TRANSIENT STABILITY ANALYSIS OF MULTIMACHINE POWER SYSTEM
VIA
LYAPUNOV'S DIRECT METHOD

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PREFACE

In this thesis, we are concerned with an application of Lyapunov's direct method to the transient stability analysis of multimachine power systems.

Power systems have been steadily growing larger and larger with increase in the demand for electric power, and the analyses on their transient stability also have been becoming more complicated. With the advent of high-powered electronic computers, the size and detail of the system representation have been improved very much, but the analyses with such detail representations are still expensive and time consuming tasks. In system planning and operations, many cases must be studied under a range of operating conditions, or for different system configurations. With the shift from conventional dispatch centers to modern power control centers in which the security control concept is introduced, the need for on-line transient stability analyses has recently been recognized. The conventional method based on simulations is not suitable for those studies. In view of this situation, new effective methods have been searched for.

Lyapunov's direct method has been developed as one of promissing methods for the last decade. It enables us to assess the first swing stability of power systems more directly and effectively than the conventional method based on simulations. The aim of this thesis is to develope Lyapunov's direct method to the level where it is a useful tool for transient stability analyses of power systems. It is also implicitly aimed to produce a theoretical background to the transient stability of power systems which have been blindly studied with simulations.

Since we lay stress on the developement of practical procedures for the transient stability analysis, multimachine power systems are mainly dealt with throughout this thesis. Many researches have been made on

Lyapunov's direct method with one-machine connected to an infinite bus systems, and several construction methods and important knowledges have been derived for power systems in which system equipments are represented in detail. Most of the construction methods are incompetent to construct Lyapunov functions for multimachine systems, however, so our attention is restricted only to a few of them which are also applicable to multimachine power systems. It will justify this attitude that the computational superiority of the direct method is not so much compared with the conventional method based on simulations which always able to handle more detail system representations in such low dimensional systems.

The direct method has not reached the practical level yet in spite of many researcher's effort over the last decade. It is prevented from practical applications owing to three obstacles. The first is the well-known conservative nature, that is, this method often yields very conservative results compared with the actual stability, especially in large power systems. The second is that it takes long computational time to calculate the critical value which represents an extent of the transient stability region, and as a result, the computational advantage of this method is cancelled. The third is that this method needs highly simplified system representations, such as, generators represented by constant voltages behind transient reactances. Our specified objective is to remove these restrictions as much as possible.

The present work has been initiated after the period when a systematic construction method based on a generalized Popov criterion was proposed by J.L. Willems in 1970, and this method had been refined and established by several researchers. Before this period, several Lyapunov functions had been derived by some researchers, e.g., P.D. Aylett and A. H. El-Abiad through physical considerations based on the system energy. On the other hand, some investigations on the critical value had also been initiated in order to remove the the obstacles mentioned above.

Firstly, we have started investigations on the ciritical value, have introduced a new critical value, and have developed a simple method of calculating this critical value. We have succeeded in removing the conservative nature and the computational difficulty related with the critical value. Next, we have proceeded to improve the system representaion. We have derived a generalized Popov criterion which is more general than that derived by J.B. Moore and B.D.O. Anderson. On the basis of this criterion, we have constructed Lyapunov functions for two power system representations; one in which dynamics of field flux linkages of generators are incorporated, and one in which automatic voltage regulators and thyristor exciters are installed in generators. The transient stability of these systems have been successfully analyzed by Lyapunov's direct method.

We hope that the present work will be a help for Lyapunov's direct method to come to be prevailingly used in transient stability analyses on various stages of system planning and operations in the future, and that it would stimulate further studies on this direct method.

CONTENTS

Chapter I

GENERAL INTRODUCTION

As a general introduction of the present thesis, we begin in this chapter with a short description of trends in present-day power systems to clarify practical problems faced by power utility industries. Power system stabiliities which are important items in system design and operation are then briefly explained paying a special attention to the transient stability. Lyapunov's direct method of analyzing the transient stability, is then introduced.

§1. Trends in power systems [1,2]

Demands for electric power are steadily increasing every year. It is forecasted that the gross power demand will be twice as large as the present one after a decade. Large power stations and transmission lines have been builded in order to meet these demands, and power systems are growing larger and larger.

The shift from thermal power plants to nuclear power plants have been obliging power utility industories to build power stations at places far from load areas owing to environmental problems. Sites for power stations are severely limited, so many power stations with capacities more than 1,000 kw will be concentrically builded in several allowed areas. Similarly, loads are maldistributed in big cities and industrial areas. In those areas, the growth of power demand is getting slow because of shortage in industrial water and environmental problems, but it is considerably rapid in their environs. In the future, exisiting load areas and their environs will form very wide and high-density load areas.

Since power generation areas and load areas are separated very much from each other, transmission lines connectiong those areas extend as well. Their length which is usually 50 ∿ 100 km, will reach 200 ∿ 600 km in the near future. The transmission capacity is mainly determined by power system stabilities if transmission lines are longer than 100 km, and is inversely proportional to the transmission distance, that is, it decreases with increase in the transmission distance. On the other hand, sites for transmission lines are also severely restricted because of nature preservations. This state of affairs claim to transfer bulk power through limited transmission routes. In order to meet this claim, trunk transmission lines with 500 kv of transmission voltage have been builded in our country since 1973. Presently, UHV (Ultra High Voltage) transmission with 1,000 kv of transmission voltage is under study. Besides, several stabilization methods have been developed and practically used as well such as, series condensers, braking resisters, high initial response exciters, high initial response governors, fast circuit breakers. As a result, capacities of these trunk transmission lines will come to occupy several ∿ several 10 percents of total system capacities, so outages of these lines have significant influence on the whole system, and their influence extends to wide areas. In order to operate those transmission lines with high-reliability, better system configurations, including bus configurations and protective relaying systems, must be developed.

Transmission lines are connected in grids to supply power with high-reliability to wide and high-density load areas. Electrical connections among all points are strengthened, but short-circuit currents are increased at the same time. The increase in short-circuit currents brings undesirable problems such as, shortage in breaking capacities of circuit breakers, spreads of damages due to faults to wide areas, and increase in induced noises in communication links. In order to solve these problems, changes in system configurations by adopting higher transmission voltages, introductions of direct current power transmissions, and developements of circuit breakers with larger capacities must be investigated.

Interconnections between power systems are enlarged and tightened for the sake of saving system reserves, econimically operating large power stations, eliminating redundancies of system equipments, and improving reliabilities. On the other hand, this growth of interconnection has possibilities of increasing short-circuit currents, extending disturbances represented by lightnings on transmission lines to wide areas, and promoting negative damping oscillations between power systems. It is necessary to develope such system configurations and operations that take advantages of interconnections avoiding these problems.

Summing up these trends, present-day power systems are characterized by bulky capacities along with their individual components, enormarous number of system equipments, and huge size of their interconnections. In order to keep their sound growth, many aspect of studies on power systems are indispensable in their designs and operations. The key item in those studies is power system stabilities, which all systems should possess as their characteristic without fail if they will fulfil their functions constantly. In the next section, we describe on analyses of power system stabilities along with their uses in power system design and operation.

§2. Power system stabilities and analysis

Power system stabilities are generously classified into two groups as follows [3]:

1) Dynamic stability

The dynamic stability is concerned with small disturbances which power systems suffer under normal operating conditions such as, small changes in generation and load outputs, and rotory and static condenser outputs. It is characteristic of instability in this sense that it is caused by particular unstable modes which individual power system potentially may have, and that it does not matter how large disturbances are applied. Since small disturbances are dealt with, all nonlinear elements, e.g., saturations, dead zones, limiters in exciters and governors, synchronous torques, and loads, are negligible in analyses of this stability. Many analytical methods have been developed on a basis of linear system theories and from physical considerations. Results of those analyses are used in determining future configurations of trunk and subordinate transmission systems, the highest transmission voltage of trunk transmission systems, distributions of power flows on outer rim of transmission lines and lower voltages of transmission lines, and allowable operating ranges of power stations. The dynamic stability must be kept for power systems to operate stably under normal operating conditions.

2) Transient stability

The transient stability is concerned with relatively large disturbances which power system suffer owing to various faults of transmission lines, switchings of transmission lines, and sudden changes in generation load outputs, e.t.c. Since applied disturbances are large, all nonlinear characteristics of system components come to take part in this stability. The transient stability analyses are usually classified into two groups according to the terms with which they are concerned as follows:

i) Short term analysis

The short term means 0 ∿ 2 seconds after a occurrence of a fault, in which rotors of generators are accellated, and show the first swings. Typical disturbances which injure synchronized operations of interconnected generators in this term are severe faults followed by actions of primary protective relaying system, such as short-circuits and ground faults of transmission lines. It is characteristics of the instability in this term that synchronizing torques which vary with relative rotor angles of generators play an important role in their developements. Since the term is very short, all the system components whose responses are relatively slow, are neglected, and simple but adequately accurate system representations are used in transient stability analyses in order that many case of system configurations, or operating conditions, can be studied. Results of those analyses are used in determining future configurations of trunk and subordinate transmission systems, the highest transmission voltage of trunk transmission systems, methods of transferring bulk power from large power stations to outer rim of transmission systems, distributions of power flow on outer rim of transmission lines and lower voltages of transmission lines, operation policies on interconnections with other power utilities, operation policies on irregular power systems in repairing system components, etc. Thus, the transient stability analysis occupies an important position in routine power system designs and operations.

ii) Long term analysis

The long term means 0 ∿ 20 seconds after a occurrence of a fault, in which system variables oscillate several ∿ several ten times. Typical disturbances which are concerned with instabilities in this term, are relatively large disturbances, such as switchings of transmission lines and medium size of changes in generation and load outputs. It is characteristic of instabilities in this term that those oscillations last without damping, or grow for long time to injure synchronized operations of interconnected generators. Under these undamped oscillations of system variables, power systems are not able to transfer power constantly, and have to stop their operations once in order to recover normal operations. Since applied disturbances are relatively large, nonlinear characteristics of system components have possibilities to cause produced oscillations to last or grow. In analyses of this stability, dynamics of generators, exciters, governors, and voltage and frequency characteristics of loads are represented in detail in order that their results could keep adequate accuracies over the long term. Results of those analyses are used in investigating stabilization methods, such as series condensers, braking resisters, intermediate switching stations, etc., and their protective relaying systems, in studying methods of preventing faults from spreads, such as various back-up protective relaying systems, system separation systems, and in elucidating unusual phenomena and serious accidents, etc.

The uses of these stability analyses are summarized in Table 1. They all have several uses in power system planning and operations. It should be noted that the transient stability analysis in the long term has flexible uses which vary according to problems at hand. The transient stability was originally that in the short term. However, with the developement of electronic computers, the size and detail of the system representation has been improved very much, which makes long-term simulations possible. Simulations yield time responses of all the variables in the system representation, and provide answers on the stability. Their object is, accordingly, not necessarily limited to the transient stability, but the dynamic stability can be also treated.

There are three concrete items which are investigated in usual stability analyses as follows:

[Item 1] Is a power system stable or not for a given disturbance ?

[Item 2] How much power can a power system transfer through transmission lines without losing synchronism for a given disturbance ?

[Item 3] How do system variables vary with time for a given disturbance ?

Results on these items are combined according to their uses in power system planning and operations as shown in Table 2. The first item is the most basic question in the stability analysis. The stability is one of basic qualities which the system must have in order to perform its functions stably. The second item is necessary in determining stable operat-

Table 1. Stability analysis and its objects

	Object of analysis	Dynamic stability	Transient stability Short-term	Transient stability Long-term
System planning	Configuration of future trunk transmission system	◎	◎	○
	Protective relaying system of future system	◎	◎	
	Future interconnection of power systems	◎	◎	
	Confirmation of functions of designed power systems	○	◎	
System operation	Operation policy on trunk transmission system	◎	◎	○
	Operation policy on system interconnection		◎	○
	Operation policy on irregular transmission systems	○	◎	○
	Protective relaying systems	○		◎
	Methods of improving system stabilities	○	○	◎
	Simple stability supervision		◎	
Operation of power station	Operating limits of power stations	◎	○	
	Optimal setting of control equipments		○	◎
	Shocks on generators due to system faults		○	◎

(◎: mainly performed, ○: partially performed)

Table 2. Stability analysis and its contents

	Object of analysis	Judgement of Stability	Stability power limits	System behaviors
System planning	Configuration of future trunk transmission system	○	○	○
	Protective relaying system of future system	○	○	
	Future interconnection of power systems	○	○	
	Confirmation of functions of designed power systems	○		
System operation	Operation policy on trunk transmission system		○	
	Operation policy on system interconnection	○	○	○
	Operation policy on irregular transmission systems	○		
	Protective relaying systems			○
	Methods of improving system stabilities	○		
	Simple stability supervision	○		
Operation of power station	Operating limits of power stations	○		
	Optimal setting of control equipments			○
	Shocks on generators due to system faults			○

ing areas of generators, and stable power limits of transmission lines. This item is determined by iterating the first item for a range of load flow conditions, so these two items are the same from a computational point of view. The third item is useful for getting physical insights into system behaviors in the transient period. For a given disturbance, the time responses of all the variables included in the system representation are calculated, so simulations are useful in studying this item. Simulations are also able to answer the first two items, but it is somewhat indirect because the time responses of system variables must be observed for this purpose.

While the transient stability analyses have several uses that can differ with stages of power system planning and operation, its determination is almost exclusively made by simulations. Simulations are able to include very wide range of system components into the system representation, and to investigate their influence on the stability. However, the use of simulations should be limited because of its indirect nature in determining the stability of a system.

The detail and size of the system representation both directly affect the computational cost of simulations. The continued growth of interconnections as well as their increased used for bulk power transfer has increased the necessary system representation size to the point that simulation cost is prohibitory in many applications. Because of the large inertias involved and the long ties, swings must be carried out for several seconds or more. This indeed amounts to a lot of expensive computer time. One example of the effort which this situation has stimulated is the coherence based dynamic equivalents approach [4-7]. This approach significantly increases simulation efficiency by reducing the system representation size.

In addition to computation cost, other application-dependent disadvantages of simulation result from the fact that it is an indirect approach to transient stability analyses. As an example of this, consider the need in some planning studies for a relative figure of merit or stability measure. Due to the nature of simulation analyses, this need has, almost by necessity, been met with the critical clearing time. In order to calculate the critical clearing time, simulation must be made for many initial conditions, so it can be a very time consuming and expensive task.

Throughout the electric utility industries in the world today, traditional dispatcher's offices are giving way to modern system control centers [8-10]. This change from the old to the new is not merely one of modernization of dispatching and supervisory equipment, although there are indeed many new centers which provide little more than what used to be done with old-style equipment. What is significant is the change from a limited concept of generation dispatching or supervisory control to a more comprehensive and integrated approach to monitoring and controlling a power system. This broader concept is referred as the security control concept [8]. For this purpose, the transient analysis must be made in on-line mode. The approach envisioned is to periodically scan the required steady-state variables of the power system from which the transiently stable behavior of the system when subjected to specified disturbances could be predicted or calculated. This digital system is continually updated by actual readings from the real power system. For on-line stability analysis, possibly 50 to 100 transient stability studies of the existing system are required to be run periodically, possibly every five or ten minutes during peak load periods [11]. This requirement is by far beyond the faculty of simulations, and some direct and more efficient methods must be developed.

In the short-term transient stability analyses, simulation is not necessarily useful method of judging whether a system is stable or not, and of calculating stability power limits. In the next section, we introduce a direct method of analyzing the transient stability which is referred as Lyapunov's direct method.

§3. Lyapunov's direct method

Lyapunov's direct method is based on a theorem which was proved by A.A. Lyapunov [12]. In this section, we begin with its description.

The nonlinear systems considered here are those whose dynamics are described by

$$\frac{dx}{dt} = f(x) \tag{1.1}$$

where x is an n-dimensional state variable vector, and f(x) is an n-dimensional vector consisting of continuous functions, such that

$$f(x) = 0 \qquad \text{for } x = 0 \qquad (1.2)$$

Namely, the origin is the equilibrium point of (1.1). The stablity criterion for those systems is given as follows:

[Theorem 1]

If there exists a real scalar function $V(x)$ which is continuous together with its partial derivatives, such

1) $V(x)$ is positive definite in a closed region Ω,
2) One of surfaces yielded by $V(x) = C$ (constant) bounds Ω,
3) The time derivative of V, denoted by dV/dt, is negative definite or negative semidefinite in Ω, and not identically equal to zero on a solution of the system other than at the origin,

then, the system described by (1.1) is asymptotically stable in Ω.

In this theorem, conditions 1) and 2) imply that, for $V < C$, V is equal to a constant which defines a closed hypersurfaces surrounding the origin, which condition 3) implies that all trajectories originating within Ω approach the origin as time goes to infinity. The function $V(x)$ which satisfies all the conditions 1-3) is called Lyapunov function. This function plays an important role in applications of Lyapunov's direct methods to transient stability analyses.

The transient stability analysis is concerned with a system's ability to remain in synchronism following major disturbances such as sudden changes in load or generation powers, or in transmission system configurations due to faults and line switchings. A fault occurs somewhere in a system, which disturbs its operating conditions. This triggers a sequence of events; the fault is cleared and the system is restored to a healthy condition. The state of the system after fault clearing is in general not the desired equilibrium state of the post-fault system. The question is whether or not the system will converge to this equlibrium state. This is a typical example where asymptotic stability in the sense of Lyapunov is prime importance. It must be noted that the stability problem [13] as en-

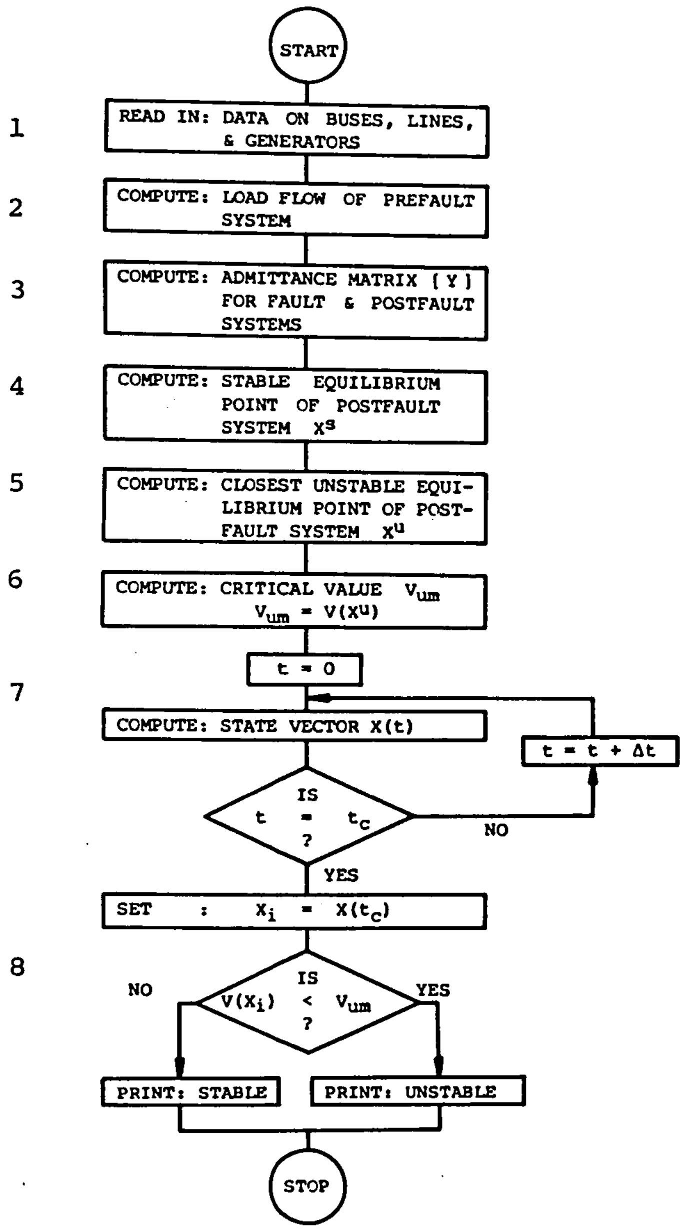


Fig. 1 Flow chart for judging transient stability by Lyapunov's direct method.

countered in power systems is not a problem of global stability, but a problem of estimating the domain of attraction of an equilibrium state of the system. The power systems are never asymptotically stable in the large. The aim of transient stability studies is to compute regions of asymptotic stability of equilibrium solutions.

The general approach used when Lyapunov's direct methods are applied is as follows: The transient stability region of the post-fault power system or an estimate of it is computed by means of a direct method. Then the system is simulated on a computer, and the post-fault initial state is obtained by integrating the system differential equations during fault conditions. It is checked whether or not the state lies within the region of stability region. The procedure for establishing the region of stability and judging whether the system is stable or not for a given disturbance is shown in the flow chart of Fig.1. The following are the main steps [14]:

Step 1: Read the necessary data on the system, i.e., those on the transmission lines, the buses, and the generators.

Step 2: Compute the load flow for the prefault state.

Step 3: Compute the reduced admittance matrices between the generators by eliminating the buses without generator for the fault and postfault states.

Step 4: Compute the stable equilibrium point for the postfault state.

Step 5: Compute the unstable equilibrium point closest to the stable equilibrium point found in step 4.

Step 6: Compute the critical value V_{um}, which is equal to the value of V at the closest unstable equilibrium point found in step 5.

Step 7: Integrate step by step the fault system equations, and compute the initial state x_i of the postfault state.

Step 8: Compare $V(x_i)$ with V_{um}. If $V(x_i)$ is smaller than V_{um}, then the system is classified as stable.

This is one of typical applications of Lyapunov's direct method to transient stability analyses. It is easy to see its computational superiority to conventional methods based on simulations. In this application, it is only necessary to calculate an initial system state when the system reaches

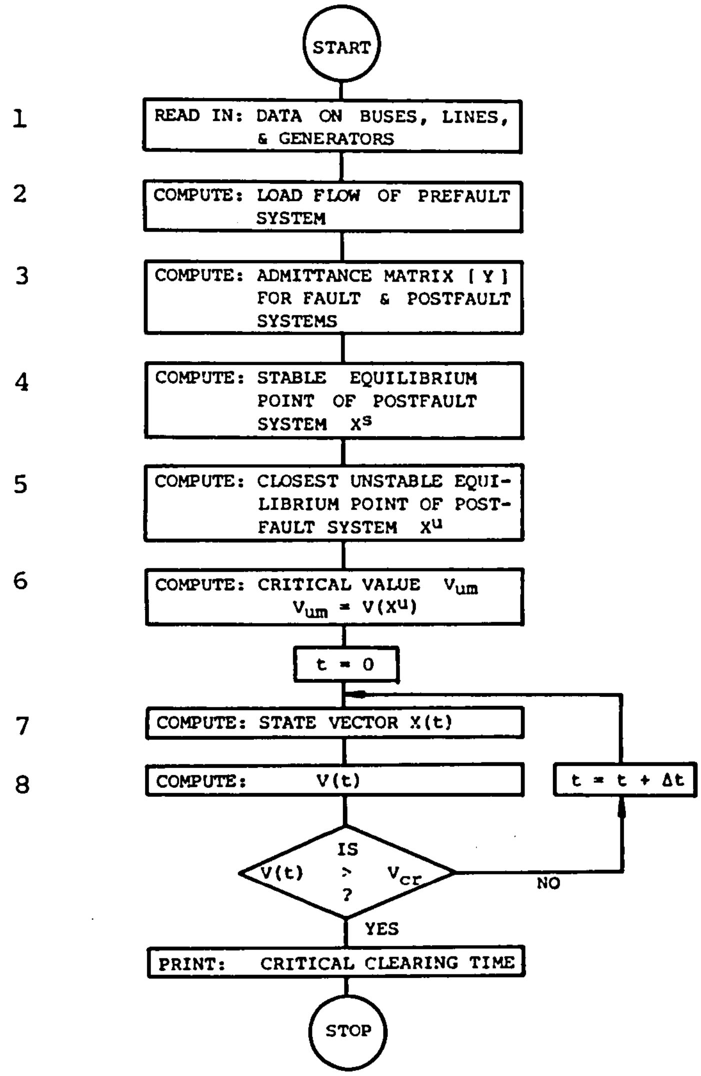


Fig.2 Flow chart for determining critical fault clearing time by Lyapunov's direct method.

the postfault state. This leads to much savings of computational time and costs compared with the indirect approaches based on simulations.

One of important characteristics concerning the transient stability of a power system is its ciritical clearing time. If a fault occurs in the system a transient phenomenon is started; after a fault clearance, it will converge to its stable equilibrium is the state at the instant of fault clearing is within the stability region of the postfault system. The critical clearing time is defined as the maximum allowed fault duration for transient stability. If this figure is to be calculated by simulations, the system equations must be integrated for many clearing times. This is a time consuming and expensive task. One important application of Lyapunov's direct method can be found in the calculation of this stability measure. Lyapunov's direct method can easily produce this figure with little additional expense of computation time to that for judging system stability. The procedures for this calculation is shown in the flow chart of Fig.2. Step 1 ∿ 6 are the same as those in Fig. 1.

Step 7: Compute state variables x(t) by integrating the fault system equations with initial state variable $x(t-\Delta t)$, where Δt is the integration step length.

Step 8: Compute V(t) with x(t) found in step 7, and compare it with the critical value V_{um}. If V(t) is greater than V_{um}, then the system has reached the boundary of the stability region.

In this application, the critical clearing time can be calculated by integrating the fault system equations only once. Computational time and costs are both saved very much in comparison with the method based on simulations.

It should be emphasized, however, that Lyapunov's direct method does not exclude simulations. The theorem of Lyapunov is concerned with autonomous systems, so Lyapunov's direct method is applicable to the postfault system for power system stability analyses. It needs to compute the system variables at the instant of fault clearing in order to determine the value of Lyapunov function at that instant. In general, Lyapunov's direct method is combined with simulations in its applications. It only substitutes for simulations as far as the computation of the transient stability of the postfault system is concerned. This is the most important and by far the most time consuming part of a stability analysis by computer simulation, however.

§4. Background of the present work

4.1 Present work objectives

The overall objectives of the present work is to develope Lyapunov's direct method to the point where it is a useful tool for transient stability analyses of power systems. It offers the opportunity of assessing the first swing stability of power systems more directly and effectively than the conventional approach based on simulation. For example, it allows critical clearing times to be directly calculated from a single solution. More fundamentally and, in terms of potential applications, more significantly it also provides a quantitative measure of how stable or unstable a particular fault case may be.

A brief description on the present-day status of Lyapunov's direct method in transient stability analyses will serve to clarify the critical problems which should be solved in this work. To its present state of developement, Lyapunov's direct method can be summarized by saying that it has been of little practical value to date due to three troublesom problems as follows:

1) The usual Lyapunov's method yields sufficient but not necessary conditions for stability. These conditions are usually much too conservative to be useful, particularly, for systems with more than three or for generators.
2) The computational requirements have made studies of large power systems infeasible. The magnitude of this effort can be appreciated by noting that it needs the closest unstable equilibrium point which is determined through calculations of 2^{n-1} unstable equilibrium points, where n is the number of generators.
3) The method requires the system representation to be highly simplified, e.g., generators are represented by constant voltages behind transient reactances, and loads are represented by constant impedances.

The specific objectives of the present work have been to eliminate these limitations as much as possible. The problem 3) and a part of the problem

1) are concerned with Lyapunov functions. The problem 2) and the most part of the problem 1) are concerned with the critical value of Lyapunov function which corresponds with boundary of the transient stability region of the system. Studies on Lyapunov functions and their critical values are the key in solving these problems. Concerning to Lyapunov functions, systematic methods of constructing them for various level of system representaions have been developed although most of them are adressed to one machine connected to an infinite bus systems [15-30]. Research along this direction will lead to more detail system representation. As for the critical value, more physical considerations on system behaviors in the transient period should be made although theoretical investigations are, of course, indispensable. The usual critical value has been determined by straightly applying Lyapunov's theorem to power systems without paying no attention to their features. The third problem may not be always critical one because much useful information can be obtained from analyses with this simplified system representation if Lyapunov's direct method is considered as complementing rather than replacing simulation which will always be able to handle more detailed system representations.

4.2 Previous works on Lyapunov's direct method

Since the present work deals with investigations on Lyapunov's direct method applied to transient stability analyses of multimachine power systems, a review of the previous works on this subject will well describe a background of this work.

The application of Lyapunov's direct method to the transient stability analysis has been attracting many researchers for a long time since the first work was published in 1948 [31]. Reviews on plenty of works have been made by Willems [13] and Pavella [32], and a list of them, not complete, is also provided in the last part of this thesis. Those works are roughly classified into two groups: the first is theoretical with one-machine connected to an infinite bus systems, and the other is practical works with multimachine power systems. These two groups are complementing each other.

Among them, the works which played a fuse to the present work are as follows; works by Willems [33,34], Pai & Murthy [35], Henner [36], and Gudaru [37] on a systematic construction method of Lyapunov functions based a generalized Popov criterion; works by Luders [38], Willems [25], Prabhakara & El-Abiad [39], and Gupta & El-Abiad [40] on methods of determining the critical value of Lyapunov function; works by Siddiqee [23], De Sarkar & Daharma Rao [27], Pai & Rai [28], and Willems [41] on applications of Lyapunov's direct method to transient stability analyses of power systems in which dynamics of field flux linkages and, moreover, voltage regulators are incorporated to their representations. Of course, all the other references given in the last part of the present thesis gave more or less influence to us. Especially several books regarding with a general mathematical treatment of the linear and nonlinear theories on the stability [42-46], have been providing us necessary knowledge through all stages of the present work. The paper by El-Abiad & Nagappan gave us an outline for applying Lyapunov's method to transient stability analyses, and the suggestions for the future works by Willems [13] helped us determine the direction of our research on this subject.

The works contained in the present thesis have been performed during a period from 1976 to 1979 so that excellent recent works have been published partly during the period and partly more recently. They are listed up here for references; Athay & Virmani [47] on transient energy direct method of practical significance, Bergen & Hill [48] on new representation of loads, Pai & Narayana [49] and Jocic, Pavella & Siljac [50] on a construction of Lyapunov function based on the concept of vector Lyapunov function. Those works deal with different subjects on Lyapunov's direct method, and will help Lyapunov's direct method be a useful tool in transient stability analyses.

4.3 Contribution of the present work

In chapter II, we are concerned with the transient stability analysis of multimachine power systems in which generators are represented by constant voltages behind transient reactances, and loads are by constant impedances. In usual power system designs and operations, transient stability analyses are performed with this system representation. Three important problems are investigated; firstly, how can we construct an appropriate Lyapunov function? Section 2 and 3 are adressed to this problem. In §2, a generalized Popov criterion is derived, and in §3, a Lur'e type Lyapunov function is systematically constructed on the basis of the criterion.

Secondly, how can we determine a suitable critical value of the Lyapunov function? It is desirable that it can yield accurate results in stability studies, and that is computed in adequately short time. In §4, a new critical value is introduced from physical considerations of power system behaviors in the transient period, and a simple method of computing the critical value is developed. Thirdly, how can we take influence of transfer conductances into account? This problem is rather subtle compared with the foregoing two problems, but it is very important in practical analyses. §5 is adressed to this problem, and one method of counting their influence is developed. Lastly, Lyapunov's direct method is applied to a 10-machine power system, and its effectiveness is verified in §6.

In chapter III, we are concerned with the transient stability analysis of multimachine power systems in which dynamics of field flux linkages are incorporated. The field flux linkages generally decrease in the transient period owing to the armature reaction, and deteriorate the transient stability. In §2, a Lur'e type Lyapunov function is systematically constructed on the basis of the generalized Popov criterion. In §3, one method of determining its critical value is developed, and it is also shown that there is a possibility for a new instability to occur. In §4, the method of taking account of transfer conductances is generalized to the system under investigation. Lastly, a 10-machine power system is analyzed by Lyapunov's direct method, and its effectiveness is verified in §5.

In chapter IV, we are concerned with the transient stability analysis of multimachine power systems in which generators are installed with automatic voltage regulators and thyristor exciters. Thyrister exciters have very fast response, and are widely used to improve the transient stability. In §2, a Lyapunov function is systematically constructed based on the generalized Popov criterion. In §3, it is shown that automatic voltage regulator gains must be somewhat low compared with actual values in order that the Lyapunov function could be constructed according to the criterion. In §4, Lyapunov's direct method is generalized to be applicable to power systems with high gain voltage regulators by introducing a pseudo-Lyapunov function. In §5, Lyapunov's direct method is applied to a 10-machine power system, and its effectiveness is verified.

In chapter V, some summary and conclusions of the present thesis are described.

Chapter II

TRANSIENT STABILITY ANALYSIS OF MULTIMACHINE POWER SYSTEM VIA LYAPUNOV'S DIRECT METHOD: CONVENTIONAL SYSTEM MODEL

§1. Introduction

In this chapter, we are concerned with the transient stability analysis of multimachine power systems in which generators are represented by constant voltages behind transient reactances.

This system representation has been used in the transient stability analysis of multimachine power systems for long time. With the advent of high-powered electronic computers, more detail system representations have come to be used recently, but their uses are still limited because of computational time and cost. In practical system planning and its operation, many cases must be studied in short time and with adequate accuracy, so this system representation is mainly used in usual. In view of the uses of Lyapunov's direct method in system planning and operations, the transient stability analysis with this system representation carries weight in the research on this method.

Lyapunov's direct method consists of two main parts, that is, the construction of the Lyapunov function and the determination of its critical value.

Firstly, the former part is investigated. The systematic method of constructing Lyapunov functions have been searched for long time. Lyapunov functions had been constructed on physical considerations untill J.L. Willems [33] proposed a systematic method based on a generalized Popov criterion derived by J.B. Moore and B.D.O. Anderson [51] in 1970. However, the application of this method is limited to this system representation because of the criterion used as the basis. In §2, this criterion is generalized in order that it would be applicable to more general system models.

In §3, a Lur'e type Lyapunov function is constructed on the basis of this new generalized Popov criterion, and its variation with parameters contained in it is investigated in order to search for the proper one to the transient stability analysis.

Secondly, the latter part is investigated. The critical value of the Lyapunov function has significant influence on the accuracy of the estimated result of the critical fault clearing time. In usual analyses, the critical value has been given by the minimum value among the values of potential energy at all unstable equilibrium points. It is well-known, however, that this critical value often yields very conservative results, especially, in large power systems, and that it inherently needs calculations of extremely many unstable equilibrium points. These two fact have been obstructing the practical application of Lyapunov's direct method to the transient stability analysis. In §3, the cause of this conservative nature is clarified through physical considerations on the power system behaviors in the transient period. From the consideration, a new critical value is introduced, and a simple method of computing the critical value is developed.

Thirdly, we make some investigations on the influence of the transfer conductances contained in the reduced admittance matrices which are obtained after the elimination of the nodes without generators. The Lyapunov function which can completely incorporate the transfer conductances has not been constructed yet in spite of many researcher's efforts. The transfer conductances increases with system loads, and accordingly, they are not negligible in practical power systems. In §5, their influence on the Lyapunov function is investigated, and some methods of counting in their influence are developed.

Lastly, in §6, the transient stability analysis of a 10-machine power system is made by Lyapunov's direct method incorporated with the methods developed in this chapter, and their effectiveness is verified by comparing the results by the direct method with those by the conventional method based on simulations.

§2. Generalized Popov criterions

In this section, two generalized Popov criterion are introduced. The

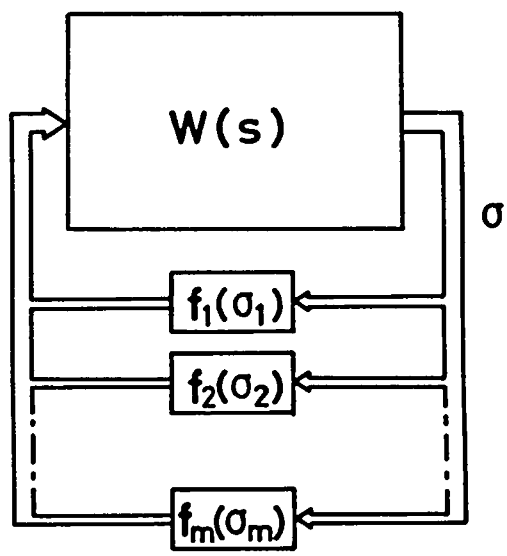


Fig.3. Nonlinear system model.

method used in this thesis in order to construct Lyapunov functions is based on these criterions. One of them was derived by J.B. Moore and B.D.O. Anderson, in 1968, [51], and was first applied by J.L. Willems in order to construct a Lyapunov function for a multimachine power system represented by the conventional model in 1970 [33]. The other was derived in a form different from that introduced here by C.A. Desoer and M.Y. Wu in 1969, [52]. and was first applied by M.A. Pai and V. Rai in order to construct a Lyapunov function for a one-machine connected to an infinite bus system in which a voltage regulator is installed to the generator. in 1974 [28].

2.1 Generalized Popov criterion by Moore & Anderson

The nonlinear systems considered here are those whose form is shown in Fig. 3. The Lyapunov stability is considered, and so the inputs are not indicated. The matrix W(s) is an $m \times m$ matrix of stable rational transfer functions, assumed to be such that

$$W(\infty) = 0 \qquad (2.1)$$

The nonlinearity $F(\sigma)$ is an m dimensional vector, assumed to satisfy the conditions as follows:

$$0 \le f_i(\sigma_i)\sigma_i \le k_i\sigma_i^2. \qquad (2.2)$$

and

$$f_i(\sigma_i) = 0 \qquad \text{for } \sigma_i = 0 \qquad (2.3)$$

where k_i is a scalor satisfying

$$k_i \ge 0 \qquad (2.4)$$

The k_i define the matrix $K = \mathrm{diag}(k_1, k_2, \ldots, k_m)$, and the output σ_i define the output vector $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_m)'$, where the superscript " ' " denotes the transposition of the matrix or the vector. The stability criterion for the above system is given as follows:

[Theorem 2]

If there exist real diagonal matrices N and Q such that

$$Z(s) = NK^{-1} + (N + Qs)\, W(s) \tag{2.5}$$

is positive real, then the system shown in Fig.2 is stable, where $N = \mathrm{diag}(n_1, n_2, \ldots, n_m)$ and $Q = \mathrm{diag}(q_1, q_2, \ldots, q_m)$, with $n_i \geq 0$, $q_i \geq 0$ and $n_i + q_i > 0$, and $-n_i/q_i$ is not a pole of any of the ith row elements of W(s).

Before proving this theorem, it is necessary to introduce a lemma of B.D. O. Anderson [53].

[Lemma 1]

Let Z(s) be a matrix of rational transfer functions such that Z(s) is finite and Z(s) has poles which lie in Re s < 0, or are simple on Re s = 0. Let (A, B, C) be a minimal realization of $Z(s) - Z(\infty)$. The Z(s) is positive real if and only if there exist a symmetric positive definite matrix P and matrices W_o and L such that

$$\begin{aligned} PA + A'P &= -LL' \\ PB &= C - LW_o \\ W_o'W_o &= Z(\infty) + Z'(\infty) \end{aligned} \tag{2.6}$$

With the aid of this lemma, the theorem is proved.

[Proof of theorem]

A transfer function W(s) satisfying (2.1) possesses a minimal realization (A, B, C) which is a set of three constant matrices satisfying

$$W(s) = C'(sI - A)^{-1}B \tag{2.7}$$

An expansion of Z(s) in terms of A, B, and C gives

$$\begin{aligned} Z(s) &= NK^{-1} + (N + Qs)W(s) \\ &= NK^{-1} + NC'(sI - A)^{-1}B + QC'[(sI - A) + A](sI - A)^{-1}B \end{aligned}$$

$$= (NK^{-1} + QC'B) + (NC' + QC'A)(sI - A)^{-1}B$$

Since Z(s) is positive real, lemma 1 can be applied to the triple (A, B, CN' + A'CQ'), which is a minimal realization of Z(s) - Z(∞). Thus, there exist a positive definite symmetric matrix P, and matrices L and W_o such that

$$\begin{aligned} PA + A'P &= - LL' \\ PB &= CN' + A'CQ' - LW_o \\ W_o'W_o &= 2NK^{-1} + QC'B + B'CQ' \end{aligned} \tag{2.8}$$

Consider as a tentative Lyapunov function for the system of Fig.2:

$$V(x) = x'Px + 2\int_0^{\sigma} F'(\rho)\,Q\,d\rho \tag{2.9}$$

where x is the state vector of the system. The positive definiteness of P, the positive semidefiniteness of Q, and the restriction on F(σ) of (2.2) ensure that V is positive for all nonzero x. Differentiating (2.9) gives

$$\mathring{V}(x) = \mathring{x}'Px + x'P\mathring{x} + 2F'(\sigma)\mathring{\sigma} \tag{2.10}$$

The time derivatives may be expressed in terms of A, B, C, and F(σ) by noting that the system of Fig.2 may be represented as a linear system using state space notation, i.e., from (2.7)

$$\begin{aligned} \dot{x} &= Ax - BF(\sigma) \\ \sigma &= C'x \end{aligned} \tag{2.11}$$

Thus,

$$\begin{aligned} \mathring{V}(x) &= [x'A' - F'(\sigma)B']Px + x'P[Ax - BF(\sigma)] \\ &\quad + 2F'(\sigma)QC'[Ax - BF(\sigma)] \\ &= x'(PA + A'P)x - 2x'(PB - A'CQ')F(\sigma) \\ &\quad - F(\sigma)'(QC'B + B'CQ')F(\sigma) \\ &= - x'LL'x + 2x'LW_oF(\sigma) - 2x'CN'F(\sigma) \end{aligned}$$

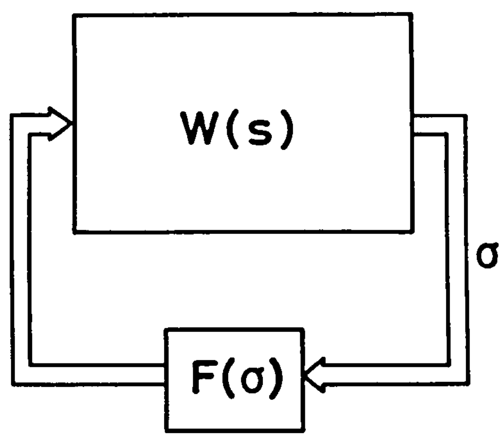


Fig.4. Nonlinear system model.

$$- F(\sigma)'(W_o'W_o - 2NK^{-1})F(\sigma)$$

$$= - [x'L - F'(\sigma)W_o'][L'x - W_oF(\sigma)]$$

$$- 2F'(\sigma)N[\sigma - K^{-1}F(\sigma)] \qquad (2.12)$$

where (2.8) is applied in deriving (2.12). The first term is plainly non-positive, and the nonnegative nature of N together with the nonlinearity condition (2.2) ensure the nonpositivity of the second term. Accordingly, $\dot{V}(x)$ proves to be positive. Since there exists a Lyapunov function for the system in Fig.2, this system is stable according to Theorem 1. This completes the proof.

2.2 Generalized Popov criterion by Desoer and Wu [72]

The nonlinear systems considered here are those whose form is shown in Fig.4. The Lyapunov stability is considered, and so the inputs are not indicated. The matrix W(s) is an m × m matrix of stable rational transfer functions, assumed to be such that

$$W(\infty) = 0 \qquad (2.13)$$

The nonlinearity $F(\sigma)$ is an m dimensional vector, assumed to satisfy the conditions as follows [52]:

1) $F(\sigma)$ is continuous and maps R^m into R^m.
2) For some constant real matrix N,

$$F(\sigma)'N\sigma \geq 0 \qquad \text{for all } \sigma \text{ in } R^m \qquad (2.14)$$

and

$$F(\sigma) = 0 \qquad \text{if } \sigma = 0 \qquad (2.15)$$

3) There is a function $V_1 \in C^1$ mapping R^m into R such that

$$V_1(\sigma) \geq 0 \qquad \text{for all } \sigma \text{ in } R^m \qquad (2.15)$$

and

$$V_1(\sigma) = 0 \qquad \text{if } \sigma = 0 \qquad (2.16)$$

and for some constant real matrix Q

$$\nabla V_1(\sigma) = Q'F(\sigma) \qquad \text{for all } \sigma \text{ in } R^m \tag{2.17}$$

holds. The stability criterion for the above system is given as follows [72]:

[Theorem 3]

If there exist real matrices N and Q such that

$$Z(s) = (N + Qs)\, W(s) \tag{2.18}$$

is positive real, then the system in Fig.3 is stable, where $(N + Qs)$ does not cause pole-zero cancellations with $W(s)$.

[Proof of theorem]

The transfer matrix $W(s)$ possesses a minimal realization (A, B, C), which is a set of three constant matrices satisfying

$$W(s) = C'(sI - A)^{-1}B \tag{2.19}$$

An expansion of $Z(s)$ in terms of A, B, and C gives

$$\begin{aligned} Z(s) &= (N + Qs)W(s) \\ &= NC'(sI - A)^{-1}B + QC'[(sI - A) + A](sI - A)^{-1}B \\ &= QC'B + (NC' + QC'A)(sI - A)^{-1}B \end{aligned}$$

Since $Z(s)$ is positive real, Lemma 1 can be applied to the triple $(A, B, CN' + A'CQ')$, which is a minimal realization of $Z(s) - Z(\infty)$. Thus, there exist a positive definite symmetric matrix P, and matrices L and W_o such that

$$\begin{aligned} PA + A'P &= -LL' \\ PB &= CN' + A'CQ' - LW_o \\ W_o'W_o &= QC'B + B'CQ' \end{aligned} \tag{2.20}$$

Consider as a tentative Lyapunov function for the system of Fig.3:

$$V(x) = x'Px + 2V_1(\sigma) \tag{2.21}$$

where x is the state vector of the system. The positive definiteness of P and the positive semi-definiteness of $V_1(\sigma)$ ensure that $V(x)$ is positive for every nonzero x. Differentiating $V(x)$ gives

$$\begin{aligned}\dot{V}(x) &= \dot{x}'Px + x'P\dot{x} + 2\nabla V_1'(\sigma)\dot{\sigma}\\ &= [x'A' - F(\sigma)'B']Px + x'P[Ax - BF(\sigma)]\\ &\quad + 2F(\sigma)'QC'[Ax - BF(\sigma)]\\ &= x'(PA + A'P)x - 2x'(PB - A'CQ')F(\sigma)\\ &\quad - F(\sigma)'(QC'B + B'CQ')F(\sigma)\\ &= - x'LL'x + 2x'LW_oF(\sigma) - 2x'CN'F(\sigma) - F(\sigma)'W_o'W_oF(\sigma)\\ &= - [x'L - F(\sigma)'W_o'][L'x - W_oF(\sigma)] - 2F(\sigma)'N\sigma \end{aligned} \tag{2.22}$$

where (2.11) and (2.20) are applied in deriving (2.22). The first term is plainly non-positive, and the non-negative nature of $F(\sigma)'N\sigma$ ensures the non-positivity of the second term. Accordingly, $\dot{V}(x)$ proves to be non-positive. Since there exists a Lyapunov function for the system in Fig.3, this system is stable according to Theorem 1. This completes the proof.

2.3 Use of criterions

These generalized Popov criterions can be used as a basis of constructing Lyapunov functions for multimachine power systems. Some power systems can be formulated as such multivariable nonlinear feedback dynamical systems as in Fig.3 or 4. If a power system proves to be stable according to Theorem 2 or 3, then there exists a Lur'e type Lyapunov function given by (2.9) or (2.21). The function is obtained by solving the matrix equations in (2.8) or (2.20). Thus we can systematically construct Lyapunov functions for multimachine power systems. In the followings, we construct some Lyapunov functions for a multimachine power system which is represented in different levels of detail. Theorem 3 is used instead of Theorem 2 as its base because the former is superior to the latter in applicability. The obtained functions play important roles in Lyapunov's direct method to determine the domain of attraction of the system.

§3. Construction of Lyapunov function

In Lyapunov's direct method, it occupies a very important part to construct Lyapunov functions which are suitable to systems under studies, and many investigations have been made on this subject. Lyapunov functions were derived through physical considerations at the initial stage of developement of this direct method, partially because Theorem 1 derived by A.A. Lyapunov does not offer any information on the method of constructing Lyapunov functions, and partially because power system engineers have been using some direct methods in transient stability analyses before Lyapunov's direct method became popular among them. Early direct methods are based on the energy concept. The equal-area criterion [42], and the energy integral criterion [54] were derived on this basis. Lyapunov's direct method was first applied to transient stability analyses of multimachine power systems by A.H. El-Abiad & K. Nagappan in 1966. Following them, several Lyapunov functions were derived, but those were not different so much from the energy integral function constructed by P.D. Aylett in essence although some improvement were made. In 1971, a systematic construction method was proposed by J.L. Willems [33]. This method is based on a generalized Popov criterion derived by J.B. Moore & B.D.O. Anderson, that is, Theorem 2 in §2. It is superior to other method in a sense that it can systematically construct Lyapunov functions for multimachine power systems, and as a result, it may be applicable to more detail power system models in which physical considerations will be no help in deriving their Lyapunov functions.

In this section, another systematic construction method based on Theorem 3 in §2 is introduced. This theorem is more general than that used by J.L. Willems in a sense, so this new construction method has wider applications than that proposed by J.L. Willems. This new method is shown in a process of constructing a Lyapunov function for a power system represented by the conventional model, that is, in which each generator is modeled as a constant voltage behind a transient reactance. Some detail investigation is made on the obtained Lyapunov function by varying the parameters contained in it for the purpose of getting an appropriate Lyapunov function for transient stability analyses.

3.1 System equation

In usual transient stability analyses of power systems, some basic assumptions are made in modeling multimachine power systems as follows:

1) Each synchronous machine is represented by a constant voltage behind its transient reactance. In other words, its field flux linkages are assumed to be constant during the transient period, and regulation of its terminal voltage is not taken into consideration.
2) Mechanical power input to each generator is constant, and no governor action is taken into account.
3) Damping power of each generator is proportional to its slip velocity, and it is assumed to be mainly due to its asynchronous torque.
4) Inertia coefficient of each generator is constant, and its variation owing to deviation of rotor speed from the synchronous speed is assumed to be negligible.
5) Each synchronous machine is a round-rotor machine.
6) Each load is represented by a constant impedance.

Under these assumptions, the mothion of the ith generator is described as follows:

$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = P_{mi} - \sum_{j=1}^{n} E_i E_j Y_{ij} \sin(\delta_{ij} + \theta_{ij}) \qquad (2.26)$$

for i=1,2, ...,n,

where, for generator i,

P_{mi} : mechanical power input,
m_i : angular momentum constant,
d_i : damping power coefficient,
$Y_{ij}\angle\phi_{ij}$: post-fault transfer admittance between the ith and the jth generator nodes (obtained after reduction of a network retaining only generator nodes),
θ_{ij} : complement of ϕ_{ij}, i.e., $\theta_{ij} = \pi/2 - \phi_{ij}$,
$E_i\angle\delta_i$: internal voltage,
δ_{ij} : $\delta_i - \delta_j$.

In order to construct a Lyapunov function, it is necessary to assume that the transfer conductances in the reduced admittance matrix are negligible.

Under this assumption, (2.26) changes as follows:

$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij}) \qquad \text{for } i = 1, 2, \ldots, n, \tag{2.27}$$

where

$$B_{ij} = Y_{ij}\cos\theta_{ij}$$

and the superscript " o " denotes the stable equilibrium point of the post-fault system, so (2.27) applies to the post-fault state.

Eq.(2.27) in the state space notation is given as follows:

$$\begin{aligned} \dot{x} &= Ax - BF(\sigma) \\ \sigma &= C'x \end{aligned} \tag{2.28}$$

where

$$A = \begin{bmatrix} 0_{nn} & I_{nn} \\ 0_{nn} & -M^{-1}D_{nn} \end{bmatrix} \qquad B = \begin{bmatrix} 0_{nm} \\ M^{-1}T_{nm} \end{bmatrix} \qquad C = \begin{bmatrix} T_{nm} \\ 0_{nm} \end{bmatrix} \tag{2.29}$$

and in which

$$\begin{aligned} M_{nn} &= \text{diag}(m_1, m_2, \ldots, m_n) \\ D_{nn} &= \text{diag}(d_1, d_2, \ldots, d_n) \\ T_{nm} &= \begin{bmatrix} 1_{1(n-1)} & 0_{1(m-n+1)} \\ -I_{(n-1)(n-1)} & T_{(n-1)(m-n+1)} \end{bmatrix}, \quad T_{21} = [1, -1]' \end{aligned} \tag{2.30}$$

The matrices 1 and 0 in (2.29) and (2.30) have all their elements equal to unity and zero, respectively. The number m is defined by

$$m = n(n-1)/2 \tag{2.31}$$

The state vector x is a 2n dimensional vector consisting of two vectors as follows:

$$x = [\ \delta^{*\prime} \quad \omega'\ ]' \tag{2.32}$$

where δ^* and ω are n dimensional vectors defined by

$$\delta_i^* = \delta_i - \delta_i^o$$

$$\omega_i = \dot{\delta}_i \qquad \text{for } i=1,2,\ldots,n. \qquad (2.33)$$

The nonlinearity $F(\sigma)$ is an m dimensional vector defined as follows:

$$f_k(\sigma_k) = B_{ij}E_iE_j[\sin(\sigma_k + \delta_{ij}^o) - \sin\delta_{ij}^o] \qquad (2.34)$$

$$\text{for } i=1,2,\ldots,n-1, \quad j=i+1,\ldots,n,$$

$$k=1,2,\ldots,m$$

where k is related with i and j by

$$k = (i-1)n - i(i+1)/2 + j \qquad (2.35)$$

The output σ is an m dimensional vector defined as follows:

$$\sigma_k = \delta_{ij} - \delta_{ij}^o \qquad \text{for } k=1,2,\ldots,m, \qquad (2.36)$$

where k is related with i and j by (2.35). Eq.(2.28) describes the multi-machine power system as a multivariable dynamical system of the form as shown in Fig.4.

3.2 Stability check of system

The transfer matrix $W(s)$ for the linear part of the system is written as follows:

$$W(s) = C'(sI - A)^{-1}B$$

$$= T'[s(sI + M^{-1}D)]^{-1}M^{-1}T \qquad (2.37)$$

For the system to be stable, there must exist matrices N and Q such that $Z(s)$ defined by (2.18) is positive real. In this problem, N is chosen as follows:

$$N = (1/q)I_{mm} \qquad (2.38)$$

The inequality in (2.14) is equivalent to the following inequalities:

$$f_k(\sigma_k)\sigma_k \geq 0 \qquad \text{for all } \sigma_k \text{ in } R \text{ and } k=1,2,\ldots,m. \qquad (2.39)$$

However, the above inequalities are satisfied not for all σ_k in R, but for a range of σ_k as follows:

$$- \pi - 2\delta_{ij}^{o} \leq \sigma_k \leq \pi - 2\delta_{ij}^{o} \qquad (2.40)$$

As is observed from (2.22), $F(\sigma)'N\sigma$ has influence on the time derivative of $V(x)$. Since (2.14) is not satisfied for σ which does not satisfies (2.40), it is desirable to make its influence zero by letting $q \to \infty$. However, this selection of q causes a pole zero cancellation between $(N + Qs)$ and $W(s)$ because $W(s)$ has a pole at $s = 0$. In order to avoid the pole-zero cancellation, we give q a finite value in the process of construction.

The function $V_1(\sigma)$ in (2.15) is chosen as follows:

$$V_1(\sigma) = \sum_{k=1}^{m} \int_0^{\sigma} f_{1k}(\sigma_k)d\sigma_k$$
$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij}E_iE_j[\cos\delta_{ij}^{o} - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^{o})\sin\delta_{ij}^{o}] \qquad (2.41)$$

It is observed from (2.41) that $V_1(\sigma)$ is positive not for all σ in R^m, but for a range of σ about $\sigma = 0$. Accordingly, the global stability of the system can not be concluded with this function. However, it is possible to estimate the domain of attraction by using the Lyapunov function obtained with this function. The partial derivatives of $V_1(\sigma)$ are given as follows:

$$\frac{\partial V_1}{\partial \sigma_k} = B_{ij}E_iE_j[\sin(\sigma_k + \delta_{ij}^{o}) - \sin\delta_{ij}^{o}] \qquad (2.42)$$

for $k=1,2,\ldots,m$,

that is,

$$\nabla V_1(\sigma) = I_{mm}F(\sigma) \qquad (2.43)$$

Hence Q in (2.17) is given by

$$Q = I_{mm} \qquad (2.44)$$

By substituting (2.38) and (2.44) into (2.18), Z(s) is given as follows:

$$Z(s) = (1/q + s)T'[s(sI + M^{-1}D)]^{-1}M^{-1}T \qquad (2.45)$$

The conditions for $Z(s)$ to be positive real are

1) $Z(s)$ has all elements which are analytic for Re $s > 0$,
2) $Z^*(s) = Z(s^*)$ for Re $s > 0$,
3) $Z'(s^*) + Z(s)$ is positive semi-definite for Re $s > 0$.

Since the first two conditions clearly hold for $Z(s)$, so it is only necessary for $Z(s)$ to satisfy the condition 3). In this case, it is sufficient to show that $Z(j\omega) + Z'(-j\omega)$ is positive semi-definite for each scalor ω. After some manipulation, it is found out that

$$Z(j\omega) + Z'(-j\omega) = 2T'\mathrm{diag}\left(\frac{d_i - m_i/q}{m_i^2\omega^2 + d_i^2} \right)T \tag{2.46}$$

holds. If the following inequalities are satisfied;

$$q > m_i/d_i \qquad \text{for } i=1,2,\ldots,n \tag{2.47}$$

then the right hand term in (2.46) is positive semi-definite. Hence $Z(s)$ is positive real, the system is stable according to Theorem 3.

3.3 Minimal realization of $W(s)$

The transfer matrix $W(s)$ can be expanded as follows:

$$W(s) = \frac{1}{s} T'D^{-1}T + \sum_{i=1}^{n} \frac{1}{s + d_i/m_i} T'H_iT \tag{2.48}$$

where

$$H_i = \mathrm{diag}(\overset{1}{0},\ldots,0,\overset{i}{-d_i},0,\ldots,\overset{n}{0}) \tag{2.49}$$

if relative dampings of generators are non-uniform, that is, d_i/m_i takes different values for all generators. The degree of $W(s)$ denoted by $\delta[W(s)]$ is equal to the minimal order of realizations of $W(s)$, and it is given as follows:

$$\delta[W(s)] = \text{rank of } [T'D^{-1}T] + \sum_{i=1}^{n} \text{rank of } [T'H_iT] \tag{2.50}$$

Since

$$\text{rank of } [T'D^{-1}T] = n-1, \qquad \text{rank of } [T'H_iT] = 1$$

hold, the degree $\delta[W(s)]$ is given by

$$\delta[W(s)] = 2n - 1 \; . \tag{2.51}$$

Therefore the minimal order of realizations of $W(s)$ is $(2n-1)$. In this section, non-uniform damping cases are treated because other cases where relative dampings are not completely non-uniform can be regarded as particular cases of these cases.

The minimal realization of $W(s)$ is given as follows:

$$\begin{aligned} \dot{x} &= Ax - BF(\sigma) \\ \sigma &= C'x \end{aligned} \tag{2.52}$$

where

$$A = \begin{bmatrix} 0 & K'_{n(n-1)} \\ 0 & -M^{-1}D_{nn} \end{bmatrix} \qquad B = \begin{bmatrix} 0_{(n-1)m} \\ M^{-1}T_{nm} \end{bmatrix} \qquad C = \begin{bmatrix} G_{(n-1)m} \\ 0_{nm} \end{bmatrix} \tag{2.53}$$

and in which

$$K_{n(n-1)} = \begin{bmatrix} 1_{1(n-1)} \\ -I_{(n-1)(n-1)} \end{bmatrix} \tag{2.54}$$

$$G_{(n-1)m} = [\, I_{(n-1)(n-1)} \quad -T_{(n-1)(m-n+1)} \,]$$

There is a relation among matrices T, K, and G as follows:

$$T_{nm} = K_{n(n-1)}G_{(n-1)m} \tag{2.55}$$

The state vector x is a $(2n-1)$ dimensional vector consisting of two vectors as follows:

$$x = [\, \delta'_r \quad \omega' \,]' \tag{2.56}$$

where δ_r and ω are $(n-1)$ and n dimensional vectors defined by

$$\begin{aligned} \delta_{ri} &= \delta_{1(i+1)} - \delta^{o}_{1(i+1)} \qquad \text{for } i=1,2,\ldots,n-1, \\ \omega_i &= \dot{\delta}_i \qquad \text{for } i=1,2,\ldots,n. \end{aligned} \tag{2.57}$$

The nonlinearity $F(\sigma)$ and the output σ are the same as those defined by (2.33) and (2.36).

3.4 Solution of matrix equations

Since the system is stable, there exists a Lyapunov function as follows:

$$V(x) = x'Px + 2V_1(\sigma) \tag{2.58}$$

where P is a (2n-1) × (2n-1) symmetric positive definite matrix satisfying the following equations:

$$\begin{aligned} PA + A'P &= -LL' \\ PB &= CN' + A'CQ' - LW_o \\ W_o'W_o &= QC'B + B'CQ' \end{aligned} \tag{2.59}$$

in which L and W_o are (2n-1) × m and m × m matrices. From (2.53), C'B = 0 holds, so (2.59) reduces to

$$\begin{aligned} PA + A'P &= -LL' \\ PB &= CN' + A'CQ' \end{aligned} \tag{2.60}$$

P and L are partitioned as follows:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \qquad L = \begin{bmatrix} L_{11} \\ L_{12} \end{bmatrix} \tag{2.61}$$

where P_{11}, P_{12}, P_{21}, P_{22}, L_{11} and L_{12} are (n-1) × (n-1), (n-1) × n, n × (n-1), n × n, (n-1) × m, and m × m matrices, respectively. Substituting (2.53) and (2.61) into (2.60) gives

$$0 = -L_{11}L_{11}' \tag{2.62}$$

$$P_{11}K' - P_{12}M^{-1}D = 0 \tag{2.63}$$

$$P_{21}K' + KP_{12} - P_{22}M^{-1}D - DM^{-1}P_{22} = -L_{12}L_{12}' \tag{2.64}$$

$$P_{12}M^{-1}T = (1/q)G \tag{2.65}$$

$$P_{22}M^{-1}T = T \tag{2.66}$$

From (2.63) and (2.65),

$$(D^{-1}KP_{11}K'D^{-1} - D^{-1}/q)T = 0 \tag{2.67}$$

is obtained. Since $(D^{-1}KP_{11}K'D^{-1} - D^{-1}/q)$ is a symmetric matrix, its solution is given as follows (Appendix A):

$$D^{-1}KP_{11}K'D^{-1} - D^{-1}/q = \rho U \qquad (2.68)$$

where ρ is a scalar, and U is an $n \times n$ matrix with all elements equal to unity. Eq.(2.68) gives

$$KP_{11}K' = D/q + \rho DUD . \qquad (2.69)$$

From (2.63) and (2.69), we obtain

$$KP_{12} = M/q + \rho DUM \qquad (2.70)$$

Eq.(2.66) is transformed as follows:

$$(M^{-1}P_{22}M^{-1} - M^{-1})T = 0 \qquad (2.71)$$

Since $(M^{-1}P_{22}M^{-1} - M^{-1})$ is symmetric, its solution is given as follows:

$$M^{-1}P_{22}M^{-1} - M^{-1} = \mu U \qquad (2.72)$$

where μ is a scalar. Eq.(2.72) gives

$$P_{22} = M + \mu MUM \qquad (2.73)$$

Thus the matrix equations (2.59) was solved, and their solutions are summed up as follows:

$$\begin{aligned} KP_{11}K' &= (1/q)D + \rho DUD \\ KP_{12} &= (1/q)M + \rho DUM \\ P_{22} &= M + \mu MUM \end{aligned} \qquad (2.74)$$

The scalars ρ and μ must satisfy

$$\begin{aligned} \rho &\geq -(1/q)\sum_{i=1}^{n} d_i \\ \mu - \rho &\geq -1/\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} \end{aligned} \qquad (2.75)$$

for P to be positive definite matrix (Appendix A). Substituting (2.74) into (2.64) gives

$$2(D - M/q) + (\mu - \rho)(DUM + MUD) \geq 0 \tag{2.76}$$

This inequality is satisfied if

$$(\mu^*)^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(d_i m_j - d_j m_i)^2}{4(d_i - m_i/q)(d_j - m_j/q)} - \mu^* \sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} - 1 \leq 0 \tag{2.77}$$

is satisfied, where

$$\mu^* = \mu - \rho \tag{2.78}$$

Accordingly, μ^* must lie in the interval as follows:

$$\mu_1^* \leq \mu^* \leq \mu_2^* \tag{2.79}$$

where μ_1^* and μ_2^* are the two roots of the quadratic equation defined by (2.77), (Appendix A).

3.5 Lyapunov function

An expression for the Lyapunov function can be obtained by substituting (2.56) and (2.74) into (2.58) as follows:

$$\begin{aligned} V(x) &= [\delta_r' \ \omega'] \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} \delta_r \\ \omega \end{bmatrix} + 2V_1(\sigma) \\ &= \delta_r' P_{11} \delta_r + 2\delta_r' P_{12}\omega + \omega' P_{22}\omega + 2V_1(\sigma) \\ &= \delta'(D/q + \rho DUD)\delta + 2\delta'(M/q + \rho DUM)\omega \\ &\quad + \omega'(M + \mu MUM)\omega + 2V_1(\sigma) \end{aligned} \tag{2.80}$$

Substituting (2.41) into (2.80), and expanding and rearranging terms in (2.80) gives the following expression:

$$\begin{aligned} V(x) &= (1/2 \sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2 \\ &+ (\mu^* - \mu_o)(\sum_{i=1}^{n} m_i \omega_i)^2 \\ &+ (1/q) \sum_{i=1}^{n} (d_i \delta_i^* + 2m_i \omega_i)\delta_i^* \end{aligned}$$

$$+ \rho [\sum_{i=1}^{n} (d_i \delta_i^* + m_i \omega_i)]^2$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} E_i E_j [\cos\delta_{ij}^o - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^o) \sin\delta_{ij}^o]$$

(2.81)

where μ_o is a scalar defined by

$$\mu_o = - 1/ \sum_{i=1}^{n} m_i \tag{2.82}$$

From (2.22), the time derivative of V(x) is obtained as follows:

$$\begin{aligned}
\mathring{V}(x) &= - [x'L - F(\sigma)'W_o'][L'x - W_o F(\sigma)] \\
&\quad - 2F(\sigma)'N\sigma \\
&= - \omega' [2(D - M/q) + (\mu - \rho)(DUM + MUD)]\omega \\
&\quad - (2/q) \sum_{k=1}^{m} f_k(\sigma_k)\sigma_k \\
&= - 2 \sum_{i=1}^{n} (d_i - m_i/q)\omega_i^2 - \mu^* \sum_{i=1}^{n} d_i \omega_i \sum_{i=1}^{n} m_i \omega_i \\
&\quad - (1/q) \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} E_i E_j (\sin\delta_{ij} - \sin\delta_{ij}^o)(\delta_{ij} - \delta_{ij}^o)
\end{aligned}$$

(2.83)

Now, a general expression of V(x) has been derived for multimachine power system with constant field flux linkages and damping torques of generators. V(x) in (2.81) contains three parameters q, ρ, and μ* which are subject to (2.47), (2.75), and (2.76), respectively. We can get an infinite number of Lyapunov functions by varying these parameters. In the next section, we investigate on several particular Lyapunov functions which can be obtained by chosing the values of these parameters appropriately.

3.6 Selection of parameters q, ρ, μ*

Three parameters, q, ρ, and μ*, are contained in V(x) expressed by (2.81). These parameters have to satisfy certain conditions for V(x) to be a Lyapunov function. Firstly, if damping torques are all zero, then

$$q \to \infty$$

$$\rho > 0 \qquad (2.84)$$

$$\mu^* > \mu_0$$

must hold. Secondly, if damping torques are uniform, that is, d_i/m_i takes the same value for all generators, then

$$q > \lambda_0$$

$$\rho > \rho_0 \qquad (2.85)$$

$$\mu^* > \mu_0^*$$

must hold, where λ, ρ_0, and μ_0^* are defined as follows:

$$\lambda_0 \equiv m_i/d_i$$

$$\rho_0 \equiv -1/q \sum_{i=1}^{n} d_i \qquad (2.86)$$

$$\mu_0^* \equiv -1/\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q}$$

Lastly, if damping torques are non-uniform, then

$$q > \lambda_1$$

$$\rho > \rho_0 \qquad (2.87)$$

$$\mu_1^* < \mu^* < \mu_2^*$$

must hold, where λ_1 is defined as follows:

$$\lambda_1 \equiv \max_i (m_i/d_i) \qquad (2.88)$$

The parameters q, ρ, and μ^* can take any values satisfying (2.84), (2.85), or (2.87). Then, how does V(x) vary with these parameters?

For illustrative purpose, a 4-machine power system is considered. A line diagram of the system is shown in Fig.5, and its generator parameters are provided in Table 3. It is assumed that the system is disturbed by a 3-phase short-circuit which occurs at a point near no.2 generator, and is cleared by opening the line connecting no.1 and 2 generators at both terminals after a certain lapse of time. Three set of damping torques in Table 4 are adopted, which are referred as no damping, uniform damping, and non-uniform damping, respectively. The values ρ_0, μ_0^*, μ_1^*, and μ_2^* vary with the parameter q, as shown in Fig.6. In the case of uniform damping, μ_0^* and

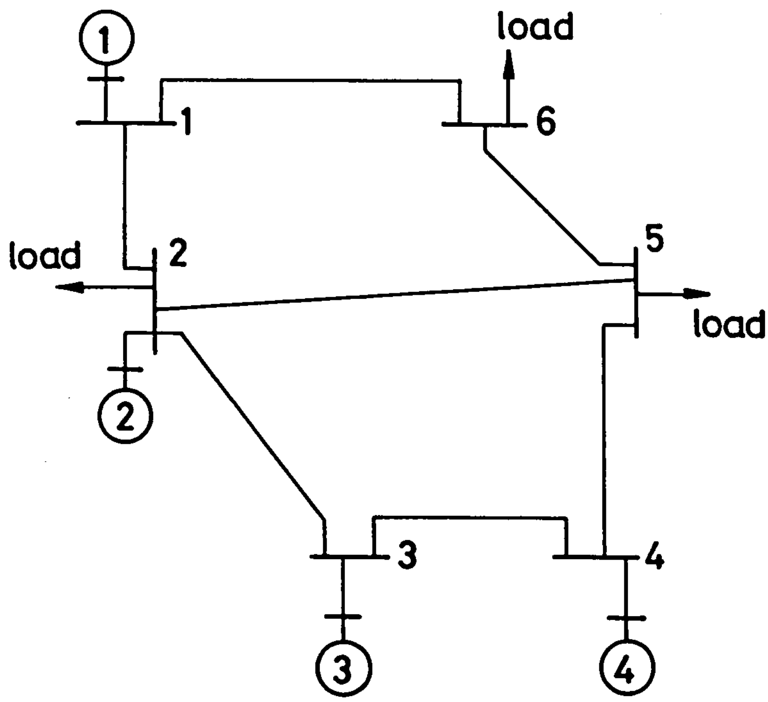


Fig.5 Configuration of 4-machine system

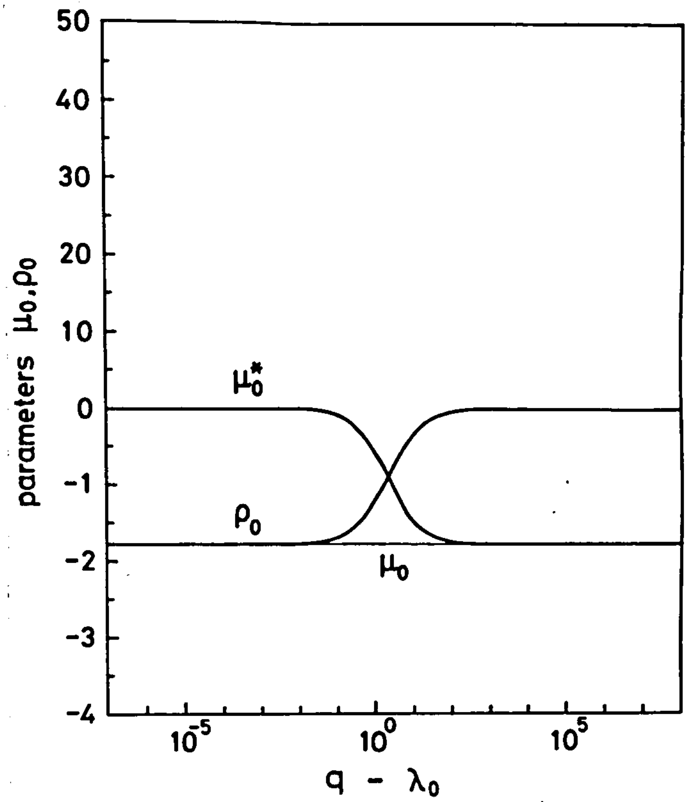
Table 3. Parameters of 4-machine system

No	H	x_d'
1	100.0	0.004
2	1.5	1.000
3	3.0	0.500
4	2.0	0.400

Table 4. Three cases of damping torques

No	no	uniform	non-uniform
1	0.0	100.0	20.0
2	0.0	1.5	1.5
3	0.0	3.0	4.0
4	0.0	2.0	1.0

(a) uniform damping



(b) non-uniform damping

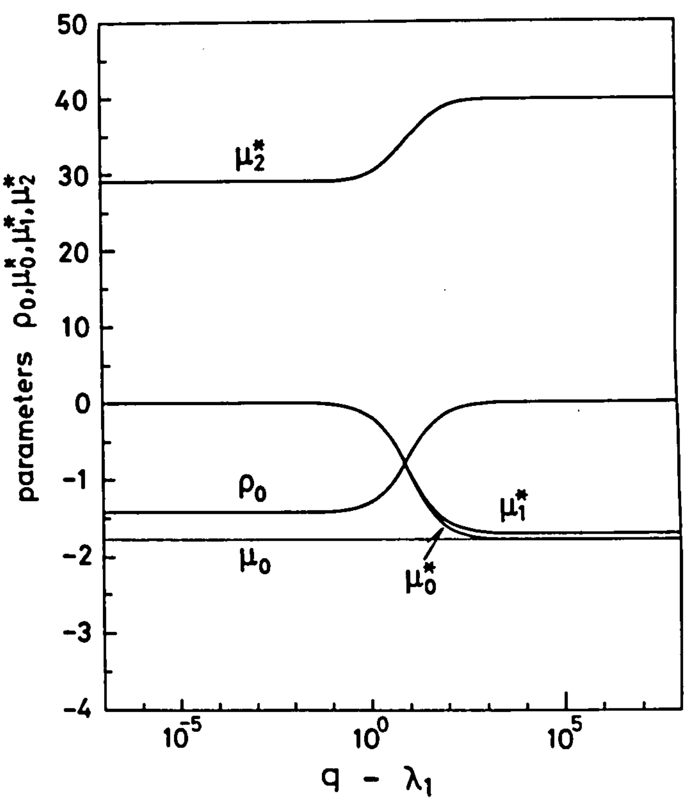


Fig.6. Variations of parameters: ρ_0, μ_0^*, μ_1^*, μ_2^* with q.

ρ_0 which are both negative, monotonously decreases and increases, respectively. Both of them approach certain values when q approaches ∞ and λ_0 as follows:

$$\begin{aligned} \mu_0^* &\to \mu_0 && \text{if} \quad q \to \infty \\ &\to 0 && \phantom{\text{if}} \quad q \to \lambda_0 \end{aligned} \qquad (2.89)$$

and

$$\begin{aligned} \rho_0 &\to 0 && \text{if} \quad q \to \infty \\ &\to \mu_0 && \phantom{\text{if}} \quad q \to \lambda_0 \end{aligned} \qquad (2.90)$$

It is observed from (2.89) and (2.90) that if we let $q \to \infty$, then the inequalities in (2.85) approaches to those in (2.84). In the case of non-uniform damping, the variations of μ_0^* and ρ_0 are similar to those in the uniform damping case except that ρ_0 approaches a value which is different from μ_0 as follows:

$$\begin{aligned} \mu_0^* &\to \mu_0 && \text{if} \quad q \to \infty \\ &\to 0 && \phantom{\text{if}} \quad q \to \lambda_1 \end{aligned} \qquad (2.91)$$

and

$$\begin{aligned} \rho_0 &\to 0 && \text{if} \quad q \to \infty \\ &\to \rho_\lambda && \phantom{\text{if}} \quad q \to \lambda_1 \end{aligned} \qquad (2.92)$$

The values μ_1^* and μ_2^* which are negative and positive, respectively, approach certain values when q approaches ∞ and λ_1 as follows:

$$\begin{aligned} \mu_1^* &\to \mu_1 && \text{if} \quad q \to \infty \\ &\to 0 && \phantom{\text{if} \quad q} \to \lambda_1 \end{aligned} \qquad (2.93)$$

and

$$\begin{aligned} \mu_2^* &\to \mu_2 && \text{if} \quad q \to \infty \\ &\to \mu_\lambda && \phantom{\text{if}} \quad q \to \lambda_1 \end{aligned} \qquad (2.94)$$

As is observed from Fig.6(b), μ_1^* always takes greater values than μ_0^*, and accordingly, μ^* must be greater than μ_1^* in stead of μ_0^*. Thus, the outlines of the area in which the parameters have to stay for V(x) to be a Lyapunov function, are now drawn. In this area, those parameters can take any values.

Three interesting Lyapunov functions can be obtained by appropriately chosing the parameters q, ρ, and μ*. Two of them are well-know functions, and the remaining one is a new function somewhat different from the first two functions.

(a) El-Abiad function

If we choose the parameters as follows:

$$q \to \infty$$
$$\rho \to 0 \qquad (2.95)$$
$$\mu^* \to 0$$

then, (2.81) reduces as follows:

$$V(x) = (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2$$
$$+ (1/\sum_{i=1}^{n} m_i)(\sum_{i=1}^{n} m_i\omega_i)^2$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}E_iE_j[\cos\delta_{ij}^{o} - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^{o})\sin\delta_{ij}^{o}]$$
$$= \sum_{i=1}^{n} m_i\omega_i^2$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}E_iE_j[\cos\delta_{ij}^{o} - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^{o})\sin\delta_{ij}^{o}] \qquad (2.96)$$

This function was first derived by El-Abiad & Nagappan [14] and Gless [55] in 1966. The first term represents kinetic energy, and it depends only on absolute angular velocities. The second term represents potential energy, and it is stored in the system owing to some deviations of rotor angles from those at the stable equilibrium point. Fig.7 shows the time variations of V(x) for the three damping cases, where the fault is cleared at the critical fault clearing time. The critical clearing times are 0.60, 0.65, and 0.64 sec, respectively. It is inferred from the variation of V(x) for the no damping case that V_{kc} which represents the kinetic energy of the center of inertia defined by

$$\bar{\delta} = \sum_{i=1}^{n} m_i\delta_i / \sum_{i=1}^{n} m_i \qquad (2.97)$$

(a) no damping

(b) uniform damping

(c) non-uniform damping

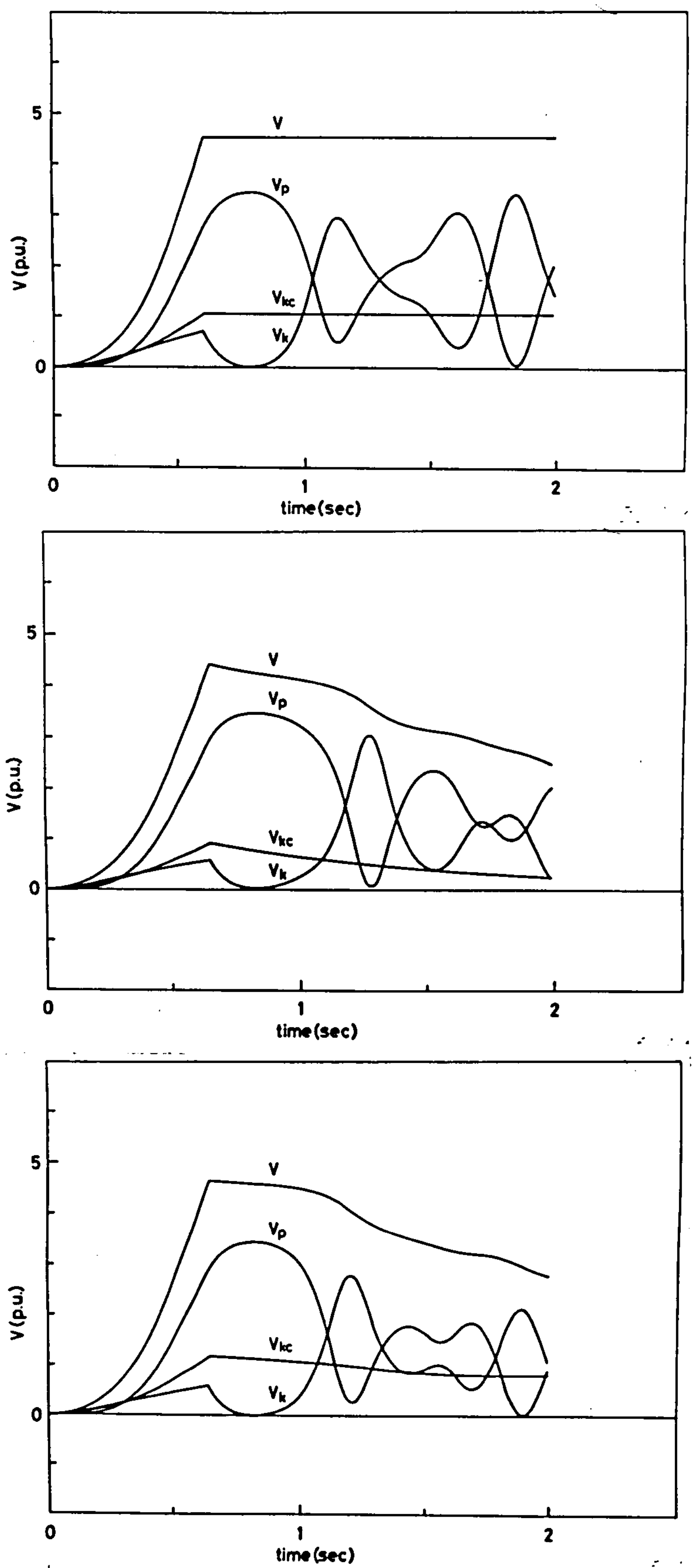


Fig.7. Time variations of El-Abiad function for three cases of damping torques.

has no influence on the stability of the system in view of the facts that this energy is kept constant after the instant of fault clearing, and that it only increases the magnitude of V(x) which is kept constant after the instant, too. Hence, it is desirable to make its value as small as possible by approaching μ^* to μ_0.

(b) Energy integral function

If we choose the parameters as follows:

$$\begin{aligned} q &\to \infty \\ \rho &\to 0 \\ \mu^* &\to \mu_0 \end{aligned} \tag{2.98}$$

then, the second, the third, and the fourth terms in (2.81) disappear, and (2.81) reduces as follows:

$$V(x) = (1/2 \sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} E_i E_j [\cos\delta_{ij}^{o} - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^{o}) \sin\delta_{ij}^{o}] \tag{2.99}$$

This function was first derived by Aylett [54] in 1958. It consists of two terms. The first term represents kinetic energy, and it depends only on relative angular velocities. The second term represents potential energy. The value μ^* can take μ_0 only in the cases where dampings are zero or uniform. In the non-uniform damping case, μ_1 is adopted as μ^* in stead of μ_0, then (2.81) reduces to

$$V(x) = (1/2 \sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2 + (\mu_1 - \mu_0) (\sum_{i=1}^{n} m_i \omega_i)^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} E_i E_j [\cos\delta_{ij}^{o} - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^{o}) \sin\delta_{ij}^{o}] \tag{2.100}$$

As observed from Fig.6(b), the difference between μ_0 and μ_1 is small, so the second term in (2.100) takes negligible values compared with other

(a) no damping

(b) uniform damping

(c) non-uniform damping

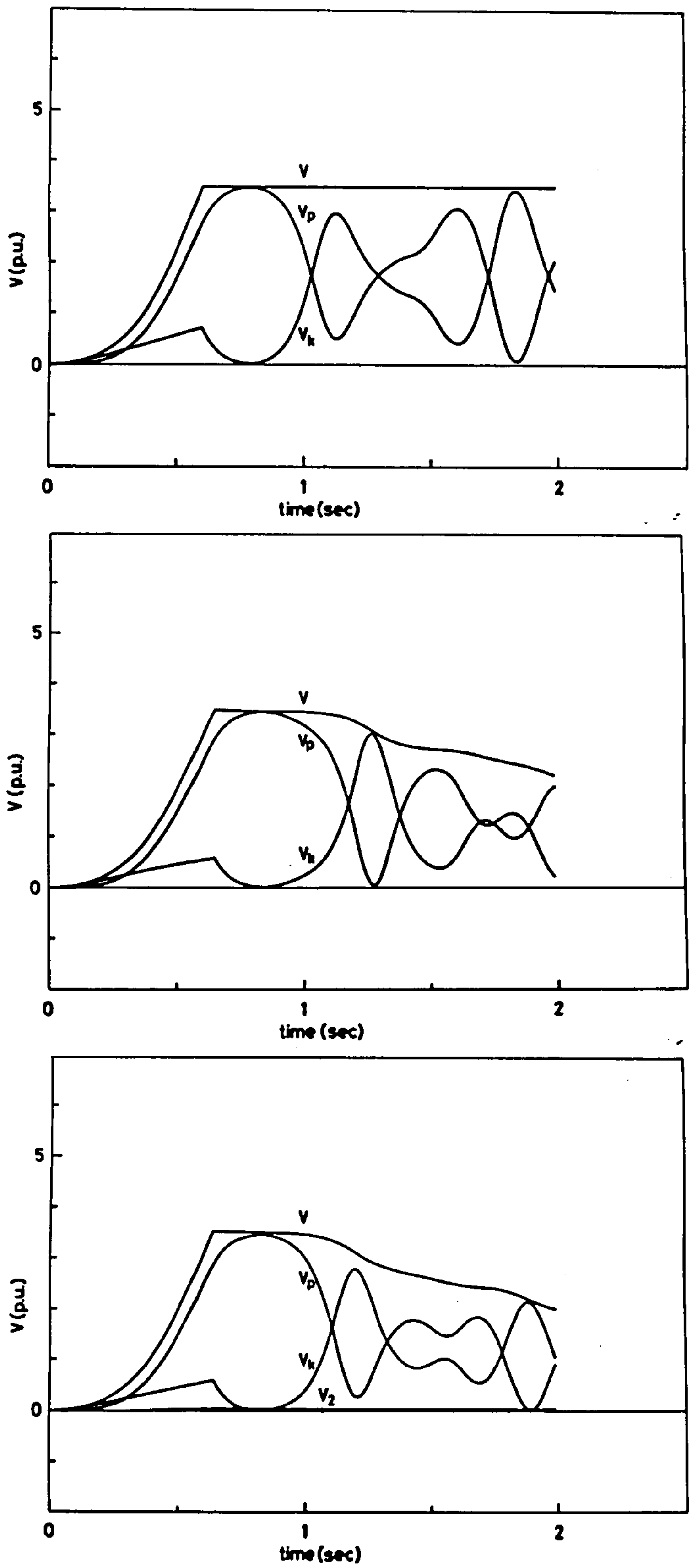


Fig.8. Time variations of Energy integral function for three cases of damping torques.

terms. Fig. 8 shows the time variations of V(x) for the three damping cases. In the case of no-damping, the total energy V(x) is kept constant after the instant of fault clearance. The kinetic and the potential energy are exchanging energy between them. If there are some dampings, then V(x) decreases with time. Its damping rate is conspicuous in the intervals when the kinetic energy is dominant, which corresponds with the fact that the total energy is dissipated through damping torques. It should be noted that V(x) takes almost the same values at the critical fault clearing time for the three cases while the critical clearing time itself differs with the cases. Namely, there exists a critical energy which is common to all damping cases. This fact is very important when we search for the critical value of V(x) for a given fault.

(c) New function

If we choose the parameters as follows:

$$
\begin{aligned}
q &\to \lambda_1 \text{ or } \lambda_2 \\
\rho &\to \rho_0 \\
\mu^* &\to 0
\end{aligned}
\qquad (2.101)
$$

then (2.81) reduces as follows:

$$
\begin{aligned}
V(x) = {} & (1/2 \sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2 \\
& + (1/ \sum_{i=1}^{n} m_i) (\sum_{i=1}^{n} m_i \omega_i)^2 \\
& + (1/\lambda) \sum_{i=1}^{n} (d_i \delta_i^* + 2m_i \omega_i) \delta_i^* \\
& - (1/\lambda \sum_{i=1}^{n} d_i) [\sum_{i=1}^{n} (d_i \delta_i^* + m_i \omega_i)]^2 \\
& + \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} E_i E_j [\cos\delta_{ij}^o - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^o) \sin\delta_{ij}^o]
\end{aligned}
\qquad (2.102)
$$

This function contains all the terms in (2.81), especially the third and the fourth terms to their maximum extent. These terms disappear in the no damping case, however, and V(x) is equivalent to El-Abiad function. Fig. 9 shows the time variations of V(x) for the three damping cases. In the

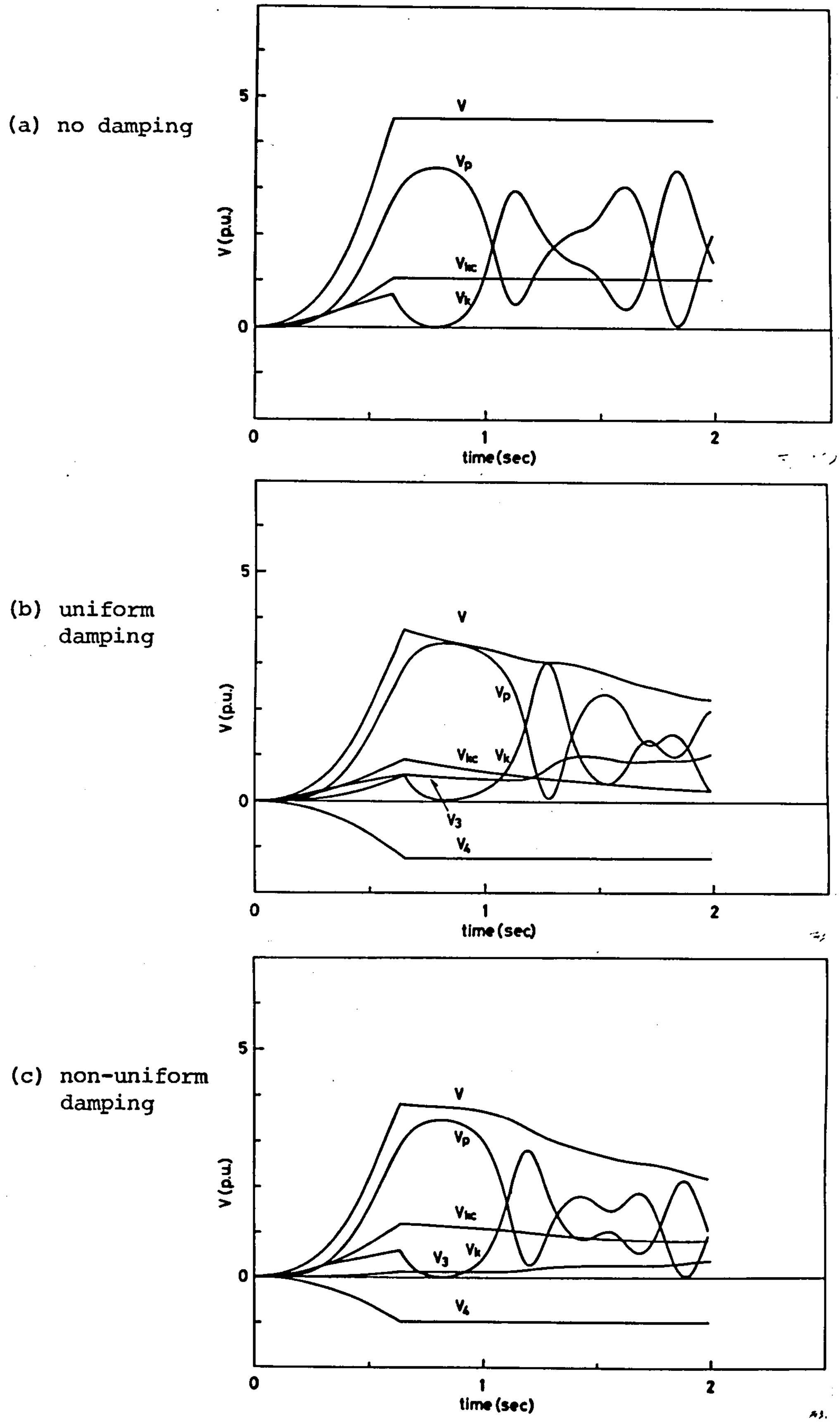


Fig.9. Time variations of new function for three cases of damping torques.

case of non-uniform damping, the second term cancells out with the sum of the third and the fourth term. The total of them has an effect of averaging the damping rate of V(x). In the non-uniform damping case, the third term is relatively small compared that in the uniform damping case. As a result, the total of those three terms takes positive value at each time. It has an effect of shifting the sum of the first and the fifth terms which is equivalent to the energy integral function, upwards as a whole. It is observed that there is no consistency of behavior in V(x) defined by (2.102) for all different damping cases. V(x) takes different values at the instant of fault clearance for these cases, which makes it difficult to determine the critical value of V(x).

3.7 Conclusions

In this section, we have constructed a Lyapunov function with a systematic method based on a generalized Popov criterion for multimachine power systems represented by the conventional model. The nonlinearities contained in the system do not satisfy the necessary conditions for all their variables, but for a range around the stable equilibrium point, so the system is not globally stable. The problem is how we can get an appropriate Lyapunov function to accurately estimate the domain of attraction. Three parameters are contained in the obtained Lyapunov function, and an infinite number of different Lyapunov functions can be obtained by varying these parameters. Among them, three interesting Lyapunov functions were chosen and investigated. It is concluded that the energy integral function is suitable to determining the critical value for a given fault.because it has a critical energy for the stability which is common to all cases of damping torques. On the other hand, the new function may have potential superiority over the energy integral function, but it will not be used in the following investigations because the determination of its critical value is somewhat complicated compared with the energy integral function.

§4. Critical value of Lyapunov function

In the preceding section, a Lur'e type Lyapunov function was derived. It can be regarded as a kind of energy consisting of kinetic energy and potential energy. The kinetic energy is one which is stored to rotors of generators, and depends on relative angular velocities. The potential energy is one which is stored to a network owing to some deviations of rotor angles from those at the stable equilibrium point. There are interactions between them, that is, energy moves from kinetic energy to potential energy according to variations of system variables. However, the total energy is kept constant all the time if there is no damping torque. The basic idea of Lyapunov's direct method is that a system is stable for a given contigency if its total energy which is stored during a fault-on period is smaller than a certain critical value.

It is the key to a good success in analyzing the transient stability of multimachine power systems whether we can precisely determine the critical value of the Lyapunov function. Many studies have been made on this subject for the last decade [38-41]. It is common to them with a few exceptions to choose a value of potential energy at an unstable equilibrium point which is closest (in terms of energy) to the stable equilibrim point as the critical value. This selection of the critical value brings two troublesome problems to Lyapunov's direct method. One of them is that this critical value frequently yields results that are very conservative in multimachine transient stability analyses. This conservative nature of Lyapunov's method is well-known and taken for an inherent characteristic of this direct method because of a simple reason that its basic theorem i.e. Theorem 1 is only sufficient but not necessary, and more detailed investigations of the conservative nature have not been made. The other problem is that it takes long computational time to calculate the critical value because there are $[2^{n-1} - 1]$ independent equilibrium points at maximum, where n is the number of generators, for example, if n is equal to 20, then the number of equilibrium points is 524,286. The above two problems have severely limited the practical application of the direct method to multimachine power system analyses.

In order to solve these problems, several researches have been made [38 - 40]. On the latter problem, Prabhakara and El-Abiad [39] proposed a simplified method of determing the closest unstable equilibrium point.

Its basic idea is that the Lur'e type Lyapunov function does not vary so much around the equilibrium points, so it is possible to get sufficiently accurate results by approximating those points appropriately. By using this method, the computational time will be shortened to an extent, but it will be useless if generators increase in number because the number of equilibrium points increases exponentially. Moreover, the conservative nature of the direct method still remains. It is impossible to get rid of the conservative nature as long as we use the usual critical value. It is necessary to introduce a new critical value for this purpose. G. A. Lüders suggested in his paper as follows [38]: when a power system loses its synchronism owing to some fault, it splits into two groups of generators at the first instant; these groups defined the step-out mode; the step-out mode corresponds with a particular unstable equilibrium point; we can adopt the value of the potential energy at the equilibrium point as the critical value for the fault.

In this section, the above mentioned idea is investigated in detail in order to get rid of the conservative nature. If it is possible, then we can shorten the computational time for the critical value at the same time because we only have to determine the step-out mode for a given fault.

4.1 Model and basic equations

If damping torques of generators and transfer conductances of reduced admittance matrices are zero, then the motion of the ith generator is described as follows:

$$m_i \frac{d^2\delta_i}{dt^2} = \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij}) \qquad (2.103)$$

$$\text{for } i=1,2,\ldots,n,$$

where the superscript " o " denotes the stable equilibrium point of the post-fault system, so (2.103) applies to the post-fault state.

A Lur'e type Lyapunov function has been derived for this system in the preceding section as follows:

$$V(x) = (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}E_iE_j[\cos\delta_{ij}^o - \cos\delta_{ij} + (\delta_{ij} - \delta_{ij}^o)\sin\delta_{ij}^o]$$

$$= V_k(\omega) + V_p(\delta) \qquad (2.104)$$

The first term in (2.104) represents kinetic energy of generators, and it is a function of relative angular velocities of rotors. The second term represnts potential energy which is stored in networks owing to deviations of rotor angles from those at the stable equilibrium point, and it is a function of relative angles of rotors. V_k and V_p denotes kinetic energy and potential energy, respectively. The time derivatives of V_k and V_p are given as follows:

$$\frac{dV_k}{dt} = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij})(\omega_i - \omega_j) \qquad (2.105)$$

$$\frac{dV_p}{dt} = -\sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij})(\omega_i - \omega_j) \qquad (2.106)$$

The right hand terms in (2.105) and (2.106) are of the same magnitude and of the opposite signs of each other, which implies that there exists exchange of energy between kinetic energy and potential energy. These terms do not contribute to the damping rate of V, and the time derivative of V is given as follows:

$$\frac{dV}{dt} = 0 \qquad (2.107)$$

Hence, if a value is given to V at an initial time, then it is kept constant afterwards. This Lyapunov function can be regarded as a kind of energy, and our discussion will be developed on this basis from now on.

4.2 Transinet stability region

From (2.104), it is observed that the potential energy V_p depends only on relative rotor angles. If δ_1 is chosen as reference, V_p can be treated as a function of (n-1) dimensional vector δ_r defined as follows:

$$V_p = V_p(\delta_r) \tag{2.108}$$

where

$$\delta_r = (\delta_{21}, \delta_{31}, \ldots, \delta_{n1}) \tag{2.109}$$

Fig.10 shows an example of $V_p(\delta_r)$ in an (n-1) dimensional relative angular space for a 3-machine power system. The curves C_1, C_2, ... are equipotential curves yielded by

$$V_p(\delta_r) = C_i \qquad i = 1,2,\ldots. \tag{2.110}$$

The function V_p takes the minimum value at the point S. The points U_1, U_2, ... are saddle points. The curves O_1, O_2, ... are those which go through U_1, U_2, ..., and are orthogonal to equipotential curves, respectively. If C_i takes samll values, then the corresponding equipotential curves are closed, and surround the points S. With increase in magnitude of C_i, equipotential curve goes outside, and reaches the lowest saddle point U_1 when C_i takes the value defined as follows:

$$V_{u1} = V_p(\delta_{u1}) \tag{2.111}$$

where δ_{u1} is the relative angle vector at U_1. If C_i is greater than V_{u1}, then the corresponding curve is not closed any more. With more increase in magnitude of C_i, the equipotential curve reaches the saddle points U_2, U_3, ... in sequence according to

$$\begin{aligned} V_{u2} &= V_p(\delta_{u2}) \\ V_{u3} &= V_p(\delta_{u3}) \\ &\vdots \end{aligned} \tag{2.112}$$

where δ_{u2}, δ_{u3}, ... are the relative angle vectors at U_2, U_3,..., respectively.

From (2.103), it is observed that each generator receives a torque expressed as follows:

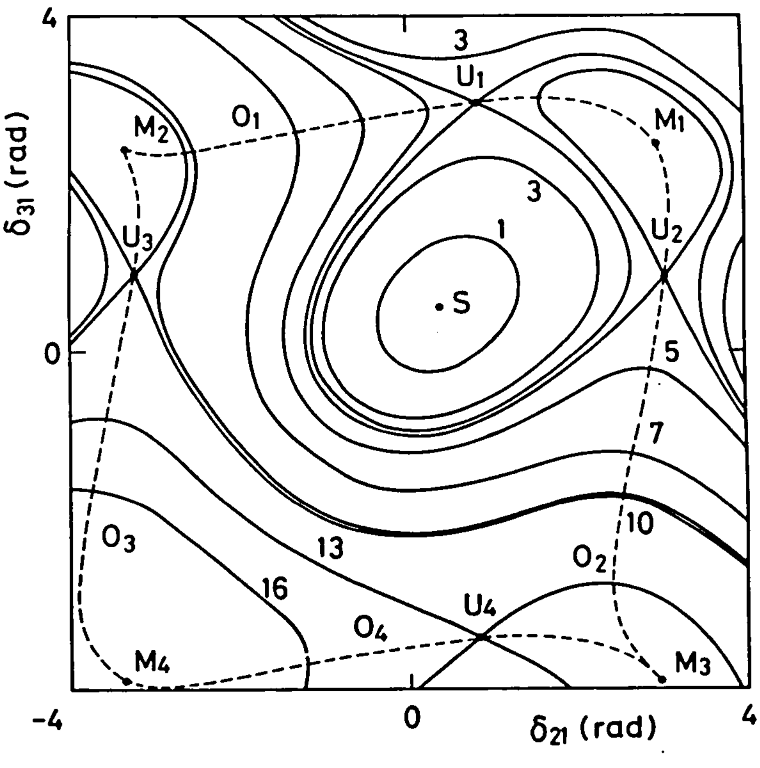


Fig.10. Equipotential curves of 3-machine system.

$$f_i = \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta^o_{ij} - \sin\delta_{ij}) \tag{2.113}$$

The f_i defines an n dimensional vector f

$$f = [\, f_1, f_2, \ldots, f_n \,]' \tag{2.114}$$

The sum of all torques denoted by $\bar{f}$ is given as follows:

$$\begin{aligned} \bar{f} &= \sum_{i=1}^{n} f_i \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta^o_{ij} - \sin\delta_{ij}) \\ &= 0 \end{aligned} \tag{2.115}$$

Eq.(2.115) implies that the center of angular velocities $\bar{\omega}$ defined by

$$\bar{\omega} = \sum_{i=1}^{n} m_i\omega_i / \sum_{i=1}^{n} m_i \tag{2.116}$$

does not receive any torque, and accordingly, it is kept constant all the time, that is,

$$\bar{\omega} = \text{constant} \tag{2.117}$$

This fact implies that each torque does not contribute to the accellation of the center of angular velocities, but that each torque has only influence on relative behaviors of generators. There is a relation between the torque f and the potential energy function V_p as follows; the partial derivative of V_p with respect to relative angles are given by

$$\begin{aligned} \frac{\partial V_p}{\partial \delta_{i1}} &= 2\sum_{j=1}^{n} B_{ij}E_iE_j(\sin{}_{ij} - \sin{}^{o}_{ij}) \\ &= -2f_i \qquad\qquad i \neq 1 \end{aligned} \tag{2.118}$$

which gives

$$\frac{\partial V_p}{\partial \delta_r} = -2f_r \qquad (2.119)$$

where f_r is a reduced torque of (n-1) dimension defined by

$$f_r = [\, f_2, f_3, \ldots, f_n \,] \qquad (2.120)$$

Since the direction of $(\partial V_p/\partial \delta_r)$ is orthogonal to equipotential curves, and its magnitude is proportional to the gradient of equipotential curves, (2.119) shows that the system receives the torque which always acts orthogonally to equipotential curves.

In Fig.10, f_r is parallel with the curves O_1, O_2, ... from the definition of those curves. Those curves enclose the region in which the stable equilibrium point S exists. In this region, f_r acts on the system in such way that it will confine the system in this region. On the other hand, f_r acts in such way that it will separates the system from this region in the outside of the region. The system will lose synchronism if it crosses one of curves O_1, O_2,... from the inside to the outside of the region, because f_r will acts in such way that it will separate the system from the curve, afterwards. It is conclueded that the region enclosed by the curves O_1, O_2, ... can be regarded as the transient stability region in the wide sense that the system receives the synchronizing torque in the region. We define the transient stability region by this region.

There are several equilibrium points on the boundary of the transient stability region, for example, U_1 and M_2. At these points, the partial derivatives of V_p is zero, that is,

$$\frac{\partial V_p}{\partial \delta_r} = 0 \qquad (2.121)$$

From (2.119) and (2.121),

$$f_r = 0 \qquad (2.122)$$

holds, and equivalently from (2.115)

$$f = 0 \tag{2.123}$$

holds. Namely, the solutions of (2.121) do not depend on the selection of the reference generator. All equilibrium points can be obtained by solving (2.122). Since f_r is a vector consisting of periodic functions, there exist an infinite number of solutions. Let δ_r be a solution of (2.122), then δ_r' defined by

$$\begin{aligned} \delta_{21}' &= \delta_{21} + 2k_2\pi \\ \delta_{31}' &= \delta_{31} + 2k_3\pi \\ &\vdots \\ \delta_{n1}' &= \delta_{n1} + 2k_n\pi \end{aligned} \tag{2.124}$$

is also a solution of (2.122), where $k_1, k_2, \ldots, k_n$ are arbitrary integers. The number of all independent solutions of (2.122) denoted by N_m is

$$N_m = 2^{n-1} \tag{2.125}$$

at maximum, and they are obtained numerically with initial approximations as follows:

$$\delta_u = [\ \xi_{21}, \xi_{31}, \ldots, \xi_{n1}\ ] \tag{2.126}$$

where

$$\xi_{i1} = 0 \quad \text{or} \quad \pi \qquad\qquad \text{for } i=2,3,\ldots,n.$$

It is clear from (2.126) that 2^{n-1} different initial approximations can be yielded by this equation. In Fig.1, four independent solutions exist, that is, the saddle points U_1 and U_2, the maximum point M_1, and the stable equilibrium point S. These points are corresponding to the following initial approximations:

$$S \rightarrow [\ 0, 0\ ]$$

$$U_1 \rightarrow [\ \pi, 0\ ]$$

$$U_2 \rightarrow [\ 0,\ \pi\ ] \tag{2.127}$$

$$M_2 \rightarrow [\ \pi,\ \pi\ ]$$

Other points U_3, U_4, M_2, ... can be obtained by applying (2.124). Among all saddle and maximum points, a few of them are on the boundary of the transient stability region, e.g., U_1, U_2, U_3, U_4, M_1, M_2, M_3, and M_4 in Fig.10. These points are all obtained from initial approximations made as follows:

$$\delta_u = [\ \zeta_{21},\ \zeta_{31},\ \ldots,\ \zeta_{n1}\ ] \tag{2.128}$$

where

$$\zeta_{i1} = -\pi,\ 0,\ \pi, \qquad \text{for } i = 2,3,\ldots,n.$$

Accordingly, the number of them denoted by N_c, is given as follows:

$$N_c = 3^{n-1} - 1 \tag{2.129}$$

N_c is equal to 8 in the case of Fig.10. The transient stability region is bounded by the curves (or surfaces) which go through this number of saddle and maximum points. If n is 10, there exist 19,682 saddle and maximum points, and if n is 20, then there exist 1,162,381,466 saddle and maximum points at maximum on the boundary.

4.3 Stability conditions

Some geometrical interpretation of the potential field in Fig.10 is tried in order to get physical insights into qualitative characteristics of the system. The area in the center of Fig.10 can be regarded as a basin surrounded by mountains whose tops are corresponding to the maximum points M_1, M_2, ... Equipotential curves are treated as contour lines which represent the height of place. Saddle points are regarded as mountain passes surrounding the basin. The curves O_1, O_2, ... are ridges which links mountain tops and passes. Lastly, the system is regarded as a traveler. He is now going to get out of this basin. He has some energy. Since (2.107) holds, his energy is kept constant all the time. It can takes two form, that is, kinetic energy and potential energy. At the point S, his kinetic energy is maximum, and he begins to climb a slope to a certain direction. He always receives a force which will pull him down to S, and his kinetic energy is transformed to potential energy as he climbs. When he reaches the height which corresponds to his total energy, all of his kinetic energy is lost, and he can not climb any more. Consequently, he must begin to descend afterwards. If the contour line corresponding to his energy is closed, then he can not get out the area which is enclosed by the line, forever. In order to get out of the basin, he must have more energy than V_{u1} at least, where V_{u1} is defined by (2.111). The pass U_1 is the lowest one among all points on the boundary of the transient stability region. If he has more energy than V_{u1}, he is able to go over the pass U_1. However, if his energy is smaller than V_{u1}, there is no possibility for him to get out of the basin because the contour line corresponding to his energy is closed. Summing up these observations, we can obtain a stability condition as follows:

[Stability condition 1]

If a system satisfies

$$V < V_{um} \tag{2.130}$$

then the system is stable, where V_{um} is the value of $V_p(\delta_r)$ at the lowest saddle point among all saddle points.

This stability condition is associated with the lowest saddle point. It says that if one wants to get out of the basin, then he must have at least more energy than V_{u1}. However, he does not always want to go over the lowest pass. There are many passes on the boundary of the basin. If he intends to go over the pass U_2, for example, according to his destination, then he has to have more energy than V_{u2}. V_{u2} is, of course, greater than V_{u1}. If his energy is smaller than V_{u2}, he is not able to go over the pass U_2 while he may be able to go over the pass U_1. Namely, there exists a particular condition which is related with each pass. It is described as follows:

[Stability condition 2]

If a system satisfies

$$V < V_{ui} \qquad \text{for } i=1,2,\ldots, \qquad (2.131)$$

then the system is stable around the saddle point U_i, where V_{ui} is the value of $V_p(\delta_r)$ at U_i.

If (2.131) is satisfied, he can not go over the pass U_i, but he may be able to go over the passes lower than U_i. On the other hand, if (2.130) is satisfied, he is not able to go over any passes on the boundary of the basin. In this sense, Stability condition 1 is absolute while Stability condition 2 is local. Those stability conditions are both concerning with the cases where he intends to get out of the basin through some mountain pass. Since V_p takes a local minimum value at the pass if V_p is restricted on the boundary, it is easier for him to go over the pass than other points around it. However, if he has enough energy to go over those points, then there is no obligation on him to go through the pass. Namely, he can go over any point if he has enough energy to go over it. From these considerations, we can obtain the most general stability condition as follows:

[Stability condition 3]

If a system satisfies

$$V < V_c \qquad (2.132)$$

then the system is stable for the points where V_p takes greater values than V_c, where V_c is the value of $V_p(\delta_r)$ at the point c on the boundary of the stability region.

This condition is somewhat different from the preceding two conditions. The latter conditions are based on the passes. V_p takes locally minimum values at the passes if V_p is limited on the boundary, so there is no point in the vicinity of the passes which is on the boundary, and is lower than them. Accordingly, the two conditions are also applicable to all boundary points around the pass under consideration. On the other hand, if he has energy V_c, then he is able to go over any point lower than c, and such points infinitely exist in the vicinity of c. Namely, Stability condition 3 can not guarantee the stability for all points around c, but for a half of them. In order to apply this condition, it must be known beforehand that he intends to go over a boundary point higher than c.

Three stability conditions are now at hand. These conditions are related with each other as illustrated in Fig.11. The condition for the system to be stable becomes looser in sequence of Stability condition 1,2, and 3. These conditions should be used according to the information which is available. If there is no information on the boundary point which he is going to go over, it is necessary to apply Stability condition 1 in order to prevent him from getting out of the basin. If it is certain that he is going to go over some boundary point around a pass U_i, we can apply Stability condition 2. Lastly, if the boundary point where he is going to go over is known, then we can apply Stability condition 3. Thus the applicable stability conditions becomes looser with increase in the available information. In other words, we can use looser stability condition with increase in information on the system behavior in the transient period. This fact leads to improvement in the accuracy of Lyapunov's direct method applied to the transient stability analysis. In the following section, it is investigated which stability condition is appropriate for the transient stability analysis in the light of power system's behaviors in the transient period.

4.4 Selection of stability condition

In the applications of Lyapunov's direct method to the transient stability analysis of power systems, it has been the usual method for long time to apply Stability condition 1. This condition is absolute in the sense that it can gurantee the system's stability without detail information on the system's behavior in the transient period. In this sense, it is superior to the other two conditions. This condition is useful in the studies in which no information on the system's behavior is available, and only rough estimation on the stability is necessary. In spite of these advantages, however, there are two troublesome problem in applying this condition as follows: there is no effective method of determining the lowest

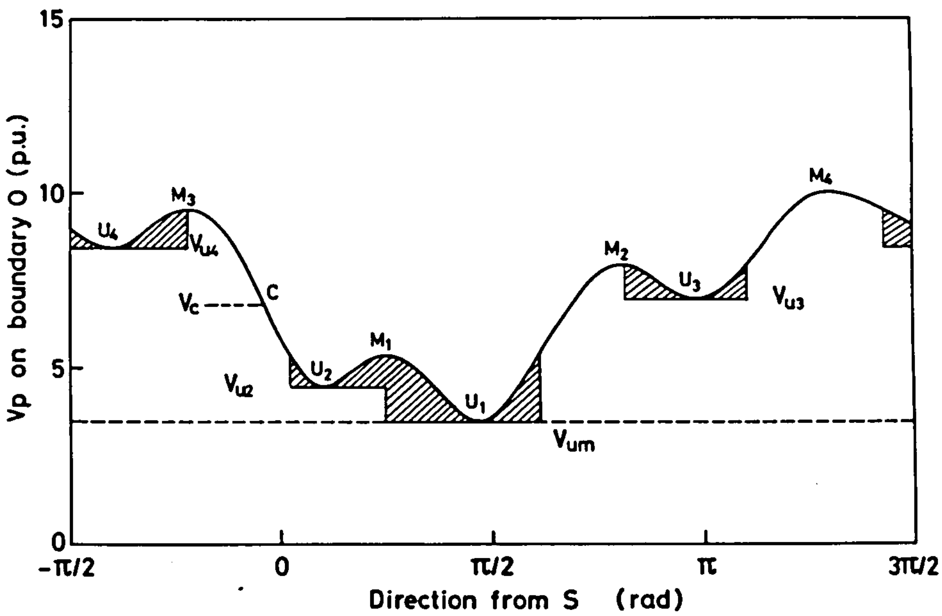
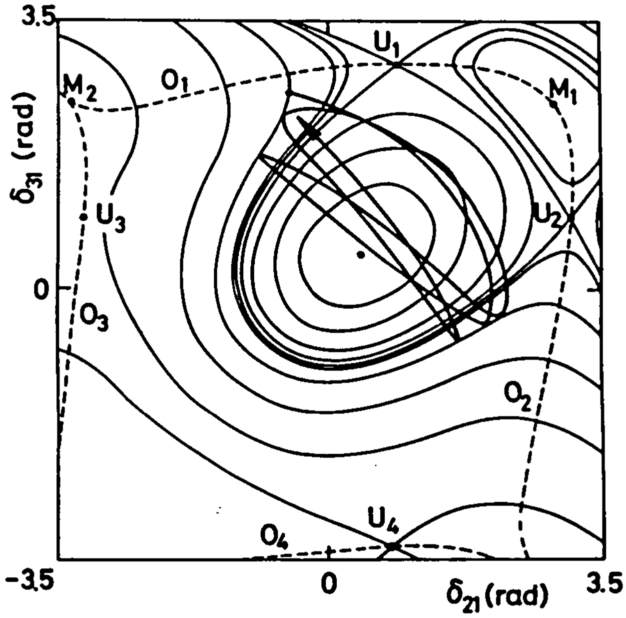


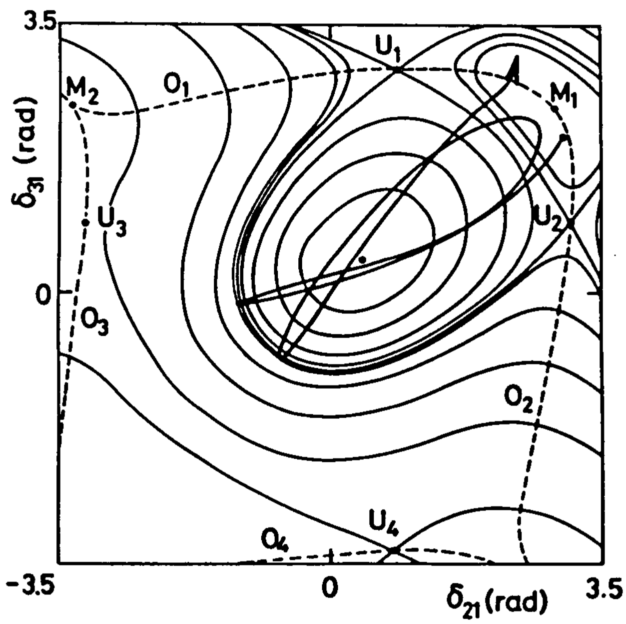
Fig.11. Variation of potential energy $V_p(\delta)$ on boundary of transient stability region & Stability conditions.

saddle point in reasonably short time. From (2.129), there are 1,048,572 saddle and maximum points on the boundary of the transient stability region if there are 20 generators in the system. In order to determine the lowest saddle point, it is necessary to calculate all of these points. It becomes a formidable task with increase in generators. The other problem is that the stability condition often yields results which are very conservative. This tendancy is conspicuous in large power systems. These two problem have severely limited the application of Lyapunov's direct method to the transient stability analysis of multimachine power systems.

In practical transient stability analyses, a system is classified as stable if it remains in synchronism during the first few swings, and the critical fault clearing time is determined on this basis. Namely, these analyses deal with the stability not in long time, but in short time. A system does not necessary keep its synchronism for long time, and it only has to keep its synchronism for several seconds in order that it is classified as stable. In accord with this definition, there are several features of the system which support it. If a fault occurs in a power system, it produces some discrepancies between inputs and outputs of generators. As a result, some generators which are near the fault location are accellated, and are separated from the remaining generators. If the fault is cleared in time, then a synchronizing toque acts between these two groups of generators, and the system can keep its synchronism. If not, it will go over the boundary of the transient stability region, and will lose its synchronism, afterwards. Fig.12 shows some examples of trajectories for a 3-machine power system. The initial angular velocities are assumed to be zero, and the initial state is put on the relative angular space. In Fig.12(a), the initial point δ_0 is (-0.5, 2.5) rad., and the total energy is 5.03, which is enough for the system to go over the saddle point U_1 and U_2. In spite of this fact, the system stays in the stability region for three oscillations. In Fig.12(b), starting from the point δ_0 = (3.0, 2.0) rad., the system crosses the boundary O_2 after two oscillations. Its total energy is 4.27, and is enough for the system to go over U_1 and U_2, too. These two cases can be regarded as those which illustrate the system's behavior in the transient period after the first swing. The mode of the system's oscillations do not very so much during the first several swings. This inclination becomes conspicuous in large power systems. In those sys-



(a) δ_0 = (-0.5, 2.5) rad.



(b) δ_0 = (3.0, 2.0) rad.

Fig.12. Trajectories in relative angular space for 3-machine system.

tems, their oscillation modes vary very slowly, and as a result, they remain in synchronism for long time if they are stable for the first swing. Moreover, the damping torques of generators which are neglected in the present investigation, dissipate their energy as illustrated in Fig.8(b), (c), and help the systems to remain in synchronism. In view of the above mentioned facts, it is clear that whether a power system stays in synchronism or not for the first swing has significant part in the practical analyses. Therefore, we limit out attention to the stability of power systems for the first swing, and select the stability condition on this basis.

Stability condition 1 is too strict to apply it to judge the first swing stability because it does not take account of the system's oscillation mode in the transient period. Now, two stability conditions remain as the candidate. Stability condition 3 is looser than Stability condition 2. Selection between these conditions depends on the information which is available. In Lyapunov's direct method, it is usual to numerically simulate system behaviors under a given fault. Fig.13 illustrates an example of system trajectory in the relative angular space for a case where a fault continues. The trajectory crosses the boundary O of the transient stability region at the point c. Total energy increases with time in the fault-on period. In order to keep the system in synchronism, the fault must be cleared in an appropriate time. Since it is known that the system is going to go over the point c, we are able to apply Stability condition 3 for this purpose. Thus if we limit our attention to the stability for the first swing, we can easily obtain the information necessary to apply Stability condition 3. This condition can be rewritten in a suitable form as follows;

[Stability condition 4]

If a system satisfies

$$V < V_{cr} \tag{2.133}$$

then it is stable for the first swing, where V_{cr} is the value of $V_p(\delta_r)$ at the point c.

In the derivation of this condition, we have implicitly assumed that the system goes toward the point c without fail even in those cases where the

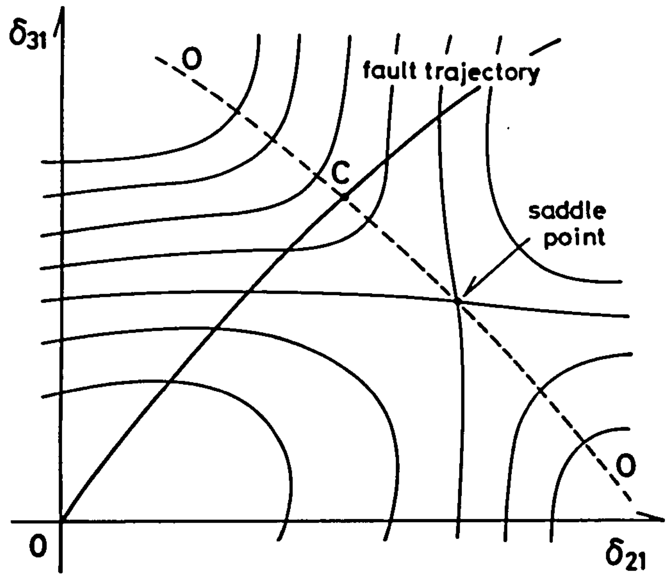


Fig.13. Trajectory in relative angular spa space for 3-machine system.

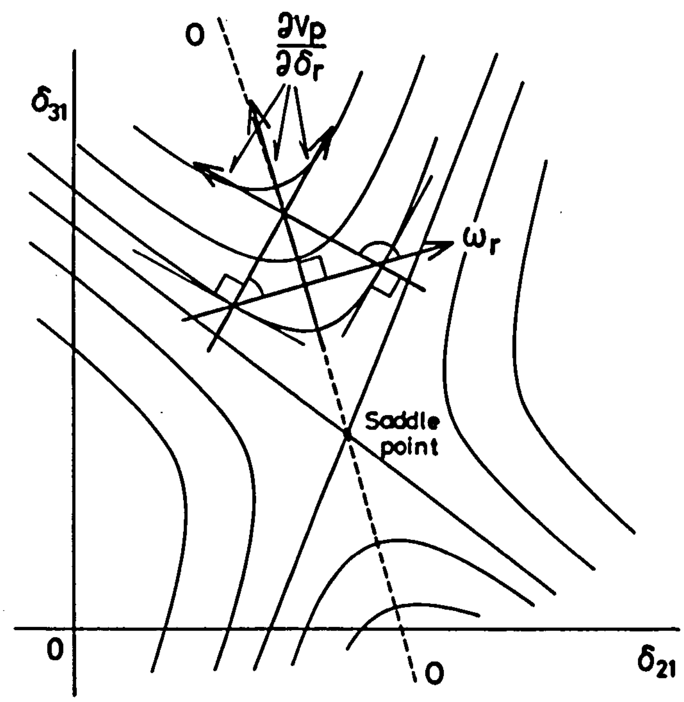


Fig.14. Relation between ω_r and $\partial V_p/\partial \delta_r$ when system crosses boundary O_x.

fault does not continue, in other words, it is cleared at some instants. This assumption is, however, satisfied not strictly, but approximately, because the system trajectory deviates a little from that for the sustained fault in those cases. If the deviation is adequately small, then we can use the above condition as it is, but it is not negligible, some correction must be made. In the next section, we will describe a method of determining the critical value V_{cr} as well as this problem.

4.5 Method of determining critical value

It is necessary to look for the point c in order to apply Stability condition 4. The time derivatives of V_p and V_k in (2.105) and (2.106) can be rewritten as follows:

$$\frac{dV_p}{dt} = \frac{\partial V_p}{\partial \delta_r} \omega_r \tag{2.134}$$

$$\frac{dV_k}{dt} = - \frac{\partial V_p}{\partial \delta_r} \omega_r \tag{2.135}$$

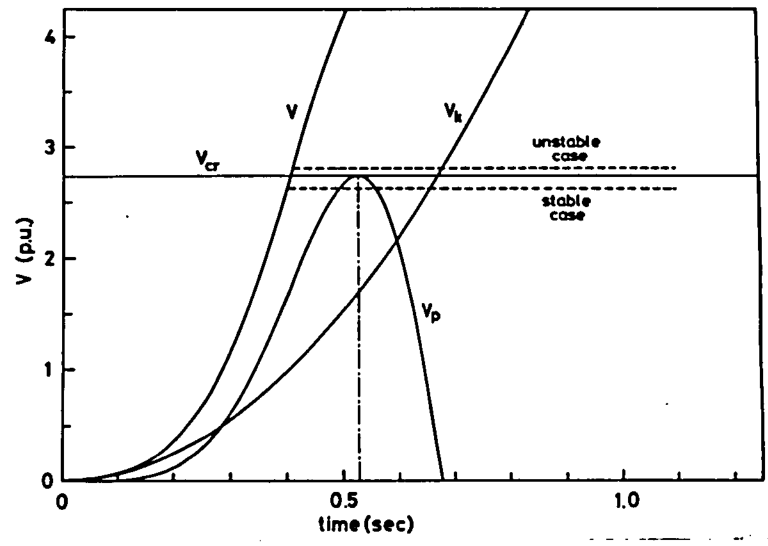
The signs of these time derivatives depends on the angle which is made by the two vectors $(\partial V_p/\partial \delta_r)$ and ω_r. Fig.14 illustrates the relation between the relative angular velocity ω_r and the partial derivative $(\partial V_p/\partial \delta_r)$ at the instant when the system crosses the boundary of the transient stability region. The vector $(\partial V_p/\partial \delta_r)$ is always orthogonal to the equipotential curves. From the definition, the boundary O is also orthogonal to the equipotential curves. It is assume in this figure that ω_r is perpendicular to the boundary O at the point c. In the inside of the boundary, $(\partial V_p/\partial \delta_r)$ and ω_r make an accute angle, so

$$\frac{dV_p}{dt} > 0 \qquad \frac{dV_k}{dt} < 0 \tag{2.136}$$

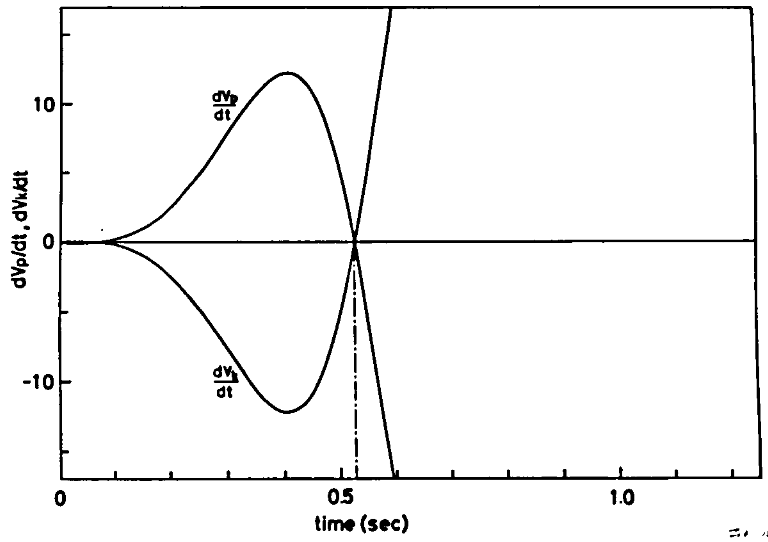
hold, that is, V_p increases, and V_k decreases in the stability region. On the point c, the two vectors make a right angle, so

$$\frac{dV_p}{dt} = 0 \qquad \frac{dV_k}{dt} = 0 \tag{2.137}$$

(a) V and its components



(b) Time derivative of V_p & V_k



(c) Swing curves

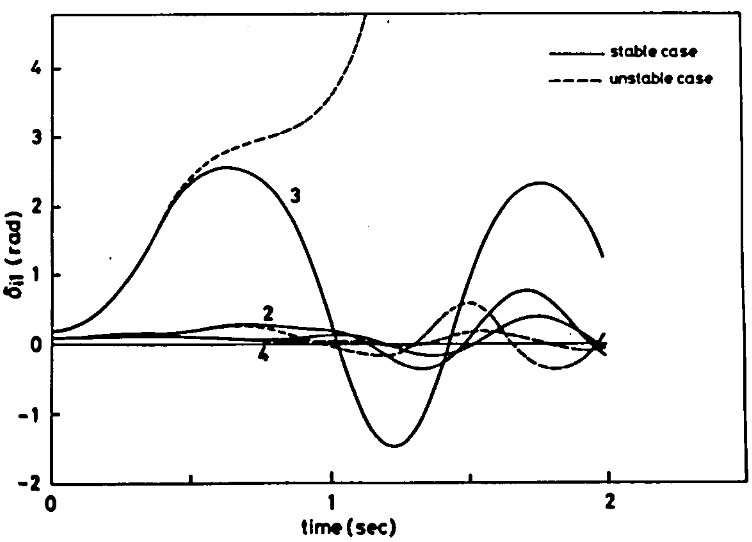


Fig.15. Method of determining critical value of V & its verification.

hold, that is, V_p and V_k stop increasing and decreasing at this point, respectively. In the outside of the boundary, the two vectors make an obtuse angle, so

$$\frac{dV_p}{dt} < 0 \qquad\qquad \frac{dV_k}{dt} > 0 \qquad (2.138)$$

hold, that is, V_p and V_k begin to decrease and to increase afterwards, respectively. Thus the time derivatives of V_p and V_k changes their signs from positive to negative, and from negative to positive, respectively, at the instant when the system crosses the boundary. Hence we can easily obtain the point c by examining the signs of these time derivatives, and can easily calculate the critical value V_{cr}. In practical cases, ω_r is not precisely perpendicular to the boundary O, and a little discrepancy exists between the point c and the point where the time derivatives of V_p and V_k vanish. However, the direction of $(\partial V_p/\partial \delta_r)$ rapidly varies, and the value of V_p varies very slowly along ω_r, around the boundary of the stability region, so the discrepancy has little influence on V_{cr}.

An example will serve to clarify the above mentioned method. The 4-machine power system shown in Fig. 5 is cited again. A 3-phase short-circuit occurs at a point near no. 3 generator, and is cleared by opening the line connecting no.3 and 4 generators at both terminals after a certain lapse of time. Fig.15(a) and (b) shows the time variations of V, V_p, V_k, (dV_p/dt), and (dV_k/dt) for a case where the fault continues. From Fig.15(b), it is observed that (dV_p/dt) and (dV_k/dt) are of the same magnitude, and of opposite signs with each other, and that they change their signs from positive to negative and from negative to positive at 0.53 sec, respectively. This instant coincides with that when the system crosses the boundary of the stability region. The potential energy V_p varies as shown in Fig.15(a). According to the above mentioned method, the critical value V_{cr} is obtained by taking the value of V_p at 0.53 sec. V_{cr} is equal to 2.725 in this case. Since the value of V is 2.621 and 2.792 at 0.40 and 0.41 sec, respectively, it is guessed that the critical fault clearing time exists between 0.40 and 0.41 sec. In Fig.15(c), the time variations of relative rotor angles are shown for the two cases where the fault is cleared at 0.40 and 0.41 sec. The figure verifies the above guess. Namely, if the fault is cleared at 0.40 sec, then the system keep synchronism, and conversely, if the fault is cleared at 0.41 sec, then the system loses synchronism. In this case, Lyapunov's direct method yields very accurate result.

However, in those cases where there is much difference between the trajectory for a sustained fault and that for the fault which is cleared at some instant, the stability condition 4 can not be applied as it is, and some correction must be made. If the stability condition 4 is satisfied, then it can not happen that the system goes over the point which higher than the point c. If the synchronism is to be lost, it is through the point which lower than c. On this basis, we adopt a method of correcting the critical value iteratively as follows:

[Step 1] Determine V_{cr} with the method described in the above paragraph, and treat it as the first approximation V_{cr}^1 of the actual critical value for a given fault.

[Step 2] Clear the fault at the instant when V reaches V_{cr}^1. If V_{cr}^1 is greater than the actual critical value, then the system will crosses the boundary of the stability region.

[Step 3] Determine the second approximation V_{cr}^2 of the actual critical value with the same method as that for V_{cr}^1.

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By iterating the above steps, a sequence of approximate critical values V_{cr}^1, V_{cr}^2, V_{cr}^3, ... are obtained. V_{cr}^2, V_{cr}^3, ... are closer to the actual critical value than V_{cr}^1, V_{cr}^2, ..., respectively. However, the iteration should be stopped in two or three times in order to keep the advantage of Lyapunov's direct method. It will be shown later that two or three iterations yield sufficient accurate results.

4.6 Conclusions

In this section, we have developed a method of determing the critical value. The transient stability region was defined in a wide sense that the synchronizing torque acts on the power system in it. From the considration of the system behavior based on the energy concept, the three stability coditions were derived. Selection of the basic condition for determining the critical value among these conditions depends on the information which is available. If no information is available on the behavior of the power system in the transient period, then we have to rely on Stability condition 1. However, it brings the well-known conservative nature as well as the computational difficulty to Lyapunov's direct method. By investigating the definition of the transient stability and the system behavior in the transient period, it gets clear that whether the system remains in synchronism or not for the first swing, is practically very important. Hence, the stability is limited to that for the first swing. It is easy to get the information necessary in applying Stability condition 3. By using this condition, we are able to get rid of the conservative nature from Lyapunov's direct method. One simple method of determing the critical value was presented. Since this method does not need any calculation of the saddle point, it is able to shorten the computing time for the critical value. In §6, this method will be applied to a 10-machine power system, and its effectiveness will be verified.

§5. Influence of transfer conductances

Transfer conductances have been neglected in constructing the Lyapunov function and in determining its critical value. These conductances represent real parts of reduced admittance matrices obtained after elimination of nodes without generators. Loads in power systems are modeled as constant impedances, and those are equivalently incorporated to reduced admittance matrices. Transfer conductances get larger with increase in loads. In usual, power systems have loads which are comparable with their capacity, so transfer conductances can not be neglected.

There are two basic ways of taking account of transfer conductances. One of them is to construct new Lyapunov functions which are applicable to power systems with transfer conductances. Several researches have been made along this direction, but those have not reached results of practical significance, yet. The other is to correct existing Lyapunov functions appropriately. This way is chosen in this section because we have already had an excellent Lyapunov function at hand. Its time derivatives is zero for power systems with no transfer conductances, and its value is kept constant all the time. This nature was used as the base of determining the critical value in the preceding section. It is injured by transfer conductances, however. The time derivative of the Lyapunov function deviates from zero to negative, and to positive with time. Consequently, the accuracy of estimation of critical fault clearing times is degraded for power system with transfer conductances. Our basic idea of correcting the Lyapunov function is to keep its value as constant as possible after the instant of fault clearance, or at least during the first swing.

In this section, firstly, the influence of transfer conductances on the Lyapunov function is investigated. Secondly, an outline of correcting the Lyapunov function is described. Lastly, two modified Lyapunov functions are derived.

5.1 Model and basic equations

Since transfer conductances of reduced admittance matrices are taken into account, the motion of the ith generator is described as follows:

$$\begin{aligned} m_i \frac{d^2\delta_i}{dt^2} &= \sum_{j=1}^{n} Y_{ij}E_iE_j[\sin(\delta_{ij}^o + \theta_{ij}) - \sin(\delta_{ij} + \theta_{ij})] \\ &= \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij}) \\ &+ \sum_{j=1}^{n} G_{ij}E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \end{aligned}$$

$$\text{for } i=1,2,\ldots,n \qquad (2.139)$$

where B_{ij} and G_{ij} are transfer susceptance and transfer conductance between the ith and the jth generators defined by

$$B_{ij} = Y_{ij}\cos\theta_{ij} \qquad G_{ij} = Y_{ij}\sin\theta_{ij} \qquad (2.140)$$

respectively. The superscript " o " denotes the stable equilibrium point of the post-fault system, so (2.139) applies to the post-fault state.

For the power systems without transfer conductances, the following Lyapunov function has been constructed and used in the preceding sections:

$$V(x) = (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}E_iE_j[\cos\delta_{ij}^o - \cos\delta_{ij} - (\delta_{ij} - \delta_{ij}^o)\sin\delta_{ij}^o]$$

$$= V_k(\omega) + V_p(\delta) \tag{2.141}$$

where V_k and V_p denote kinetic energy and potential energy, respectively. From (2.139), the time derivatives of V_k and V_p are given as follows:

$$\frac{dV_k}{dt} = 2\sum_{i=1}^{n} \sum_{j=1}^{n} G_{ij}E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij})(\omega_i - \bar{\omega}) + \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij})(\omega_i - \omega_j) \tag{2.142}$$

$$\frac{dV_p}{dt} = -\sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}E_iE_j(\sin\delta_{ij}^o - \sin\delta_{ij})(\omega_i - \omega_j) \tag{2.143}$$

where $\bar{\omega}$ is defined by

$$\bar{\omega} = \sum_{i=1}^{n} m_i\omega_i / \sum_{i=1}^{n} m_i \tag{2.144}$$

The first term in (2.142) is due to transfer conductances, and is not sign-definite. The second term of (2.142) and the first term of (2.143) are of the same magnitude, and of the opposite signs of each other. These terms cancell out with each other, and do not contribute to the damping rate of V. Consequently, the time derivative of V is given by

$$\frac{dV}{dt} = 2\sum_{i=1}^{n} \sum_{j=1}^{n} G_{ij}E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij})(\omega_i - \bar{\omega}) \tag{2.145}$$

Since the right hand term of (2.145) is not sign-definite, V in (2.141) is not a Lyapunov function for the power systems with transfer conductances. Some correction must be made on V in order to use it as Lyapunov function.

5.2 Determination of critical value

In §4, a method of determining the critical value of V was developed for the power systems without transfer conductances. This method is based on the time derivatives of V_k and V_p. These time derivatives changes their signs at the instant when the system crosses the boundary of the stability region. However, if there are transfer conducntances, the expression of the time derivative of V_k changes from (2.105) to (2.142). Hence, a little correction must be made on the method of determining the critical value.

Define g_i and g_r as follows:

$$g_i(\delta_r) = \sum_{j=1}^{n} G_{ij}E_iE_j(\cos\delta_{ij}^{o} - \cos\delta_{ij}) \tag{2.146}$$

$$g_r(\delta_r) = [\ g_2(\delta_r),\ g_3(\delta_r),\ \ldots,\ g_n(\delta_r)\ ]' \tag{2.147}$$

then (2.142) and (2.143) can be rewritten as follows:

$$\frac{dV_k}{dt} = [\ Lg_r(\delta_r) - \frac{\partial V_p}{\partial \delta_r}\ ]'\omega_r \tag{2.148}$$

$$\frac{dV_p}{dt} = \frac{\partial V_p{}'}{\partial \delta_r}\omega_r \tag{2.149}$$

or, from (2.119),

$$\frac{dV_k}{dt} = 2[\ f_r(\delta_r) + Lg_r(\delta_r)\ ]'\omega_r \tag{2.150}$$

$$\frac{dV_p}{dt} = -\ 2f_r(\delta_r)'\omega_r \tag{2.151}$$

where L is an (n-1)×(n-1) matrix defined as follows:

$$\ell_{ij} = \begin{cases} 1 - m_{(i+1)}/\sum_{k=1}^{n} m_k & \text{for } i=j \\ -\ m_{(i+1)}/\sum_{k=1}^{n} m_k & \text{for } i\neq j \end{cases} \tag{2.152}$$

The torque always acts on the system in the direction that is orthogonal to the equipotential curves, and in such way that it will synchronize the systems in the transient stability region. In the power systems without transfer conductances, only this torque exists, and it correspond (dV_k/dt) to $-(dV_p/dt)$ completely. This correspondence is lost in the power systems with transfer conductances, however, because of the new term Lg_r in (2.148) or (2.150). The torque applied to the system in the relative angular space is expressed by $(f_r + Lg_r)$. Hence, the transient stability region must be defined on the basis of $(f_r + Lg_r)$ instead of f_r. Fig. 16 illustrate an example of a trajectory in the relative angular space for the case where a fault continues. The trajectory crosses the boundary O of the transient stability region which is defined based on f_r. The angular velocity ω_r is assumed to be orthogonal to the boundary O at the point c. The torque f_r makes a right angle with ω_r at the point c, so (dV_p/dt) in (2.151) vanishes. This is the same as for the power systems without transfer conductances. The existence of transfer conductances has no influence on (dV_p/dt). However, it does not apply to (dV_k/dt). The sign of (dV_k/dt) is determined by the angle which is made by the two vectors $(f_r + Lg_r)$ and ω_r. Assume that Lg_r has a component $Lg_{r//}$ which is parallel with ω_r as shown in Fig.16. The direction of $(f_r + Lg_r)$ deviates from that of f_r, so the time derivative (dV_k/dt) vanishes at the point c' where $(f_r + Lg_r)$ makes a right angle with ω_r. The torque $(f_r + Lg_r)$ makes an obtuse angle with ω_r, and accordingly, (dV_k/dt) takes negative values before the system reaches the point c', which implies that the torque acts in such way that it will keep the system in synchronism. On the other hand, $(f_r + Lg_r)$ makes an accute angle with ω_r, and (dV_k/dt) takes positive values after the system goes over the point c', which implies that the torque acts in such way that it will separate the system into two groups. Consequently, it is derived that the point c' is on the boundary of the actual transient stability region.

The critical value of V for the first swing is defined by the value of $V_p(\delta_r)$ at the point c' instead of the point c. An example will serve to clarify the correction which is made on the method of determing the critical value. The 10-machine power system shown in Fig.17 is used. The system is disturbed by a 3-phase short-circuit which occurs at a point near the bus 11, and is cleared by opening the line connecting the buses 11 and 12 at both terminals after a certain lapse of time. Fig.18(a) and (b) show the time variations of V, V_k, V_p, (dV/dt), (dV_k/dt), and (dV_p/dt) for the case where the fault continues. As shown in Fig.18(b), (dV_k/dt) and (dV_p/dt) are not

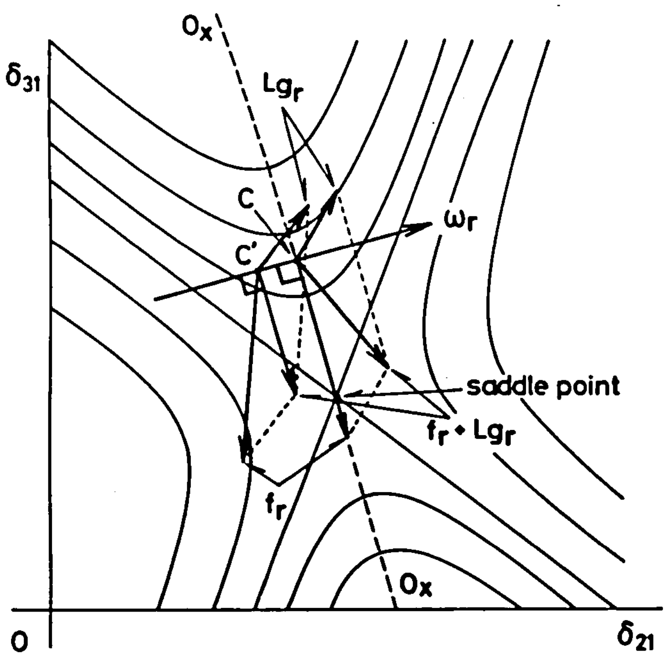


Fig.16. Relation between ω_r, f_r, and Lg_r when system crosses boundary O_x.

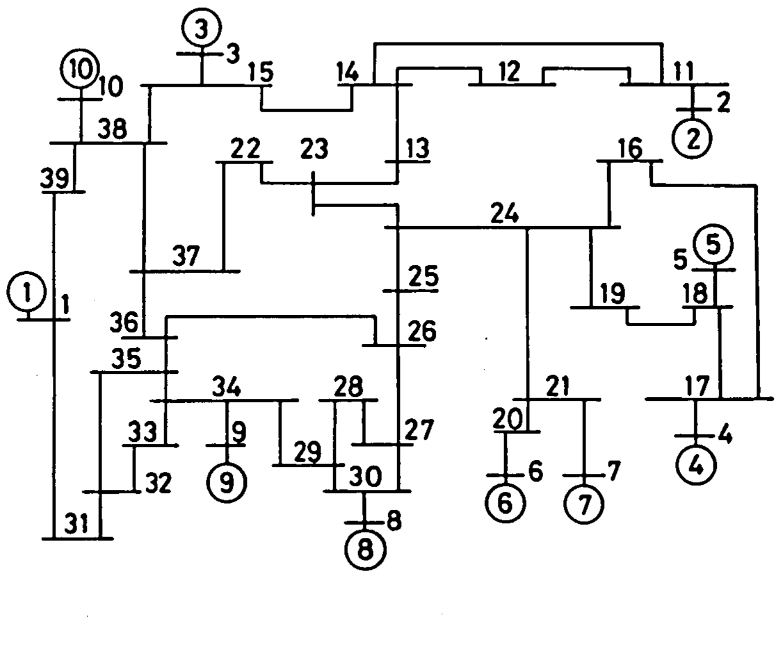
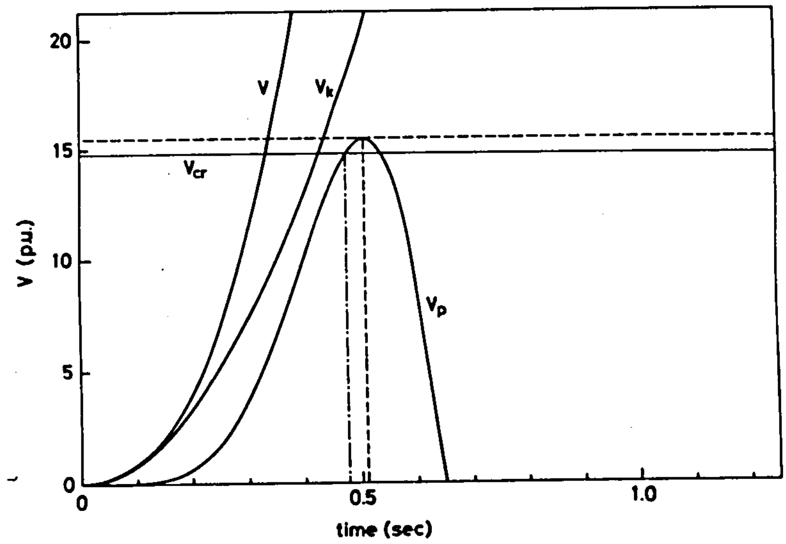
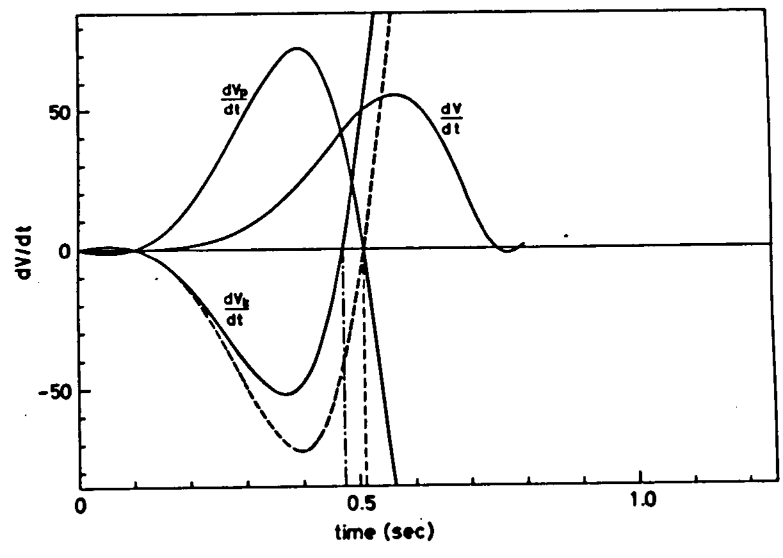


Fig.17. Configuration of 10-machine system.

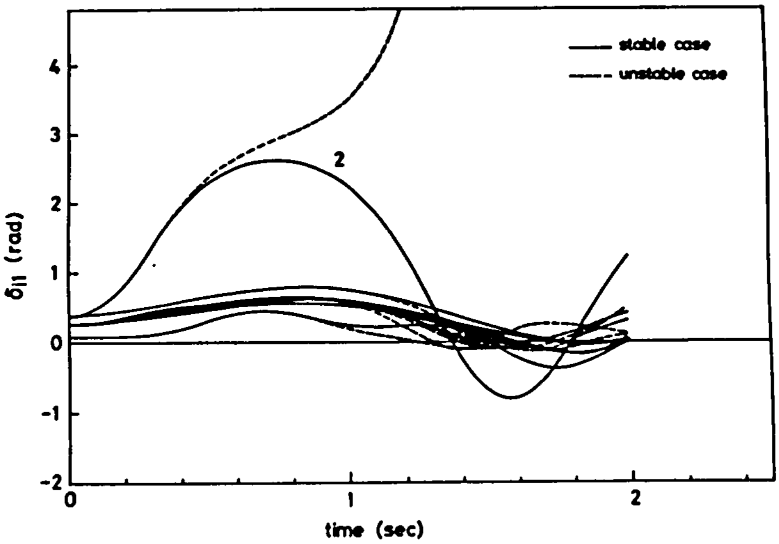


(a) V and its components

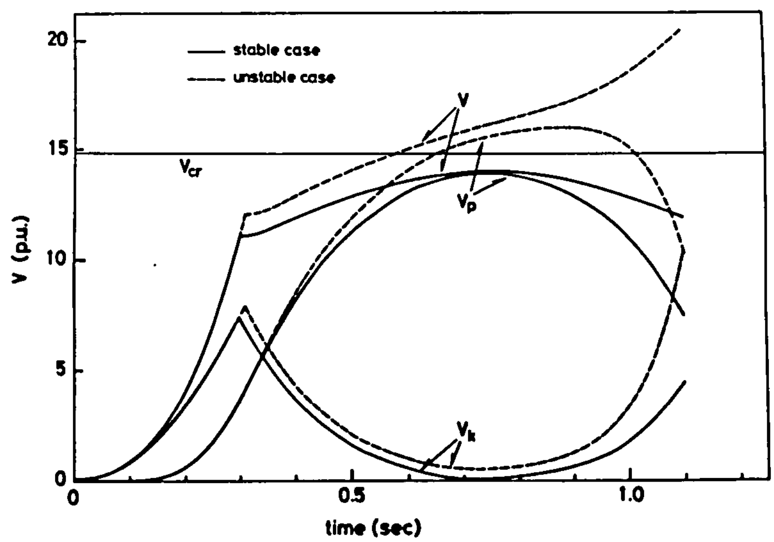


(b) Time derivatives of V
and its components

Fig.18. Method of determining critical value of V in case where transfer conductances are not negligible.



(a) Rotor angles



(b) V and its components

Fig.19. Influence of transfer conductances on V and its compoents.

of the same magnitude, and accordingly, (dV/dt) is not zero. This is due to the new term in (2.148) or (2.150). If this term is neglected, then (dV_k/dt) and (dV_p/dt) are of the same magnitude and of the opposite signs of each other, and they change their signs from negative to positive, and from positive to negative at 0.51 sec. The value of V_p at this time will be adopted as the critical value of V if the system has no transfer conductances. However, the system contains transfer conductances in actual, and (dV_k/dt) changes its sign from negative to positive at 0.47 sec. The value of V_p at this instant is used as the critical value, and it is equal to 14.753 in this case. Thus the critical value of V is obtained for the system with transfer conductances.

5.3 Correction of V

In the preceding section, a method of determining the critical value was derived for power systems with transfer conductances. It can not yields results of good accuracy by itself, however. For example, the critical value was found out to be 14.735 for the case in Fig. 18. Since V takes 14.221 at 0.33 sec, and 15.357 at 0.34 sec, it is guessed that the critical fault clearing time exists between 0.33 and 0.34 sec. However, the actual critical clearing time exists between 0.30 and 0.31 sec as shown in Fig.19(a). If the fault is cleared at 0.31 sec, the the no.2 generator steps out. The discrepancy between the estimated and the actual critical clearing times is due to transfer conductances, again.

Fig.19(b) shows the time variations of V, V_k and V_p for the case where the fault is cleared at 0.30 sec (stable case), and at 0.31 sec (unstable case). V increases monotonously during the fault-on period. In both cases, the fault is cleared before V reaches the critical value V_{cr}. In spite of this fact, the system remains in synchronism in one case, and it loses synchronism in the other case. V slowly increases after the instant of fault clearance. In the stable case, V reaches a peak, and decreases afterwards. It should be noted that the peak value is smaller than the critical value V_{cr}. On the other hand, in the unstable case, V increases monotonously, and it becomes greater than V_{cr} after the instant 0.58 sec. These facts suggest that the fault must be cleared in such way that V does not become greater than V_{cr} after fault clearance. In Fig.19(a), V takes 10.960 at 0.30 sec, and 14.000 (peak value) at 0.75 sec in the stable case. The difference between them is 3.040. Namely, this energy is generated in the period between 0.30 and 0.75 sec. If it is possible to know this value beforehand, then we can add it to V in estimating the critical clearing time. By comparing this V with the critical value V_{cr}, we can get an accurate estimation of the critical clearing time. This is the basic idea for correcting V for the power systems with transfer conductances.

From (2.148) and (2.149), the time derivative of V is rewritten as follows:

$$\frac{dV}{dt} = 2Lg_r(\delta_r)'\omega_r \tag{2.153}$$

Hence, ΔV, the increment of V in a time interval, can be given as follows:

$$\begin{aligned} \Delta V &= \int_{t_s}^{t_e} \frac{dV}{dt}\, dt \\ &= \int_{t_s}^{t_e} 2Lg_r(\delta_r)'\omega_r dt \\ &= \int_{\delta_s}^{\delta_e} 2Lg_r(\delta_r)' d\delta_r \\ &\equiv \Delta V(\delta_s \sim \delta_e) \end{aligned} \tag{2.154}$$

where t_s and t_e are both ends of the time interval, and δ_s and δ_e are the values of δ_r at the instants t_s and t_e, respectively. The increment ΔV is determined only by the trajectory in the relative angular space, and it does not matter how long it takes the system to moves from δ_s to δ_e. Lets return to Fig.16. This figure illustrates the trajectory in the relative angular space for the fault continues. The point c' is on the boundary of the actual stability region. If the system goes over this point, then the system will lose synchronism, afterwards. The value of V_p at the point c' is used as the critical value of V. If the fault is critically cleared, the system moves along the trajectory, and reaches the point c', then a relation holds as follows:

$$V(t_{cr}) + \Delta V(\delta_{cr} \sim \delta_{c'}) = V_{cr} \tag{2.155}$$

where t_{cr} is the critical clearing time, and δ_{cr} and $\delta_{c'}$ are the values of δ_r at the instant t_{cr} and the point c', respectively. If the fault is cleared before t_{cr}, then the system can not reach the point c', that is, stable. Conversely, if the fault is cleared after t_{cr}, then the system goes over the point c', that is, unstable. Hence, two inequalities hold for these two cases as follows:

$$\begin{aligned} V(t) + \Delta V(\delta_r \sim \delta_{c'}) &< V_{cr} \qquad \text{for } t < t_{cr} \\ V(t) + \Delta V(\delta_r \sim \delta_{c'}) &> V_{cr} \qquad \text{for } t > t_{cr} \end{aligned} \tag{2.156}$$

where t is the clearing time, and δ_r denotes one at the instant t. Lets define a function $V_\alpha(t)$ as follows:

$$V_\alpha(t) = V(t) + \Delta V(\delta_r \sim \delta_{c'}) \tag{2.157}$$

Since (2.155) or (2.156) holds for each clearing time, it is possible to get the critical clearing time by comparing $V_\alpha(t)$ with the critical value V_{cr}. Fig.20 shows the time variations of V_α and its components for the same case as in Fig.19. V_α is greater than V by ΔV as is cleare from the definition. ΔV takes some value at the initial time, but decreases with time, and vanishes at the time when the system crosses the boundary of the stability region. As a result, V_α approaches V with time. The critical value V_{cr} is 14.753. Since V_α takes 13.964 at 0.29 sec, and 14.830 at 0.30 sec, the critical clearing time is estimated to be 0.299 sec. The actual critical clearing time exists between 0.30 and 0.31 sec, so the above result is adequately accurate.

One method of correcting V have been derived, now. It is able to take account of transfer conductances in estimating critical clearing times. It is not desirable, however, that there exist large discrepancies between V and V_α in the initial period if we were to treat V_α as Lyapunov function, which causes us to derive another method of correcting V.

The increments of V_k and V_p denoted by ΔV_k and ΔV_p are given as follows:

$$\Delta V_k = \int_{t_s}^{t_e} \left(Lg_r - \frac{\partial V_p}{\partial \delta_r}\right)' \omega_r \, dt$$

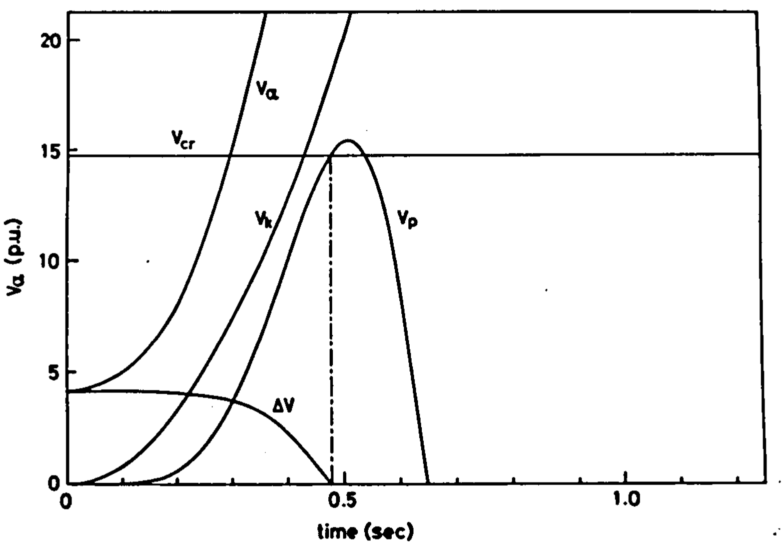


Fig.20. Time variations of V_α and its components: sustained fault.

$$= \Delta V(\delta_s \sim \delta_e) - [V_p(\delta_e) - V_p(\delta_s)] \qquad (2.158)$$

$$\Delta V_p = V_p(\delta_e) - V_p(\delta_s) \qquad (2.159)$$

It is observed from these equations that ΔV_k is determined by the trajectory in the relative angular space, and the ΔV_p is determined by only δ_s and δ_e. Define a ratio γ of ΔV_p to ΔV_k as follows:

$$\gamma \equiv |\Delta V_p/\Delta V_k| \qquad (2.160)$$

By substituting (2.158) and (2.159) into (2.160), γ is expressed as follows:

$$\gamma = \frac{1}{1 - \Delta V/\Delta V_p} \qquad (2.161)$$

The ratio γ varies with $\Delta V/\Delta V_p$ as follows:

$$\begin{aligned} \gamma &> 1 \qquad \text{for} \quad \Delta V/\Delta V_p > 0 \\ \gamma &= 1 \qquad \text{for} \quad \Delta V/\Delta V_p = 0 \qquad (2.162) \\ \gamma &< 1 \qquad \text{for} \quad \Delta V/\Delta V_p < 0 \end{aligned}$$

If transfer conductances are zero, the second equation in (2.162) holds, and ΔV_k is equal to $-\Delta V_p$. On the other hand, if transfer conductances are not zero, γ deviates from 1, and the first or the third inequalities hold. As is observed from Fig.19(b), V_k decreases, and V_p increases after the instant of fault clearance. At the instant when V reaches the peak value, the most part of V is occupied by V_p, and V_k is nearly zero. Let t_c and t_p denote the clearing time and the time when V reaches the peak value, respectively. Energy is transferred from V_k to V_p in the period between t_c and t_p. The ratio γ is, of course, greater than 1 in this period. Since V is the sum of V_k and V_p, V increases. Assume that all of V_k is transferred to V_p, then the peak value can be given as follows:

$$V(t_p) = V_p(t_c) + \gamma V_k(t_c) \qquad (2.163)$$

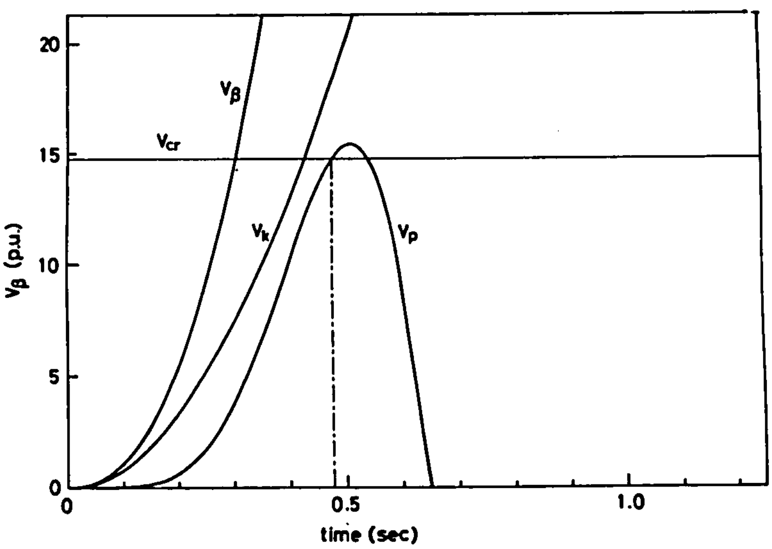


Fig.21. Time variations of V_β and its components: sustained fault.

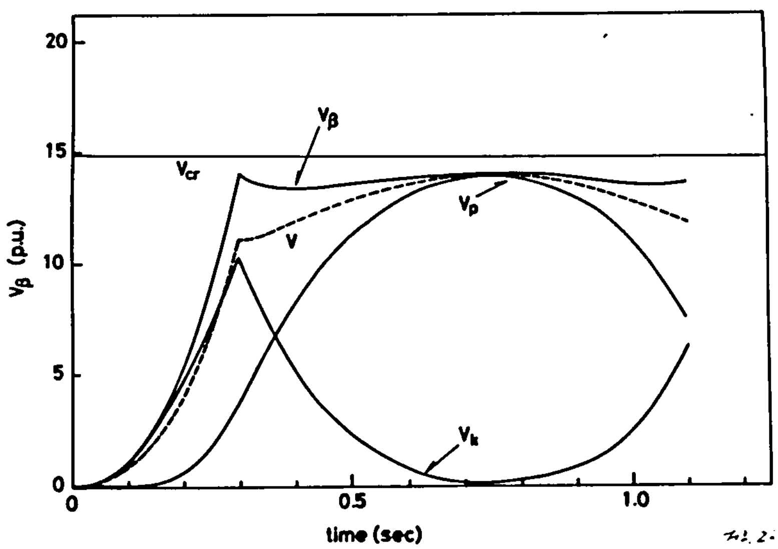


Fig.22. Time variations of V_β and its components: critically cleared fault.

where γ is that in the period between t_c and t_p. The fault must be cleared in such way that the following inequality is satisfied:

$$V(t_p) < V_{cr} \tag{2.164}$$

Eq.(2.163) and (2.164) suggests that it is useful to make a correction on V as follows:

$$V_\beta(t) = V_p(t) + \gamma V_k(t) \tag{2.165}$$

in estimating the critical fault clearing time. It is desirable to use the same value of γ as in (2.163), but it is impracticable because t_c is, of course, unknown in the stage of estimation. It is clear from (2.161) that γ is determined by the trajectory in the relative angular space. If it is assumed that the system moves along the trajectory for the case where the fault continues, in the period after the fault clearance, too, then γ takes the same values for the two cases in which the fault continues, and the fault is cleared at a certain time. Lets return to Fig.18(b). The time derivative (dV_k/dt) takes negative values in the period between t_s and t_c'. On the other hand, (dV_p/dt) takes positive values in this period. Namely, V_k decreases, and V_p increases. The value of γ in (2.165) is defined as follows:

$$\gamma \equiv \left| \int_{t_s}^{t_c'} (dV_p/dt)dt \Big/ \int_{t_s}^{t_c'} (dV_k/dt)dt \right| \tag{2.166}$$

The time t_s generally does not coincides with the critical clearing time t_{cr}, so the values of γ in (2.163) and (2.165) are not equal. Fig.21 shows the time variations of V_β and its components for the same case as in Fig.19. The value of γ is 1.399. V_k is initially zero, so V and V_β take the same value at 0.0 sec. The difference between them becomes large with increase in V_k. The critical value V_{cr} is 14.753. V_β takes 14.062 at 0.30 sec, and 15.242 at 0.31 sec, so the critical clearing time is estimated to be 0.306 sec. The actual critical clearing time exists between 0.30 and 0.31 sec, so this result is accurate. Fig.22 shows the time variations of V_β and its components for the case where the fault is cleared at 0.30 sec. The value of γ is that in Fig.21, i.e., 1.399. V_β varies a little after the fault clearance, but its extent is small compared with V. V_β takes 14.062 at the clearing time. It is very close to the peak value of V, 14.000, which implies that V is actually corrected according to our basic idea.

5.4 Conclusions

In this section, we have developed a method of taking account of transfer conductances into the Lyapunov function derived in §3 under the assumption that transfer conductances are negligible. In practical systems, generators receive additional torques due to transfer conductances. In order to take into account these torques, some corrections have been made in determining the critical value. Namely, it is given by the value of the potential energy V_p at the instant when the time derivative of the kinetic energy V_k changes its sign from negative to positive under a given fault. On the other hand, the time derivative of V is not zero. It deviates from zero to positive, and to negative with time. In order to eliminate this deviation, we have made some correction on V in such the way that V is kept constant after the instant of fault clearance. As a result, two new functions have been derived by adding some terms to the original function V. We have obtained some results of good accuracy in estimating the critical clearing time for the example cases.

§6. Numerical example

In this section, the transient stability of a 10-machine power system is studied. Two different methods are used to analyze the transient stability. Firstly, a reasonably comprehensive series of classical stability analyses using the accepted method of step-by-step simulation. Next, Lyapunov's direct method developed in the preceding sections is applied. Lastly, their results are compared. These two techniques differ markedly in their approaches as well as in the kind of information which they provide; the systems variables in a simulation are explicit functions of time, for example, whereas in the transient energy method they are not. Both methods can be used to compute critical clearing times however, so this familiar transient stability performance measure is used as a basis of comparison.

The line diagram of the system is shown in Fig.23, again. The data on its transmission lines and generators are shown in Table 5 and 6, res-

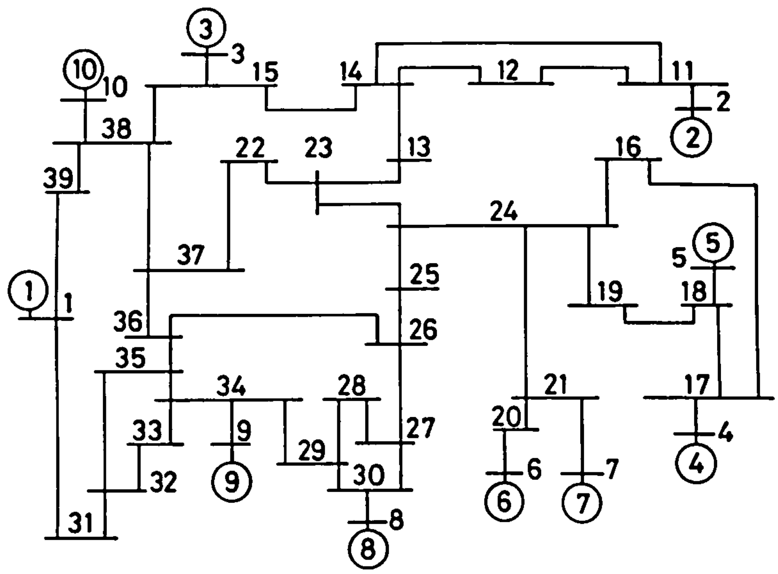


Fig.23. Configuration of 10-machine system

Table 5. Generator parameters of 10-machine system

No.	H	X_d'
1	500.0	0.0060
2	34.5	0.0570
3	24.3	0.0570
4	26.4	0.0490
5	34.8	0.0500
6	26.0	0.1320
7	28.6	0.0436
8	35.8	0.0531
9	30.3	0.0697
10	42.0	0.0310

Table 6. Transmission line constants of 10-machine system.

BUS	BUS	R	X	S	TAP
1	31	0.0010	0.0250	1.2000	1.0
1	39	0.0010	0.0250	0.7500	1.0
2	11	0.0008	0.0156	0.0	1.025
3	15	0.0006	0.0232	0.0	1.025
4	17	0.0005	0.0272	0.0	1.0
5	18	0.0	0.0143	0.0	1.025
6	20	0.0009	0.0180	0.0	1.009
7	21	0.0007	0.0142	0.0	1.070
8	30	0.0	0.0200	0.0	1.070
9	34	0.0	0.0250	0.0	1.070
10	38	0.0	0.0181	0.0	1.025
11	12	0.0014	0.0151	0.2490	1.0
11	14	0.0057	0.0625	1.0290	1.0
12	14	0.0043	0.0474	0.7802	1.0
13	14	0.0014	0.0147	0.2396	1.0
13	23	0.0013	0.0173	0.3216	1.0
14	15	0.0032	0.0323	0.5130	1.0
15	38	0.0070	0.0086	0.1460	1.0
16	17	0.0022	0.0350	0.3610	1.0
16	24	0.0003	0.0059	0.0680	1.0
17	18	0.0006	0.0096	0.1846	1.0
18	19	0.0008	0.0140	0.2565	1.0
19	24	0.0008	0.0135	0.2548	1.0
20	21	0.0007	0.0138	0.0	1.060
21	24	0.0016	0.0195	0.3040	1.0
22	23	0.0007	0.0082	0.1319	1.0
22	37	0.0011	0.0133	0.2138	1.0
23	24	0.0007	0.0089	0.1342	1.0
24	25	0.0009	0.0094	0.1710	1.0
25	26	0.0018	0.0217	0.3660	1.0
26	27	0.0009	0.0101	0.1723	1.0
26	36	0.0008	0.0129	0.1382	1.0
27	28	0.0016	0.0435	0.0	1.006
27	30	0.0004	0.0043	0.0729	1.0
29	28	0.0016	0.0435	0.0	1.006
29	30	0.0004	0.0043	0.0729	1.0
29	34	0.0007	0.0082	0.1389	1.0
31	32	0.0023	0.0363	0.3804	1.0
32	33	0.0004	0.0046	0.0780	1.0
32	35	0.0008	0.0112	0.1476	1.0
33	34	0.0006	0.0092	0.1130	1.0
34	35	0.0002	0.0026	0.0434	1.0
35	36	0.0008	0.0126	0.1342	1.0
36	37	0.0013	0.0213	0.2214	1.0
37	38	0.0013	0.0151	0.2572	1.0
38	39	0.0035	0.0411	0.6987	1.0

Table 7. Load conditions of 10-machine system

(a) light load

BUS	V	ANGLE	P_g (MW)	Q_g (MVA)	P_ℓ (MW)	Q_ℓ (MVA)
1	1.0300	0.000	500.00	44.00	557.86	323.41
2	1.0265	8.461	415.00	-126.87	0.0	0.0
3	1.0635	5.900	270.00	-86.83	0.0	0.0
4	1.0635	8.642	280.00	-15.23	0.0	0.0
5	1.0493	7.198	325.00	38.08	0.0	0.0
6	1.0123	5.782	254.00	67.76	0.0	0.0
7	0.9972	6.135	316.00	-22.47	0.0	0.0
8	0.9831	5.599	325.00	10.14	0.0	0.0
9	0.9820	4.704	281.60	23.38	4.60	2.30
10	1.0475	3.138	125.00	29.47	0.0	0.0
11	1.0696	5.112	0.0	0.0	141.70	13.40
12	1.0766	3.751	0.0	0.0	103.00	13.80
13	1.0768	1.083	0.0	0.0	140.50	37.70
14	1.0839	2.035	0.0	0.0	69.50	8.50
15	1.0728	2.696	0.0	0.0	112.00	23.60
16	1.0713	1.681	0.0	0.0	154.30	46.09
17	1.0684	4.796	0.0	0.0	123.70	42.40
18	1.0713	4.887	0.0	0.0	0.0	0.0
19	1.0672	2.761	0.0	0.0	137.00	57.50
20	1.0082	3.271	0.0	0.0	340.00	51.50
21	1.0688	3.873	0.0	0.0	0.0	0.0
22	1.0691	0.751	0.0	0.0	79.00	15.00
23	1.0716	1.147	0.0	0.0	0.0	0.0
24	1.0687	1.623	0.0	0.0	164.70	16.10
25	1.0607	0.956	0.0	0.0	160.00	76.50
26	1.0548	1.121	0.0	0.0	0.0	0.0
27	1.0523	1.895	0.0	0.0	0.0	0.0
28	1.0484	1.839	0.0	0.0	4.20	44.00
29	1.0501	1.842	0.0	0.0	0.0	0.0
30	1.0518	2.230	0.0	0.0	0.0	0.0
31	1.0473	-0.093	0.0	0.0	0.0	0.0
32	1.0424	-0.154	0.0	0.0	261.00	88.30
33	1.0431	0.080	0.0	0.0	117.00	42.00
34	1.0478	1.101	0.0	0.0	0.0	0.0
35	1.0476	0.773	0.0	0.0	0.0	0.0
36	1.0506	0.249	0.0	0.0	250.00	92.00
37	1.0655	0.635	0.0	0.0	161.00	1.20
38	1.0689	2.009	0.0	0.0	0.0	0.0
39	1.0560	0.761	0.0	0.0	0.0	0.0

(b) heavy load

BUS	V	ANGLE	P_g (MW)	Q_g (MVA)	P_ℓ (MW)	Q_ℓ (MVA)
1	1.0300	0.000	1000.00	88.00	1095.07	260.25
2	1.0265	17.042	830.00	15.56	0.0	0.0
3	1.0278	11.779	540.00	-6.75	0.0	0.0
4	1.0635	17.637	560.00	95.39	0.0	0.0
5	1.0493	14.705	650.00	207.65	0.0	0.0
6	1.0123	11.612	508.00	161.26	0.0	0.0
7	0.9972	12.317	632.00	105.15	0.0	0.0
8	0.9831	11.428	650.00	198.49	0.0	0.0
9	0.9820	9.587	563.30	203.60	9.20	4.60
10	1.0475	6.260	250.00	143.30	0.0	0.0
11	1.0508	10.323	0.0	0.0	283.50	269.00
12	1.0511	7.567	0.0	0.0	206.00	276.00
13	1.0397	2.042	0.0	0.0	281.00	75.50
14	1.0535	4.058	0.0	0.0	139.00	17.00
15	1.0586	5.327	0.0	0.0	224.00	47.20
16	1.0398	3.278	0.0	0.0	308.60	92.20
17	1.0463	9.794	0.0	0.0	247.50	84.80
18	1.0515	9.991	0.0	0.0	0.0	0.0
19	1.0340	5.557	0.0	0.0	274.00	115.00
20	0.9924	6.519	0.0	0.0	680.00	103.00
21	1.0522	7.768	0.0	0.0	0.0	0.0
22	1.0335	1.336	0.0	0.0	158.00	30.00
23	1.0361	2.172	0.0	0.0	0.0	0.0
24	1.0345	3.159	0.0	0.0	329.40	323.00
25	1.0185	1.751	0.0	0.0	320.00	153.00
26	1.0156	2.140	0.0	0.0	0.0	0.0
27	1.0186	3.779	0.0	0.0	0.0	0.0
28	1.0049	3.661	0.0	0.0	8.50	88.00
29	1.0172	3.669	0.0	0.0	0.0	0.0
30	1.0217	4.481	0.0	0.0	0.0	0.0
31	1.0298	-0.267	0.0	0.0	0.0	0.0
32	0.9999	-0.559	0.0	0.0	522.00	176.60
33	1.0011	-0.066	0.0	0.0	233.80	84.00
34	1.0120	2.101	0.0	0.0	0.0	0.0
35	1.0095	1.413	0.0	0.0	0.0	0.0
36	1.0076	0.271	0.0	0.0	500.00	184.00
37	1.0327	1.096	0.0	0.0	322.00	2.40
38	1.0500	3.961	0.0	0.0	0.0	0.0
39	1.0482	1.488	0.0	0.0	0.0	0.0

pectively. This system is one of the IEEE standard power systems, and has been used by several researchers, so a comprehensive amount of knowledge of this system has been accumulated until now. In our studies, it is assumed that this system is disturbed by a 3-phase short-circuit which occurs at a terminal x of a transmission line x-y, and is cleared by opening the line at both terminals. The fault location is changed in order that its effect on the critical clearing time can be investigated. The critical clearing time will vary according to the fault location, of course. It is interesting whether Lyapunov's direct method developed in the preceding section is able to well take account of the change in the fault location. The studies are made under two different load conditions which are shown in Table 7 (a) and (b). One of them is severe than the other, and they are refered as heavy load and light load, respectively. With increase in loads, it is expected that the critical clearing time will be short, and the transient stability of the system will be deteriorated.

6.1 Procedure of estimation

The procedure for estimating the critical clearing time is shown in the flow chart of Fig.24. The main steps are as follows:

[Step 1] Read the necessary data on the system, i.e., those on the transmission lines, the buses, and the generators.

[Step 2] Compute the load flow for the prefault state.

[Step 3] Compute the reduced admittance matrices between the generators by eliminating the buses without generators, for the fault and the post-fault states.

[Step 4] Compute the stable equilibrium point for the post-fault state.

[Step 5] Integrate the fault system equations step by step, and compute the rotor angles and their speeds; $\delta(t)$ and $\omega(t)$.

[Step 6] Examine whether the system has reached the boundary of the stability region by observing the sign of the time derivative of the kinetic energy, i.e., (dV_k/dt). If the system has reached the boundary, then go to Step 7, and if not, then return to Step 5.

[Step 7] Compute the critical value V_{cr}, which is equal to the value of the potential energy V_p at the instant when the system reaches

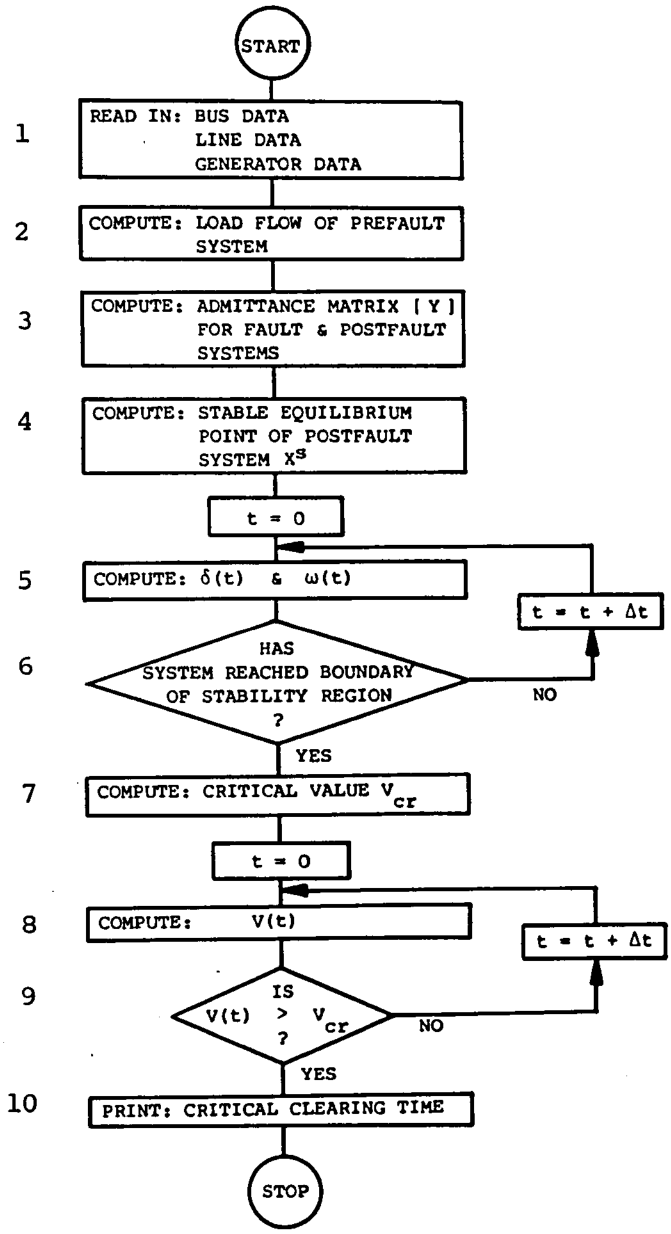


Fig.24. Flow chart for estimating critical fault clearing time by Lyapunov's direct method.

the boundary of the stability region.

[Step 8] Compute the value of the Lyapunov function V(t).

[Step 9] Compare V(t) and V_{cr}. If V(t) is greater than V_{cr}, go to Step 10, and if not, then return to Step 8.

[Step 10] Print the critical clearing time.

Some comments on those steps should be made. The integration time step length in Step 5 is 0.01 sec in this study although more rough step length might be adequate to the study. The method of determing the critical value V_{cr} has been developed in §4. Steps 6 and 7 serve to determine the critical value. In Step 8, V is used as Lyapunov function, but it will be replaced by V_α or V_β for those cases where transfer conductances are not negligible. In this study, they are, of course, not negligible, so V_α and and V_β are also used, and the results with these functions are compared with those with the original function V. The method of producing those functions have been described in §5.

6.2 Results by simulations

Firstly, the conventional approach based on simulations are applied to the transient stability analysis of the 10-machine power system. The system equation (2.26) is integrated step by step to yield swing curves of generators, where damping torques of generators are assumed to be zero in this study. In this method, a given fault is cleared at an appropriate instant, and the corresponding swing curves are calculated. The clearing time is adjusted by observing the swing curves. If the system is stable for the clearing time, then it is delayed , and if the system is unstable, then the clearing time is advanced. By iterating these manipulations, the critical clearing time is obtained. It takes 4 or 5 times in general for the iteration to converge. Fig. 25 and 26 show the swing curves for the eight cases of fault locations under the light load and the heavy load conditions, respectively. In each figure, only two cases of swing curves are shown, that is, the cases near the critical one. The difference of clearing time between the two case is 0.01 sec, and the critical clearing time exists between the two clearing times. It is observed from the figures that the step-out generators vary with fault location.. For example, no.2 generator steps out for fault 11-12 while no.3 generator steps out for

(a) Stable Case (b) Unstable Case

(1) Fault 11-12

(2) Fault 15-14

(3) Fault 17-18

(4) Fault 18-17

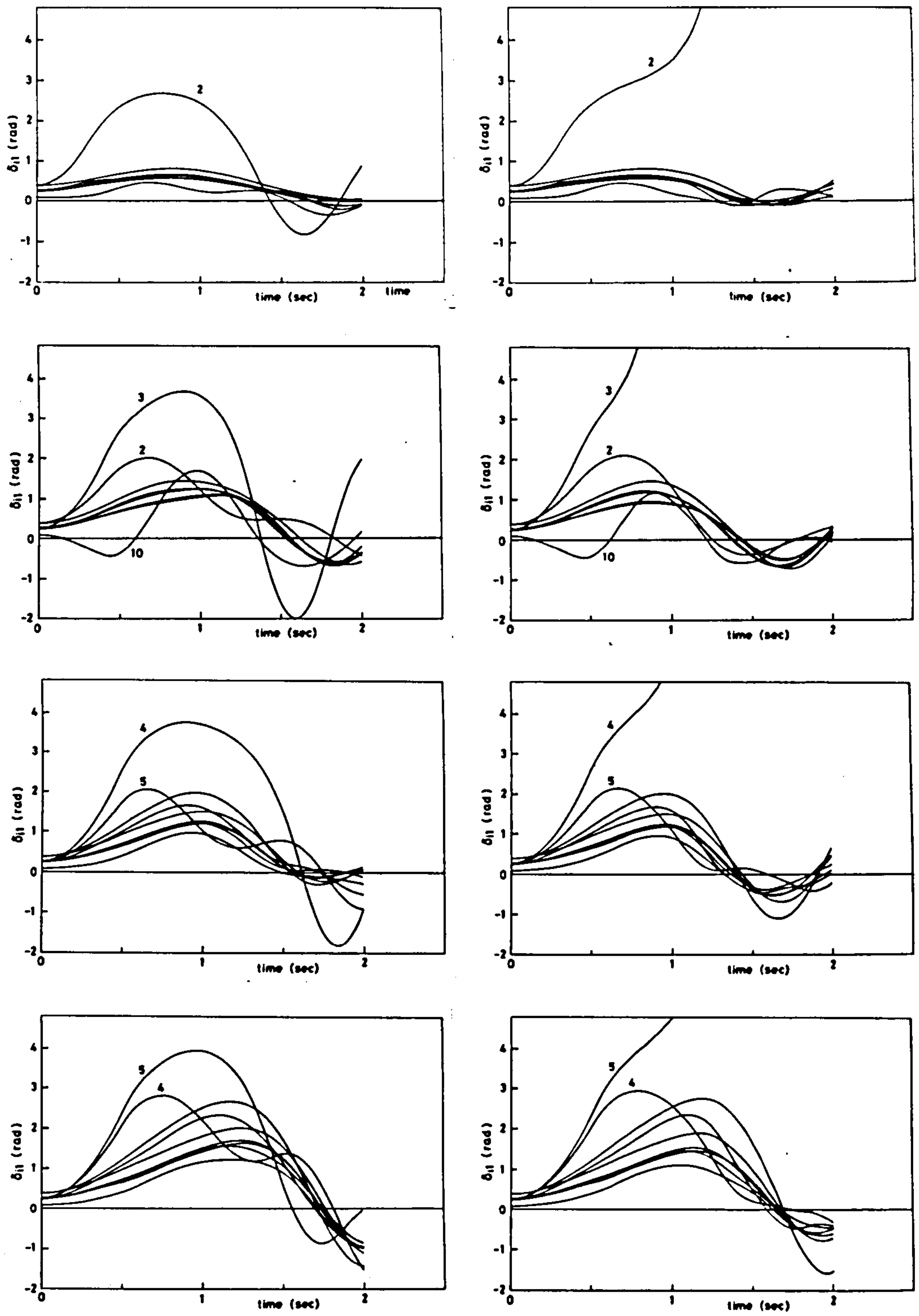
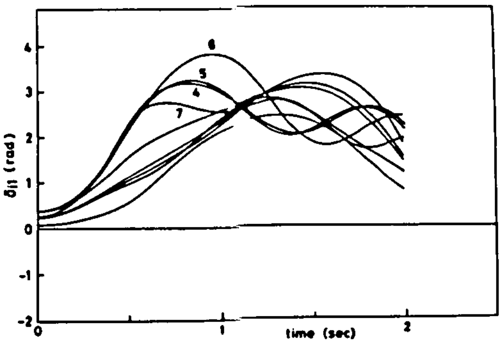
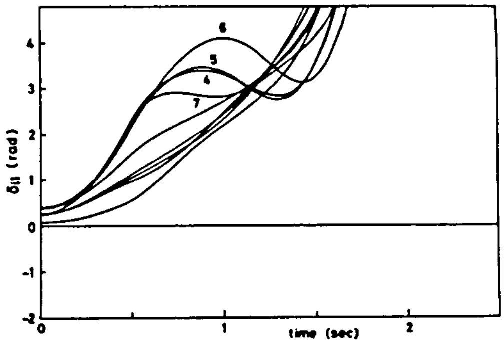


Fig.25 Swing curves of generators for eight cases of fault location: light load

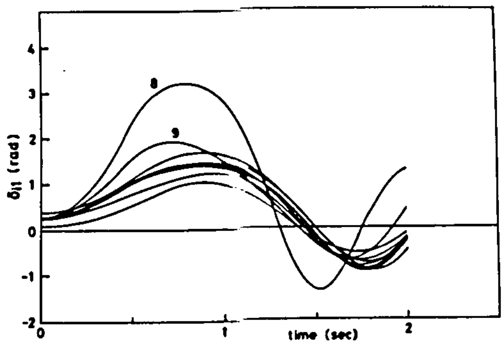
(a) Stable Case (b) Unstable Case

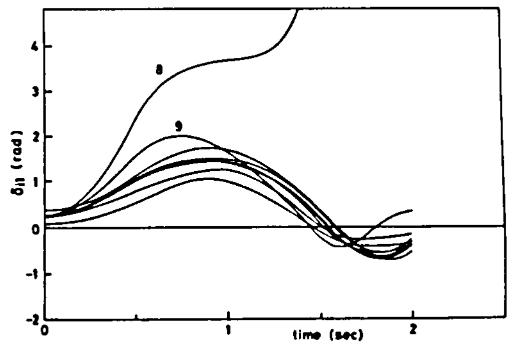
(5) Fault 24-16



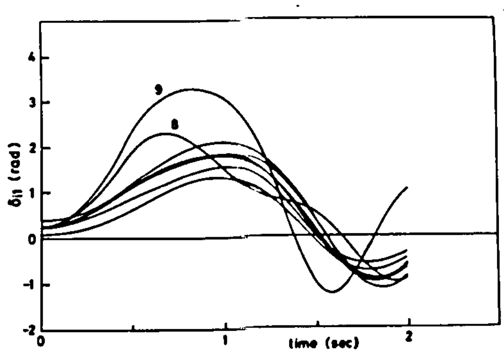


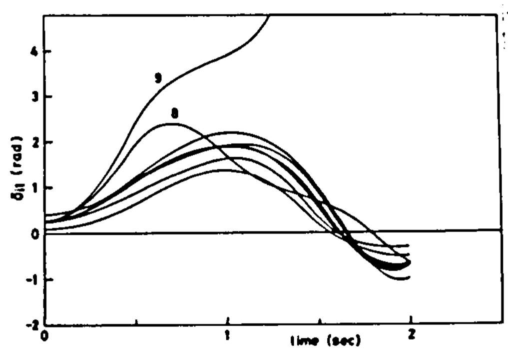
(6) Fault 30-27



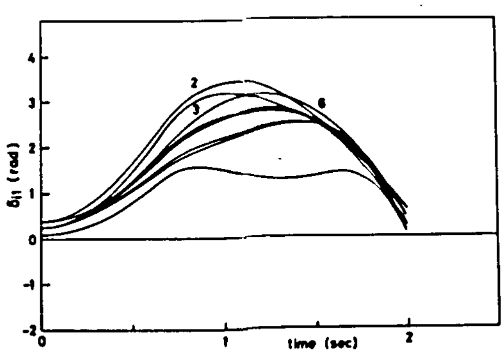


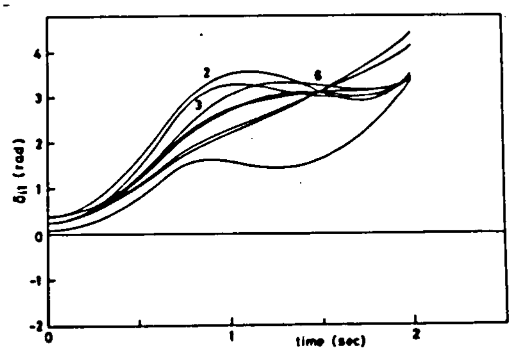
(7) Fault 34-27





(8) Fault 38-15





(a) Stable Case (b) Unstable Case

(1) Fault 11-12

(2) Fault 15-14

(3) Fault 17-18

(4) Fault 18-17

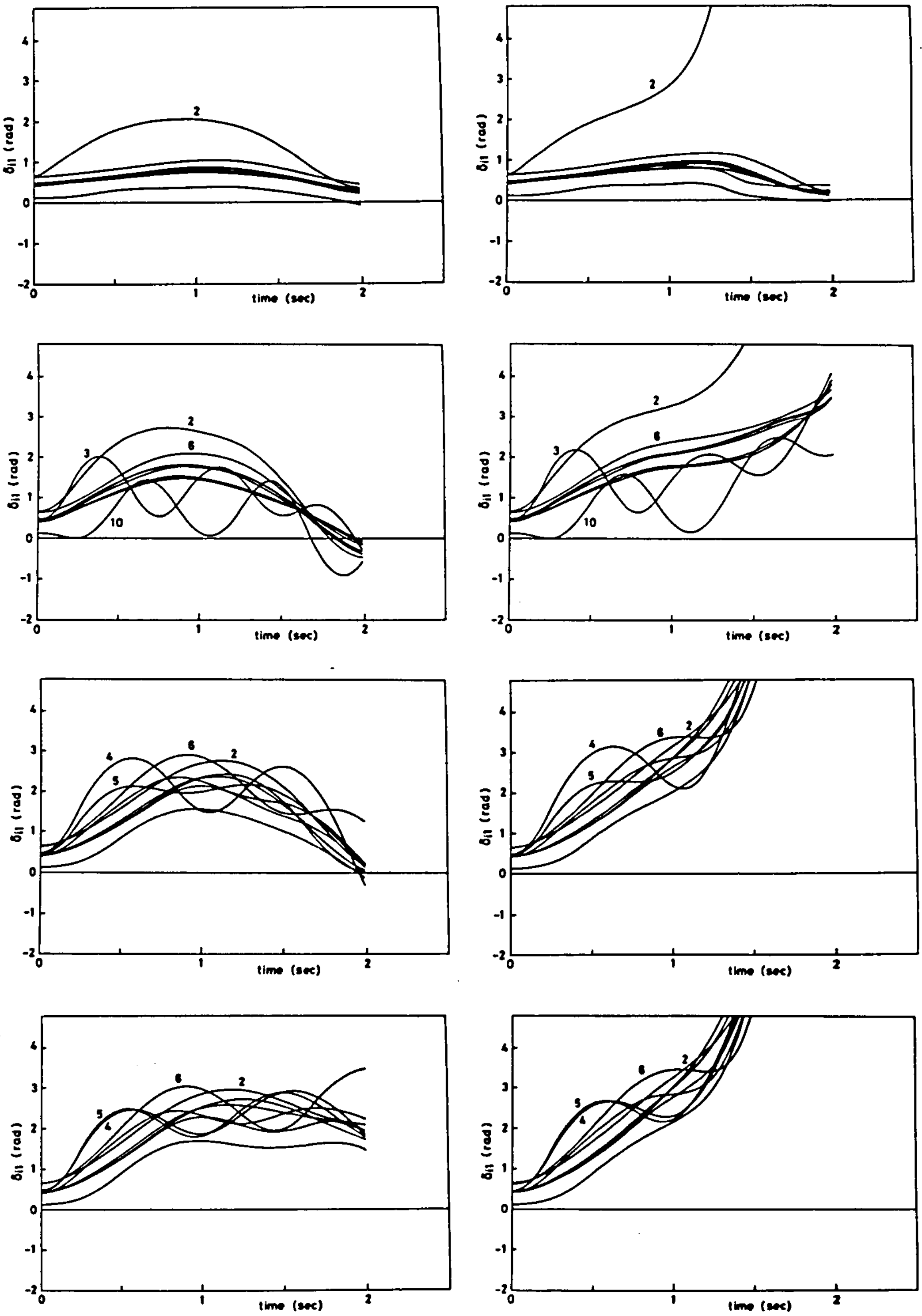


Fig.26 Swing curves of generators for eight cases of fault location: heavy load

(a) Stable Case (b) Unstable Case

(5) Fault 24-16

(6) Fault 30-27

(7) Fault 34-29

(8) Fault 38-15

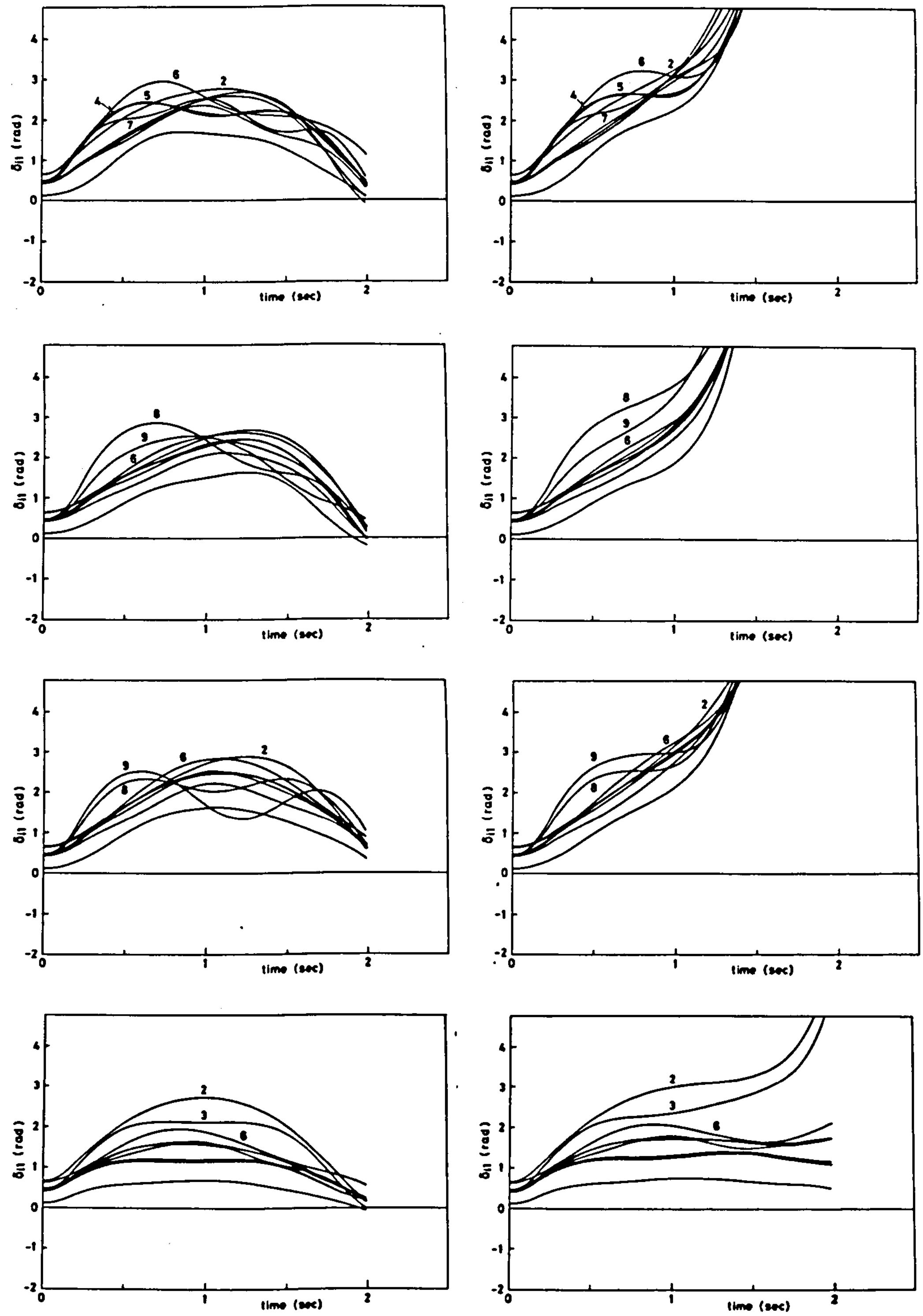


Table 8. Critical fault clearing times obtained by simulations

(a) light load

Fault	Clearing times(sec) stable	unstable	Unstable generators
11 - 12	0.30	0.31	2
15 - 14	0.41	0.42	3
17 - 18	0.47	0.48	4
18 - 17	0.50	0.51	5
24 - 16	0.45	0.46	1
30 - 27	0.47	0.48	8
34 - 29	0.46	0.47	9
38 - 15	0.65	0.66	1

(b) heavy load

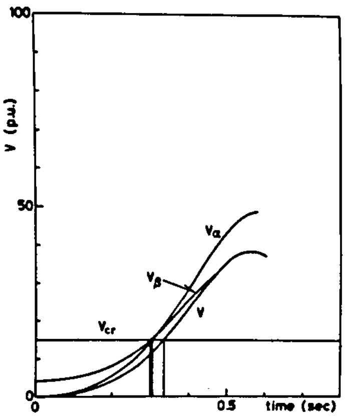
Fault	Clearing times(sec) stable	unstable	Unstable generators
11 - 12	0.06	0.07	2
15 - 14	0.20	0.21	1
17 - 18	0.22	0.23	1
18 - 17	0.22	0.23	1
24 - 16	0.18	0.19	1
30 - 27	0.23	0.24	1
34 - 29	0.22	0.23	1
38 - 15	0.17	0.18	1

fault 15-14. In general, the generators near the fault location suffer large disturbances even though all of them do not necessarily step out. There are several cases where all generators in the system except no.1 generator which represents equivalently an adjacent power system, are separated from no.1 generator. The cases of fault 24-16 and 38-15 under the light load condition, and all faults except 11-12 under heavy load condition fall into such the cases. In these cases, the interconnection between the power systems is disjoined because of some fault occured in on them. It should be noted that only one generator near the fault location mainly steps out under the light load condition while one of the power system is mainly separated from the other under the heavy load condition. It is guessed that large power is supplied into this system from the adjacent system, and as a result, the stability margin between them becomes low, and some disturbance occured in this system causes them to separate easily. The results by simulations are summarized in Table 8(a) and (b). The critical clearing time exists in ranges of 0.30 ∿ 0.66 and 0.06 ∿ 0.24 sec for the cases of light load and heavy load, respectively. With increase in loads, the critical clearing time proves to be shortened very much. In practical transient stability analyses, many cases of load conditions are studied in order to determine the operating policies of the power system.

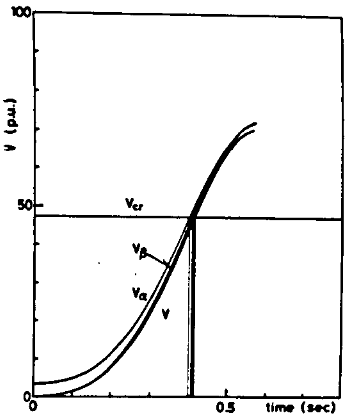
6.3 Results by Lyapunov's direct method

Secondly, Lyapunov's direct method is applied to the transient stability analysis of the 10-machine power system. In this method, the system equation (2.26) is integrated step by step for a given fault, where the fault is not cleared, to yield the time variations of the Lyapunov function V. In this study, transfer conductances are not negligible, so the functions V_α and V_β are also calculated. The instants when these functions reach the critical value V_{cr} are adopted as the estimations of the critical fault clearing time. The critical value V_{cr} is defined as the value of the potential energy V_p at the instant when the time derivative of the kinetic energy V_k changes its sign from negative to positive. Hence, the integration of (2.26) is continued till that instant.

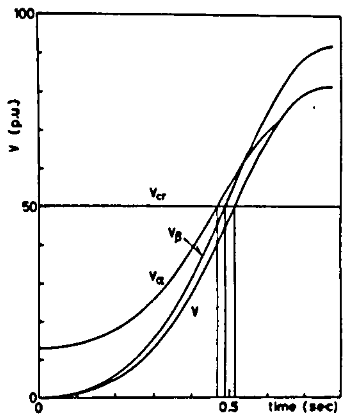
Fig.27 and 28 show the time variations of V, V_α, and V_β for the eight cases of fault locations under the light load and the heavy load conditions,



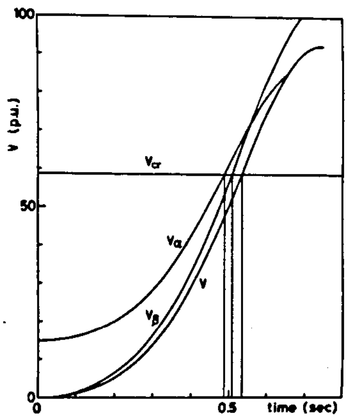
(1) Fault 11-12



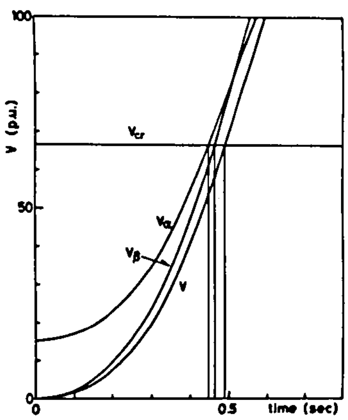
(2) Fault 15-14



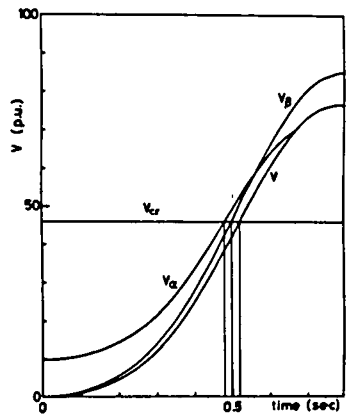
(3) Fault 17-18



(4) Fault 18-17

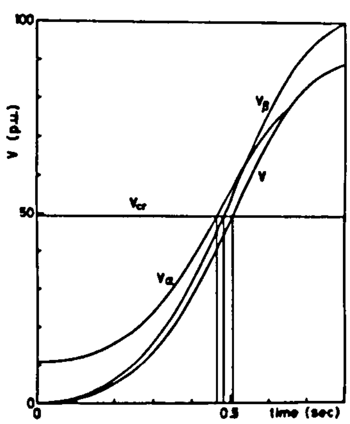


(5) Fault 24-16

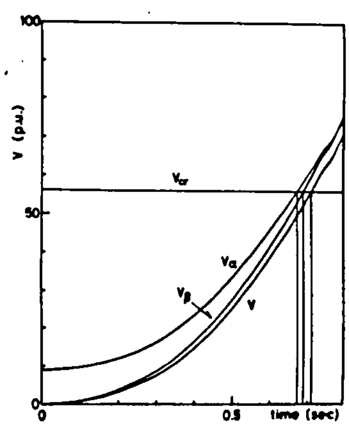


(6) Fault 30-27

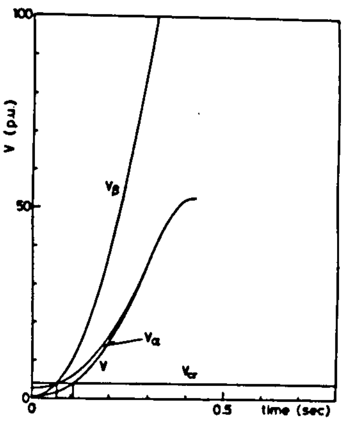
Fig.27 Estimation of critical clearing time by Lyapunov's direct method: light load



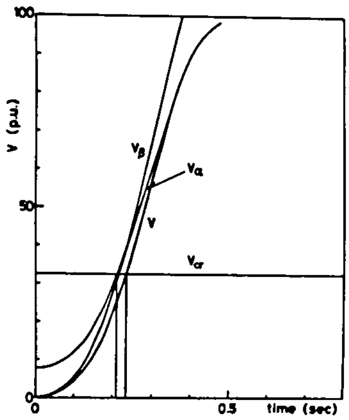
(7) Fault 34-29



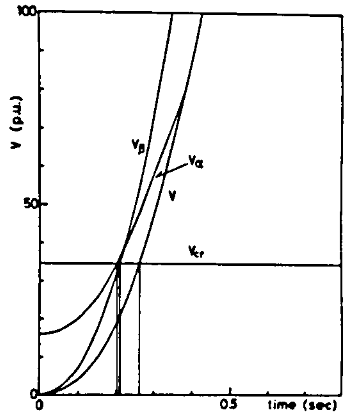
(8) Fault 38-15



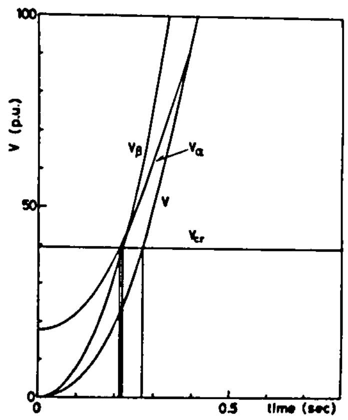
(1) Fault 11-12



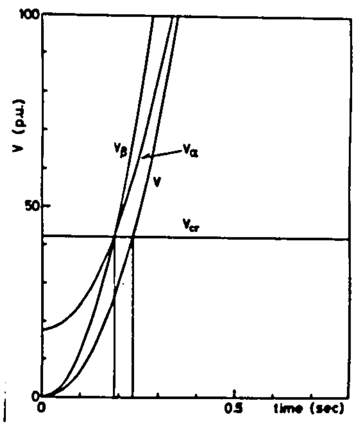
(2) Fault 15-14



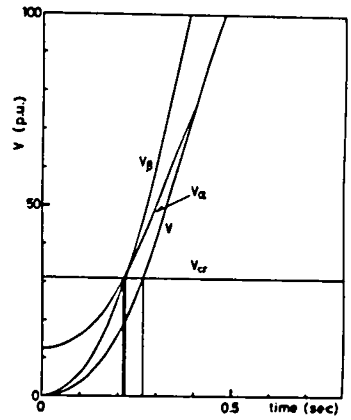
(3) Fault 17-18



(4) Fault 18-17

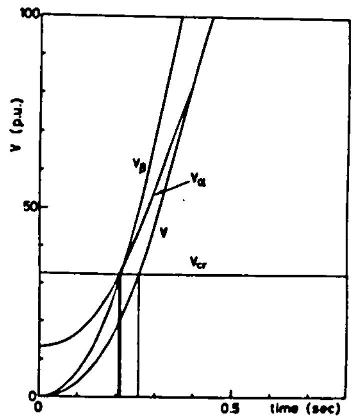


(5) Fault 24-16

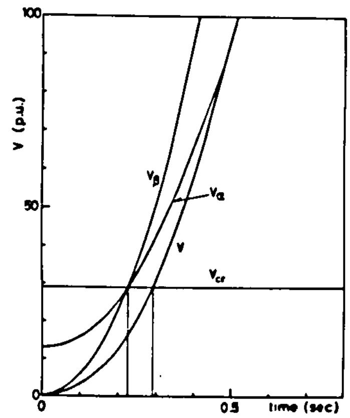


(6) Fault 30-27

Fig.28 Estimation of critical clearing time by Lyapunov's direct method: heavy load



(7) Fault 34-29



(8) Fault 38-15

Table 9. Critical fault clearing times estimated by Lyapunov's direct method (sec).

(a) light load

Fault	γ	V_{cr}	V	V_α	V_β	T_{cr}
11-12	1.398	14.75	0.33	0.29	0.30	0.30
15-14	1.077	47.11	0.42	0.40	0.41	0.41
17-18	1.351	50.17	0.51	0.47	0.49	0.47
18-17	1.347	58.61	0.53	0.48	0.50	0.50
24-16	1.305	66.41	0.49	0.44	0.46	0.45
30-27	1.275	45.89	0.51	0.47	0.49	0.47
34-29	1.281	49.17	0.50	0.46	0.48	0.46
38-15	1.189	56.12	0.71	0.67	0.69	0.65

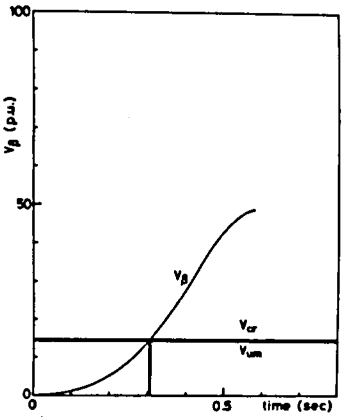
(b) heavy load

Fault	γ	V_{cr}	V	V_α	V_β	T_{cr}
11-12	2.719	3.91	0.10	0.06	0.06	0.06
15-14	1.318	32.46	0.23	0.21	0.21	0.20
17-18	1.880	34.38	0.26	0.20	0.21	0.22
18-17	1.857	39.18	0.27	0.21	0.21	0.22
24-16	1.763	41.96	0.23	0.18	0.18	0.18
30-27	1.703	31.01	0.26	0.21	0.22	0.23
34-29	1.731	32.33	0.25	0.20	0.21	0.22
38-15	1.841	28.78	0.28	0.22	0.22	0.17

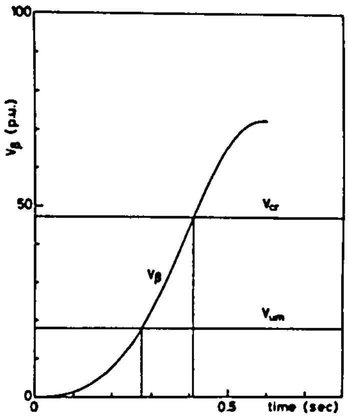
T_{cr}: critical clearing time obtained by simulations.

respectively. The function V is nearly equal to zero at the instant when each fault occurs. However, it increases monotonously during the fault-on period. Namely, the total energy stored in the system increases with time. In order to keep the system in synchronims, the fault must be cleared before the total energy reaches the critical value V_{cr}. In the case of fault 11-12, for example, V reaches V_{cr} between 0.33 and 0.34 sec which implies that the system is stable if the fault is cleared at 0.33 sec, and that it is unstable if the fault is cleared at 0.34 sec. Thus we can get an estimation of the critical clearing time. However, the influence of transfer conductances is not taken into account by using the function V, so V_α and V_β are used in order to take account of transfer conductances. V_α approaches V with time although it takes some amount of positive value at the instant when the fault occurs. On the other hand, V_β takes the same value as V at that instant, but separates from V with time. As a whole, both V_α and V_β take greater values than V all the time. In the case of fault 11-12, for example, V_α and V_β reach the critical value V_{cr} at the instants between 0.29 and 0.30, and between 0.30 and 0.31 sec, respectively. These results are very close to that by the conventional approach based on simulations. Table 9(a) and (b) shows the results by Lyapunov's direct method for the cases of the light load and the heavy load condition, respectively, along with those by simulations. The results with V_α and V_β are closer to those by simulations than V. In all the fault cases, V yields optimistic results compared with the actual stability by 0.01 $\sim$ 0.06 sec under the light load condition, and by 0.03 $\sim$ 0.11 sec under the heavy load condition. Its extent is more conspicuous in the heavy load case than in the light load case. This fact seems to be related with the fact that transfer conductances take larger values in the former case than in the latter case. In both the cases, V_α and V_β yield the results whose differeneces with those by simulations are within 0.02 sec, except the case of fault 11-12. It should be noted that the results with V_α are in general smaller than those with V_β by 0.00 $\sim$ 0.02 sec. As a whole, we can get adequately accurate results by using any of these two functions, but it is desirable to use V_α rather than V_β because the former function yields more safe results than the latter.

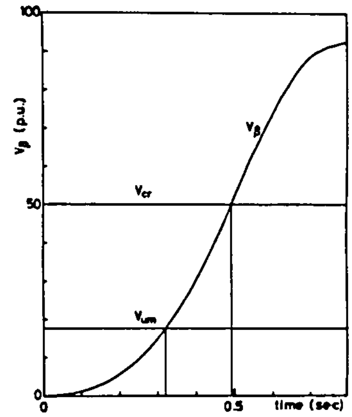
An example should serve to clarify the superiority of the new critical value V_{cr} introduced in §4 to the usual critical value V_{min} in the accuracy of estimation of the critical clearing time. As the basis of comparison, V_β



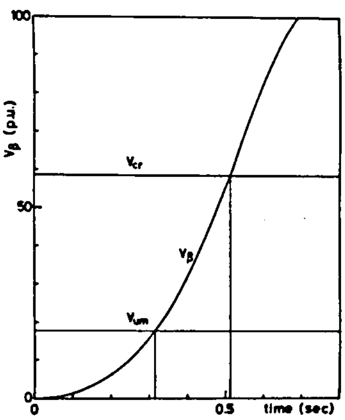
(1) Fault 11-12



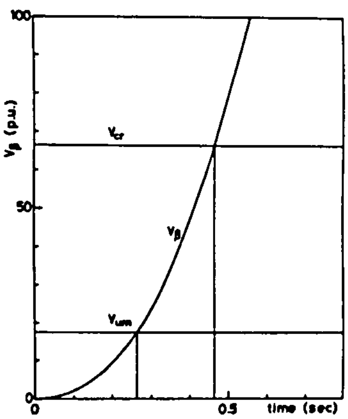
(2) Fault 15-14



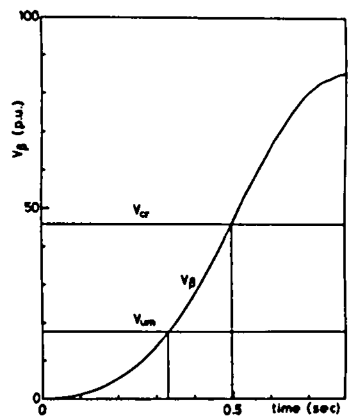
(3) Fault 17-18



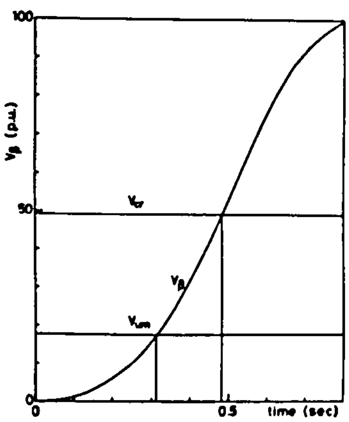
(4) Fault 18-17



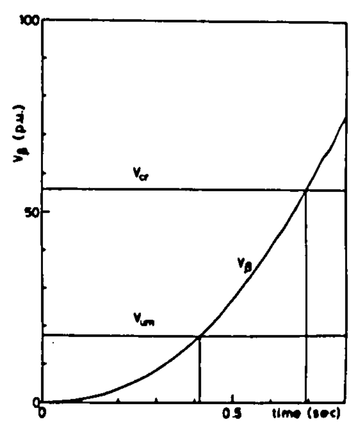
(5) Fault 24-16



(6) Fault 30-27

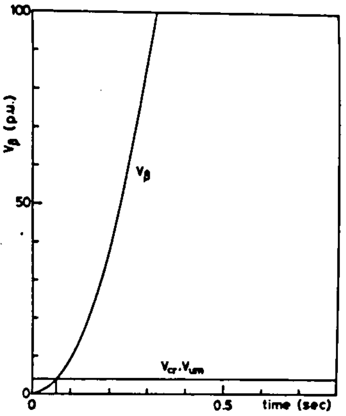


(7) Fault 34-29

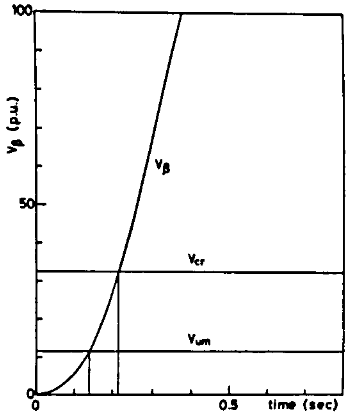


(8) Fault 38-15

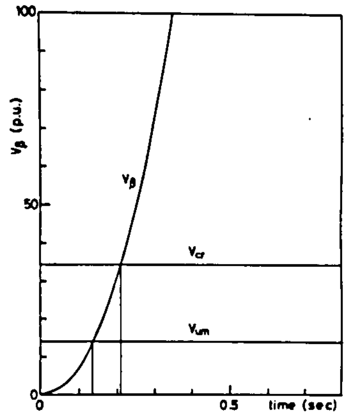
Fig.29 Influence of critical value on estimation by the direct method: light load



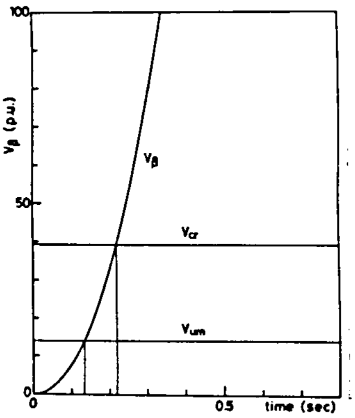
(1) Fault 11-12



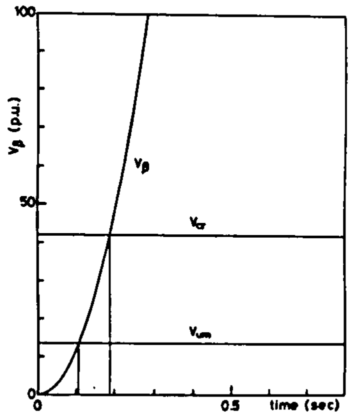
(2) Fault 15-14



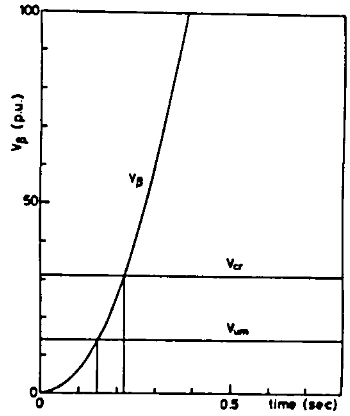
(3) Fault 17-18



(4) Fault 18-17

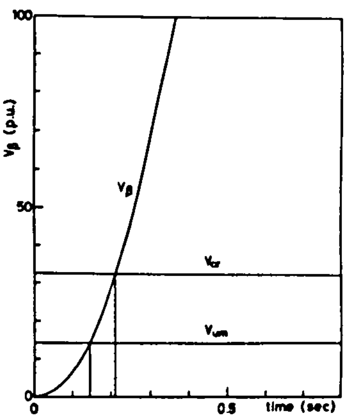


(5) Fault 24-16

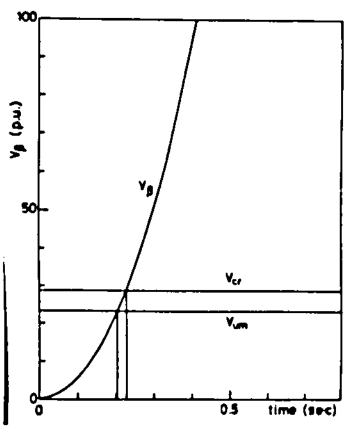


(6) Fault 30-27

Fig.30 Influence of critical value on estimation by the direct method: heavy load



(7) Fault 34-29



(8) Fault 38-15

Table 10. Influence of critical value on estimation of critical fault clearing time by Lyapunov's direct method.

(a) light load

Fault	Critical values		Critical clearing times(sec)		
	V_{um}	V_{cr}	V_{um}	V_{cr}	T_{cr}
11 - 12	14.22	14.75	0.30	0.30	0.30
15 - 14	17.91	47.11	0.27	0.41	0.41
17 - 18	17.67	50.17	0.32	0.49	0.47
18 - 17	17.67	58.61	0.31	0.50	0.50
24 - 16	17.48	66.41	0.26	0.46	0.45
30 - 27	17.77	45.89	0.33	0.49	0.47
34 - 29	17.67	49.17	0.31	0.48	0.46
38 - 15	17.69	56.13	0.41	0.69	0.65

(b) heavy load

Fault	Critical values		Critical clearing times(sec)		
	V_{um}	V_{cr}	V_{um}	V_{cr}	T_{cr}
11 - 12	4.09	3.90	0.06	0.06	0.06
15 - 14	11.63	32.52	0.14	0.21	0.20
17 - 18	14.02	34.38	0.13	0.21	0.22
18 - 17	14.02	39.19	0.13	0.21	0.22
24 - 16	13.57	41.93	0.10	0.18	0.18
30 - 27	14.04	31.04	0.15	0.22	0.23
34 - 29	14.24	32.35	0.14	0.21	0.22
38 - 15	13.80	28.70	0.15	0.22	0.17

T_{cr}: critical clearing time obtained by simulations.

is used as Lyapunov function. Fig.29 and 30 show the time variations of V_β for the eight cases of fault location under the light load and the heavy load conditions, respectively. In the figures, the critical values V_{cr} and V_{um} are shown, too. It is observed from those figures that there is an amount of difference between V_{cr} and V_{um} for all the fault cases except fault 11-12. V_{um} takes almost the same value irrespective of fault location, which is due to the fact that the lowest saddle point does not vary so much with fault location. Accordingly, the estimated value of the critical value deviates a little with fault location mainly owing to some changes in the time variations of V_β. On the other hand, V_{cr} varies very much with fault location. Namely, it can take account of fault location. The instants when V_β reaches V_{cr} and V_{um} are adopted as the respective estimations of the critical clearing time. In the case of fault 15-14, for example, V_β reaches V_{um} and V_{cr} at the instants between 0.27 and 0.28 sec, and between 0.41 and 0.42 sec, respectively. Since the actual critical clearing time exists between 0.41 and 0.42 sec, the result with V_{um} is very conservative whereas V_{cr} yields the result very close to the actual one. Table10(a) and (b) show the results of estimation for the cases of the light load and the heavy load conditions, respectively. In the case of fault 11-12, V_{um} and V_{cr} take almost the same values, and the estimated values of the critical clearing time is 0.30 and 0.06 sec under the light load and the heavy load conditions, respectively. This fault case is only one in which V_{um} and V_{cr} give the same results. In the other cases, the two critical values yields different results. V_{um} takes values in ranges of 17.48 ∿ 17.91, and 11.63 ∿ 14.24 under the two load conditions while V_{cr} takes values in comparatively wide ranges of 45.89 ∿ 66.41, and 28.70 ∿ 41.93. The estimated value of the critical clearing time with V_{um} is smaller than the actual one by 0.14 ∿ 0.24 sec under the light load condition, and by 0.02 ∿ 0.09 sec under the heavy load conditions, respectively. The above discrepancy is somewhat large in view of the fact that the esimated value with V_{cr} falls into a range of 0.02 and 0.01 sec about the actual value under the light load and the heavy load conditions, respectively, except for the fault 38-15. Hence, it is concluded that V_{cr} is superior to V_{um} in the estimation accuracy.

Lastly, we should make some investigation on the case of fault 38-15. In this case, V_α and V_β yield the results which are relatively larger than

Table 11. Iterations of estimation of critical fault clearing time for fault 38-15 (sec).

(a) light load

No.	V_{cr}	V_α	V_β	γ
1	56.12	0.67	0.69	1.189
2	53.06	0.65	0.69	1.176

(b) heavy load

No.	V_{cr}	V_α	V_β	γ
1	28.70	0.22	0.22	1.831
2	21.92	0.19	0.19	1.890
3	19.73	0.18	0.18	1.891

the actual critical clearing times. It is guessed that there is large difference between the sustained fault trajectory and the fault trajectory cleared at some instant, and that the critical value V_{cr} is different from the actual one to an extent. One method of improving the critical value has been described in §4.5. In this method, V_{cr} is updated by iteratively clearing the fault at the clearing time which is estimated to be critical in the previous estimation. The results are shown in Table 11(a) and (b) for the two cases of load condition. In the light load case, V_{cr} changes from 56.12 to 53.06. With this new critical value, the crtical clearing time is estimated to be 0.65 and 0.67 sec by using V_α and V_β, respectively. On the other hand, in the heavy load case, V_{cr} changes from 28.70 to 21.92, and from 21.92 to 19.73. With these new critical values, the estimated value changes from 0.22 to 0.19 sec, and from 0.19 to 0.18 sec by using any of V_α and V_β. Thus, we can get adequately accurate estimations by updating V_{cr} one or two times. In the tables, the value of γ is also shown. It does not vary so much with iteration, and has little influence on the time variation of V_β. By iterating the above procedure, more accurate results will be obtained, but the iteration should be limited at most to one time in order that the computational advantages are not lost.

6.4 Conclusions

In this section, we have made some transient stability analysis of a 10-machine power system. The Lyapunov function, and the methods of determining the critical value, and of taking account of transfer conductances developed in §3, 4, and 5 have been applied to this analysis. The results are summarized as follows:

1) The original Lyapunov function V can not take account of transfer conductances, so it has yielded somewhat optimistic results compared with the actual stability.
2) The functions V_α and V_β have been able to take account of transfer conductances to some extent, and they have yielded adequately accurate results whose difference with the actual critical clearing time is within 0.02 sec except one fault case.
3) The usual critical value V_{um} has yielded very conservative results compared with the actual critical clearing time.

4) The new critical value V_{cr} has accurately showed the boundary of the transient stability region for almost all the fault cases.
5) In the cases where V_{cr} is different from the actual critical value to some extent, more accurate critical value has been obtained by iterating the procedure in §4.5 one or two times.

Thus, we have get rid of the conservative nature of Lyapunov's direct method by introducing the new critical value. Since the method of determining this critical value does not need any calculation of unstable equilibrium points, so we have removed the computational difficulty which has been pointed out in the calculation of the usual critical value, at the same time. Besides, we have take account of transfer conductances which are not negligible in practical power systems by introducing the two functions. By using these functions, and the new critical value, results of practical significance will be obtained for other multimachine power systems with actual loads.

Chapter III

TRANSIENT STABILITY ANALYSIS OF MULTIMACHINE POWER SYSTEM VIA LYAPUNOV'S DIRECT METHOD: DYNAMICS OF FIELD FLUX LINKAGES

§1. Introduction

In this chapter, we are concerned with the transient stability analysis of multimachine power systems in which dynamics of field flux linkages of generators are incorporated in their system representation.

In Chapter II, we have made some investigations on Lyapunov's direct method applied to the transient stability analysis of power systems in which each generator is represented by a constant voltages behind a transient reactance, and have developed it to the point where it can yield results of practical significance in the analysis. As a natural extension, we proceed to some investigations on the direct method applied to the transient stability analysis of power systems in which dynamics of field flux linkages in the transient period are taken into account. These flux linkages generally decrease in the period owing to the armature reactions of generators, and as a result, the transient stability of the systems is deteriorated to some extent compared with the results obtained under the assumption that those linkages are kept constant in the transient period. By taking account of their dynamics, we are able to get the results which are closer to the actual stability of the systems than usual. It should be noted, however, that dynamics of control equipments, such as automatic voltage regulators and governors, are not taken into account, so that the systems has some stability margins compared with the results which are obtained with the system representation.

Lyapunov's direct method consists of two main parts, that is, the construction of the Lyapunov function and the determination of its crtical value.

Firstly, the former part is investigated. There is a systematic method of constructing Lyapunov functions based on the generalized Popov criterion derived by J.B. Moore and B.D.O. Anderson, i.e. Theorem 2 [51]. It was proposed by J.L. Willems in a process of constructing a Lyapunov function for a multimachine power system, where it is represented by the conventional model, in 1970 [33]. The obtained Lyapunov function is equivalent to the energy integral function [54]. It is suitable for estimating the transient stability of the system. His method was applied and refined by several researchers [35, 36, 37]. It is not applicable to the systems in which dyanmics of field flux linkages are taken into account, because Theorem 2 used as the basis applies to systems with single-variable nonlinear elements whereas those systems belong to a set of multivariable dynamical systems with multivariable nonlinear elements. In 1974, M.A. Pai and V. Rai derived another generalized Popov criterion which is applicable to those systems. [28]. They applied it to a one-machine connected to an infinite bus system in which dynamics of field flux linkage is taken into account, and constructed a Lyapunov function. Their criterion is applicable also to multimachine power systems, but it needs a particular transformation of system equations, and as a result, it makes it difficult to apply the systematic method proposed by J.L. Willems. It is necessary to derive a new criterion applicable to more general form of systems, and was derived by us in 1979, i.e., Theorem 3. §2 is adressed to the construction of a new Lyapunov function based on this new criterion.

Secondly, the latter part, that is, the determination of the critical value is investigated. Several investigations have been made on the critical value [38-40]. In the usual method, the value of the potential energy at the saddle point closest to the stable equilibrium point has been adopted as the critical value, and it was the main cause of the conservative nature of the direct method. This conservative nature has been removed by introducing the new critical value which corresponds with the first swing stability [70, 47]. For the systems in which dyanmics of field flux linkages are taken into account, a few investigations have been made until now. [41, 64]. J.L. Willems pointed out that there is a new saddle point which is closest to the stable equilibrium point[41], but the critical value given on the basis of this saddle point yields very conservative results even for one-machine connected to an infinite bus systems. H. Sasaki treated field flux linkages as parameters, and proposed to use the critical value which varies with those parameters [64]. This method has a possibility of yielding results of practical significance. In §3, we make some considerations on equipotential curves, define the transient stability region, and develop one method of determining the critical value of Lyapunov function.

Thirdly, we generalize the method of taking account of transfer conductances developed in the previous chapter in order to make it applicable to the system under investigation in §4.

Lastly, we make a transient stability analysis of a 10-machine power system by Lyapunov's direct method, and verify its effectiveness in §5.

§2. Construction of Lyapunov function [72,73]

In this section, a Lyapunov function is systematically constructed on the basis of Theorem 3 in §2 of chap. II. The outline of the method has been shown in the process of constructing a Lyapunov function for the system represented with the conventional model. We naturally follow it with some changes which are characteristic of the system in which dynamics of field flux linkages of generators are taken into account. We begin this section with a derivation of the system equations of the system.

2.1 System equation

In transient stability analyses, an n-machine power system in which dynamics of field flux linkages of generators are taken into account, is usually described as follows:

$$m_i \frac{d^2\delta_i'}{dt^2} + d_i \frac{d\delta_i'}{dt} = P_{mi} - \sum_{j=1}^{n} Y_{ij}E_iE_j\sin(\delta_{ij} + \theta_{ij}) \tag{3.1}$$

and

$$T_{doi}' \frac{dE_{qi}'}{dt} = E_{fdi} - E_{qi}' - (x_{di} - x_{di}')i_{di} \tag{3.2}$$

where, for generator i,

P_{mi} : mechanical power input,

m_i : angular momentum constant,

d_i : damping power coefficient,

$Y_{ij}\angle\phi_{ij}$: post-fault transfer admittance between the ith and the jth generator nodes (obtained after reduction of a network retaining only generator nodes),

θ_{ij} : complement of ϕ_{ij}, i.e., $\theta_{ij} = \pi/2 - \phi_{ij}$,

$E_i\angle\delta_i$: internal voltage,

δ_{ij} : $\delta_i - \delta_j$,

$E'_{qi}\angle\delta'_i$: voltage related with the internal voltage as in Fig. 31, and δ'_i indicates a rotor angle relative to a reference frame rotating at synchronous speed,

E_{fdi}: excitation voltage,

i_{di} : d-axis current,

x_{di}, x'_{di}: d-axis synchronous, transient reactances,

T'_{doi}: d-axis transient open-circuit time constant.

In order to construct a Lyapunov function, two basic assumptions are necessary:

1) Each internal voltage lags behind the q-axis of each generator by a constant angle ϕ_i all the time [64].
2) The transfer conductances in the reduced admittance matrix are negligible.

Under the assumption 1), (3.1) and (3.2) change to the equations which describe the variations of the internal voltages as follows:

$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = P_{mi} - \sum_{j=1}^{n} Y_{ij}E_iE_j\sin(\delta_{ij} + \theta_{ij}) \tag{3.3}$$

and

$$T'_{doi}\frac{dE_i}{dt} = \frac{E_{fdi}}{\cos\phi_i} - E_i + (x_{di} - x'_{di})\sum_{j=1}^{n} Y_{ij}E_j\cos(\delta_{ij} + \theta_{ij})$$
$$- \tan\phi_i \cdot (x_{di} - x'_{di})\sum_{j=1}^{n} Y_{ij}E_j\sin(\delta_{ij} + \theta_{ij}) \tag{3.4}$$

In the cases where ϕ_i is small, the last term in (3.4) can be neglected.

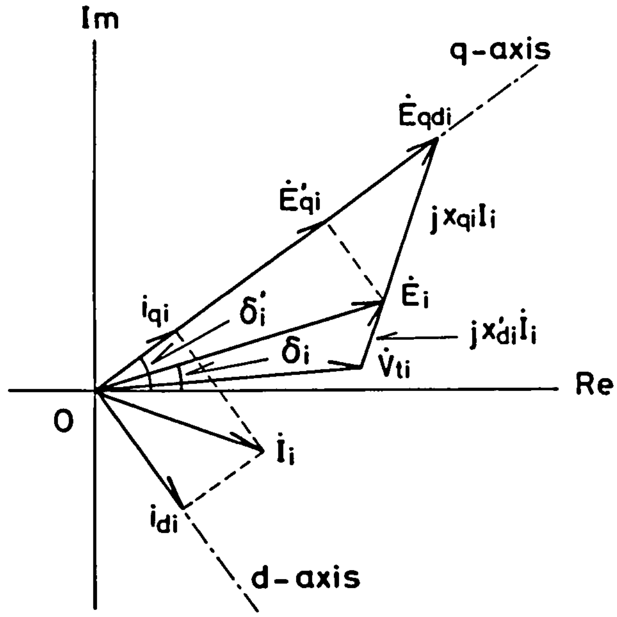


Fig.31. Relations of generator variables.

Under the assumption 2), (3.3) and (3.4) change as follows (Appendix B):

$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \tag{3.5}$$

and

$$\frac{dE_i}{dt} = -[1 - (x_{di} - x'_{di})B_{ii}]/T'_{doi} \cdot (E_i - E_i^o) - (1/T'_{doi})(x_{di} - x'_{di}) \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \tag{3.6}$$

for $i=1,2,\ldots,n$

where

$$B_{ij} = Y_{ij}\cos\theta_{ij}$$

The superscript " o " denotes the stable equilibrium point of the post-fault system, so (3.5) and (3.6) apply to the post-fault state.

Eqs.(3.5) and (3.6) in the state space notation are given as follows:

$$\begin{aligned} \dot{x} &= Ax - BF(\sigma) \\ \sigma &= C'x \end{aligned} \tag{3.7}$$

where

$$A = \begin{bmatrix} 0 & K'_{n(n-1)} & 0 \\ 0 & -M^{-1}D_{nn} & 0 \\ 0 & 0 & -\alpha_{nn} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ M^{-1}T_{nm} & 0 \\ 0 & \beta_{nn} \end{bmatrix}$$

$$C = \begin{bmatrix} G_{(n-1)m} & 0 \\ 0 & 0 \\ 0 & I_{nn} \end{bmatrix} \tag{3.8}$$

in which

$$M_{nn} = \text{diag}(m_1, m_2, \ldots, m_n)$$

$$D_{nn} = \text{diag}(d_1, d_2, \ldots, d_n)$$

$$\alpha_{nn} = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

$$\beta_{nn} = \text{diag}(\beta_1, \beta_2, \ldots, \beta_n)$$

$$K_{n(n-1)} = \begin{bmatrix} 1_{1(n-1)} \\ -I_{(n-1)(n-1)} \end{bmatrix} \qquad (3.9)$$

$$G_{(n-1)m} = [\ I_{(n-1)(n-1)} \quad -T_{(n-1)(m-n+1)}\ ]$$

$$T_{nm} = \begin{bmatrix} 1_{1(n-1)} & 0_{1(m-n+1)} \\ -I_{(n-1)(n-1)} & T_{(n-1)(m-n+1)} \end{bmatrix}$$

The matrices 1 and 0 in (3.8) and (3.9) have all their elements equal to unity and zero, respectively. The number m is defined by

$$m = n(n-1)/2 \qquad (3.10)$$

The state vector x is a (3n-1) dimensional vector consisting of three vectors as follows:

$$x = [\ \delta_r' \quad \omega' \quad \Delta E'\ ]' \qquad (3.11)$$

where δ_r, ω, and ΔE are defined by

$$\delta_{ri} = \delta_{1(i+1)} - \delta^o_{1(i+1)} \qquad \text{for } i=1,2,\ldots,n-1,$$

$$\omega_i = \dot{\delta}_i \qquad \text{for } i=1,2,\ldots,n, \qquad (3.12)$$

$$\Delta E_i = E_i - E^o_i \qquad \text{for } i=1,2,\ldots,n.$$

The nonlinearity $F(\sigma)$ is an $(m+n)$ dimensional vector consisting of two vectors as follows:

$$F(\sigma) = [\ f_1(\sigma)' \quad f_2(\sigma)'\ ]' \qquad (3.13)$$

where $f_1(\sigma)$ is an m dimensional vector defined by

$$f_{1k}(\sigma) = B_{ij}[E_iE_j\sin(\sigma_k + \delta^o_{ij}) - E^o_iE^o_j\sin\delta^o_{ij}] \qquad (3.14)$$

$$\text{for } i=1,2,\ldots,n-1, \quad j=i+1,\ldots,n,$$
$$k=1,2,\ldots,m,$$

where k is related with i and j by

$$k = (i-1)n - i(i+1)/2 + j, \qquad (3.15)$$

and $f_2(\sigma)$ is an n dimensional vector defined by

$$f_{2i}(\sigma) = \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^{o} - \cos\delta_{ij}) \qquad (3.16)$$

for $i=1,2,\ldots,n$.

The output σ is an (m+n) dimensional vector defined by

$$\begin{aligned} \sigma_k &= \delta_{ij} - \delta_{ij}^{o} && \text{for } k=1,2,\ldots,m, \\ \sigma_k &= E_i - E_i^{o} && \text{for } k=m+1,\ldots,m+n, \end{aligned} \qquad (3.17)$$

where k is related with i and j by (3.15) for $k=1,2,\ldots,m$. Eq.(3.7) describes the multimachine power system as a multivariable dynamical system of the form as shown in Fig.4.

2.2 Stability check of system

The transfer matrix W(s) for the linear part of the system is written as follows:

$$\begin{aligned} W(s) &= C'(sI - A)^{-1}B \\ &= \begin{bmatrix} T'[s(sI + M^{-1}D)]^{-1}M^{-1}T & 0 \\ 0 & I(sI + \alpha)^{-1}\beta \end{bmatrix} \\ &= \begin{bmatrix} W_1(s) & 0 \\ 0 & W_2(s) \end{bmatrix} \end{aligned} \qquad (3.18)$$

For the system to be stable, there must exist matrices N and Q such that Z(s) defined by (2.18) is positive real. In this problem, N is chosen as follows:

$$N = \begin{bmatrix} (1/q)I_{mm} & 0_{mn} \\ 0_{nm} & 0_{nn} \end{bmatrix} \qquad (3.19)$$

The inequality in (2.14) is equivalent to the following inequalities:

$$f_{1k}(\sigma)\sigma_k \geq 0 \qquad \text{for all } \sigma_k \text{ in } R \text{ and } k=1,2,\ldots,m. \qquad (3.20)$$

However, the inequalities are satisfied not for all σ_k in R, but for ranges of σ_k as follows:

$$\sigma_{min} \leq \sigma_k \leq \sigma_s \quad \text{and} \quad 0 \leq \sigma_k \leq \sigma_{max} \qquad \text{for } \sigma_s \leq 0$$

or (3.21)

$$\sigma_{min} \leq \sigma_k \leq 0 \quad \text{and} \quad \sigma_s \leq \sigma_k \leq \sigma_{max} \qquad \text{for } \sigma_s \geq 0$$

where

$$\sigma_{min} = -\pi - (\delta_{ij}^o + \delta_{ij}^s)$$

$$\sigma_{max} = \pi - (\delta_{ij}^o + \delta_{ij}^s)$$

$$\sigma_s = \delta_{ij}^s - \delta_{ij}^o$$

and

$$\delta_{ij}^s = \sin^{-1}(E_i^o E_j^o \sin\delta_{ij}^o / E_i E_j)$$

As observed from (2.22), $F(\sigma)'N\sigma$ has an influence on the time derivative of $V(x)$. Hence, it is desirable to make its influence zero by letting $q \to \infty$. However, this selection of q causes a pole-zero cancellation between $(N+Qs)$ and $W(s)$ because $W_1(s)$ has a pole at $s = 0$. In order to avoid the pole-zero cancellation, we give q a finite value in constructing the Lyapunov function, and once it is obtained, we let $q \to \infty$.

The function $V_1(\sigma)$ in (2.15) is chosen as follows:

$$V_1(\sigma) = \sum_{k=1}^{m} \int_0^{\sigma} f_{1k}(\sigma) d\sigma_k$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij}[E_i E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o) E_i^o E_j^o \sin\delta_{ij}^o] \qquad (3.22)$$

The function $V_1(\sigma)$ is not positive for all σ, but for a range of $\sigma = 0$. Accordingly, the global stability of the system can not be concluded with this function. It is possible, however, to estimate the domain of attraction by using the Lyapunov function obtained with this function. It should be noted that $V_1(\sigma)$ can take negative values in the vicinity of the origin if $E_i = E_i^o$ is not satisfied for all E_i, for $i=1,2,\ldots,n$. This fact may have significant influence on the stability of the system, but its influence is assumed to be negligible.

The partical derivatives of $V_1(\sigma)$ are given as follows:

$$\frac{\partial V_1}{\partial \sigma_k} = B_{ij}[E_iE_j\sin(\sigma_k + \delta_{ij}^o) - E_i^oE_j^o\sin\delta_{ij}^o] \qquad \text{for } k=1,2,\ldots,m,$$

$$\frac{\partial V_1}{\partial \sigma_k} = \sum_{\substack{j=1\\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \qquad \text{for } k=m+1,\ldots,m+n, \tag{3.23}$$

that is,

$$\nabla V_1(\sigma) = I_{(m+n)(m+n)}\,F(\sigma) \tag{3.24}$$

Accordingly, Q is given by

$$Q = I_{(m+n)(m+n)} \tag{3.25}$$

By substituting (3.19) and (3.25) into (2.18), Z(s) is given as follows:

$$Z(s) = \begin{bmatrix} (1/q+s)T'[s(sI+M^{-1}D)]^{-1}M^{-1}T & 0 \\ 0 & s(sI+\alpha)^{-1}\beta \end{bmatrix}$$

$$= \begin{bmatrix} Z_1(s) & 0 \\ 0 & Z_2(s) \end{bmatrix} \tag{3.26}$$

The conditions for Z(s) to be positive real are

1) Z(s) has elements which are analytic for Re s > 0,

2) $Z^*(s) = Z(s^*)$ for Re s > 0,

3) $Z'(s^*) + Z(s)$ is positive semi-definite for Re s > 0.

Since Z(s) is a direct sum of $Z_1(s)$ and $Z_2(s)$, those are investigated independently of each other. The first two conditions clearly hold for both $Z_1(s)$ and $Z_2(s)$. For condition 3) to be satisfied, it is sufficient in this case to show that $Z_i(j\omega) + Z_i'(-j\omega)$ is positive semi-definite for each scalor ω, where $i=1,2$. After some manipulation, those are found out to be as follows:

$$Z_1(j\omega) + Z_1'(-j\omega) = 2T'\mathrm{diag}\left(\frac{d_i - m_i/q}{m_i^2\omega^2 + d_i^2}\right)T \tag{3.27}$$

$$Z_2(j\omega) + Z_2'(-j\omega) = 2\text{diag}\left(\frac{\beta_i \omega^2}{\omega^2 + \alpha_i^2} \right) \tag{3.28}$$

If the following inequalities are satisfied;

$$q > m_i/d_i \qquad \text{for } i=1,2,\ldots,n, \tag{3.29}$$

$$\beta_i > 0 \qquad \text{for } i=1,2,\ldots,n, \tag{3.30}$$

then both the right hands in (3.27) and (3.28) are positive semi-definite. Hence $Z_1(s)$ and $Z_2(s)$ are positive real, $Z(s)$ is positive real, too. From theorem 3, the system is stable.

2.3 Solution of matrix equations

Since the system is stable, there exists a Lyapunov function as follows:

$$V(x) = x'Px + 2V_1(\sigma) \tag{3.31}$$

where P is a $(3n-1)\times(3n-1)$ positive definite symmetric matrix satisfying the following equations:

$$\begin{aligned} PA + A'P &= -LL' \\ PB &= CN' + A'CQ' - LW_o \\ W_o'W_o &= QC'B + B'CQ' \end{aligned} \tag{3.32}$$

L and W_o in (3.32) are $(3n-1)\times(m+n)$ and $(m+n)\times(m+n)$ matrices, and there are some relations concerning L and W_o [53]. $Z(s) + Z'(-s)$ is factorized as follows:

$$Z(s) + Z'(-s) = Y'(-s)Y(s) \tag{3.33}$$

where $Y(s)$ is an $(m+n)\times(m+n)$ matrix. $Y(s)$ has a minimal realization (A, B, L), that is,

$$Y(s) - Y(\infty) = L'(sI - A)^{-1}B \tag{3.34}$$

and W_o is equal to $Y(\infty)$, i.e.,

$$W_o = Y(\infty) \tag{3.35}$$

Since $Z(s)$ is a direct sum of $Z_1(s)$ and $Z_2(s)$, P is given as a direct sum of P_1 and P_2, where P_1 and P_2 are $(2n-1)\times(2n-1)$ and $n\times n$ matrices, and they correspond to $Z_1(s)$ and $Z_2(s)$, respectively. Namely,

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \tag{3.36}$$

The transfer matrix W (s) is rewritten as follows:

$$W_1(s) = C_1'(sI - A_1)^{-1}B_1 \tag{3.37}$$

where

$$A_1 = \begin{bmatrix} 0 & K' \\ 0 & -M^{-1}D \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ M^{-1}T \end{bmatrix} \qquad C_1 = \begin{bmatrix} G \\ 0 \end{bmatrix} \tag{3.38}$$

Since $C_1'B_1 = 0$ holds, (2.20) reduces to

$$\begin{aligned} P_1A_1 + A_1'P_1 &= - L_1L_1' \\ P_1B_1 &= C_1N_1' + A_1'C_1Q_1' \end{aligned} \tag{3.39}$$

P_1 and L_1 are partitioned as follows:

$$P_1 = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \qquad L_1 = \begin{bmatrix} L_{11} \\ L_{12} \end{bmatrix} \tag{3.40}$$

where P_{11}, P_{12}, P_{21}, and P_{22} are $(n-1)\times(n-1)$, $(n-1)\times n$, $n\times(n-1)$, and $n\times n$ matirces, and L_{11} and L_{12} are $(n-1)\times m$ and $n\times m$ matrices, respectively. Substituting (3.38) and (3.40) into (3.39) gives the following equations:

$$0 = - L_{11}L_{11}' \tag{3.41}$$

$$P_{11}K' - P_{12}M^{-1}D = 0 \tag{3.42}$$

$$P_{21}K' + KP_{12} - P_{22}M^{-1}D - DM^{-1}P_{22} = - L_{12}L_{12}' \tag{3.43}$$

$$P_{12}M^{-1}T = (1/q)G \tag{3.44}$$

$$P_{22}M^{-1}T = T \tag{3.45}$$

These equations are the same as (2.62) ∿ (2.66), and their solutions are given as follows:

$$\begin{aligned} KP_{11}K' &= (1/q)D + \rho DUD \\ KP_{12} &= (1/q)M + \rho DUM \\ P_{22} &= M + \mu MUM \end{aligned} \tag{3.46}$$

where U is an $n \times n$ matrix with all elements equal to 1. The scalars ρ and μ must satisfy

$$\begin{aligned} \rho &\geq -(1/q)\sum_{i=1}^{n} d_i \\ \mu - \rho &\geq -1/\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} \end{aligned} \tag{3.47}$$

for P_1 to be positive definite matrix, and satisfy

$$(\mu^*)^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(d_i m_j - d_j m_i)^2}{4(d_i - m_i/q)(d_j - m_j/q)} - \mu^* \sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} - 1 \leq 0 \tag{3.48}$$

for (3.43) to be satisfied, where μ^* is defined by

$$\mu^* = \mu - \rho . \tag{3.49}$$

The transfer matrix $W_2(s)$ is rewritten as follows:

$$W_2(s) = C_2'(sI - A_2)^{-1}B_2 \tag{3.50}$$

where

$$A_2 = -\alpha_{nn}, \quad B_2 = \beta_{nn}, \quad C_2 = I_{nn} \tag{3.51}$$

Since $Z_2(s)$ is positive real, $Z_2(s) + Z_2'(-s)$ is factorized as follows:

$$Z_2(s) + Z_2'(-s) = Y_2'(-s)Y_2(s) \tag{3.52}$$

where

$$Y_2(s) = \operatorname{diag}_i\left(\frac{s\sqrt{2\beta_i}}{s + \alpha_i} \right) \tag{3.53}$$

By solving (3.34) with (3.51) and (3.53), we obtain

$$L_2 = -\ \mathrm{diag}_i(\sqrt{2}\alpha_i/\sqrt{\beta_i}) \tag{3.54}$$

P_2 and L_2 are related with each other as follows:

$$P_2A_2 + A_2'P_2 = -\ L_2L_2' \tag{3.55}$$

By substituting (3.51) and (3.54) into (3.55), and solving it, P_2 is given as follows:

$$P_2 = \alpha\beta^{-1} \tag{3.56}$$

Thus P_1 and P_2, and accordingly, P are obtained.

2.4 Lyapunov function

An expression for the Lyapunov function can be obtained by substituting (3.11), (3.46), and (3.56) into (3.31) as follows:

$$\begin{aligned} V(x) &= [\delta_r', \omega', \Delta E'] \begin{bmatrix} P_{11} & P_{12} & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_2 \end{bmatrix} \begin{bmatrix} \delta_r \\ \omega \\ \Delta E \end{bmatrix} + 2V_1(\sigma) \\ &= \delta_r'P_{11}\delta_r + 2\delta_r'P_{12}\omega + \omega'P_{22}\omega + \Delta E'P_2\Delta E + 2V_1(\sigma) \\ &= \delta'(D/q + \rho DUD)\delta + 2\delta'(M/q + \rho DUM)\omega + \omega'(M + \mu MUM)\omega \\ &\quad + \Delta E'\alpha\beta^{-1}\Delta E + 2V_1(\sigma) \end{aligned} \tag{3.57}$$

Now the Lyapunov function is obtained, so we let $q \to \infty$ because q is introduced only in order not to cause a pole-zero cancellation between $(N + Qs)$ and $W(s)$ as mentioned before. Substituting (3.22) into (3.57), and expanding and rearranging the terms in (3.57), we obtain the following expression:

$$\begin{aligned} V(x) &= (1/2 \sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2 \\ &\quad + (\mu^* - \mu_o)(\sum_{i=1}^{n} m_i\omega_i)^2 \\ &\quad + \rho\{\sum_{i=1}^{n} [d_i(\delta_i - \delta_i^o) + m_i\omega_i]\}^2 \end{aligned}$$

$$+ \sum_{i=1}^{n} (\alpha_i/\beta_i)(E_i - E_i^o)^2$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o] \tag{3.58}$$

where μ_o is a scalar defined by

$$\mu_o = - 1/\sum_{i=1}^{n} m_i \tag{3.59}$$

The first and the second terms in (3.58) represent kinetic energy. If damping torques of generators are uniform, then we can choose μ^* to be equal to μ_o. The scalar ρ in the third term is an arbitrary non-negative scalar. It is chosen to be zero because the term narrows and complicates the estimation of the transient stability of the system. The fourth term is the new term which represents a magnitude of deviations in field flux linkages. If field flux linkages are constant, this new term disappears. The fifth term represents potential energy which is stored in the system owing to some deviations of rotor angles of generators from those at the stable equilibrium point. The potential energy plays an important role in defining the transient stability region of the system.

In those cases where damping torques are uniform or zero, (3.58) reduces to

$$V(x) = (1/2\sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_im_j(\omega_i - \omega_j)^2$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o]$$

$$+ \sum_{i=1}^{n} (\alpha_i/\beta_i)(E_i - E_i^o)^2$$

$$= V_k(\omega) + V_p(\delta, E) + V_f(E) \tag{3.60}$$

where ρ is chosen to be zero. V_k and V_p are kinetic energy and potential energy, respectively. The time derivatives of V_k, V_p, and V_f are written as follows:

$$\frac{dV_k}{dt} = -(1/\sum_{i=1}^{n} d_i)\sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j(\omega_i - \omega_j)^2$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \qquad (3.61)$$

$$\frac{dV_p}{dt} = -\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j)$$
$$+ 2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \qquad (3.62)$$

$$\frac{dV_f}{dt} = -2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})$$
$$- 2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)^2 \qquad (3.63)$$

The first term of (3.61) is due to damping torques of generators, and it is non-positive. A part of the kinetic energy is dissipated by damping torques. The second term of (3.61) and the first term of (3.62) are the same magnitudes and of the opposite signs of each other, which implies that there is some exchange of energy between the kinetic energy and the potential energy. These terms do not contribute to the damping rate of V. Similarly, the second term of (3.62) and the first term of (3.63) are of the same magnitude and of the opposite signs of each other. There is some exchange of energy between V_p and V_f, too. The second term of (3.63) is in proportion to squares of (dE_i/dt), and it is non-positive regardless of whether internal voltages E_i are decreasing or increasing. As a whole, V dampens according to

$$\frac{dV}{dt} = -(1/\sum_{i=1}^{n} d_i)\sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j(\omega_i - \omega_j)^2$$
$$- 2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)^2 \qquad (3.64)$$

while V_k and V_p, V_p and V_f are interacting with each other, respectively.

2.5 Conclusions

In this section, we have constructed a Lyapunov function with a systematic method based on a generalized Popov criterion for the multimachine power system in which dynamics of field flux linkages are taken into account. There is some difference between this new Lyapunov function and the Lyapunov function derived in §3 of the previous chapter for the power system in which field flux linkages are assumed to be constant in the transient period, in several point as follows:

1) The new function consists of three terms, that is, kinetic energy V_k, potential energy V_p, and a new term V_f.
2) The kinetic energy V_k is the same in both the functions.
3) The potential energy V_p varies with some deviations of field flux linkages as well as relative rotor angles of generators.
4) The new term V_f is associated with some deviations of field flux linkages. It does not appear for the systems represented with the conventional model.
5) The damping rate of V consists of two terms. One of them is due to damping torques of generators, and the other is due to the time derivatives of field flux linkages. The latter term does not appear for the conventionally modeled systems.

Thus, the obtained Lyapunov function is characteristic of the power systems in which dynamics of field flux linkages are taken into account. The transient stability of the systems is investigated on the basis of this new functions in the following sections.

§3. Critical value of Lyapunov function [73]

In this section, we make some investigations on the critical value of the Lyapunov function constructed in the preceding section. For the power systems in which field flux linkages of generators are assumed to be constant all the time, we have defined the transient stability region, and introduced the new critical value for the first swing stability in order to get rid of the conservative nature of Lyapunov's direct method. In these systems under investigation, field flux linkages vary with time, and as a result, the equipotentical curves also vary with time because the potential energy function V_p defined by (3.60) is a function of field flux linkages as well as relative rotor angles of generators. Accordingly, the transient stability region defined in the previous chapter does not apply to these systems as it is, and some corrections must be made on it and the critical value as well.

3.1 Model and basic equations

If damping torques of generators and transfer conductances of reduced admittance matrices are zero, then the motion of the ith generator is described as follows:

$$m_i \frac{d^2\delta_i}{dt^2} = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \qquad (3.65)$$

and

$$\frac{dE_i}{dt} = -\alpha_i(E_i - E_i^o) - \beta_i \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \qquad (3.66)$$

where

$$\alpha_i = [1 - (x_{di} - x'_{di})B_{ii}]/T'_{doi}$$

$$\beta_i = (x_{di} - x'_{di})/T'_{doi}$$

The superscript " o " denotes the stable equilibrium point of the post-fault system, so (3.65) and (3.66) apply to the post-fault state.

A Lur'e type Lyapunov function has been derived for this system in the preceding section as follows:

$$V(x) = (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o]$$

$$+ \sum_{i=1}^{n}(\alpha_i/\beta_i)(E_i - E_i^o)^2$$

$$= V_k(\omega) + V_p(\delta, E) + V_f(E) \qquad (3.67)$$

The first term in (3.67) represents kinetic energy of generators, and it is a function of relative angular velocities of rotors. The second term represents potential energy which is stored in networks owing to some deviations of rotor angles from those at the stable equilibrium point. It must be noted that V_p varies with internal voltages, too. The third term V_f represents a magnitude of deviations in internal voltages. The time derivatives of V_k, V_p, and V_f are given as follows:

$$\frac{dV_k}{dt} = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \qquad (3.68)$$

$$\begin{aligned} \frac{dV_p}{dt} = &- \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \\ &+ 2 \sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \end{aligned} \qquad (3.69)$$

$$\begin{aligned} \frac{dV_f}{dt} = &- 2 \sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \\ &- 2 \sum_{i=1}^{n} (1/\beta_i)(dE_i/dt)^2 \end{aligned} \qquad (3.70)$$

The right hand term of (3.68) and the first term of (3.69) are of the same magnitude and of the opposite signs of each other, which implies that there exists exchange of energy between V_k and V_p. Similarly, the second term of (3.69) and the first term of (3.70) are of the same magnitude and of the opposite signs of each other. There exists exchange of energy between V_p and V_f, too. Those terms do not contribute to the damping rate of V. As a whole, V dampens according to

$$\frac{dV}{dt} = - 2 \sum_{i=1}^{n} (1/\beta_i)(dE_i/dt)^2 \qquad (3.71)$$

The right hand term of (3.71) is in proportion to squares of the time derivatives of internal voltages. It takes non-positive values all the time regardless of whether internal voltages are increasing or decreasing. The total energy of the system is dissipated through this term.

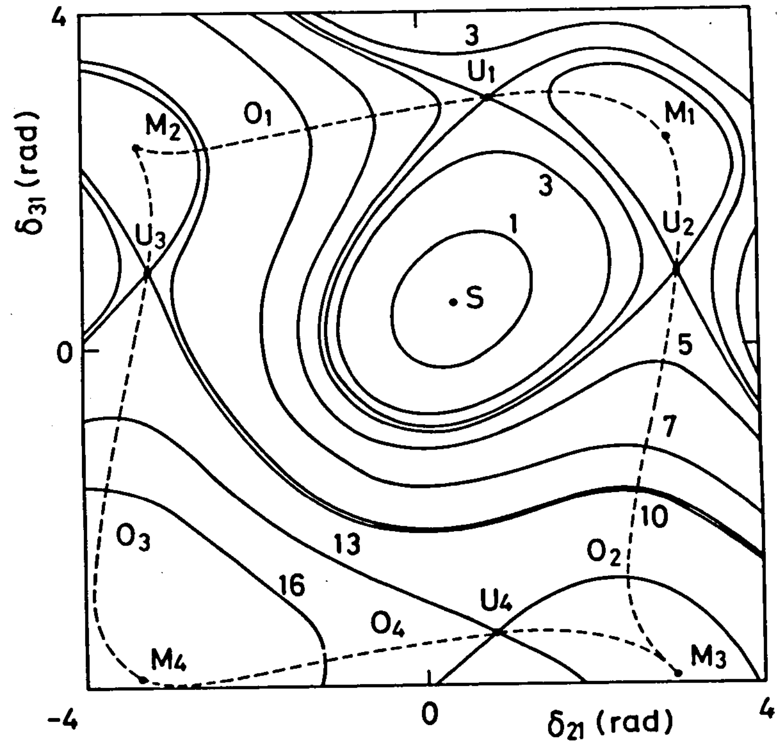


Fig.32. Equipotential curves of 3-machine system: E = (1.0, 1.0, 1.0) p.u..

3.2 Transient stability region

It is observed from (3.67) that the potential energy V_p depends on relative rotor angles and internal voltages. If δ_1 is chosen as reference, V_p can be treated as a function of a (n-1) dimensional vector δ_r and a n dimensional vector E as follows:

$$V_p = V_p(\delta_r, E) \tag{3.72}$$

where

$$\delta_r = (\delta_{21}, \delta_{31}, \ldots, \delta_{n1})' \tag{3.73}$$

Fig.32 shows an example of $V_p(\delta_r, E)$ in a (n-1) dimensional relative angular space for a 3-machine power system. The curves C_1, C_2, ... are equipotential curves yielded by

$$V_p(\delta_r, E) = C_i \qquad i = 1,2,\ldots \tag{3.74}$$

where internal voltages are treated as parameters, and E = (1.0, 1.0, 1.0)' in this figure. This figure is the same as Fig.10. The function V_p takes the minimum value at the point S. The points U_1, U_2, ... are saddle points. The curves O_1, O_2, ... are those which go through U_1, U_2, ..., and are orthogonal to equipotential curves, respectively. If C_i takes small values, then the corresponding equipotential curves are closed, and surround the point S. With increase in magnitude of C_i, equipotential curves goes outside, and reaches the lowest saddle point U_1 when C_i takes the value defined as follows:

$$V_{u1} = V_p(\delta_{u1}, E) \tag{3.75}$$

where δ_{u1} is the relative angle vector at U_1. If C_i is greater than V_{u1}, then the corresponding curve is not closed any more. With more increase in magnitude of C_i, the equipotential curve reaches the saddle points U_2, U_3, ... in sequence according to

$$V_{u2} = V_p(\delta_{u2}, E)$$

$$V_{u3} = V_p(\delta_{u3}, E) \tag{3.76}$$

$$\vdots$$

where δ_{u2}, δ_{u3}, ... are the relative angle vectors at U_2, U_3, ..., respectively.

It is observed from (3.65) that each generator receives a torque expressed as follows:

$$f_i = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \tag{3.77}$$

The f_i defines an n dimensional vector f

$$f = [\ f_1,\ f_2,\ \ldots,\ f_n\ ]' \tag{3.78}$$

The sum of all torques denoted by $\bar{f}$ is given as follows:

$$\begin{aligned} \bar{f} &= \sum_{i=1}^{n} f_i \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \\ &= 0 \end{aligned} \tag{3.79}$$

Eq.(3.79) implies that the center of angular velocities $\bar{\omega}$ defined by

$$\bar{\omega} = \sum_{i=1}^{n} m_i \omega_i / \sum_{i=1}^{n} m_i \tag{3.80}$$

does not receive any torque, and accordingly, it is kept constant all the time, that is,

$$\bar{\omega} = \text{constant} \tag{3.81}$$

This fact implies that each torque does not contribute to the accellation of the center of angular velocities, but that each torque has only influence on relative behaviors of generators. There is a relation between the torque f and the potential energy function V_p as follows; the partial derivatives of V_p with respect to relative angles are given by

$$\frac{\partial V_p}{\partial \delta_{i1}} = 2\sum_{j=1}^{n} B_{ij}(E_i E_j \sin\delta_{ij} - E_i^o E_j^o \sin\delta_{ij}^o)$$

$$= -2f_i \qquad i \neq 1 \tag{3.82}$$

which gives

$$\frac{\partial V_p}{\partial \delta_r} = -2f_r \tag{3.83}$$

where f_r is a reduced torque of (n-1) dimension defined as follows:

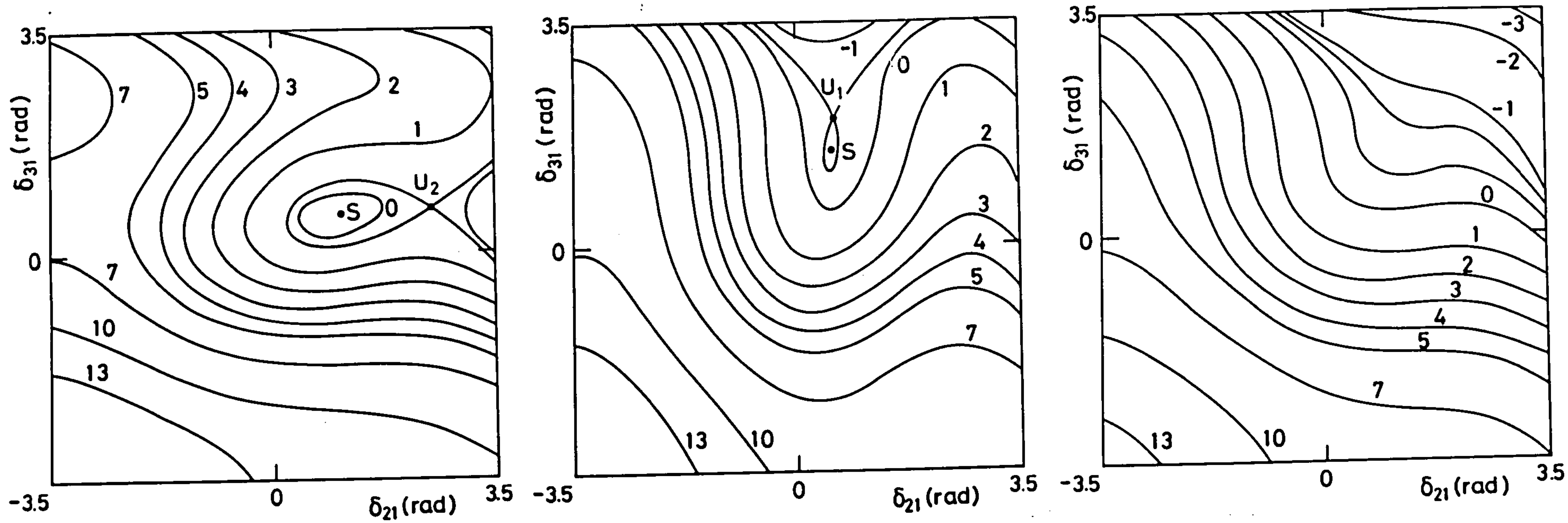
$$f_r = [\ f_2,\ f_3,\ \ldots,\ f_n\ ]' \tag{3.84}$$

The direction of $(\partial V_p/\partial \delta_r)$ is orthogonal to equipotential curves, and its magnitude is proportional to the gradient of equipotential curves. Eq.(3.83) shows that the system receives the torque which always acts orthogonally to equipotential curves.

In Fig.32, f_r is parallel with the curves O_1, O_2, ... on these curves. Those curves enclose the region in which the stable equilibrium point S exists. In this region, f_r acts on the system in such way that it will confine the system in this region. The system will lose synchronism if it crosses one the curves O_1, O_2, ... from the inside to the outside of the region, because f_r will act in such way that it will separate the system from the curve afterwards. The transient stability region is defined as the region which is bounded by the curves O_1, O_2,... in a wide sense that the system receives the synchronizing torque in it. Thus the transient stability region is defined in the same way as for the power systems represented by the conventional model.

3.3 Variation of equipotential curves

Since the potential energy function V_p contains internal voltages as its variables, equipotential curves yielded by (3.74) varies with internal voltages. Fig.33 shows some examples of equipotential curves for the same system as in Fig.32. In Fig.33(a), E_2 is 0.4 p.u. whereas E_1 and E_3 are both 1.0 p.u. In this case only the saddle point U_2 remains, and other saddle points disappear. The value of V_p at U_2 is 0.17, which is very small compared with the case where E_1, E_2, and E_3 are all 1.0 p.u. (Fig.32). The region enclosed by the equipotential curve which goes through U_2 is narrow.



(a) E = (1.0, 0.4, 1.0) p.u.

(b) E = (1.0, 1.0, 0.4) p.u.

(c) E = (1.0, 0.4, 0.4) p.u.

Fig.33. Variations of equipotential curves with internal voltages in 3-machine system.

Relatively small disturbances are able to cause no.2 generator to step out. Fig.33(b) shows the equipotential curves for the case where E_3 is 0.4 p.u., and E_1 and E_2 are 1.0 p.u. In this case, only the saddle point U_1 remains. The region enclosed by the equipotential curve which goes through U_1 is very narrow, so no.3 generator will step out even for small disturbances. In Fig.33(c), E_2 and E_3 are both 0.4 p.u., and only E_1 is 1.0 p.u.. In this case, there is no saddle point, and accordingly, no transient stability region. The system can not keep synchronism any more in such cases. Thus the equipotential curves vary with internal voltages. These variations have significant influence on the stability of the system. Next, we make some investigation on their influence.

3.3.1 Time variations of internal voltages

As is observed from Fig.33(a) and (b), when some internal voltages decrease in magnitude, the equipotential curves vary in such way that the corresponding generators will step out easily. Namely, when E_2 decreases from 1.0 p.u. to 0.4 p.u., no.2 generator becomes liable to step out, and when E_3 decreases from 1.0 p.u. to 0.4 p.u., then no.3 generator becomes liable to step out. Thus the magnitude of the internal voltage is closely related with the stability of each individual generator. Hence it will be useful to make some investigation on qualitative nature of their time variations.

Internal voltages vary with time according to (3.66), i.e.,

$$\frac{dE_i}{dt} = -\alpha_i(E_i - E_i^o) - \beta_i \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \qquad (3.66)$$

for $i=1,2,\ldots,n$

The first term of (3.66) prevents the ith internal voltage from deviating its stable value. The second term is due to some deviations of relative rotor angles from those at the stable equilibrium point. It causes the ith internal voltage to leave from its stable value. As shown in Fig.34, $(\cos\delta_{ij}^o - \cos\delta_{ij})$ takes positive values in section A and C, and negative values in section B. Assume that the ith generator is accellated with respect to other generators, and that δ_{ij} stays in C for all j ($j = 1 \sim n, \neq i$), then E_i decreases very much compared with other internal voltages. The rate of decrease in E_i is maximum when δ_{ij} is equal to π rad. It is concluded from

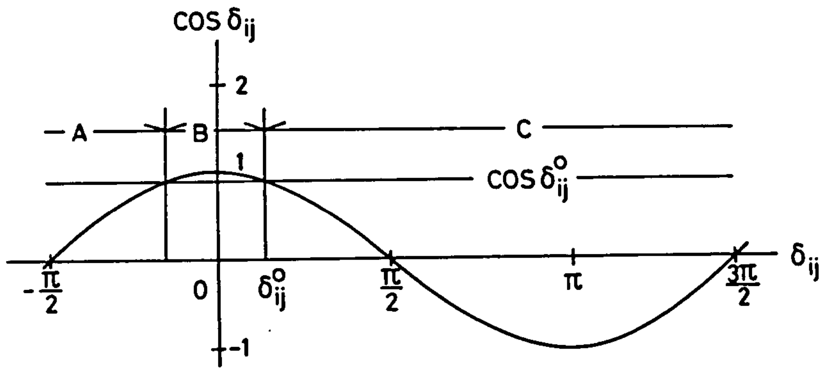


Fig.34. Cos δ_{ij} curve.

these investigations that the internal voltages decrease much in magnitude for the generators whose rotors are accelerated with respect to other generators. Such generators are in usual corresponding to those which are near fault locations.

Combining the fact observed from Fig.33(a) and (b) and the investigations made in the above paragraph, we can draw a picture as follows: when some generators suffer large disturbances owing to a fault, then their rotors are accelerated with respect to other generators. With separation of their rotor angles from those of other generators, their internal voltages decrease much compared with other generators. The equipotential curves of the system vary with the variations of the internal voltages in such way that the corresponding generators will easily step out. As a result, the stability of the system will be degraded much compared with that which is evaluated under the assumption that the internal voltages are kept constant all the time.

3.3.2 Movement of saddle points

In Fig.33(a), there exists only one saddle point U_2, and other saddle points disappear. Let us assume that no.2 generator suffers large disturbance owing to some fault, and that its internal voltage E_2 begins to decrease from 1.0 p.u., and reaches 0.4 p.u. at some instant. The saddle point U_2 moves from the point shown in Fig.32 to the point shown in Fig.33(a) during the period. If the internal voltage continues to decrease, then U_2 approaches the point S, and at last, join to it. Both of them will vanish at the next instant, and the transient stability region also vanish at the same time. On the other hand, if the internal voltage E_2 stop decreasing before U_2 and S join, then the transient stability region remains. Some investigations are made on this latter case.

Following the discussions made in §4.3 of the previous chapter, we can get a stability condition as follows:

[Stability condition 5]

If a system satisfies

$$V_k + V_p < V_u \qquad (3.85)$$

then the system is stable around a saddle point U, where V_u is the value of $V_p(\delta_r, E)$ at U.

This stability condition corresponds to Stability condition 2 in the previous chapter. As is clear from the definition of V_u, V_u is not constant,but varies with internal voltages. Namely, the inequality in (3.85) is time-dependent. Since V_f in (3.67) takes positive values, it is possible to replace $(V_k + V_p)$ by V. However, this replacement will yield a condition more strict than (3.85). In Fig.33(a), there exists only one saddle point U_2. If $(V_k + V_p)$ is smaller than V_{u2} all the time, then the system is stable, where V_{u2} is the value of $V_p(\delta_r, E)$ at the saddle point U_2. Bearing this case in mind, let us compare the time variations of V_u and $(V_k + V_p)$. The time derivative of V_u is given as follows:

$$\frac{dV_u}{dt} = (\partial V_p/\partial E)_{\delta_u} \cdot \frac{dE}{dt} + (\partial V_p/\partial \delta_r)_{\delta_u} \cdot \frac{d\delta_u}{dt} \qquad (3.86)$$

Since $(\partial V_p/\partial \delta_r) = 0$ holds at the saddle point U, (3.86) reduces to

$$\begin{aligned}\frac{dV_u}{dt} &= (\partial V_p/\partial E)_{\delta_u} \cdot \frac{dE}{dt} \\ &= 2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^{o} - \cos\delta_{ij}^{u}) \end{aligned} \qquad (3.87)$$

On the other hand, the time derivative of $(V_k + V_p)$ is given from (3.68) and (3.69) as follows:

$$\frac{d(V_k + V_p)}{dt} = 2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^{o} - \cos\delta_{ij}) \qquad (3.88)$$

By subtracting (3.88) from (3.87), we obtain the following equation:

$$\frac{d[V_u - (V_k + V_p)]}{dt} = 2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij} - \cos\delta_{ij}^{u}) \qquad (3.89)$$

If the right hand term of (3.89) takes positive values, $[V_u - (V_k + V_p)]$ increases, and if it takes negative values, then $[V_u - (V_k + V_p)]$ decreases.

The term is not sign-definite, however, so both the cases can happen. Now assume that only the ith generator is accellated owing to some fault, and that the fault is cleared at an instant t_c in such way that

$$V_u - (V_k + V_p) = V_r > 0 \tag{3.90}$$

holds, that is, (3.85) holds. V_r is regarded as stability margin. If it takes positive values at each instant after t_c, then the system is stable according to Stability condition 5. In usual, the following inequalities hold during the first swing:

$$\frac{dE_i}{dt} < 0 , \quad \frac{dE_j}{dt} \simeq 0 \qquad \text{for } j = 1 \sim n, \neq i \tag{3.91}$$

and

$$\cos\delta_{ij} - \cos\delta_{ij}^{u} > 0 \qquad \text{for } j = 1 \sim n, \neq i \tag{3.92}$$

By substituting (3.91) and (3.92) into (3.89), we can obtain an inequality as follows:

$$\frac{d[V_u - (V_k + V_p)]}{dt} < 0 \tag{3.93}$$

Accordingly, the stability margin V_r decreases with time. In order that it takes positive values all the time after the instant of fault clearance, some positive value must be given to V_r at the instant of fault clearance.

It will be useful to make a rough evaluation on the necessary stability margin. Firstly, let us assume as follows:

$$\frac{dE_i}{dt} = -\varepsilon , \quad E_j = 1.0 \text{ p.u.}, \qquad \delta_{ij} = \pi/2 \text{ rad}, \quad \delta_{ij}^{u} = \pi \text{ rad}, \qquad \text{for } j = 1 \sim n, \neq i \tag{3.94}$$

then (3.89) reduces to

$$\frac{d[V_u - (V_k + V_p)]}{dt} = -2\varepsilon \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} \simeq 2\varepsilon \cdot B_{ii} \tag{3.95}$$

If we assume that

$$B_{ii} = -10.0, \qquad \varepsilon = 0.2 \text{ p.u./sec} \tag{3.97}$$

hold, then we obtain from (3.96) an equation as follows:

$$\frac{d[V_u - (V_k + V_p)]}{dt} \simeq 4.0 \tag{3.98}$$

Namely, the stability margin V_r decreases at the rate of 4.0 p.u./sec. If it is assumed that the differences of rotor angles between the ith generator and other generators become maximum at some instant in the interval of 0.0 ∿ 1.0 sec, the stability margin V_r should be 4.0 at the clearing time t_c. Next, V_u is roughly evaluated. Let us assume as follows:

$$\begin{array}{lll} E_j^o = 1.0 \text{ p.u.}, & E_j = 1.0 \text{ p.u.}, & \text{for } j = 1 \sim n, \\ \delta_{ij}^o = 0.0 \text{ rad}, & \delta_{ij}^u = \pi \text{ rad}, & \text{for } j = 1 \sim n, \neq i, \\ \delta_{jk}^o = 0.0 \text{ rad}, & \delta_{jk}^u = 0.0 \text{ rad}, & \text{for } j,k = 1 \sim n, \neq i, \end{array} \tag{3.99}$$

Since V_u is defined by the value of $V_p(\delta_r, E)$ at the saddle point U, it is given by substituting the relations in (3.99) into (3.67) as follows:

$$V_u = 4 \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} \simeq -4 \cdot B_{ii} \tag{3.100}$$

The ratio of $d[V_u - (V_k + V_p)]/dt$ to V_u is given from (3.95) and (3.100) as follows:

$$\chi \equiv \frac{d[V_u - (V_k + V_p)]/dt}{V_u} = -\varepsilon/2 \tag{3.101}$$

If we substitute (3.97) into (3.100) and (3.101), then we obtain

$$V_u = 40, \qquad |\chi| = 0.1 \tag{3.102}$$

Accordingly, the stability margin V_r which is necessary at the clearing time, proves to be 10 percent of V_u. In order to satisfy (3.85) during the first swing, it is necessary to clear the fault before $(V_k + V_p)$ reaches $(V_u - V_r)$ instead of V_u, e.g., 36 instead of 40 in the above case.

Thus it is clarified that some stability margin must be reserved to Stability condition 5. In a particular case where only one machine suffers large accellation owing to a fault, we have made a rough evaluation of the necessary stability margin. However, it is very difficult to extend the above discussion to general cases where several generators suffer large disturbances at the same time. We close this discussion only by noting the necessity of such stability margin.

3.3.3 Vanishment of transient stability region

As observed from Fig.33(c), there is no saddle point in the case where E_2 and E_3 are both decreased from 1.0 p.u. to 0.4 p.u. Considering that f_r in (3.84) is always orthogonal to the equipotential curves, it is clear that the system is accellated to one direction in this case, and as a result, that the system will lose synchronism. In other words, the transient stability region does not exist. For the transient stability region to exist, the internal voltages must remain in some area. Fig. 35 shows such a area for the system in Fig.32. The shaded area indicates one where the transient stability region does not exist. If the internal voltages decreases owing to a fault, and if they fall into this shaded area, then the transient stability region vanishes. This vanishment will have significant influence on the stability of the system.

Consider a one-machine connected to an infinite bus system shown in Fig.36. The system equation in (3.65) and (3.66) reduce in this case as follows:

$$m \frac{d^2\delta}{dt^2} = BE_\infty E(\sin\delta^o - \sin\delta) \qquad (3.103)$$

and

$$\frac{dE}{dt} = -\alpha(E - E^o) - \beta BE(\cos\delta^o - \cos\delta) \qquad (3.104)$$

where δ is the rotor angle of the generator, and E_∞ is the voltage of the infinite bus. The Lyapunov function V in (3.67) reduces for this system as follows:

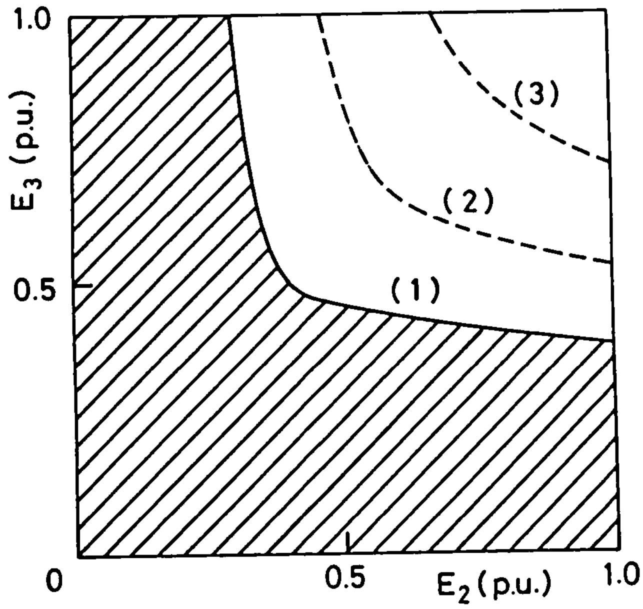


Fig.35. Region of E where transient stability region vanishes.

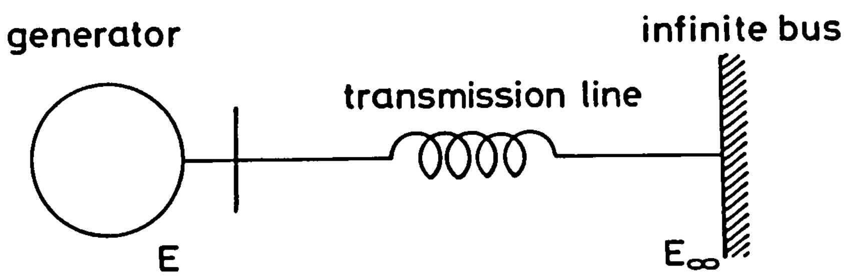


Fig.36. One machine connected to an infinite bus system.

$$V(x) = 2V^*(x) \qquad (3.105)$$

where

$$V^*(x) = (1/2)m\omega^2 + BE_\infty[E(\cos\delta^o - \cos\delta) - (\delta - \delta^o)E^o\sin\delta^o]$$
$$+ (\alpha/2\beta)(E - E^o)^2$$

$V^*(x)$ is equivalent to the Lyapunov function derived by M.W. Siddiqee [23]. Fig.37 shows some examples of the curves which are yielded by

$$V^*(x) = C \text{ (constant)} \qquad (3.106)$$

In Fig.37(a), (b), and (c), $(E - E^o)$ is treated as constant, and is given as follows:

$$E - E^o = 0.0 \text{ p.u.} \quad \cdots\cdots \quad (a)$$
$$= -0.5 \text{ p.u.} \quad \cdots\cdots \quad (b)$$
$$= -1.0 \text{ p.u.} \quad \cdots\cdots \quad (c)$$

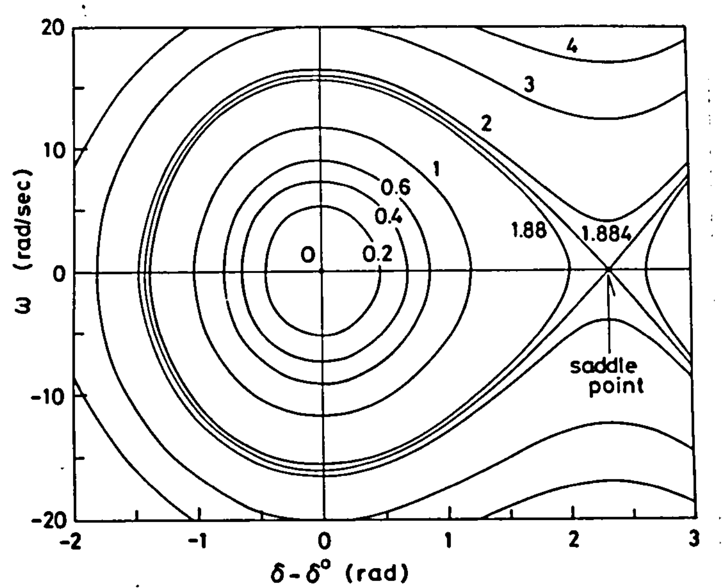
If it is assumed that the internal voltage E is fixed all the time, then the system moves along the curve determined by (3.106) when a constant is given to $V^*(x)$. If the curve is closed, then the system keeps synchronism. Otherwise, the system loses synchronism. In the case of Fig.37(a), the curves corresponding to

$$V^*(x) < 1.884 \qquad (3.107)$$

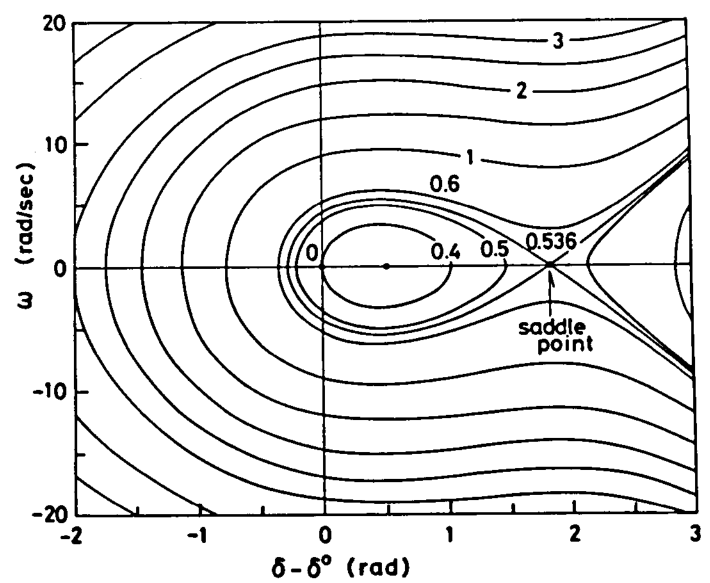
are all closed, and the system is stable if (3.107) is satisfied. Similarly, in the case of Fig.37(b), the curves corresponding to

$$V^*(x) < 0.536 \qquad (3.108)$$

are all closed, and the system is stable if (3.108) is satisfied. The conditions (3.107) and (3.108) are equivalent to the condition (3.85) if $(\alpha/2\beta) \times (E - E^o)^2$ is subtracted from both the sides in (3.107) and (3.108). The values 1.884 and 0.536 are corresponding to the value V_u in (3.85). V_u varies with the internal voltages as discussed in §3.3.3. Hence, (3.85)

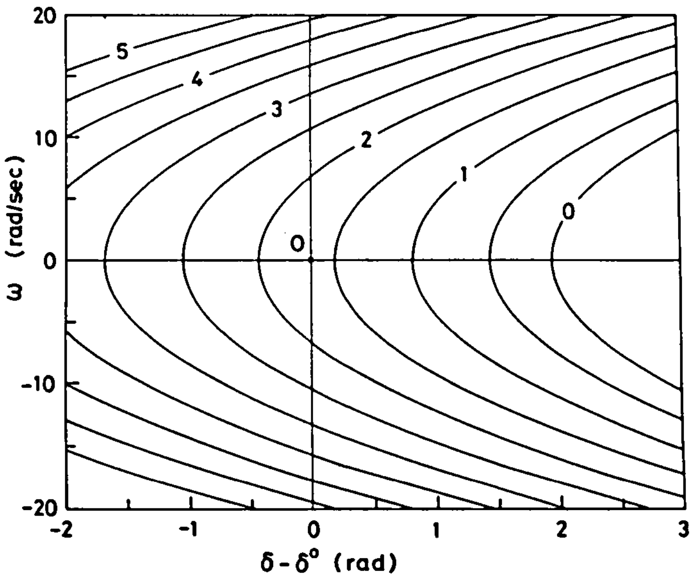


(a) E - E^{O} = 0.0 p.u.

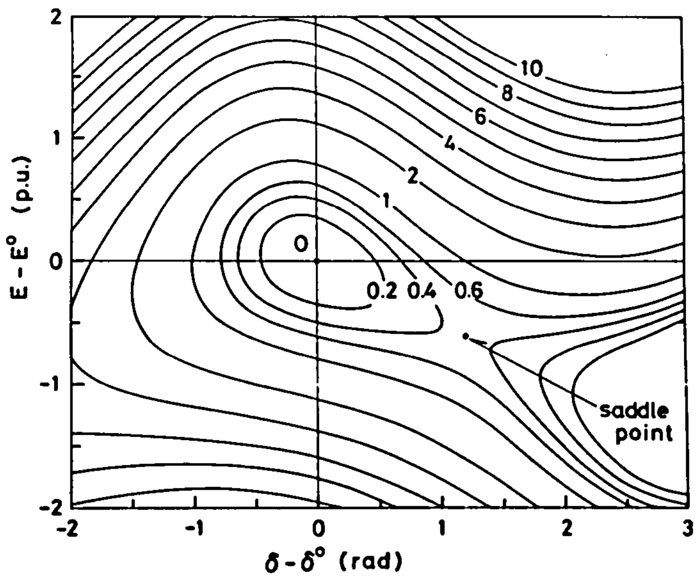


(b) E - E^{O} = - 0.5 p.u.

Fig. 37. Equipotential curves in ($\delta - \delta^{O}$, ω, $E - E^{O}$) space for one-machine system.



(c) E - E^{O} = - 1.0 p.u.



(d) ω = 0 rad/sec

is satisfied all the time, then the system is stable. However, in the case of Fig.37(c), there is no closed curve, so the system can not keep synchronism. In this case, we can not get such a condition as those in (3.107) and (3.108). For such conditions to be obtained,

$$(E - E^{o}) \;>\; -\,0.60 \text{ p.u.} \tag{3.109}$$

must be satisfied. Otherwise, there is no closed curve, and accordingly, we can not get such a condition as (3.107). If $(E - E^{o})$ is equal to - 0.60 p.u., we can obtain a condition as follows:

$$V^{*}(x) \;<\; 0.484 \tag{3.110}$$

This a limit condition which is obtained by decreasing $(E - E^{o})$. If $(E - E^{o})$ becomes smaller than - 0.60 p.u., we can not get such condition any more. Fig.37(d) shows the curves yielded by (3.106), where the rotor velocity ω is set to be zero. The curves which satisfy (3.110) are all closed. Hence, it is concluded from Fig.37(a), (b), and (c), that those curves are also closed in the space of $(\delta - \delta^{o}, \omega, E - E^{o})$. If (3.110) is satisfied, then the system is stable regardless of the variation of the internal voltage E. In this sense, (3.110) is an absolue condition whereas (3.107) and (3.108) are temporary ones.

The same discussion hold for the system in Fig.32 and 35, and we can get conditions corresponding to (3.109) and (3.110). In the shaded area of Fig.35, the transient stability region does not exist. The condition (3.109) is corresponding to Fig.35. By calculating the maximum value V'_u of $V(x)$ in (3.60) under the constraint that x does not violate the shaded area, we can get a condition corresponding (3.110) as follows:

$$V(x) \;<\; V'_u \tag{3.111}$$

If this condition is satisfied, then the transient stability region does not vanish, and the system is able to keep synchronism. However, it is not desirable to adopt (3.111) in evaluating the stability of the system because of two defects as follows: Firstly, it yields very conservative results when internal voltages do not decrease enough to cause the vanishment of the transient stability region. As an example, consider the one-machine system. If the internal voltage E is constant, (3.107) is satisfactory for the system to be stable. There is a large discrepancy between (3.107) and (3.110), and if the latter condition is adopted, then it will yield very conservative result as is clear from Fig.37(a). Secondly, it takes very much computation to get the boundary of the shaded area, and accordingly, to get the condition (3.111). This difficulty will be appreciated if one consider the case where a system contains many generators, for example, 20 generators. In this case, the boundary forms a 19-dimensional surface in a 20-dimensional space. Hence it is infeasible to obtain the condition (3.111). Hence, in the following, we treat the internal voltages as parameter, and develope our discussion on this basis.

The system usually loses synchronism by getting out of the transient stability region. In the previous chapter, only this phenomenon has been dealt with. However, there is a possibility for the system to lose synchronism owing to the vanishment of the transient stability region in the cases where the dynamics of field flux linkages are taken into account as described above. In order to distinguish between these two types of instabilities, we temporarily refer them as the ususal type of instability and the second type of instability, respectively. The usual type of instability is dominant in the cases where field flux linkages do not decrease so much. We have to pay attention also to the second type of instability in the cases where the decrease in internal voltages are not negligible. The second type of instability seems not to occur in the case shown in Fig.35 because the internal voltages must decrease much, i.e. to about 0.4 p.u. for the transient stability region to vanish. However, with increase in the electric power output P_2 and P_3, the shaded area becomes wider as indicated by the dotted lines, and accordingly, relatively small decrease in the internal voltages become able to cause the vanishment of the transient stability region. Thus the second type of instability is prevalent in the practical multimachine power systems which have heavy loads.

3.4 Method of determining critical value

In §4.4 of the previous chapter, we have derived Stability condition 4 which was adopted as the basis of determining the critical value of Lyapunov function for the first swing stability of the system. Following the procedure of deriving it, we can formally get a condition as follows:

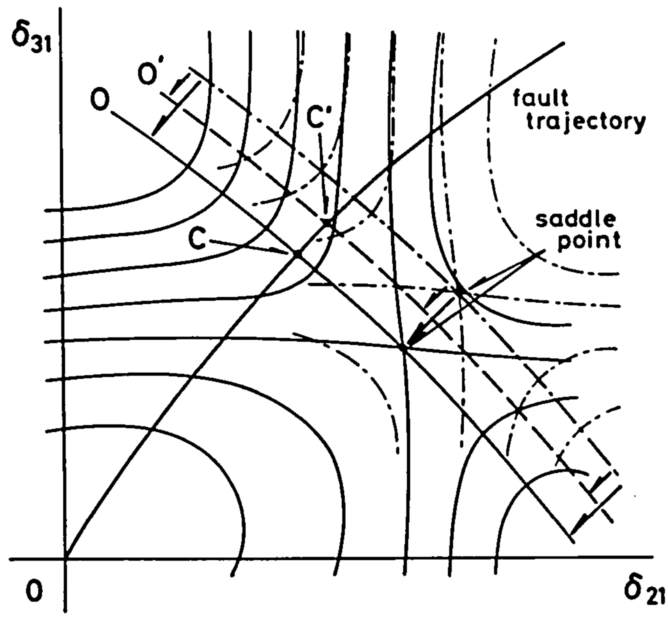


Fig.38. Trajectory in relative angular space for 3-machine system.

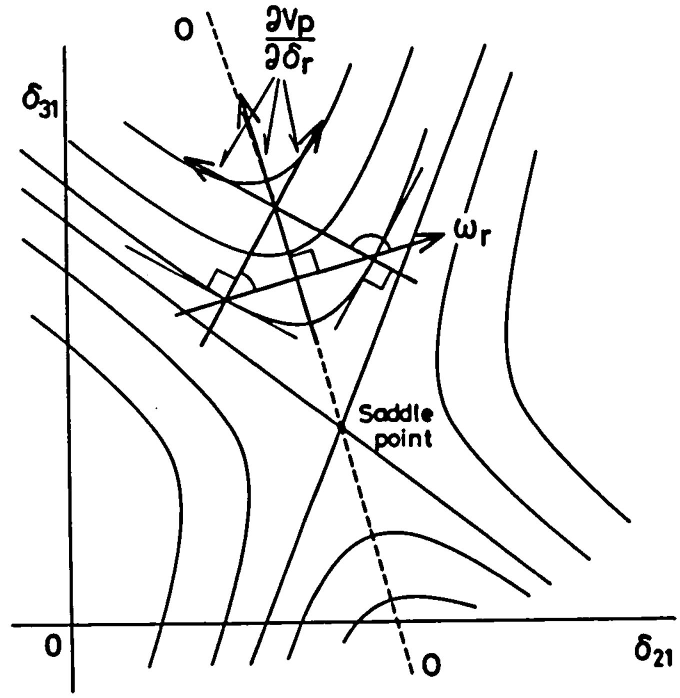


Fig.39. Relation between ω_r and $\partial V_p/\partial \delta_r$ when system crosses boundary O_x.

[Stability condition 6]

If a system satisfies

$$V < V_{cr} \tag{3.112}$$

then it is stable for the first swing, where V_{cr} is the value of $V_p(\delta_r, E)$ at the point c where the sustained fault trajectory crosses the boundary of the transient stability region.

Since the transient stability region varies with internal voltages, some comments on its influence should be necessary. Fig.38 shows an example of system trajectory in a relative angular space for a case where a fault continues. The system crosses the boundary of the transient stability region at the point c. Over this point, the torque f_r defined by (3.84) acts in such way that it will accelerate the system, and will separate it from the stability region. In order that the system can remain in synchronism, the fault must be cleared in such way that the system can not go over the point c. Thus we obtain the condition in (3.112). It may be possible to replace V by $(V_k + V_p)$ because the second term in (3.69) usually takes negative values during the first swing, and accordingly, $(V_k + V_p)$ decreases with time. It should be noted, however, that we have implicitly made two assumptions in deriving the condition in (3.112) as follows:

1) The system moves along the sustained fault trajectory even in the period after the fault clearance.
2) The internal voltages vary in the same way as for the sustained fault case even in the period after the fault clearance.

The former assumption is the same as one made in deriving Stability condition 4, and it is satisfied to some extent in actual power systems. The latter assumption is not satisfied so well as the former one. In usual, the internal voltages do not decreases in the period after the fault clearance so much as in the fault-on period. Since the transient stability region becomes narrower with decrease in internal voltages, so its boundary will move as shown in Fig.38. Consider a case where a fault is critically cleared. If both the above assumptions are satisfied, then the system will just reach the point c, and the boundary will just move to O. On the other hand, if the internal voltages do not decrease so much as in the sustained fault case, then the system will not reach the point c, and the boundary will not move to O, but to O' in actual, too. This fact implies that the system still has a little stability margin, namely, that the condition (3.112) is somewhat conservative compared with the actual stability of the system. However, this conservative nature proves to be acceptable if we consider a case where the system just stay on the point c'. In this case, f_r has no component along the sustained fault trajectory at this point, so the system will stay there for long time. The internal voltages decrease much there as described in §3.3.1, and the transient stability region becomes narrower. If its boundary moves from O' toward O, as a result, the system gets out of the stability region, and will lose synchronism. Thus it proves to be necessary to reserve some stability margin, which has been already incorporated in the condition (3.112) by adopting V instead of (V_k + V_p), and by making the assumption 2).

It is necessary to look for the point c in order to apply Stability condition 6. The time derivative of V_k in (3.68) can be rewritten as follows:

$$\frac{dV_k}{dt} = -\frac{\partial V_p}{\partial \delta_r}\omega_r \qquad (3.113)$$

The sign of this time derivative depends on the angle which is made by the two vectors ($\partial V_p/\partial \delta_r$) and ω_r. Fig.39 illustrates the relation between the relative angular velocity ω_r and the partial derivative ($\partial V_p/\partial \delta_r$) at the instant when the system crosses the boundary of the transient stability region. The vector ($\partial V_p/\partial \delta_r$) is always orthogonal to the equipotential curves. From the definition, the boundary O is also orthogonal to the equipotential curves. It is assumed in this figure that ω_r is perpendicular to the boundary O at the point c. In the inside of the boundary, ($\partial V_p/\partial \delta_r$) and ω_r make an accute angle, so

$$\frac{dV_k}{dt} < 0 \qquad (3.114)$$

holds, namely, V_k decreases in the stability region. At the point c, the two vectors make a right angle, so

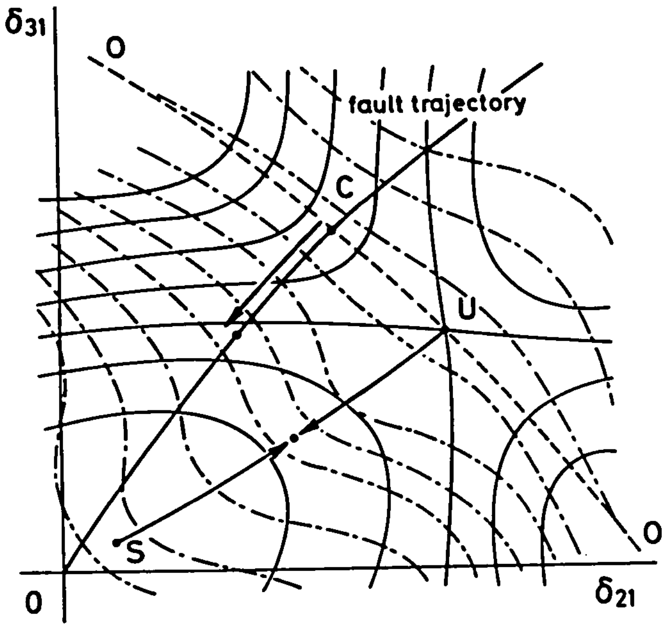
$$\frac{dV_k}{dt} = 0 \qquad (3.115)$$

holds, that is, V_k stops decreasing at this point. In the outside of the boundary, the two vectors make an obtuse angle, so

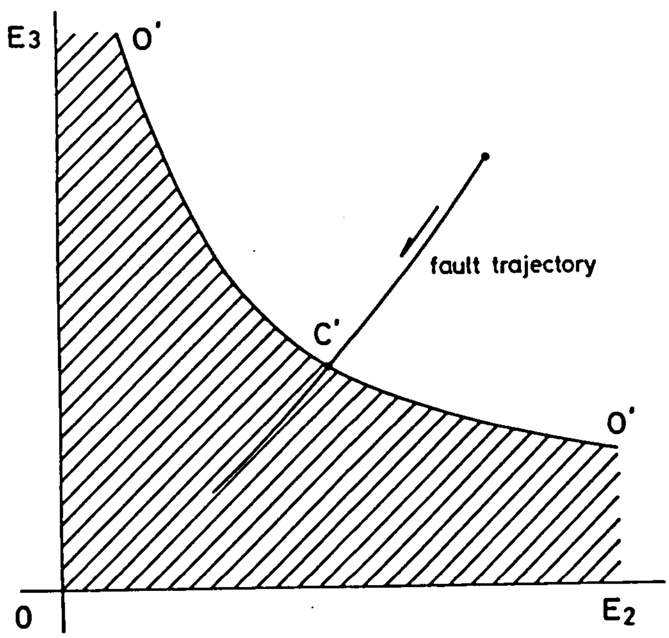
$$\frac{dV_k}{dt} > 0 \qquad (3.116)$$

holds, namely, V_k begins to increase, afterwards. Thus the time derivative of V_k changes its sign from negative to positive at the instant when the system crosses the boundary. Hence we can easily obtain the point c, and accordingly, the critical value V_{cr} by examining the sign of the time derivative. In practical cases, however, ω_r is not precisely perpendicular to the boundary O, and a little discrepancy exists between the point c and the point where the time derivative vanishes. If we consider that the value of V_p varies slowly along ω_r while the direction of $(\partial V_p/\partial \delta_r)$ varies rapidly, then this discrepancy proves to be negligible. Thus we can get the point c with adequate accuracy in those cases, too. This method does not need nay calculation of saddle points, so it can get rid of the difficulty associated with their calculation.

Hitherto, we have developed our discussion under the assumption that the transient stability region always exists. However, there are several cases where the transient stability region vanishes owing to decreases in internal voltages. We should make some comments on these cases. Consider a case where a system does not crosses the boundary of the transient stability region before it vanishes under a sustained fault. Fig.40 shows an example of system trajectory in a relative angular space(a), and in an internal voltage space(b), for a case where a fault continues. The internal voltage E decreases as in Fig.40(b), and it enter the shaded area in which the transient stability region does not exist. The equipotential curves initially are ones as shown by the continuous lines. V_p takes its minimum value at the point S. U denotes a saddle point. The points S and U are separated. However, with decrease in some internal voltages, S and U aproach each other, and at last, join when E crosses the boundary O' at the



(a) Trajectory in δ_r space



(b) Trajectory in E space

Fig.40. Trajectory in (δ_r, E) space under sustained fault which occurs vanishment of transient stability region.

point c'. The equipotential curves at this instant are such ones as shown by dotted lines. There exists no transient stability region any more. The system is accelerated afterwards, and will lose synchronism. Since, before the instant, the transient stability region exists, and the system does not crosses its boundary, we obtain a stability condition as follows:

[Stability condition 7]

If a system satisfies

$$V < V'_{cr} \tag{3.117}$$

then it is stable for the first swing, where V'_{cr} is the value of $[V_p(\delta_r, E) + V_f(E)]$ at the instant when the transient stability region vanishes under the sustained fault.

This condition is, of course, derived under the assumptions 1) and 2) made in deriving Stability condition 6. It is corresponding to the condition in (3.111). If it is satisfied, the transient stability region does not vanish, and accordingly, the second type of instability does not occur during the first swing. In order to reserve some necessary stability margin as described in the preceding paragraph, V'_{cr} may be replaced by V_{cr} although this replacement may result in too conservative condition because $V_f(E)$ has relatively large values in the cases where the second type instability can occur. The instant when the transient stability region vanishes can be obtained with the same method as one of detecting the point c where the system crosses the boundary of the transient stability region. Since the transient stability region exists, and the system does not crosses its boundary before that instant, so

$$\frac{dV_k}{dt} < 0 \tag{3.118}$$

holds, that is, V_k decreases. However, after the instant, the equipotential curves becomes as shown by the dotted lines. The vector $(\partial V_p/\partial \delta_r)$ and ω_r make an obtuse angle, so

$$\frac{dV_k}{dt} > 0 \qquad (3.119)$$

holds, and V_k begins to increase. Namely, the time derivative of V_k changes its sign from negative to positive at the instant when the transient stability region vanishes. Hence we can get the critical value V'_{cr} by examining the sign of the time derivative of V_k under the sustained fault.

If we replace V_{cr} by V'_{cr} in (3.112), or replace V'_{cr} by V_{cr} in (3.117), then Stability condition 6 and 7 are equivalent to each other. Namely, we can use the same method in determining the critical value for both the cases where the transient stability region remains all the time, and the cases in which the transient stability region vanishes owing to some decreases in internal voltages.

3.5 Conclusions

In this section, we have made some investigations on the critical value of the Lyapunov function constructed in §2. The transient stability region was defined on the basis of torque applied to the system by regarding the internal voltages as parameters. There are several natures which are characteristic of multimachine power systems in which the dynamics of field flux linkages are taken into account, as follows:

1) When some internal voltages decrease in magnitude owing to a fault, the transient stability region varies in such way that the corresponding generators will step out easily.
2) Conversely, when some generators are accellated owing to a fault, and their rotor angles are separated from other generators, their internal voltages decreases much.
3) When we use a value of V_p at a saddle point U, denoted by V_u, as the critical value of V, some stability margin V_r must be reserved because $[V_u - (V_k + V_p)]$ usually decreases during the first swing.
4) When some internal voltages decrease much in magnitude, there are several cases where the transient stability region vanishes, and as a result, the system loses synchronism. We temporally refer this instability as the second type of instability.

Bearing these natures in mind, we have derived two stability conditions. One of them is associated with the case where the transient stability region remains regardless of the variations of internal voltages. The other is associated with the case where the transient stability region vanishes owing to some decrease in internal voltages. When we use any of them as the basis for determining the critical value, it can be obtained by examining the sign of the time derivative of V_K under a sustained fault. It need no calculation of saddle points, so we can get rid of the difficulty which we experience in calculation of these points.

§4. Influence of transfer conductances

In this section, we make some investigations on the influence of the transfer conductances. Transfer conductances have been neglected in constructing the Lyapunov function and in determining its critical value. In usual, loads in power systems are represented by constant impedances, and they are equivalently incorporated to reduced admittance matrices. Transfer conductances denote real parts of reduced admittance matrices. Hence they get larger with increase in loads. Practical power systems have loads which are comparable with their capacity, so transfer conductances are usually not negligible. Since we have made some investigations on their influence in the previous chapter, we describe only on a little difference between the power system which is represented by the conventional model and the power system in which the dynamics of field flux linkages is taken into account.

4.1. Model and basic equations

Since transfer conductances of reduced admittance matrices are taken into account, the motion of the ith generator is described as follows:

$$m_i \frac{d^2\delta_i}{dt^2} = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) + \sum_{j=1}^{n} G_{ij}(E_i^o E_j^o \cos\delta_{ij}^o - E_i E_j \cos\delta_{ij}) \quad \text{for } i=1,2,\dots,n \tag{3.120}$$

and

$$\frac{dE_i}{dt} = -\alpha_i(E_i - E_i^o) - \beta_i \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})$$

$$+ \beta_i \sum_{\substack{j=1 \\ j\neq i}}^{n} G_{ij}E_j(\sin\delta_{ij}^o - \sin\delta_{ij}) \qquad (3.121)$$

$$\text{for } i=1,2,\ldots,n$$

where B_{ij} and G_{ij} are the transfer susceptance and the transfer conductance between the ith and the jth generators, respectively, which are defined by

$$B_{ij} = Y_{ij}\cos\theta_{ij} \qquad G_{ij} = Y_{ij}\sin\theta_{ij} \qquad (3.122)$$

and

$$\alpha_i = [1 - (x_{di} - x'_{di})B_{ii}]/T'_{doi}$$

$$\beta_i = (x_{di} - x'_{di})/T'_{doi} \qquad (3.123)$$

The superscript " o " denotes the stable equilibrium point of the post-fault system, so (3.120) and (3.121) applies to the post-fault state.

For the power system without transfer conductances, the following Lyapunov function has been constructed:

$$V(x) = (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o]$$

$$+ \sum_{i=1}^{n}(\alpha_i/\beta_i)(E_i - E_i^o)^2$$

$$= V_k(\omega) + V_p(\delta, E) + V_f(E) \qquad (3.124)$$

where V_k, V_p, and V_f denote kinetic energy, potential energy, and one associated with internal voltages. From (3.120) and (2.121), the time derivatives of V_k and V_p are given as follows:

$$\frac{dV_k}{dt} = 2\sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}(E_i^o E_j^o \cos\delta_{ij}^o - E_i E_j \cos\delta_{ij})(\omega_i - \bar{\omega}) + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \tag{3.125}$$

$$\frac{dV_p}{dt} = -\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) + 2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \tag{3.126}$$

$$\frac{dV_f}{dt} = -2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) + 2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} G_{ij}E_j(\sin\delta_{ij}^o - \sin\delta_{ij}) - 2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)^2 \tag{3.127}$$

where $\bar{\omega}$ is defined by

$$\bar{\omega} = \sum_{i=1}^{n} m_i\omega_i / \sum_{i=1}^{n} m_i \tag{3.128}$$

The first term in (3.125) is due to transfer conductances, and is not sign-definite. The second term of (3.125) and the first term of (3.126) are of the same magnitude, and of the opposite signs of each other. Similarly, the second term of (3.126) and the first term of (3.127) are of the same magnitude, and of opposite signs of each other. Those terms cancell out with each other, and do not contribute to the damping rate of V. Consequently, the time derivative of V is given as follows:

$$\frac{dV}{dt} = 2\sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}(E_i^o E_j^o \cos\delta_{ij}^o - E_i E_j \cos\delta_{ij})(\omega_i - \bar{\omega}) + 2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} G_{ij}E_j(\sin\delta_{ij}^o - \sin\delta_{ij}) - 2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)^2 \tag{3.129}$$

The right hand term of (3.129) is not sign-definite, so V in (3.124) is not a Lyapunov function for the power systems with transfer conductances. Some correction must be made on V in order to use it as Lyapunov function.

4.2 Time derivative of kinetic energy V_k

In §3, we have developed a method of determining the critical value of V. This method is based on the time derivative of kinetic energy V_k. The time derivative changes its sign from negative to positive when a system crosses the boundary of the transient stability region, or when the transient stability region vanishes. In the cases where transfer conductances are taken into account, the time derivative changes from (3.68) to (3.125), however.

Define g_i and g_r as follows:

$$g_i(\delta_r, E) = \sum_{j=1}^{n} G_{ij}(E_i^o E_j^o \cos\delta_{ij}^o - E_i E_j \cos\delta_{ij}) \tag{3.130}$$

$$g_r(\delta_r, E) = [\ g_2(\delta_r, E),\ g_3(\delta_r, E),\ \ldots,\ g_n(\delta_r, E)\ ] \tag{3.131}$$

then (3.125) can be rewritten as follows:

$$\frac{dV_k}{dt} = [\ 2Lg_r(\delta_r, E) - \frac{\partial V_p}{\partial \delta_r}\ ]'\omega_r \tag{3.132}$$

or from (3.83),

$$\frac{dV_k}{dt} = 2[\ f_r(\delta_r, E) + Lg_r(\delta_r, E)\ ]'\omega_r \tag{3.133}$$

where $L = (\ell_{ij})$ is an $(n-1)\times(n-1)$ matrix defined as follows:

$$\ell_{ij} = \begin{cases} 1 - m_{(i+1)} / \sum_{k=1}^{n} m_k & \text{for } i=j \\ - m_{(i+1)} / \sum_{k=1}^{n} m_k & \text{for } i\neq j \end{cases} \tag{3.134}$$

The torque f_r always acts on the system in the direction which is orthogonal

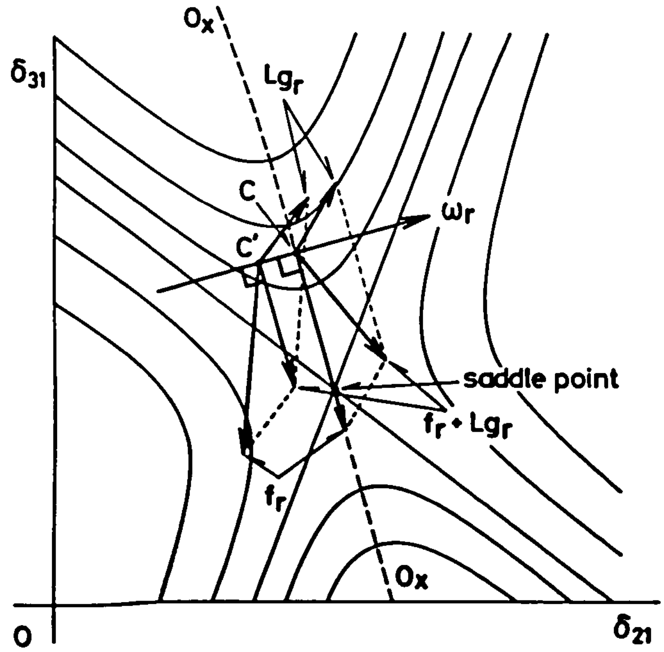


Fig.41. Relations between ω_r, f_r, and Lg_r when system crosses boundary O_x.

to the equipotential curves, and in such way that it will synchronize the system. Only this torque exists in the power systems without transfer conductances. However, there exists a new term Lg_r as shown in (3.132) or (3.133) in the cases where transfer conductances are taken into account. The sign of (dV_k/dt) is determined by the angle which is made by the two vectors ω_r and $(f_r + Lg_r)$, namely, f_r is replaced by $(f_r + Lg_r)$. Fig.41 illustrates an example of a trajectory in a relative angular space for a case where a fault continues. The trajectory crosses the boundary O of the transient stability region which defined on the basis of f_r. The angular velocity ω_r is assumed to be orthogonal to the boundary O at the point c. Since f_r make a right angle with ω_r at the point c, (dV_k/dt) vanishes if Lg_r has no component along ω_r. In general, Lg_r has a components along ω_r, denoted by $Lg_{r//}$, however, so the direction of $(f_r + Lg_r)$ deviates from that of f_r as shown in Fig.41. The time derivative (dV_k/dt) vanishes at the point c' at which $(f_r + Lg_r)$ make a right angle with ω_r. Before the system reaches this point, $(f_r + Lg_r)$ and ω_r make obtuse angles, and accordingly, (dV_k/dt) takes negative values. Namely, kinetic energy V_k decreases, which implies that the torque acts in such way that it will keep the system in synchronism. On the other hand, $(f_r + Lg_r)$ and ω_r make accute angles, and accordingly, (dV_k/dt) takes positive values after the system goes over the point c'. Hence kinetic energy V_k begins to increase, which implies that the torque acts in such way that it will separate the system into two groups. Summing up the above discussion, the time derivative of V_k changes its sign from negative to positive at the instant when the system goes over the point c' which is usually different from the point c. Since the kinetic energy begins to increase if the system goes over the point c', we conclude that the point c' is on the boundary of the actual transient stability region, and adopt the value of $V_p(\delta_r, E)$ at this point as the critical value of V for the first swing.

An example will serve to clarify the correction which is made. The 10-machine power system shown in Fig.42 is studied. The system is disturbed by a 3-phase short-circuit which occurs at a point near the bus 11, and is cleared by opening the line connecting the buses 11 and 12 at both terminals. Fig.43 shows the time variations of V, its components, and (dV_k/dt) for the case where the fault continues. As shown in Fig.43(b), (dV_k/dt) changes its sign from negative to positive at 0.43 sec. The value of V_p at this time is

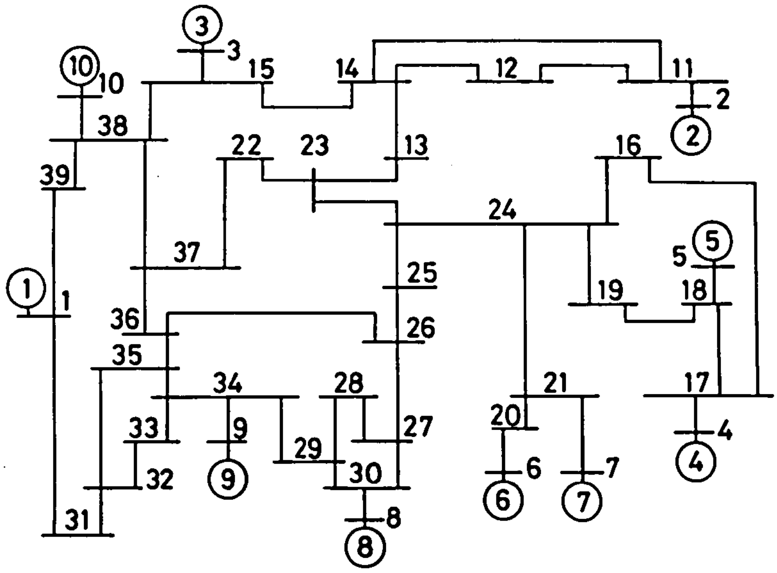
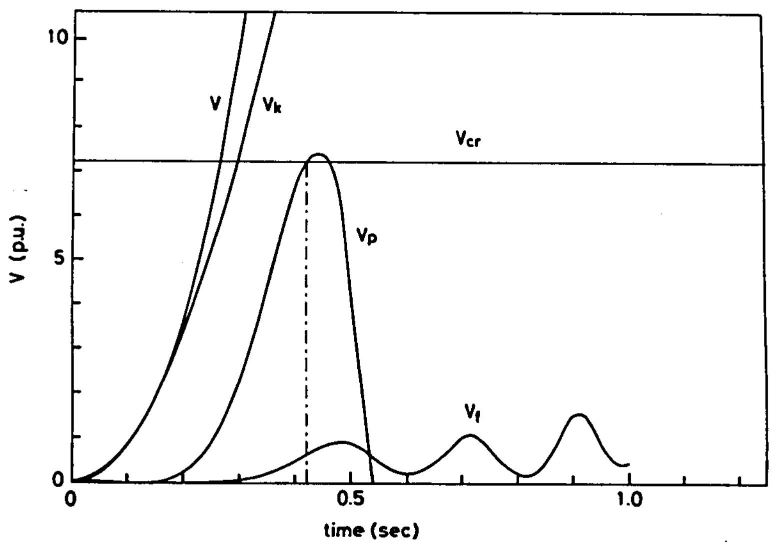
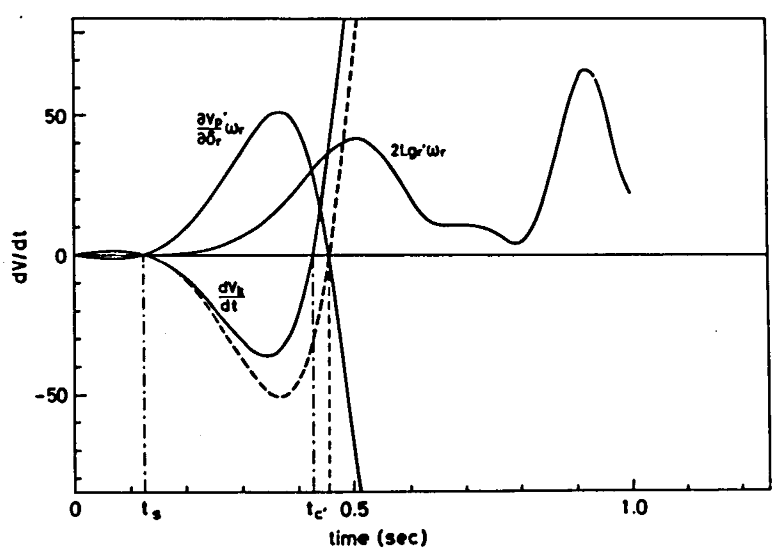


Fig.42. Configuration of 10-machine system.



(a) V and its components



(b) Time derivative of V_k

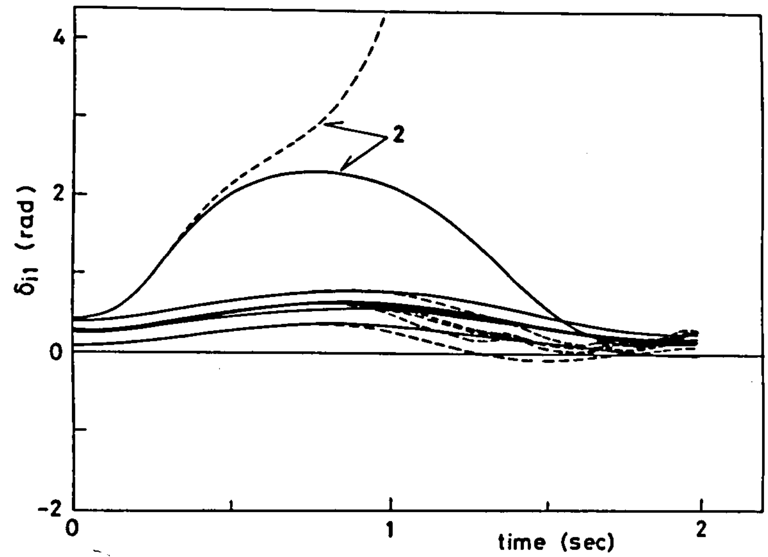
Fig.43. Method of determining critical value of V in case where field flux decays are taken into account.

7.225. On the other hand, if the new term Lg_r is neglected, then (dV_k/dt) varies as shown by the dotted line. In this case, (dV_k/dt) changes its sign from negative to positive at 0.46 sec, and V_p takes 7.304 at this instant. If transfer conductances are negligible, then this value will be used as the critical value. However, the system contains transfer conductances, so 7.225 is adopted as the critical value. Thus the critical value of V is determined for the systems with transfer conductances.

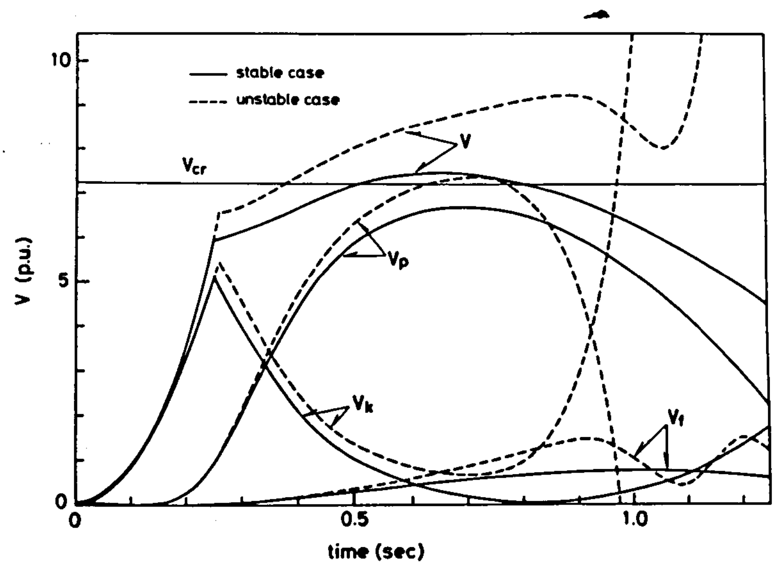
4.3 Time derivative of V

In the preceding section, we have derived a method of determining the critical value for the power systems with transfer conductances. As the basis of this method, we have used Stability condition 6 and 7 which have been derived under the assumption that the time derivative of V is given by (3.71). However, transfer conductances are not negligible in practical power systems, and the time derivative of V is given by (3.129). Hence the method of determining the critical value is not able to yield results of good accuracy by itself.

An example will serve to clarify this fact. In the case of Fig.43, the critical value is found out to be 7.225. Since V takes 6.667 at 0.26 sec, and 7.341 at 0.27 sec, it is guessed that the critical fault clearing time exists between 0.26 and 0.27 sec. However, the actual critical clearing time exists between 0.25 abd 0.26 sec as observed from Fig.44(a). In Fig.44(b), the time variations of V and its components are shown for the cases where the fault is cleared at 0.25 sec (stable case), and 0.26 sec (unstable case). V increases monotonously during the fault-on period. In both cases, the fault is cleared before V reaches the critical value V_{cr}. In spite of this fact, the system remains in synchronism in one case, and loses synchronism in the other case. V slowly increases after the instant of fault clearance. In the stable case, V reaches a peak, and decreases, afterwards. On the other hand, in the unstable case, V increases monotonously, and it becomes greater than V_{cr} after the time 0.39 sec. It should be noted that V becomes greater than V_{cr} at several times even in the stable case. V takes its peak value 7.456 at 0.65 sec, and this value is certainly greater than V_{cr}. However, $(V_k + V_p)$ takes 6.951 at this instant. If we remember that Stability condition 6 has been derived by comparing $(V_k + V_p)$ with V_{cr}, then the system remains in synchronism because $(V_k + V_p)$ is



(a) Rotor angles



(b) V and its components

Fig.44. Influence of transfer conductances on V and its components.

smaller than V_{cr} in the stable case. In any case, it is not desirable that V increases after the fault clearance because we can not get an a-curate estimation in such case.

In order to explain such time variation of V, let us consider the time derivative of V. It is given by (3.129). The first term is associated with V_k, and the second and the third terms are with V_f. The first term is not sign-definite, and it takes positive values after the fault clearance. The increse in V is owing to this term. Similarly, the second term takes positive values during the first swing, and it seems to increase V as well as the first term. However, we can rewrite (3.129) as follows:

$$\begin{aligned}\frac{dV}{dt} &= 2\sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}(E_i^o E_j^o \cos\delta_{ij}^o - E_i E_j \cos\delta_{ij})(\omega_i - \bar{\omega}) \\ &- 2\sum_{i=1}^{n}(dE_i/dt)_o \sum_{j=1}^{n} G_{ij}E_j(\sin\delta_{ij}^o - \sin\delta_{ij}) \qquad (3.135) \\ &- 2\sum_{i=1}^{n}(1/\beta_i)(dE_i/dt)_o^2\end{aligned}$$

where $(dE_i/dt)_o$ is defined as follows:

$$(dE_i/dt)_o = -\alpha_i(E_i - E_i^o) - \beta_i \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \qquad (3.136)$$

Namely, the right hand of (3.136) is the same as that of (3.66). Consider a case where some generators are accellated owing to a fault, then for these generators the following inequalities hold during the first swing:

$$\begin{aligned}&(dE_i/dt)_o < 0 \\ &\sum_{j=1}^{n} G_{ij}E_j(\sin\delta_{ij}^o - \sin\delta_{ij}) < 0\end{aligned} \qquad (3.137)$$

and for other generators the following relations hold:

$$(dE_i/dt)_o \simeq 0$$

$$\sum_{j=1}^{n} G_{ij}E_j(\sin\delta^o_{ij} - \sin\delta_{ij}) \underset{\sim}{-} 0 \tag{3.138}$$

From (3.137) and (3.138), the second term of (3.135) takes negative values during the first swing. The third term is, of course, negative semi-definite. Thus it is concluded that the increase in V in some interval after the fault clearance, is due to only the first term of (3.135), and that we only have to take the first term in consideration.

In §5 of the previous chapter, we have introduced the function V_β in order to take account of transfer conductances. Following this function, we will derive a function in the following. We can rewrite (3.125) and (3.126) as follows:

$$\frac{dV_k}{dt} = 2Lg_r(\delta_r, E)'\omega_r - \frac{\partial V_p'}{\partial\delta_r}\omega_r \tag{3.139}$$

$$\frac{dV_p}{dt} = \frac{\partial V_p'}{\partial\delta_r}\omega_r + \frac{\partial V_p'}{\partial E}\frac{dE}{dt} \tag{3.140}$$

Hence, the increments of V_k and V_p in a time interval are given as follows:

$$\begin{aligned}\Delta V_k &= \int_{t_s}^{t_e} \frac{dV_k}{dt}dt \\ &= \Delta V_g(\delta_s \sim \delta_e, E_s \sim E_e) - \Delta V_{k\to p}(\delta_s \sim \delta_e, E_s \sim E_e)\end{aligned} \tag{3.141}$$

$$\begin{aligned}\Delta V_p &= \int_{t_s}^{t_e} \frac{dV_p}{dt}dt \\ &= \Delta V_{k\to p}(\delta_s \sim \delta_e, E_s \sim E_e) - \Delta V_{p\to f}(\delta_s \sim \delta_e, E_s \sim E_e)\end{aligned} \tag{3.142}$$

where

$$\Delta V_g(\delta_s \sim \delta_e, E_s \sim E_e) \equiv \int_{\delta_s}^{\delta_e} 2Lg_r(\delta_r, E)'d\delta_r \tag{3.143}$$

$$\Delta V_{k\to p}(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv \int_{\delta_s}^{\delta_e} \frac{\partial V_p'}{\partial \delta_r} d\delta_r \qquad (3.144)$$

$$\Delta V_{p\to f}(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv -\int_{E_s}^{E_e} \frac{\partial V_p'}{\partial E} dE \qquad (3.145)$$

t_s and t_e are both ends of the time interval, and δ_s, δ_e, E_s, and E_e are the values of δ_r and E at the instants t_s and t_e, respectively. The term ΔV_g denotes the energy which is generated owing to transfer conductances, $\Delta V_{k\to p}$ denotes the energy which is transfered from V_k to V_p, and $V_{p\to f}$ denotes the energy which is transfered from V_p to V_f. These terms are determined by the trajectory in (δ_r, E)-space, and it does not matter how long it takes the system to move from (δ_s, E_s) to (δ_e, E_e). Let us define a ratio γ as follows:

$$\gamma(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv |\Delta V_{k\to p}/\ \Delta V_k| = \frac{1}{1 - \Delta V_g/\Delta V_{k\to p}} \qquad (3.146)$$

This ratio is one of $\Delta V_{k\to p}$ to ΔV_k, and implies that kinetic energy V_k is transfered to potential energy V_p at rate of γ when the system moves from (δ_s, E_s) to (δ_e, E_e). It varies with $\Delta V_g/\Delta V_{k\to p}$ as follows:

$$\begin{aligned} \gamma &> 1 \quad &\text{for}\quad \Delta V_g/\Delta V_{k\to p} &> 0 \\ \gamma &= 1 \quad & \Delta V_g/\Delta V_{k\to p} &= 0 \\ \gamma &< 1 \quad & \Delta V_g/\Delta V_{k\to p} &< 0 \end{aligned} \qquad (3.147)$$

If transfer conductances are zero, then the second equation in (3.147) holds, and accordingly, γ is equal to 1. Namely, ΔV_k is transfered from V_k to V_p at the rate 1. On the other hand, if transfer conductances are not zero, γ deviates from 1, that is, the first or the third inequalities hold. Lets return to Fig.41. This figure illustrates a trajectory in a relative angular space for a case where a fault continues. The point c' is on the boundary of the actual stability region. If the system goes over this point, then the system will lose synchronism. According to Stability condition 6, the value of V_p at this point, denoted by V_{cr}, is adopted as the critical value of V. Assume that the system moves along the trajectory, reaches the point c', and stops there when the fault is critically cleared, then the following equation holds.

$$\begin{aligned} 0 &= V_k(t_{c'}) \\ &= V_k(t_{cr}) - \Delta V_k(\delta_{cr} \sim \delta_{c'}, E_{cr} \sim E_{c'}) \end{aligned} \tag{3.148}$$

$$\begin{aligned} V_{cr} &= V_p(t_{c'}) \\ &= V_p(t_{cr}) + \Delta V_{k\to p}(\delta_{cr} \sim \delta_{c'}, E_{cr} \sim E_{c'}) \\ &\qquad - \Delta V_{p\to f}(\delta_{cr} \sim \delta_{c'}, E_{cr} \sim E_{c'}) \end{aligned} \tag{3.149}$$

where (δ_{cr}, E_{cr}) and $(\delta_{c'}, E_{c'})$ are the values of (δ_r, E) at the critical clearing time t_{cr} and the time $t_{c'}$ when the system reaches the point c', respectively. From (3.146), (3.148), and (3.149), we obtain

$$\begin{aligned} & V_p(t_{cr}) + \gamma(\delta_{cr} \sim \delta_{c'}, E_{cr} \sim E_{c'})V_k(t_{cr}) \\ &\quad = V_{cr} + \Delta V_{p\to f}(\delta_{cr} \sim \delta_{c'}, E_{cr} \sim E_{c'}) \\ &\quad \geq V_{cr} \end{aligned} \tag{3.150}$$

where we have used in deriving the last inequality the fact that $\Delta V_{p\to f}$ takes a negative value during the first swing. Now, let us define a function as follows:

$$V_\beta(t) = V_p(t) + \gamma V_k(t) + V_f(t) \tag{3.151}$$

where

$$\gamma \equiv \gamma(\delta_r \sim \delta_{c'}, E \sim E_{c'}) \tag{3.152}$$

in which (δ_r, E) is not determined yet. If (δ_{cr}, E_{cr}) is substituded into

(δ_r, E) in (3.152), then

$$V_\beta(t_{cr}) \geq V_{cr} \tag{3.153}$$

holds from (3.150) and (3.151) because V_f always takes non-negative values as is clear from (3.124). If a fault is critically cleared, then (3.153) always holds. Hence, we reach a stability condition for the first swing stability given as follows:

[Stability condition 8]

If a system satisfies

$$V_\beta < V_{cr} \tag{3.154}$$

then it is stable for the first swing, where V_{cr} is the value of $V_p(\delta_r, E)$ at the point c' where the system crosses the boundary of the transient stability region under a sustained fault.

It should be noted that the same assumptions as in Stability condition 6 and 7 have been made in deriving this condition, too. Besides, it is observed from (3.153) and (3.154) that this condition may yield results conservative compared with actual stability. Lastly, it is impractical to use γ which is obtained by substituting (δ_{cr}, E_{cr}) into (δ_r, E) in (3.152) because t_{cr} is evidently unknown in the stage of estimation. Now let us return to Fig.43(b). In the period between t_s and $t_{c'}$, (dV_k/dt) takes negative values while $(\partial V_p/\partial \delta_r)'\omega_r$ takes positive values, which implies that V_k decreases, and that its energy is transformed into V_p. In the stage of estimation, we make an approximation of γ as follows:

$$\gamma \simeq \gamma(\delta_s \sim \delta_{c'}, E_s \sim E_{c'}) \tag{3.155}$$

The instant t_s is not generally equal to the critical clearing time t_{cr}, so there is some difference between this approximation and γ used in (3.153) and (3.154). Fig.45 shows some time variations of V_β and its components for the same case in Fig.43. The value of γ is 1.388. V_k is first

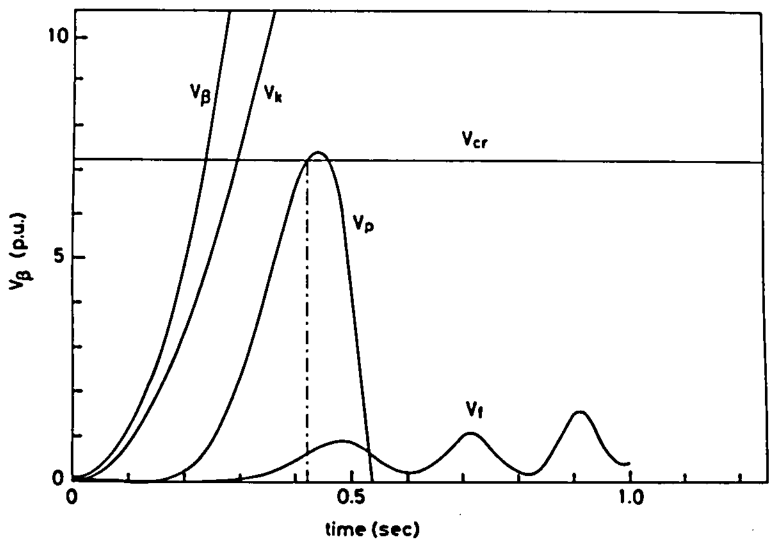


Fig.45. Time variations of V_β and its components: sustained fault.

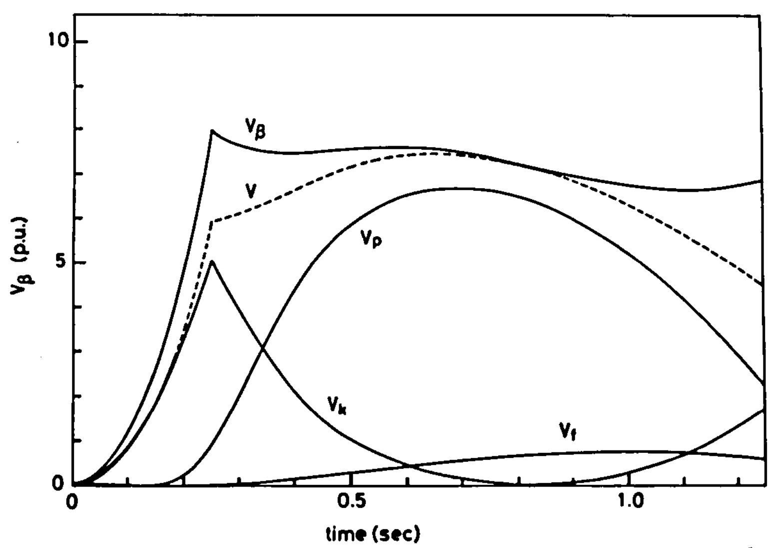


Fig.46. Time variations of V_β and its components: critically cleared fault.

zero, so V and V_β take the same value at 0.0 sec. The difference between them become larger with increase in V_k. The critical value V_{cr} is 7.225. V_β takes 6.609 at 9.23 sec, and 7.297 at 0.24 sec, and accordingly, the critical clearing time is estimated to be 0.239 sec. Since the actual critical clearing time exists between 0.24 and 0.25 sec, the estimation result is adequately accurate. Fig.46 shows the time variations of V_β and its components for the case where the fault is cleared at 0.24 sec. The value of γ is the same as that in Fig.45, i.e., 1.388. The function V designated by the dotted line increases after the fault clearance. On the other hand, V_β decreases monotonously after the clearing time. These facts imply that the transfer conductances have been taken into account well by using the function V_β instead of V.

4.4 Conclusions

In this section, we have developed some methods of taking account of transfer conductances which have been neglected in constructing V and in determining its critical value. If there are some transfer conductances, generators receive additional torques owing to them. In order to correct these torques, some corrections have been made in determining the critical value of V. The time derivative of V_k changes its sign from negative to positive at the point where the system crosses the boundary of the transient stability region defined on the basis of the equipotential curves if transfer conductances are negligible, but it changes its sign at a point different from the above point if transfer conductances are not negligble. We adopt the value of V_p at this point as the critical value of V. On the other hand, V increases after the instant of fault clearance owing to transfer conductances, again. Since this increase has wrong influence on the accuracy of estimations of the critical fault clearing time, we have introduced a new function V_β in order to remove this influence. It is generated by multiplying some ratio on V_k in V. As an example, we have applied these method to an estimation of critical fault clearing time in a case where a fault occurs in a 10-machine power system, and have obtained an good result.

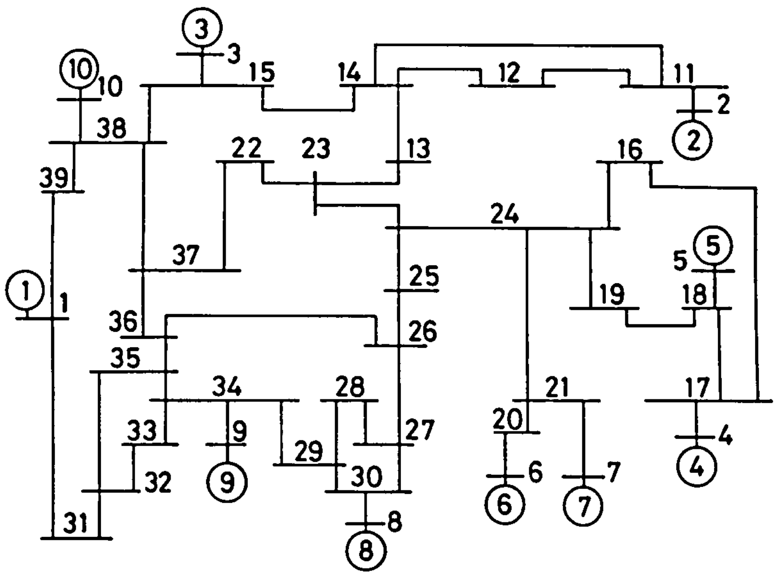


Fig.47. Configuration of 10-machine system.

Table 12. Generator parameters of 10-machine system.

Unit	H	x_d	x_d'	x_q	T_{do}'	
					Case 1	Case 2
1	500.0	0.0200	0.0060	0.0190	7.00	1.75
2	34.5	0.2106	0.0570	0.2050	4.79	1.20
3	24.3	0.2900	0.0570	0.2800	6.70	1.68
4	26.4	0.2950	0.0490	0.2920	5.66	1.42
5	34.8	0.2540	0.0500	0.2410	7.30	1.82
6	26.0	0.6700	0.1320	0.6200	5.40	1.35
7	28.6	0.2620	0.0436	0.2580	5.69	1.42
8	35.8	0.2495	0.0531	0.2370	5.70	1.42
9	30.3	0.2950	0.0697	0.2820	6.56	1.64
10	42.0	0.1000	0.0310	0.0690	10.20	2.55

§5. Numerical example

In this section, the transient stability of a 10-machine power system is studied. The line diagram of the system is shown in Fig.47, and the data on its generators are provided in Table 12. It is assumed that this system is disturbed by a 3-phase short-circuit which occurs at a terminal x of a transmission line x-y, and is cleared by opening the line at both the terminals. Two different methods are used to analyze the transient stability, that is, the conventional method based on simulations and Lyapunov's direct method. These methods differs markedly in their approaches, but both of them can be used to compute the critical fault clearing time. Hence, this familiar transient stability measure is used as a basis of comparison. Two sets of d-axis transient open-circuit time constants are used in order to make some investigations on the influence of internal voltages on the stability. The time constants are shown in Table 12. The sets are refered as Case 1 and Case 2.

5.1 Procedure of estimation

The procedure for estimating the critical clearing time is shown in the flow chart of Fig.48. The main steps are as follows:

[Step 1] Read the necessary data on the system, i.e., those on transmission lines, buses, and generators.

[Step 2] Compute the load flow for the prefault system.

[Step 3] Compute the reduced admittance matrices between the generators by eliminating the buses without generators, for the fault and the post-fault systems.

[Step 4] Compute the stable equilibrium point for the post-fault system.

[Step 5] Integrate the fault system equations step by step, and compute the rotor angles, the rotor speeds, and the internal voltages: $\delta(t)$, $\omega(t)$, and $E(t)$.

[Step 6] Examine whether the system has or not reached the boundary of the transient stability region, whether the transient stability region has or not vanished by checking the sign of the time derivative of kinetic energy V_k. If the time derivative changes its sign from negative to positive, then go to Step 7, and if not, return to Step 5.

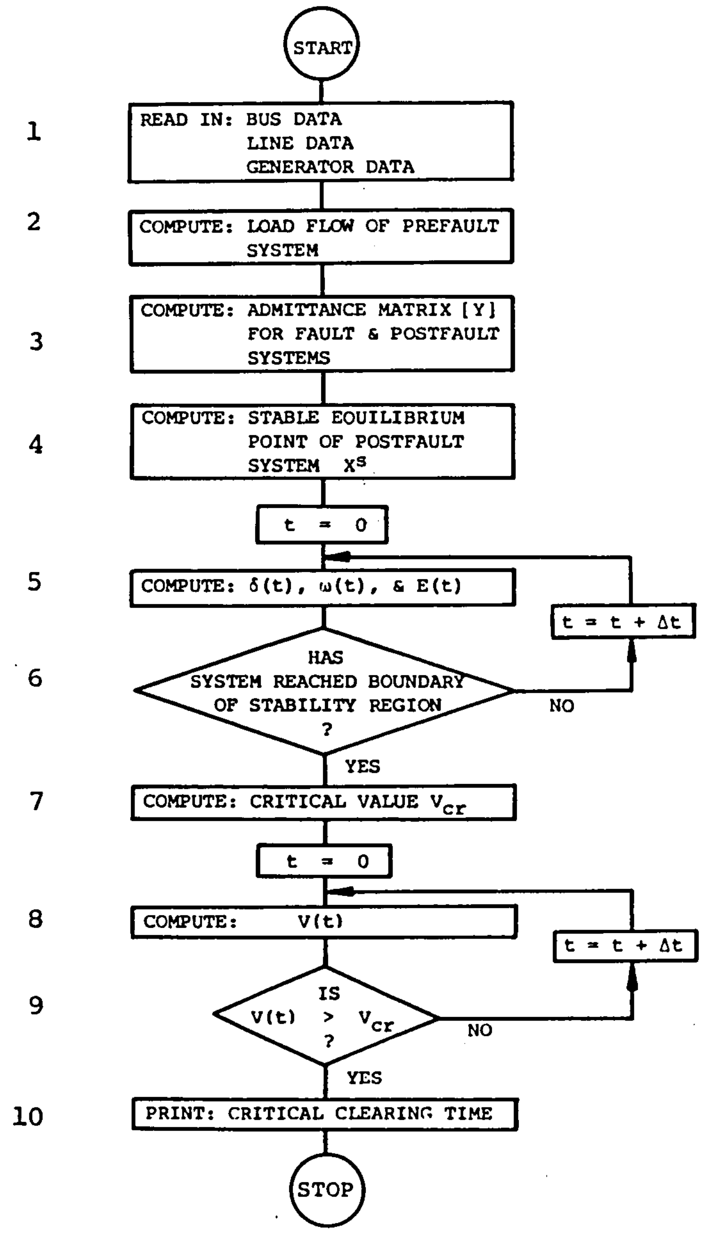


Fig.48. Flow chart for estimating critical fault clearing time by Lyapunov's direct method.

[Step 7] Compute the critical value V_{cr}, which is the value of the potential energy V_p at the instant when the time derivative (dV_k/dt) changes its sign.

[Step 8] Compute the value of the Lyapunov function V(t).

[Step 9] Compare V(t) and V_{cr}. If V(t) is greater than V_{cr}, go to Step 10, and if not, then return to Step 8.

[Step 10] Print the critical clearing time.

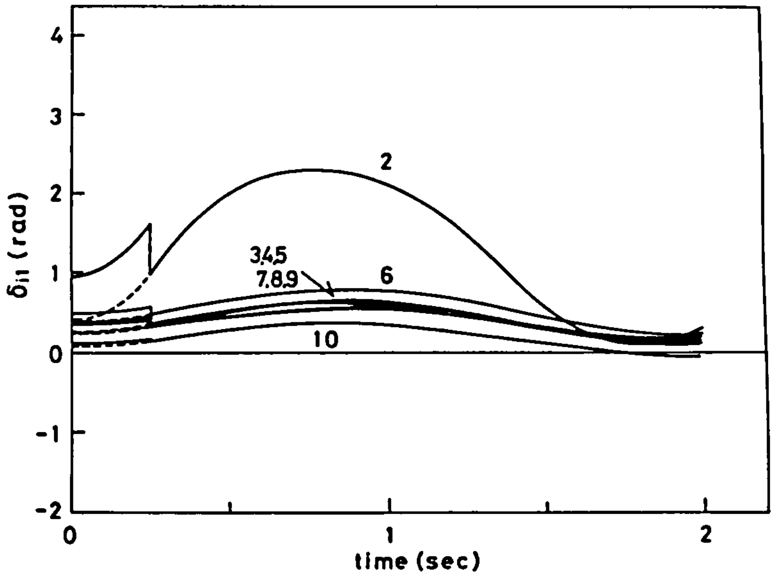
We can get an estimation of the critical clearing time for a fault through these steps. The first three steps need no explanation. The stable equilibrium point of the post-fault system is necessary in calculating the value of V. It is computed in Step 4 by solving the following equations:

$$P_{mi} - \sum_{j=1}^{n} Y_{ij}E_iE_j\sin(\delta_{ij} + \theta_{ij}) = C \text{ (constant)} \tag{3.156}$$

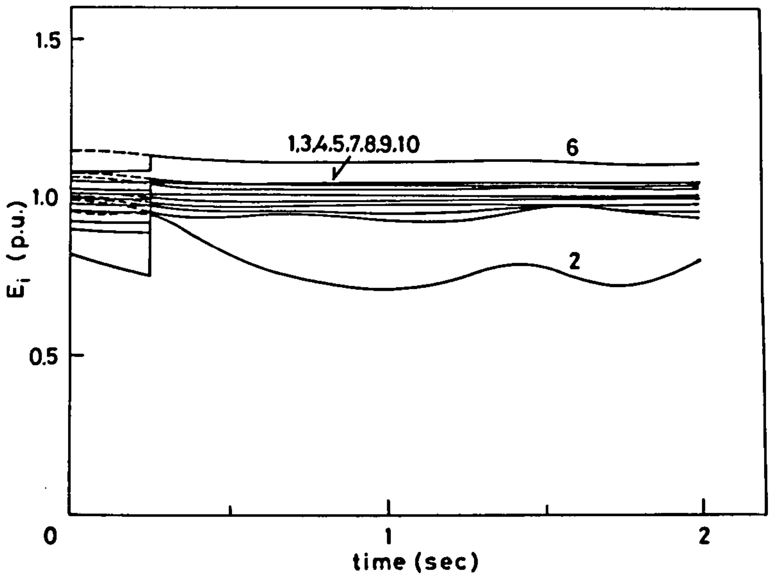
$$E_{fdi} - E'_{qi} - (x_{di} - x'_{di})i_{di} = 0 \tag{3.157}$$

for i=1,2,...,n

These equations are nonlinear, and are solved with Newton-Raphson method iteratively (Appendix C). Four or five iterations can yield results of good accuracy. In Step 5, the system equations (3.1) and (3.2) are numerically integrated step by step, and the values of $\delta'(t)$, $\omega(t)$, and $E'_{qi}(t)$ are calculated, where the integration step length Δt is 0.01 sec in our studies. From these values, $\delta(t)$, $\omega(t)$, and $E(t)$ are obtained, where δ, E, δ', and E'_{qi} are related with each other as shown in Fig. 31. Fig. 49 shows the time variations of the internal voltages for a fault 11-12 in the 10-machine system, where the fault is cleared at t = 0.25 sec. The internal voltages are not the state variables, so their variations are discontinuous at the clearing time when the admittance matrix changes from one for the fault system to one for the post-fault system. This discontinuousness is not desirable because the value of V which calculated with δ, ω, and E, also varies discontinuously at the clearing time. In order to remove this discontinuousness, we calculate them with the post-fault admittance matrix even in the fault-one period. After this manipulation, the internal voltages vary continuously as denoted by the dotted lines. In constructing the Lyapunov function V, we have assumed that each internal



(a) Angles of internal voltages



(b) Magnitudes of internal voltages

Fig.49. Time variations of internal voltages of generators for fault 11-12 in 10-machine system.

voltage lags behind the q-axis of each generator by constant angle ϕ_i all the time. This assumption is satisfied to an extent. Each internal voltage angle deviates a little from a constant angle ϕ_i, and correspondingly, E_i oscillates a little. However, these deviations have little influence on the time variation of V, hence we can use this function as Lyapunov function so far as transfer conductances are negligible. In Step 7, as the critical value of V, we use V_{cr} which denotes the value of V_p at the instant when the time derivative (dV_k/dt) changes its sign from negative to positive under a sustained fault. We can replace V_{cr} by V'_{cr} defined in (3.117), but we use V_{cr} in order to reserve some stability margin mentioned in §3.4. In Step 8, V is used as Lyapunov function, but it will be replaced by V_β for those cases where transfer conductances are not negligible. In this study, transfer conductances are not negligible, so V_β is used as well as V. The method of producing V_β has been described in §4.

5.2 Results by simulations

Firstly, the conventional approach based on simulations are applied to the transient stability analysis of the 10-machine system. The system equations (3.1) and (3.2) are integrated step by step to yield the time variations of the internal voltages, where damping torques of generators are assumed to be zero in this study. By observing these time variations we jugde whether the system is stable or not for a given fault. If the system is stable, the fault clearing time is delayed, and if not, then it is advanced. By iterating these manipulations, the critical clearing time is obtained. It usually takes 4 or more times for the iteration to converge.

Fig.50 and 51 show the time variations of the internal voltages for eight fault locations and two cases of time constants, respectively. In each figure, two cases of time variations in which a fault is cleared at an instant nearest to the critical one, are shown. The difference of clearing time between these two cases is 0.01 sec. In one of them, the system keeps synchronism for the first swing, and in the other case, the system loses synchronism. We call them stable case and unstable case, and denote by real lines and dotted lines, respectively. From these figures, we can observe several features as follows:

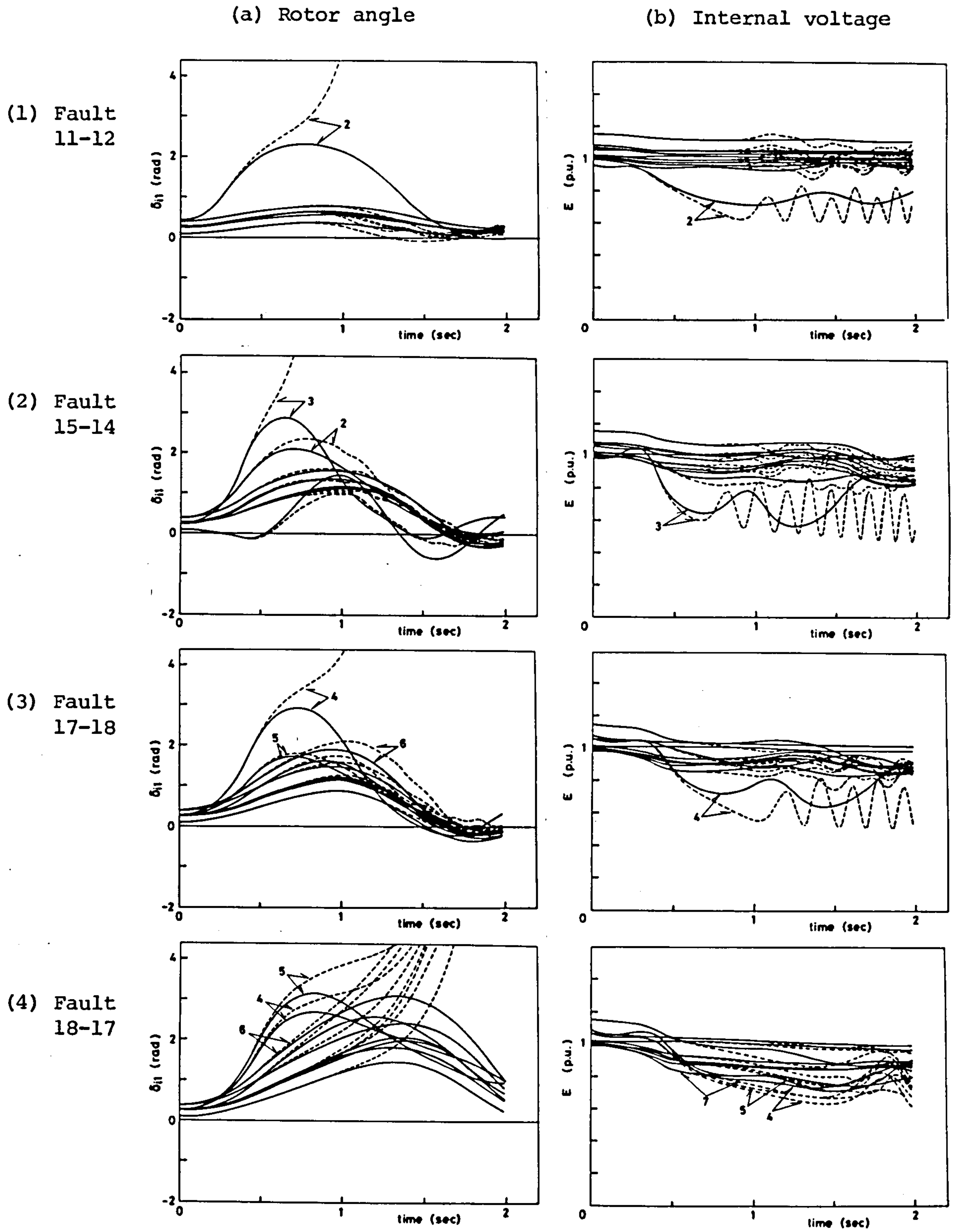


Fig.50 Time variations of rotor angles and internal voltages for eight cases of fault locations: T'_{doi}; normal value

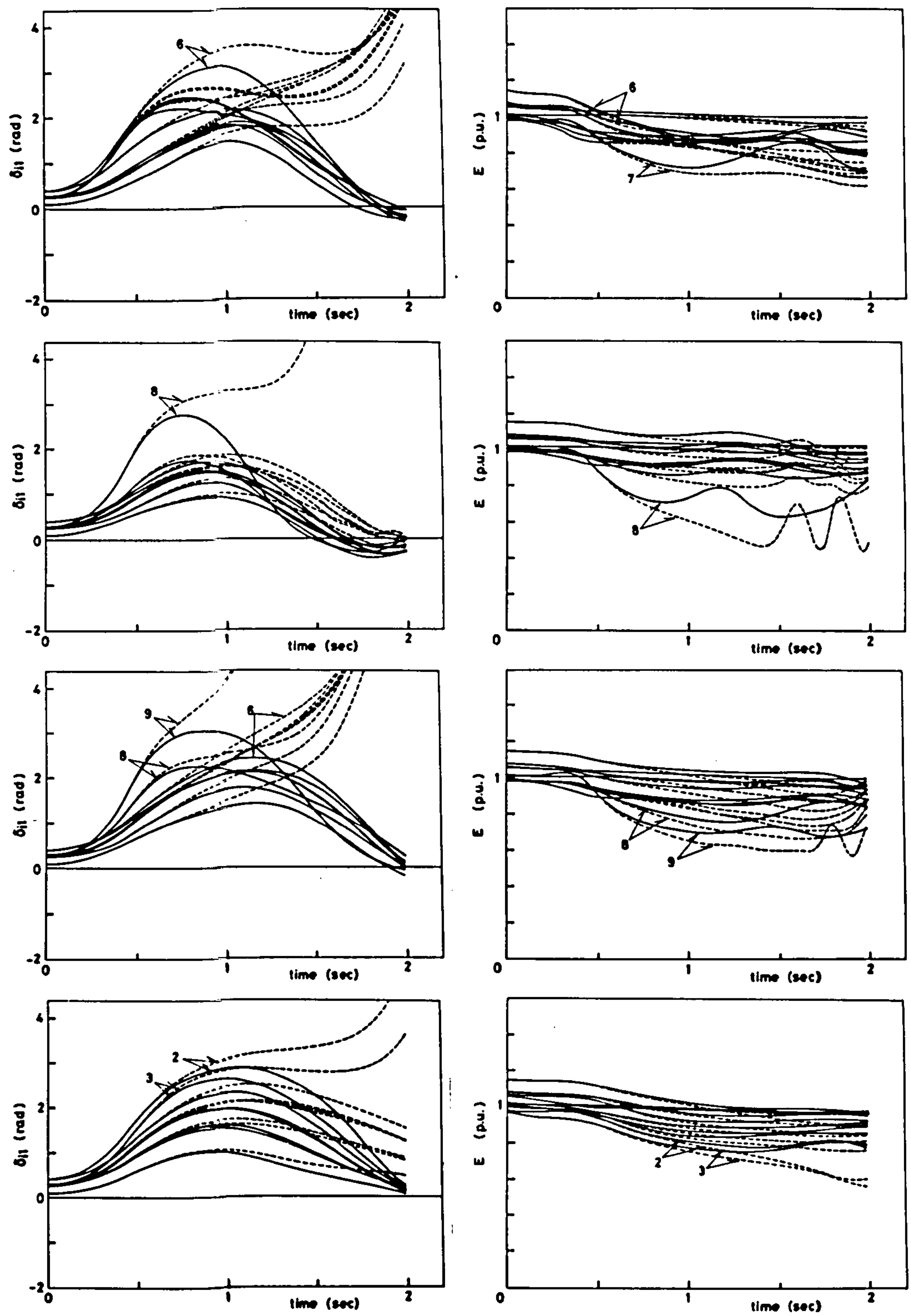
(a) Rotor angle (b) Internal voltage

(5) Fault 24-16

(6) Fault 30-27

(7) Fault 34-29

(8) Fault 38-15



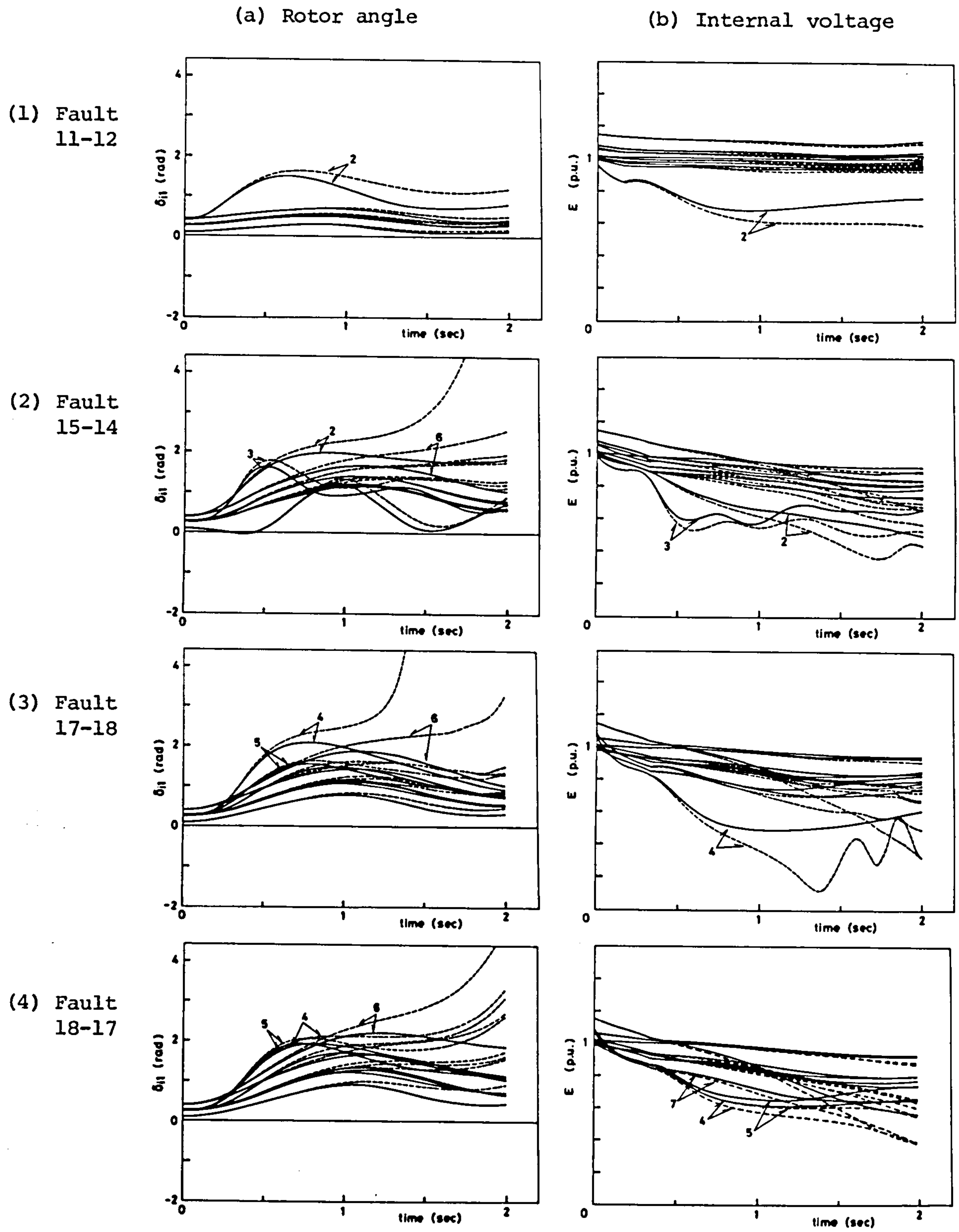


Fig.51 Time variations of rotor angles and internal voltages for eight cases of fault locations: T'_{doi}; a quater of normal value

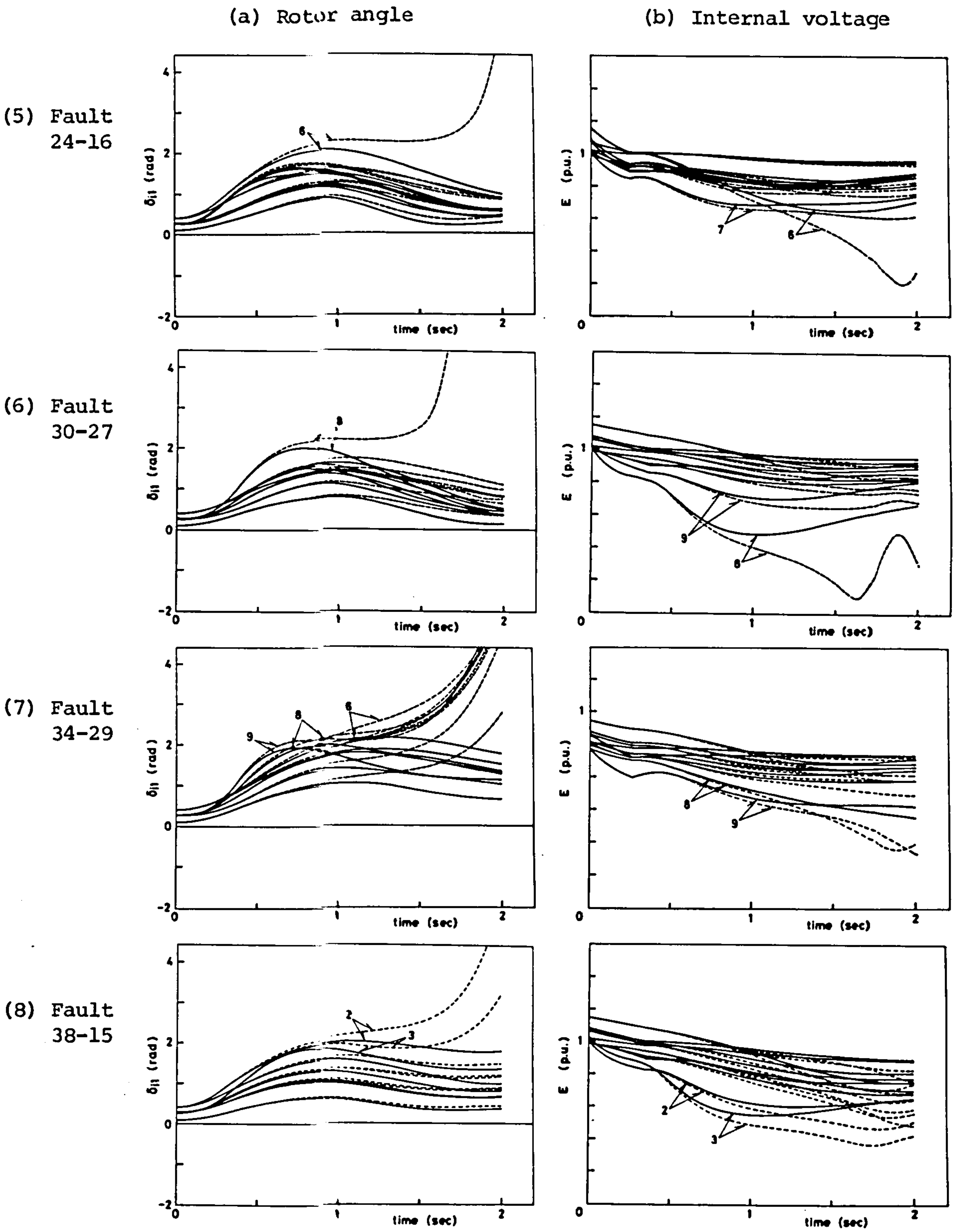

(a) Rotor angle
(b) Internal voltage
(5) Fault 24-16
(6) Fault 30-27
(7) Fault 34-29
(8) Fault 38-15
δij (rad)
E (p.u.)
time (sec)

Table 13. Critical fault clearing times obtained by simulations.

(a) T'_{doi}: normal value

Fault	Clearing times(sec) stable	unstable	Unstable generators
11 - 12	0.25	0.26	2
15 - 14	0.38	0.39	3
17 - 18	0.42	0.43	4
18 - 17	0.46	0.47	1
24 - 16	0.37	0.38	1
30 - 27	0.44	0.45	8
34 - 29	0.44	0.45	1
38 - 15	0.44	0.45	1

(b) T'_{doi}: a quater of normal value

Fault	Clearing times(sec) stable	unstable	Unstable generators
11 - 12	0.17	0.18	2
15 - 14	0.32	0.33	1
17 - 18	0.33	0.34	4,6
18 - 17	0.34	0.35	1
24 - 16	0.24	0.25	6
30 - 27	0.35	0.36	8
34 - 29	0.35	0.36	1
38 - 15	0.26	0.27	1

1) Step-out generators vary with fault location, for instance, no.2 generator steps out for fault 11-12 while no.3 generator steps out for fault 15-14.

2) Step-out generators vary also with values of d-axis transient open-circuit time constants T'_{doi}, for instance, only no.4 generator steps out in Case 1 while no.4 and 6 generators step out in Case 2 for fault 17-18.

3) Decrease in magnitudes of internal voltages are conspicuous in the generators which step out in the unstable case, for instance, E_2 decreases very much for fault 11-12.

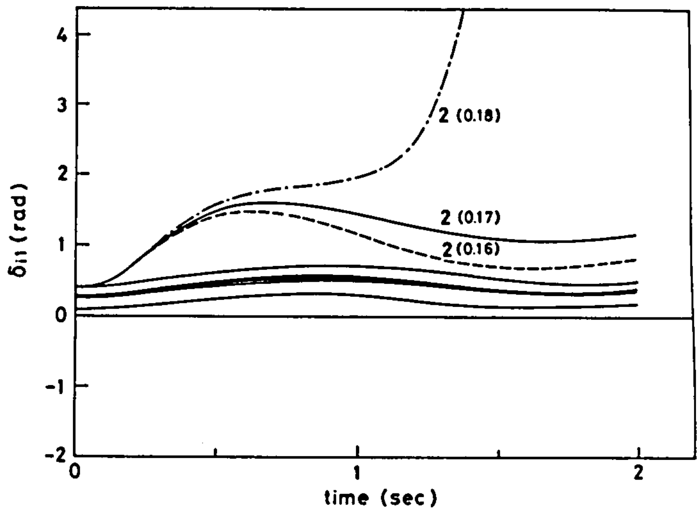
4) Decrease in magnitudes of internal voltages are more conspicuous in Case 2 than in Case 1, for instance, E_2 decreases from 0.961 to 0.712 in Case 1, and to 0.598 in Case 2, where a fault is cleared at 0.25 sec and 0.17 sec, respectively.

5) The critical clearing time is shorter in Case 2 than in Case 1, for instance, it is 0.25 sec in Case 1, and is 0.17 sec in Case 2 for fault 11-12.

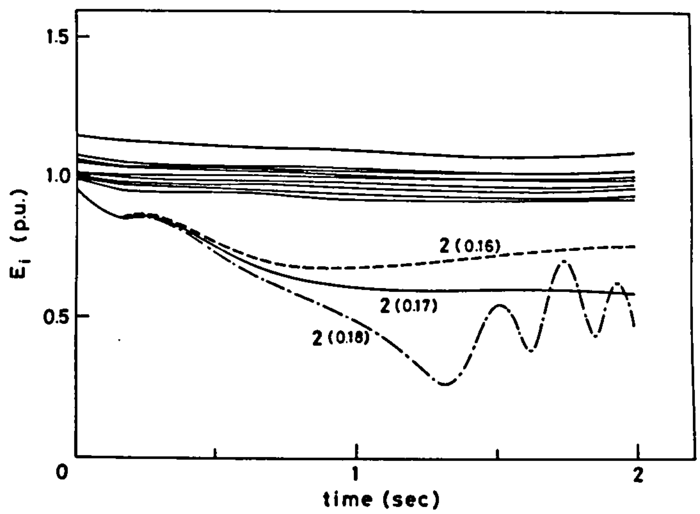
The results by simulations are summarized in Table 13(a) and (b). The critical clearing time exists in ranges of 0.25 ∿ 0.46 sec and 0.17 ∿ 0.35 sec in Case 1 and 2 respectively. In the tables, the critical clearing times also shown for the case where field flux linkages are assumed to be constant. This case can be interpreted as one where all T'_{doi} are infinitely large. With decreases in T'_{doi}, the critical clearing time is shortened very much.

In Case 2, we can see the second type of instability for almost all faults indicated in Table 12. Namely, some internal voltages decrease much in magnitude to such extent that the transient stability region vanishes. As an example, we take the case of fault 11-12, and observe the progress of the second type of instability. Under this fault, no.2 generator suffers large disturbance, and its internal voltage decreases much in magnitude. In Fig.52, the time variations of internal voltages are shown. Fig.52(c) shows the time variations of δ_{21} at the stable equilibrium point and the unstable equilibrium point which corresponds to the first swing step-out mode. The difference of δ_{21} between these two points is an index which represents an extent of the transient stability region. If these points join, then the transient stability region vanishes. Fig.52(a) and (b) show the time vari-

(a) Variations of angles



(b) Variations of magnitudes



(c) Variations of equilibrium points

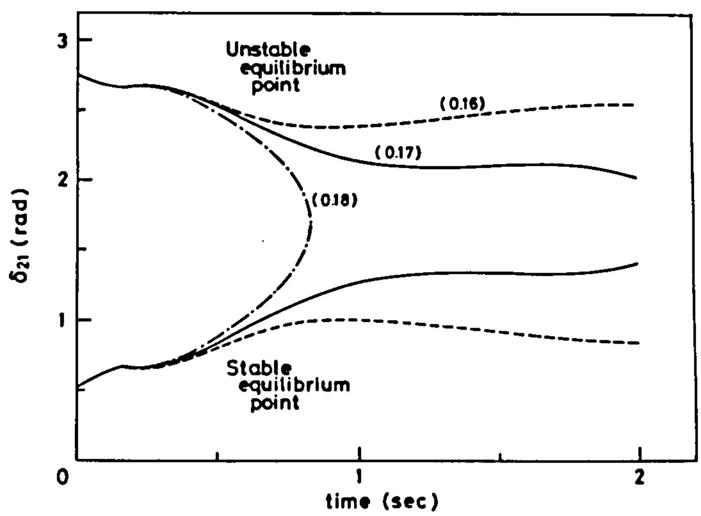


Fig.52. Time variations of internal voltages: second type of instability.

ations of the angles and magnitudes of the internal voltages for the case where the fault 11-12 is cleared at 0.17 sec. In this case, no.2 generator seems to remain in synchronism with other generators during the first swing. The angle δ_{21} increases, takes a peak value 1.626 rad at 0.71 sec, and decreases, afterwards. However, δ_{21} decreases only to 1.093 rad by 1.68 sec, and begins to increase again. Similarly, the magnitude E_2 decreases from 0.961 p.u. to 0.598 p.u. by 1.28 sec, and increases, afterwards. However, E_2 increases only to 0.600 p.u. by 1.61 sec, and begins to decrease again. With decrease in E_2, the stable and the unstable equilibrium points approach each other as is observed from Fig.52(c). The difference of δ_{21} between these points is initially 2.238 rad, but it decreases to 0.768 rad by 1.28 sec. It increases to 0.780 rad by 1.61 sec with increase in E_2, but it begins to decrease, afterwards. The stable and the unstable equilibrium point join after a while, and accordingly, the transient stability region vanishes. Before that instant, no.2 generator receives some synchronizing torque, but afterwards, it is accellated to step out. Thus the system loses synchronism owing to the second type of instability. The progress of the instability is very slow in this case, so we have erroneously judged that the system is stable in the case where the fault is cleared at 0.17 sec. In order that the system keeps synchronism, the fault must be cleared at 0.16 sec. In the case where the fault is cleared at 0.18 sec, the progress of the instability is rapid. The magnitude E_2 decreases rapidly, and it reaches 0.558 p.u. at 0.84 sec, when the transient stability region vanishes. After this instant, no.2 generator is accellated, and steps out. It should be investigated for other faults whether the second type of instability does occur or not. Table 14 shows the time when the vanishment of the transient stability occurs. From the table, it is observed that the transient stability region vanishes in the unstable case for all faults, which implies that the second type of instability is dominant in Case 2 of d-axis transient open-circuit time contants. Besides, it is observed that the transient stability region vanishes for five faults of all faults even in the stable case. The vanishment of the transient stability region means that the system will lose its synchronism soon, so the system is not stable for the fault clearing time. In order that the system is stable, the fault should be cleared at a time earlier than it, and accordingly, some correction must be made on Table 13.

Table 14. Vanishment of transient stability region for eight cases of fault location where T'_{doi} is normal for all generators.

Fault	Stable case		Unstable case	
	vanish?	when?	vanish?	when?
11 - 12	yes	> 2.00	yes	0.84
15 - 14	yes	1.52	yes	1.16
17 - 18	no	-	yes	0.98
18 - 17	yes	1.74	yes	1.28
24 - 16	no	-	yes	1.32
30 - 27	no	-	yes	1.14
34 - 29	yes	> 2.00	yes	1.22
38 - 15	yes	1.48	yes	1.22

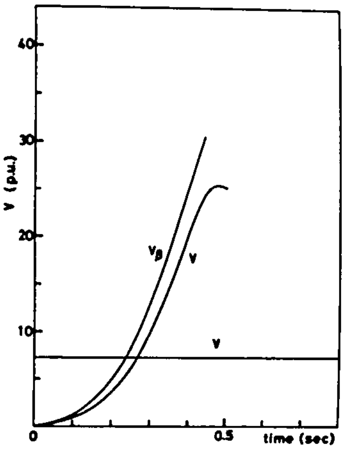
(Time: sec)

5.3 Results by Lyapunov's direct method

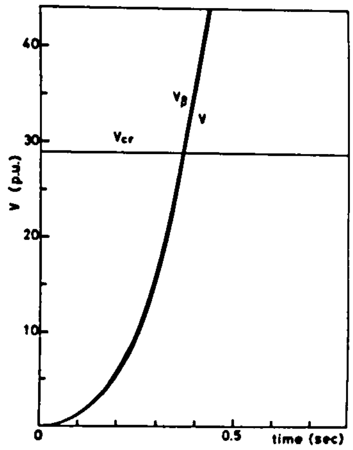
Secondly, Lyapunov's direct method is applied to the transient stability analysis of the 10-machine power system. In this method, the system equations (3.1) and (3.2) are integrated step by step for a given fault, where the fault is not cleared, to yield the time variation of the Lyapunov function V. In this study, transfer conductances are not negligible, so V_β is calculated as well as V. The instants when these functions reach the critical value V_{cr} are adopted as the estimations of the critical fault clearing time. The critical value V_{cr} is defined as the value of the potential energy V_p at the instant when the time derivative of the kinetic energy V_k changes its sign from negative to positive. The integration of (3.1) and (3.2) is continued till that instant.

Fig.53 and 54 show the time variations of V and V_β, and the critical value V_{cr} for the eight cases of fault location in Case 1 and 2 of transient open-circuit time constants, respectively. The functions V and V_β are nearly equal to zero at the instant when each fault occurs. They increase monotonously during the fault-on period. Namely, the total energy stored in the system increases with time. In order that the system keeps synchronism, the fault must be cleared before these functions reach V_{cr}. From these figures, we can observe several features as follows:

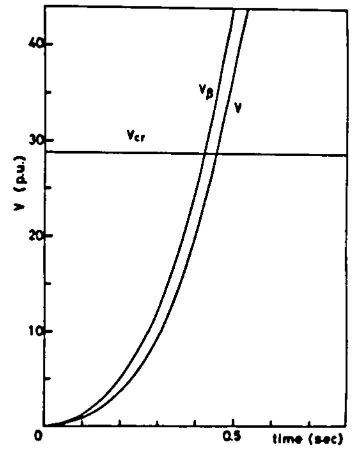
1) The critical value varies with fault locations, for instance, V_{cr} is 7.225 for fault 11-12, and 28.246 for fault 15-14, in Case 1.
2) The critical value varies also with values of d-axis transient open-circuit time constants T'_{doi}, for instance, V_{cr} for fault 11-12 is 7.225 in Case 1, and 3.116 in Case 2. V_{cr} usually decreases with decrease in T'_{doi}.
3) V_β takes greater values at each time than V in this study, which is owing to the fact that γ defined by (3.146) and (3.155) takes values greater than 1, for instance, γ is 1.388 for fault 11-12 in Case 1.
4) As a result of 3), V_β yields smaller estimations of the critical fault clearing time than V, for instance, V_β yields 0.239 sec while V yields 0.268 sec for fault 11-12 in Case 1.
5) The difference of estimation between V and V_β is more conspicuous in Case 2 than in Case 1, for instance, the difference for fault 11-12 is 0.029 sec in Case 1, and 0.047 sec in Case 2, which is owing to the



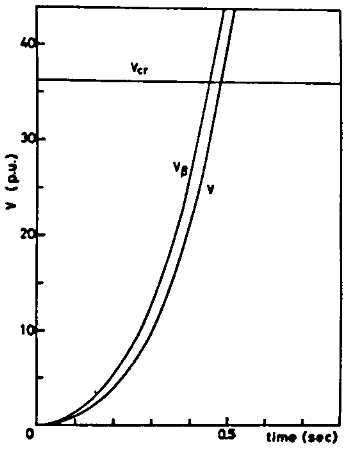
(1) Fault 11-12



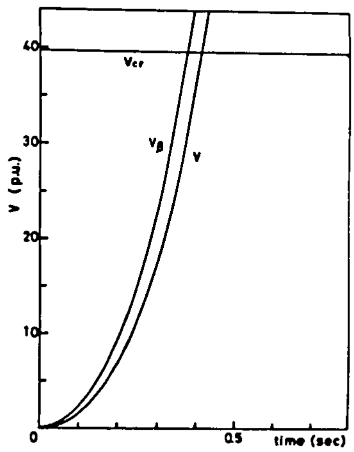
(2) Fault 15-14



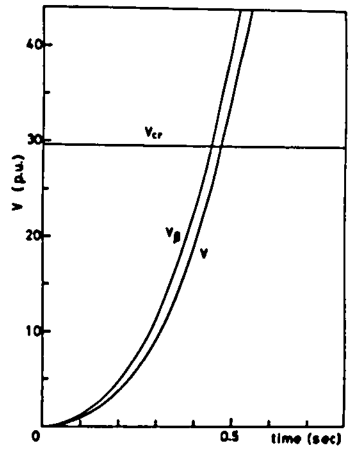
(3) Fault 17-18



(4) Fault 18-17

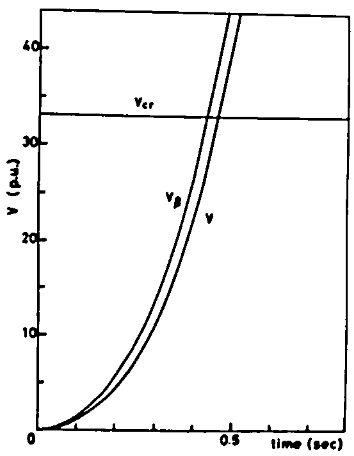


(5) Fault 24-16

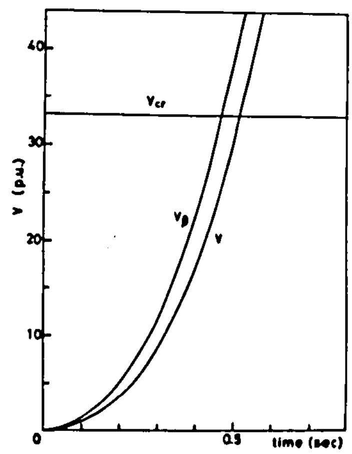


(6) Fault 30-27

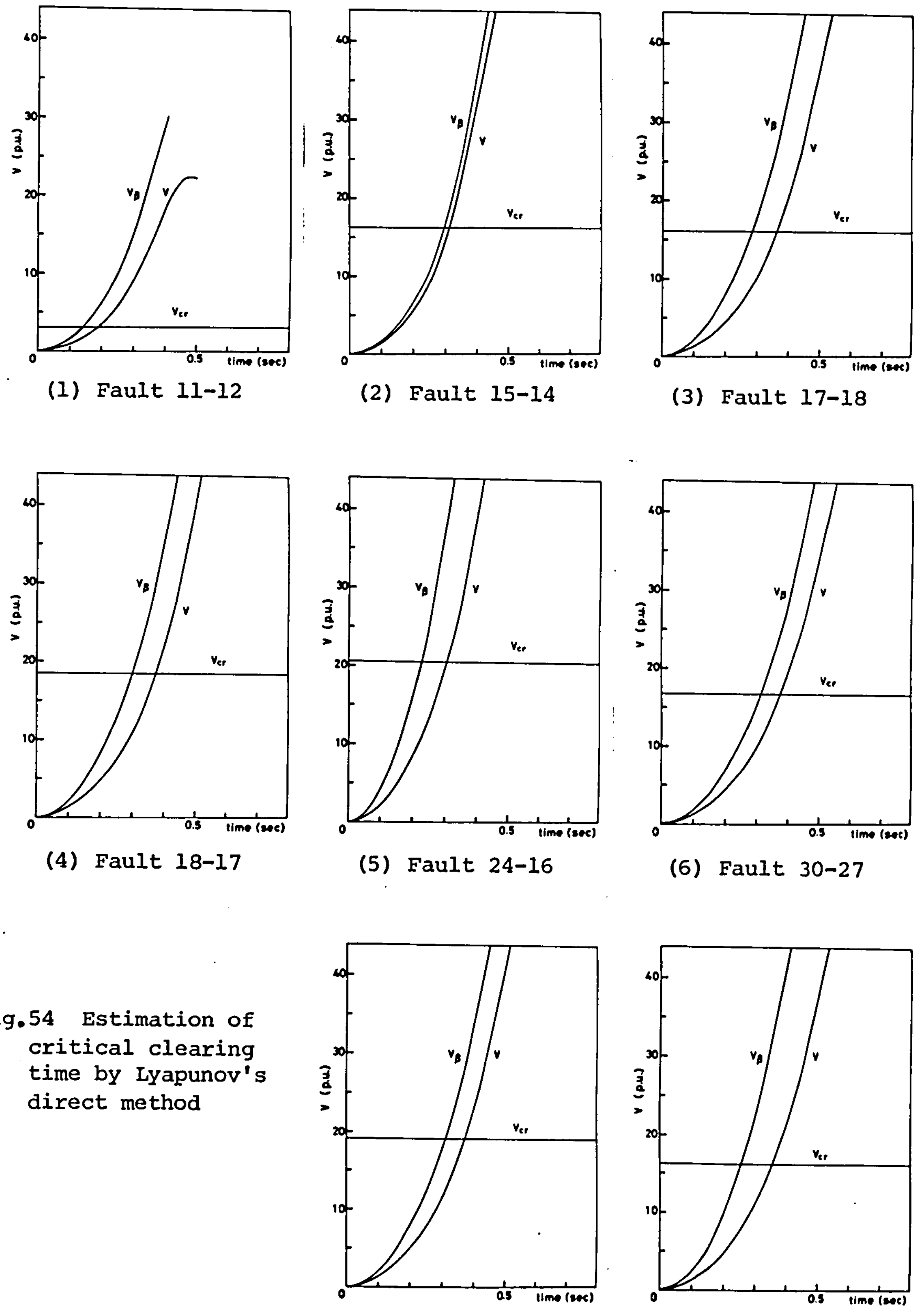
Fig.53 Estimation of critical clearing time by Lyapunov's direct method



(7) Fault 34-29



(8) Fault 38-15



(1) Fault 11-12
(2) Fault 15-14
(3) Fault 17-18
(4) Fault 18-17
(5) Fault 24-16
(6) Fault 30-27
(7) Fault 34-29
(8) Fault 38-15

Fig.54 Estimation of critical clearing time by Lyapunov's direct method

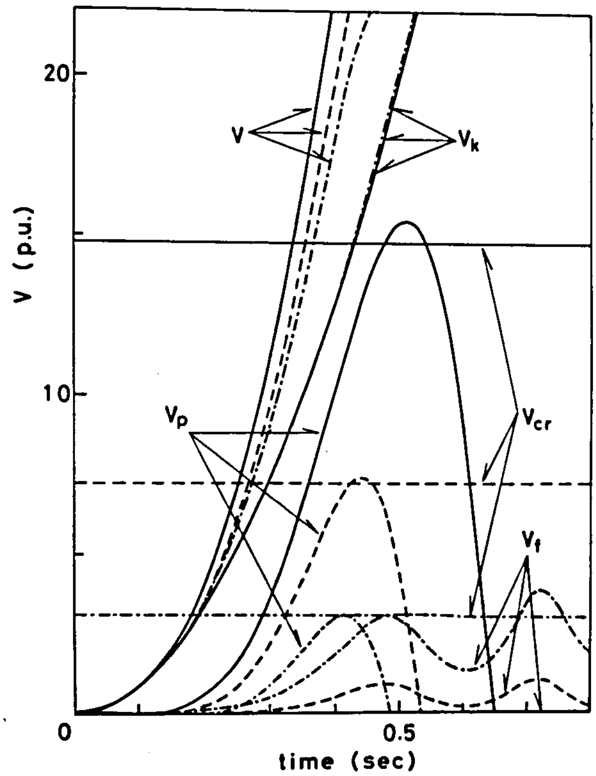


Fig.55. Time variations of V and its components for three cases of transient open-circuit time constants T'_{doi}:
———— infinitively large,
– – – – – normal value,
—·—·— a quater of normal value.

fact that γ takes 1.388 in Case 1, and 1.814 in Case 2.

The first feature corresponds with the first one observed in §5.2 from the time variations of the internal voltages, that is, the critical value varies with the step-out generators. As for the second feature, an example will serve to clarify its cause. Fig.55 shows the time variations of V and its components for three cases of transient open-circuit time constants, where the fault is 11-12. Two of them are Case 1 and Case 2. The remaining one is the case where all the time constants are infinitely large, in other words, all the internal voltages are constant. We refer this case as Case 0. The internal voltages decrease more rapidly with decrease in the time constants. The kinetic energy V_k increases monotonously with time, and it takes almost the same values at each time for all the cases, which implies that all generators receive almost the same acceleration. On the other hand, the variations of V_p and V_f differ markedly with the cases. The potential energy V_p initially increases monotonously with time, but it takes a peak value, and decreases, afterwards. The peak value decrease from 15.471 to 7.417, and from 7.417 to 3.132, and the time when V_p takes the peak value decreases from 0.50 sec to 0.44 sec, and 0.44 sec to 0.41 sec with the changes from Case 0 to Case 1, and from Case 1 to Case 2. These facts mean that the transient stability region becomes narrower and shallower with decrease in the time constants. Consequently, the critical value V_{cr} decreases from 14.753 to 7.225, and from 7.225 to 3.115 with the changes of the time constants. The term V_f initially increases with time, and takes a peak value, but it oscillates, afterwards. Its value at each time becomes larger in sequence of Case 0, 1, and 2. Since it represents the magnitude of the deviations in internal voltages, the above fact means that the internal voltages decrease more in Case 1 than in Case 0, and in Case 2 than in Case 1. In Case 2, V_f takes values comparable with V_p. The remaining three features are concerned with transfer conductances. In general V yields optimistic results, and V_β yields pessimistic results compared with the actual transient stability.

Table 15(a) and (b) summarizes the results by Lyapunov's direct method. Several features on this method are observed from these tables as follows:

1) The function V which does not take account of the transfer conductances, yields optimistic results for all faults in both Case 1 and 2. Its ex-

Table 15. Critical fault clearing times estimated by Lyapunov's direct method (sec).

(a) T'_{doi}: normal value

Fault	γ	V_{cr}	T_{es}	T_{cr}	T^*_{cr}
11-12	1.388	7.23	0.24	0.25	0.30
15-14	1.102	28.85	0.37	0.38	0.41
17-18	1.420	28.77	0.42	0.42	0.47
18-17	1.403	36.17	0.45	0.46	0.50
24-16	1.413	39.71	0.38	0.37	0.45
30-27	1.317	29.49	0.44	0.44	0.47
34-29	1.314	33.00	0.43	0.44	0.46
38-15	1.429	33.14	0.47	0.44	0.65

(b) T'_{doi}: a quater of normal value

Fault	γ	V_{cr}	T_{es}	T_{cr}	T^*_{cr}
11-12	1.814	3.12	0.14	0.17	0.30
15-14	1.227	16.26	0.30	0.32	0.41
17-18	2.044	16.06	0.28	0.33	0.47
18-17	1.936	18.47	0.30	0.34	0.50
24-16	2.205	20.52	0.22	0.24	0.45
30-27	1.723	16.73	0.31	0.35	0.47
34-29	1.685	19.11	0.31	0.35	0.46
38-15	2.345	16.20	0.25	0.26	0.65

T_{cr}: critical clearing time obtained by simulations.

T^*_{cr}: critical clearing time for cases where T'_{doi} is infinitively large, that is, field flux linkages are kept constant.

T_{es}: critical clearing time estimated by the direct method.

tent is more conspicuous in Case 2 than in Case 1. For fault 38-15, for example, the result is greater than that by simulations by 0.07 sec in Case 1, and by 0.10 sec in Case 2.

2) The function V_β which takes account of the transfer conductances is able to yield results of good accuracy compared with V. In Case 1, the results are very accurate, and are different from those by simulations only by 0.01 sec except for fault 38-15. In Case 2, V_β has yielded somewhat conservative results compared with those by simulations, but the difference between them still remains within 0.05 sec even in this case.
3) The critical value V_{cr} varies with fault location. It exists in the ranges of 7.23 ∿ 39.71 and 3.12 ∿ 20.52 in Case 1 and 2, respectively. It also varies with transient open-circuit time constants, namely, in Case 2, V_{cr} decreases to about a half of that in Case 1.
4) The ratio γ varies with transient open-circuit time constants. In Case 2, its value is beyond 2.0 for several faults whereas it remains under 1.5 for all faults in Case 1.

The conservative nature observed in 2) is due to Stability condition 6 used as the basis of determining the critical value V_{cr}. In Case 2, the internal voltages decrease to such an extent that the second type of instability occurs. The conservative nature is conspicuous in such cases as this. It should be emphasized, however, that the direct method has yielded relatively accurate results in this case.

5.4 Conclusions

In this section, we have made some transient stability analysis of a 10-machine power system. The Lyapunov function, and the methods of determining the critical value, and of taking account of transfer conductances developed in §2, 3, and 4 have been applied to this analysis. The results are summarized as follows:

1) The original Lyapunov function V can not take account of the transfer conductances, so it has yielded somewhat optimistic results compared with the actual stability in the two cases of transient open-circuit time constants.
2) The function V_β has been able to take account of the transfer conduc-

tances, and has yielded adequately accurate results whose difference with the actual critical clearing time is within 0.01 sec except one fault case in Case 1 of transient open-circuit time constants.

3) With decrease in transient open-circuit time constants, the internal voltages come to decrease more rapidly, and as the result, the transient stability of the system is degraded markedly.
4) In Case 1 of transient open-circuit time constants, the internal voltages do not decrease so much to cause the second type of instability under the load condition used in this study. In this case, the usual type of instability is dominant, and the direct method has yielded results of good accuracy.
5) In Case 2 of transient open-circuit time constants, the internal voltages decrease enough to cause the second type of instability. The direct method has somewhat conservative results compared with the actual stability of the system. This conservative nature is due to Stability condition 6 which is used as the basis of determining the critical value of the Lyapunov function, and it is conspicuous in the case where the internal voltages decrease rapidly.
6) The progress of the second type of instability is sometimes slow, and accordingly, the system seems to be stable for the first swing. However, it may constitute one cause of the phenomenon where the system loses synchronism for the second or latter swings while it keeps synchronism for the first swing.

Thus the variations of field flux linkages of generators have significant influences on the transient stability of the system, so it is very important to take account of their influence in the transient stability analysis. We have applied Lyapunov's direct method to this analysis, and have obtained some results of practical significance.

Chapter IV

TRANSIENT STABILITY ANALYSIS OF MULTIMACHINE POWER SYSTEM VIA LYAPUNOV'S DIRECT METHOD: DYNAMICS OF AVR & EXCITATION SYSTEMS

§1. Introduction

In this chapter, we are concerned with the transient stability analysis of multimachine power systems in which dynamics of automatic voltage regulators and excitation systems of generators are incorporated in their system representation.

In Chapter III, we have made some investigations on Lyapunov's direct method applied to the transient stability analysis of power systems, where only dynamics of field flux linkages of generators have been incorporated in their system representations, and have developed it to the point where it can yield results of practical significance. The transient stability evaluated with this representation has proved to fall to some extent compared with that evaluated with the conventional system representation in which each generator is represented by a constant voltage behind a transient reactance. This transient stability is concerned with native generators in which no control equipment is installed. Some stabilizing control equipments have recently come into action to enhance the transient stability , however, and accordingly, actual systems have some stability margin besides the stability evaluated with the above system representation. This fact moves us to analyze how much is the transient stability enhanced by such stabilizing equipments. In this chapter, we incorporate dynamics of automatic voltage regulators and excitation systems of generators in the system representation, and develope Lyapunov's direct method based on this representaion.

Lyapunov's direct method consists of two main parts, that is, the construction of Lyapunov functions,and the determination of its critical value.

Firstly, the former part is investigated. There are several works on the method of constructing Lyapunov functions for power systems in which some voltage regulators are installed in generators. In 1968, M.W. Siddiqee found a Lyapunov function for a one machine connected to an infinite bus system in which the generator is installed with a forced voltage regulator through trial and error [23]. M.A. Pai and V. Rai constructed the same function with a systematic method based on a generalized Popov criterion in 1974 [28]. This type of regulators do not use the terminal voltages as feedback signals. V.K. Verma et al. constructed a Lyapunov function for a one machine power system in which automatic voltage regulator is installed in the generator with the variable gradient method in 1975 [29]. T. Taniguch and H. Miyagi constructed another Lyapunov function with a method based on a Lagrangean function in 1977 [30]. However, it is very difficult to apply their construction methods to multimachine power systems. In §2, we systematically construct a Lyapunov function on the basis of Theorem 3 for a multimachine power system in which generators are all installed with automatic voltage regulators and excitation systems modeled with third order transfer functions.

Secondly, the latter part, i.e., the determination of the critical value is investigated. There has been few works made on this subject for the system under consideration. In §3, we first make some investigation on the equipotential curves of the system, define the transient stability region, and observe how does the region vary with internal voltages. Next, we investigate how high automatic voltage regulator gains are allowed according to the generalized Popov criterion. Lastly we check the applicability of the method of determing the critical value developed in the preceding chapters.

Thirdly, we try one generalization of Lyapunov's direct method to power system in which generators are installed with general and different types of excitation system in §4. This trial is motivated by the fact that the generalized Popov criterion imposes somewhat strict constraint on automatic voltage regulator gains. We propose a pseude-Lyapunov function, and make some considerations on its applicability to the transient stability analysis.

Lastly, a transient stability analysis of a 10-machine power system is made by Lyapunov's direct method developed in this chapter, and its effectiveness is investigated by comparing results obtained by the direct method with those obtained by the conventional method.

§2. Construction of Lyapunov function [74,75]

In this section, a Lyapunov function is systematically constructed on the basis of Theorem derived in §2 of chapter II. The outline of the method was shown in the process of constructing a Lyapunov function for the system represented by the conventional model. We follow the method, of course, with some changes which are necessary to the system in which dynamics of automatic voltage regulators and excitation systems are taken into account. We begin in this section with a derivation of the system equations of the system.

2.1 System equation

In transient stability analysis, an n-machine power system in which dynamics of automatic voltage regulators and excitation systems are taken into account, is described as follows [65,66]:

$$m_i \frac{d^2\delta_i'}{dt^2} + d_i \frac{d\delta_i'}{dt} = P_{mi} - \sum_{j=1}^{n} Y_{ij}E_iE_j\sin(\delta_{ij} + \theta_{ij}) \tag{4.1}$$

$$\frac{dE_{qi}'}{dt} = (1/T_{doi}')[E_{fdi} - E_{qi}' - (x_{di} - x_{di}')i_{di}] \tag{4.2}$$

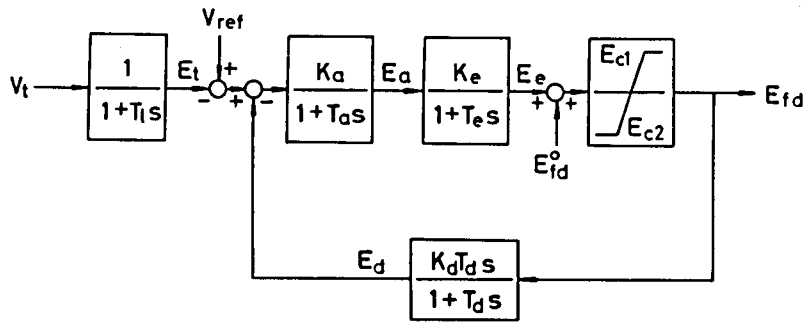
$$\frac{dE_{ai}}{dt} = -(1/T_{ai})[E_{ai} + K_{ai}E_{di} - K_{ai}(V_{refi} - V_{ti})] \tag{4.3}$$

$$\frac{dE_{ei}}{dt} = (1/T_{ei})[K_{ei}E_{ai} - E_{ei}] \tag{4.4}$$

$$\frac{dE_{di}}{dt} = (1/T_{ei})[K_{di}K_{ei}E_{ai} - K_{di}E_{ei} - (T_{ei}/T_{di})E_{di}] \tag{4.5}$$

for $i = 1,2,\ldots,n$

where, for generator i,



$K_a = 1.0$ $\quad K_e = 1.0$ $\quad K_d = 0.004$

$T_a = 0.02$ $\quad T_e = 0.04$ $\quad T_d = 0.5$

$T_l = 0.0$ $\quad E_{c1} = 7.0$ $\quad E_{c2} = 0.0$

Fig.56. Block diagram of Excitation system.

P_{mi} : mechanical power input,

m_i : angular momentum constant,

d_i : damping power coefficient,

$Y_{ij}\angle\phi_{ij}$: post-fault transfer admittance between the ith and the jth generator nodes (obtained after reduction of a network retaining only generator nodes),

θ_{ij} : complement of ϕ_{ij}, i.e., $\theta_{ij} = \pi/2 - \phi_{ij}$,

$E_i\angle\delta_i$: internal voltage,

δ_{ij} : $\delta_i - \delta_j$,

$E'_{qi}\angle\delta'_i$: voltage related with the internal voltage as in Fig.31, and δ'_i indicates a rotor angle relative to a reference frame rotating at synchronous speed,

E_{fdi}: excitation voltages,

i_{di} : d-axis current,

x_{di}, x'_{di}: d-axis synchronous, transient reactances,

T'_{doi}: d-axis transient open-circuit time constant,

E_{ai}, E_{ei}, E_{di}: state variables of excitation system shown in Fig.56,

K_{ai}, K_{ei}, K_{di}: gains of automatic voltage regulator, excitator, and damping circuit, respectively,

T_{ai}, T_{ei}, T_{di}: time constants of automatic voltage regulator, excitor, and damping circuit, respectively,

V_{refi}: reference voltage of automatic voltage regulator,

and T_1, the time constant of detective circuit, is assumed to be zero. The terminal voltage of the ith generator V_{ti} is represented as follows:

$$\begin{aligned} \dot{V}_{ti} &= \dot{E}_i - jx'_{di}\dot{I}_i \\ &= \dot{E}_i - jx'_{di}\sum_{j=1}^{n} \dot{Y}_{ij}\dot{E}_j \qquad (4.6) \end{aligned}$$

for $i = 1,2,\ldots,n$

In order to construct a Lyapunov function, three basic assumptions are necessary as follows:

1) Each internal voltage lags behind the q-axis of each generator by a constant angle ϕ_i all the time [64].
2) The transfer conductances in the reduced admittance matrix are negligible.

3) The magnitude of the ith terminal voltage can be approximately expressed by

$$V_{ti} \simeq E_i + x'_{di} \sum_{j=1}^{n} Y_{ij} E_j \cos(\delta_{ij} + \theta_{ij}) \tag{4.7}$$

where this equation is derived in Appendix D.

Under these assumptions, (4.1) ∿ (4.5) change to the following equations:

$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \tag{4.8}$$

$$\begin{aligned} \frac{dE_i}{dt} = {} & (1/T'_{doi}\cos{}_i)(E_{fdi} - E_{fdi}^o) \\ & - (1/T'_{doi})[1 - (x_{di} - x'_{di})B_{ii}](E_i - E_i^o) \\ & - (1/T'_{doi})(x_{di} - x'_{di}) \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \end{aligned} \tag{4.9}$$

$$\begin{aligned} \frac{dE_{ai}}{dt} = {} & -(1/T_{ai})[(E_{ai} - E_{ai}^o) + K_{ai}(E_{di} - E_{di}^o) \\ & + (1 + x'_{di}B_{ii})K_{ai}(E_i - E_i^o)] \\ & + (K_{ai}/T_{ai})x'_{di} \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij}(\cos\delta_{ij}^o - \cos\delta_{ij}) \end{aligned} \tag{4.10}$$

$$\frac{dE_{ei}}{dt} = (1/T_{ei})[K_{ei}(E_{ai} - E_{ai}^o) - (E_{ei} - E_{ei}^o)] \tag{4.11}$$

$$\begin{aligned} \frac{dE_{di}}{dt} = {} & (1/T_{ei})[K_{ei}K_{di}(E_{ai} - E_{ai}^o) - K_{di}(E_{ei} - E_{ei}^o) \\ & - (T_{ei}/T_{di})(E_{di} - E_{di}^o)] \end{aligned} \tag{4.12}$$

for $i = 1,2,\ldots,n$

where the superscript " o " denotes the stable equilibrium point of the post-fault system, and accordingly, (4.8) ∿ (4.12) apply to the post-fault system. Eqs.(4.8)∿(4.12) in the state space notation are given as follows:

$$\begin{aligned} \overset{\circ}{x} &= Ax - BF(\sigma) \\ \sigma &= C'x \end{aligned} \tag{4.13}$$

where

$$A = \begin{bmatrix} 0 & K'_{n(n-1)} & 0 & 0 & 0 & 0 \\ 0 & -M^{-1}D_{nn} & 0 & 0 & 0 & 0 \\ 0 & 0 & -H_1 & 0 & H_2 & 0 \\ 0 & 0 & -H_3 & -H_4 & 0 & -H_5 \\ 0 & 0 & 0 & H_6 & -H_7 & 0 \\ 0 & 0 & 0 & H_8 & -H_9 & -H_{10} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -M^{-}\ T_{nm} & 0 \\ 0 & \alpha_{nn} \\ 0 & -\beta_{nn} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} G_{(n-1)m} & 0 \\ 0 & 0 \\ 0 & I_{nn} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.14}$$

in which

$$\begin{aligned} M &= \mathrm{diag}(m_1,\ m_2,\ \ldots,\ m_n) \\ D &= \mathrm{diag}(d_1,\ d_2,\ \ldots,\ d_n) \\ K_{n(n-1)} &= \begin{bmatrix} 1_{1(n-1)} \\ -I_{(n-1)(n-1)} \end{bmatrix} \\ G_{(n-1)m} &= [\ I_{(n-1)(n-1)} \quad -T_{(n-1)(m-n+1)}\ ] \\ T_{nm} &= \begin{bmatrix} 1_{1(n-1)} & 0_{1(m-n+1)} \\ -I_{(n-1)(n-1)} & T_{(n-1)(m-n+1)} \end{bmatrix} \end{aligned} \tag{4.15}$$

and $H_i\,(i=1,2,\ldots,10)$, α, and β are $n \times n$ matrix of diagonal form whose elements are defined as follows:

$$h_{1i} = (1/T'_{doi})[1 - (x_{di} - x'_{di})B_{ii}]$$
$$h_{2i} = 1/T'_{doi}\cos\phi_i$$
$$h_{3i} = K_{ai}(1 + x'_{di}B_{ii})/T_{ai}$$
$$h_{4i} = 1/T_{ai} \tag{4.16}$$
$$h_{5i} = K_{ai}/T_{ai} \qquad h_{8i} = K_{di}K_{ei}/T_{ei}$$
$$h_{6i} = K_{ei}/T_{ei} \qquad h_{9i} = K_{di}/T_{ei}$$
$$h_{7i} = 1/T_{ei} \qquad h_{10i} = 1/T_{di}$$
$$\alpha_i = (x_{di} - x'_{di})/T'_{doi} \qquad \beta_i = K_{ai}x'_{di}/T_{ai}$$

for $i = 1,2,\ldots,n$

The row vectors 1 and 0 in (4.15) have all their elements equal to unity and zero, respectively. The number m is defined by

$$m = n(n-1)/2 \tag{4.17}$$

The state vector x is a (6n-1) dimensional vector consisting of six vectors as follows:

$$x = [\ \delta'_r,\ \omega',\ \Delta E',\ \Delta E'_a,\ \Delta E'_e,\ \Delta E'_d\ ]' \tag{4.18}$$

where the elements of these vectors are defined by

$$\delta_{ri} = \delta_{1(i+1)} - \delta^o_{1(i+1)} \qquad \text{for } i = 1,2,\ldots,n-1$$
$$\omega_i = \dot{\delta}_i \qquad \text{for } i = 1,2,\ldots,n$$
$$\Delta E_i = E_i - E^o_i \qquad \text{for } i = 1,2,\ldots,n$$
$$\Delta E_{ai} = E_{ai} - E^o_{ai} \qquad \text{for } i = 1,2,\ldots,n \tag{4.19}$$
$$\Delta E_{ei} = E_{ei} - E^o_{ei} \qquad \text{for } i = 1,2,\ldots,n$$
$$\Delta E_{di} = E_{di} - E^o_{di} \qquad \text{for } i = 1,2,\ldots,n$$

The nonlinearity $F(\sigma)$ is a (m+n) dimensional vector consisting of two vectors as follows:

$$F(\sigma) = [\ f_1(\sigma)',\ f_2(\sigma)'\ ]' \tag{4.20}$$

where $f_1(\sigma)$ and $f_2(\sigma)$ are m and n dimensional vectors defined by

$$f_{1k}(\sigma) = B_{ij}[E_i E_j \sin(\sigma_k + \delta_{ij}^o) - E_i^o E_j^o \sin\delta_{ij}^o] \tag{4.21}$$

for $i=1,2,\ldots,n-1, \quad j=i+1,\ldots,n,$
$k=1,2,\ldots,m,$

where k is related with i and j by

$$k = (i-1)n - i(i+1)/2 + j, \tag{4.22}$$

and $f_2(\sigma)$ is an n dimensional vector defined by

$$f_{2i}(\sigma) = \sum_{\substack{j=1\\ j\neq i}}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \tag{4.23}$$

for $i=1,2,\ldots,n.$

The output σ is an (m+n) dimensional vector defined by

$$\begin{aligned} \sigma_k &= \delta_{ij} - \delta_{ij}^o \qquad \text{for } k=1,2,\ldots,m, \\ \sigma_k &= E_i - E_i^o \qquad \text{for } k=m+1,\ldots,m+n, \end{aligned} \tag{4.24}$$

where k is related with i and j by (4.22) for $k=1,2,\ldots,m$. Eq.(4.13) describes the multimachine power system as a multivariable dynamical system of the form as shown in Fig.1.

2.2 Stability check of system

The transfer matrix W(s) for the linear part of the system is written as follows:

$$\begin{aligned} W(s) &= C'(sI - A)^{-1}B \\ &= \begin{bmatrix} T'[s(sI + M^{-1}D)]^{-1}M^{-1}T & 0 \\ 0 & \Delta^{-1}(s^3\varepsilon_1 + s^2\varepsilon_2 + s\varepsilon_3 + \varepsilon_4) \end{bmatrix} \\ &= \begin{bmatrix} W_1(s) & 0 \\ 0 & W_2(s) \end{bmatrix} \end{aligned} \tag{4.25}$$

where Δ and $\varepsilon_i (i=1,2,3,4)$ are $n\times n$ matrices defined as follows:

$$\Delta(s) = s^4 I + s^3\gamma_1 + s^2\gamma_2 + s\gamma_3 + \gamma_4 \tag{4.26}$$

$$\gamma_1 = H_1 + H_4 + H_7 + H_{10}$$

$$\gamma_2 = H_1(H_4 + H_7 + H_{10}) + H_4(H_7 + H_{10}) + H_7H_{10} + H_5H_8$$

$$\gamma_3 = H_1(H_4H_7 + H_7H_{10} + H_{10}H_4 + H_5H_8) + H_4H_7H_{10} + H_2H_3H_6$$

$$\gamma_4 = (H_1H_4H_7 + H_2H_3H_6)H_{10}$$

and

$$\varepsilon_1 = \alpha$$

$$\varepsilon_2 = \alpha(H_4 + H_7 + H_{10})$$

$$\varepsilon_3 = \alpha(H_4H_7 + H_7H_{10} + H_{10}H_4 + H_5H_8) - \beta H_2H_6 \qquad (4.27)$$

$$\varepsilon_4 = (\alpha H_4H_7 - \beta H_2H_6)H_{10}$$

For the system to be stable, there must exist matrices N and Q such that Z(s) defined by (2.18), that is,

$$Z(s) = (N + Qs)W(s) \qquad (4.28)$$

is positive real. In this problem, N is chosen as follows:

$$N = \begin{bmatrix} (1/q)I_{mm} & 0_{mn} \\ 0_{nm} & 0_{nn} \end{bmatrix} \qquad (4.29)$$

The inequality in (2.14) is equivalent to the following inequalities:

$$f_{1k}(\sigma)\sigma_k \geq 0 \qquad \text{for all } \sigma_k \text{ in R and } k=1,2,\ldots,m. \qquad (4.30)$$

However, the inequalities are satisfied not for all σ_k in R, but for some ranges of σ_k as follows:

$$\sigma_{min} \leq \sigma_k \leq \sigma_s \text{ and } 0 \leq \sigma_k \leq \sigma_{max} \qquad \text{for } \sigma_s \leq 0$$

or $\qquad (4.31)$

$$\sigma_{min} \leq \sigma_k \leq 0 \text{ and } \sigma_s \leq \sigma_k \leq \sigma_{max} \qquad \text{for } \sigma_s \geq 0$$

where

$$\sigma_{min} = -\pi - (\delta^o_{ij} + \delta^s_{ij})$$

$$\sigma_{max} = \pi - (\delta^o_{ij} + \delta^s_{ij})$$

$$\sigma_s = \delta^s_{ij} - \delta^o_{ij}$$

and

$$\delta_{ij}^{s} = \sin^{-1}(E_i^o E_j^o \sin\delta_{ij}^o / E_i E_j)$$

As observed from (2.22), $F(\sigma)'N\sigma$ has an influence on the time derivative of $V(x)$. Hence, it is desirable to make its influence zero by letting $q \to \infty$. However, this selection of q causes a pole-zero cancellation between $(N + Qs)$ and $W(s)$ because $W_1(s)$ has a pole at $s = 0$. In order to avoid the pole-zero cancellation, we give q a finite value in constructing the Lyapunov function, and once it is obtained, we let $q \to \infty$.

The function $V_1(\sigma)$ in (2.15) is chosen as follows:

$$\begin{aligned} V_1(\sigma) &= \sum_{k=1}^{m} \int_0^{\sigma} f_{1k}(\sigma) d\sigma_k \\ &= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij} [E_i E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \\ &\qquad - (\delta_{ij} - \delta_{ij}^o) E_i^o E_j^o \sin\delta_{ij}^o] \end{aligned} \tag{4.32}$$

The function $V_1(\sigma)$ is not positive for all σ, but for a range of σ. Accordingly, the global stability of the system can not be concluded with this function. It is possible, however, to estimate the domain of attraction by using the Lyapunov function obtained with this function. It should be noted that $V_1(\sigma)$ can take negative values in the vicinity of the origin if $E_i = E_i^o$ is not satisfied for all E_i, where $i=1,2,\ldots,n$. This fact may have significant influence on the stability of the system, but its influence is assumed to be negligible. The partial derivative of $V_1(\sigma)$ are given as follows:

$$\begin{aligned} \frac{\partial V_1}{\partial \sigma_k} &= B_{ij} [E_i E_j \sin(\sigma_k + \delta_{ij}^o) - E_i^o E_j^o \sin\delta_{ij}^o] \quad \text{for } k=1,2,\ldots,m, \\ \frac{\partial V_1}{\partial \sigma_k} &= \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \quad \text{for } k=m+1,\ldots,m+n, \end{aligned} \tag{4.33}$$

that is,

$$\nabla V_1(\sigma) = I_{(m+n)(m+n)} F(\sigma) \tag{4.34}$$

From this equation, we obtain

$$Q = I_{(m+n)(m+n)} \tag{4.35}$$

By substituting (4.29) and (4.35) into (4.28), Z(s) is given as follows:

$$Z(s) = \begin{bmatrix} (1/q+s)T'[s(sI+M^{-1}D)]^{-1}M^{-1}T & 0 \\ 0 & s(sI+\alpha)^{-1}\beta \end{bmatrix}$$

$$= \begin{bmatrix} Z_1(s) & 0 \\ 0 & Z_2(s) \end{bmatrix} \tag{4.36}$$

The conditions for Z(s) to be positive real are

1) Z(s) has elements which are analytic for Re $s > 0$,
2) $Z^*(s) = Z(s^*)$ for Re $s > 0$,
3) $Z'(s^*) + Z(s)$ is positive semi-definite for Re $s > 0$.

Since Z(s) is a direct sum of $Z_1(s)$ and $Z_2(s)$, those are investigated independently of each other. The first two conditions clearly hold for both $Z_1(s)$ and $Z_2(s)$. For condition 3) to be satisfied, it is sufficient in this case to show that $Z_i(j\omega) + Z_i'(-j\omega)$ is positive semi-definite for each scalar ω, where $i = 1,2$. After some manipulation, those are found out to be as follows:

$$Z_1(j\omega) + Z_1'(-j\omega) = 2T'\mathrm{diag}\left(\frac{d_i - m_i/q}{m_i^2\omega^2 + d_i^2}\right)T \tag{4.37}$$

$$Z_2(j\omega) + Z_2'(-j\omega) = 2\omega^2\mathrm{diag}\left[\frac{\xi_{1i}\omega^6 + \xi_{2i}\omega^4 + \xi_{3i}\omega^2 + \xi_{4i}}{\Delta_i(j\omega)\Delta_i(-j\omega)}\right] \tag{4.38}$$

where ξ_i ($i=1 \sim 4$) is an $n \times n$ matrix defined by

$$\begin{aligned} \xi_1 &= \varepsilon_1 \\ \xi_2 &= -\varepsilon_1\gamma_2 + \varepsilon_2\gamma_1 - \varepsilon_3 \\ \xi_3 &= \varepsilon_1\gamma_4 - \varepsilon_2\gamma_3 + \varepsilon_3\gamma_2 - \varepsilon_4\gamma_1 \\ \xi_4 &= -\varepsilon_3\gamma_4 + \varepsilon_4\gamma_3 \end{aligned} \tag{4.39}$$

Both the right hands of (4.37) and (4.38) are positive semi-definite if the following conditions are satisfied:

$$q > m_i/d_i \qquad \text{for } i=1,2,\ldots,n, \tag{4.40}$$

and

$$\xi_{1i} \geq 0, \quad \xi_{2i} \geq 0, \quad \xi_{3i} \geq 0, \quad \xi_{4i} \geq 0, \qquad \text{for } i=1,2,\ldots,n. \tag{4.41}$$

Under these conditions, $Z_1(s)$, $Z_2(s)$, and accordingly, $Z(s)$ are positive real, and the system is stable according to Theorem 3.

2.3 Solution of matrix equations

If the inequalities in (4.40) and (4.41) are satisfied, then the system is stable, and there exists a Lyapunov function as follows:

$$V(x) = x'Px + 2V_1(\sigma) \tag{4.42}$$

where P is a $(6n-1)\times(6n-1)$ positive definite symmetric matrix satisfying the following matrix equations:

$$\begin{aligned} PA + A'P &= -LL' \\ PB &= CN' + A'CQ' - LW_o \\ W_o'W_o &= QC'B + B'CQ' \end{aligned} \tag{4.43}$$

where L and W_o are $(6n-1)\times(m+n)$ and $(m+n)\times(m+n)$ matrices. Since $Z(s)$ is a direct sum of $Z_1(s)$ and $Z_2(s)$, P can be expressed as follows:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \tag{4.44}$$

where P_1 and P_2 are $(2n-1)\times(2n-1)$ and $4n\times 4n$ matrices, and are related with $Z_1(s)$ and $Z_2(s)$, respectively.

The transfer matrix $W_1(s)$ is rewritten as follows:

$$W_1(s) = C_1'(sI - A_1)^{-1}B_1 \tag{4.45}$$

where

$$A_1 = \begin{bmatrix} 0 & K' \\ 0 & -M^{-1}D \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ M^{-1}T \end{bmatrix} \qquad C_1 = \begin{bmatrix} G \\ 0 \end{bmatrix} \tag{4.46}$$

Since $C_1'B_1 = 0$ holds, (2.20) reduces to

$$P_1A_1 + A_1'P_1 = -L_1L_1' \quad . \qquad (4.47)$$
$$P_1B_1 = C_1N_1' + A_1'C_1Q_1'$$

P_1 and L_1 are partitioned as follows:

$$P_1 = \begin{bmatrix} P_{11} & P_{12} \\ P_{21}. & P_{22} \end{bmatrix} \qquad L_1 = \begin{bmatrix} L_{11} \\ L_{12} \end{bmatrix} \qquad (4.48)$$

where P_{11}, P_{12}, P_{21}, P_{22}, L_{11} and L_{12} are $(n-1)\times(n-1)$, $(n-1)\times n$, $n\times(n-1)$, $n\times n$, $(n-1)\times m$, and $n\times m$ matrices, respectively. Substituting (4.46) and (4.48) into (4.47) gives

$$0 = -L_{11}L_{11}' \qquad (4.49)$$

$$P_{11}K' - P_{12}M^{-1}D = 0 \qquad (4.50)$$

$$P_{21}K' + KP_{12} - P_{22}M^{-1}D - DM^{-1}P_{22} = -L_{12}L_{12}' \qquad (4.51)$$

$$P_{12}M^{-1}T = (1/q)G \qquad (4.52)$$

$$P_{22}M^{-1}T = T \qquad (4.53)$$

These equations are the same as (2.62) ∿ (2.66), and their solutions are given as follows:

$$\begin{aligned} KP_{11}K' &= (1/q)D + \rho DUD \\ KP_{12} &= (1/q)M + \rho DUM \\ P_{22} &= M + \mu MUM \end{aligned} \qquad (4.54)$$

where U is an $n\times n$ matrix with all elements equal to 1. The scalars ρ and μ must satisfy

$$\rho \geq -(1/q)\sum_{i=1}^{n} d_i \quad .$$
$$\mu - \rho \geq -1/\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} \qquad (4.55)$$

for P_1 to be positive definite matrix, and satisfy

$$(\mu^*)^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(d_i m_j - d_j m_i)^2}{4(d_i - m_i/q)(d_j - m_j/q)} - \mu^* \sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} - 1 \leq 0 \tag{4.56}$$

for (4.51) to be satisfied, where μ^* is defined by

$$\mu^* = \mu - \rho. \tag{4.57}$$

The transfer matrix $W_2(s)$ is rewritten as follows:

$$W_2(s) = C_2'(sI - A_2)^{-1}B_2 \tag{4.58}$$

where

$$A_2 = \begin{bmatrix} -H_1 & 0 & H_2 & 0 \\ -H_3 & -H_4 & 0 & -H_5 \\ 0 & H_6 & -H_7 & 0 \\ 0 & H_8 & -H_9 & -H_{10} \end{bmatrix} \quad B_2 = \begin{bmatrix} \alpha \\ -\beta \\ 0 \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.59}$$

Since $Z_2(s)$ is positive real, $Z_2(s) + Z_2'(-s)$ is factorized as follows:

$$Z_2(s) + Z_2'(-s) = Y_2'(-s)Y_2(s) \tag{4.60}$$

where

$$Y_2(s) = \operatorname*{diag}_i\left[\frac{\sqrt{2\xi_{1i}}\, s(s + \zeta_{1i})(s + \zeta_{2i})(s + \zeta_{3i})}{\Delta_i(s)} \right] \tag{4.61}$$

in which ζ_{1i}, ζ_{2i}, and ζ_{3i} are determined by

$$\xi_{1i}\omega^6 + \xi_{2i}\omega^4 + \xi_{3i}\omega^2 + \xi_{4i} = \xi_{1i}(\omega^2 + \zeta_{1i}^2)(\omega^2 + \zeta_{2i}^2)(\omega^2 + \zeta_{3i}^2) \tag{4.62}$$

$Y_2(s)$ has a minimal realization (A_2, B_2, L_2), that is,

$$Y_2(s) = L_2(sI - A_2)^{-1}B_2 \tag{4.63}$$

where L_2 is an $4n \times n$ matrix consisting of four matrices as follows:

$$L_2 = (L_{21}', L_{22}', L_{23}', L_{24}')' \tag{4.64}$$

The matrices L_{21}, L_{22}, L_{23}, and L_{24} are $n \times n$ diagonal matrices defined by

$$L_{21} = \operatorname*{diag}_i(\ell_{1i}) \qquad L_{22} = \operatorname*{diag}_i(\ell_{2i})$$

$$L_{23} = \operatorname*{diag}_{i}(\ell_{3i}) \qquad L_{24} = \operatorname*{diag}_{i}(\ell_{4i}) \tag{4.65}$$

and their elements $\ell_{1i} \sim \ell_{4i}$ are determined by solving the following equation:

$$\lambda_i \ell_i = \tau_i \qquad \text{for } i=1,2,\ldots,n, \tag{4.66}$$

where λ_i, ℓ_i, and τ_i are 4×4 matrix and 4 dimensional vectors, respectively, defined as follows:

$$\begin{aligned}
\lambda_{i11} &= \alpha_i \\
\lambda_{i12} &= -\beta_i \\
\lambda_{i13} &= 0 \\
\lambda_{i14} &= 0 \\
\lambda_{i21} &= \alpha_i(h_{4i} + h_{7i} + h_{10i}) \\
\lambda_{i22} &= -\alpha_i h_{3i} - \beta_i(h_{1i} + h_{7i} + h_{10i}) \\
\lambda_{i23} &= -\beta_i h_{6i} \\
\lambda_{i24} &= -\beta_i h_{8i} \\
\lambda_{i31} &= \alpha_i(h_{4i}h_{7i} + h_{7i}h_{10i} + h_{10i}h_{4i} + h_{5i}h_{8i}) - \beta_i h_{2i}h_{6i} \\
\lambda_{i32} &= -\alpha_i h_{3i}(h_{7i} + h_{10i}) - \beta_i(h_{1i}h_{7i} + h_{7i}h_{10i} + h_{10i}h_{1i}) \\
\lambda_{i33} &= -\alpha_i h_{3i}h_{6i} - \beta_i h_{6i}(h_{1i} + h_{10i}) \\
\lambda_{i34} &= -(\alpha_i h_{3i} + \beta_i h_{1i})h_{8i} \\
\lambda_{i41} &= (\alpha_i h_{4i}h_{7i} - \beta_i h_{2i}h_{6i})h_{10i} \\
\lambda_{i42} &= -(\alpha_i h_{3i} + \beta_i h_{1i})h_{7i}h_{10i} \\
\lambda_{i43} &= -(\alpha_i h_{3i} + \beta_i h_{1i})h_{6i}h_{10i} \\
\lambda_{i44} &= 0
\end{aligned} \tag{4.67}$$

$$\begin{aligned}
\tau_{1i} &= \sqrt{2\xi_{1i}}\,(\zeta_{1i} + \zeta_{2i} + \zeta_{3i} - \gamma_{1i}) \\
\tau_{2i} &= \sqrt{2\xi_{1i}}\,(\zeta_{1i}\zeta_{2i} + \zeta_{2i}\zeta_{3i} + \zeta_{3i}\zeta_{1i} - \gamma_{2i}) \\
\tau_{3i} &= \sqrt{2\xi_{1i}}\,(\zeta_{1i}\zeta_{2i}\zeta_{3i} - \gamma_{3i}) \\
\tau_{4i} &= \sqrt{2\xi_{1i}}\,(-\gamma_{4i})
\end{aligned} \tag{4.68}$$

$$\ell_i = (\ell_{1i}, \ell_{2i}, \ell_{3i}, \ell_{4i}) \tag{4.69}$$

for $i=1,2,\ldots,n$.

The matrix L_2 is related with P_2 by

$$P_2A_2 + A_2'P_2 = -L_2L_2' \tag{4.70}$$

By solving this equation, we obtain

$$P_2 = \begin{bmatrix} P_{33} & P_{34} & P_{35} & P_{36} \\ P_{43} & P_{44} & P_{45} & P_{46} \\ P_{53} & P_{54} & P_{55} & P_{56} \\ P_{63} & P_{64} & P_{65} & P_{66} \end{bmatrix} \tag{4.71}$$

where P_{ij} (i,j = 3,4,5,6) is an $n \times n$ diagonal matrix defined by

$$P_{ij} = \underset{k}{\mathrm{diag}}(p_{ijk}) \qquad \text{for } i,j=3,4,5,6,\ k=1,2,\ldots,n, \tag{4.72}$$

and its elements are determined by solving the following equations:

$$\Gamma_k p_k = \nu_k \qquad \text{for } k=1,2,\ldots,n, \tag{4.73}$$

in which Γ_k, p_k, and ν_k are 10×10 matrix and 10 dimensional vectors, respectively, defined as follows:

$$\Gamma_k = \begin{bmatrix} 2h_1 & 2h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_1+h_4 & -h_6 & -h_8 & h_3 & 0 & 0 & 0 & 0 & 0 \\ -h_2 & 0 & h_1+h_7 & h_9 & 0 & h_3 & 0 & 0 & 0 & 0 \\ 0 & h_5 & 0 & h_1+h_{10} & 0 & 0 & h_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2h_4 & -2h_6 & -2h_8 & 0 & 0 & 0 \\ 0 & -h_2 & 0 & 0 & 0 & h_4+h_7 & h_9 & -h_6 & -h_8 & 0 \\ 0 & 0 & 0 & 0 & h_5 & 0 & h_4+h_{10} & 0 & -h_6 & -h_8 \\ 0 & 0 & -2h_2 & 0 & 0 & 0 & 0 & 2h_7 & 2h_9 & 0 \\ 0 & 0 & 0 & -h_2 & 0 & h_5 & 0 & 0 & h_7+h_{10} & h_9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2h_5 & 0 & 0 & 2h_{10} \end{bmatrix}_k$$

$$p_k = (p_{33k},\ p_{34k},\ p_{35k},\ p_{36k},\ p_{44k},\ p_{45k},\ p_{46k},\ p_{55k},\ p_{56k},\ p_{66k})$$

$$\nu_k = (\ell_{1k}\ell_{1k},\ \ell_{1k}\ell_{2k},\ \ell_{1k}\ell_{3k},\ \ell_{1k}\ell_{4k},\ \ell_{2k}\ell_{2k},\ \ell_{2k}\ell_{3k},\ \ell_{2k}\ell_{4k},\ \ell_{3k}\ell_{3k},\ \ell_{3k}\ell_{4k},\ \ell_{4k}\ell_{4k})$$

$$\text{for } i=1,2,\ldots,n \tag{4.74}$$

Thus P_1 and P_2, and accordingly, P are obtained.

2.4 Lyapunov function

An expression for the Lyapunov function can be obtained by substituting (4.18), (4.48), and (4.71) into (4.42) as follows:

$$V(x) = [\delta_r', \omega', \Delta E', \Delta E_a', \Delta E_e', \Delta E_d'] \begin{bmatrix} P_{11} & P_{12} & 0 & 0 & 0 & 0 \\ P_{21} & P_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{33} & P_{34} & P_{35} & P_{36} \\ 0 & 0 & P_{43} & P_{44} & P_{45} & P_{46} \\ 0 & 0 & P_{53} & P_{54} & P_{55} & P_{56} \\ 0 & 0 & P_{63} & P_{64} & P_{65} & P_{66} \end{bmatrix} \begin{bmatrix} \delta_r \\ \omega \\ \Delta E \\ \Delta E_a \\ \Delta E_e \\ \Delta E_d \end{bmatrix}$$

$$+ 2V_1(\sigma)$$

$$= \delta_r' P_{11} \delta_r + 2\delta_r' P_{12} \omega + \omega' P_{22} \omega + 2V_1(\sigma)$$

$$+ \Delta E' P_{33} \Delta E + \Delta E_a' P_{44} \Delta E_a + \Delta E_e' P_{55} \Delta E_e + \Delta E_d' P_{66} \Delta E_d$$

$$+ 2(\Delta E' P_{34} \Delta E_a + \Delta E' P_{35} \Delta E_e + \Delta E' P_{36} \Delta E_d$$

$$+ \Delta E_a' P_{45} \Delta E_e + \Delta E_a' P_{46} \Delta E_d + \Delta E_e' P_{56} \Delta E_d) \tag{4.75}$$

Now the Lyapunov function is obtained, we let $q \to \infty$ because q is introduced only in order not to cause a pole-zero cancellation between $(N + Qs)$ and $W(s)$ as mentioned before. Substituting (4.32) into (4.75), and expanding and rearranging the terms in (4.75), we obtain the following expression:

$$V(x) = (1/2 \sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2$$

$$+ (\mu^* - \mu_o)(\sum_{i=1}^{n} m_i \omega_i)^2 + \rho\{\sum_{i=1}^{n} [d_i(\delta_i - \delta_i^o) + m_i \omega_i]\}^2$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} [E_i E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o) E_i^o E_j^o \sin\delta_{ij}^o]$$

$$+ \sum_{i=1}^{n} \{p_{33i}(E_i - E_i^o)^2 + p_{44i}(E_{ai} - E_{ai}^o)^2$$

$$+ p_{55i}(E_{ei} - E_{ei}^o)^2 + p_{66i}(E_{di} - E_{di}^o)^2$$

$$+ 2(E_i - E_i^o)[p_{34i}(E_{ai} - E_{ai}) + p_{35i}(E_{ei} - E_{ei}^o) + p_{36i}(E_{di} - E_{di}^o)]$$

$$+ 2(E_{ai} - E_{ai}^o)[p_{45i}(E_{ei} - E_{ei}^o) + p_{46i}(E_{di} - E_{di}^o)]$$

$$+ 2p_{56i}(E_{ei} - E_{ei}^o)(E_{di} - E_{di}^o)\} \tag{4.76}$$

where

$$\mu_o = -1/\sum_{i=1}^{n} m_i \tag{4.77}$$

The first and the second terms in (4.76) represent kinetic energy. If damping torques of generators are uniform, then we can choose μ^* to be equal to μ_o. The scalar ρ in the third term is an arbitrary non-negative scalar. It is chosen to be zero because the term narrows and complicates the estimation of the transient stability of the system. The fourth term represents potential energy which is stored in the system owing to some deviations of rotor angles of generators from those at the stable equilibrium point. The potential energy plays an important role in defining the transient stability region of the system. The fifth term is a new term which is related with the field flux linkages and the variables of the excitation systems.

In those cases where damping torques are uniform or zero, (4.76) reduces to

$$\begin{aligned} V(x) = & (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2 \\ & + \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o] \\ & + \sum_{i=1}^{n} [p33i(E_i - E_i^o)^2 + p44i(E_{ai} - E_{ai}^o)^2 \\ & \quad + p55i(E_{ei} - E_{ei}^o)^2 + p66i(E_{di} - E_{di}^o)^2 \\ & \quad + 2p34i(E_i - E_i^o)(E_{ai} - E_{ai}^o) + 2p35i(E_i - E_i^o)(E_{ei} - E_{ei}^o) \\ & \quad + 2p36i(E_i - E_i^o)(E_{di} - E_{di}^o) + 2p45i(E_{ai} - E_{ai}^o)(E_{ei} - E_{ei}^o) \\ & \quad + 2p46i(E_{ai} - E_{ai}^o)(E_{di} - E_{di}^o) + 2p56i(E_{ei} - E_{ei}^o)(E_{di} - E_{di}^o)] \\ = & \; V_k(\omega) + V_p(\delta, E) + V_f(E, E_a, E_e, E_d) \end{aligned} \tag{4.78}$$

where ρ is chosen to be zero. V_k, V_p, and V_f denote the kinetic energy, the potential energy, and the new term, respectively. The time derivatives of V_k, V_p, and V_f are written as follows:

$$\frac{dV_k}{dt} = -(1/\sum_{i=1}^{n} d_i)\sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j(\omega_i - \omega_j)^2$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \tag{4.79}$$

$$\frac{dV_p}{dt} = -\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j)$$
$$+ 2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \tag{4.80}$$

$$\frac{dV_f}{dt} = -2\sum_{i=1}^{n}(dE_i/dt)\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})$$
$$-\sum_{i=1}^{n}[\ell_{1i}(E_i - E_i^o) + \ell_{2i}(E_{ai} - E_{ai}^o) + \ell_{3i}(E_{ei} - E_{ei}^o) + \ell_{4i}(E_{di} - E_{di}^o) - \sqrt{2\xi_{1i}}\, f_{2i}(\sigma)]^2 \tag{4.81}$$

The first term of (4.79) is due to damping torques of generators, and it is non-positive. A part of the kinetic energy is dissipated by damping torques. The second term of (4.79) and the first term of (4.80) are of the same magnitude and of the opposite signs of each other, which implies that there is some exchange of energy between the kinetic energy and the potential energy. These terms do not contribute to the damping rate of V. Similarly, the second term of (4.80) and the first term of (4.81) are of the same magnitude and of the opposite signs of each other. There is some exchange of energy between V_p and V_f, too. The second term of (4.81) is due to field flux linkages and excitation system variables. It is non-positive regardless of their time variations. As a whole, V dampens according to

$$\frac{dV}{dt} = -(1/\sum_{i=1}^{n} d_i)\sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j(\omega_i - \omega_j)^2$$
$$-\sum_{i=1}^{n}[\ell_{1i}(E_i - E_i^o) + \ell_{2i}(E_{ai} - E_{ai}^o) + \ell_{3i}(E_{ei} - E_{ei}^o) + \ell_{4i}(E_{di} - E_{di}^o) - \sqrt{2\xi_{1i}}\, f_{2i}(\sigma)]^2 \tag{4.82}$$

while V_k and V_p, V_p and V_f are interacting with each other, respectively.

2.5 Conclusions

In this section, we have constructed a Lyapunov function with a systematic method based on a generalized Popov criterion for the multimachine power systems in which dynamics of automatic voltage regulators and exciters are taken into account. There is some difference between this new Lyapunov function and the Lyapunov function derived in §2 of chapter III for the power system in which only dynamics of field flux linkages are taken into account, in several points as follows:

1) The new function consists of three terms, that is, kinetic energy V_k, potential energy V_p, and a new term V_f.
2) The kinetic energy V_k is same in both the functions.
3) The potential energy V_p is same in both the functions, and it is a function of field flux linkages as well as relative rotor angles of generators.
4) The new term V_f is a function not only of field flux linkages, but also of state variables in the excitation systems.
5) The damping rate of V consists of two terms. One of them is due to damping torques of generators, and the other is due to some deviations in excitation system state variables.

Thus the obtained Lyapunov function is characteristic of the power systems in which dynamics of excitation systems are taken into account. In the following sections, the transient stability of the system is investigated on a basis of this function.

§3. Critical value of Lyapunov function

In this section, we make some investigation on the critical value of the Lyapunov function constructed in the preceding section. We can develop almost the same discussion as in chapter III. We begin in this section with a description of basic equations.

3.1 Model and basic equations

If damping torques of generators and transfer conductances of reduced admittance matrices are zero, then the motion of the ith generator is described as follows:

$$m_i \frac{d^2\delta_i}{dt^2} = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})$$

$$\frac{dE_i}{dt} = (1/T'_{doi}\cos\phi_i)(E_{fdi} - E_{fdi}^o) - (1/T'_{doi})[1-(x_{di}-x'_{di})B_{ii}](E_i - E_i^o)$$
$$- (1/T'_{doi})(x_{di}-x'_{di})\sum_{\substack{j=1\\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})$$

$$\frac{dE_{ai}}{dt} = -(1/T_{ai})[(E_{ai}-E_{ai}^o) + K_{ai}(E_{di}-E_{di}^o) + (1+x'_{di}B_{ii})K_{ai}(E_i - E_i^o)]$$
$$- (K_{ai}/T_{ai})x'_{di}\sum_{\substack{j=1\\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})$$

$$\frac{dE_{ei}}{dt} = (1/T_{ei})[K_{ei}(E_{ai}-E_{ai}^o) - (E_{ei}-E_{ei}^o)]$$

$$\frac{dE_{di}}{dt} = (1/T_{ei})[K_{ei}K_{di}(E_{ai}-E_{ai}^o) - K_{di}(E_{ei}-E_{ei}^o) - (T_{ei}/T_{di})(E_{di}-E_{di}^o)]$$

$$\text{for } i=1,2,\ldots,n \qquad (4.83)$$

The superscript " o " denotes the stable equilibrium point of the post-fault system, and (4.83) applies to the post-fault system.

A Lur'e type Lyapunov function has been derived for the system in (4.83) in the preceding section as follows:

$$V(x) = (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i - \omega_j)^2$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - (\delta_{ij} - \delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o]$$
$$+ \sum_{i=1}^{n}[p_{33i}(E_i - E_i^o)^2 + p_{44i}(E_{ai}-E_{ai}^o)^2$$
$$+ p_{55i}(E_{ei}-E_{ei}^o)^2 + p_{66i}(E_{di}-E_{di}^o)^2$$
$$+ 2p_{34i}(E_i-E_i^o)(E_{ai}-E_{ai}^o) + 2p_{35i}(E_i-E_i^o)(E_{ei}-E_{ei}^o)$$
$$+ 2p_{36i}(E_i-E_i^o)(E_{di}-E_{di}^o) + 2p_{45i}(E_{ai}-E_{ai}^o)(E_{ei}-E_{ei}^o)$$
$$+ 2p_{46i}(E_{ai}-E_{ai}^o)(E_{di}-E_{di}^o) + 2p_{56i}(E_{ei}-E_{ei}^o)(E_{di}-E_{di}^o)]$$
$$= V_k(\omega) + V_p(\delta, E) + V_f(E, E_a, E_e, E_d) \qquad (4.84)$$

where V_k, V_p, and V_f denote kinetic energy, potential energy, and the new term related with internal voltages and excitation system state variables, respectively. Their time derivatives are given as follows:

$$\frac{dV_k}{dt} = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \tag{4.85}$$

$$\begin{aligned}\frac{dV_p}{dt} = & - \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \\ & + 2 \sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij})\end{aligned} \tag{4.86}$$

$$\begin{aligned}\frac{dV_f}{dt} = & - 2 \sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \\ & - 2 \sum_{i=1}^{n} [\ell_{1i}(E_i - E_i^o) + \ell_{2i}(E_{ai} - E_{ai}^o) + \ell_{3i}(E_{ei} - E_{ei}^o) \\ & + \ell_{4i}(E_{di} - E_{di}^o) - \sqrt{2\xi_{1i}}\, f_{2i}(\sigma)]^2\end{aligned} \tag{4.87}$$

The right hand term of (4.85) and the first term of (4.86) are of the same magnitude and of opposite signs of each other, which implies that there exists some exchange of energy between V_k and V_p. Similarly, the second term of (4.86) and the first term of (4.87) are of the same magnitude and of opposite signs of each other. There exists some exchange of energy between V_p and V_f, too. Those terms do not contribute to the damping rate of V. As a whole, V dampens according to

$$\begin{aligned}\frac{dV}{dt} = & - 2 \sum_{i=1}^{n} [\ell_{1i}(E_i - E_i^o) + \ell_{2i}(E_{ai} - E_{ai}^o) + \ell_{3i}(E_{ei} - E_{ei}^o) \\ & + \ell_{4i}(E_{di} - E_{di}^o) - \sqrt{2\xi_{1i}}\, f_{2i}(\sigma)]^2\end{aligned} \tag{4.88}$$

while V_k and V_p, V_p and V_f are interacting with each other, respectively.

3.2 Transient stability region

If δ_1 is chosen as reference, V_p in (4.84) can be treated as a function of an (n-1) dimensional vector δ_r and an n dimensional vector E as follows:

$$V_p = V_p(\delta_r, E) \tag{4.89}$$

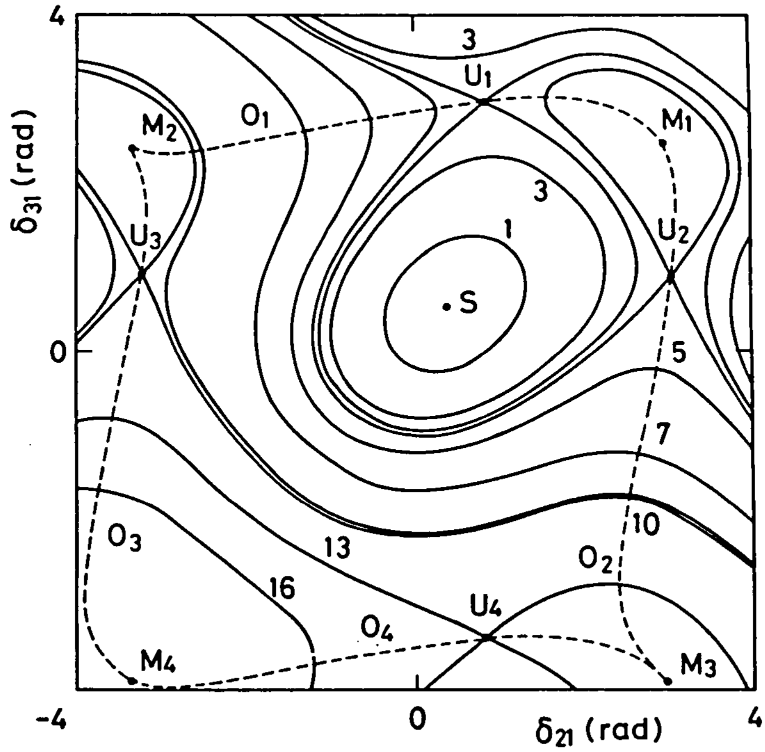


Fig.57. Equipotential curves of 3-machine system: E = (1.0, 1.0, 1.0) p.u..

where

$$\delta_r = (\delta_{21}, \delta_{31}, \ldots, \delta_{n1})$$

$$E = (E_1, E_2, \ldots, E_n)$$

Fig.57 shows an example of $V_p(\delta_r, E)$ in an (n-1) dimensional relative angular space for a 3-machine power systems. The curves C_1, C_2, ... are equipotential curves yielded by

$$V_p(\delta_r, E) = C_i \qquad i=1,2,\ldots,n \tag{4.90}$$

where E is treated as parameters, and E = (1.0, 1.0, 1.0)' in this figure. The function V_p takes the minimum value at the point S. The points U_1, U_2, ... are saddle points. The curves O_1, O_2, ... are those which go through U_1, U_2,...., and are orthogonal to equipotential curves, respectively. If C_i takes small values, then the corresponding equipotential curves are closed, and surround the point S. With increase in magnitude of C_i, equipotential curve goes outside, and reaches the lowest saddle point U_1 when C_i takes the value defined as follows:

$$V_{u1} = V_p(\delta_{u1}, E) \tag{4.91}$$

where δ_{u1} is the relative angle vector at U_1. If C_i is greater than V_{u1}, then the corresponding curve is not closed any more. With more increase in magnitude of C_i, the equipotential curve reaches the saddle points U_2, U_3, ... in sequence according to

$$\begin{aligned} V_{u2} &= V_p(\delta_{u2}, E) \\ V_{u3} &= V_p(\delta_{u3}, E) \\ &\vdots \end{aligned} \tag{4.92}$$

where δ_{u2}, δ_{u3}, ... are the relative angle vectors at U_2, U_3, ..., respectively.

From (4.83), each generator receives a torque as follows:

$$f_i = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \tag{4.93}$$

The f_i defines an n dimensional vector f

$$f = [f_1, f_2, \ldots, f_n]' \tag{4.94}$$

The sum of all torques denoted by $\overline{f}$ is given as follows:

$$\begin{aligned} \overline{f} &= \sum_{i=1}^{n} f_i \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij}) \\ &= 0 \end{aligned} \tag{4.95}$$

Eq.(4.95) implies that the center of angular velocities $\overline{\omega}$ defined by

$$\overline{\omega} = \sum_{i=1}^{n} m_i \omega_i / \sum_{i=1}^{n} m_i \tag{4.96}$$

does not receive any torque, and accordingly, it is kept constant all the time, that is,

$$\overline{\omega} = \text{constant} \tag{4.97}$$

This fact implies that each torque does not contribute to the acceleration of the center of angular velocities, but that each torque has only influence on relative behaviors of generators. There is a relation between the torque f and the potential energy function V_p as follows; the partial derivarives of V_p with respect to relative angles are given by

$$\begin{aligned} \frac{\partial V_p}{\partial \delta_{i1}} &= 2 \sum_{j=1}^{n} B_{ij}(E_i E_j \sin\delta_{ij} - E_i^o E_j^o \sin\delta_{ij}^o) \\ &= -2f_i \qquad\qquad i \neq 1 \end{aligned} \tag{4.98}$$

which gives

$$\frac{\partial V_p}{\partial \delta_r} = -2f_r \tag{4.99}$$

where f_r is a reduced torque of (n-1) dimension defined as follows:

$$f_r = [\ f_2,\ f_3,\ \ldots,\ f_n\ ] \tag{4.100}$$

The direction of $(\partial V_p/\partial \delta_r)$ is orthogonal to the equipotential curves, and its magnitude is proportional to the gradient of the equipotential curves. Eq.(4.99) shows that the system receives the torque which always acts on it in the direction orthogonal to the equipotential curves.

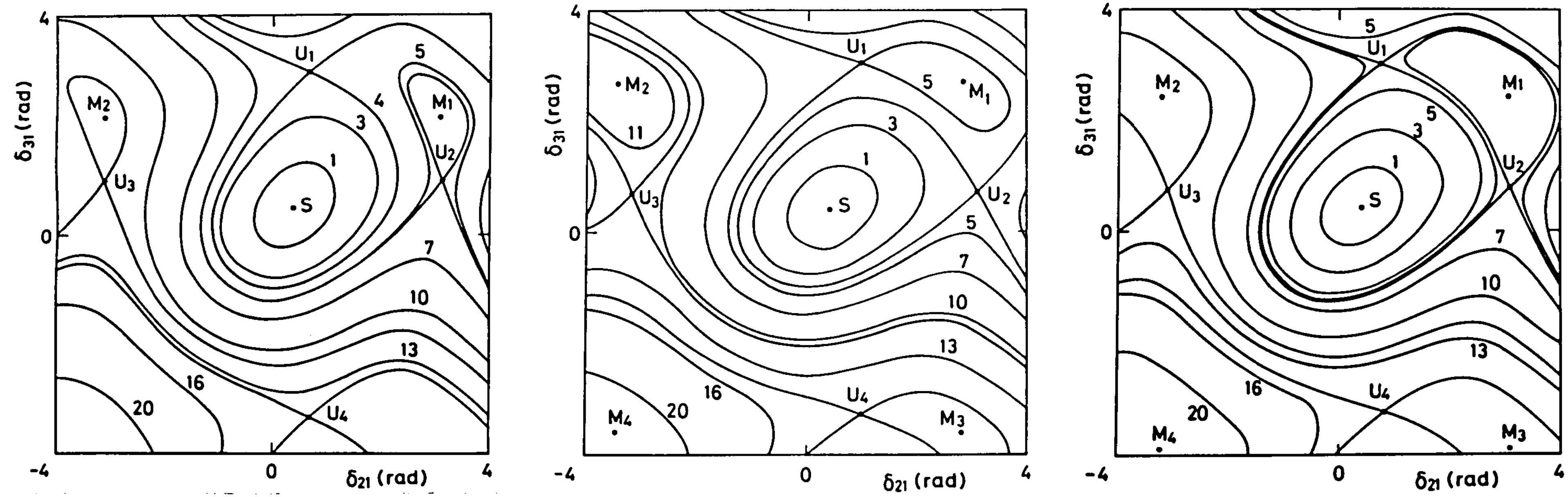
Let us return to Fig.57. The torque f_r is parallel with the curves O_1, O_2, ... on them. Those curves enclose the region in which the stable equilibrium point S exists. In this region, f_r acts on the system in such way that it will confine the system in this region. The system will lose synchronism if it crosses one of the curves O_1, O_2, ... from the inside to the outside of the region because f_r will acts in such way that it will separate the system from the curve, afterwards. The transient stability region is defined by the region which is bounded by the curves O_1,O_2, ... in a wide sense that the system receives the synchronizing torque in it. Thus the transient stability region is defined in the same way as for the power systems which were investigated in the preceding chapters.

3.3 Variation of equipotential curves

The potential energy function V_p contains internal voltages as its variables, so the equipotential curves yielded by (4.90) vary with internal voltages. Fig.58 shows some examples of the equipotential curves for the 3-machine power system in Fig.57, where some of E_1, E_2, and E_3 are increased from 1.0 p.u. to 1.2 p.u. Fig.58(a) shows the equipotential curves in the case where only E_2 is increased to 1.2 p.u. The equipotential curves move inwards while the saddle points U_1, U_2, U_3, and U_4 shift a little outwards. The value of V_p at each saddle point varies as follows:

$$\begin{aligned} V_{u1} &: \quad 3.608 \rightarrow 4.010 \\ V_{u2} &: \quad 3.854 \rightarrow 5.117 \\ V_{u3} &: \quad 10.139 \rightarrow 11.401 \\ V_{u4} &: \quad 13.032 \rightarrow 13.437 \end{aligned} \tag{4.101}$$

while V_p is nearly equal to zero at S. Thus V_{ui} increases with the



(a) E = (1.0, 1.2, 1.0) p.u.

(b) E = (1.0, 1.0, 1.2) p.u.

(c) E = (1.0, 1.2, 1.2) p.u.

Fig.58. Variations of equipotential curves with internal voltages in 3-machin system.

increase in E_2. In particular, the increase in V_{ui} is conspicuous for U_2 and U_3 which are corresponding to the step-out of no.2 generator. This fact implies that the increase in E_2 makes it difficult for no.2 generator to step out by raising the saddle point which corresponds its step-out. In Fig.58(b), only E_3 is increased to 1.2 p.u. Similarly, V_{ui} varies as follows:

$$\begin{aligned} V_{u1} &: \quad 3.608 \rightarrow 5.022 \\ V_{u2} &: \quad 3.854 \rightarrow 4.288 \\ V_{u3} &: \quad 10.139 \rightarrow 10.572 \\ V_{u4} &: \quad 13.032 \rightarrow 14.801 \end{aligned} \tag{4.102}$$

while V_p takes a value nearly equal to zero at S. In this case, the increase in V_{ui} is conspicuous for U_1 and U_4 which are corresponding to the step-out of no.3 generator, and makes it difficult for no.3 generator to step out. In Fig.58(c), E_2 and E_3 are increased to 1.2 p.u. while E_1 is kept at 1.0 p.u. In this case, V_{ui} varies as follows:

$$\begin{aligned} V_{u1} &: \quad 3.608 \rightarrow 5.523 \\ V_{u2} &: \quad 3.854 \rightarrow 5.650 \\ V_{u3} &: \quad 10.139 \rightarrow 11.934 \\ V_{u4} &: \quad 13.032 \rightarrow 14.946 \end{aligned} \tag{4.103}$$

The increase in V_{ui} is almost same for all saddle points U_1, U_2, U_3, and U_4, and makes it difficult for both no.2 and 3 generators to step out. Summing up the above observations, we conclude that some increase in a internal voltage enhance the transient stability of the corresponding generator. On the other hand, in the cases where some internal voltages are decreased, the transient stability region gets narrower and shallower as observed from Fig.33 in chapter III.

Whether some internal voltages are increased or decreased depends on the gains of the automatic voltage regulators installed in the generators of the system. If the gains are low, then the internal voltages will decrease owing to some fault, and conversely, if the gains are high enough, the internal voltages will increase. Then how high gains are allowed for the system in (4.83) by the generalized Popov criterion ?

For the system in (4.83) to be judged to be stable according to the generalized Popov criterion, the inequalities in (4.41) should be satisfied. Those inequalities are rewritten as follows:

$$\xi_{1i} = \alpha_i \geq 0 \qquad (4.104\text{-a})$$

$$\xi_{2i} = \eta_{1i}K_{ai} + \eta_{2i} \geq 0 \qquad (4.104\text{-b})$$

$$\xi_{3i} = \eta_{3i}K_{ai}^2 + \eta_{4i}K_{ai} + \eta_{5i} \geq 0 \qquad (4.104\text{-c})$$

$$\xi_{4i} = \eta_{6i}K_{ai}^2 + \eta_{7i}K_{ai} + \eta_{8i} \geq 0 \qquad (4.104\text{-d})$$

where

$$\eta_{1i} = \beta_i' h_{2i}h_{6i} - 2\alpha_i h_{5i}' h_{8i}$$

$$\eta_{2i} = \alpha_i(h_{4i}^2 + h_{7i}^2 + h_{10i}^2)$$

$$\eta_{3i} = (\alpha_i h_{5i}' h_{8i} - \beta_i' h_{2i}h_{6i})h_{5i}' h_{8i}$$

$$\eta_{4i} = \alpha_i[2h_{5i}' h_{8i}(h_{4i}h_{7i} + h_{7i}h_{10i} + h_{10i}h_{4i}) - h_{2i}h_{3i}' h_{6i} \times (h_{4i} + h_{7i})] + \beta_i'(h_{10i}^2 - h_{1i}h_{4i} - h_{4i}h_{7i} - h_{7i}h_{1i})h_{2i}h_{6i}$$

$$\eta_{5i} = \alpha_i(h_{4i}^2 h_{7i}^2 + h_{7i}^2 h_{10i}^2 + h_{10i}^2 h_{4i}^2)$$

$$\eta_{6i} = -(\alpha_i h_{3i}' + \beta_i' h_{1i})h_{2i}h_{5i}' h_{6i}h_{8i}h_{10i}$$

$$\eta_{7i} = -[\alpha_i(h_{4i} + h_{7i})h_{3i}' + \beta_i'(h_{1i}h_{4i} + h_{4i}h_{7i} + h_{7i}h_{1i})] \times h_{2i}h_{6i}h_{10i}^2$$

$$\eta_{8i} = \alpha_i(h_{4i}h_{7i}h_{10i})^2$$

and

$$h_{3i}' = h_{3i}/K_{ai} = (1 - x_{di}' B_{ii})/T_{doi}'$$

$$h_{5i}' = h_{5i}/K_{ai} = 1/T_{ai}$$

$$\beta_i' = \beta_i/K_{ai} = x_{di}'/T_{ai}$$

The inequalities in (4.104-a) and (4.104-b) are satisfied for normal generators, so we only have to investigate the remaining inequalities. From (4.104-c) and (4.104-d), we obtain two conditions as follows:

$$\xi_{3i\min} \leq K_{ai} \leq \xi_{3i\max} \qquad (4.105\text{-a})$$

and

$$\xi_{4i\min} \leq K_{ai} \leq \xi_{4i\max} \qquad (4.105\text{-b})$$

where

$$\xi_{3imin} \simeq - \frac{(x_{di}T^2_{di} - x'_{di}T_{ai}T'_{doi})T_{ei} + (x_{di}T_{ai} + x'_{di}T'_{doi})T^2_{di}}{x'_{di}K_{di}K_{ei}T'_{doi}T^2_{di}}$$

$$\xi_{3imax} \simeq \frac{(x_{di} - x'_{di})(T^2_{ai} + T^2_{ei} + T^2_{di})T'_{doi}\cos\phi_i}{K_{ei}[(x_{di}T^2_{di} - x'_{di}T_{ai}T'_{doi})T_{ei} + (x_{di}T_{ai} + x'_{di}T'_{doi})T^2_{di}]}$$

$$\xi_{4imin} \simeq - \frac{x_{di}T_{ai} + x'_{di}T'_{doi} + x_{di}T_{ei}}{x_{di}K_{ei}K_{di}T_{di}}$$

$$\xi_{4imax} \simeq \frac{(x_{di} - x'_{di})T'_{doi}\cos\phi_i}{K_{ei}(x_{di}T_{ai} + x'_{di}T'_{doi} + x_{di}T_{ei})}$$

In (4.105), ξ_{3imin} and ξ_{4imin} are both negative for normal generators, so the left hand inequalities are satisfied because K_{ai} is usually positive. On the other hand, ξ_{3imax} and ξ_{4imax} are both positive, and accordingly, K_{ai} should be smaller than these values. As an example, let us take a 10-machine power system in Fig.59, where its genrator parameters are provided in Table 16. The variations of ξ_{3imax} and ξ_{4imax} for no.2 generator are shown in Fig.60. The value of ξ_{3imax} increases monotonously with the time constant T_{ai}, and conversely, the value of ξ_{4imax} decreases monotonously. As is clear from the figure, ξ_{4imax} is always smaller than ξ_{3imax}. From the above considerations, (4.105) reduces to

$$0 \leq K_{ai} \leq \xi_{4imax} \tag{4.106}$$

Since ξ_{4imax} takes its maximum value when T_{ai} and T_{ei} are both zero, let T_{ai}, $T_{ei} \to 0$, then (4.106) is simplified as follows:

$$0 \leq K_{ai} \leq K_{aimax}\cos\phi_i \tag{4.107}$$

where

$$K_{aimax} = \frac{x_{di} - x'_{di}}{x'_{di}}$$

The coefficient K_{aimax} is the maximum value of K_{ai} allowed by the genera-

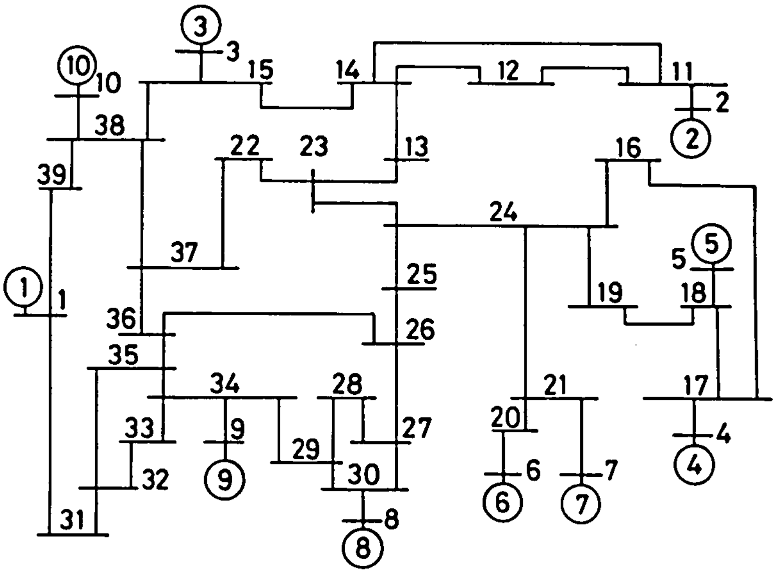


Fig. 59. Configuration of 10-machine system.

Table 16. Generator parameters of 10-machine system.

Unit	H	x_d	x'_d	x_q	T'_{do}	Exciter & AVR
1	500.0	0.0200	0.0060	0.0190	7.00	K_a = 1.000
2	34.5	0.2106	0.0570	0.2050	4.79	K_e = 1.000
3	24.3	0.2900	0.0570	0.2800	6.70	K_d = 0.004
4	26.4	0.2950	0.0490	0.2920	5.66	T_a = 0.020
5	34.8	0.2540	0.0500	0.2410	7.30	T_e = 0.040
6	26.0	0.6700	0.1320	0.6200	5.40	T_d = 0.500
7	28.6	0.2620	0.0436	0.2580	5.69	T_1 = 0.0
8	35.8	0.2495	0.0531	0.2370	5.70	E_{c1}= 7.0
9	30.3	0.2950	0.0697	0.2820	6.56	E_{c2}= 0.0
10	42.0	0.1000	0.0310	0.0690	10.20	

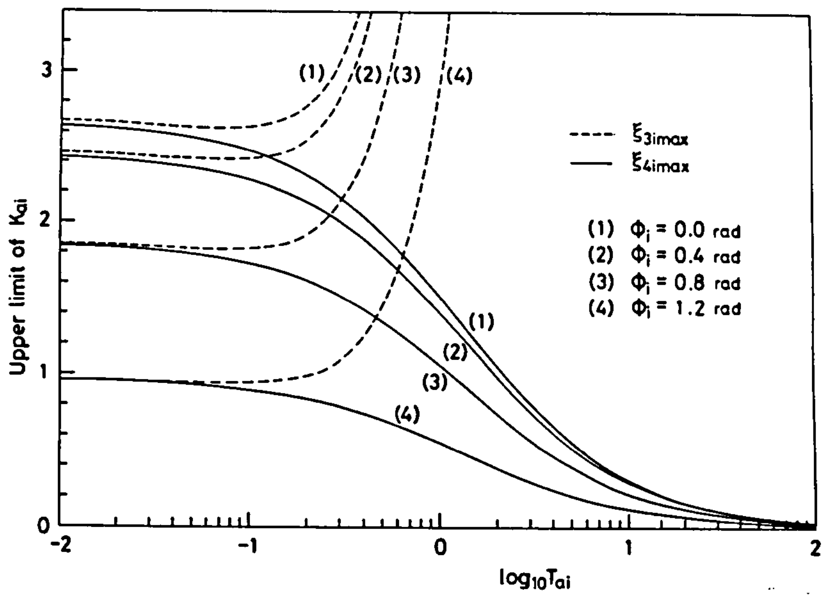


Fig.60. Variation of ξ_{3imax} and ξ_{4imax} with time constant T_{ai}: i = 2.

Table 17. Value of K_{aimax}.

Unit	K_{aimax}
1	2.333
2	2.695
3	4.088
4	5.020
5	4.080
6	4.076
7	5.009
8	3.699
9	3.232
10	2.226

lized Popov criterion, and it is determined only by d-axis synchronous and transient reactances x_{di}, x'_{di}. Table 17 shows the value of K_{aimax} for all generators in the system. The maximum allowable gain K_{aimax} varies with generator, but remains in a range of 2.0 ∿ 6.0. These values are somewhat unsatisfactory. In practical power systems, the gain of automatic voltage regulator is sometimes set at an value higher than these values. In those cases, the inequality in (4.107) is not satisfied, and accordingly, we can not produce the Lyapunov function given by (4.84).

It may be useful to elucidate from what the inequality in (4.107) originates in order to know whether the internal voltages are increased or not by the automatic voltage regulators with the gains allowed by the generalized Popov criterion. First of all, let T_{ai}, $T_{ei} \to 0$, then the variations of the terminal voltages of the generators are directly transmitted to the excitation voltages, and (4.83) reduces as follows:

$$m_i \frac{d^2\delta_i}{dt^2} = \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})$$
$$\frac{dE_i}{dt} = -\alpha_i^*(E_i - E_i^o) - \beta_i^* \sum_{\substack{j=1\\ j\neq i}}^{n} B_{ij} E_j (\cos\delta_{ij}^o - \cos\delta_{ij}) \qquad (4.108)$$

where

$$\alpha_i^* = [1 - (x_{di} - x'_{di})B_{ii} + (1 + x'_{di}B_{ii})K_{ai}K_{ei}/\cos\phi_i]/T'_{doi}$$
$$\beta_i^* = (x_{di} - x'_{di} - x'_{di}K_{ai}K_{ei}/\cos\phi_i)/T'_{doi}$$

Eq.(4.108) is formally equivalent to (3.65) and (3.66), and is equivalent to them if K_{ai} is zero. With increase in K_{ai}, β_i^* decreases monotonously, and at last, it comes to take negative values while α_i^* always takes positive values. It is clear from (3.30) that the following condition should be satisfied for the system to be judged to be stable according to the generalized Popov criterion:

$$\beta_i^* \geq 0 \qquad (4.109)$$

This condition is clearly equivalent to the following condition:

$$K_{ai} \leq \frac{x_{di} - x'_{di}}{x'_{di}} \cos\phi_i \tag{4.110}$$

Thus we obtain the same condition as (4.107). Namely, (4.107) originates from (3.30).

Next let us investigate whether the internal voltages are increased or not by the automatic voltage regulators with the gains satisfying (4.107). We can rewrite the second equation in (4.108) as follows:

$$\begin{aligned} \frac{dE_i}{dt} = & - \alpha_i(E_i - E_i^o) - \beta_i \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \\ & - \alpha'_i(E_i - E_i^o) + \beta'_i \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \end{aligned} \tag{4.111}$$

where

$$\alpha'_i = K_{ai}K_{ei}(1 + x'_{di}B_{ii})/T'_{doi}\cos\phi_i$$

$$\beta'_i = K_{ai}K_{ei}x'_{di}/T'_{doi}\cos\phi_i$$

The first and the second term are the same as those in (3.66). The second term is due to the armature reaction of the ith generator, and it causes the internal voltages to decrease during the first swing as investigated in detail in the preceding chapter. The third and the fourth terms are due to the voltage regulation of the ith generator. The second and the fourth terms are of opposite signs, so the voltage regulation cancells out with the armature reactions, and suppresses the decrease in the internal voltages owing to the armature reactions. If K_{ai} is zero, α'_i and β'_i are both zero, and only the first two terms exist. With increase in K_{ai}, β'_i gets greater, but as far as (4.107) is satisfied,

$$\beta'_i \leq \beta_i \tag{4.112}$$

holds, and the internal voltages decreases during the first swing owing to the armature reactions, yet. In order that the internal voltages increase during the first swing,

$$\beta_i' \quad > \quad \beta_i \tag{4.113}$$

or equivalently,

$$K_{ai} \quad > \quad \frac{x_{di} - x_{di}'}{x_{di}'} \cos\phi_i \tag{4.114}$$

should be satisfied, where K_{ei} is set 1.0. This condition is contrary to (4.107). Namely, the internal voltages are not increased during the first swing so far as (4.107) is satisfied. This conclusion will hold in the cases where T_{ai} and T_{ei} are not zero because (4.106) is more strict than (4.107), that is, because K_{ai} allowed by (4.106) is lower than that by (4.107).

3.4 Method of determining critical value

The automatic voltage regulator is classified into two groups referred as low gain regulator and high gain regulator according to whether its gain satisfies (4.106) or not. In the cases where all generator in a system are installed with low gain regulators, almost the same discussions as those in §3.3 of chapter III hold to this system.

The time derivatives of V_k and V_p in (4.85) and (4.86) are rewritten as follows:

$$\frac{dV_k}{dt} = -\frac{\partial V_p}{\partial \delta_r} \omega_r \tag{4.115}$$

$$\frac{dV_p}{dt} = \frac{\partial V_p}{\partial \delta_r} \omega_r + \frac{\partial V_p}{\partial E} \frac{dE}{dt} \tag{4.116}$$

Hence the increments of V_k and V_p in a time interval are given as follows:

$$\begin{aligned} \Delta V_k &= \int_{t_s}^{t_e} \frac{dV_k}{dt} dt \\ &= -\Delta V_{k\to p}(\delta_s \sim \delta_e, E_s \sim E_e) \end{aligned} \tag{4.117}$$

and

$$\Delta V_p = \int_{t_s}^{t_e} \frac{dV_p}{dt} dt \tag{4.118}$$

$$= \Delta V_{k\to p}(\delta_s \sim \delta_e,\ E_s \sim E_e) - \Delta V_{p\to f}(\delta_s \sim \delta_e,\ E_s \sim E_e)$$

where

$$\Delta V_{k\to p}(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv \int_{\delta_s}^{\delta_e} \frac{\partial V_p}{\partial \delta_r} d\delta_r \ , \tag{4.119}$$

$$\Delta V_{p\to f}(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv -\int_{E_s}^{E_e} \frac{\partial V_p}{\partial E} dE \ , \tag{4.120}$$

t_s and t_e are both ends of the time interval, and δ_s, δ_e, E_s, and E_e are the values of δ_r and E at the instants t_s and t_e, respectively. The term $\Delta V_{k\to p}$ denotes the energy which is transfered from V_k to V_p, and the term $\Delta V_{p\to f}$ denotes the energy which is transfered from V_p to V_f in the time interval. These terms are determined by the trajectory of the system in the (δ_r, E)-space, and it does not matter how long it takes the system to move from (δ_s, E_s) to (δ_e, E_e).

Fig.61 shows an example of system trajectory in a relative angular space for a case where a fault continues. The system crosses the boundary of the transient stability region at the point c. Over this point, the torque f_r which is related with V_p by (4.99) acts in such way that it will accellate the system, and will separate it from the transient stability region. In order that the system stays in synchronism during the first swing, the fault must be cleared in such time that the system can not go over the point c. We make here two assumptions as follows:

1) The system moves along the sustained fault trajectory even in the period after the fault clearance.
2) The internal voltages vary in the same way as for the sustained fault case even in the period after the fault clearance.

The former assumption is satisfied to some extent in large power systems. On the other hand, the latter assumption is not satisfied so well as the former one. If a fault is cleared, the internal voltages do not decrease so much as in the case where the fault is on, so this assumption gives a somewhat strict stability condition as will become clear in the followings.

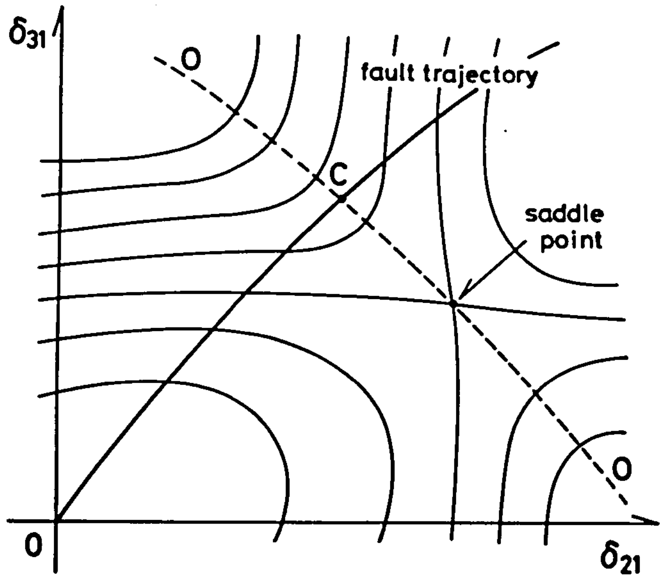


Fig.61. Trajectory in relative angular space: sustained fault.

Under these assumptions, let us consider a case where a given fault is cleared at an instant. If the fault is cleared at an instant later than the critical clearing time, the system will move along the sustained fault trajectory, and go over the point c. Conversely, if the fault is cleared at an instant earlier than the critical clearing time, then the system will move along the trajectory, but will not reach the point c. Now, let us consider the case where the fault is cleared just at the critical clearing time. The system will move along the trajectory, reach the point c, and stop there. Hence, in this case, we obtain two relations as follows:

$$\begin{aligned} V_k(t_c) &= V_k(t_{cr}) - \Delta V_{k\to p}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \\ &= 0 \end{aligned} \tag{4.121}$$

$$\begin{aligned} V_p(t_c) &= V_p(t_{cr}) + \Delta V_{k\to p}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \\ &\quad - \Delta V_{p\to f}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \end{aligned} \tag{4.122}$$

where (δ_{cr}, E_{cr}) and (δ_c, E_c) are the values of (δ_r, E) at the critical clearing time t_{cr} and the time t_c when the system reaches the point c, respectively. From (4.121) and (4.122), we obtain

$$V_k(t_{cr}) + V_p(t_{cr}) = V_{cr} + \Delta V_{p\to f}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \tag{4.123}$$

where

$$\begin{aligned} V_{cr} &\equiv V_p(t_c) \\ &= V_p(\delta_c, E_c) \end{aligned} \tag{4.124}$$

Since the internal voltages decrease during the first swing, the second term in (4.123) takes some positive value, i.e.,

$$\Delta V_{p\to f}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \geq 0 \tag{4.125}$$

From (4.123) and (4.125), we get

$$V_k(t_{cr}) + V_p(t_{cr}) \geq V_{cr} \tag{4.126}$$

This inequality suggests us that the system remains in synchronism for the first swing if the fault is cleared in such way that

$$V_k(t) + V_p(t) < V_{cr} \tag{4.127}$$

holds, and leads us to a stability condition as follows:

[Stability condition 8]

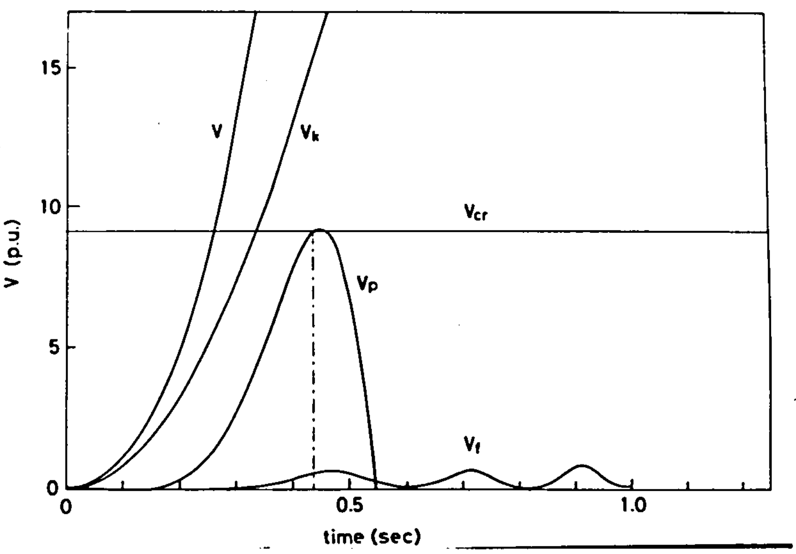
If a system satisfies

$$V < V_{cr} \tag{4.128}$$

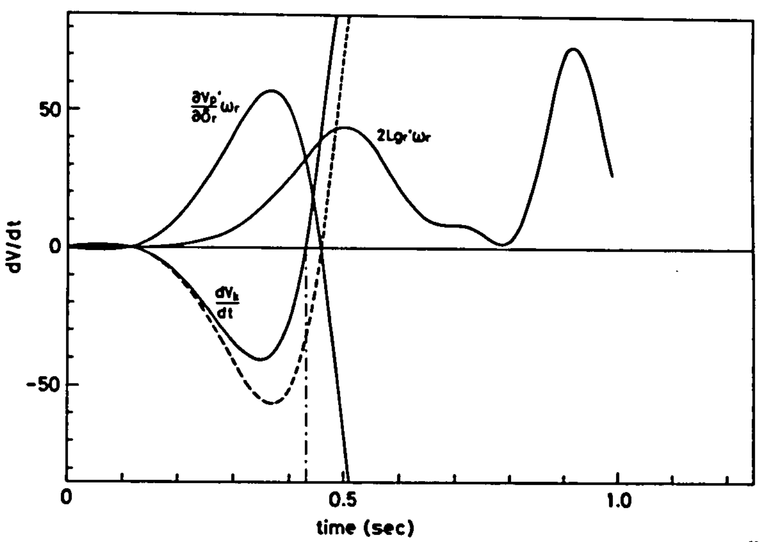
then it is stable for the first swing, where V_{cr} is the value of $V_p(\delta_r, E)$ at the point c where the sustained fault trajectory crosses the boundary of the transient stability region.

This condition is similar to the stability condition 6 which was derived in §3.4 of chapter III. In (4.128), $(V_k + V_p)$ has been replaced by V in order to reserve some stability margin mentioned in deriving the stability condition 6. We can use this condition as the basis of estimating the critical fault clearing time by using V_{cr} as the critical value of the Lyapunov function V.

In order to apply the stability condition 8, we have to look for the point c. The time derivative of V_k is given by (4.115), which is the same as (3.113). Hence the same discussions as those after (3.113) in §3.4 in the previous chapter hold true. Namely, the time derivative of V_k changes its sign from negative to positive at the instant when the system crosses the boundary of the transient stability under a sustained fault. Accordingly, the critical value V_{cr} can be obtained by calculating the value of $V_p(\delta_r, E)$ at that instant. This method of determining the critical value is very simple, and needs no calculation of saddle points, so it is able to get rid of the usual difficulty accompanying its calculation. As an example, let us take a case where a 3-phase short-circuit occurs at a point near the bus 11 in the 10-machine power system shown by Fig.59. The fault is cleared by opening the line connecting the buses 11 and 12 at both terminals. Fig.62 shows the time variations of V, its components, and $(dV_k/$



(a) V and its components



(b) Time derivative of V_k.

Fig.62. Method of determining critical value of V in case where voltage regulator gains are low.

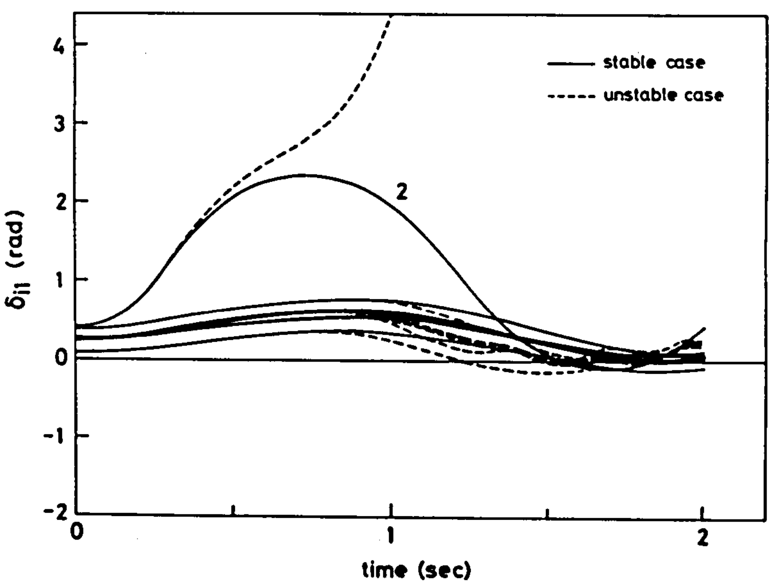


Fig.63. Swing curves of generators for case where voltage regulator gains are low: fault 11-12
——— cleared at 0.26 sec,
- - - - - cleared at 0.27 sec.

dt) for the case where the fault lasts, where V has been modified in order to take account of the transfer conductances as follows:

$$V(t) = V_p(t) + \gamma V_k(t) + V_f(t) \tag{4.129}$$

in which γ was defined by (3.146) and (3.155). The influence of the transfer conductances has been already investigated in detail in §4 of chapter III, and the discussions and results in the section apply as it is to the system under study only by replacing V_f in (3.124) by that in (4.84). As shown in Fig. 62(b), (dV_k/dt) changes its sign from negative to positive at 0.44 sec. This instant coincides with that when the system crosses the boundary of the transient stability region. The critical value V_{cr} is obtained by taking the value of V_p at the instant. V_{cr} is 9.123, in this case. Since V defined by (4.129) takes 9.116 and 9.997 at 0.26 and 0.27 sec, respectively, it is guessed that the critical fault clearing time exists between 0.26 and 0.27 sec. In Fig. 63, the time variations of relative rotor angles are shown for the two cases where the fault is cleared at 0.26 sec and 0.27 sec. This figure verifies the above guess. Namely, if the fault is cleared at 0.26 sec, the system keeps synchronism, and conversely, if the fault is cleared at 0.27 sec, then the system loses synchronism. Thus Lyapunov's direct method yields very accurate result in this case.

The stability condition 8 certainly applies to the systems where their generators are all installed with low gain regulators satisfying (4.106). It should be noted, however, that this condition is based only on (4.125) except for the two assumptions made on the system behaviors, and that it does not matter whether the time derivative of V is negative semi-definite or not if V_f is some positive definite (or semi-definite) function of the internal voltages and the excitation system variables. If (4.125) is satisfied, then the stability condition 8 holds true even in the cases where automatic voltage regulators do not necessarily satisfy (4.106). On this basis, we will try one generalization of Lyapunov's direct method to power systems which are installed with high gain voltatage regulators in the next section.

3.5 Conclusions

In this section, we have some investigations on the critical value of of the Lyapunov function constructed in §2. The transient stability region was defined on the basis of the torque applied to the system by treating the internal voltages as parameters. By investigating the transient stability region and the behaviors of the internal voltages, we have obtained some results as follows:

1) The transient stability region gets wider and deeper with increase in the internal voltages, and conversely, gets narrower and shallower with decrease in the internal voltages.
2) In order that V in (4.84) can be yielded, automatic voltage regulator gains should be smaller than certain values which are determined by d-axis synchronous and transient reactances x_{di}, x'_{di}.
3) The internal voltages are not increased during the first swing if all generators in the system are installed with low gain regulators satisfying (4.106).

Bearing these natures in mind, we have derived a stability condition concering with the first swing stability. This condition is similar to the stability condition 6 derived in the previous chapter. On this basis, the critical value V_{cr} has been introduced. It is defined by the value of potential energy V_p at the point where the system crosses the boundary of the transient stability region, which is obtained by checking the sign of the time derivative of kinetic energy V_k under a sustained fault. Since it needs no calculation of saddle points, it is able to avert the difficulty which we sometimes experience in calculating those points.

§4. One generalization of Lyapunov's direct method to power systems with high gain regulators

In the preceding sections, we have constructed a Lyapunov function for the power system described by (4.1) $\sim$ (4.5) on the basis of the generalized Popov criterion, and have determined its critical value concerning with the first swing stability. However, automatic voltage regulator gains should be set somewhat low values for the system to be judged to be stable according to the generalized Popov criterion, and for V in (4.84) to be calculated. The transient stability of the system can not be so much so far as we use automatic voltage regulators with gains allowed by the criterion. In view of these facts, we are moved to generalize Lyapunov's direct method to power systems in which some or all generators are installed with high gain regulators.

4.1 Model and basic equations

An n-machine power system in which each generator is installed with some automatic voltage regulators, is described as follows:

$$m_i \frac{d^2\delta_i'}{dt^2} + d_i \frac{d\delta_i'}{dt} = P_{mi} - \sum_{j=1}^{n} Y_{ij}E_iE_j \sin(\delta_{ij} + \theta_{ij}) \tag{4.130}$$

$$\frac{dE'_{qi}}{dt} = (1/T'_{doi})[E_{fdi} - E'_{qi} - (x_{di} - x'_{di})i_{di}] \tag{4.131}$$

for i=1,2,...,n

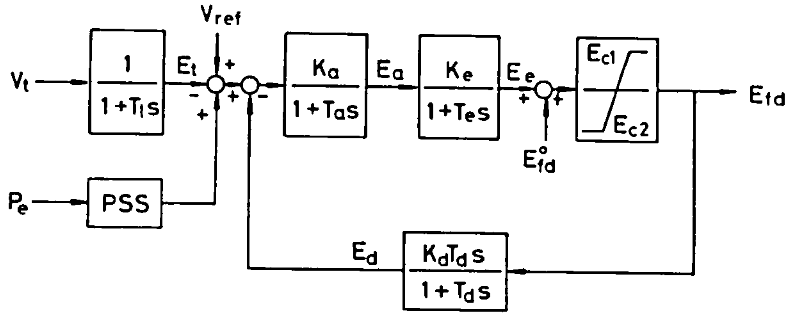
Eqs. (4.130) and (4.131) are the same as (4.1) and (4.2). If each generator is installed with automatic voltage regulators shown by Fig.54, then each excitation voltage is controlled by (4.3) ∿ (4.5). In this section, we use excitation systems as shown in Fig.64. Each gain of automatic voltage regulator is enlarged to 100.0, where we add power system stabilizers as shown in Fig.65 in order to keep the system dynamically stable. The dynamics of the excitation systems are described as follows:

$$\frac{dE_{ai}}{dt} = -(1/T_{ai})[E_{ai} + K_{ai}E_{di} - K_{ai}E_{pi} - K_{ai}(V_{refi} - V_{ti})] \tag{4.132}$$

$$\frac{dE_{ei}}{dt} = (1/T_{ei})[K_{ei}E_{ai} - E_{ei}] \tag{4.133}$$

$$\frac{dE_{di}}{dt} = (1/T_{ei})[K_{di}K_{ei}E_{ai} - K_{di}E_{ei} - (T_{ei}/T_{di})E_{di}] \tag{4.134}$$

$$\frac{dE_{fli}}{dt} = -(1/T_{fli})[E_{fli} + (P_{ei} - P_{refi})] \tag{4.135}$$



K_a = 100.0
T_a = 0.02
T_1 = 0.0

K_e = 1.0
T_e = 0.04
E_{c1} = 7.0

K_d = 0.004
T_d = 0.50
E_{c2} = 0.0

Fig.64. Block diagram of excitation system with PSS installed.

Fig.65.
Block diagram of PSS:

T_{f1} = 0.04

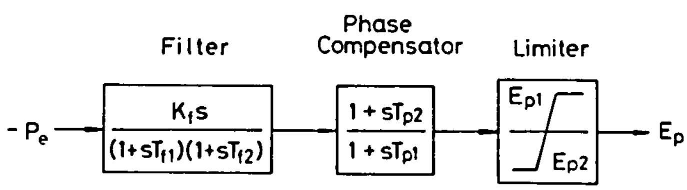
T_{f2} = 0.50

T_{p1} = 0.25

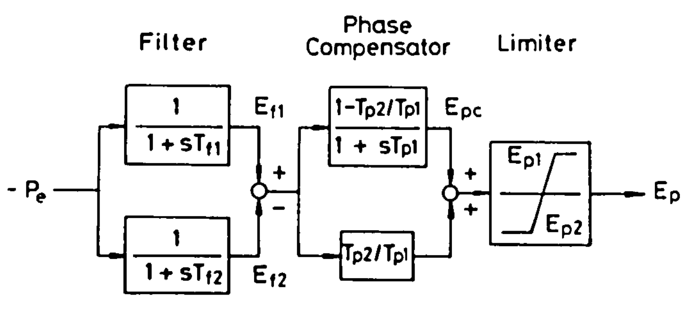
T_{p2} = 0.00

E_{p1} = +0.5

E_{p2} = -0.5



(a) PSS



(b) Equivalent circuit

$$\frac{dE_{f2i}}{dt} = -(1/T_{f2i})[E_{fdi} + (P_{ei} - P_{refi})] \tag{4.136}$$

$$\frac{dE_{pci}}{dt} = [(T_{p1i} - T_{p2i})/T^2_{p1i}](E_{f1i} - E_{f2i}) - (1/T_{p1i})E_{pci} \tag{4.137}$$

where

$$E_{fdi} = \begin{cases} E_{c1i} & \text{if } \sigma_{3i} > E_{c1i} \\ E_{c2i} & \sigma_{3i} < E_{c2i} \\ \sigma_{3i} & \text{otherwise} \end{cases}$$

$$E_{pi} = \begin{cases} E_{p1i} & \text{if } \sigma_{4i} > E_{p1i} \\ E_{p2i} & \sigma_{4i} < E_{p2i} \\ \sigma_{4i} & \text{otherwise} \end{cases} \tag{4.138}$$

in which

$$\sigma_{3i} = E_{fdoi} + E_{ei}$$
$$\sigma_{4i} = (T_{p2i}/T_{p1i})(E_{f1i} - E_{f2i}) + E_{pci} \tag{4.139}$$

The ceiling value E_{c1i} has significant influence on the transient stability of the system. The transient stability is improved very much with increase in E_{c1i} in a range 3.0 ∿ 10.0 p.u., but not so much beyond 10.0 p.u. In this section, we set $E_{c1i} = 7.0$ p.u. As in §1, let us make three basic assumptions as follows:

1) Each internal voltage lags behind the q-axis of each generator by a constant angle ϕ_i all the time.
2) The transfer conductances in the reduced admittance matrix are negligible.
3) The magnitude of the ith terminal voltage can be approximately expressed by

$$V_{ti} \simeq E_i + x'_{di}\sum_{j=1}^{n} Y_{ij}E_j\cos(\delta_{ij} + \theta_{ij}) \qquad (4.140)$$

Under these assumptions, (4.130) ∿ (4.137) change to the following equations:

$$m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} = \sum_{j=1}^{n} B_{ij}(E_i^oE_j^o\sin\delta_{ij}^o - E_iE_j\sin\delta_{ij}) \qquad (4.141)$$

$$\begin{aligned}\frac{dE_i}{dt} &= (1/T'_{doi}\cos\phi_i)(E_{fdi} - E_{fdi}^o) \\ &- (1/T'_{doi})[1 - (x_{di} - x'_{di})B_{ii}](E_i - E_i^o) \\ &- (1/T'_{doi})(x_{di} - x'_{di})\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})\end{aligned} \qquad (4.142)$$

$$\begin{aligned}\frac{dE_{ai}}{dt} &= (1/T_{ai})[-(E_{ai} - E_{ai}^o) - K_{ai}(E_{di} - E_{di}^o) + K_{ai}(E_{pi} - E_{pi}^o) \\ &- K_{ai}(1 - x'_{di}B_{ii})(E_i - E_i^o) - K_{ai}x'_{di}\sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij})]\end{aligned} \qquad (4.143)$$

$$\frac{dE_{ei}}{dt} = (1/T_{ei})[K_{ei}(E_{ai} - E_{ai}^o) - (E_{ei} - E_{ei}^o)] \qquad (4.144)$$

$$\frac{dE_{di}}{dt} = (1/T_{ei})[K_{ei}K_{di}(E_{ai} - E_{ai}^o) - K_{di}(E_{ei} - E_{ei}^o) - (T_{ei}/T_{di})(E_{di} - E_{di}^o)] \qquad (4.145)$$

$$\frac{dE_{f1i}}{dt} = -(1/T_{f1i})[(E_{f1i} - E_{f1i}^o) - \sum_{j=1}^{n} B_{ij}(E_i^oE_j^o\sin\delta_{ij}^o - E_iE_j\sin\delta_{ij})] \qquad (4.146)$$

$$\frac{dE_{f2i}}{dt} = -(1/T_{f2i})[(E_{f2i} - E_{f2i}^o) - \sum_{j=1}^{n} B_{ij}(E_i^oE_j^o\sin\delta_{ij}^o - E_iE_j\sin\delta_{ij})] \qquad (4.147)$$

$$\begin{aligned}\frac{dE_{pci}}{dt} &= [(T_{p1i} - T_{p2i})T_{p2i}^2][(E_{f1i} - E_{f1i}^o) - (E_{f2i} - E_{f2i}^o)] \\ &- (1/T_{p1i})(E_{pci} - E_{pci}^o)\end{aligned} \qquad (4.148)$$

for i=1,2,...,n

where the superscript " o " denotes the stable equilibrium point of the post-fault system, and accordingly, (4.141) ∿ (4.148) apply to the post-fault state.

By rewriting (4.141) ∿ (4.148) in the state space notation, and by applying the generalized Popov criterion, the system in (4.141) ∿ (4.148) proves to be stable under some conditions. However, the conditions on automatic voltage regulator gains expressed by (4.41) are included in the conditions, so such high gains used in this section can not be allowed by the generalized Popov criterion, again, and accordingly, a Lur'e type Lyapunov function in (2.21) can not be constructed.

4.2 Pseudo-Lyapunov function

Since it is impossible to construct a Lyapunov function on a basis of the generalized Popov criterion, let us introduce a pseudo-Lyapunov function as follows:

$$\begin{aligned} V(x) &= (1/2\sum_{i=1}^{n} m_i) \sum_{i=1}^{n} \sum_{j=1}^{n} m_i m_j (\omega_i - \omega_j)^2 \\ &+ \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} [E_i E_j (\cos\delta_{ij}^{o} - \cos\delta_{ij}) \\ &\qquad - (\delta_{ij} - \delta_{ij}^{o}) E_i^{o} E_j^{o} \sin\delta_{ij}^{o}] \\ &+ \varepsilon \sum_{i=1}^{n} (\alpha_i/\beta_i)(E_i - E_i^{o})^2 \\ &= V_k(\omega) + V_p(\delta, E) + V_f(E) \end{aligned} \tag{4.149}$$

where V_k, V_p, and V_f denote the first, the second, and the third terms, respectively. The first term represents kinetic energy, and depends on relative rotor angles. The second term represents potential energy which is stored in the system owing to some deviations of rotor angles from those at the stable equilibrium point. The third term represents a magnitude of deviations in internal voltages. The ε is a scalar. If ε is 1.0, then V(x) in (4.149) reduces to that in (3.67) which has been derived for the system in which no voltage regulation is taken into account. For the time being, let us set $\varepsilon = 1$ for brevity's sake. The time derivatives of V_k, V_p, and V_f

are given as follows:

$$\frac{dV_k}{dt} = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) \quad (4.150)$$

$$\frac{dV_p}{dt} = - \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin\delta_{ij}^o - E_i E_j \sin\delta_{ij})(\omega_i - \omega_j) + 2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \quad (4.151)$$

$$\frac{dV_f}{dt} = - 2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) - 2\sum_{i=1}^{n} (1/\beta_i)(dE_i/dt)(dE_i/dt)_f \quad (4.152)$$

where damping torques of generators are assumed to be zero, and $(dE_i/dt)_f$ is defined by

$$(dE_i/dt)_f = - \alpha_i(E_i - E_i^o) - \beta_i \sum_{j=1}^{n} B_{ij}E_j(\cos\delta_{ij}^o - \cos\delta_{ij}) \quad (4.153)$$

The right hand term of (4.150) and the first term of (4.151), the second term of (4.151) and the first term of (4.152) indicate that there are some exchange of energy between V_k and V_p, V_p and V_f, respectively. As a whole, the time derivative of V is given as follows:

$$\frac{dV}{dt} = - 2\sum_{i=1}^{n} (1/\beta_i)(dE_i/dt)(dE_i/dt)_f \quad (4.154)$$

If there is no voltage regulation, (4.142) reduces to (4.153), and the right hand term of (4.154) becomes negative semi-definite. However, this term is usually not negative semi-definite but non-sign definite. Hence, V(x) in (4.149) is not Lyapunov function. This fact has no significant influence in applying V(x) in (4.149) to Lyapunov's direct method as will get clear in the followings.

4.3 Method of determining critical value

Since $V_p(\delta, E)$ in (4.149) is the same as one in (4.84), the discussions made in §3.2 holds as it is for the system described by (4.141) ∿ (4.148). We define the transient stability region in the same way as in the section. Namely, in Fig.55, the transient stability region is defined by the region which is bounded by the curves O_1, O_2, O_3, and O_4. In this region, the torque f_r defined by (4.100) acts in such way that it will confine the system in the region. The transient stability region varies with internal voltages E as illustrated by Fig.56. It gets wider and deeper with increases in the internal voltages.

Similarly, the discussions made in §3.4 hold as it is in this study, too. Let us follow them in order to clarify the base on which we depend in using V(x) in (4.149). The time derivatives of V_k and V_p in (4.150) and (4.151) are rewritten as follows:

$$\frac{dV_k}{dt} = -\frac{\partial V_p}{\partial \delta_r}\omega_r \qquad (4.155)$$

$$\frac{dV_p}{dt} = \frac{\partial V_p}{\partial \delta_r}\omega_r + \frac{\partial V_p}{\partial E}\frac{dE}{dt} \qquad (4.156)$$

From these equations, the increments of V_k and V_p in a time interval are given as follows:

$$\begin{aligned}\Delta V_k &= \int_{t_s}^{t_e} \frac{dV_k}{dt}\,dt \\ &= -\Delta V_{k\to p}(\delta_s \sim \delta_e,\ E_s \sim E_e)\end{aligned} \qquad (4.157)$$

$$\begin{aligned}\Delta V_p &= \int_{t_s}^{t_e} \frac{dV_p}{dt}\,dt \\ &= \Delta V_{k\to p}(\delta_s \sim \delta_e,\ E_s \sim E_e) - \Delta V_{p\to f}(\delta_s \sim \delta_e,\ E_s \sim E_e)\end{aligned} \qquad (4.158)$$

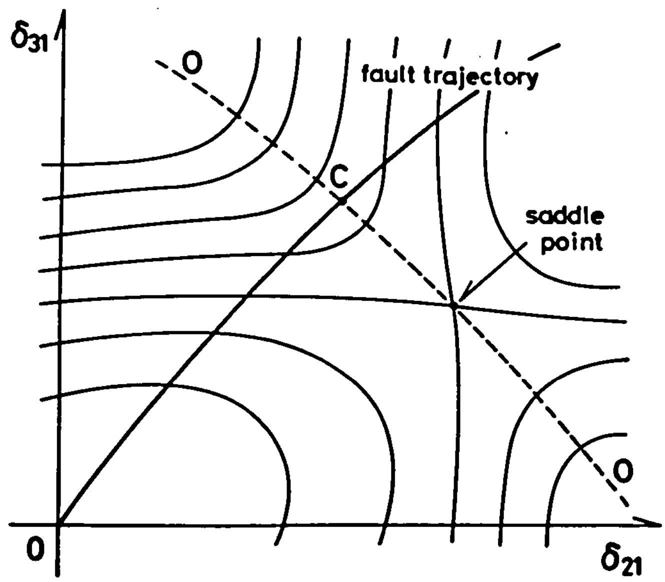


Fig.66. Trajectory in relative angular space: sustained fault.

where

$$\Delta V_{k\to p}(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv \int_{\delta_s}^{\delta_e} \frac{\partial V_p}{\partial \delta_r} d\delta_r \ , \qquad (4.159)$$

$$\Delta V_{p\to f}(\delta_s \sim \delta_e,\ E_s \sim E_e) \equiv -\int_{E_s}^{E_e} \frac{\partial V_p}{\partial E} dE \ , \qquad (4.160)$$

t_s and t_e are both ends of the time interval, and δ_s, δ_e, E_s, and E_e are the values of δ_r and E at the instants t_s and t_e, respectively. The term $\Delta V_{k\to p}$ denotes the energy which is transfered from V_k to V_p, and the term $\Delta V_{p\to f}$ denotes the energy which is transfered from V_p to V_f in the time interval. These terms are determined by the trajectory of the system in the $(\delta_r$, E)-space, and it does not matter how long does it takes the system to move from (δ_s, E_s) to (δ_e, E_e).

Fig.66 shows an example of system trajectory in a relative angular space for a case where a fault continues. The system crosses the boundary of the transient stability region at the point c. Over this point, the torque f_r which is related with V_p by (4.99) acts in such way taht it will accelerate the system, and will separate it from the stability region. In order that the system stays in synchronism during the first swing, the fault must be cleared in such time that the system can not go over the point c. We make here two assumptions as follows:

1) The system moves along the sustained fault trajectory even in the period after the fault clearance.
2) The internal voltage vary in the same way as for the sustained fault case even in the period after the fault clearance.

The former assumption is satisfied to some extent in large power systems. On the other hand, the latter assumption is not satisfied so well as the former one, particularly, in the cases where the excitation voltages are intensively controlled through such high gain automatic voltage regulators as in this study, which brings some uncertainty to a stability condition derived under the above assumptions. Let us consider a case where a given fault is cleared at an instant. If the fault is cleared at an instant later than the critical clearing time, the system will move along the sustained fault trajectory, and go over the point c. Conversely, if the fault is cleared at an instant earlier than the critical clearing time, then the system will move along the trajectory, but will not reach the point c. Now let us consider the case where the fault is cleared just at the critical clearing time. The system will move along the trajectory, reach the point c, and stop there. In this case, the following two relations hold:

$$
\begin{aligned}
V_k(t_c) &= V_k(t_{cr}) - \Delta V_{k\to p}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \\
&= 0
\end{aligned}
\tag{4.161}
$$

$$
\begin{aligned}
V_p(t_c) &= V_p(t_{cr}) + \Delta V_{k\to p}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \\
&\quad - \Delta V_{p\to f}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c)
\end{aligned}
\tag{4.162}
$$

where (δ_{cr}, E_{cr}) and (δ_c, E_c) are the values of (δ_r, E) at the critical clearing time t_{cr} and the time t_c when the system reaches the point c, respectively. From these equations, we obtain

$$
V_k(t_{cr}) + V_p(t_{cr}) = V_{cr} + \Delta V_{p\to f}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \tag{4.163}
$$

where

$$
\begin{aligned}
V_{cr} &\equiv V_p(t_c) \\
&= V_p(\delta_c,\ E_c)
\end{aligned}
\tag{4.164}
$$

We assume here that

$$
\Delta V_{p\to f}(\delta_{cr} \sim \delta_c,\ E_{cr} \sim E_c) \geq 0 \tag{4.165}
$$

holds. This inequality implies that some energy is dissipated from the sum $[V_k + V_p]$ after the clearance of the fault, and that $[V_k + V_p]$ can be treated as Lyapunov function during the first swing. From (4.163) and (4.165), we obtain a inequality as follows:

$$
V_k(t_{cr}) + V_p(t_{cr}) \geq V_{cr} \tag{4.166}
$$

Since $[V_k + V_p]$ monotonously increases during the fault-on period, and the inequality in (4.166) holds in the case where the fault is critically cleared, the system reamins in synchronism for the first swing if the fault is cleared in such time that

$$V_k(t) + V_p(t) < V_{cr} \tag{4.167}$$

holds. On this basis, we get a stability condition as follows:

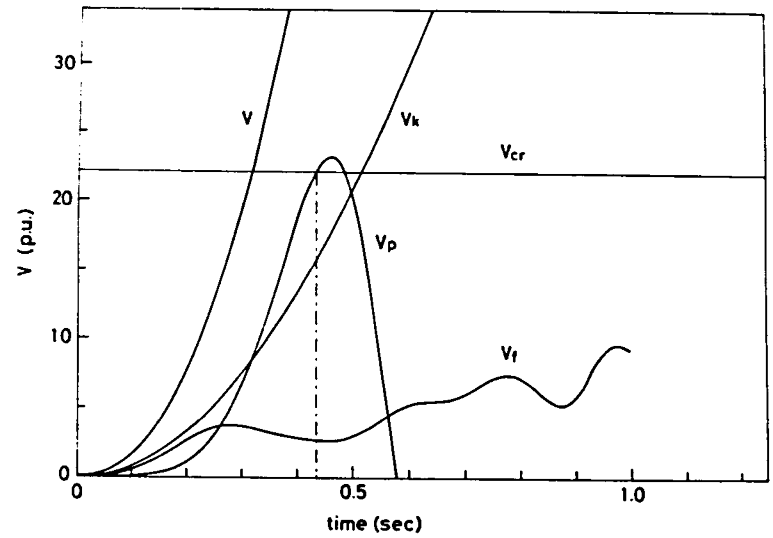
[Stability condition 9]

If a system satisfies

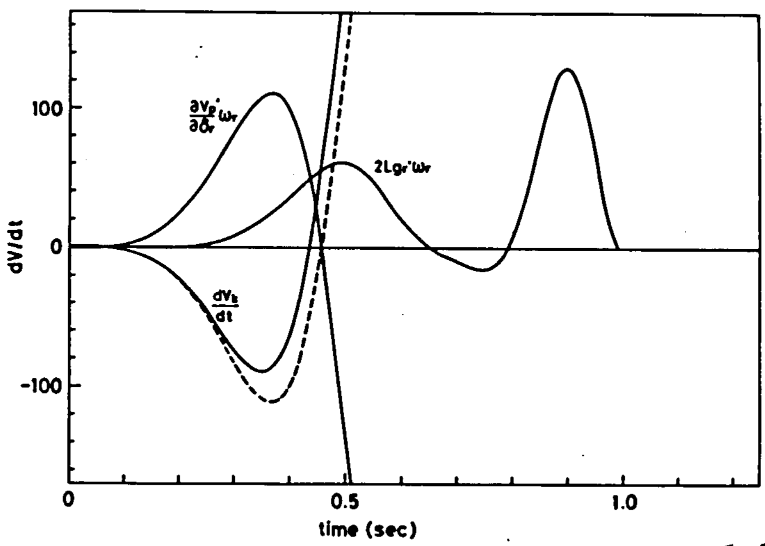
$$V < V_{cr} \tag{4.168}$$

then, it is stable for the first swing, where V_{cr} is the value of $V_p(\delta_r, E)$ at the point c where the sustained fault trajectory crosses the boundary of the transient stability region.

The inequality in (4.168) is more strict than that in (4.167) by εV_f which can be adjusted by varying ε. The second assumption made on the system behavior is not satisfied so well as the first one. Namely, there is some difference of variation in the internal voltages between the case where the fault is sustained and the case where the fault is critically cleared. The internal voltages are not so much enhanced in the latter case as in the former case, so the critical value defined by (4.164) gives somewhat optimistic results compared with actual transient stabilities. The term εV_f has been introduced in order to cancel a part of critical value V_{cr} which is brought in excess owing to the above fact. We now have no systematic method of determining an appropriate value of ε at hand, yet. Some systematic method should be developed, but it is left as a future problem. In this study, we set $\varepsilon = 1$ for trial, and apply the stability condition 9 to some transient stability studies. The critical value V_{cr} can be approximately given by the value of V_p at the instant when the time derivative of V_k changes its sign from negative to positive under a sustained fault. As an example, let us take a case where a 3-phase short-circuit occurs at a point near the bus 11 in the 10-machine power system shown by Fig.59, where all generators are installed with the excitation system in Fig.64. The fault is



(a) V and its components



(b) Time derivative of V_k

Fig.67. Method of determining critical value of $\dot{V}$ in case where voltage regulator gains are high.

cleared by opening the line connecting the buses 11 and 12 at both terminals. Fig. 67 shows the time variations of V, its components, and (dV_k/dt) for the case where the fault lasts, where V has been modified in order to take account of the transfer conductances as follows:

$$V(t) = V_p(t) + \gamma V_k(t) + \varepsilon V_f(t) \qquad (4.169)$$

in which γ has been defined by (3.146) and (3.155), and is equal to 1.222 in this case. As shown in Fig. 67(b), (dV_k/dt) changes its sign from negative to positive at 0.43 sec. This instant coincides with that when the system crosses the boundary of the transient stability region. The critical value V_{cr} is given by the value of V_p at the instant, and is equal to 22.101 in this case. V defined by (4.169) takes 20.921 at 0.31 sec, and 22.540 at 0.32 sec, so the critical clearing time is estimated to be 0.317 sec. In Fig. 68, the time variations of relative rotor angles are shown for two cases where the fault is cleared at 0.32 sec and 0.33 sec, respectively. Since the system keeps synchronism in the former case while it loses synchronism in the latter case, the actual critical clearing time exists between 0.32 sec and 0.33 sec. Hence the above estimation proves to be adequately accurate. In the next section, we apply the stability condition 9 to transient stability studies for a range of faults which occur in the 10-machine power system, and investigate its applicability.

4.4 Conclusions

In this section, we have tried one generalization of Lyapunov's direct method to power systems in which all generators are installed with high gain automatic voltage regulators. Since it is impossible to construct a Lyapunov function on the basis of the generalized Popov criterion, we have introduced a pseudo-Lyapunov function V. This function is characterized as follows:

1) It consists of three terms, that is, kinetic energy V_k, potential energy V_p, and a new term εV_f.
2) The kinetic energy V_k and the potential energy V_p are the same as those for the power system in which all generators are installed with low gain voltage regulators.
3) The new term εV_f is a function of field flux linkages only.

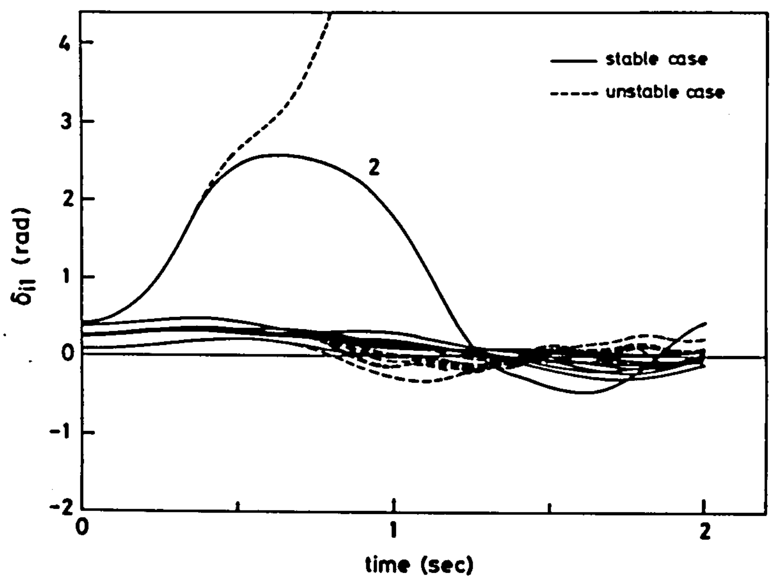


Fig.68. Swing curves of generators for case where voltage regulator gains are high: fault 11-12
——— cleared at 0.32 sec,
- - - - - cleared at 0.33 sec.

4) The time derivative of V is given by (4.154), which is not sign-definite.

5) Hence V is not a Lyapunov function for the system under investigation.

In spite of these facts, it is possible to use this function in judging the first swing stability of the system if

1) The system moves along the sustained fault trajectory even in the period after the fault clearance,

2) The internal voltages vary in the same way as for the sustained fault case even in the period after the fault clearance,

3) The inequality in (4.165) holds,

are satisfied. The last two assumptions are not satisfied so much as the first one, however. Bearing this fact in mind, we have derived the stability condition 9. V in (4.168) can be adjusted by varying ε in order to cancel some uncertainty of the above three assumptions. The critical value V_{cr} can be obtained by taking the value of V_p at the instant when the time derivative of V_k in (4.150) changes its sign from negative to positive under a sustained fault. We have not got an appropriate method of determining ε, yet, so some method should be developed in the future. In the next section, we set $\varepsilon = 1$ for trial, and apply the stability condition 9 to a transient stability study of a 10-machine power system.

§5. Numerical example

In this section, the transient stability of a 10-machine power system is studied. The line diagram of the system is shown in Fig.69, and the data on its generators are provided in Table 18. It is assume that this system is disturbed by a 3-phase short-circuit which occrus at a terminal x of a transmission line x-y, and is cleared by opening the line at both the terminals. Two different method, i.e., the conventional method based on simulations and Lyapunov's direct method, are used to compute the critical fault clearing time. This familiar transient stability measure is used as a basis of comparison. Two sets of automatic voltage regulator gains, refered as Case 1 and Case 2, are used in order to make their influence on the transient stability clear. All the automatic voltage regulator gains are set 1.0 and 100.0 in Case 1 and Case 2, respectively, where power system stabilizing signals are added to all the excitation systems in the latter case in order to keep the system dynamically stable.

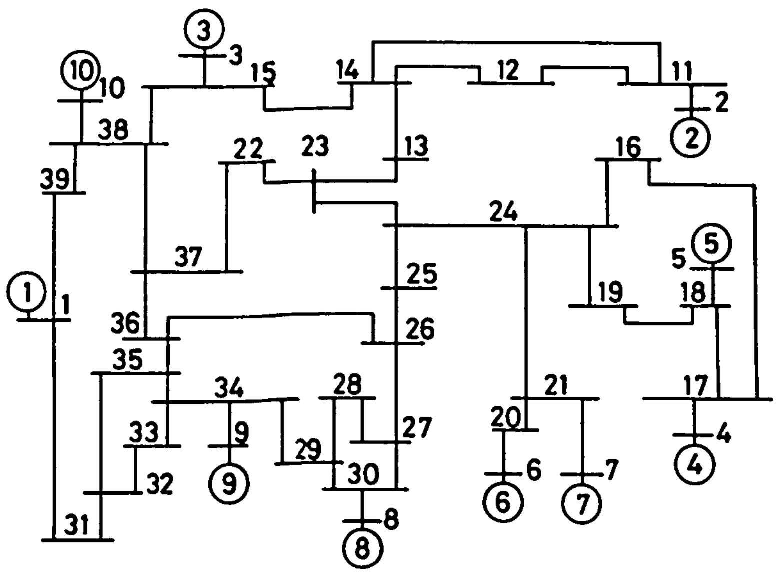


Fig.69. Configuration of 10-machine system.

Table 18. Generator parameters of 10-machine system.

Unit	H	x_d	x_d'	x_q	T_{do}'
1	500.0	0.2200	0.0060	0.0190	7.00
2	34.5	0.2106	0.0570	0.2050	4.79
3	24.3	0.2900	0.0570	0.2800	6.70
4	26.4	0.2950	0.0490	0.2920	5.66
5	34.8	0.2540	0.0500	0.2410	7.30
6	26.0	0.6700	0.1320	0.6200	5.40
7	28.6	0.2620	0.0436	0.2580	5.69
8	35.8	0.2495	0.0531	0.2370	5.70
9	30.3	0.2950	0.0697	0.2820	6.56
10	42.0	0.1200	0.0310	0.0690	10.20

5.1 Procedure of estimation

The procedure for estimating the critical clearing time is shown in the flow chart of Fig.70. The main steps are as follows:

[Step 1] Read the necessary data on the system, i.e., those on transmission lines, buses, and generators.

[Step 2] Compute the load flow for the prefault system.

[Step 3] Compute the reduced admittance matrices between the generators by eliminating the buses without generators, for the fault and the post-fault system.

[Step 4] Compute the stable equilibrium point for the post-fault system.

[Step 5] Integrate the fault system equations step by step, and compute the rotor angles, the rotor speeds, the internal voltages, and the excitation system state variables: $\delta(t)$, $\omega(t)$, $E(t)$, $E_a(t)$, $E_d(t)$, $E_e(t)$.

[Step 6] Examine whether the system has or not reached the boundary of the transient stability region by checking the sign of the time derivative of kinetic energy V_k. If the time derivative changes its sign from negative to positive, then go to Step 7, and if not, return to Step 5.

[Step 7] Compute the critical value V_{cr}, which is the value of the potential energy V_p at the instant when the time derivative (dV_k/dt) changes its sign.

[Step 8] Compute the value of the Lyapunov function V(t).

[Step 9] Compare V(t) and V_{cr}. If V(t) is greater than V_{cr}, go to Step 10, and if not, then return to Step 8.

[Step 10] Print the critical clearing time.

We can get an estimation of critical clearing time for a fault through these steps. The first three steps need no explanation. The stable equilibrium point of the post-fault system is necessary in calculating the value of V. It is computed in Step 4 by solving the following equations:

$$P_{mi} - \sum_{j=1}^{n} Y_{ij}E_iE_j\sin(\delta_{ij} + \theta_{ij}) = C \text{ (constant)} \tag{4.170}$$

$$E_{fdi} - E'_{qi} - (x_{di} - x'_{di})i_{di} = 0 \tag{4.171}$$

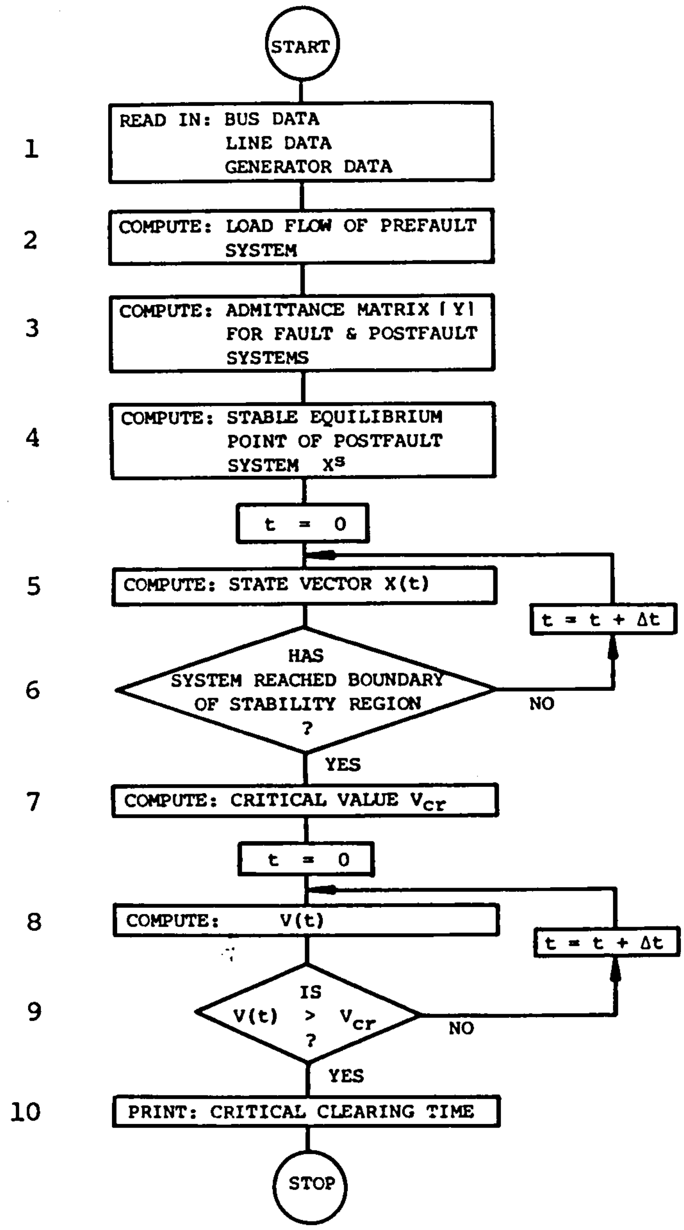


Fig.70. Flow chart for estimating critical fault clearing time by Lyapunov's direct method.

where

$$E_{fdi} = E^{o}_{fdi} + K_{ai}K_{ei}(V_{refi} - V_{ti})$$

for i=1,2,...,n

These equations are nonlinear, and can be solved with Newton-Raphson method iteratively (Appendix C). Four or five iterations can yield results of good accuracy. In Step 5, the system equations are numerically integrated with Runge-Kutta-Gill(R.K.G.) method, where the integration step length Δt is set 0.001 sec because each excitation system includes some elements with short time constants such as 0.02 and 0.04 sec. Eqs. (4.1) ∿ (4.5) and (4.130) ∿ (4.137) are used as system equations in Case 1 and Case 2, respectively. Steps 6 and 7 serve to determine the critical value V_{cr}. Step 8 serve to compute a Lyapunov function, where V in (4.129) and (4.169) are adopted as Lyapunov function in Case 1 and Case 2, respectively.

5.2 Results by simulations

Firstly, the conventional approach based on simulations are applied to the transient stability analysis of the 10-machine power system. The system equations (4.1) ∿ (4.5) are integrated step by step in Case 1, and (4.130) ∿ (4.137) are in Case 2 in order to yield the time variations of the internal voltages, where damping torques of generators are assumed to be zero in this study. By observing these time variations we judge whether or not the system is stable for a given fault. If the system is stable for a clearing time, then it is delayed, and if not, it is advanced. By iterating these manipulations, the critical fault clearing time is obtained. It usuallly takes 4 or more times for the iteration to converge.

Fig.71 and 72 show the time variations of the system variables for two cases of automatic voltage regulator gains. In each figure, results for 8 different fault locations are shown, where each fault has been cleared at such an instant that if it is delayed by 0.01 sec, then the system gets unstable. From these figure, we can observe several features as follows:

1) System variables vary conspicuously in generators near each fault location: for example, for fault 11-12,
   a) Rotor angle δ_2 swings very much with respect to other generators;
   b) Internal voltage E_2 deviates much compared with other generators;

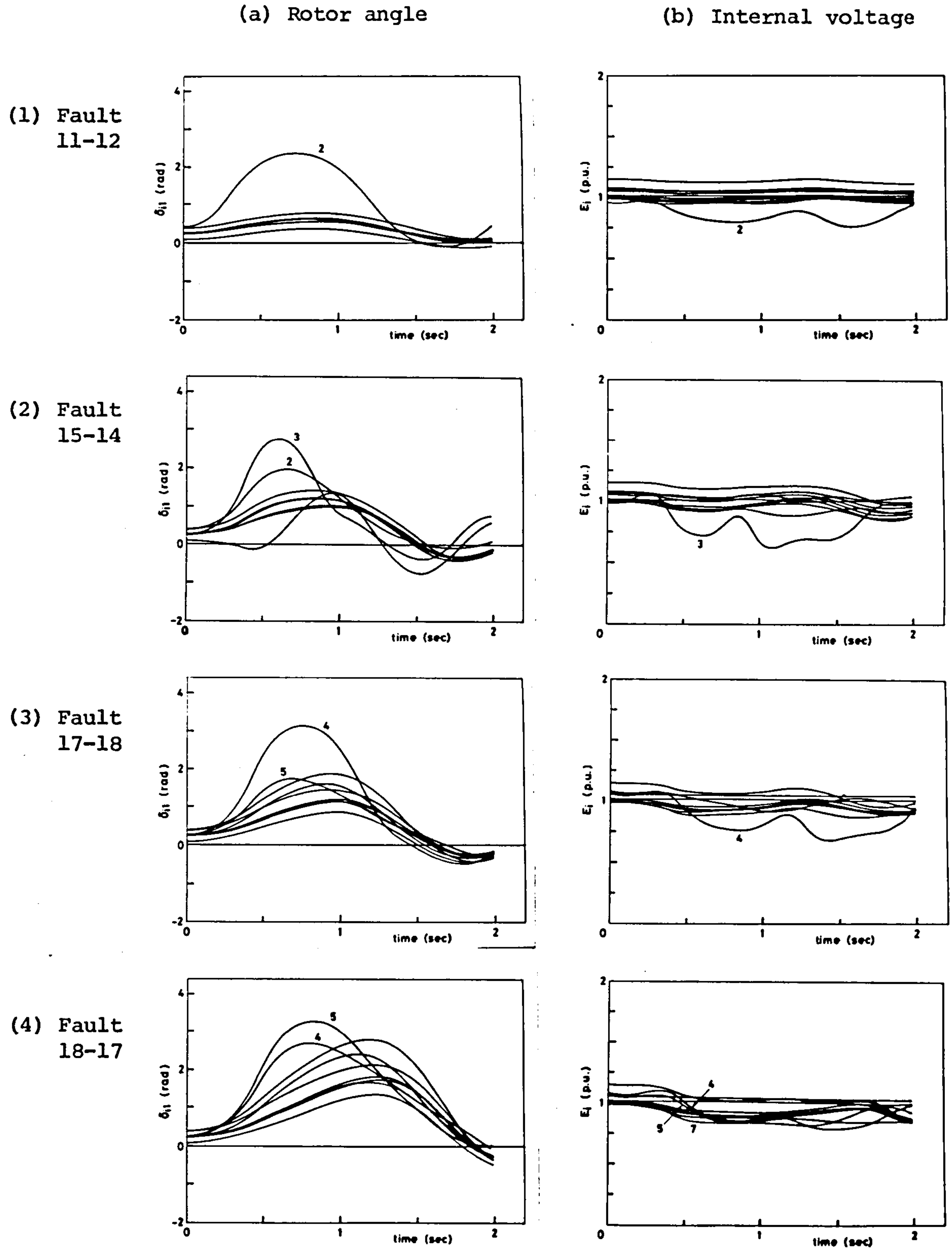


Fig.71 Time variations of generator variables for eight cases of fault location: $K_{ai} = 1.0$

(c) Terminal voltage

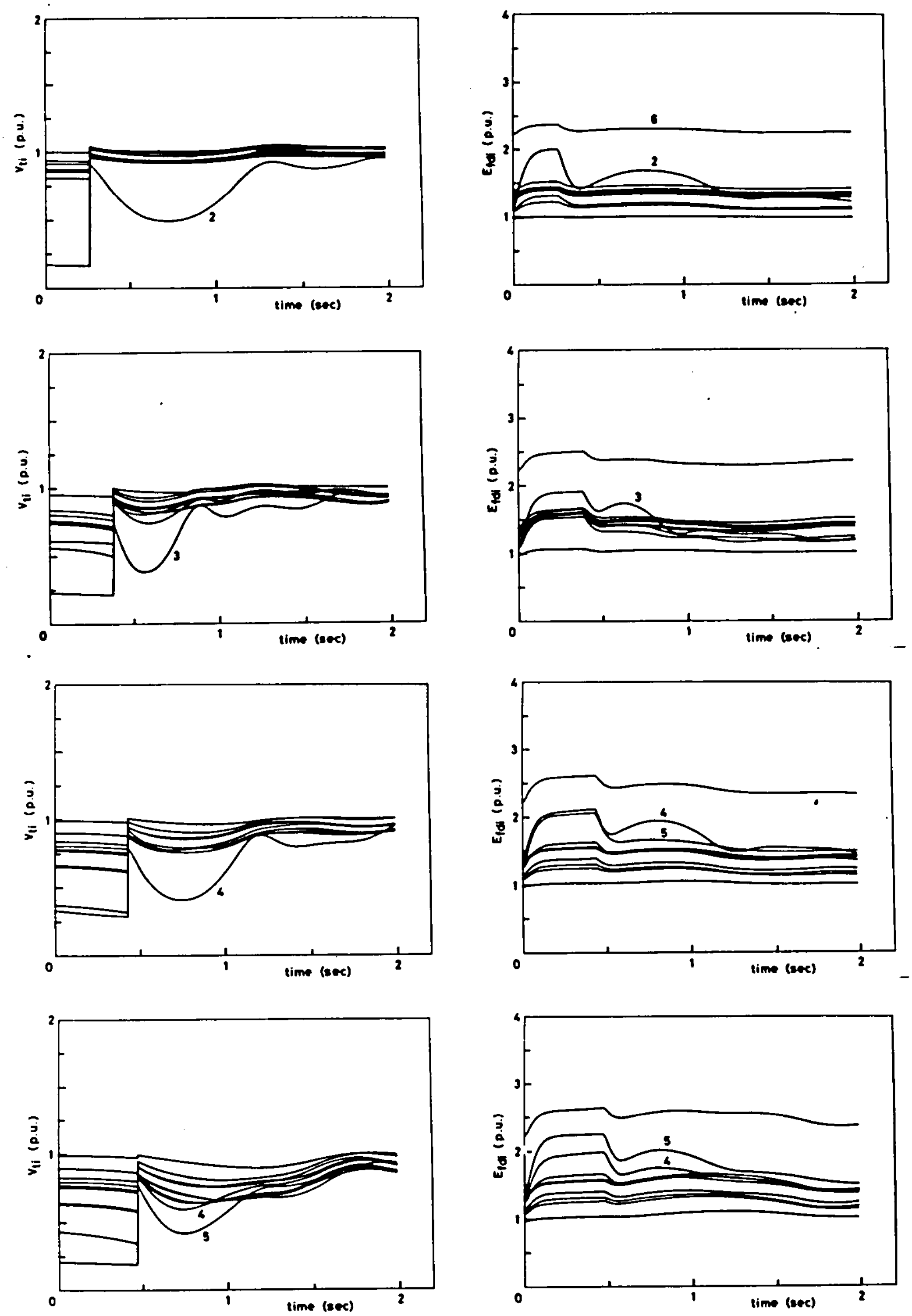
(d) Excitation voltage

(1) Fault 11-12

(2) Fault 15-14

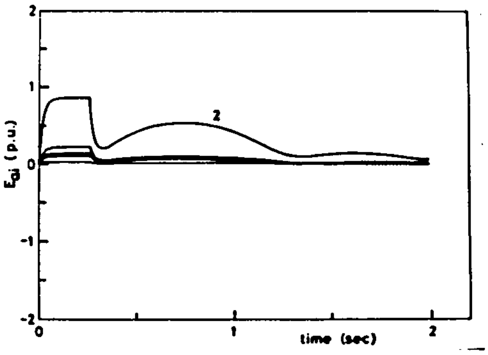
(3) Fault 17-18

(4) Fault 18-17

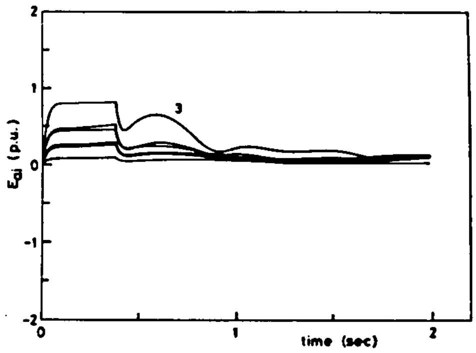


(e) AVR output

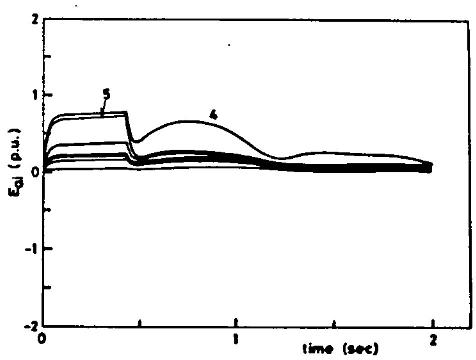
(1) Fault 11-12



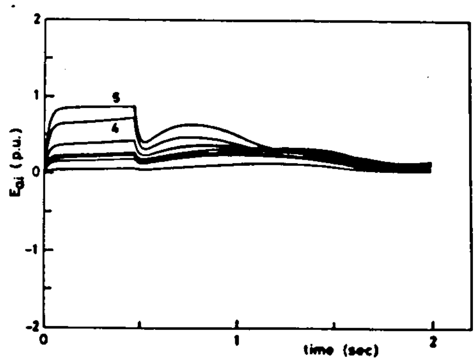
(2) Fault 15-14



(3) Fault 17-18

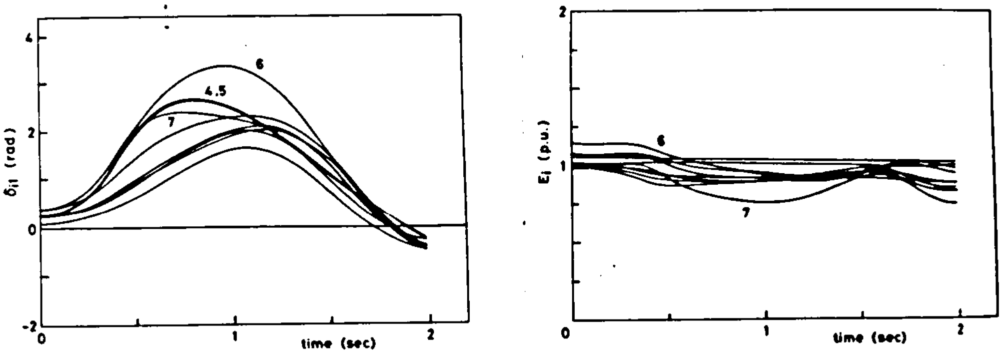


(4) Fault 18-17

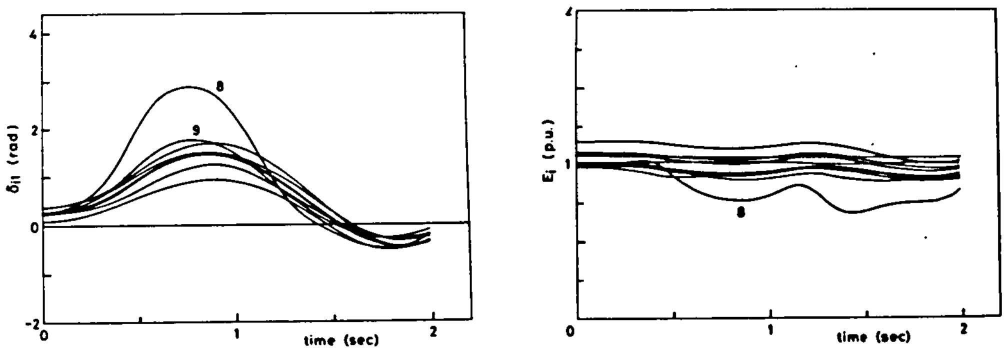


(a) Rotor angle (b) Internal voltage

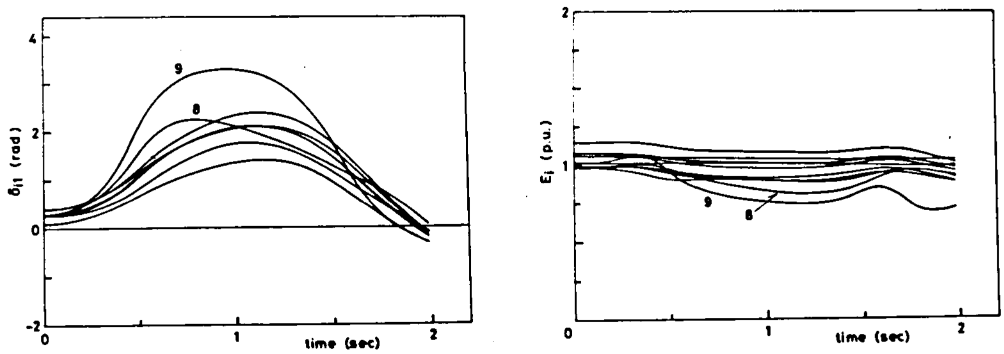
(5) Fault 24-16



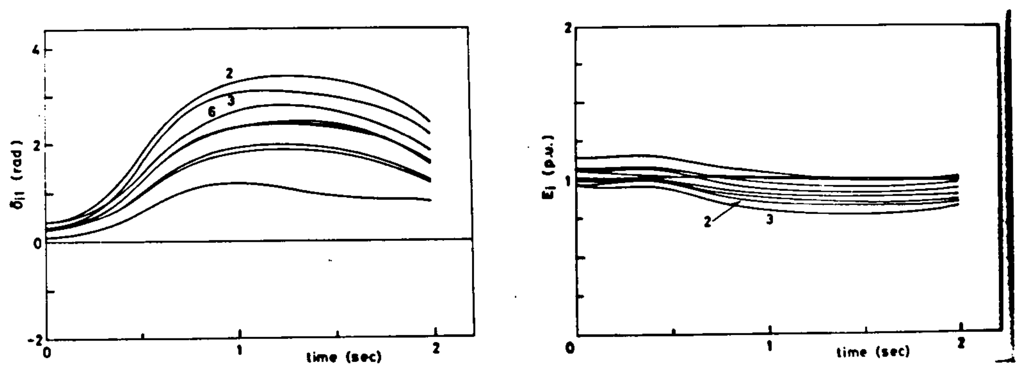
(6) Fault 30-27



(7) Fault 34-29



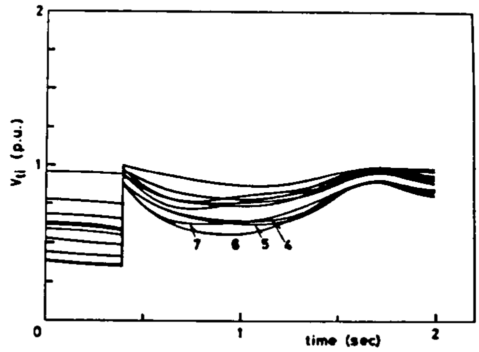
(8) Fault 38-15

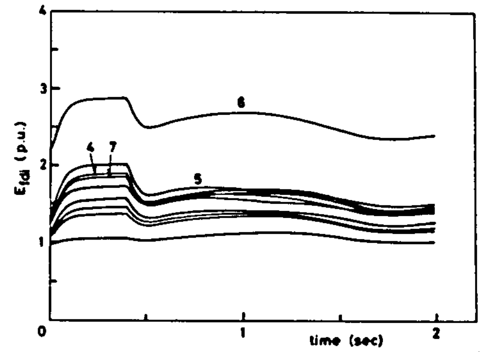


(c) Terminal voltage

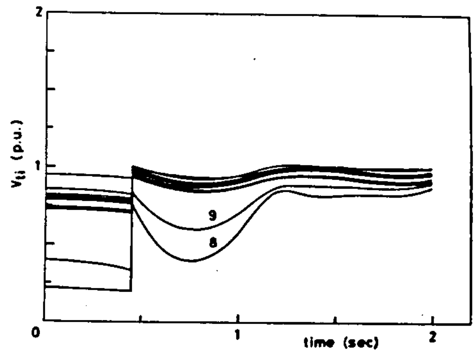
(d) Excitation voltage

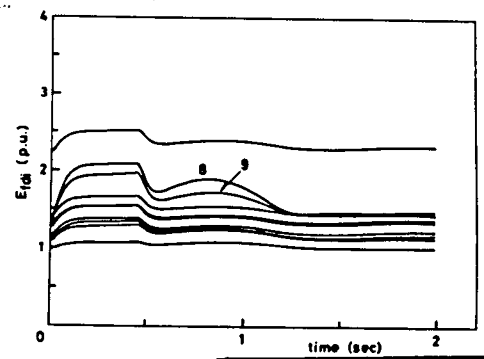
(5) Fault 24-16



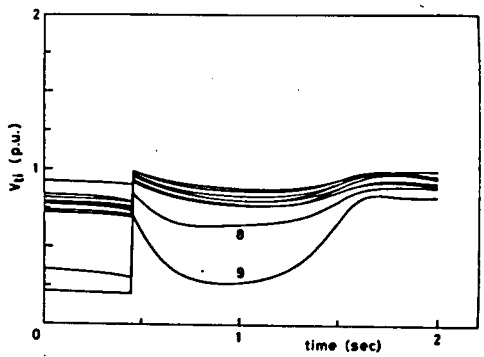


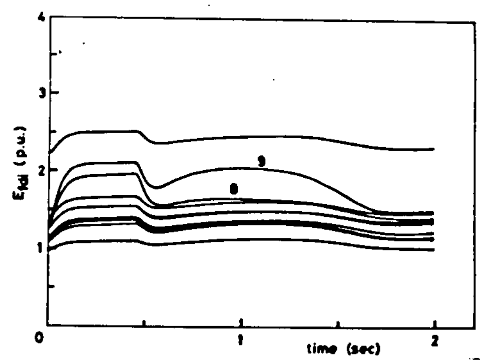
(6) Fault 30-27



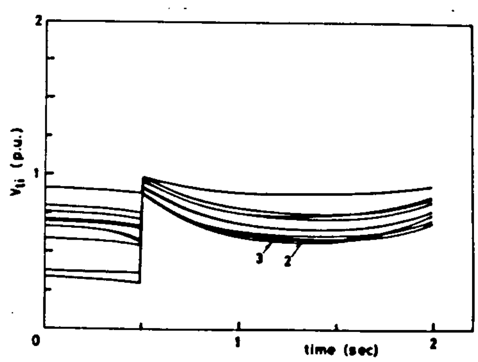


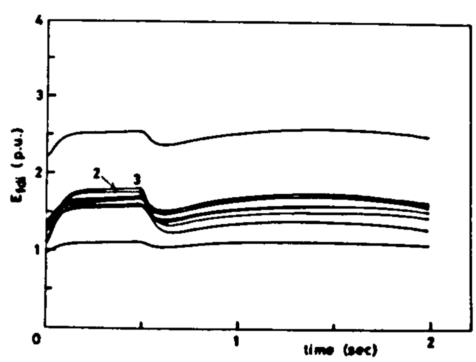
(7) Fault 34-29





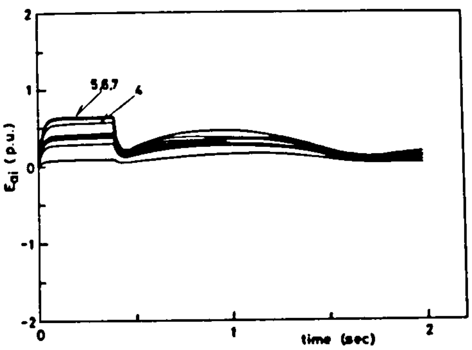
(8) Fault 38-15



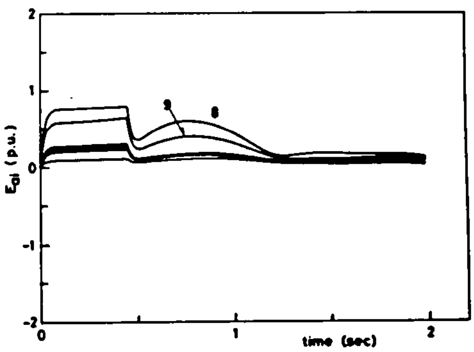


(e) AVR output

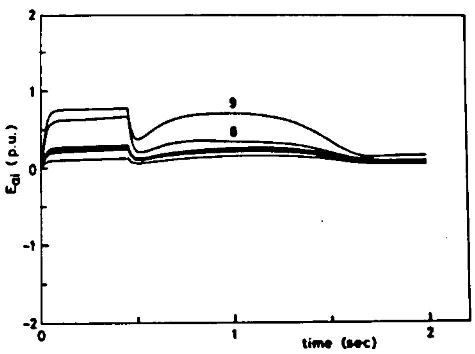
(5) Fault 24-16



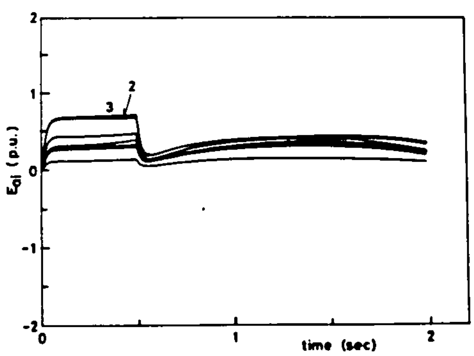
(6) Fault 30-27



(7) Fault 34-29



(8) Fault 38-15



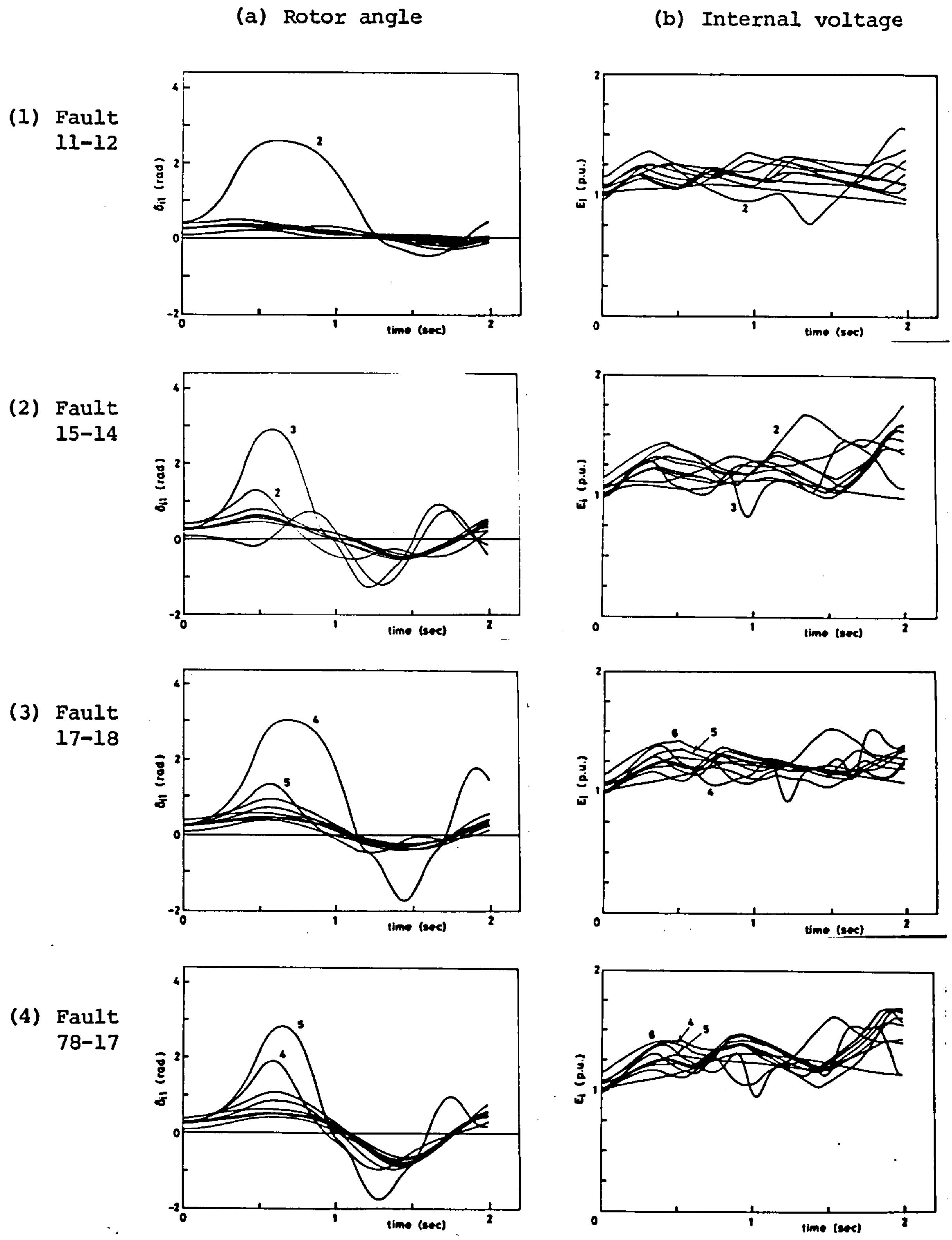


Fig.72 Time variations of generator variables for eight cases of fault locations: K_{ai} = 100

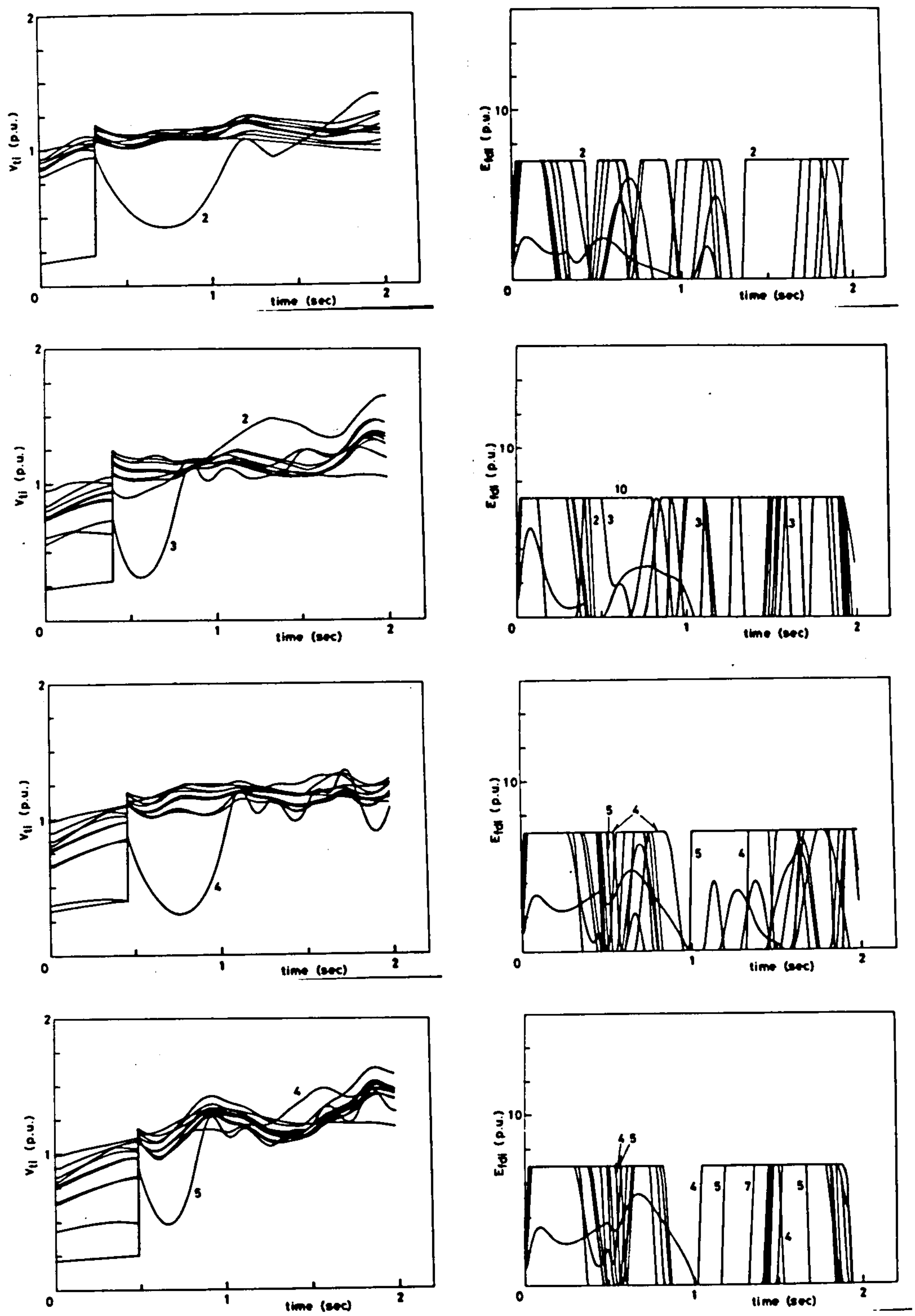
(c) Terminal voltage | (d) Excitation voltage

(1) Fault 11-12

(2) Fault 15-14

(3) Fault 17-18

(4) Fault 18-17



(e) AVR output

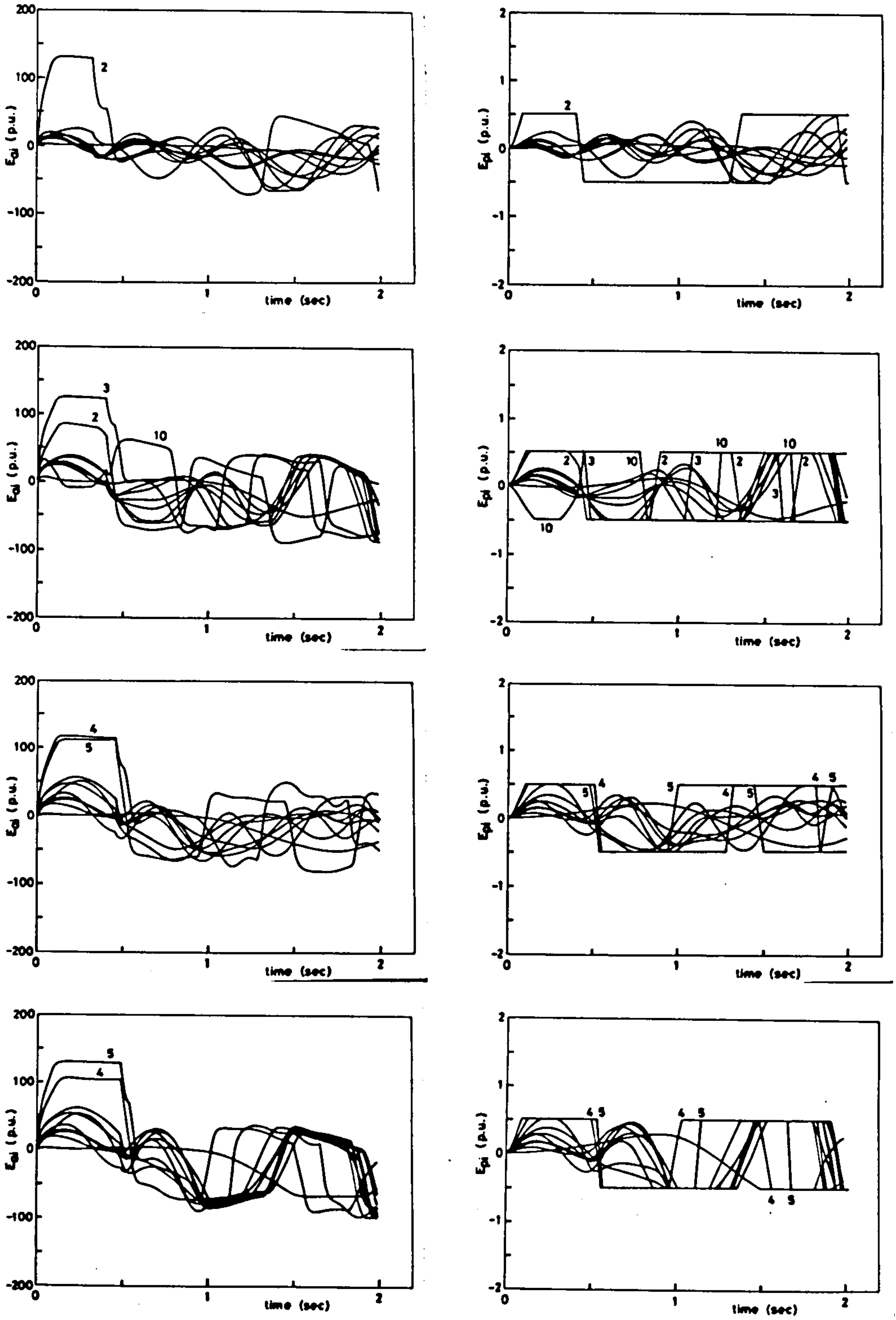
(f) PSS output

(1) Fault 11-12

(2) Fault 15-14

(3) Fault 17-18

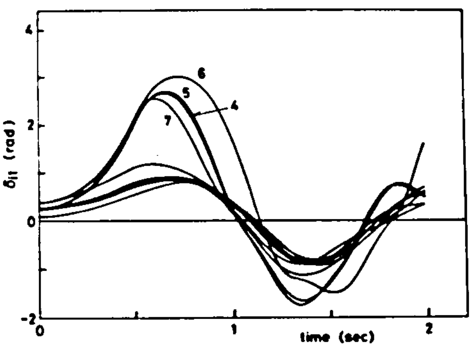
(4) Fault 18-17

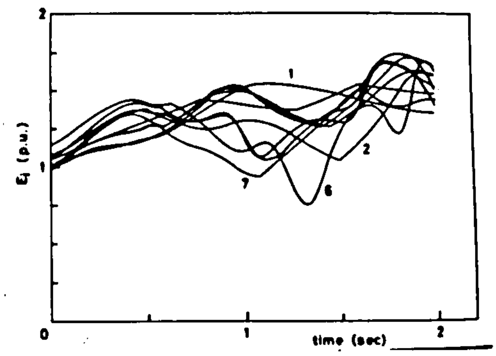


(a) Rotor angle

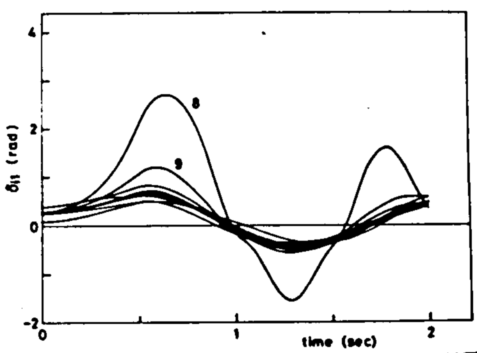
(b) Internal voltage

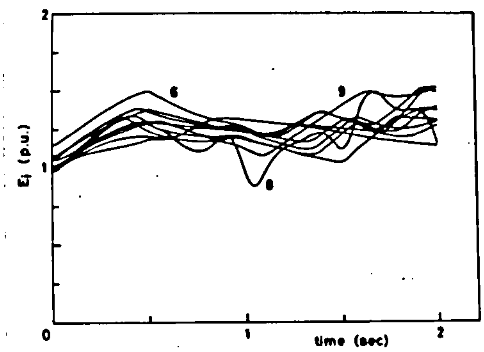
(5) Fault 24-16



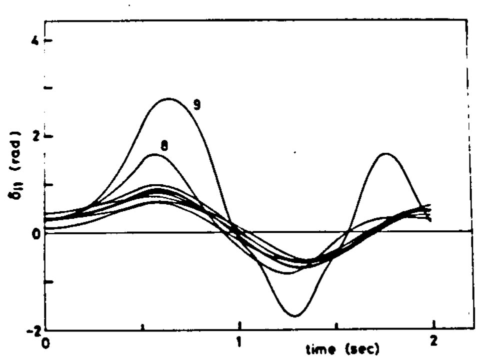


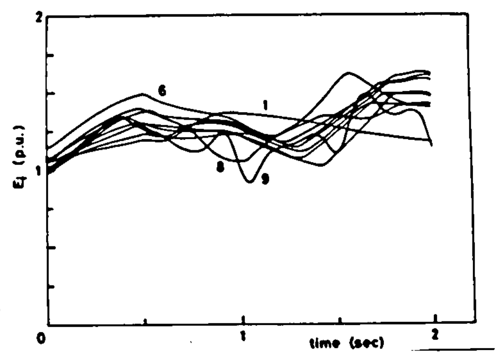
(6) Fault 30-27



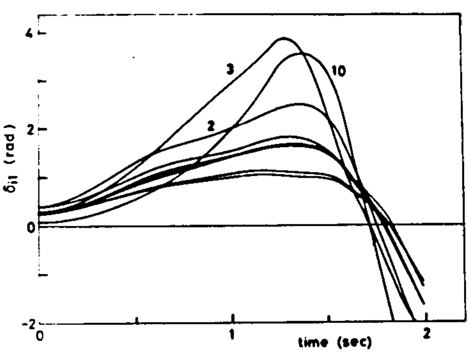


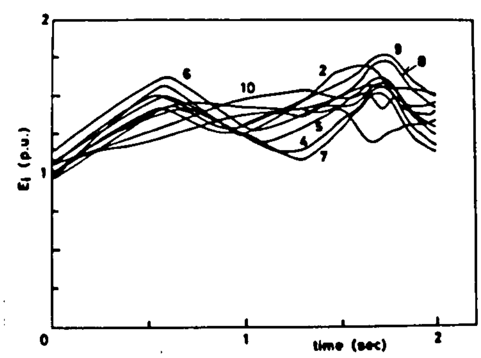
(7) Fault 34-29





(8) Fault 38-15

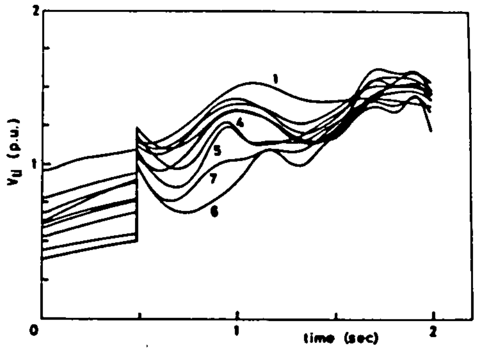


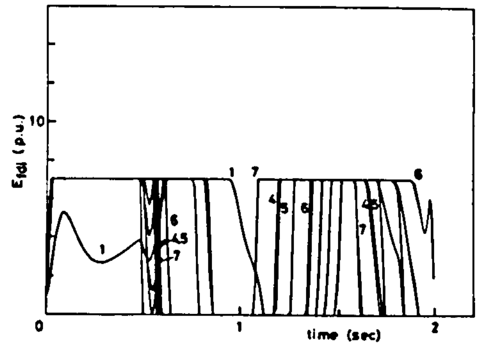


(c) Terminal voltage

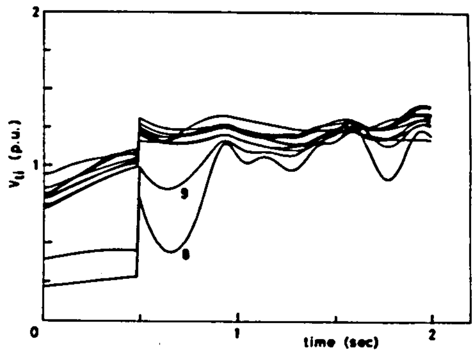
(d) Excitation voltage

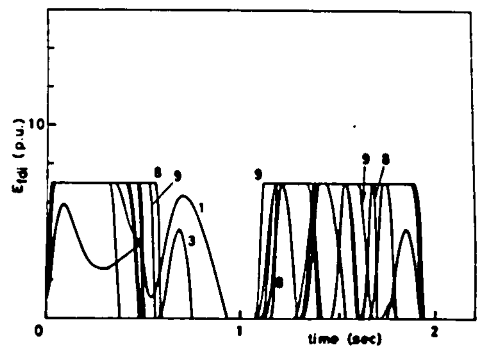
(5) Fault 24-16



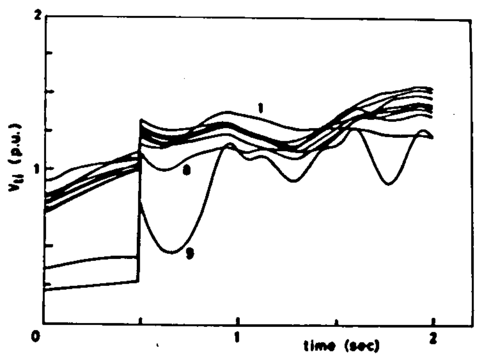


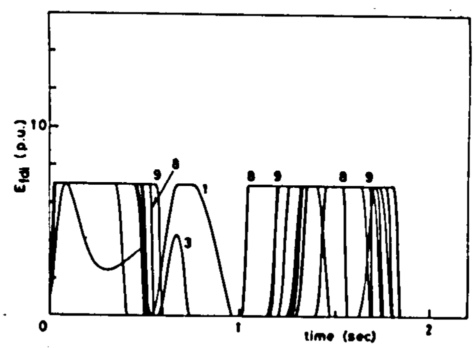
(6) Fault 30-27



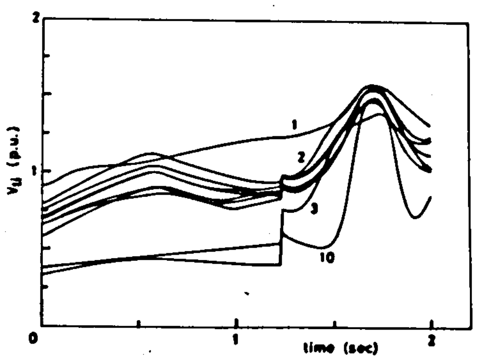


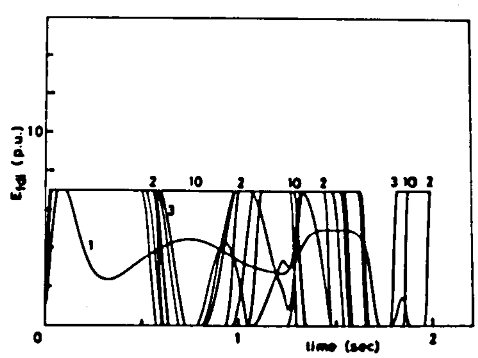
(7) Fault 34-29





(8) Fault 38-15

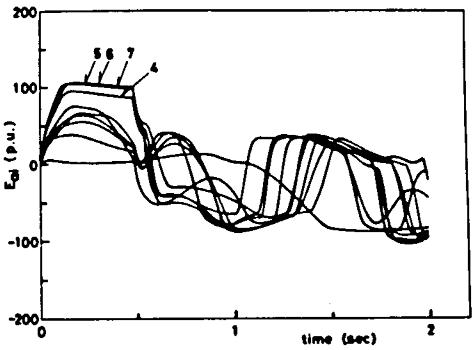


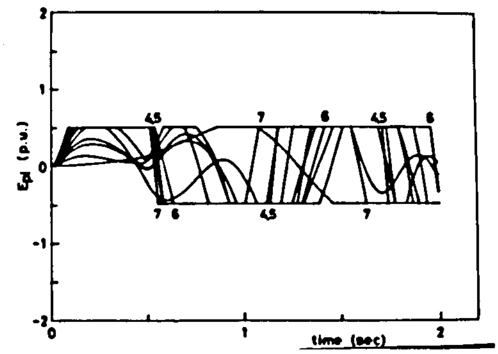


(e) AVR output

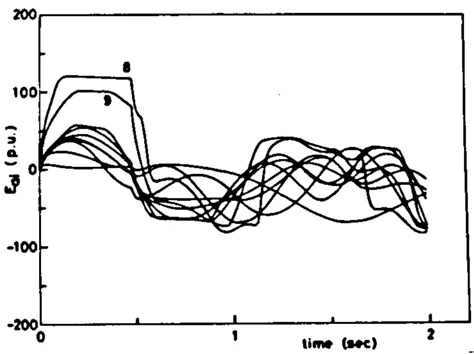
(f) PSS output

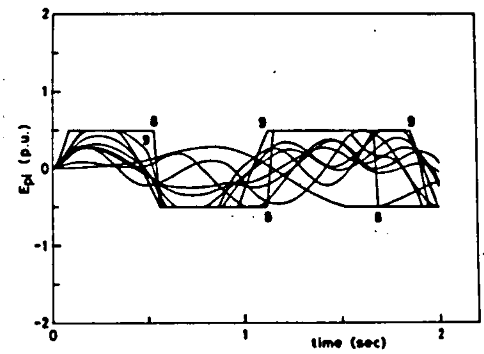
(5) Fault
24-16



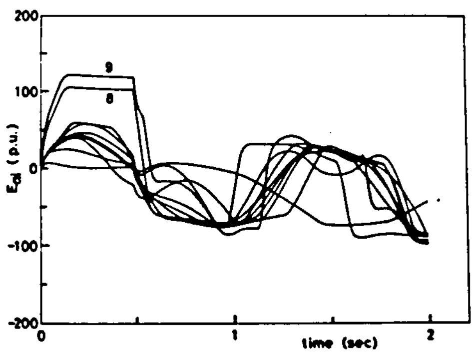


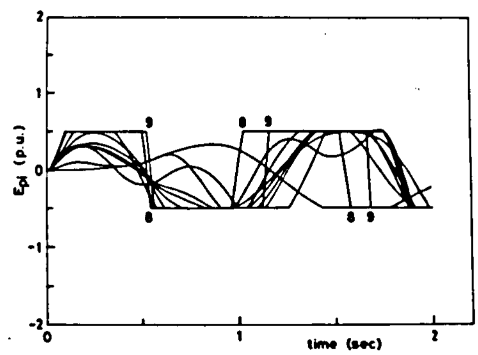
(6) Fault
30-27



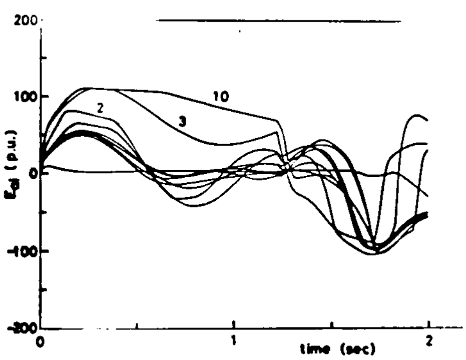


(7) Fault
34-29





(8) Fault
38-15



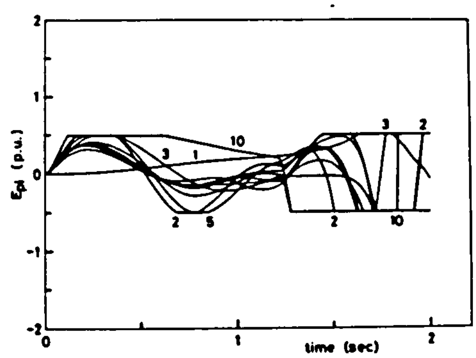


Table 19. Critical fault clearing times obtained by simulations.

(a) K_{ai}: 1.0

Fault	Clearing times (sec) stable	unstable	Unstable generators
11 - 12	0.26	0.27	2
15 - 14	0.38	0.39	3
17 - 18	0.43	0.44	4
18 - 17	0.47	0.48	5
24 - 16	0.39	0.40	1
30 - 27	0.45	0.46	8
34 - 29	0.45	0.46	9
38 - 15	0.49	0.50	1

(b) K_{ai}: 100.0

Fault	Clearing times (sec) stable	unstable	Unstable generators
11 - 12	0.32	0.33	2
15 - 14	0.40	0.41	3
17 - 18	0.47	0.48	4
18 - 17	0.49	0.50	5
24 - 16	0.48	0.49	1
30 - 27	0.48	0.49	8
34 - 29	0.48	0.49	9
38 - 15	1.22	1.23	1

c) Terminal voltage V_{t2} deviates very much while those of other generators are kept almost constant;

d) Similarly, excitation system variables E_{a2}, E_{fd2}, and E_{p2} deviate very much compared with those of other generators.

2) Step-out generators vary with fault location although those do not vary so much between Case 1 and Case 2, for example, no.2 generator steps out for fault 11-12 while no.3 generator steps out for fault 15-14.

3) There are some differences of time variations in internal voltages, terminal voltages, and excitation system variables between Case 1 and Case 2: for example, for fault 11-12,

a) The internal voltages except E_2 almost constant in Case 1, but those vary violently with time in Case 2, namely, increase initially, decrease subsequently, and increase again, ... ;

b) The terminal voltages except for V_{t2} are almost constant in Case 1 while those vary serratedly corresponding to the internal voltages;

c) The outputs of automatic voltage regulators E_{ai} are violently varied in Case 2, namely E_{a2} does not reach even 1.0 p.u. in Case 1 while it takes values greater than 100.0 p.u., which is corresponding to the fact that E_2 and V_{t2} increase even in the fault-on period;

d) The excitation voltages E_{fdi} do not vary so much in Case 1 while those vary very much and reach the ceiling values E_{cli} in Case 2;

e) The stabilizing signals E_{pi} are added to the automatic voltage regulators in Case 2. E_{p2} also reaches the limit values 0.5 p.u. and -0.50 p.u. almost all the time.

These facts are corresponding to the fact that the first swing stability has been more improved in Case 2 than in Case 1.

The critical fault clearing time obtained by simulations are shown in Table 19, where those for the case where no voltage regulation is made, refered as Case 0, are also shown. From this table, we can derive some features as follows:

1) The critical clearing time varies with fault location:

a) It exists in a range of $0.25 \sim 0.46$ sec in Case 0;

b) It exists in a range of $0.26 \sim 0.49$ sec in Case 1;

c) It exists in a range of $0.32 \sim 1.12$ sec in Case 2.

2) The critical clearing time increases with increases in automatic voltage regulator gains:

a) It increases for all fault locations; for example, for fault 11-12, it increases from 0.25 to 0.26 sec in Case 1, and from 0.25 to 0.32 sec in Case 2.

b) The increase in critical clearing time is not so much, and remains in a range of 0.00 ∿ 0.05 sec for all faults in Case 1.

c) The increase in critical clearing time is relatively much, and extends to a range of 0.02 ∿ 0.87 sec for all faults in Case 2.

We have thus obtained the critical fault clearing times for all fault locations in both Case 1 and Case 2. These results are used as a basis of evaluating results obtained by Lyapunov's direct method.

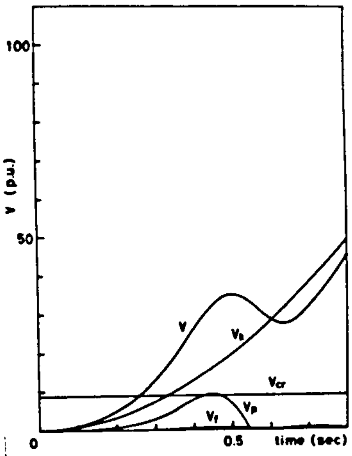
5.3 Results by Lyapunov's direct method

Secondly, Lyapunov's direct method is applied to the transient stability analysis of the 10-machine power system. In this method, the system equations (4.1) ∿ (4.5) or (4.130) ∿ (4.137) are integrated step by step for a given fault, where the fault is not cleared, to yield the time variation of the Lyapunov function V in (4.129) ∿ (4.169). The instants when V reaches the critical value V_{cr} is adopted as an estimation of the critical fault clearing time. The critical value V_{cr} is defined as the value of the potential energy V_p at the instant when the time derivative of the kinetic energy V_k changes its sign from negative to positive. Hence, the integration of the system equations is continued till that instant.

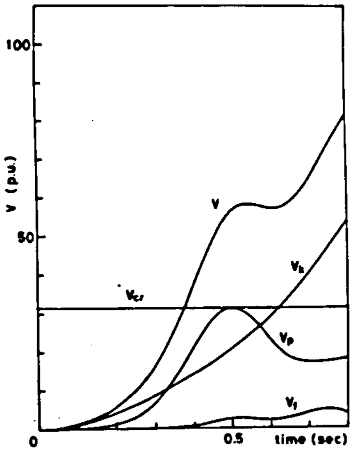
Fig. 73 and 74 show the time variations of V and its components, and V_{cr} for the eight cases of fault location in Case 1 and Case 2 of automatic voltage regulator gains, respectively, where V in (4.129) and (4.169) has been used in Case 1 and 2, respectively. The function takes a value nearly equal to zero at the instant when each fault occurs. It increases monotonously during the fault-on period. Namely, a total energy stored in the system increases with time. In order that the system keeps synchronism, the fault must be cleared before V reaches V_{cr}. From these figures, we can observe several features as follows:

1) There are some differences of time variations of V and its components between Case 1 and Case 2:

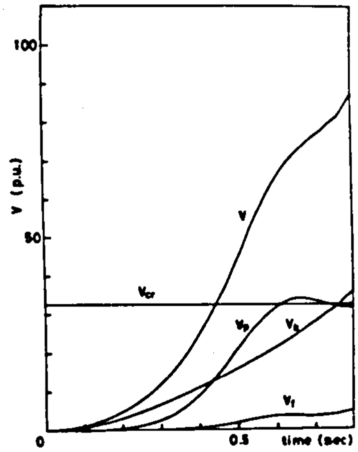
a) V_k increases monotonously with time. It generally takes smaller value at each time in Case 2 than in Case 1, which implies that



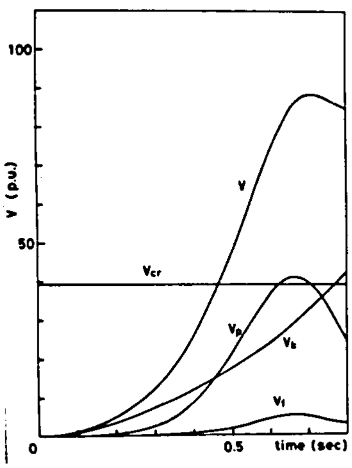
(1) Fault 11-12



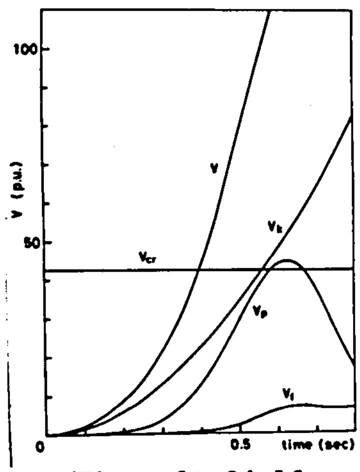
(2) Fault 15-14



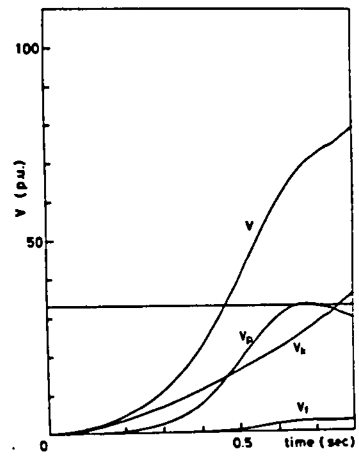
(3) Fault 17-18



(4) Fault 18-17

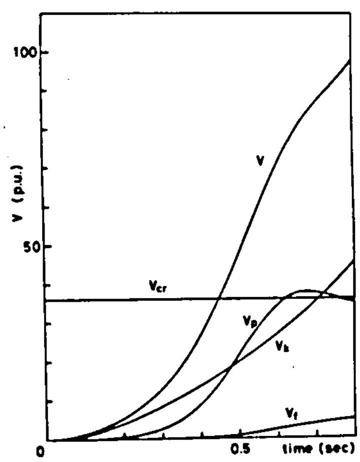


(5) Fault 24-16

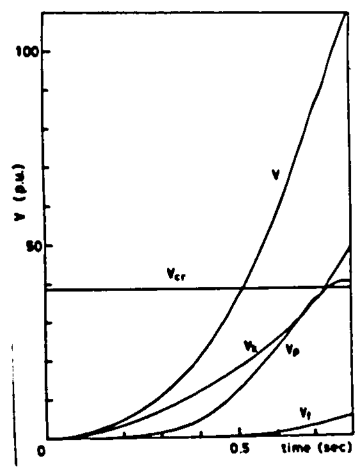


(6) Fault 30-27

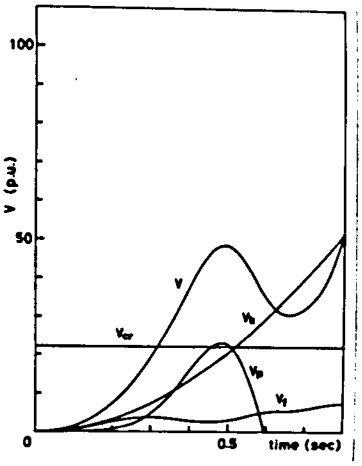
Fig.73 Estimation of critical clearing time by Lyapunov's direct method



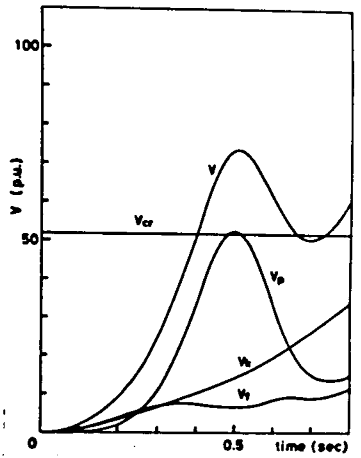
(7) Fault 34-29



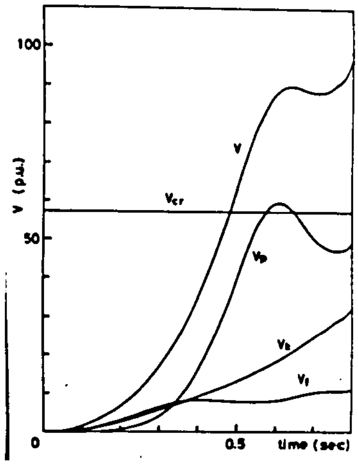
(8) Fault 38-15



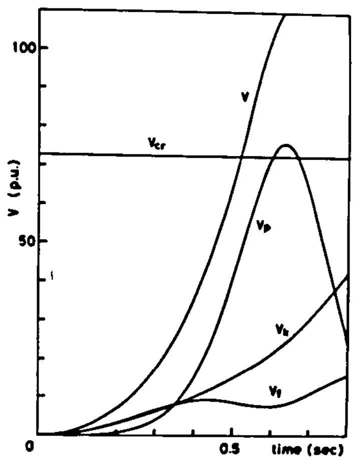
(1) Fault 11-12



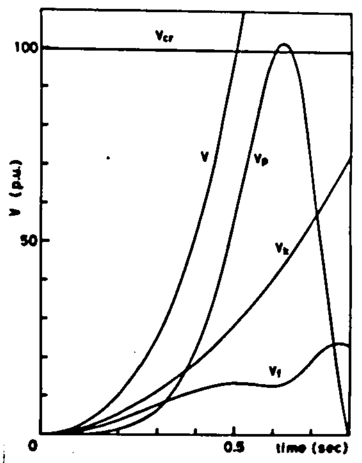
(2) Fault 15-14



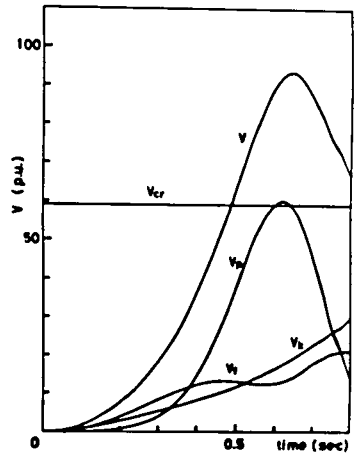
(3) Fault 17-18



(4) Fault 18-17

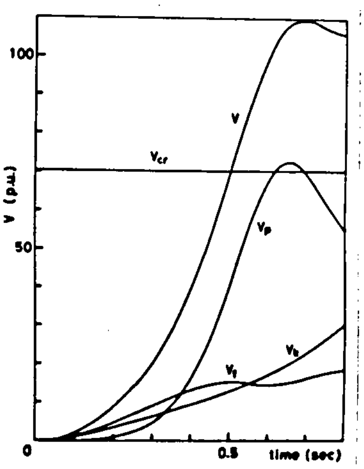


(5) Fault 24-16

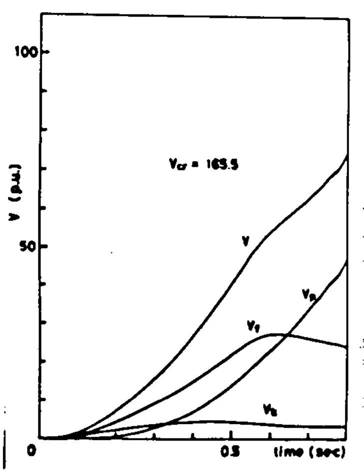


(6) Fault 30-27

Fig.74 Estimation of critical clearing time by Lyapunov's direct method



(7) Fault 34-29



(8) Fault 38-15

the energy which will split the system is suppressed by increasing the regulator gains.

b) V_p takes greater value at each time in Case 2 than in Case 1, which implies that the transient stability region gets deeper with increase in the regulator gains.

c) V_f does not take so much values, and has no significant influence on V in Case 1. On the other hand, it takes relatively large values, and its influence on V is not negligible.

V, as a whole, takes greater value at each time in Case 2 than in Case 1.

2) There are two features concerning with the critical value:

a) V_{cr} varies with fault location; for example, it takes 9.12 for fault 11-12, and 31.37 for fault 15-14, in Case 1.

b) V_{cr} varies with automatic voltage regulator gains; for example, it takes 9.12 in Case 1, but 22.10 in Case 2 for fault 11-12.

3) In Case 2, V reaches V_{cr} at an instant later than in Case 1 for all faults.

These features clarify how is the transient stability of the system improved by increasing the automatic voltage regulator gains.

Table 20(a) and (b) summarizes the results by Lyapunov's direct method. Several features on this method are observed from these tables as follows:

1) The direct method yields results very close to those obtained by the simulations:

a) The difference between the results by the two methods remains in a range of 0.00∿0.02 sec for all faults in Case 1.

b) The difference between the results by the two methods remains in a range of 0.00∿0.02 sec for all faults except for fault 38-15, in Case 2.

c) There are four faults for which the direct method yields optimistic results by 0.02 sec in Case 2.

2) The critical value varies with fault location:

a) V_{cr} exists in a range of 9.12∿42.66 in Case 1.

b) V_{cr} exists in a range of 22.10∿165.45 in Case 2.

3) The ratio γ varies with fault location, too:

Table 20. Critical fault clearing times estimated by Lyapunov's direct method (sec).

(a) K_{ai}: 1.0

Fault	γ	V_{cr}	T_{es}	T_{cr}	T^*_{cr}
11-12	1.398	9.12	0.26	0.26	0.25
15-14	1.085	31.37	0.37	0.38	0.38
17-18	1.396	32.58	0.44	0.43	0.42
18-17	1.345	39.23	0.47	0.47	0.46
24-16	1.327	42.66	0.39	0.39	0.37
30-27	1.272	32.60	0.46	0.45	0.44
34-29	1.265	35.93	0.45	0.45	0.44
38-15	1.349	38.56	0.51	0.49	0.44

(b) K_{ai}: 100.0

Fault	γ	V_{cr}	T_{es}	T_{cr}	T^*_{cr}
11-12	1.222	22.10	0.31	0.32	0.25
15-14	0.997	51.65	0.40	0.40	0.38
17-18	1.116	57.33	0.48	0.46	0.42
18-17	1.087	72.26	0.51	0.49	0.46
24-16	0.997	99.73	0.50	0.48	0.37
30-27	1.068	59.21	0.48	0.48	0.44
34-29	1.041	70.29	0.50	0.48	0.44
38-15	0.967	165.45	1.17	1.22	0.44

T_{es}: critical clearing time estimated by the direct method. method.

T_{cr}: critical clearing time obtained by simulations.

T^*_{cr}: critical clearing time for cases where K_{ai} is zero, that is, no voltage regulation is made.

a) γ exists in a range of 1.085 ∿ 1.398 in Case 1.

b) γ exists in a range of 0.967 ∿ 1.222 in Case 2.

c) γ decreases with increase in the regulator gains; there are four faults for which γ takes values smaller than 1 in Case 2 while it takes values greater than 1 for all faults in Case 1.

These features verify that the direct method works well, and can take account of the improvement of the transient stability due to automatic voltage regulators to an extent. We have no systematic method of determining ε, however, and we set $\varepsilon = 1$ in Case 2 for trial. Table 21 shows how does the result vary with ε, which is summarized as follows:

4) The result gets smaller with increase in ε for all faults;

   a) The difference between the results by the two methods remains in a range of 0.00 ∿ 0.02 sec for all faults except for fault 38-15, and there exists only one fault for which the direct method gives an optimistic result when ε is set 1.5.

   b) The difference between the results by the two methods extends to a range of 0.00 ∿ 0.04 sec for all faults except for fault 38-15, but there exists no fault for which the direct method gives an optimistic result.

   c) It seems better to set $\varepsilon = 1.5$ in stead ob 1.0 in this study.

The results thus vary much according to ε. Since the above feature holds only to this system, so it is indispensable to develope a systematic method of determining an optimal ε.

5.4 Conclusions

In this section, we have made some transient stability analysis of a 10-machine power system, where all generators in the system are installed with automatic voltage regulators. Two Lyapunov functions and the method of determining their critical value developed in §2, 3, and 4 have been applied to this analysis. The results are summarized as follows:

1) The transient stability of the system is improved by installing automatic voltage regulators:

   a) The improvement is not so much if all regulator gains are set 1, and remains in a range of 0.00 ∿ 0.05 sec for all faults.

Table 21. Influence of ε on estimation of critical fault clearing time by Lyapunov's direct method. (sec).

Fault	$\varepsilon = 1.0$	$\varepsilon = 1.5$	$\varepsilon = 2.0$
11-12	0.31	0.30	0.29
15-14	0.40	0.39	0.37
17-18	0.48	0.47	0.45
18-17	0.51	0.50	0.49
24-16	0.50	0.48	0.47
30-27	0.48	0.46	0.44
34-29	0.50	0.48	0.45
38-16	1.17	1.12	1.08

b) The improvement is comparatively much, and extends to a range of $0.02 \sim 0.87$ sec for all faults if all regulator gains are set 100.

2) The direct method yields in general results very close to those obtained by the simulations:

a) The difference between the results by the two methods remains in a range of $0.00 \sim 0.02$ sec for all faults in the case where all regulator gains are set 1.

b) The difference between the results by the two methods remains in a range of $0.00 \sim 0.02$ sec for all faults except one fault in the case where all regulator gains are set 100.

where

3) Two different Lyapunov functions have been used:

a) V in (4.129) has been used in the case where all regulator gains are set 1, and has worked well in the analysis.

b) V in (4.169) has been used in the case where all regulator gains are set 100, and has worked well to an extent in the analysis, where ε has been set 1.

c) The result by the direct method varies much for all faults when ε varies from 1 to 1.5, and to 2 in the latter case so its selection has significant influence on the accuracy of the direct method.

Automatic voltage regulators have thus significant influence on the transient stability of power systems. We have applied Lyapunov's direct method to its analysis, and have obtained some results of practical significance.

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Chapter V

SUMMARY AND CONCLUSIONS

In this thesis, we have made some investigations on Lyapunov's direct method with a view to applying it to the transient stability analysis of multimachine power systems.

In Chapter II, three basic problems associated with Lyapunov's direct method have been studied with the power system representation in which each generator is represented by a constant voltage behind a transient reactance. Firstly, a systematic method of constructing an appropriate Lyapunov function has been developed. A generalized Popov criterion has been derived. It is applicable to more general power system model than that by Moore and Anderson. After the method proposed by Willems, a Lur'e type Lyapunov function has been systematically constructed on the basis of our generalized Popov criterion. Three parameters contained in it have been varied, and corresponding Lyapunov functions have been investigated. From the investigation, they have been so chosen that it yields the energy integral function derived by Aylette, and it has been adopted as Lyapunov function in our transient stability studies. Secondly, some investigations on the critical value of the Lyapunov function have been made. From the physical consideration on the system behavior in the transient period, it has been found out that it is useful to adopt a critical value which corresponds to the first swing stability, and that it is possible to get rid of the well-known conservative nature of the direct method by using this critical value. One method of determining this critical value has been developed. It need no calculation of unstable equilibrium points, and can calculate the critical value in adequately short time. As a result, the problem associated with the calculation of the usual critical value has been solved as well. Thirdly, the influence of transfer conductances on the Lyapunov function and its critical value has been studied. It has significant influence on them, and accordingly, on the estimation of critical fault clearing time. The method of counting in their influence have been developed. Lastly, the transient stability analysis of a 10-machine power system has been made by Lyapunov's direct method, and its results have been compared with those obtained by simulations. The results have been very close to those by simulations. The methods developed in this chapter have worked very well. It has been concluded that the direct method is very useful for the transient stability analysis of the power systems which is represented as in this chapter.

In Chapter III, some investigations have been made on Lyapunov's direct method applied to the power system representation in which dynamics of field flux linkages of generators are incorporated. A Lur'e type Lyapunov function has been constructed on the basis of the generalized Popov criterion. The function is similar to that obtained in the preceding chapter but for a few points. The transient stability region has been defined in the same way as for the system with constant field flux linkages. Its variation with interanl voltages has been studied. The transient stability region gets narrow with decrease in internal voltages, and the system gets liable to lose synchronism. There is a possibility for the system to lose synchronism owing to the vanishment of the transient stability region itself, and the second type of instability has been introduced. The methods developed in Chapter II of determining the critical value for the first swing stability and of counting in the influence of transfer conductances have been generalized a little in order that those get applicable to this system representation. The total of them has yielded the results of good accuracy in the transient stability analysis of the 10-machine power system.

In Chapter IV, some investigations have been made on Lyapunov's direct method applied to the transient stability analysis of the power system in which automatic voltage regulators and thyristor exciters are installed in generators. The generalized Popov criterion has guaranteed that Lyapunov functions could be contructed for those cases where automatic voltage regulator gains are relatively low, such as $0 \sim 10$. Under this constraint, a Lur'e type Lyapunov function has been systematically constructed, and it has been applied to the transient stability analysis of a 10-machine power system. Lyapunov's direct method has yielded results very close to those obtained by simulations. The transient stability is improved very much by using high gain voltage regulators, but those are often greater than those allowed by the generalized Popov criterion. We have generalized the direct method to high gain systems by introducing a pseudo Lyapunov function and by applying our method of determining the critical value for the first swing stability. The results of the transient stability analysis have been acceptable.

In conclusion, we have succeeded in getting rid of the three troublesome problems pointed out in Chapter I from Lyapunov's direct method to some extent. Its conservative nature and the difficulty accompanying the calculation of the critical value have been almost completely removed. The constraint on the system representation has been released to an extent, that is, the dynamics of field flux linkages have been almost completely incorporated, and the dynamics of automatic voltage regulators and excitation systems have been incorporated in those cases where automatic voltage regulator gains are relatively low. We hope that this work would contribute more or less to the development of Lyapunov's direct method, and that it would be a help in implementing this method as a useful tool of analizing the transient stability of practical power systems at various stages of system planning and operation.

Appendix A

Derivation of Eqs.(2.68), (2.75), and (2.77)

§1. Derivation of (2.68)

Consider a matrix equation as follows:

$$XT = 0_{nm} \tag{A1}$$

where X is an unknown symmetric $n \times n$ matrix, T is an $n \times m$ matrix defined in (2.30), and 0_{nm} is an $n \times m$ matrix with all zero elements. The matrix T contains $m = n(n-1)/2$ columns, each of which only contains two nonzero elements, 1 on the ith row and -1 on the jth row, so that any (i,j) pair is included. Hence, all elements on the same row of X are equal. Moreover, since X is symmetric, it follows thal all elements of X are equal. Accordingly, a necessary and sufficient condition for a symmetric matrix X to be a solution of (A1) is that it has the following form:

$$X = \chi U \tag{A2}$$

where χ is a scalar constant, and U is an $n \times n$ matrix with all its elements equal to 1 [33].

§2. Derivation of (2.75)

Consider a matrix as follows:

$$P^* = \begin{bmatrix} D/q + \rho DUD & M/q + \rho DUM \\ M/q + \rho MUD & M + \mu MUM \end{bmatrix} \tag{A3}$$

where P^* is a $2n \times 2n$ matrix. If μ and ρ are zero, P^* is positive definite because the following inequalities hold:

$$[\,\delta',\ \omega'\,]\begin{bmatrix} D/q & M/q \\ M/q & M \end{bmatrix}\begin{bmatrix} \delta \\ \omega \end{bmatrix}$$

$$= \sum_{i=1}^{n} [(d_i/q)\delta_i^2 + (2m_i/q)\delta_i\omega_i + m_i\omega_i^2]$$

$$\geq \sum_{i=1}^{n} m_i(\delta_i/q + \omega_i)^2$$

$$\geq 0 \tag{A4}$$

and

$$\det P^* = \prod_{i=1}^{n} (m_i/q)(d_i - m_i/q)$$

$$> 0 \tag{A5}$$

P* may not be positive definite if μ and ρ are not zero, however. After some manipulation, we obtain

$$\det P^* = - \frac{\prod_{i=1}^{n} m_i(d_i - m_i/q)}{q\sum_{i=1}^{n} d_i \sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q}} \left(\rho + \frac{1}{q\sum_{i=1}^{n} d_i}\right)\left(\rho - \mu - \frac{1}{\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q}}\right) \tag{A6}$$

As observed from this equation, det P* is in a quadratic form of ρ, and if

$$-\frac{1}{q\sum_{i=1}^{n} d_i} < \rho < \mu + \frac{1}{\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q}} \tag{A7}$$

is satisfied, then det P* is positive. Since P* is positive definite if μ and ρ are zero, it is positive definite also for μ and ρ which satisfy (A7). Eq.(A7) is equivalent to the following inequalities:

$$\rho > -\frac{1}{q\sum_{i=1}^{n} d_i}$$

$$\mu - \rho \geq -\frac{1}{\sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q}} \tag{A8}$$

§3. Derivation of (2.77)

Consider a matrix as follows:

$$Z^* = 2(D - M/q) + \mu^*(DUM + MUD) \tag{A9}$$

where Z* is an n × n matrix. If μ* is zero, then Z* is positive definite because (2.47) holds. The determinant of Z* is given by

$$\det Z^* = 2^n \prod_{i=1}^{n} (d_i - m_i/q) \tag{A10}$$

which is, of course, positive. Z* may not be positive definite for non-zero μ*. After some manipulation, we obtain

$$\det Z^* = A(\mu^*)^2 + B\mu^* + C \tag{A11}$$

where

$$A = -2^n \prod_{i=1}^{n} (d_i - m_i/q) \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(d_i m_j - d_j m_i)^2}{4(d_i - m_i/q)(d_j - m_j/q)}$$

$$B = 2^n \prod_{i=1}^{n} (d_i - m_i/q) \sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q}$$

$$C = 2^n \sum_{i=1}^{n} (d_i - m_i/q)$$

As observed from this equation, det Z* is in a quadratic form of μ*, and if

$$A(\mu^*)^2 + B\mu^* + C > 0 \tag{A12}$$

is satisfied, then det Z* is positive. Since Z* is positive definite if μ* is zero, it is positive definite also for μ* which satisfies (A12). Eq.(A12) is equivalent to the following inequality:

$$(\mu^*)^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(d_i m_j - d_j m_i)^2}{4(d_i - m_i/q)(d_j - m_j/q)} - \mu^* \sum_{i=1}^{n} \frac{d_i m_i}{d_i - m_i/q} - 1 \leq 0 \tag{A13}$$

Eq.(2.77) is thus derived [36].

Appendix C

Calculation of Equilibrium Point

In applying Lyapunov's direct method to transient stability analyses, it is necessary to calculate a stable equilibrium point of a system in a post-fault operating state. The equilibrium point is obtained by soving the following equations:

$$g_{1i} \equiv (P_{m1} - P_{e1})/m_1 - [P_{m(i+1)} - P_{e(i+1)}]/m_{(i+1)} = 0 \quad \text{for } i=1,2,\ldots,n-1, \tag{C1}$$

$$g_{2i} \equiv E_{fdi} - E'_{qi} - (x_{di} - x'_{di})i_{di} = 0 \tag{C2}$$

$$g_{3i} \equiv E_{ai} + K_{ai}E_{di} - K_{ai}(V_{refi} - V_{ti}) = 0 \tag{C3}$$

$$g_{4i} \equiv K_{ei}E_{ai} - E_{ei} = 0 \tag{C4}$$

$$g_{5i} \equiv (K_{di}K_{ei}/T_{ei})E_{ai} - (K_{di}/T_{ei})E_{ei} - (1/T_{di})E_{di} = 0 \quad \text{for } i=1,2,\ldots,n, \tag{C5}$$

where damping torques of generators are neglected. From (C3) ∿ (C5), we obtain

$$E_{ai} = K_{ai}(V_{refi} - V_{ti}) \tag{C6}$$

$$E_{ei} = K_{ai}K_{ei}(V_{refi} - V_{ti}) \tag{C7}$$

$$E_{di} = 0 \tag{C8}$$

and

$$g_{2i} \equiv [E^{o}_{fdi} + K_{ai}K_{ei}(V_{refi} - V_{ti})] - E'_{qi} - (x_{di} - x'_{di})i_{di} = 0 \tag{C9}$$

Hence the stable equilibrium point can be obtained by solving (C1) and (C9). In the followings, we derive necessary relations for solving these equations.

Voltages and currents of generators are related by

$$\mathring{I}_i = \sum_{j=1}^{n} \mathring{Y}'_{ij}\dot{E}_{qdj} \quad \text{for } i=1,2,\ldots,n \tag{C10}$$

where $\mathring{Y}'$ is the admittance matrix which relates $\dot{I}$ and $\mathring{E}_{qd}$ behind q-axis reactance x_q. Since $\dot{E}_{qdi}$ is parallel with the q-axis of the ith machine,

$\dot{I}_i'$ in the ith machine frame is given as follows:

$$\begin{aligned}\dot{I}_i' &= \dot{I}_i e^{j(\pi/2 - \delta_i)} \\ &= \sum_{j=1}^{n} Y_{ij}' E_{qdi} \angle \pi - (\delta_{ij} + \theta_{ij})\end{aligned} \tag{C11}$$

From this equation, we get the d- and q-axis components of the ith machine as follows:

$$\begin{aligned}i_{di} &= \text{Re } \dot{I}_i' \\ &= - \sum_{j=1}^{n} Y_{ij}' E_{qdi} \cos(\delta_{ij} + \theta_{ij})\end{aligned} \tag{C12}$$

$$\begin{aligned}i_{qi} &= \text{Im } \dot{I}_i' \\ &= \sum_{j=1}^{n} Y_{ij}' E_{qdi} \sin(\delta_{ij} + \theta_{ij})\end{aligned} \tag{C13}$$

Since E_{qdi} is defined by

$$E_{qdi} = E_{qi}' + (x_{qi} - x_{di}') i_{di} \tag{C14}$$

we can get two equations by substituting (C14) into (C12) and (C13) as follows:

$$Ai_d = f_d \tag{C15}$$

$$i_q = Bi_d + f_q \tag{C16}$$

where A, B are $n \times n$ matrices, and f_d, f_q are n dimensional vectors defined by

$$\begin{aligned}A_{ij} &= Y_{ij}'(x_{qj} - x_{dj}')\cos(\delta_{ij} + \theta_{ij}) \qquad \text{for } j \neq i \\ A_{ii} &= Y_{ii}'(x_{qi} - x_{di}')\cos\theta_{ii} + 1 \\ B_{ij} &= Y_{ij}'(x_{qj} - x_{dj}')\sin(\delta_{ij} + \theta_{ij}) \\ f_{di} &= - \sum_{j=1}^{n} Y_{ij}' E_{qj}' \cos(\delta_{ij} + \theta_{ij})\end{aligned} \tag{C17}$$

$$f_{qi} = \sum_{j=1}^{n} Y'_{ij} E'_{qdj} \sin(\delta_{ij} + \theta_{ij})$$

Currents i_d and i_q are obtained by solving (C15) and (C16). The terminal voltage of the ith generator is defined by

$$\mathring{V}_{ti} = \mathring{E}_{qdi} - jx_{qi}\mathring{I}_i \quad \text{(C18)}$$

and the d- and the q-axis of components of the voltage are given as follows:

$$v_{di} = \text{Re}\,\mathring{V}_{ti} = x_{qi} i_{qi} \quad \text{(C19)}$$

$$v_{qi} = \text{Im}\,\mathring{V}_{ti} = E'_{qi} - x'_{di} i_{di} \quad \text{(C20)}$$

The magnitude of the terminal voltage is given by

$$V_{ti} = (v_{di}^2 + v_{qi}^2)^{1/2} \quad \text{(C21)}$$

The active power of the ith generator is given by

$$\begin{aligned} P_{ei} &= E_{qdi} i_{qi} \\ &= [E'_{qi} + (x_{qi} - x'_{di}) i_{di}] i_{qi} \end{aligned} \quad \text{(C22)}$$

We now have derived all necessary equations, so we next linearize them in the followings. Eqs.(C15) and (C16) are linearized as follows:

$$(\Delta A) i_d + A(\Delta i_d) = \Delta f_d \quad \text{(C23)}$$

$$\Delta i_q = (\Delta B) i_d + B(\Delta i_d) + \Delta f_q \quad \text{(C24)}$$

where

$$\begin{aligned} (\Delta A) i_d &= S_1 \Delta\delta_r \\ (\Delta B) i_d &= S_2 \Delta\delta_r \\ \Delta f_d &= S_3 \Delta\delta_r - T_1 \Delta E'_q \\ \Delta f_q &= S_4 \Delta\delta_r - T_2 \Delta E'_q \end{aligned} \quad \text{(C25)}$$

In (C25), S_1, S_2, S_3, and S_4 are $n \times (n-1)$ matrices, and T_1, T_2 are $n \times n$ matrices defined as follows:

$$S_{1i(j-1)} = -\sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}(x_{qk} - x'_{dk})\sin(\delta_{ik} + \theta_{ik})i_{dk} \qquad \text{for } j = i$$

$$S_{1i(j-1)} = Y'_{ij}(x_{qj} - x'_{dj})\sin(\delta_{ij} + \theta_{ij})i_{dj} \qquad \text{for } j \neq i$$

$$S_{2i(j-1)} = \sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}(x_{qk} - x'_{dk})\cos(\delta_{ik} + \theta_{ik})i_{dk} \qquad \text{for } j = i$$

$$S_{2i(j-1)} = -Y'_{ij}(x_{qj} - x'_{qj})\cos(\delta_{ij} + \theta_{ij})i_{dj} \qquad \text{for } j \neq i$$

$$S_{3i(j-1)} = \sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}E'_{qk}\sin(\delta_{ik} + \theta_{ik}) \qquad \text{for } j = i$$

$$S_{3i(j-1)} = -Y'_{ij}E'_{qj}\sin(\delta_{ij} + \theta_{ij}) \qquad \text{for } j \neq i$$

$$S_{4i(j-1)} = \sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}E'_{qk}\cos(\delta_{ik} + \theta_{ik}) \qquad \text{for } j = i$$

$$S_{4i(j-1)} = -Y'_{ij}E'_{qj}\cos(\delta_{ij} + \theta_{ij}) \qquad \text{for } j \neq i$$

$$T_{1ij} = Y'_{ij}\cos(\delta_{ij} + \theta_{ij})$$

$$T_{2ij} = Y'_{ij}\sin(\delta_{ij} + \theta_{ij}) \qquad \text{(C26)}$$

where $i=1,2,\ldots,n$,
$j=2,3,\ldots,n$

By substituting (C25) into (C23) and (C24), we obtain

$$\Delta i_d = U_1\Delta\delta_r - U_2\Delta E'_q \qquad \text{(C27)}$$

$$\Delta i_q = U_3\Delta\delta_r - U_4\Delta E'_q \qquad \text{(C28)}$$

where

$$U_1 = A^{-1}(S_3 - S_1) \qquad U_3 = S_2 + S_4 + BU_1$$
$$U_2 = A^{-1}T_1 \qquad U_4 = T_2 - BU_2 \qquad \text{(C29)}$$

Eqs.(C19) ∿ (C21) are linearized as follows:

$$\Delta v_{di} = x_{qi}\Delta i_{qi} \tag{C30}$$

$$\Delta v_{qi} = \Delta E'_{qi} - x'_{di}\Delta i_{di} \tag{C31}$$

$$\Delta V_{ti} = (v_{di}\Delta v_{di} + v_{qi}\Delta v_{qi})V_{ti} \tag{C32}$$

By substituting (C27) and (C28) into (C30) ∿ (C32), we obtain

$$\Delta V_t = U_5\Delta\delta_r + U_6\Delta E'_q \tag{C33}$$

where

$$\begin{aligned} U_5 &= \mathrm{diag}(x_{qi}v_{di}/V_{ti})U_3 - \mathrm{diag}(x'_{di}v_{qi}/V_{ti})U_1 \\ U_6 &= \mathrm{diag}(x_{qi}v_{di}/V_{ti})U_4 + \mathrm{diag}(x'_{di}v_{qi}/V_{ti})U_2 + \mathrm{diag}(v_{qi}/V_{ti}) \end{aligned} \tag{C34}$$

Eq.(C22) is linearized as follows:

$$\begin{aligned} \Delta P_{ei} = {} & [\Delta E'_{qi} + (x_{qi} - x'_{di})\Delta i_{di}]i_{qi} \\ & + [E'_{qi} + (x_{qi} - x'_{di})i_{di}]\Delta i_{qi} \end{aligned} \tag{C35}$$

From this equation, we obtain

$$\Delta P_e = U_7\Delta\delta_r + U_8\Delta E'_q \tag{C36}$$

where

$$U_7 = \mathrm{diag}[(x_{qi} - x'_{di})i_{qi}]U_1 + \mathrm{diag}[E'_{qi} + (x_{qi} - x'_{di})i_{di}]U_3$$

$$\begin{aligned} U_8 = {} & \mathrm{diag}(i_{qi}) - \mathrm{diag}[(x_{qi} - x'_{di})i_{qi}]U_2 \\ & + \mathrm{diag}[E'_{qi} + (x_{qi} - x'_{di})i_{di}]U_4 \end{aligned} \tag{C37}$$

After all these manipulations, (C1) and (C2) are linearized as follows:

$$\Delta g_1 = (\partial g_1/\partial\delta_r)\Delta\delta_r + (\partial g_1/\partial E'_q)\Delta E'_q \tag{C38}$$

$$\Delta g_2 = (\partial g_2/\partial\delta_r)\Delta\delta_r + (\partial g_2/\partial E'_q)\Delta E'_q \tag{C39}$$

where

$$\partial g_1/\partial\delta_r = - K'M^{-1}U_7$$

$$\partial g_1/\partial E_q' = -K'M^{-1}U_8$$

$$\partial g_2/\partial \delta_r = -\operatorname{diag}(x_{di} - x_{di}')U_1 - \operatorname{diag}(K_{ai}K_{ei})U_5 \qquad \text{(C40)}$$

$$\partial g_2/\partial E_q' = -I + \operatorname{diag}(x_{di} - x_{di}')U_2 - \operatorname{diag}(K_{ai}K_{ei})U_6$$

Since (C38) and (C39) are obtained, it is easy to solve (C1) and (C9) iteratively by the well-known Newton-Raphson method. The value of δ_r and E_q' at the stable equilibrium point can be obtained by interating the following equation:

$$\begin{bmatrix} \delta_r \\ E_q' \end{bmatrix}_{(i+1)} = \begin{bmatrix} \delta_r \\ E_q' \end{bmatrix}_{(i)} - \begin{bmatrix} \partial g_1/\partial \delta_r & \partial g_1/\partial E_q' \\ \partial g_2/\partial \delta_r & \partial g_2/\partial E_q' \end{bmatrix}^{-1} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}_{(i)} \qquad \text{(C41)}$$

where the subscript " (i) " denotes the iteration number. The initial values of δ_r and E_q' are set as follows:

$$\delta_r = 0_{(n-1)1}$$

$$E_q' = 1_{n1} \qquad \text{(C42)}$$

where $0_{(n-1)1}$ and 1_{n1} are (n-1) and n dimensional column vectors with all the elements equal to zero and unity, respectively. A sufficiently accurate solution of the stable equilibrium point can be obtained in 4 or 5 iterations.

Appendix D

Approximation of Terminal Voltage

The terminal voltage of the ith generator is given as follows:

$$\begin{aligned} \dot{V}_{ti} &= \dot{E}_i - jx'_{di}\dot{I}_i \\ &= \dot{E}_i - jx'_{di} \sum_{j=1}^{n} \dot{Y}_{ij}\dot{E}_j \qquad \text{for } i=1,2,\ldots,n \end{aligned} \tag{D1}$$

From this equation, it is observed that $\dot{V}_{ti}$ is in a circle with its center at $\dot{E}_i$ and radius r_i as in Fig.A, where

$$r_i = x'_{di} \sum_{j=1}^{n} Y_{ij}E_j \qquad \text{for } i=1,2,\ldots,n \tag{D2}$$

If r_i is small, then the magnitude of $\dot{V}_{ti}$ can be approximated by projecting $\dot{V}_{ti}$ onto $\dot{E}_i$ as follows:

$$V_{ti} \simeq E_i + x'_{di} \sum_{j=1}^{n} Y_{ij}E_j\cos(\delta_{ij} + \theta_{ij}) \tag{D3}$$

Fig.B shows an example of variation of V_{ti} and its approximation by (D3) with ψ_i, where E_i and r_i are set 1.0 and 0.2 p.u., respectively. The maximum approximation error is given by $(r_i^2/2)$, and is equal to 0.02 p.u. in this case, which is adequately small. Eq.(D3) can thus yield an good approximation of V_{ti} in those cases where r_i is small.

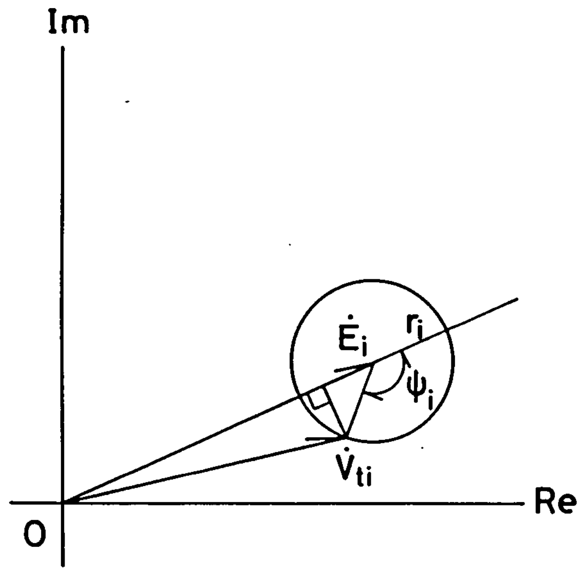

Fig.A. Approximation of terminal voltage.

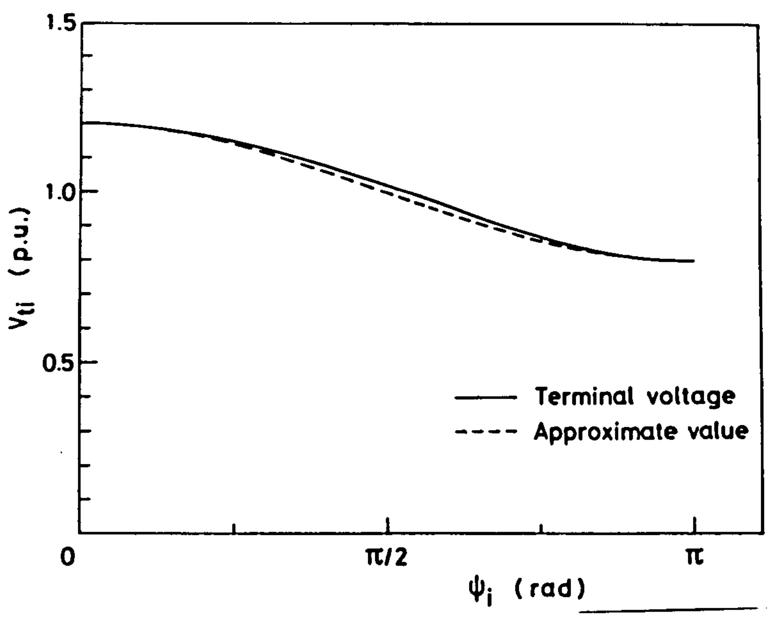

Fig.B. Variation of terminal voltage with ψ_i.

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