Electronic Supplementary Data

Electronic stress tensor of chemical bond

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Appendix – SA

We may first quickly review basic mathematics.[17] The coordinate $\boldsymbol{x}$ with the contravariant components $\boldsymbol{x}$ and the covariant components $\boldsymbol{x}$ and the metric tensor $=$ of the Minkowski space, together with the inner product of two 4-vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ written as $\boldsymbol{A}\cdot\boldsymbol{B}$ as well as the inner product of the Dirac gamma matrices and a 4-vector $\boldsymbol{A}$ written as the Dirac slash $\boldsymbol{A}$ are defined as follows:

$$\boldsymbol{x} = \left(\boldsymbol{x}^0, \boldsymbol{x}^k\right) = \left(\boldsymbol{x}^0, \boldsymbol{x}^1, \boldsymbol{x}^2, \boldsymbol{x}^3\right) = \left(\boldsymbol{ct}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\right) = \left(\boldsymbol{ct}, \vec{\boldsymbol{r}}\right) = \left(\boldsymbol{ct}, \vec{\boldsymbol{x}}\right) \quad (1)$$

$$\boldsymbol{x} = \quad \boldsymbol{x} = \left(\boldsymbol{x}_0, \boldsymbol{x}_k\right) = \left(\boldsymbol{x}_0, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3\right) = \left(\boldsymbol{ct}, -\boldsymbol{x}, -\boldsymbol{y}, -\boldsymbol{z}\right) = \left(\boldsymbol{ct}, -\vec{\boldsymbol{r}}\right) = \left(\boldsymbol{ct}, -\vec{\boldsymbol{x}}\right) \quad (2)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \quad , \qquad = \mathit{\Delta} \quad = \begin{cases} 1, & = \\ 0, & \neq \end{cases} \quad (3)$$

$$\partial \; = \frac{\partial}{\partial \boldsymbol{x}} = \left(\frac{\partial}{\partial \boldsymbol{x}^0}, \frac{\partial}{\partial \boldsymbol{x}^1}, \frac{\partial}{\partial \boldsymbol{x}^2}, \frac{\partial}{\partial \boldsymbol{x}^3}\right) = \left(\frac{1}{\boldsymbol{c}}\frac{\partial}{\partial \boldsymbol{t}}, \vec{\nabla}\right) = \left(\frac{1}{\boldsymbol{c}}\frac{\partial}{\partial \boldsymbol{t}}, \mathrm{grad}\right) \quad (4)$$

$$\partial \; = \quad \frac{\partial}{\partial \boldsymbol{x}} = \left(\frac{\partial}{\partial \boldsymbol{x}^0}, -\frac{\partial}{\partial \boldsymbol{x}^1}, -\frac{\partial}{\partial \boldsymbol{x}^2}, -\frac{\partial}{\partial \boldsymbol{x}^3}\right) = \left(\frac{1}{\boldsymbol{c}}\frac{\partial}{\partial \boldsymbol{t}}, -\vec{\nabla}\right) = \left(\frac{1}{\boldsymbol{c}}\frac{\partial}{\partial \boldsymbol{t}}, -\mathrm{grad}\right) \quad (5)$$

$$\boldsymbol{A}\cdot\boldsymbol{B} = \quad \boldsymbol{A} \; \boldsymbol{B} \; = \boldsymbol{A}^0\boldsymbol{B}^0 - \vec{\boldsymbol{A}}\bullet\vec{\boldsymbol{B}}, \quad \vec{\boldsymbol{A}}\bullet\vec{\boldsymbol{B}} = \boldsymbol{A}_x\boldsymbol{B}_x + \boldsymbol{A}_y\boldsymbol{B}_y + \boldsymbol{A}_z\boldsymbol{B}_z \quad (6)$$

$$\boldsymbol{A} = \quad \boldsymbol{A} \; = \; {}^0\boldsymbol{A}^0 - \vec{\;}\bullet\vec{\boldsymbol{A}}, \quad \vec{\;}\bullet\vec{\boldsymbol{A}} = \; {}^1\boldsymbol{A}_x + \; {}^2\boldsymbol{A}_y + \; {}^3\boldsymbol{A}_z \quad (7)$$

$$\square \; = \partial^2 = \left(\frac{1}{\boldsymbol{c}}\frac{\partial}{\partial \boldsymbol{t}}\right)^2 - \Delta, \quad \Delta = \left(\vec{\nabla}\right)^2 \quad (8)$$

$$\left\{\boldsymbol{A}, \boldsymbol{B}\right\} = \boldsymbol{AB} + \boldsymbol{BA} = \left[\boldsymbol{A}, \boldsymbol{B}\right]_+ ; \quad \left[\boldsymbol{A}, \boldsymbol{B}\right] = \boldsymbol{AB} - \boldsymbol{BA} = \left[\boldsymbol{A}, \boldsymbol{B}\right]_- \quad (9)$$

where the Einstein summation convention is used.

The spinor $\boldsymbol{\Psi}(\boldsymbol{x})$ in the chiral representation $\boldsymbol{\Psi}_{\mathrm{chiral}}(\boldsymbol{x})$ is constructed by the undotted spinor $\boldsymbol{\Psi}_{\boldsymbol{R}}(\boldsymbol{x}) = \;^{\boldsymbol{A}}(\boldsymbol{x})$ with right-handed chirality and the dotted spinor $\boldsymbol{\Psi}_{\boldsymbol{L}}(\boldsymbol{x}) = \;_{\dot{\boldsymbol{U}}}(\boldsymbol{x})$ with left-handed chirality as

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}_{\mathrm{chiral}} = \begin{pmatrix} \boldsymbol{\Psi}_{\boldsymbol{R}} \\ \boldsymbol{\Psi}_{\boldsymbol{L}} \end{pmatrix} = \begin{pmatrix} ^{\boldsymbol{A}} \\ _{\dot{\boldsymbol{U}}} \end{pmatrix} \quad (10)$$

$$^{\boldsymbol{A}} = \begin{pmatrix} ^1 \\ ^2 \end{pmatrix}, \qquad _{\dot{\boldsymbol{U}}} = \begin{pmatrix} _{\dot{1}} \\ _{\dot{2}} \end{pmatrix} \quad (11)$$

The undotted and dotted capital Latin letters run from 1 to 2 and change position by using the antisymmetric matrix as

$$_{\boldsymbol{A}} = \;^{\boldsymbol{B}}\;_{\boldsymbol{BA}}, \qquad ^{\dot{\boldsymbol{U}}} = \;^{\dot{\boldsymbol{U}}\dot{\boldsymbol{V}}}\;_{\dot{\boldsymbol{V}}} \quad (12)$$

$$^{\boldsymbol{A}} = \;^{\boldsymbol{AB}}\;_{\boldsymbol{B}}, \qquad _{\dot{\boldsymbol{U}}} = \;^{\dot{\boldsymbol{V}}}\;_{\dot{\boldsymbol{V}}\dot{\boldsymbol{U}}} \quad (13)$$

$$_{\boldsymbol{AB}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \;^{\boldsymbol{AB}}, \qquad ^{\dot{\boldsymbol{U}}\dot{\boldsymbol{V}}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \;_{\dot{\boldsymbol{U}}\dot{\boldsymbol{V}}} \quad (14)$$

where the Einstein summation convention is used.

The Pauli matrix with the contravariant components and the covariant components

$$=\left(\ ^{0},\ ^{k}\right)=\left(\ ^{0},\ ^{1},\ ^{2},\ ^{3}\right)=\left(1,\ _{x},\ _{y},\ _{z}\right)=\left(1,\vec{\ }\right) \tag{15}$$

$$= \quad =\left(\ _{0},\ _{k}\right)=\left(\ _{0},\ _{1},\ _{2},\ _{3}\right)=\left(1,-\ _{x},-\ _{y},-\ _{z}\right)=\left(1,-\vec{\ }\right) \tag{16}$$

(note the use of 1 as the unit matrix) are cast into the MTW (Misner-Thorne-Wheeler) representation as[21]

$$\begin{aligned}
\left(\ _{0}\right)^{A\dot{U}} &= \left(\ ^{0}\right)_{\dot{V}B}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\ ^{0}\\
\left(\ _{1}\right)^{A\dot{U}} &= \left(\ ^{1}\right)_{\dot{V}B}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}=\ _{x}\\
\left(\ _{2}\right)^{A\dot{U}} &= \left(\ ^{2}\right)_{\dot{V}B}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}=\ _{y}\\
\left(\ _{3}\right)^{A\dot{U}} &= \left(\ ^{3}\right)_{\dot{V}B}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}=\ _{z}
\end{aligned} \tag{17}$$

Also, the Dirac gamma matrices and the chiral matrix $_{5}$

$$_{5}=i\ ^{0}\ ^{1}\ ^{2}\ ^{3} \tag{18}$$

are given in the chiral representation using the MTW representation of the Pauli matrices as

$$\begin{aligned}
^{0} &= \begin{pmatrix}0 & \left(\ _{0}\right)^{A\dot{U}}\\ \left(\ ^{0}\right)_{\dot{V}B} & 0\end{pmatrix}=\begin{pmatrix}0 & ^{0}\\ ^{0} & 0\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\\
^{k} &= \begin{pmatrix}0 & -\left(\ _{k}\right)^{A\dot{U}}\\ \left(\ ^{k}\right)_{\dot{V}B} & 0\end{pmatrix}=\begin{pmatrix}0 & -\ ^{k}\\ ^{k} & 0\end{pmatrix}\\
_{5} &= \begin{pmatrix}\left(\ ^{0}\right)^{A}{}_{B} & 0\\ 0 & -\left(\ ^{0}\right)_{\dot{U}}{}^{\dot{V}}\end{pmatrix}=\begin{pmatrix}^{0} & 0\\ 0 & -\ ^{0}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}=-\ ^{5}
\end{aligned} \tag{19}$$

where the following MTW representation is found for the diagonal block:

$$\begin{aligned}
\left(\ ^{0}\right)^{A}{}_{B} &= \left(\ ^{0}\right)_{\dot{U}}{}^{\dot{V}}=\ ^{0}\\
\left(\ ^{1}\right)^{A}{}_{B} &= \left(\ ^{1}\right)_{\dot{U}}{}^{\dot{V}}=\ _{x}\\
\left(\ ^{2}\right)^{A}{}_{B} &= \left(\ ^{2}\right)_{\dot{U}}{}^{\dot{V}}=\ _{y}\\
\left(\ ^{3}\right)^{A}{}_{B} &= \left(\ ^{3}\right)_{\dot{U}}{}^{\dot{V}}=\ _{z}
\end{aligned} \tag{20}$$

The Clifford algebra of the Dirac gamma matrices should be

$$\left\{\ ,\ \right\}=2\ \begin{pmatrix}\left(\ ^{0}\right)^{A}{}_{B} & 0\\ 0 & \left(\ ^{0}\right)_{\dot{U}}{}^{\dot{V}}\end{pmatrix}=2\ \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=2 \tag{21}$$

Appendix – SB

The fundamental supersymmetry algebras are summarized.[20]

The chiral spinor representation $\boldsymbol{D}(\Lambda)$ of the Poincaré group reduces to the infinitesimal Lorentz transformation as

$$\boldsymbol{D}_{\boldsymbol{m}'\boldsymbol{m}}(1+\quad)=1-\frac{1}{2}\boldsymbol{i}\quad_{\boldsymbol{k}\ell}\left(\boldsymbol{J}^{\boldsymbol{k}\ell}\right)_{\boldsymbol{m}'\boldsymbol{m}}/\hbar,\quad \boldsymbol{m}',\boldsymbol{m}=-\boldsymbol{j},-\boldsymbol{j}+1,\cdots,\boldsymbol{j} \tag{22}$$

$$\vec{\boldsymbol{J}}=\left(\boldsymbol{J}^{23},\boldsymbol{J}^{31},\boldsymbol{J}^{12}\right)$$
$$\left[\boldsymbol{J}^{\boldsymbol{k}},\boldsymbol{J}^{\ell}\right]=\boldsymbol{i}\hbar\quad_{\boldsymbol{k}\ell\boldsymbol{n}}\boldsymbol{J}^{\boldsymbol{n}} \tag{23}$$

$$\left(\boldsymbol{J}^{1}\pm\boldsymbol{i}\boldsymbol{J}^{2}\right)_{\boldsymbol{m}'\boldsymbol{m}}=\Delta_{\boldsymbol{m}'\boldsymbol{m}}\sqrt{(\boldsymbol{j}\mp\boldsymbol{m})(\boldsymbol{j}\pm\boldsymbol{m}+1)}\hbar$$
$$\left(\boldsymbol{J}^{3}\right)_{\boldsymbol{m}'\boldsymbol{m}}=\Delta_{\boldsymbol{m}'\boldsymbol{m}}\boldsymbol{m}\hbar \tag{24}$$

$$-\left(\vec{\boldsymbol{J}}\right)^{*}_{\boldsymbol{m}',\boldsymbol{m}}=(-)^{\boldsymbol{m}-\boldsymbol{m}'}\left(\vec{\boldsymbol{J}}\right)_{-\boldsymbol{m},-\boldsymbol{m}'} \tag{25}$$

$$\left[\boldsymbol{J}^{\boldsymbol{k}},\frac{\boldsymbol{H}}{\boldsymbol{c}}\right]=0,\quad \left[\boldsymbol{J}^{\boldsymbol{k}},\boldsymbol{P}^{\ell}\right]=\boldsymbol{i}\hbar\quad_{\boldsymbol{k}\ell\boldsymbol{n}}\boldsymbol{P}^{\boldsymbol{n}} \tag{26}$$

and

$$\vec{\boldsymbol{K}}=\left(\boldsymbol{J}^{10},\boldsymbol{J}^{20},\boldsymbol{J}^{30}\right)$$
$$\left[\boldsymbol{K}^{\boldsymbol{k}},\boldsymbol{K}^{\ell}\right]=-\boldsymbol{i}\hbar\quad_{\boldsymbol{k}\ell\boldsymbol{n}}\boldsymbol{J}^{\boldsymbol{n}} \tag{27}$$

$$\left[\boldsymbol{K}^{\boldsymbol{k}},\boldsymbol{J}^{\ell}\right]=\left[\boldsymbol{J}^{\boldsymbol{k}},\boldsymbol{K}^{\ell}\right]=\boldsymbol{i}\hbar\quad_{\boldsymbol{k}\ell\boldsymbol{n}}\boldsymbol{K}^{\boldsymbol{n}} \tag{28}$$

$$\left[\boldsymbol{K}^{\boldsymbol{k}},\frac{\boldsymbol{H}}{\boldsymbol{c}}\right]=\boldsymbol{i}\hbar\boldsymbol{P}^{\boldsymbol{k}},\quad \left[\boldsymbol{K}^{\boldsymbol{k}},\boldsymbol{P}^{\ell}\right]=\boldsymbol{i}\hbar\Delta_{\boldsymbol{k}\ell}\frac{\boldsymbol{H}}{\boldsymbol{c}} \tag{29}$$

$$\vec{\boldsymbol{A}}=\frac{1}{2}\left(\vec{\boldsymbol{J}}+\boldsymbol{i}\vec{\boldsymbol{K}}\right),\quad \vec{\boldsymbol{B}}=\frac{1}{2}\left(\vec{\boldsymbol{J}}-\boldsymbol{i}\vec{\boldsymbol{K}}\right) \tag{30}$$

$$\vec{\boldsymbol{J}}=\vec{\boldsymbol{A}}+\vec{\boldsymbol{B}},\quad \vec{\boldsymbol{K}}=-\boldsymbol{i}\left(\vec{\boldsymbol{A}}-\vec{\boldsymbol{B}}\right) \tag{31}$$

$$\left[\boldsymbol{A}^{\boldsymbol{k}},\boldsymbol{A}^{\ell}\right]=\boldsymbol{i}\hbar\quad_{\boldsymbol{k}\ell\boldsymbol{n}}\boldsymbol{A}^{\boldsymbol{n}} \tag{32}$$

$$\left[\boldsymbol{B}^{\boldsymbol{k}},\boldsymbol{B}^{\ell}\right]=\boldsymbol{i}\hbar\quad_{\boldsymbol{k}\ell\boldsymbol{n}}\boldsymbol{B}^{\boldsymbol{n}} \tag{33}$$

$$\left[\vec{\boldsymbol{A}},\vec{\boldsymbol{B}}\right]=0 \tag{34}$$

The $\left(0,\frac{1}{2}\right)$- fermionic generator $\boldsymbol{Q}_{\dot{U}\boldsymbol{r}}$ is transformed under the Lorentz transformation as

$$\boldsymbol{U}\left(\Lambda^{-1}\right)\boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\boldsymbol{U}(\Lambda)=\Lambda_{\dot{U}}{}^{\dot{V}}(\Lambda)\boldsymbol{Q}_{\dot{V}\boldsymbol{r}} \tag{35}$$

$$\left[\vec{\boldsymbol{J}},\boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\right]=-\frac{\hbar}{2}\vec{\ }_{\dot{U}}{}^{\dot{V}}\boldsymbol{Q}_{\dot{V}\boldsymbol{r}}\quad,\quad \left[\vec{\boldsymbol{K}},\boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\right]=-\frac{\hbar}{2}\boldsymbol{i}\vec{\ }_{\dot{U}}{}^{\dot{V}}\boldsymbol{Q}_{\dot{V}\boldsymbol{r}} \tag{36}$$

$$\left[\vec{\boldsymbol{A}},\boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\right]=0\quad,\quad \left[\vec{\boldsymbol{B}},\boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\right]=-\frac{\hbar}{2}\vec{\ }_{\dot{U}}{}^{\dot{V}}\boldsymbol{Q}_{\dot{V}\boldsymbol{r}} \tag{37}$$

with the charge conjugation operator $\mathbf{C}$ and the complex conjugate operator $\boldsymbol{K}$,

$$\mathbf{C}\begin{pmatrix}\boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{K}\boldsymbol{Q}_{\dot{W}\boldsymbol{r}}\\ \boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\end{pmatrix}\mathbf{C}^{-1}=-\begin{pmatrix}\boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{K}\boldsymbol{Q}_{\dot{W}\boldsymbol{r}}\\ \boldsymbol{Q}_{\dot{U}\boldsymbol{r}}\end{pmatrix} \tag{38}$$

Likewise, the $\left(\frac{1}{2},0\right)$- fermionic generator $\boldsymbol{P}^{\boldsymbol{As}}$ is transformed as

$$\boldsymbol{U}\left(\Lambda^{-1}\right)\boldsymbol{P}^{\boldsymbol{As}}\boldsymbol{U}\left(\Lambda\right)=\mathit{\Lambda}^{\boldsymbol{A}}{}_{\boldsymbol{B}}\left(\Lambda\right)\boldsymbol{P}^{\boldsymbol{Bs}} \tag{39}$$

$$\left[\vec{\boldsymbol{J}},\boldsymbol{P}^{\boldsymbol{As}}\right]=-\frac{\hbar}{2}\ \vec{}\ {}^{\boldsymbol{A}}{}_{\boldsymbol{B}}\boldsymbol{P}^{\boldsymbol{Bs}} \quad , \quad \left[\vec{\boldsymbol{K}},\boldsymbol{P}^{\boldsymbol{As}}\right]=+\frac{\hbar}{2}\boldsymbol{i}\ \vec{}\ {}^{\boldsymbol{A}}{}_{\boldsymbol{B}}\boldsymbol{P}^{\boldsymbol{Bs}} \tag{40}$$

$$\left[\vec{\boldsymbol{A}},\boldsymbol{P}^{\boldsymbol{As}}\right]=-\frac{\hbar}{2}\ \vec{}\ {}^{\boldsymbol{A}}{}_{\boldsymbol{B}}\boldsymbol{P}^{\boldsymbol{Bs}} \quad , \quad \left[\vec{\boldsymbol{B}},\boldsymbol{P}^{\boldsymbol{As}}\right]=0 \tag{41}$$

$$\mathbf{C}\begin{pmatrix}\boldsymbol{P}^{\boldsymbol{As}}\\ -\boldsymbol{e}_{\dot{\boldsymbol{U}}\dot{\boldsymbol{B}}}\boldsymbol{K}\boldsymbol{P}^{\boldsymbol{Bs}}\end{pmatrix}\mathbf{C}^{-1}=-\begin{pmatrix}\boldsymbol{P}^{\boldsymbol{As}}\\ -\boldsymbol{e}_{\dot{\boldsymbol{U}}\dot{\boldsymbol{B}}}\boldsymbol{K}\boldsymbol{P}^{\boldsymbol{Bs}}\end{pmatrix} \tag{42}$$

The Haag-Lopuszanski-Sohnius theorem states that

$$\left\{\boldsymbol{Q}_{\dot{\boldsymbol{U}}\boldsymbol{r}},\boldsymbol{K}\boldsymbol{Q}_{\dot{\boldsymbol{V}}\boldsymbol{s}}\right\}=2\mathit{\Delta}_{\boldsymbol{rs}}\left(\quad\right)_{\dot{\boldsymbol{U}}\boldsymbol{V}}\boldsymbol{P} \tag{43}$$

$$\left\{\boldsymbol{Q}_{\dot{\boldsymbol{U}}\boldsymbol{r}},\boldsymbol{Q}_{\dot{\boldsymbol{V}}\boldsymbol{s}}\right\}=\boldsymbol{e}_{\dot{\boldsymbol{U}}\boldsymbol{V}}\boldsymbol{Z}_{\boldsymbol{rs}} \quad , \quad \boldsymbol{Z}_{\boldsymbol{rs}}=-\boldsymbol{Z}_{\boldsymbol{sr}} \tag{44}$$

$$\left\{\begin{pmatrix}\boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{K}\boldsymbol{Q}_{\dot{\boldsymbol{W}}\boldsymbol{r}}\\ \boldsymbol{Q}_{\dot{\boldsymbol{U}}\boldsymbol{r}}\end{pmatrix},\begin{pmatrix}\boldsymbol{e}^{\boldsymbol{BX}}\boldsymbol{K}\boldsymbol{Q}_{\dot{\boldsymbol{X}}\boldsymbol{s}}\\ \boldsymbol{Q}_{\dot{\boldsymbol{U}}\boldsymbol{s}}\end{pmatrix}^{\dagger}\ {}^{0}\right\}=2\quad \boldsymbol{P}\,\mathit{\Delta}_{\boldsymbol{rs}}-\frac{1+\ {}^{5}}{2}\boldsymbol{K}\boldsymbol{Z}_{\boldsymbol{rs}}+\frac{1-\ {}^{5}}{2}\boldsymbol{Z}_{\boldsymbol{rs}} \tag{45}$$

where $\boldsymbol{P}$ is the 4-momentum operator and $\boldsymbol{Z}_{\boldsymbol{rs}}$ are the central charges.

For simple supersymmetry, we have null $\boldsymbol{Z}_{\boldsymbol{rs}}$.

Appendix – SC

The fundamental equations of motion of the Majorana particle are summarized.

The Majorana equations are

$$\left(\boldsymbol{i}\hbar\partial\ \left(\quad\right)^{\boldsymbol{A}\dot{\boldsymbol{U}}}\pm\boldsymbol{m}_{\boldsymbol{L}}\boldsymbol{e}^{\boldsymbol{i}\mathit{\Delta}_{L}}\boldsymbol{c}\boldsymbol{e}^{\boldsymbol{AU}}\boldsymbol{K}\right)\ {}_{\dot{\boldsymbol{U}}}=0 \tag{46}$$

$$\left(\boldsymbol{i}\hbar\partial\ \left(\quad\right)_{\dot{\boldsymbol{U}}\boldsymbol{A}}\mp\boldsymbol{m}_{\boldsymbol{R}}\boldsymbol{e}^{\boldsymbol{i}\mathit{\Delta}_{\boldsymbol{R}}}\boldsymbol{c}\boldsymbol{e}_{\dot{\boldsymbol{U}}\dot{\boldsymbol{A}}}\boldsymbol{K}\right)\ {}^{\boldsymbol{A}}=0 \tag{47}$$

with the Klein-Gordon equations

$$\left(\left(\boldsymbol{i}\hbar\partial\right)^{2}-\left(\boldsymbol{m}_{\boldsymbol{L}}\boldsymbol{c}\right)^{2}\right)\ {}_{\dot{\boldsymbol{U}}}=0 \tag{48}$$

$$\left(\left(\boldsymbol{i}\hbar\partial\right)^{2}-\left(\boldsymbol{m}_{\boldsymbol{R}}\boldsymbol{c}\right)^{2}\right)\ {}^{\boldsymbol{A}}=0 \tag{49}$$

where $\boldsymbol{m}_{\boldsymbol{R},\boldsymbol{L}}$ are the real masses and $\mathit{\Delta}_{\boldsymbol{R},\boldsymbol{L}}$ are the real phases.

The charge conjugation properties are

$$\mathbf{C}\left|\mathit{\Psi}_{\mathrm{M}_1}\right\rangle=\ {}_{\mathrm{M}_1}\left|\mathit{\Psi}_{\mathrm{M}_1{}^{\boldsymbol{c}}}\right\rangle=\left|\ \mathit{\Psi}_{\mathrm{M}_1}\right\rangle \tag{50}$$

$$\mathit{\Psi}_{\mathrm{M}_1{}^{\boldsymbol{c}}}=\boldsymbol{C}\ {}^{0}\boldsymbol{K}\mathit{\Psi}_{\mathrm{M}_1}=-\ \mathit{\Psi}_{\mathrm{M}_1} \tag{51}$$

$$\mathbf{C}\left|\mathit{\Psi}_{\mathrm{M}_2}\right\rangle=\ {}_{\mathrm{M}_2}\left|\mathit{\Psi}_{\mathrm{M}_2{}^{\boldsymbol{c}}}\right\rangle=\left|\ \mathit{\Psi}_{\mathrm{M}_2}\right\rangle \tag{52}$$

$$\mathit{\Psi}_{\mathrm{M}_2{}^{\boldsymbol{c}}}=\boldsymbol{C}\ {}^{0}\boldsymbol{K}\mathit{\Psi}_{\mathrm{M}_2}=-\ \mathit{\Psi}_{\mathrm{M}_2} \tag{53}$$

$${}_{\mathrm{M}_1}=\ {}_{\mathrm{M}_2}=-1 \tag{54}$$

The Dirac spinor representations are

$$\left(\boldsymbol{i}\hbar\partial \pm \boldsymbol{m_L e^{i\Delta_L} c}\right)\Psi_{\mathrm{M}_1} = 0, \quad \Psi_{\mathrm{M}_1} = \begin{pmatrix} \boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{K} \quad \dot{\boldsymbol{W}} \\ \dot{\boldsymbol{U}} \end{pmatrix} \tag{55}$$

$$\left(\boldsymbol{i}\hbar\partial \pm \boldsymbol{m_R e^{i\Delta_R} c}\right)\Psi_{\mathrm{M}_2} = 0, \quad \Psi_{\mathrm{M}_2} = \begin{pmatrix} \boldsymbol{A} \\ -\boldsymbol{e}_{\dot{\boldsymbol{U}}\dot{\boldsymbol{B}}}\boldsymbol{K} \quad \boldsymbol{B} \end{pmatrix} \tag{56}$$

and

$$\boldsymbol{i}\hbar\partial\Psi_{\mathrm{M}} \pm \boldsymbol{m}_{\mathrm{M}}\boldsymbol{c}\left(-\right)\Psi_{\mathrm{M}^{c}} = 0 \tag{57}$$

$$\Psi_{\mathrm{M}} = \begin{pmatrix} \boldsymbol{A} \\ \dot{\boldsymbol{U}} \end{pmatrix} \tag{58}$$

$$\Psi_{\mathrm{M}^{c}} = \boldsymbol{C} \quad {}^{0}\boldsymbol{K}\Psi_{\mathrm{M}} = \left(-\right)\begin{pmatrix} \boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{K} \quad \dot{\boldsymbol{W}} \\ -\boldsymbol{e}_{\dot{\boldsymbol{U}}\dot{\boldsymbol{B}}}\boldsymbol{K} \quad \boldsymbol{B} \end{pmatrix} \tag{59}$$

$$\boldsymbol{m}_{\mathrm{M}}\boldsymbol{c} = \begin{pmatrix} \boldsymbol{m_L e^{i\Delta_L} c} & 0 \\ 0 & \boldsymbol{m_R e^{i\Delta_R} c} \end{pmatrix} \tag{60}$$

Appendix – SD

The fundamental properties of the Salam-Strathdee superfields are summarized.[20]

First, the Majorana spinors satisfy

$$\mathbf{C}\Theta = -\Theta \tag{61}$$

$$\bar{\Theta} = \Theta^{\dagger} \; {}^{0} = -{}^{t}\Theta\boldsymbol{C} = {}^{t} \; {}^{t}\Theta\boldsymbol{C} = {}^{t}\left(\boldsymbol{C} \quad \right)(\tag{62}$$

$${}^{t}\left(\frac{\partial}{\partial\bar{\Theta}}\right)\left(\bar{\Theta}\boldsymbol{M} \; \Theta\right) = 2\boldsymbol{M} \; \Theta \tag{63}$$

$$\bar{\Theta}_1\boldsymbol{M} \; \Theta_2 = \bar{\Theta}_2\boldsymbol{C}^{-1}\,{}^{t}\boldsymbol{M}\boldsymbol{C} \;\; {}_{1}\Theta \tag{64}$$

$$\Theta\bar{\Theta} = -\frac{1}{4}\left(\bar{\Theta}\Theta\right) + \frac{1}{4} \quad {}^{5}\left(\bar{\Theta} \quad {}_{5}\Theta\right)\Theta - \frac{1}{4} \quad {}^{5}\left(\bar{\Theta} \; {}_{5}\Theta\right) \tag{65}$$

A spinor is decomposed into a pair of the Majorana spinors as

$$\boldsymbol{s} = \Theta_{+} + \boldsymbol{i}\,\Theta_{-} \tag{66}$$

$$\Theta_{+} = \frac{1}{2}\left(1 - \mathbf{C}\right)\mathrm{s}, \quad \Theta_{-} = \frac{1}{2\boldsymbol{i}}\left(1 + \mathbf{C}\right)\mathrm{s} \tag{67}$$

$$\mathbf{C}\Theta_{\pm} = -\Theta_{\pm} \tag{68}$$

The Salam-Strathdee superfield is constructed by using the Majorana spinors Θ, , and Λ as

$$\begin{aligned}
\boldsymbol{S} &= \boldsymbol{C} - \boldsymbol{i}\left(\bar{\Theta}\,{}_{5} \quad \right) \\
&- \frac{1}{2}\boldsymbol{i}\left(\bar{\Theta}\,{}_{5}\,\Theta\right)\boldsymbol{M} - \frac{1}{2}\left(\bar{\Theta}\Theta\right)\boldsymbol{N} - \frac{1}{2}\left(\bar{\Theta} \; {}_{5} \quad \right)\boldsymbol{V}\Theta \\
&- \boldsymbol{i}\left(\bar{\Theta}\,{}_{5}\,\Theta\right)\left(\bar{\Theta}\left(\; \Lambda \frac{1}{2}\boldsymbol{i}\hbar\partial \quad \right)\right) - \frac{1}{4}\left(\bar{\Theta} \; {}_{5}\,\Theta\right)^{2}\left(\boldsymbol{D} - \frac{1}{2}\hbar^{2}\Box\boldsymbol{C}\right)
\end{aligned} \tag{69}$$

where the component $\boldsymbol{C}$ of $\boldsymbol{S}$ is further denoted as $\boldsymbol{C}^{\boldsymbol{S}}$, etc. Taking the ***h.c.***, we have

$$\boldsymbol{S}^\dagger = \boldsymbol{C}^\dagger - \boldsymbol{i}\left(\bar{\Theta}_5 \quad\right)$$

$$-\frac{1}{2}\boldsymbol{i}\left(\bar{\Theta}_5\Theta\right)\boldsymbol{M}^\dagger - \frac{1}{2}\left(^-\Theta\Theta\right)\boldsymbol{N}^\dagger - \frac{1}{2}\left(^-\Theta_5\quad\right)\left(\Theta\boldsymbol{V}\quad\right)^\dagger \tag{70}$$

$$-\boldsymbol{i}\left(\bar{\Theta}_5\Theta\right)\left(^-\Theta\left(\quad\mathcal{A}\frac{1}{2}\boldsymbol{i}\hbar\partial\quad\right)\right) - \frac{1}{4}\left(^-\Theta_5\right)^2\Theta\left(\boldsymbol{D}^\dagger - \frac{1}{2}\hbar^2\Box\boldsymbol{C}^\dagger\right)$$

If with the Hermitean superfield $\boldsymbol{S}^\dagger = \boldsymbol{S}$, we have

$$\boldsymbol{C}^\dagger, \boldsymbol{M}^\dagger, \boldsymbol{N}^\dagger, \boldsymbol{V}\ ^\dagger, \boldsymbol{D}^\dagger = \boldsymbol{C}, \boldsymbol{M}, \boldsymbol{N}, \boldsymbol{V}\ , \boldsymbol{D} \tag{71}$$

The infinitesimal translation $\varDelta\boldsymbol{S}$ is defined as

$$\varDelta\boldsymbol{S} = \frac{1}{\boldsymbol{i}\hbar}\left(\ -\begin{pmatrix}\boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{KQ}_{\dot{\boldsymbol{W}}\boldsymbol{r}}\\ \boldsymbol{Q}_{\dot{\boldsymbol{U}}\boldsymbol{r}}\end{pmatrix}\right)\boldsymbol{S} \tag{72}$$

$$= \left(^-\mathcal{Q}\right)\boldsymbol{S}$$

$$\left[\begin{pmatrix}\boldsymbol{e}^{\boldsymbol{AW}}\boldsymbol{KQ}_{\dot{\boldsymbol{W}}\boldsymbol{r}}\\ \boldsymbol{Q}_{\dot{\boldsymbol{U}}\boldsymbol{r}}\end{pmatrix}, \boldsymbol{S}\right\} = \boldsymbol{i}\hbar\mathcal{Q}\boldsymbol{S} \tag{73}$$

The generator $\boldsymbol{q}$ should then satisfy

$$\left\{\mathcal{Q}_{\dot{\boldsymbol{U}}}, {}^{\boldsymbol{t}}\left(\boldsymbol{C}\mathcal{Q}\right)_{\boldsymbol{V}}\right\} = -2\boldsymbol{i}\hbar\left(\quad\right)_{\dot{\boldsymbol{U}}\boldsymbol{V}}\partial \tag{74}$$

$$\left\{\mathcal{Q}_{\dot{\boldsymbol{U}}}, \mathcal{Q}_{\dot{\boldsymbol{V}}}\right\} = 0 \tag{75}$$

with the Dirac spinor representation

$$\left\{\mathcal{Q}_\ell, \bar{\mathcal{Q}}_{\bar{\ell}}\right\} = -2\boldsymbol{i}\hbar\left(\quad\right)_{\ell\bar{\ell}}\partial \tag{76}$$

Also, the generator in the superfield coordinate representation is given as

$$\mathcal{Q} = -{}^{\boldsymbol{t}}\left(\frac{\partial}{\partial\bar{\Theta}}\right) - \boldsymbol{i}\hbar\partial\Theta = {}^{\boldsymbol{t}}\boldsymbol{C}\frac{\partial}{\partial\Theta} - \boldsymbol{i}\hbar\partial\Theta \tag{77}$$

$$\bar{\mathcal{Q}} = {}^{\boldsymbol{t}}\left(\boldsymbol{C}\boldsymbol{q}\right) = {}^{\boldsymbol{t}}\left(\frac{\partial}{\partial\Theta}\right) + \boldsymbol{i}\hbar\bar{\Theta}\partial \tag{78}$$

with the Dirac spinor representation

$$\mathcal{Q}_\ell = \boldsymbol{C}_{\bar{\ell}\ell}\frac{\partial}{\partial\Theta_\ell} - \boldsymbol{i}\hbar\left(\quad\right)_{\ell\bar{\ell}}\Theta_\ell\partial \tag{79}$$

$$\bar{\mathcal{Q}}_\ell = \mathcal{Q}_{\bar{\ell}}\boldsymbol{C}_{\ell\bar{\ell}} = {}^{\boldsymbol{t}}\left(\frac{\partial}{\partial\Theta_\ell}\right) - \boldsymbol{i}\hbar\,{}^{\boldsymbol{t}}\left(\boldsymbol{C}\quad\Theta\right)_\ell\partial \tag{80}$$

The derivative in the superfield coordinate representation is defined as

$$\mathcal{D} = -{}^{\boldsymbol{t}}\left(\frac{\partial}{\partial\bar{\Theta}}\right) + \boldsymbol{i}\hbar\partial\Theta = {}^{\boldsymbol{t}}\boldsymbol{C}\frac{\partial}{\partial\Theta} + \boldsymbol{i}\hbar\partial\Theta \tag{81}$$

$$\bar{\mathcal{D}} = {}^{\boldsymbol{t}}\left(\boldsymbol{C}\mathcal{D}\right) = {}^{\boldsymbol{t}}\left(\frac{\partial}{\partial\Theta}\right) - \boldsymbol{i}\hbar\bar{\Theta}\partial \tag{82}$$

with the Dirac spinor representation

$$\left\{\mathcal{D}_\ell, \bar{\mathcal{D}}_{\bar\ell}\right\} = 2\boldsymbol{i}\hbar\left(\quad \right)_{\ell\bar\ell}\partial \tag{83}$$

We have the commutation relationships

$$\left\{\mathcal{D}, \mathcal{Q}\right\} = 0 \tag{84}$$

$$\left[\mathcal{D}, \varDelta\right] = 0 \tag{85}$$

The infinitesimal translation of the components of $\boldsymbol{S}$ should then be obtained as

$$\varDelta\boldsymbol{C} = \boldsymbol{i}\left(\bar{\ } \ {}_5 \ \right) \tag{86}$$

$$\varDelta \ = \left(-\hbar \ {}_5\partial\boldsymbol{C} - \boldsymbol{M} + \boldsymbol{i} \ {}_5\boldsymbol{N} + \boldsymbol{i}\boldsymbol{V}\right) \tag{87}$$

$$\varDelta\boldsymbol{M} = -\left(\bar{\ }\left(\varLambda - \boldsymbol{i}\hbar\partial \ \right)\right) \tag{88}$$

$$\varDelta\boldsymbol{N} = \boldsymbol{i}\left(\bar{\ } \ {}_5\left(\varLambda - \boldsymbol{i}\hbar\partial \ \right)\right) \tag{89}$$

$$\varDelta\boldsymbol{V} \ = \left(\bar{\ } \ \varLambda\right) - \boldsymbol{i}\hbar\left(\bar{\ }\partial \ \right) \tag{90}$$

$$\varDelta\varLambda = \left(\frac{1}{2}\hbar\left[\partial \ \boldsymbol{V}, \ \right] + \boldsymbol{i} \ {}_5\boldsymbol{D}\right) \tag{91}$$

$$\varDelta\boldsymbol{D} = \boldsymbol{i}\left(\bar{\ } \ {}_5\left(-\boldsymbol{i}\hbar\partial\varLambda\right)\right) \tag{92}$$

The supercurrent is defined as

$$\Theta \ = \frac{\boldsymbol{i}}{12}\sum_{\boldsymbol{n}}\left(-4\hbar^2\left(\Phi_{\boldsymbol{n}}{}^\dagger\partial \ \Phi_{\boldsymbol{n}} - \Phi_{\boldsymbol{n}}\partial \ \Phi_{\boldsymbol{n}}{}^\dagger\right) - \boldsymbol{i}\hbar\left(\left(\bar{\mathcal{D}}\Phi_{\boldsymbol{n}}{}^\dagger\right) \ \left(\mathcal{D}\Phi_{\boldsymbol{n}}\right)\right)\right)\times\boldsymbol{c} \tag{93}$$

where $\Phi_{\boldsymbol{n}}$ is the chiral superfield

$$\begin{aligned}\Phi_{\boldsymbol{n}} = {}& \varPhi_{\boldsymbol{n}} - \sqrt{2}\left(\bar\varTheta\frac{1+ \ {}_5}{2} \ \varPsi_{\boldsymbol{n}}\right) + \left(\bar\varTheta\frac{1+ \ {}_5}{2} \ \varTheta\right)\boldsymbol{F}_{\boldsymbol{n}} - \frac{1}{2}\boldsymbol{i}\hbar\left(\bar{\ }\varTheta \ {}_5\varTheta\right)\partial \ {}_{\boldsymbol{n}}\varPhi \\ & + \frac{1}{\sqrt{2}}\boldsymbol{i}\hbar\left(\bar\varTheta \ {}_5\varTheta\right)\left(\bar\varTheta\frac{1-\ {}_5}{2}\partial \ \varPsi_{\boldsymbol{n}}\right) + \frac{1}{8}\hbar^2\left(\bar{\ }\varTheta \ {}_5\varTheta\right)^2\Box \ {}_{\boldsymbol{n}}\varPhi\end{aligned} \tag{94}$$

The supercurrent conservation laws are found to be

$$\mathcal{D}\Theta \ \left(\boldsymbol{x}\right) = \mathcal{D}\boldsymbol{X}\left(\boldsymbol{x}\right) \tag{95}$$

$$\bar{\mathcal{D}}\Theta \ \left(\boldsymbol{x}\right) \ = -\bar{\mathcal{D}}\boldsymbol{X}\left(\boldsymbol{x}\right) \tag{96}$$

where $\boldsymbol{X}$ denotes the real chiral superfield

$$\begin{aligned}\boldsymbol{X} = {}& \boldsymbol{A} - \left(\bar\varTheta\varPsi\right) \\ & - \frac{1}{2}\boldsymbol{i}\left(\bar\varTheta \ {}_5\varTheta\right)\boldsymbol{G} + \frac{1}{2}\left(\bar{\ }\varTheta\varTheta\right)\boldsymbol{F} + \frac{1}{2}\hbar\left(\bar{\ }\varTheta \ {}_5 \ \varTheta\right)\partial \ \boldsymbol{B} \\ & + \left(\bar\varTheta \ {}_5\varTheta\right)\left(\bar{\ }\varTheta\left(-\frac{1}{2}\boldsymbol{i}\hbar \ {}_5\partial \ \varPsi\right)\right) + \frac{1}{8}\hbar^2\left(\bar{\ }\varTheta \ {}_5\varTheta\right)^2\Box\boldsymbol{A}\end{aligned} \tag{97}$$