Classification of the Landscape of F-theory Vacua over K3×K3 by Gauge Groups: Comparison of SO(10)-vacua and SU(5)-vacua as an Application

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In this doctoral thesis, we study the flux compactification of F-theory, and its application to landscape problem. By its nature, the present thesis is based on the papers [1, 2] in collaboration with A.P. Braun and T. Watari. Accordingly, the basic set-up of this paper is the same as [1]. The author reviews his main contributions in the papers [1, 2], then he states his original results of this doctoral thesis at the final section. We limit our attention to the case where a compactifying space Y is the product of K3 surfaces.

When four-form flux $G^{(4)}$ is turned on, both the complex structure of the space and configuration of 7-branes are determined. The presence of flux $G^{(4)}$ stabilises the complex structure moduli. The Gukov-Vafa-Witten superpotential[3]

$$W_{GVW} \propto \int_Y \Omega \wedge G^{(4)} \tag{1}$$

generates F-term scalar potential. This F-term scalar potential attains local minimum along the locus where the rank of the intersection of cohomology groups

$$\mathrm{rk}\ [H^4(Y,\mathbb{Z}) \cap H^{2,2}(Y,\mathbb{R})] \tag{2}$$

enhances. When the four-form flux $G^{(4)}$ is turned on, vacua are confined to such local minima.

When elliptic Calabi-Yau 4-fold Y, as a compactifying space, is the product of K3 surfaces, the rank (2) enhances when the Picard number of a K3 surface increases. As Picard number of a K3 surface increases, the number of flat directions inversely decreases. Resultantly, moduli particles acquire masses in accordance. Therefore, the idealistic situation to stabilise moduli space is that a K3 surface has the highest Picard number. A K3 surface whose Picard number attains highest possible, being 20, is called an *attractive*[1] K3 surface. Therefore, with the presence of flux $G^{(4)}$, the moduli stabilises to discrete points which correspond to the product of attractive K3 surfaces [4].

Turning on four-form flux $G^{(4)}$ generates the ensemble of vacua. Because 7-branes control the information of gauge groups, the presence of four-form flux $G^{(4)}$ also specifies gauge groups. Interestingly, a K3 surface, whose complex structure is fixed, admits several *distinct* elliptic fibrations, leading to different gauge groups on a 7-brane. So, distinct gauge groups resulting from different fibrations on a K3 surface, whose complex structure is fixed, correspond to different vacua over the landscape. There is a mathematical technique called *Kneser-Nishiyama method*[5, 6], to determine all the gauge groups on a K3 surface (when whose complex structure is specified). Therefore, taking statistics of the distribution of gauge groups over the landscape of vacua is possible via Kneser-Nishiyama method, when a compactifying space is the product of K3 surfaces.

The present doctoral thesis is structured as follows: we review some mathematical notions and facts needed to investigate the distribution of gauge groups over the F-theory landscape in section 2. Flux compactification of M-theory on K3×K3 was first studied in [7]. [4] imposed some physical constraints on the four-form flux to specify all possible pairs of K3 surfaces on which M-theory is compactified, so that the complex structure moduli stabilises. They find that such are all some pairs of attractive K3's. In [1], the collaborators and the present author relaxed their constraints to extend the list of pairs of attractive K3 surfaces. List of

[1] We follow the convention of the term as in [8].

K3 surfaces considered in [4], and the extension of the list as examined in [1] will be discussed in section 3. In [1], we found that there are 98 pairs of attractive K3 surfaces for F-theory compactification. Section 4 studies relationships between gauge groups and elliptic fibrations. How gauge groups can be read from elliptic fibration will be explained. This is a well-known subject, and this section serves as a review. Section 5 deals with Kneser-Nishiyama method; this section explores how mathematical technique decodes physical information such as gauge groups, on a K3 surface whose complex structure is specified. In section 6, the author studies the ratio of SO(10)-vacua and SU(5)-vacua, using Kneser-Nishiyama method. Some physical conditions are imposed on the four-form flux $G^{(4)}$, to practically perform this analysis.

References

[1] A.P. Braun, Y. Kimura, and T. Watari, *The Noether-Lefschetz problem and gauge-group-resolved landscapes: F-theory on K3×K3 as a test case*, *JHEP* **04** (2014) 050 [arXiv: 1401.5908].

[2] A.P. Braun, Y. Kimura, and T. Watari, *On the classification of Elliptic Fibrations modulo Isomorphism on K3 surfaces with large Picard Number*, [arXiv: 1312.4421].

[3] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau Four-Folds*, *Nucl. Phys.* **B584** (2000) 69–108 [hep-th/9906070].

[4] P.S. Aspinwall and R. Kallosh, *Fixing all moduli for M-theory on K3×K3*, *JHEP* **10** (2005) 001 [hep-th/0506014].

[5] K.-I. Nishiyama, *The Jacobian fibrations on some K3 surfaces and their Mordell-Weil groups*, *Japan. J. Math.* **22** (1996), 293–347.

[6] K.-I. Nishiyama, *A remark on Jacobian fibrations on K3 surfaces*, *Saitama Math. J.* **15** (1997), 67–71.

[7] K. Dasgupta, G. Rajesh and S. Sethi, *M theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [hep-th/9908088].

[8] G.W. Moore, *Les Houches lectures on strings and arithmetic*, [hep-th/0401049].