1	Spatial relationship between topography and rock uplift patterns in asymmetric
2	mountain ranges based on a stream erosion model
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22 Abstract

The spatial relationship between topography and rock uplift patterns in asymmetric 2324mountain ranges was investigated using a stream erosion model in which the asymmetric rock uplift was given and erosion rates were proportional to the *m*-th 2526power of the drainage area and the *n*-th power of the channel gradient. The model conditions were simple, and thus the effects of horizontal rock movement, diffusional 27processes, and erosion thresholds were neglected, and spatially uniform precipitation, 28lithology, and vegetation were assumed. In asymmetric mountain ranges, under 29realistic exponent conditions (m < n) and the above assumptions, the surface erosion 30 rate is faster on the steeper side and slower on the gentler side. The topographic axis 3132migrates away from the rock uplift axis toward the center of the mountain range owing 33 to the contrast in erosion rates. This migration continues until the erosion is balanced 34with rock uplift. In a dynamic steady state, the topographic pattern is independent of the rock uplift rate as indicated by an analytical solution, and is prescribed by the rock 35uplift pattern and the exponents *m* and *n*. As the asymmetry of the rock uplift pattern 36 increases, the topographic axis migrates a greater distance. The location of the 37 topographic axis is related to the location of the rock uplift axis by a simple 38logarithmic function, for a wide range of m and n. The fit of the numerical results and 3940 the logarithmic function is particularly good when m = 0.5 and n = 1.0. If the rock uplift pattern in asymmetric mountain ranges is known, the value of n-5m/4 can be 41 constrained based on the logarithmic relation, assuming a dynamic steady state. On the 42other hand, if the value of n-5m/4 is known in an asymmetric mountain range, the 43rock uplift pattern can be estimated directly from the topography. This relation was 44 applied to the Suzuka Range in central Japan, and the value of n-5m/4 was 4546 estimated for an assumed reverse fault motion.

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48 Keywords

rock uplift, topography, stream erosion model, asymmetric range, logarithmic relation

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51 **1. Introduction**

Remarkable recent developments in geodetic techniques have produced detailed 52topographic information for the entire world (e.g., Farr et al., 2007). Detailed 53elevation maps have even been obtained for some satellites and planets, such as the 5455moon (Araki et al., 2009) and Mars (Smith et al., 1999). In contrast, the data of tectonic movements are very limited in space and time. For example, even Japan's 56GEONET, one of the densest GPS networks in the world, provides crustal movement 57data at a spacing of about 20 km for a time span of less than two decades (Sagiya, 582004). Data relevant to the long-term tectonic movements $(10^3 - 10^6 \text{ years or more})$ 59that control topographic evolution are even sparser. Information of long-term tectonic 60 61 deformation is fundamentally important to deduce tectonic processes. A principal goal 62 of tectonic geomorphology would be to extract information regarding the rates and 63 patterns of tectonic deformation directly from topography.

64 The purpose of this study is to quantitatively estimate rock uplift rate patterns using the topography of asymmetric mountain ranges. Asymmetry of mountain ranges may be 65 caused by horizontal rock movements (Adams, 1980; Willett et al., 2001; Herman and 66 Braun, 2006; Miller et al., 2007), differential base levels of erosion (Ellis and Densmore, 67 2006), asymmetric distribution of rain and snow precipitation (Beaumont et al., 1992; 68 69 Willett, 1999; Mitchell and Montgomery, 2006; Anders et al., 2008), or differences in lithology (Adams, 1980). Asymmetric rock uplift, however, should also result in 70 asymmetric mountain ranges (Koons, 1989; Kooi and Beaumont, 1996; Kühni and 71Pfiffner, 2001). For example, the Suzuka Range in central Japan has been differentially 7273uplifted by a west-dipping reverse fault system at the eastern margin of the range (Ota 74and Sangawa, 1984), which results in asymmetric topography.

In this study, asymmetric vertical rock uplift is considered to be a proxy for tectonic movement. Although lateral advection may be important in some asymmetric ranges, incorporating this effect requires the addition of more model parameters, such as the dip angle and length of the fault. Furthermore, natural faults commonly have some curvature. If even more realistic models are sought, for example, the elastic thickness of

the lithosphere must also be taken into account (e.g., Thatcher and Rundle, 1984;
Fukahata and Matsu'ura, 2006). Although realistic, such complicated modeling is not
the aim of this study, which focuses on a simpler set of conditions.

To express surface erosion, a stream erosion model (e.g., Howard and Kerby, 1983; 83 84 Howard et al., 1994; Tucker and Slingerland, 1994) is used, in which the long-term erosion rate is proportional to the product of power functions for the drainage area and 85 channel gradient. The values of the exponents of the power functions have been 86 estimated in some mountain ranges from observed incision rates along river profiles 87 (Stock and Montgomery, 1999; Whipple et al., 2000; van der Beek and Bishop, 2003; 88 Harkins et al., 2007) and from the intrinsic concavity index of rivers with some 89 90 assumptions of rock uplift rates under steady state conditions (e.g., Tarboton et al., 911989; Seidl and Dietrich, 1992; Kirby and Whipple, 2001; Meade, 2010). However, the 92relation between the values of the exponents and asymmetry of topography has not been 93 elucidated.

94The response of topography to asymmetric (gabled) vertical rock uplift is examined using the stream erosion model. The location of main drainage divides (topographic 95axes) is of particular importance because these divides determine the first-order 96 topographic features and drainage patterns in mountain ranges, which prescribe the 97 98 environment and landscape of the area. If there is no erosion, the topographic axis must 99 simply coincide with the rock uplift axis. Under asymmetric rock uplift and fluvial 100 erosion, however, the erosion rate is seemingly faster on the steeper side, as long as all 101 other conditions that contribute to erosion rate, such as precipitation, lithology, and vegetation, are uniform. Asymmetric uplift and stream erosion would predict, therefore, 102 103 that the topographic axis migrates away from the rock uplift axis toward the center of 104 the mountain range. The aim of this study is to systematically quantify the distance 105 between the topographic and rock uplift axes. Because the distance also depends on the 106 exponent values of the stream erosion model, this study links to estimation of these 107 values.

108 This study employs a topographic evolution model that incorporates the effect of

rock uplift into a stream erosion model. Under conditions of asymmetric rock uplift, 109 topographic evolution is numerically simulated, and a curious relationship between 110topographic and rock uplift axes is shown. We then investigate the dependence of the 111 relation on the values of the exponents, and discuss the implications of the results for 112113real mountain ranges.

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2. Topographic evolution model

The stream erosion model is suitable for modeling fluvial erosion at a resolution of 116 1 km and was developed based on the law of open channel flow for stream incision 117 into bedrock (e.g., Howard and Kerby, 1983; Howard et al., 1994; Tucker and 118 119 Slingerland, 1994). In the stream erosion model, the erosion rate ε is expressed as 120the product of power functions for the drainage area A and channel gradient S as 121follows:

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$$\varepsilon(x, y, t) = K \left[A(x, y, t) \right]^m \left[S(x, y, t) \right]^n \tag{1}$$

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where K is the coefficient of erosion and the dimensionless exponents m and n are 125126positive constants related to basin hydrology, hydraulic geometry, and erosion process (Howard et al., 1994; Whipple and Tucker, 1999; Whipple et al., 2000). In the present 127study, surface deposition (e.g., Willgoose et al., 1991; Kooi and Beaumont, 1994; 128129Tucker and Whipple, 2002), and hillslope processes (e.g., Schmidt and Montgomery, 1995; Roering et al., 1999; Stark and Hovius, 2001; Anderson, 2002) were neglected, 130 131 and the erosion threshold that depends on the critical shear stress (e.g., Howard et al., 1994; Whipple and Tucker, 1999; Tucker and Bras, 2000; Lague et al., 2005) was not 132133employed, for simplicity. Other factors that contribute to erosion rate, such as precipitation, lithology, and vegetation, are assumed to be spatially and temporally 134uniform. 135

Rock uplift u is incorporated into the stream erosion model of Eq. (1) as in many 136

previous studies (e.g., Willgoose et al., 1991; Kirkby, 1994; Tucker and Slingerland,
1994; Kirby and Whipple, 2001; Miller et al., 2007) as

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$$\frac{d}{dt}h(x,y,t) = u(x,y,t) - K[A(x,y,t)]^m [S(x,y,t)]^n$$
(2)

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where *h* represents altitude. The drainage area *A* and the channel gradient *S* are functions of *h*. In other words, if a topographic distribution h(x, y, t) is given, A(x, y, t) and S(x, y, t) are uniquely determined. Parameter *S* is the gradient of *h*. *A* is dependent on *h* in a complex fashion, but a proportional change in *h* from h(x, y)to ch(x, y), where *c* is an arbitrary positive constant, does not affect *A*.

Given the rock uplift rate u(x, y, t), the initial height h(x, y, 0), and the boundary 147148conditions, Eq. (2) can be solved with a finite difference method. Following the 149scheme developed by Beaumont et al. (1992), the model can be discretized in space 150and time. A series of regular square poles is used to represent the discretized topography. The drainage area and channel gradient for each cell is computed using a 151steepest-descent flow accumulation algorithm (Tucker and Slingerland, 1994). In 152computing the drainage area for a particular cell, the area of that cell is included. If the 153154cell concerned is not included, the drainage divide is fixed and can not migrate because the erosion rate at the headwater cell of each stream is always zero as there is zero 155drainage area. 156

In the numerical simulation, the model area is 100×100 km, which is divided into 1×1 km cells. The *x*-axis is the east direction, and the *y*-axis is the north direction. The origin (x, y) = (0, 0) is at the center of the model area.

160 The asymmetric rock uplift rate in the x direction is given by the following 161 function:

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$$u(x, y, t) = \begin{cases} \frac{x+L}{x_{u}+L}u_{\max} & (-L \le x \le x_{u}) \\ \frac{L-x}{L-x_{u}}u_{\max} & (x_{u} \le x \le L) \end{cases}$$
(3)

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where *L* is half of the model length (L=50 km). The functional form is shown in Fig. 1. x_u represents the location of the rock uplift axis, and u_{max} represents the maximum rock uplift rate, which is equal to the rock uplift rate at the rock uplift axis x_u . The rock uplift rate is uniform in the *y* direction and constant over time.

[Insert Fig. 1]

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172The initial conditions are a flat topography at an elevation of 0 m, with white noise with maximum amplitude of 10 m. The altitudes of the eastern and western boundaries 173174are exactly set at 0 m through the computation. The water drains only through the eastern and western boundaries, and the northern and southern sides are no-flow 175boundaries. The problems associated with closed depressions (e.g., Tucker et al., 2001) 176are not included because the gradient of slopes generated by the following 177178computations is usually very steep, except for just after initiation of rock uplift. For 179closed depressions, the erosion rate is set to zero.

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181 **3. Results**

The relationship between rock uplift and topography was quantitatively investigated with the topographic evolution model introduced above. Different values for parameters x_u and u_{max} , which prescribe the rock uplift rate, were used in order to determine the dependence of topography on these parameters. In contrast, parameters *m*, *n*, and *K*, which control the erosion rate, were initially fixed at m = 0.5, n = 1.0, and $K = 1.2 \times 10^{-5}$ yr⁻¹. These values imply that the stream incision rate was modeled as a function of stream power per unit bed area (e.g., Whipple and Tucker, 189 1999). As discussed in Section 3.4, the dependence of topography on these parameters190 was also examined.

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192 **3.1 Topographic evolution under asymmetric rock uplift**

The rock uplift parameters x_u and u_{max} were initially set at 30 km and 6 mm yr⁻¹, 193respectively, to observe the topographic evolution process (Fig. 2). The results show 194that after initiation of rock uplift, the mountain grows at approximately the same rate as 195196 uplift because erosion is very weak. The topographic axis, defined here as the line of the highest mean altitude along the north-south profiles, coincides almost exactly with the 197 rock uplift axis. However, as the mountain grows, surface erosion increases, especially 198 199 on the side of the range having the steepest topographic gradient (east). The topographic 200 axis thus gradually migrates toward the center of the mountain range (Fig. 2). This 201means that the location of the topographic axis is more strongly affected by the effect of 202the channel gradient S than by the drainage area A, which is larger on the less steep side. 203A dynamic steady state is reached before t = 4.0 My. A dynamic steady state is defined as the condition under which the elevation change over a 1 My interval, averaged over 204 the model area and normalized by the maximum elevation, is less than 0.01%. Before 205reaching a steady state, the topographic axis migrates about 10 km from the rock uplift 206 207axis toward the center. The elevation of the topographic axis is slightly reduced during 208this migration.

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[Insert Fig. 2]

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212 **3.2 Relationship between rock uplift and topography**

The relation of asymmetric rock uplift to topographic evolution was simulated by changing the maximum uplift rate u_{max} and the location of the rock uplift axis x_u . Values of 2, 6, and 10 mm yr⁻¹ were used for u_{max} , and x_u was taken every 5 km from 5 to 45 km for each u_{max} . To suppress the dependence of topographic evolution on the initial random noise, the numerical simulation was performed five times for each u_{max} and x_{u} pair by changing the random noise of the initial topography. Fig. 3 shows the relation between u_{max} and the location of the topographic axis in the steady state x_{t} for various values of x_{u} . As shown in the figure, x_{t} does not depend on u_{max} irrespective of x_{u} . In the simulation of topographic evolution based on Eq. (2), the pattern of topography in the steady state is generally independent of the rate of rock uplift for the following reason. In the steady state, the left side of Eq. (2) is zero:

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$$u(x, y) = K[A(x, y)]^{m}[S(x, y)]^{n}$$
(4)

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227 Therefore, if a topographic distribution h(x, y) is a solution of Eq. (4) under a given uplift rate u(x, y), $\sqrt[\eta]{ch(x, y)}$ is also a solution for another given uplift rate with the 228same pattern cu(x, y), where c is an arbitrary positive constant. Recall that a 229proportional change in height distribution from h(x, y) to c'h(x, y) does not affect 230the drainage area A(x, y); only the channel gradient changes from S(x, y) to 231c'S(x, y). The minor fluctuation of each line in Fig. 3 is ascribed to the given random 232233noise added to the initial topography. In brief, a proportional change in u from u(x, y)to cu(x, y) results in no change in A and a proportional change in S from S(x, y) to 234 $\sqrt[n]{cS(x, y)}$ in the steady state. Here, it should be noted that u, K and m may be 235spatially variable, but *n* must be constant in space. 236

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From the theoretical relations described above, the height of the topographic axis in the steady state should be proportional to the rock uplift rate to the 1/n power. Given that *n* is taken to be unity, we show the elevation of the topographic axis in the steady state h_{max} normalized by u_{max} as a function of x_u in Fig. 4A. Each line of different rock uplift rates overlaps almost completely with each other. This means that h_{max} is indeed proportional to rock uplift rate. In addition, h_{max} decreases with x_u , which is consistent with the topographic evolution process shown in Fig. 2, where the elevation of the topographic axis decreases as the topographic axis moves away from the rock uplift axis x_u toward the center. The distance between x_u and x_t increases with x_u , as shown in Fig. 3 and Fig. 5.

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[Insert Fig. 4]

Fig. 4B shows the time required to reach a dynamic steady state for each x_u and u_{max} pair. As expected, the time to reach a steady state is longer for larger x_u . The topographic axis slowly moves from the rock uplift axis toward the center of the mountain range (Fig. 2), and the distance increases with x_u (Fig. 3). During this time, the drainage network continues to evolve and adapt. This process takes time. Comparison of Fig. 4A and B shows that the time required is most sensitive to the initial random noise.

260The time required is also independent of the maximum rock uplift rate u_{max} (Fig. 2614B). For this independence, the key is that n is taken to be unity. When a height distribution h(x, y, t) is a solution of Eq. (2) for a given uplift rate u(x, y, t) that is 262263constant in time, the height distribution ch(x, y, t) is also a solution for another given 264uplift rate with the same pattern cu(x, y, t). Recall again the drainage area A(x, y, t) is 265constant for a proportional change in height distribution from h(x, y, t) to ch(x, y, t)266and that S(x, y, t), which is the gradient of h(x, y, t), is proportional to h(x, y, t). In brief, the proportional relation of topography to rock uplift rates holds true throughout 267the time development. However, it should be noted that the proportional relation does 268not hold true when $n \neq 1$; only in the steady state, the proportional relation of 269topography to the *n*-th root of the rock uplift rate (Fig. 3 and Fig. 4A) holds true for 270271 $n \neq 1$. The independence of the elapsed time relative to the magnitude of rock uplift rate has already been noted by Kooi and Beaumont (1996) for the case of symmetric rock 272uplift. The independence is also valid for asymmetric rock uplift. 273

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275 **3.3 Logarithmic relation between topographic and rock uplift axes**

The relation between the rock uplift axis x_u and the topographic axis x_t in the steady state (Fig. 3) requires examination. Any proportional change in a given rock uplift rate, from u(x, y) to cu(x, y), does not affect the location of the topographic axis under steady-state conditions (and for transient behavior when n = 1). The numerical simulation was run five times for each u_{max} and x_u pair, and the average and standard deviation of 15 trials with different rock uplift rates were assessed as a group.

The results show that x_t increases with x_u (Fig. 5), but the rate of increase in 283 x_{t} is not as rapid as that in x_{u} . The numerical relation between x_{t} and x_{u} needs to 284be expressed with an analytical function. The function must increase monotonically 285286and the differentiation of the function should decrease monotonically; therefore, a 287logarithmic function would be appropriate. As a boundary condition, the function must pass through the origin $(x_u = x_t = 0)$ because there is no need for the topographic axis 288to move away from the center of the model region when x_u equals zero. In addition, a 289very small change in x_u from zero seems to cause the same amount of change in x_t . 290This condition is written as $dx_t / dx_u = 1$ at $x_u = 0$. The logarithmic function to fit the 291numerical relation is then expressed by 292

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$$x_{t} = f\left(x_{u}\right) = a \ln\left(\frac{x_{u}}{a} + 1\right)$$
(5)

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A natural logarithmic function is used here, but even if an apparently different base is used for the logarithmic function, the functional form is the same as Eq. (5) after transformation of the base. Eq. (5) consists of only the fitting parameter a, which is determined using the least-square method.

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303 The fit of the logarithmic function to the numerical results is statistically significant

[Insert Fig. 5]

(Fig. 5). The function passes almost exactly through the average of each trial. The average misfit to the nine data points is less than 0.2 km, which is much smaller than the grid interval of the numerical simulation. It should be noted that the result of each trial does not necessarily fit the logarithmic function, as shown by the error bar representing the standard deviation. After many trials with different initial random noise, however, the averages of the results follow the natural logarithmic function.

So far, parameters *m*, *n*, and *K*, which control erosion rates, have been fixed. Parameter *K* does not affect the functional relation between x_t and x_u because *K* has the same effect (except for the reciprocal) as *u* (Eq. (4)). However, x_t does depend on *m* and *n*. The effect of the exponents *m* and *n* on the logarithmic relation between x_t and x_u requires investigation.

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316 **3.4 Dependence of the logarithmic relation on the exponents**

317As explained by Tucker and Whipple (2002), Eq. (1) has been used to model various fluvial erosion processes, including bed shear stress $(m \simeq 0.3, n \simeq 0.7)$ (Howard and 318Kerby, 1983; Howard et al., 1994; Tucker and Slingerland, 1997), stream power per unit 319channel length $(m \simeq n = 1.0)$ (Seidl and Dietrich, 1992), and stream power per unit 320bed area $(m \simeq 0.5, n \simeq 1.0)$ (Stock and Montgomery, 1999; Whipple and Tucker, 1999; 321Kirby and Whipple, 2001). They also pointed out that the computed stream profile 322323concavity under the condition $m/n \ge 1.0$ or m/n < 0.2 is not common for most mountain drainage basins. 324

The values of the exponents m and n as well as those of x_{μ} and u_{max} are 325changed: n is set at 0.5, 1.0, and 1.5, and m/n has values that increase in increments 326 of 0.1, from 0.3 to 0.8, for each value of n. For each pair of m and n, the same 327 computation as in Fig. 5 is repeated, and the relationship between the location of the 328 topographic axis in the steady state x_t and that of the rock uplift axis x_u for each 329 pair of exponents m and n is plotted (Fig. 6). The fit of the logarithmic function is 330 again very good for most cases, indicating that the value of the fitting parameter a can 331be determined from one datum point of x_t and x_u , regardless of the values of the 332

exponents *m* and *n*. The fit appears to be best when m/n = 0.5. For smaller m/n, the numerical results appear to be slightly more rectilinear than the logarithmic function, as for example in the case of m/n = 0.3. For larger m/n, the fit is still good. This may be due to the characteristics of the fitting function. The fitting function asymptotically approaches $x_t = x_u$ as m/n increases (Fig. 6). Therefore, even if the theoretical fitting function deviated from the logarithmic function, the deviation would not be noticeable.

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[Insert Fig. 6]

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The change in the fitting function in Fig. 6 corresponds to the change in the fitting 343 parameter a in Eq. (5). For larger a, the fitting function approaches $x_t = x_u$, but for 344smaller a, the curvature of the fitting function becomes more acute. The value of a is 345346 controlled by the relative susceptibility of drainage area A and channel gradient S to 347erosion. The relative contribution of the drainage area, which is larger on the gentler 348side, increases with larger m/n. This means that for larger m/n, the topographic axis 349 x_t does not move away from the rock uplift axis x_u as much, the fitting function 350approaches $x_t = x_u$, and *a* becomes large. In contrast, for smaller m/n, the 351topographic axis migrates a substantial distance, the curvature of the fitting function is more acute, and the fitting parameter *a* becomes smaller. As shown in Fig. 6, however, 352the value of a can vary significantly for smaller m/n, even if m/n has the same 353value. In short, parameter *a* is not dependent only on m/n. 354

A contour map of parameter *a* on the *m* and *n* coordinate plane (Fig. 7A) shows that *a* is roughly constant for n - pm, where *p* is about 5/4. If *a* is then plotted relative to n - pm + q on a double logarithmic diagram (Fig. 7B) where *q* is a constant parameter, the numerical results can be fitted by a linear function expressed as

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$$\log_{10} a = r \log_{10} (n - pm + q) + s \tag{6}$$

where the optimal parameter values determined with the least-square method are 362363p=1.26, q=0.415, r=-2.55, and s=1.11. The fit is again excellent. If p=1.25is used instead of p = 1.26, the fit is nearly the same. This means that n - 5m/4 is an 364appropriate indicator of the relative susceptibility of drainage area A and channel 365 gradient S to erosion. For smaller n-5m/4, in which the contribution of the drainage 366 area is relatively high, the value of a becomes larger, and vice versa. With Eqs. (5) and 367 (6), the relation between x_t and n-5m/4 are obtained for various values of x_n 368(Fig. 8). This diagram means that we can estimate n-5m/4 or x_{μ} , if we know either 369 value of them, because the topography (i.e., x_t) is commonly well known. 370

[Insert Fig. 7], [Insert Fig. 8]

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4. Discussion

As shown in the numerical simulations, the erosion rate in asymmetric mountain 375ranges is faster on the steeper side and slower on the gentler side under realistic 376exponent conditions (m < n), as long as the other conditions that contribute to erosion 377rate are uniform. For example, in the Taiwan Central Range, where the eastern slope is 378much steeper than the western slope, significantly faster erosion is suggested for the 379 380 eastern flank as compared to the western flank, as indicated by the metamorphic grades of exposed rocks (Ho, 1986), fission track analyses (Willett et al., 2003), and heat-flow 381data (Yamano, 1995; Fukahata and Matsu'ura, 2001). 382

To address the distance migrated by the topographic axis relative to the rock uplift 383 axis, the model was applied to the Suzuka Range in central Japan as an example (Fig. 384 9A). The Suzuka Range is located in the Kinki Triangle (Huzita, 1962), an area 385386 currently under east-west compression (Huzita, 1980; Terakawa and Matsu'ura, 2010). The eastern limit of the Suzuka Range is bounded by west-dipping reverse faults of the 387 Ichishi Fault system, which have been active since the late Pliocene (Ota and Sangawa, 388 1984; Ishiyama et al., 1999). There is no significant local variation in lithology (Huzita, 389 1962; Geological Survey of Japan, AIST, 2009) and precipitation across the range. 390

[Insert Fig. 9]

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A cross-section of the topographic profile of the range (Fig. 9B) was produced by averaging the elevation data in the direction parallel to the trend of the range (approximately north-south, Fig. 9A). In the topographic profile, the horizontal distance and relative elevation of the mountain range are normalized by the width of the range $L_{\rm R}$ (20 km) and the difference between the maximum (812 m) and minimum (170 m) heights. The topographic axis of the Suzuka Range is located at about 0.20 (normalized distance) as shown in Fig. 9B.

401 The uplift and erosion rates across the Suzuka Range are not known. A seismic 402reflection survey showed that the Fumotomura Fault, one of the faults of the Ichishi 403 Fault system, has a dip angle of about 60° at shallow depth (Ishiyama et al., 1999). The 404 youngest strata, the Tokai Group, are cross-cut by the fault, but the displacement is 405very small. West of the fault, strata of the Tokai Group steeply dip eastward, indicating that deformation around the eastern end of the range is caused by a fault-propagation 406 fold (Ishiyama et al., 1999). Based on the seismic survey, we simulated the crustal 407deformation pattern in the Suzuka Range. The fault was assumed to have a dip angle of 408 409 60° at the earth's surface, which gradually changes to a horizontal detachment fault at depth (Fig. 10A). The slip rate along the fault was assumed to decrease gradually to 410 411 zero at surface from 3 km at depth because the deformation is caused by a 412 fault-propagation fold. It should be noted that the true fault geometry and slip rate distribution are not well determined from observations; the configuration described 413 414 was adopted to demonstrate the use of the model described in the previous section. In 415the computation of crustal deformation due to the dislocation across the fault plane, an elastic half-space was assumed and a value of 50 GPa was used for the bulk modulus 416 and 30 GPa for the shear modulus. 417

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Fig. 10B and C shows the profiles of vertical and horizontal displacements, 421respectively. The rock uplift axis is located at 0.32 in normalized distance (Fig. 10B). 422When $x_u = 0.32$ and $x_t = 0.20$, we can obtain the estimate of n - 5m/4 to be 423 0.38 ± 0.05 based on Fig. 8, where the estimation error is originated from the standard 424425deviation shown in Fig. 6. n-5m/4 = 0.38 almost coincides with a typically used set of the exponent values of (m,n)=(0.5,1.0). Then, n-5m/4 is 0.375. If the topography 426 of the Suzuka Range is before the dynamic steady state, the value of n-5m/4427 becomes larger, because x_t further moves toward the center of the range. 428

So far, only the purely gabled rock uplift pattern has been considered as a cause of topographic asymmetry. In actual mountain ranges, however, the other factors may significantly affect the relation between the rock uplift axis x_u and the topographic axis x_t . So, we carried out some sensitivity tests, in which $x_u = 0.32$ and (m,n)=(0.5,1.0) were used, and the topographic evolution was computed 15 times for each setting with different initial random noise.

As shown in Fig. 10B and C, the computed uplift pattern is not purely gabled and 435the horizontal displacement is not zero. Thus, the computed displacement profiles 436(solid lines in Fig. 10B and C) were used in the numerical simulation of topographic 437438 evolution, instead of the purely gabled rock uplift pattern (dotted line in Fig. 10B). As a result, 0.217 ± 0.007 was obtained for x_t . This value is close to 0.207 ± 0.010 , which 439is obtained for the purely gabled rock uplift pattern. If the horizontal displacement is 440 neglected and only the computed uplift profile (solid line in Fig. 10B) is used, 441 0.225 ± 0.012 is obtained. The effect of horizontal displacement is not significant in 442 this situation, because the horizontal displacement in the hanging wall is quite uniform 443 (Fig. 10C). The effect of asymmetrical precipitation was also investigated by giving 44420 % larger precipitation (larger A in the simulation) in one side. The topographic axis 445at each time step was used as the boundary of the different precipitation. Then, x_{t} is 446 0.179 ± 0.011 and 0.227 ± 0.007 for larger precipitation in the eastern and western sides, 447respectively. When the base level of erosion was changed 10 % in the normalized 448

height, x_t was 0.189 ± 0.014 and 0.223 ± 0.010 for the higher base level in the western and eastern sides, respectively. In either case, the calculated x_t is almost within the two sigma of the purely gabled uplift case demonstrated in the previous section.

453In the numerical simulation, the effects of thresholds for erosion and hillslope processes were omitted for simplicity. Owing to this simplification, the numerical 454model is essentially described by only three parameters, the rock uplift axis x_{u} and 455the exponents *m* and *n*, and does not depend on the rock uplift rate (Fig. 3 and Fig. 4) 456or the coefficient of erosion K. Numerical experiments in which a threshold of critical 457shear stress for stream erosion is included do not cause notable changes, probably due 458459to a rapid erosion rate in the numerical simulation. Although the model does not 460 explicitly incorporate hillslope processes, the uppermost cells of streams can be eroded 461because the area of any given cell is included in the calculation of the drainage area. 462This differs from the models of Kooi and Beaumont (1996) and Kühni and Pfiffner (2001). If we incorporate hillslope processes into the model, the migration distance of 463the topographic axis would probably be larger because the second derivative of the 464topography $\nabla^2 h$ would be reduced. The main features of the topography, however, 465 are considered to be primarily controlled by an advective stream process (Howard, 466 467 1994; Kooi and Beaumont, 1996). In fact, the numerical simulation of Miller et al. 468 (2007), in which the scale of their target (Siwalik Hills, Nepal) is about 20 km in the horizontal direction, demonstrated that diffusion does not visibly affect the asymmetry 469 of the range under normal conditions. 470

This study identifies the logarithmic relation between the topographic and rock uplift axes for a simple set of conditions (Fig. 5 and Fig. 6). The fitting parameter a of the logarithmic function was also clearly related to the exponents m and n (Fig. 7). To reveal the nature of the logarithmic relation, we tried to derive these relations analytically, but failed despite of its simplicity. This problem should be addressed in the future.

477

478 **5.** Conclusions

Asymmetric rock uplift is one of the main causes of asymmetric mountain ranges, 479such as the Suzuka Range in central Japan. In asymmetric mountain ranges, the surface 480 erosion rate is faster on the steeper side and slower on the gentler side, under realistic 481 482conditions for the exponents (m < n) and assuming that the other conditions that 483 control the erosion rate, such as precipitation, lithology, and vegetation, are uniform. 484 Therefore, the topographic axis migrates from the rock uplift axis toward the center of 485the mountain range until erosion is balanced by rock uplift (Fig. 2); this balancing takes a few million years (Fig. 4). 486

The migration distance from the rock uplift axis can be expressed by a logarithmic function (Fig. 6). The fit of the logarithmic function is especially good when (m,n) = (0.5, 1.0) (Fig. 5). A proportional change in the uplift rate does not affect this relationship (Fig. 3 and Fig. 4) because a change in the uplift rate from u(x, y) to cu(x, y) results in no change in the drainage area *A* and a proportional change in *S* from S(x, y) to $\sqrt[n]{c}S(x, y)$. Here, *u*, *K*, and *m* may be spatially variable, but *n* must be constant.

If the rock uplift pattern for an asymmetric mountain range is similar to be gabled, the value of n-5m/4 can be constrained (Fig. 8). This concept was applied to the Suzuka Range in central Japan (Fig. 9 and Fig. 10). Conversely, if a value of n-5m/4is available, the rock uplift pattern can be estimated directly from the topography in asymmetric mountain ranges.

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505 **References**

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507 Adams, J., 1980. Contemporary uplift and erosion of the Southern Alps, New Zealand.

508 Geological Society of America Bulletin 91, 1–114.

- 509 Anders, A.M., Roe, G.H., Montgomery, D.R., Hallet, B., 2008. Influence of
- 510 precipitation phase on the form of mountain ranges. Geology 36, 479–482.
- Anderson, R.S., 2002. Modeling the tor-dotted crests, bedrock edges, and parabolic
 profiles of high alpine surfaces of the Wind River Range, Wyoming.
 Geomorphology 46, 35–58.
- 514 Araki, H., Tazawa, S., Noda, H., Ishihara, Y., Goossens, S., Sasaki, S., Kawano, N.,
- 515 Kamiya, I., Otake, H., Oberst, J., Shum, C., 2009. Lunar Global Shape and Polar 516 Topography Derived from Kaguya-LALT Laser Altimetry. Science 323, 897–900.
- Beaumont, C., Fullsack, P., Hamilton, J., 1992. Erosional control of active
 compressional orogens. In: McClay, K.R. (Ed.), Thrust Tectonics. Chapman and
 Hall, London. pp. 1–18.
- Ellis, M.A., Densmore, A.L., 2006. First-order topography over blind thrusts. In:
 Willett, S.D., Hovius, N., Brandon M.T., Fisher, D. (Eds.), Tectonics, Climate, and
 Landscape Evolution. Geological Society of America Bulletin Special Paper 398, pp.
 251–266.
- Farr, T.G., Rosen, P.A., Caro, E., Crippen, R., Duren, R., Hensley, S., Kobrick, M., 524Paller, M., Rodriguez, E., Roth, L., Seal, D., Shaffer, S., Shimada, J., Umland, J., 525Werner, M., Oskin, M., Burbank, D., Alsdorf, D., 2007. The Shuttle Radar 526of 45, 527 Topography Mission. Reviews Geophysics RG2004, 528doi:10.1029/2005RG000183.
- Fukahata, Y., Matsu'ura, M., 2001. Correlation between surface heat flow and elevation
 and its geophysical implication. Geophysical Research Letters 28, 2703–2706.
- Fukahata, Y., Matsu'ura, M., 2006. Quasi-static internal deformation due to a
 dislocation source in a multilayered elastic/viscoelastic half-space and an
 equivalence theorem. Geophysical Journal International 166, 418–434.

- Geographical Survey Institute of Japan, 1997. Digital map 50 m grid (Elevation)
 Nippon III (CD-ROM).
- 536 Geological Survey of Japan, AIST, 2009. Seamless digital geological map of Japan 1:
- 537 200,000, Dec 15, 2009 version, Research Information Database DB084, Geological
- 538 Survey of Japan, National Institute of Advanced Industrial Science and Technology.
- 539 Harkins, N., Kirby, E., Heimsath, A., Robinson, R., Reiser, U., 2007. Transient fluvial
- 540 incision in the headwaters of the Yellow River, northeastern Tibet, China. Journal of
- 541 Geophysical Research 112, F03S04, doi:10.1029/2006JF000570.
- 542 Herman, F., Braun, J., 2006. Fluvial response to horizontal shortening and glaciations:
- 543 A study in the Southern Alps of New Zealand. Journal of Geophysical Research 111,
- 544 F01008, doi:10.1029/2004JF000248.
- Ho, C.S., 1986. A synthesis of the geologic evolution of Taiwan. Tectonophysics 125,
 1–16.
- Howard, A.D., 1994. A detachment-limited model of drainage basin evolution. Water
 Resources Research 30, 2261–2285.
- Howard, A.D., Kerby, G., 1983. Channel changes in badlands. Geological Society of
 America Bulletin 94, 739–752.
- Howard, A.D., Dietrich, W.E., Seidl, M.A., 1994. Modeling fluvial erosion on regional
 to continental scales. Journal of Geophysical Research 99, 13971–13986.
- 553 Huzita, K., 1962. Tectonic development of the Median Zone (Setouchi) of Southwest
- Japan since the Miocene, with special reference to the characteristic structure of
- 555 Central Kinki Area. Journal of Geoscience, Osaka City University 6, 103–144.
- 556 Huzita, K., 1980. Role of the Median Tectonic Line in the Quaternary tectonics of
- Japanese Islands. Memoirs of the Geological Society of Japan 18, 129–153.
- 558 Ishiyama, T., Takemura, K., Okada A., 1999. Structural growth rate of the eastern
- margin of the Suzuka Range during Quaternary. Journal of the Seismological
 Society of Japan 52, 229–240 (in Japanese with English abstract).
- Kirby, E., Whipple, K., 2001. Quantifying differential rock-uplift rates via stream
 profile analysis. Geology 29, 415–418.

Kirkby, M.J., 1994. Thresholds and instability in stream head hollows: A model of
magnitude and frequency for wash processes. In: Kirkby, M.J. (Ed.), Process Models
and Theoretical Geomorphology. Wiley, Chichester, pp. 295–314.

- Kooi, H., Beaumont, C., 1994. Escarpment evolution on high-elevation rifted margins:
 Insights derived from a surface processes model that combines diffusion, advection,
- and reaction. Journal of Geophysical Research 99, 12191–12209.
- Kooi, H., Beaumont, C., 1996. Large-scale geomorphology: Classical concepts
 reconciled and integrated with contemporary ideas via a surface processes model.
 Journal of Geophysical Research 101, 3361–3386.
- Koons, P.O., 1989. The topographic evolution of collisional mountain belts: A
 numerical look at the Southern Alps, New Zealand. American Journal of Science
 289, 1041–1069.
- Kühni, A., Pfiffner, O.A., 2001. Drainage patterns and tectonic forcing: a model study
 for the Swiss Alps. Basin Research 13, 169–197.
- Lague D., Hovius N., Davy P., 2005. Discharge, discharge variability, and the bedrock
 channel profile. Journal of Geophysical Research 110, F04006,
 doi:10.1029/2004JF000259.
- 580 Meade, B.J., 2010. The signature of an unbalanced earthquake cycle in Himalayan
 581 topography? Geology 38, 987–990.
- 582 Miller, S.R., Slingerland, R.L., Kirby E., 2007. Characteristics of steady state fluvial
 583 topography above fault-bend folds. Journal of Geophysical Research 112, F04004,
- 584 doi:10.1029/2007JF000772.
- 585 Mitchell, S.G., Montgomery, D.R., 2006. Influence of a glacial buzzsaw on the height
- and morphology of the Cascade Range in central Washington State, USA.
- 587Quaternary Research 65, 96–107.
- 588 Ota, Y., Sangawa, A., 1984. Active faults in the eastern foot area of the Suzuka Range,
- 589 Central Japan. Geographical Review of Japan 57, 237–262 (in Japanese with 590 English abstract).
- 591 Research Group for Active Faults of Japan, 1991. Active Faults in Japan, Sheet Maps

- and Inventories (Revised Edition). University of Tokyo Press (in Japanese).
- 593 Roering, J.J., Kirchner, J.W., Dietrich, W.E., 1999. Evidence for non-linear, diffusive
- sediment transport on hillslopes and implications for landscape morphology. Water
- 595 Resources Research 35, 853–870.
- 596 Sagiya, T., 2004. A decade of GEONET: 1994-2003 -The continuous GPS observation
- in Japan and its impact on earthquake studies-. Earth, Planets and Space 56, xxix-xli.
- 598 Schmidt, K.M., Montgomery, D.R., 1995. Limits to Relief. Science 270, 617–620.
- Seidl, M.A., Dietrich, W.E., 1992. The problem of channel erosion into bedrock.
 Catena Supplement 23, 101–124.
- 601 Smith, D.E., Zuber, M.T., Solomon, S.C., Phillips, R.J., Head, J.W., Garvin, J.B.,
- Banerdt, W.B., Muhleman, D.O., Pettengill, G.H., Neumann, G.A., Lemoine, F.G.,
- Abshire, J.B., Aharonson, O., Brown, C.D., Hauck, S.A., Ivanov, A.B., McGovern,
- P.J., Zwally, H.J., Duxbury, T.C., 1999. The global topography of Mars and
 implications for surface evolution. Science 284, 1495–1503.
- 606 Stark, C.P., Hovius, N., 2001. The characterization of landslide size distributions.
- 607 Geophysical Research Letters 28, 1091–1094.
- Stock, J.D., Montgomery, D.R., 1999. Geologic constraints on bedrock river incision
 using the stream power law. Journal of Geophysical Research 104, 4983–4994.
- Tarboton D.G., Bras, R.L., Rodriguez-Iturbe I., 1989. Scaling and elevation in river
 networks. Water Resources Research 25, 2037–2051.
- Terakawa, T., Matsu'ura, M., 2010. 3-D tectonic stress fields in and around Japan
 inverted from CMT data of seismic events. Tectonics 29, TC6008,
 doi:10.1029/2009TC002626.
- 615 Thatcher, W., Rundle, J.B., 1984. A viscoelastic coupling model for the cyclic
- 616 deformation due to periodically repeated earthquakes at subduction zones. Journal of
- 617 Geophysical Research 89, 7631–7640.
- Tucker, G.E., Bras, R.L., 2000. A stochastic approach to modeling the role of rainfall
- 619 variability in drainage basin evolution. Water Resources Research 36, 1953–1964.
- 620 Tucker, G.E., Slingerland, R.L., 1994. Erosional dynamics, flexural isostasy, and

- long-lived escarpments: A numerical modeling study. Journal of Geophysical
 Research 99, 12229–12243.
- Tucker, G.E., Slingerland, R.L., 1997. Drainage Basin Responses to Climate Change.
 Water Resources Research 33, 2031–2047.
- Tucker, G.E., Whipple, K.X., 2002. Topographic outcomes predicted by stream erosion
- models: Sensitivity analysis and intermodel comparison. Journal of Geophysical
 Research 107, 2179, doi:10.1029/2001JB000162.
- Tucker, G.E., Lancaster, S.T., Gasparini, N.M., Bras, R.L. Rybarczyk, S.M., 2001. An
 Object-Oriented Framework for Hydrologic and Geomorphic Modeling Using
 Triangulated Irregular Networks. Computers & Geosciences 27, 959–973.
- van der Beek, P., Bishop, P., 2003. Cenozoic river profile development in the Upper
 Lachlan catchment (SE Australia) as a test of quantitative fluvial incision models.
- 633 Journal of Geophysical Research 108, 2309, doi:10.1029/2002JB002125.
- Wessel, P., Smith, W. H. F., 1998. New, improved version of Generic Mapping Tool
 released. Eos, Transactions, American Geophysical Union 79, 579.
- Whipple, K.X., Tucker, G.E., 1999. Dynamics of the stream-power river incision
 model: Implications for height limits of mountain ranges, landscape response
 timescales, and research needs. Journal of Geophysical Research 104, 17661–17674.
- 639 Whipple, K.X., Hancock, G.S., Anderson, R.S., 2000. River incision into bedrock:
- 640 Mechanics and relative efficacy of plucking, abrasion, and cavitation. Geological 641 Society of America Bulletin 112, 490–503.
- Willett, S.D., 1999. Orogeny and orography: the effects of erosion on the structure of
 mountain belts. Journal of Geophysical Research 104, 28957–28981.
- Willett, S.D., Fisher, D., Fuller, C., Yeh, E.C., Lu, C.Y., 2003. Erosion rates and
 orogenic wedge kinematics in Taiwan inferred from apatite fission track
 thermochronometry. Geology 31, 945–948.
- 647 Willett, S.D., Slingerland, R., Hovius, N., 2001. Uplift, Shortening, and Steady State
- Topography in Active Mountain Belts. American Journal of Science 301, 455–485.
- 649 Willgoose, G., Bras, R.L., Rodriguez-Iturbe, I., 1991. A coupled channel network

- growth and hillslope evolution model, 1, Theory. Water Resources Research 27,1671–1684.
- 652 Yamano, M., 1995. Recent heat flow studies in and around Japan. In: Gupta M.L.,
- 653 Yamano, M. (Eds.), Terrestrial Heat Flow and Geothermal Energy in Asia. Oxford &
- IBH Publishing Co., New Dehli, pp. 173–201.

655 Figure Captions

656

Fig. 1. Profile of asymmetric rock uplift rate in the *x* (east-west) direction, equivalent to Eq. (3). x_u and u_{max} represent the location of the rock uplift axis and the maximum rock uplift rate, respectively. The rock uplift rate is uniform in the *y* (north-south) direction and constant over time.

Fig. 2. Topographic evolution process under a given asymmetric rock uplift rate. The 661 initial topography is flat with an elevation of 0 m and superimposed white-noise 662 topography with a maximum amplitude of 10 m. (A) Snapshots of topography over 663 4 My of elapsed time, with a contour interval of 1000 m. (B) East-west topographic 664 profiles averaged in the y (north-south) direction. x_t and h_{max} are the location and 665 666 elevation of the topographic axis in the steady state, respectively. The rock uplift axis x_{u} , represented by the thick broken line in each diagram, is 30 km, and the 667 maximum uplift rate u_{max} is 6 mm yr⁻¹. Water is drained only from the western 668(left) and eastern (right) boundaries. 669

Fig. 3. Relation of the location of the topographic axis in the steady state x_t to the maximum rock uplift rate u_{max} for various rock uplift axes x_u . To suppress the effect of the initial random noise, the numerical simulation was performed five times for each pair of u_{max} and x_u . The solid diamond and error bar represent the average and standard deviation of the five trials, respectively.

Fig. 4. Relation between parameters of rock uplift and topography. (A) Relation of the height h_{max} at the topographic axis in the steady state to x_u and u_{max} . In order to compare results, the height at the topographic axis is normalized by u_{max} . (B) The relation of the elapsed time to steady state versus x_u and u_{max} . The solid diamond and error bar in each diagram represent the average and standard deviation of the five trials, respectively, for each pair of x_u and u_{max} .

Fig. 5. Relation of the location of the topographic axis in the steady state x_t to the location of the rock uplift axis x_u . The numerical results are fitted by a natural logarithmic function in Eq. (5). In this figure, parameter *a* is 22.2 km. The solid

- 684 diamond and error bar represent the average and standard deviation of 15 trials, 685 respectively. For reference, $x_t = x_u$ is also shown (dotted line).
- Fig. 6. Relation between x_t and x_u for various pairs of the exponents *m* and *n*. The solid diamond and error bar represent the average and standard deviation of 15 trials, respectively, for the numerical simulation with different random noise of initial topography. The fitted logarithmic function for each pair of exponents has the form of Eq. (5), and the fitting parameter *a* is determined by the least-square method. For reference, $x_t = x_u$ is also shown (dotted line).
- Fig. 7. Relation of the log-fitting parameter a (Eq. 5) with the exponents m and n. (A) Contour map of parameter a on the m and n coordinate plane. (B) Relation between parameter a and a linear function of m and n on a double logarithmic diagram. The solid line represents the fitting function given in Eq. (6).
- Fig. 8. Relation of the location of the topographic axis in the steady state x_t to an appropriate indicator of the susceptibility of erosion n-5m/4 for various rock uplift axes x_u .
- Fig. 9. Topography of the Suzuka Range in central Japan. (A) Location map of the 699 Suzuka Range. The rectangle indicates the region where the elevation data were 700 averaged for the topographic cross-section in Fig. 9B. The area of the Kinki triangle 701 702is also shown by dotted lines (Research Group for Active Faults of Japan, 1991). The inset shows the tectonic setting of this area; EUR, NAM, and PHS stand for the 703Eurasian, North American, and Philippine Sea plates, respectively. (B) Topographic 704 cross-section for the Suzuka Range. The cross-section is calculated from the digital 705 map of Japan (Geographical Survey Institute of Japan, 1997) by averaging the 706 elevation data in the direction parallel to the trend of the range. The horizontal 707 distance and relative elevation are normalized by the width of the range $L_{\rm R}$ and the 708 difference between the maximum and minimum elevations $h_{\text{max}} - h_{\text{min}}$, respectively. 709 $L_{\rm R}$, $h_{\rm max}$, and $h_{\rm min}$ are about 20 km, 812 m, and 170 m, respectively. 710
- Fig. 10. Crustal deformation pattern caused by reverse fault movement. (A) Vertical
 cross-section of the assumed fault geometry. The slip rate gradually decreases from

- 3 km at depth to zero at the earth's surface. (B) Uplift rate profile at the earth's
- surface due to the fault motion (solid line), normalized by the maximum rock uplift
- rate. Rock uplift pattern in the form of Eq. (3) with $x_u = 32$ km is also shown
- 716 (broken line). (C) Horizontal displacement profile at the earth's surface due to the
- fault motion, normalized by the maximum rock uplift rate.



Fig. 1



Fig. 2



Fig. 3



Fig. 4



Fig. 5



Fig. 6



Fig. 7



Fig. 8



Fig. 9



Fig. 10