

Non-minimal coupling in the context of multi-field inflation

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Recent observations of the Cosmic Microwave Background (CMB) show very good consistency with inflationary models that contain non-minimal gravity sectors. In particular, the original R^2 inflation model proposed by Starobinsky, Higgs inflation and a whole class of models recently proposed by Kallosh and Linde in the context of supergravity all give the same predictions for the spectral tilt and tensor-to-scalar ratio that lie right at the centre of observational constraints. What all these models have in common is that their actions can be written in a form that contains non-minimal coupling between the field driving inflation and the Ricci Scalar. Such non-minimal coupling is well motivated in the context of quantum field theory in curved spacetime, modified gravity theories and unifying particle physics theories. Moreover, in the context of unifying theories, we expect there to be many light degrees of freedom in addition to the inflaton. Motivated by these facts, in this thesis we study multi-field models of inflation with non-minimal coupling.

There are three main parts to this thesis. We start by reviewing the need for inflation, the key distinctions between single- and multi-field models of inflation and the formulation of models with non-minimal coupling. As well as explicitly seeing how models with non-minimal coupling might arise, we also review the three specific models mentioned above. We see that the success of these models can be understood easily if one goes to the Einstein frame, where their effective potentials become stretched and flattened in the large-field limit, making them ideal for producing inflation.

After the review sections we then turn to the formulation dependence of multi-field inflation models with non-minimal coupling. It is well known that models with non-minimal coupling can be re-cast in the form of Einstein gravity via a conformal transformation of the metric. However, this is done at the expense of introducing interactions between the inflaton sector and ordinary matter, and also induces a space-time dependent rescaling of units. The original “frame” and that after the rescaling of the metric are referred to as the Jordan and Einstein frames, respectively, and there is a long-standing debate as to the level of equivalence between results obtained in the two frames. Indeed, with regard to the plausibility of the Higgs inflation model, the “frame dependence” of quantum corrections is currently a key point of contention. We focus on the difference in the curvature perturbations associated with the Jordan and Einstein frames, as these quantities are very closely linked to the temperature fluctuations of the CMB. At linear order, as a result of the isocurvature modes inherent to multi-field models of inflation, we find that the two quantities and their evolutions do not necessarily coincide. As such, the interpretation of the generation of the primordial curvature perturbation may be very different in the two frames. In particular, we find that the conservation of one of the curvature perturbations, which would usually indicate that an adiabatic limit has been reached, does not necessarily imply the conservation of the other. The fact that the two curvature perturbations are in general not equal is an indication that they cannot be directly observable, as predictions for any observable quantity should be frame-independent.

We then go beyond linear order using the δN formalism. We start by showing the con-

sistency of the δN formalism in the class of models under consideration. In comparing the Jordan and Einstein frame curvature perturbations, perhaps one of our key findings is the importance of taking into account the difference in definition of the initial flat hypersurfaces as defined in the two frames. Subsequently, the field perturbations on these hypersurfaces, which are used in the δN expansions, are also different, and it is only the flat-gauge perturbations in the Einstein frame for which we know the correlation functions. Expressing the Jordan frame flat-gauge field perturbations in terms of those in the Einstein frame we are able to find correlation functions of the curvature perturbation in the Jordan frame as well as in the Einstein frame. The second important difference is the fact that in the Jordan frame it is the time coordinate N that remains unperturbed, whereas in the Einstein frame it is \tilde{N} . Only by taking both of these differences into account are we able to confirm the equality of the two curvature perturbations when an adiabatic limit is reached.

By studying some specific example models we see that the curvature perturbations in the Jordan and Einstein frames and their statistical properties can indeed differ substantially, including their non-gaussianities. Given that non-gaussianity is a powerful tool for distinguishing between different models of inflation, this highlights the importance of determining how each of the curvature perturbations is actually related to observations. The first example consists of a minimally coupled inflaton field ϕ and a non-minimally coupled ‘spectator’ field χ . We find that by the end of inflation the Jordan and Einstein frame curvature perturbations converge to leading order in the slow-roll approximation and that it is easy to bring predictions for the observables r and n_s well within the 68% confidence contours of the recent *Planck* results. In particular, the presence of the χ field tends to reduce the tensor-to-scalar ratio as well as allowing for a wide range of spectral tilts depending on the form of the non-minimal coupling. The model also gives rise to a very small f_{NL} . The second example we consider is a non-minimally coupled extension of the multi-brid inflation model. Using the same model parameters as considered in the original model we find that the introduction of non-minimal coupling has a significant effect on the predictions of the model. In particular, we find $0.87 < n_s < 0.98$, $0.02 < r < 0.08$ and $2 < f_{NL} < 7$ for the range of non-minimal coupling parameters considered. These should be compared with the results $(n_s, r, f_{NL}) \sim (0.96, 0.04, 4.1)$ in the minimally coupled case. As such, we see that the *Planck* result $n_s = 0.9624 \pm 0.0073$ can be used to constrain the non-minimal coupling parameters. The model may also be constrained by future constraints on r and f_{NL} .

The final part of this thesis considers the reheating process in the same class of inflation models. Taking a bottom-up approach we assume that there are no direct interaction terms between the inflaton sector and the Standard Model. As such, reheating of the universe takes place via gravitationally suppressed dimension-5 interaction terms, with the decay rates of the inflatons depending on the masses of the decay products. Since all masses of the Standard Model particles are acquired from the Higgs field expectation value, we argue that the rate of gravitational particle production can be spatially modulated by the stochastic value of the Higgs condensate generated during inflation. We show that observational constraints on the curvature perturbation generated from this Higgs isocurvature mode can be used to set a lower bound on the inflaton mass during the reheating phase. Given that gravitational interactions are universal, and that we know the Higgs field to exist, this work is important in constraining the possible role that the Higgs field might play in the dynamics of the early Universe.