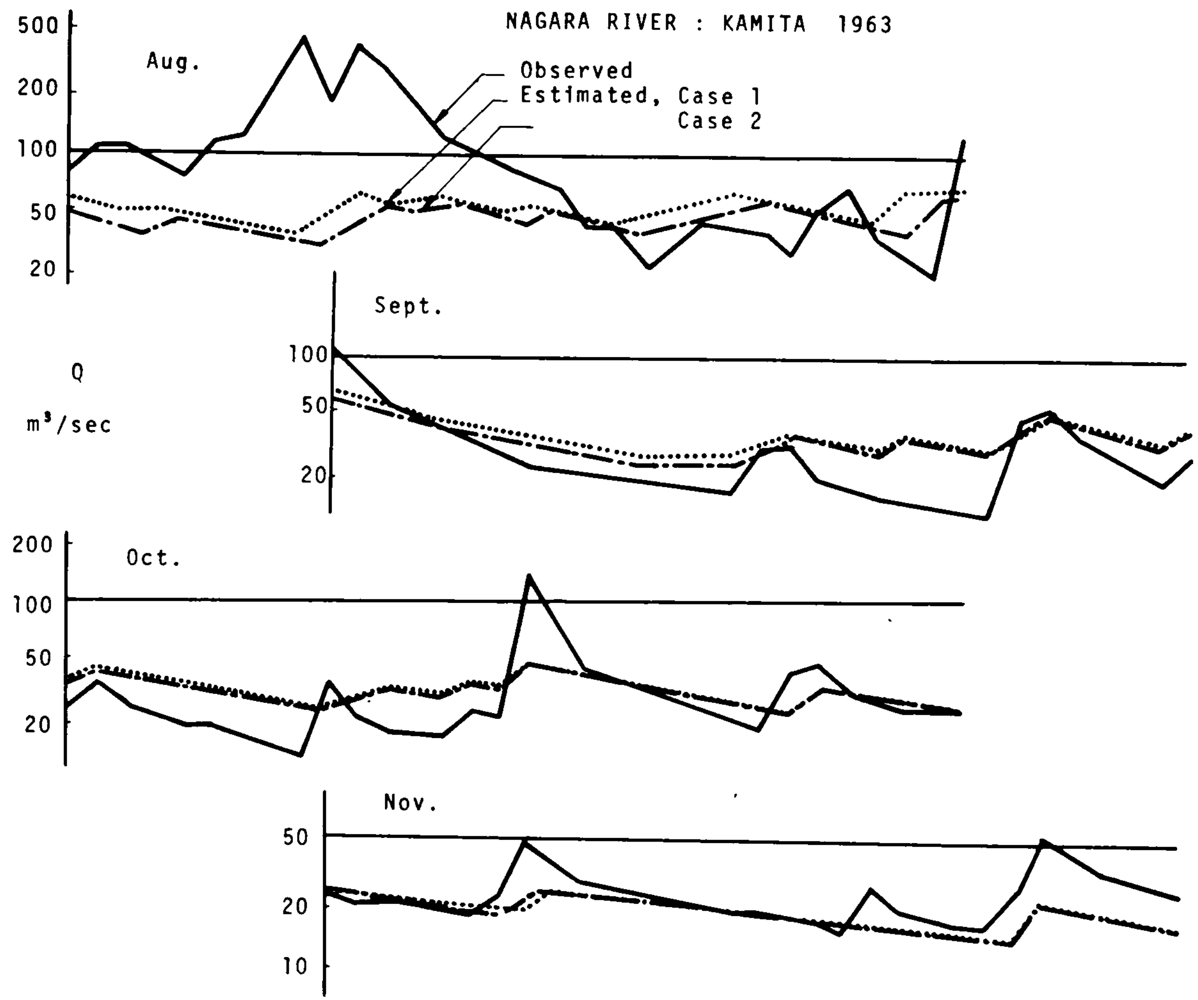

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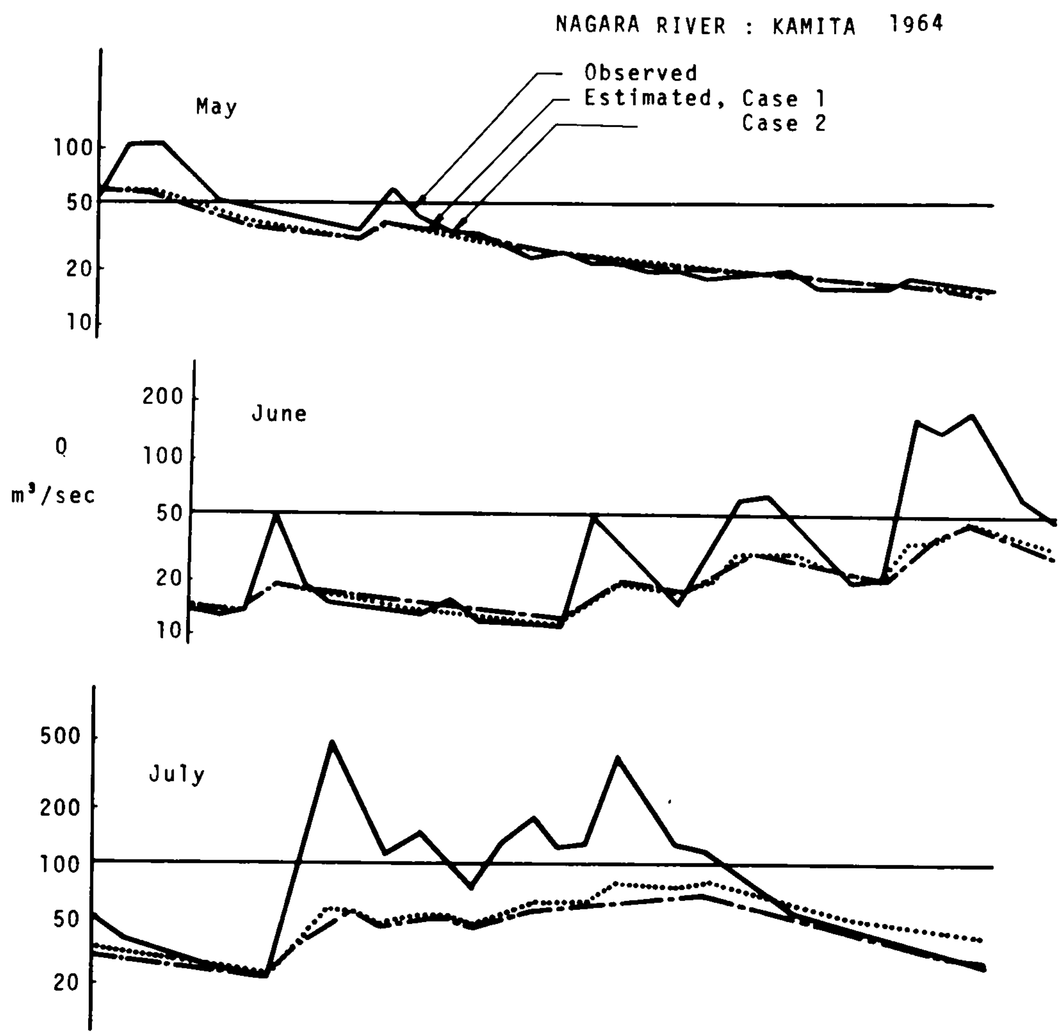


Fig. I-26 (Continued)

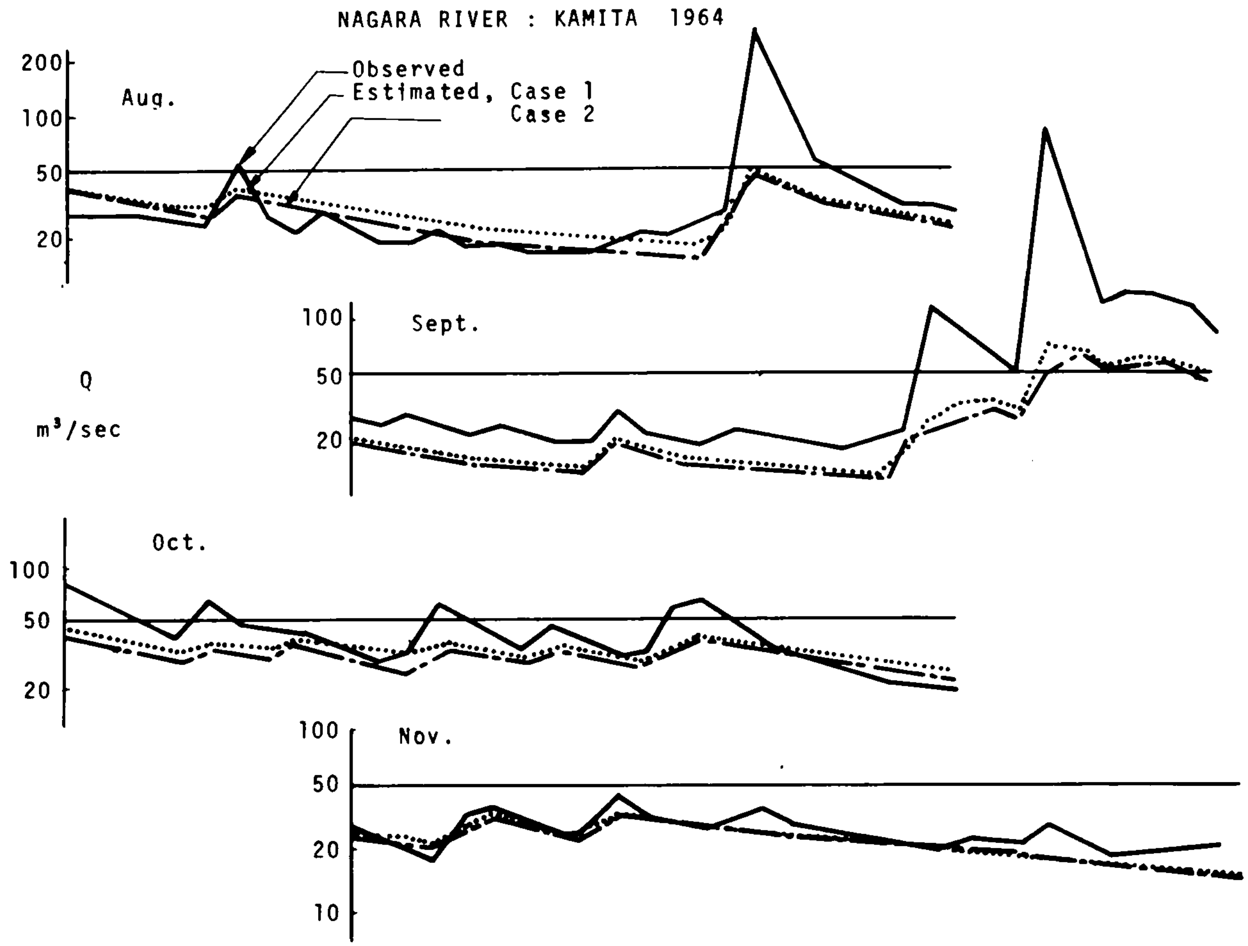


Fig. I-26 (Continued)

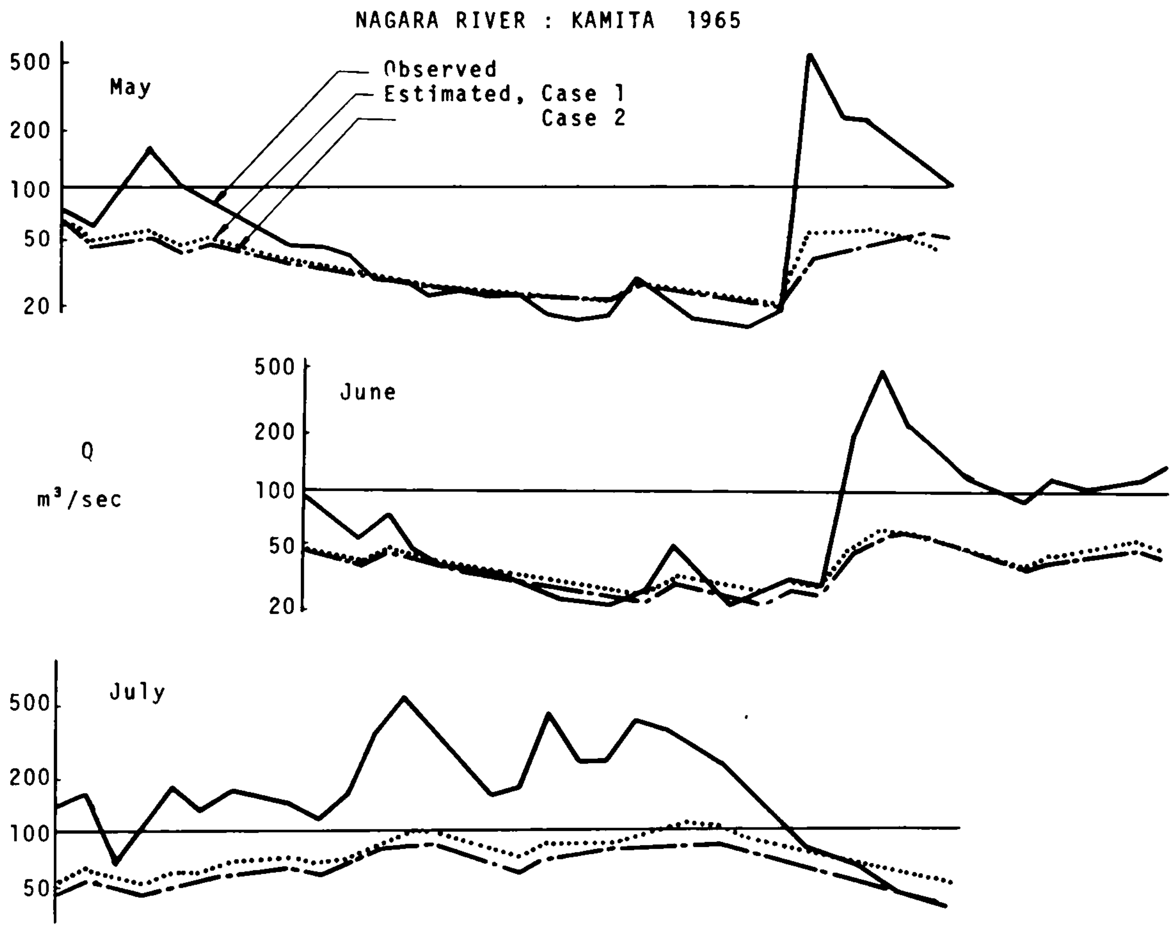


Fig. I-26 (Continued)

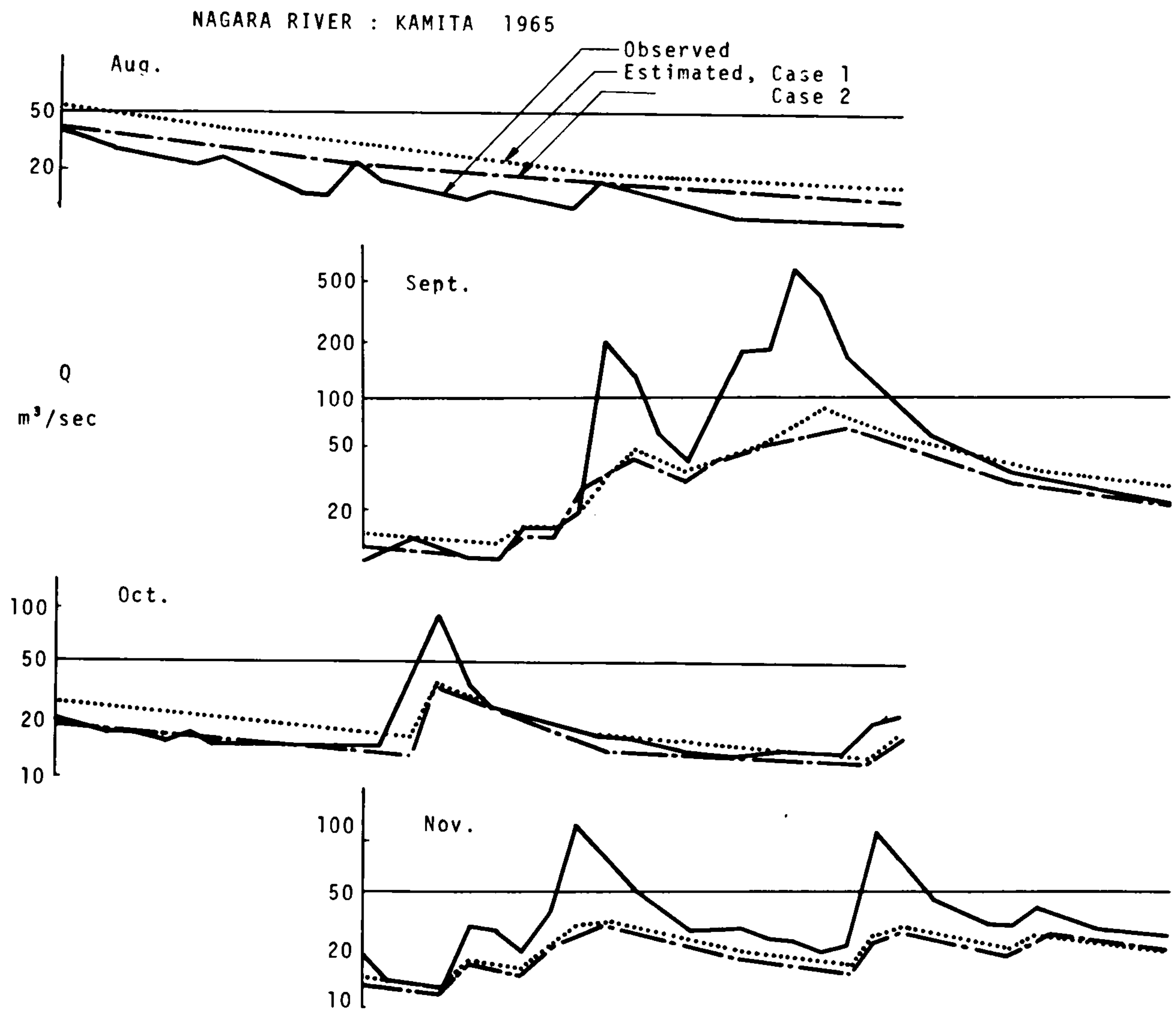
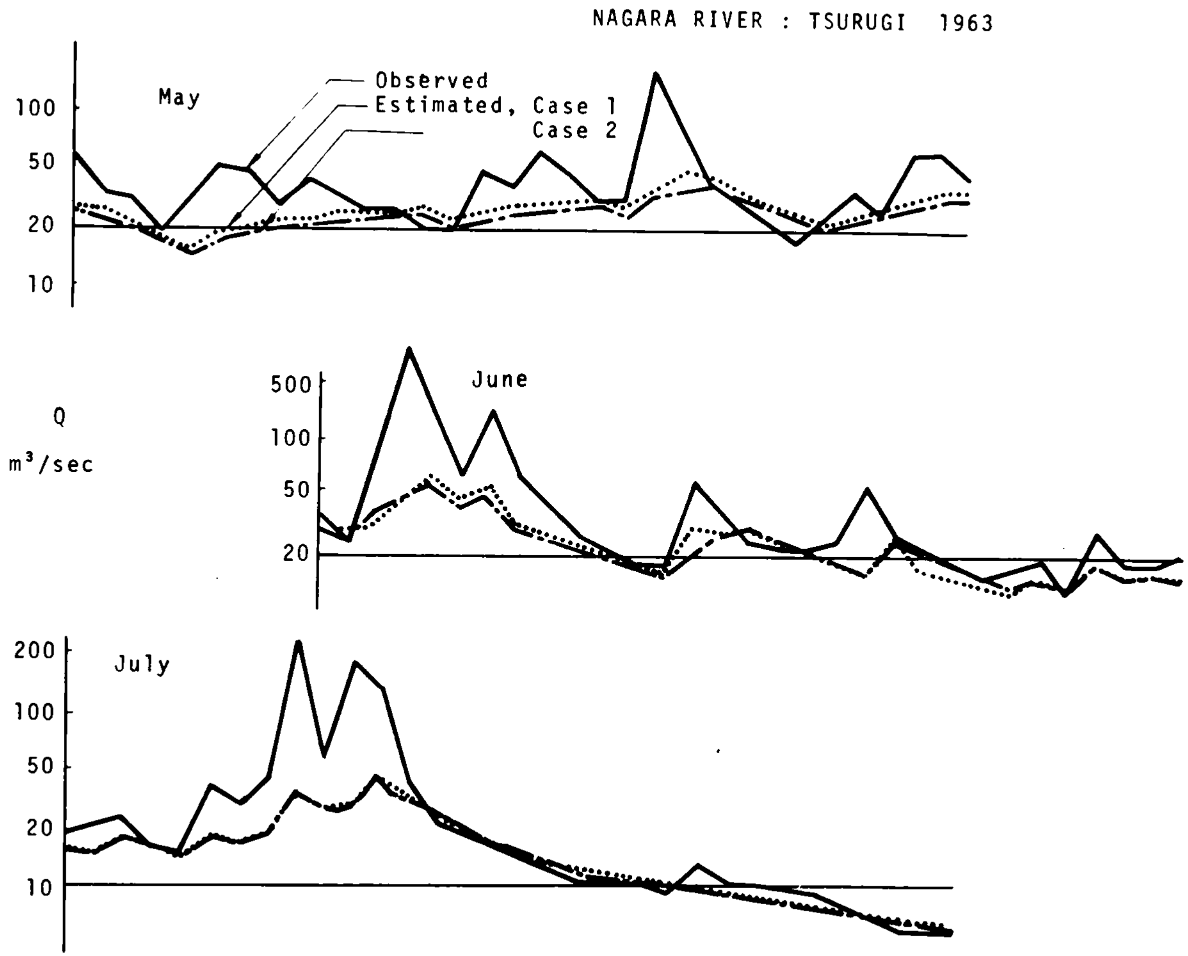


Fig. I-26 (Continued)



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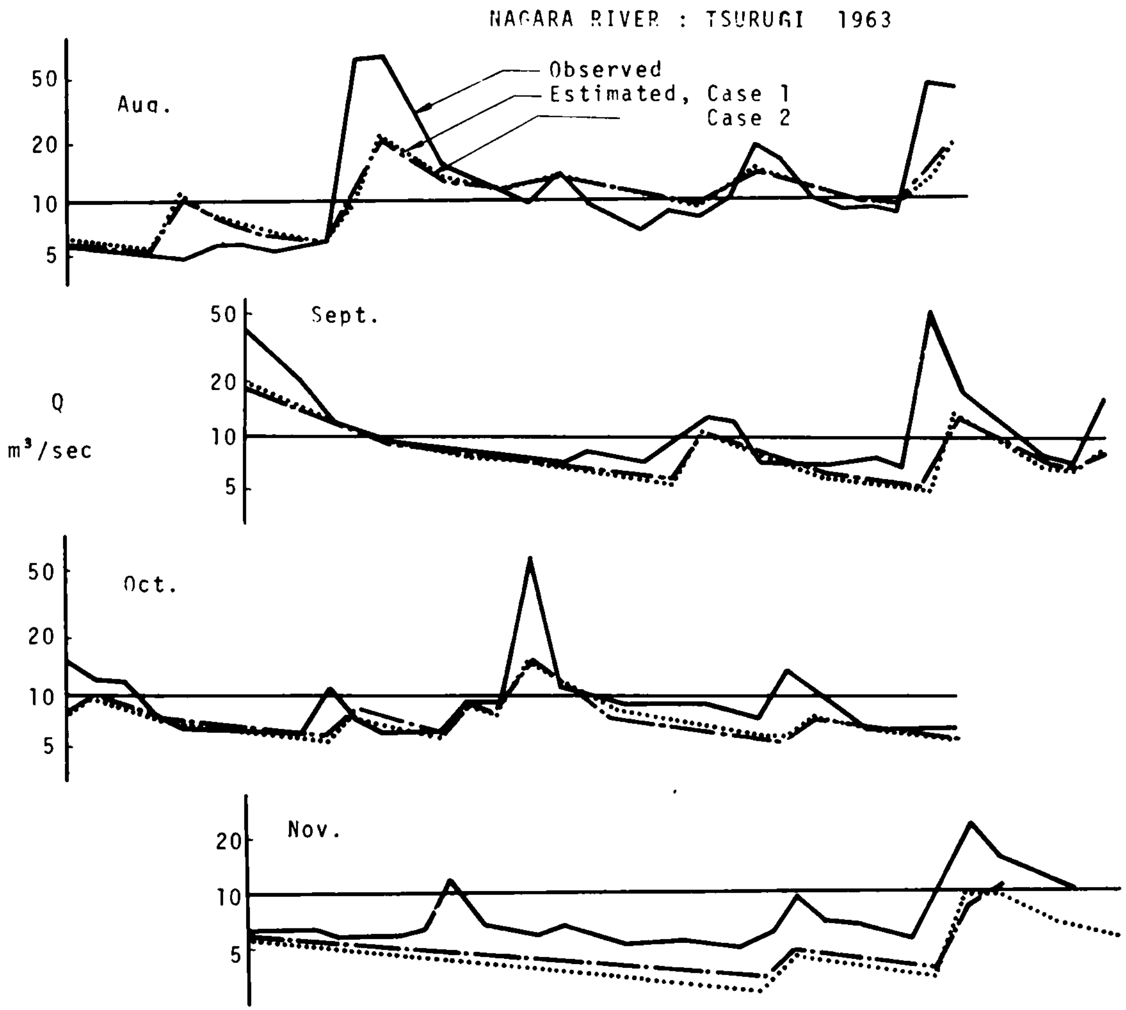

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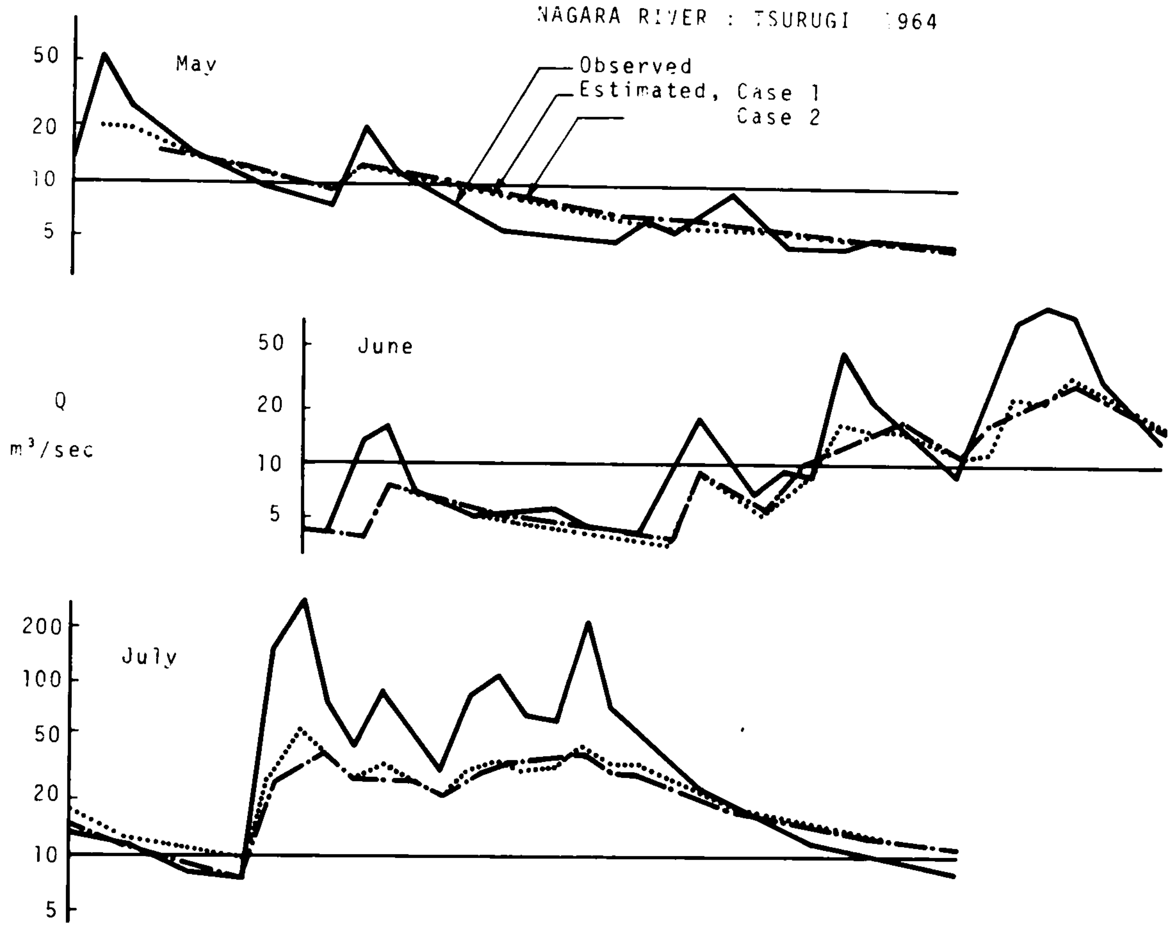

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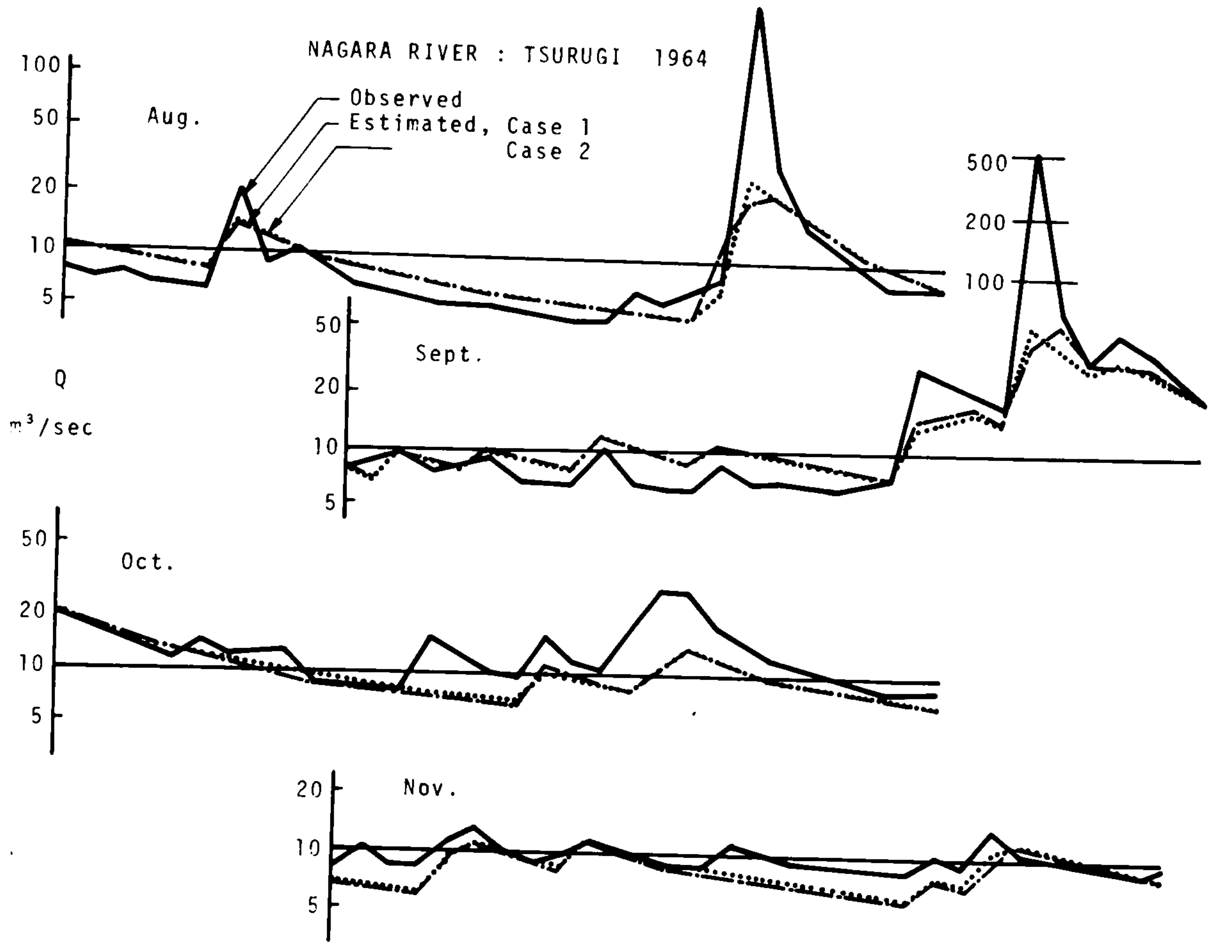

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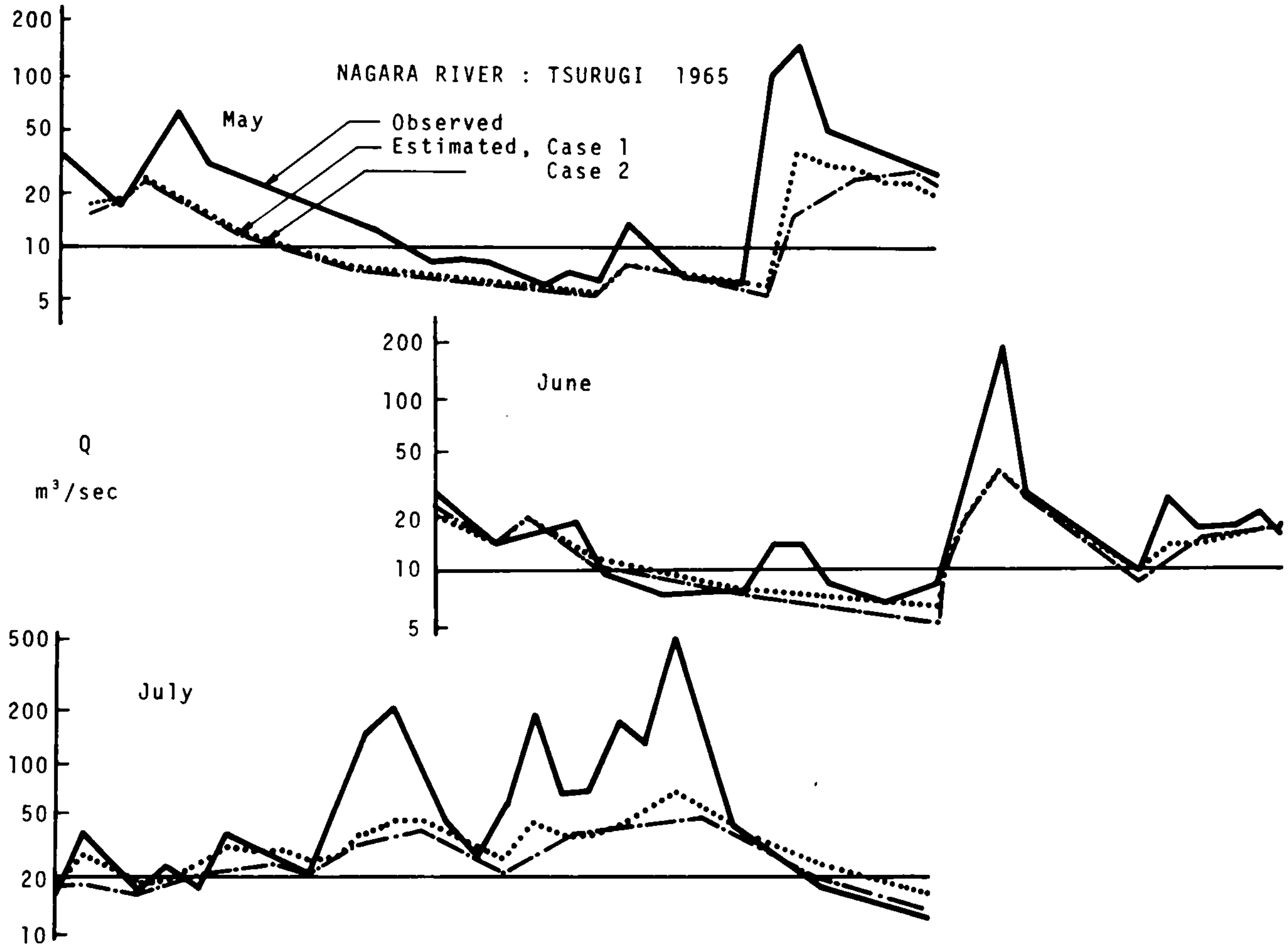

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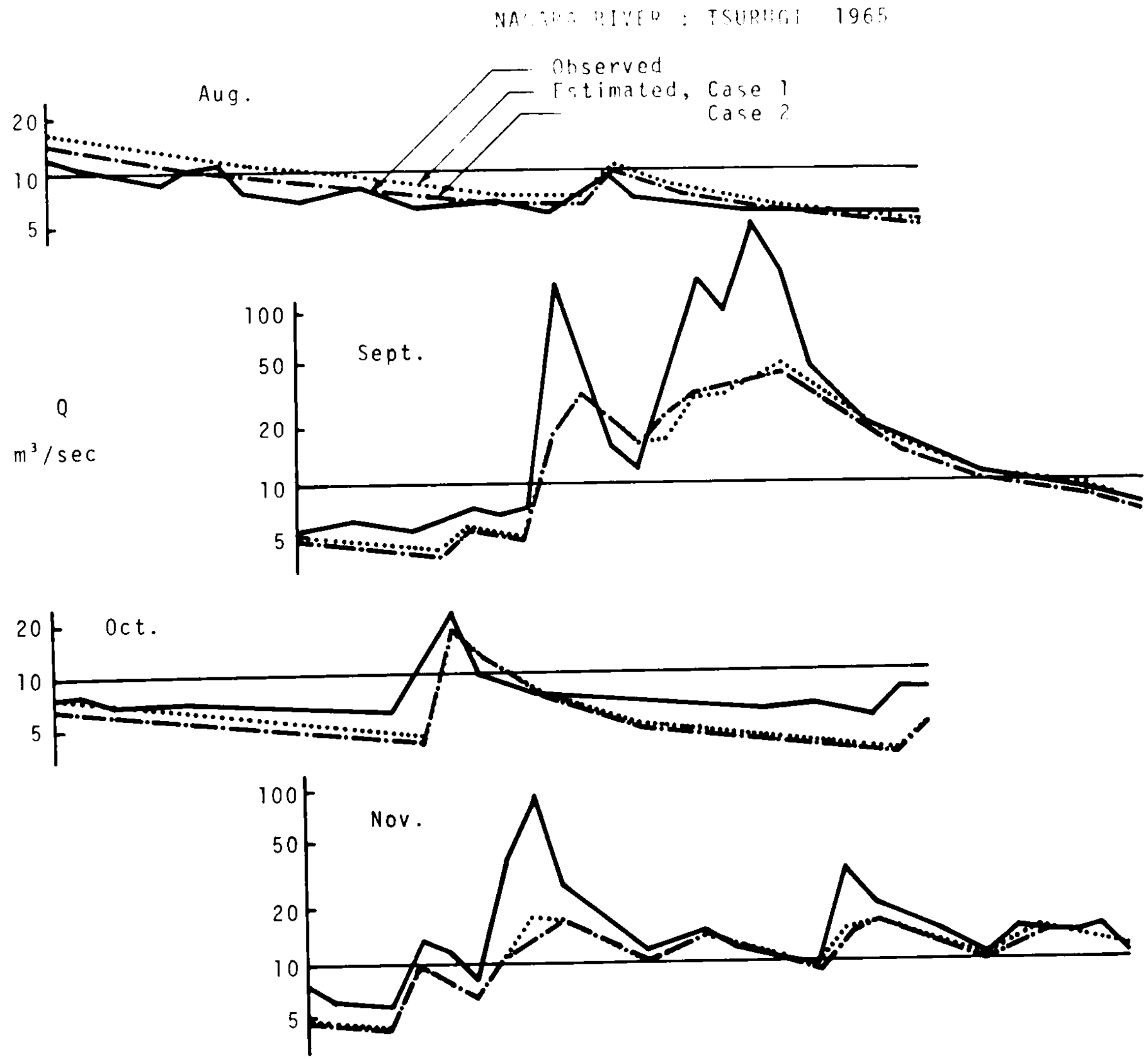
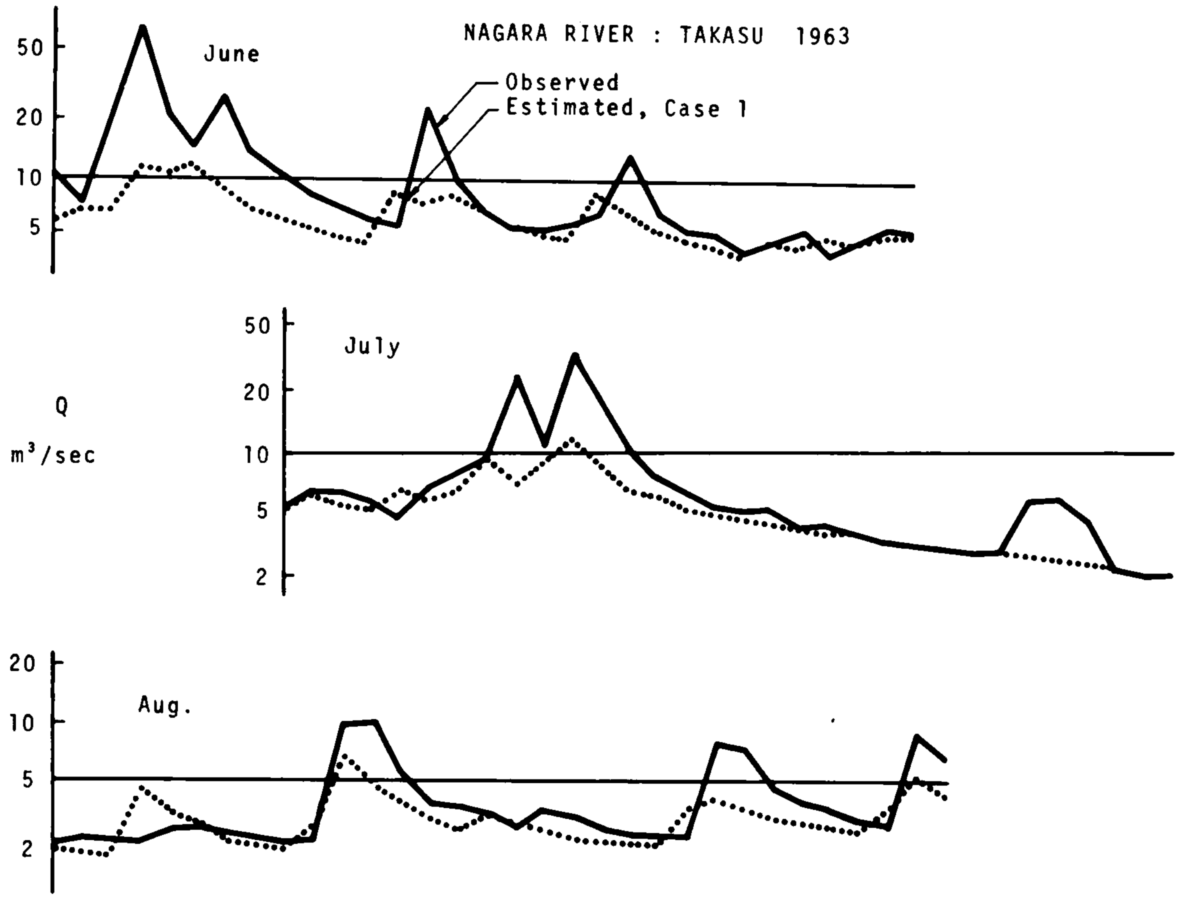


Fig. I-26 (Continued)



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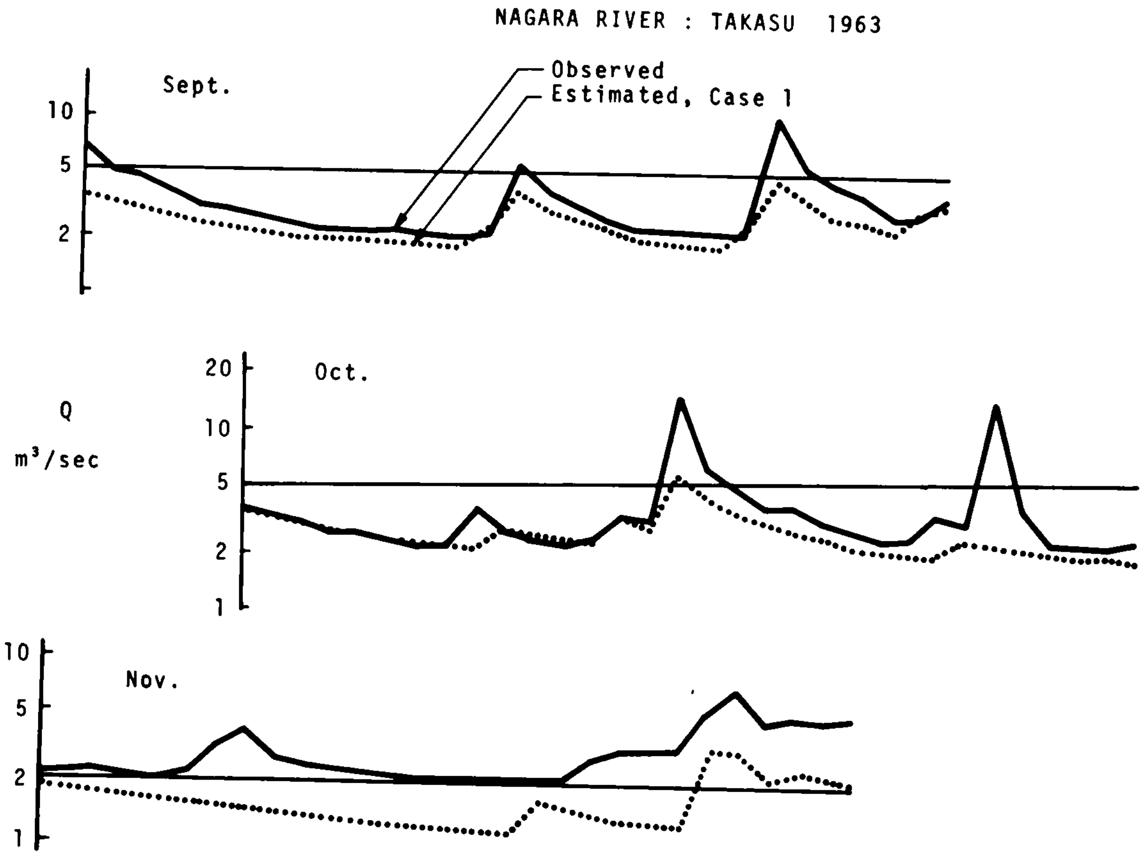

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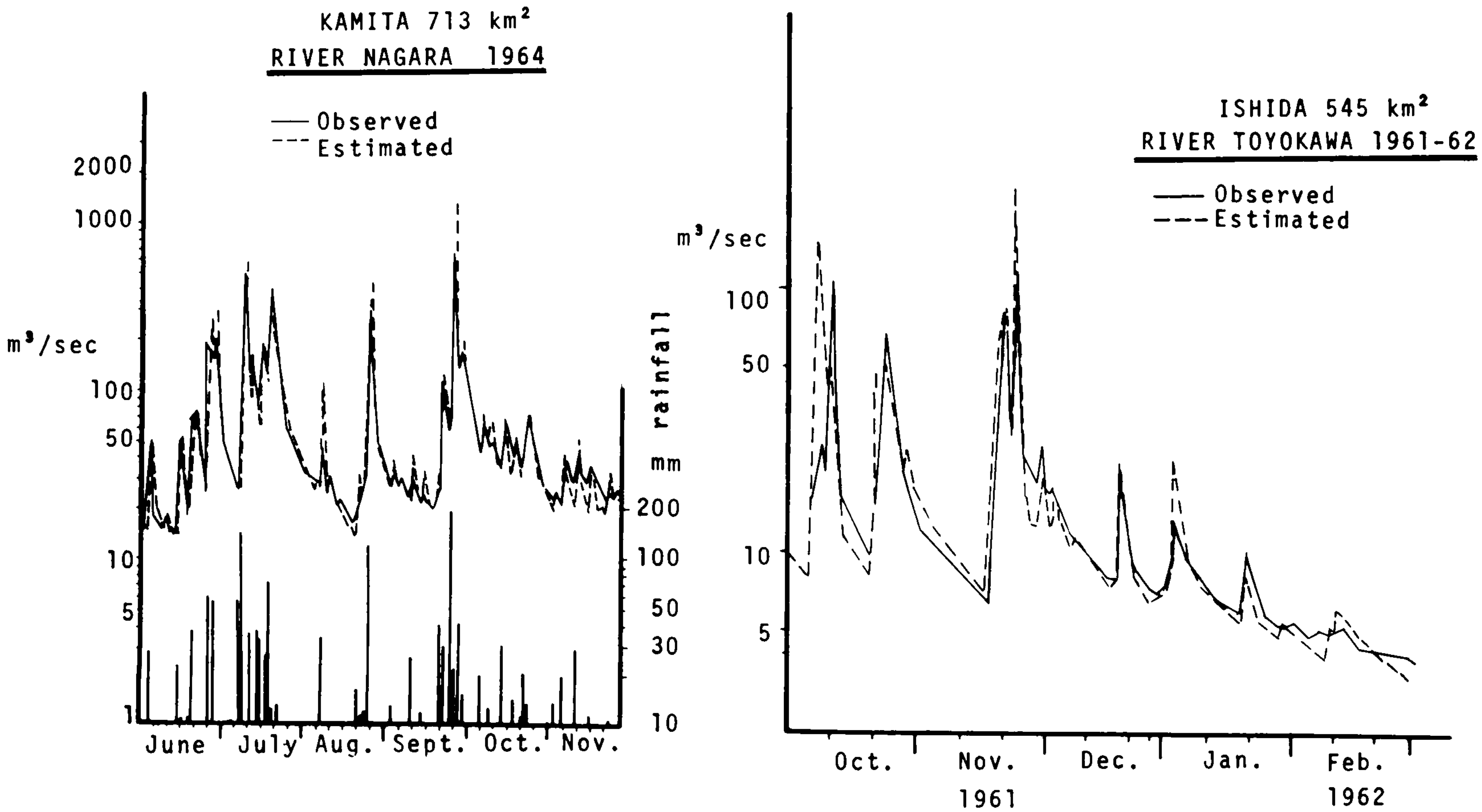

Fig. I-27 Simulated Discharge by Hosoi

(Ground water runoff is estimated by the author's method and direct runoff by the other method)

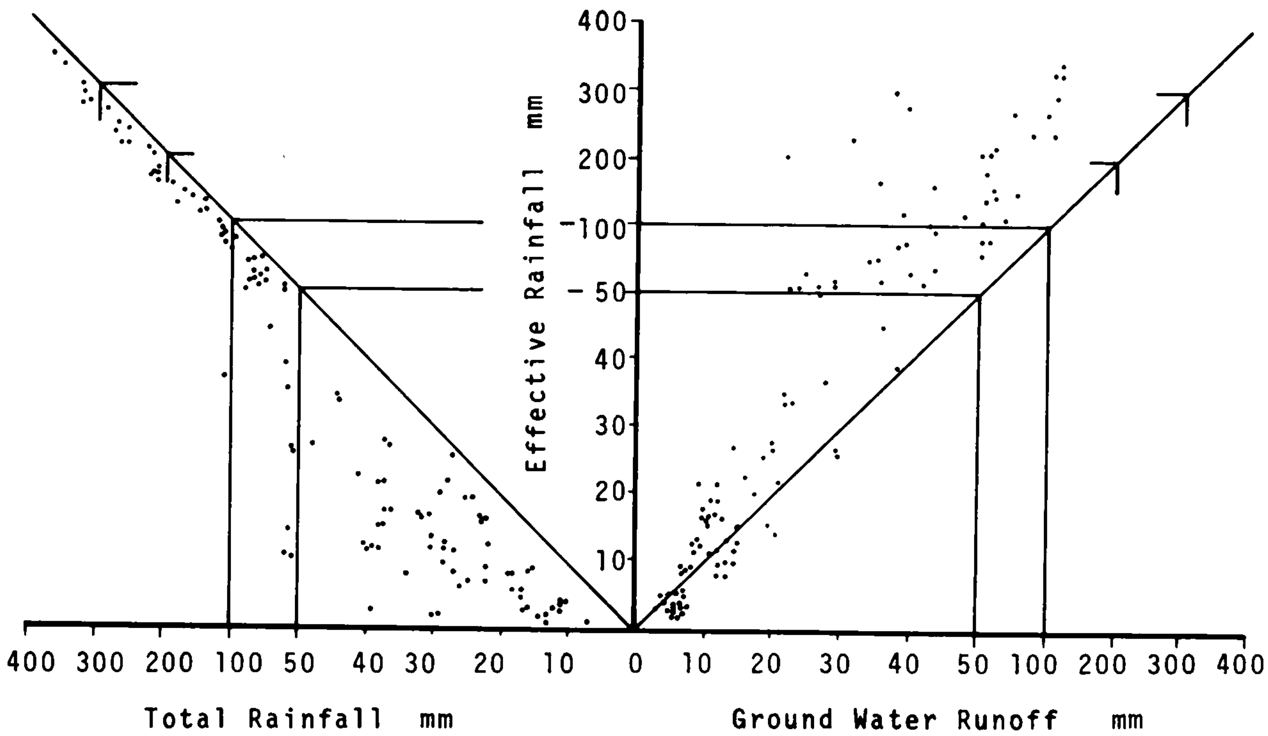

Fig. I-28 Water Balance for Each Rainfall Occasion

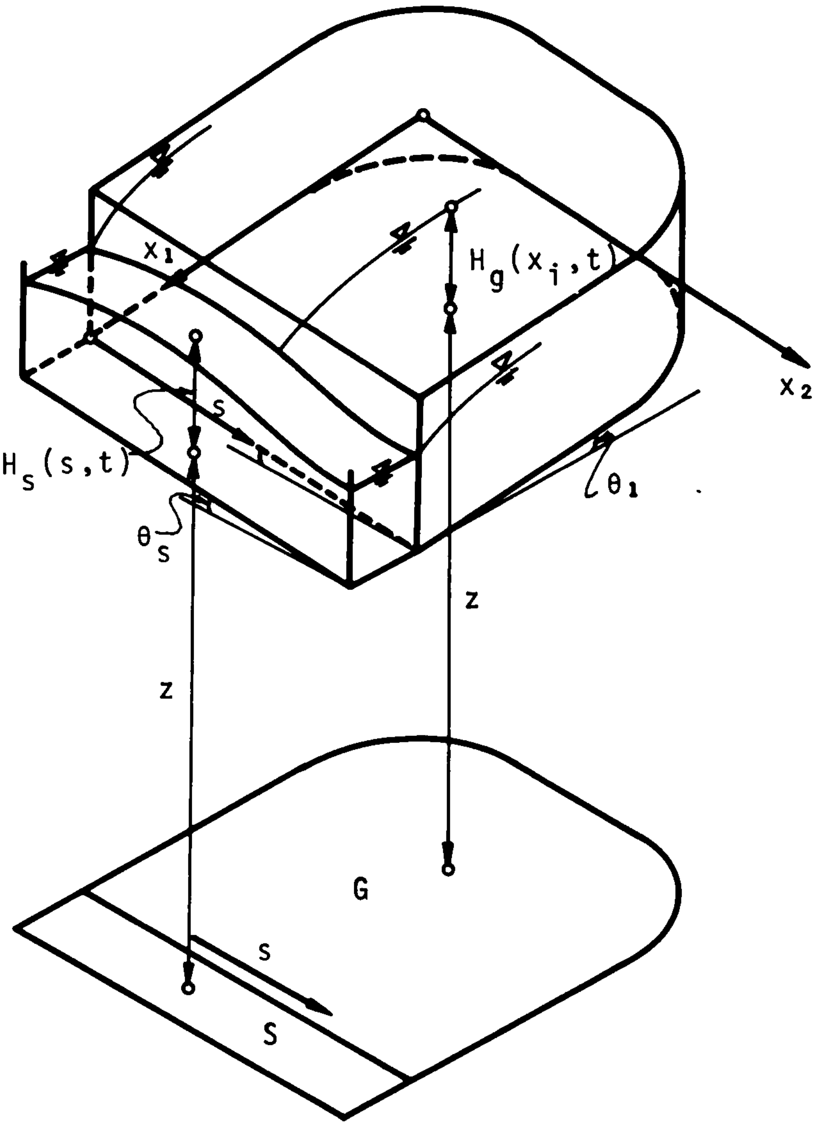

Fig. II-1 Schematic Representation of Ground Water Region and Stream

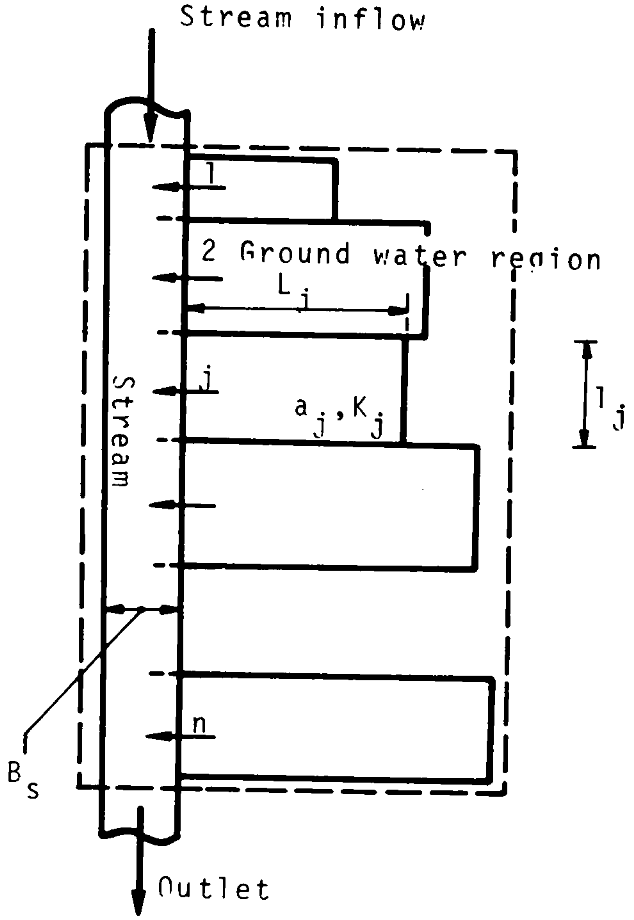

Fig. II-2 Idealized Runoff Region as a Kinematical System

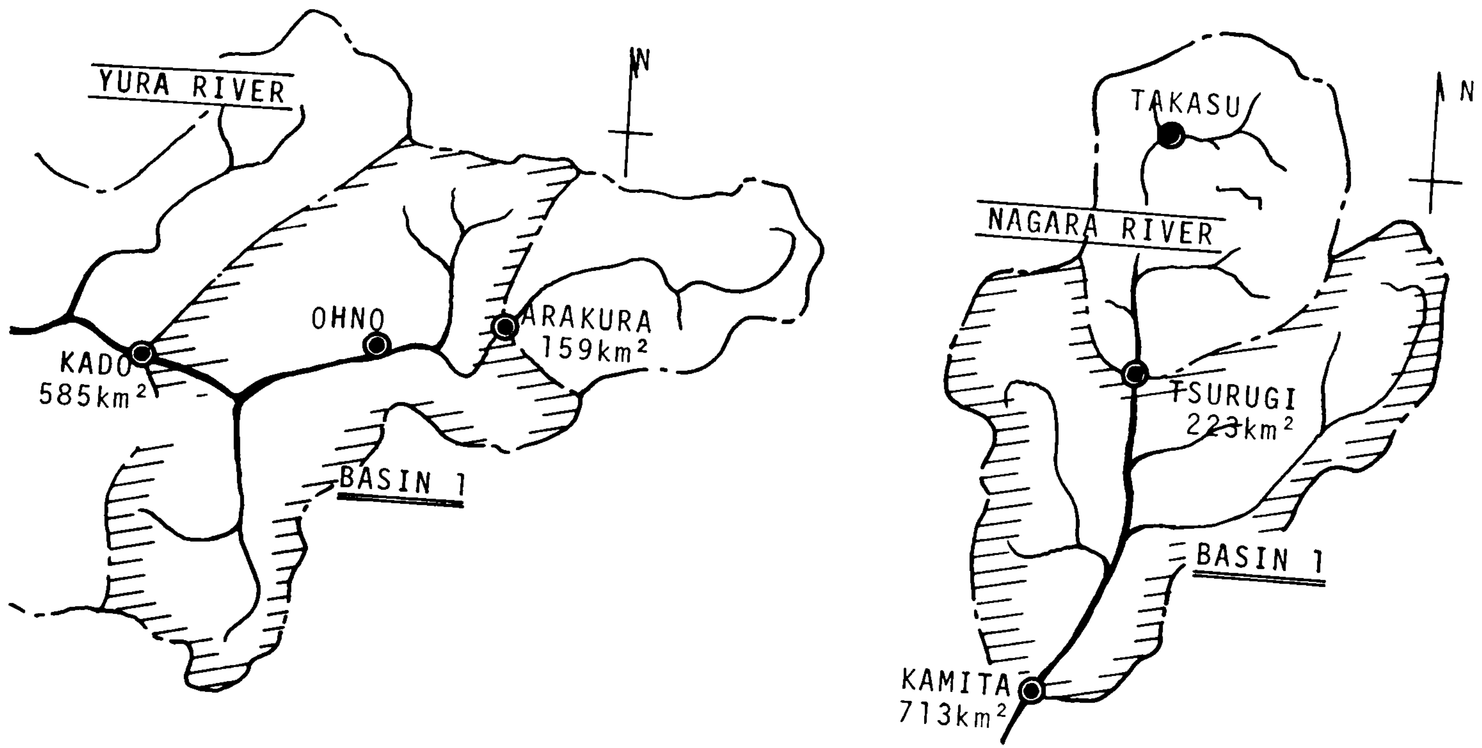

Fig. II-3 General Remarks of Watersheds

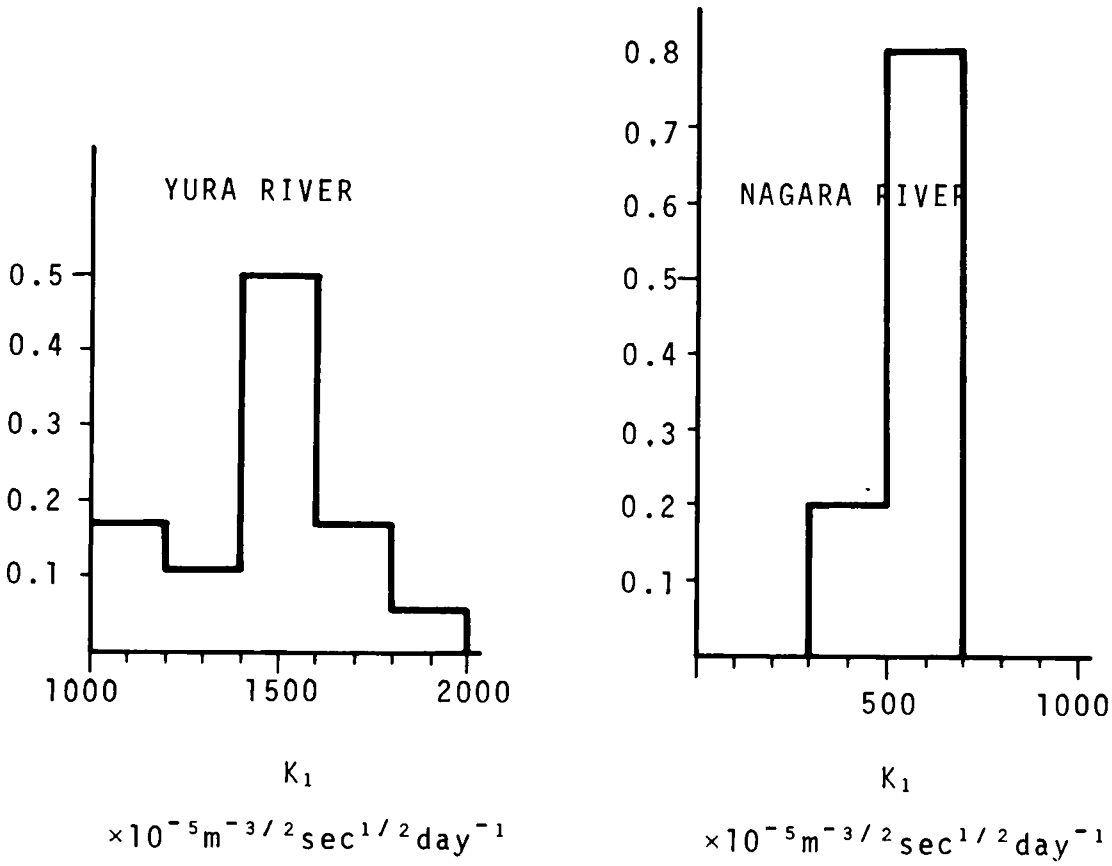

Fig. II-4 Normalized Histogram of Recession Factor K_1 at Basin 1

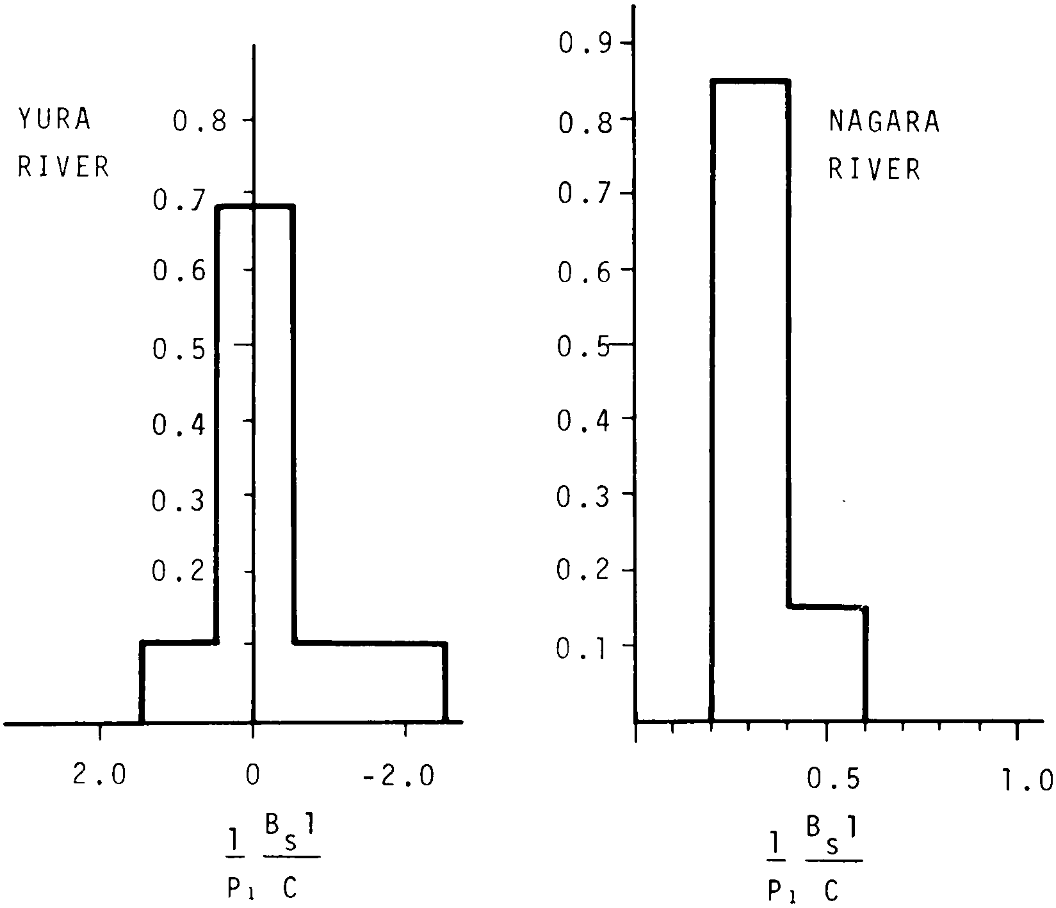

Fig. II-5 Normalized Histogram of Weight in Averaging Process of Recession Factors

NOMENCLATURE

A	catchment area of watershed,
a	recession factor of the unconfined component,
a^*	value a in a lumped system, expressed in terms of those for individual small regions,
a_j	recession factor of the unconfined component in the j-th region,
B_c	factor expressing the width of the confined aquifer,
B_u	factor expressing the width of the unconfined aquifer,
B_s	width of the stream,
B_{sj}	width of the j-th stream reach,
C	coefficient combining water depth and stream discharge,
C_j	value C in the j-th stream reach,
C'	Chézy's roughness coefficient,
D	coefficient indicating the relationship of rainfall intensity with the increment of discharge of the confined component,
f, F	cross sectional area per unit width of the confined stratum and of the tank of the confined model, respectively,
f_i	$k \sin \theta_i = - k\frac{\partial z}{\partial x_i}$,
G	ground water region
G_j	the j-th ground water region,
g	acceleration of gravity,
H_c	water depth in the tank of the confined model,
H_g	water depth of ground water,
$H_g{}^*$	macroscopic water depth of ground water,
H_{gj}	water depth in the j-th ground water region,
$H_{gj}{}^*$	macroscopic water depth in the j-th ground water region,
δH_g	small arbitrary deviation of water depth around the macroscopic water depth $H_g{}^*$ of the ground water,
H_s	water depth in the stream,

$H_s{}^*$	macroscopic water depth in the stream,
H_{sj}	water depth in the j-th stream reach,
$H_{sj}{}^*$	macroscopic water depth in the j-th stream reach,
H_s	small arbitrary deviation of water depth around the macroscopic water depth $H_s{}^*$ of the stream,
H_u	water depth in the unconfined model,
H_{uo}	initial water depth at the upstream end of the unconfined model for the recession state,
$H_{uo}{}'$	initial water depth at the upstream end of the unconfined model for the rising state,
H_{uoj}	value H_{uo} in the j-th ground water region,
h_B	hydraulic head at the point B in Fig.I-3,
h_l	head loss through the confined aquifer,
h_{uo}	initial water depth at the downstream end of the unconfined model for the recession state,
$h_{uo}{}'$	initial water depth at the downstream end of the unconfined model for the rising state,
h_{uoj}	value h_{uo} in the j-th ground water region,
i	infiltration capacity,
i_c	final infiltration capacity,
i_o	initial infiltration capacity,
i_{omax}	maximum value of i_o,
K	recession factor of the unconfined component,
K^*	value K in a lumped system, expressed in terms of those for individual small regions,
K_j	recession factor of the unconfined ground water in the j-th region,
K_1	value K in the watershed (Basin 1) as a kinematical system,
K_o	mean value of K,
k	permeability coefficient,
k_c	permeability coefficient of the confined aquifer,
k_u	permeability coefficient of the unconfined aquifer,
L_c	length of the confined aquifer,
L_u	length of the unconfined aquifer,

L_j	length of the j-th ground water region,
L_g	local potential of ground water,
L_{gj}	local potential of ground water in the j-th region,
L_s	local potential of stream water,
L_{sj}	local potential of water in the j-th stream reach,
l_j	width of the j-th ground water region or the length of the j-th stream reach,
M	water sustained in the zone of aeration,
M_o	maximum amount of water which can be sustained in the zone of aeration,
m	factor indicating initial state of water within a basin for the recession state, H_{uo}/h_{uo}.
m_j	value m in the j-th ground water region, H_{uoj}/h_{uoj}.
n	Manning's roughness coefficient,
n'	coefficient indicating the relationship of the variation rate ε and initial discharge for the rising state of the unconfined component,
p_j	coefficient of the averaging process of the recession characteristics, defined by Eq.(II-53a) for the j-th ground water region,
Q	runoff discharge,
Q_c	runoff discharge of the confined component,
Q_{co}	initial discharge of the confined component for the recession state,
Q_{cO}'	initial discharge of the confined component for the rising state,
ΔQ_c	increment of discharge of the confined component due to rainfall,
Q_s	stream discharge,
$Q_s{}^*$	macroscopic stream discharge,
Q_{sj}	discharge in the j-th stream reach,
Q_u	runoff discharge of the unconfined component,
Q_{uo}	initial discharge of the unconfined component for the recession state,
Q_{uO}'	initial discharge of the unconfined component for the rising state,

Q_{uoj}	value Q_{uo} in the j-th ground water region,
Q_{uos}	initial discharge of water flowing into a region as a kinematical system through the stream,
ΔQ_u	increment of discharge of the unconfined component due to rainfall,
q_c	runoff discharge per unit width of the confined model,
q_{co}	initial discharge per unit width of the confined model for the recession state,
q_{co}'	initial discharge per unit width of the confined model for the rising state,
Δq_c	increment of discharge per unit width of the confined component due to rainfall,
q_j	ratio of initial runoff discharge from each ground water region to that in the whole region, Q_{uoj}/Q_{uo},
q_s	ratio of initial inflow through stream to the initial runoff discharge from the whole region, Q_{uos}/Q_{uo} ,
q_u	runoff discharge per unit width of the unconfined model,
q_{uo}	initial discharge per unit width of the unconfined model for the recession state,
Δq_u	increment of discharge per unit width of the unconfined model due to rainfall,
R	daily rainfall,
R_c	recharge intensity per unit width of the confined model,
R_m	mean daily rainfall,
r	recharge intensity to the ground water table,
r_e	recharge intensity per unit area of the unconfined aquifer,
S	stream,
S_j	the j-th stream reach,
S_u	storage of the unconfined component,
s	distance along the boundary of ground water region,
T	duration of rainfall or recharge,
t	time,
t'	time from the beginning of rainfall or recharge,
v	mean velocity,
v_c	filter velocity through the confined aquifer,

v_{co}'	initial filter velocity through the confined aquifer for the rising state,
v_u	filter velocity through the unconfined aquifer,
x	distance along aquifers downwards positive,(cf.Figs. I-2 & 3),
x_i	space coordinates, x_1 and x_2, (cf. Fig. II-1),
y	space coordinate, (cf. Fig. II-2),
z	elevation of the impervious bed from the reference horizon,
α	recession factor of the confined component,
α_o	mean value of α,
β	k_u/γ_u ,
β_j	value β in the j-th ground water region,
Γ_{gj}, Γ_{sj}	quantities defined by Eq.(II-44) for the j-th ground water region and the j-th stream reach, respectively,
γ	porosity,
γ_c	porosity of the confined aquifer,
γ_u	porosity of the unconfined aquifer,
γ_j	porosity of the j-th ground water region,
δ	exponential index indicating variation rate of the confined component due to rainfall,
ε	factor indicating the variation rate of the unconfined component due to rainfall,
η	$1/\gamma_u$,
θ_i	inclinations of the impervious bed in the direction of x_i,
θ_s	bed slope of the stream, $sin\theta_s = -\frac{\partial z}{\partial s}$,
κ^2	$\beta \cdot H_{uo}' = k_u H_{uo}'/\gamma_u$,
Λ_{gj}, Λ_{sj}	quantities defined by Eq.(II-44) for the j-th ground water region and the j-th stream reach, respectively,
λ	Eigen value for the recession state of the unconfined component,
λ_j	λ-value in the j-th ground water region,
λ_s	λ-value in the stream,
μ	exponential index in the infiltration recovery curve,
Π	product notation,
σ^2	variance.

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