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“The Effects of Child Mortality Changes  
on Two Income Groups and Macroeconomics”

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# The Effects of Child Mortality Changes on Two Income Groups and Macroeconomics\*

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**Abstract:** This study considers uncertainty about the number of surviving children. Under prohibition of child labor, a higher survival probability of a child can increase opportunities for education investment as the fertility rate declines with a temporary increase in the number of surviving children. However, per capita GDP may decrease until the start of education investment. When people accumulate their human capital, the effect of an increase in the survival probability would be ambiguous. By applying panel estimation, we examine economic development with the effect of child mortality on the fertility rate and number of surviving children.

**Keywords:** Child mortality; Uncertainty about the number of surviving children; Escape from poverty by poor people; Child labor; Panel estimation.

**JEL Classification:** I10, I18, J13, O15, O50.

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## 1. Introduction

As shown in Table 1, for sub-Saharan African economies, child and adult mortality rates are still high. In these economies, with low secondary school enrollment rates, fertility rates also remain high whereas poverty head count ratios show the prevalence of poverty. Significant gaps now exist in per capita GDP between sub-Saharan African economies and East Asian and Pacific economies. The governments of developing economies including sub-Saharan African economies combat diseases that have high child mortality rates.<sup>1</sup> It would be natural to consider that uncertainty about the number of surviving children, a low opportunity cost of child rearing, and the existence of child labor cause high fertility rates, which slow the rate of economic development. Thus, when we evaluate reductions in the child mortality rates by health challenges, it would be worthwhile considering uncertainty about the number of surviving children.

This paper develops a model where parents decide on their fertility rate and whether they will make their children work as child labor or provide education for their children before the uncertainty about child mortality is realized. They will decide the level of education for their surviving children, whereas it implies the decision after the uncertainty is realized. Because we assume that the survival probability of a child depends on the income level of his or her parents, it implies a high child mortality rate of poor people and a low child mortality rate of rich people. In our model, there are income level thresholds for the start of education investment because of convex educational expenditure and the existence of child labor.

Uncertainty about the number of surviving children decreases the expected utility via the variance, while the uncertainty also affects the income of child labor. When the survival probability of a child of poor people and the productivity of child labor are high, an increase in the survival probability can increase the fertility

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<sup>1</sup> Child mortality is caused primarily by infection, pneumonia, malaria, diarrhea, and malnutrition. In developing economies, a half of child mortality under five years old is caused in a day of birth. See WHO (2013).

rate. Thus, it could prevent poor people's escape from poverty through education investment. Under the prohibition of child labor, an increase in the survival probability of a child decreases the fertility rate but temporarily increases the number of surviving children that increases the expected utility level. It can help poor people to afford education for their children. Because an increase in the number of the poor's surviving children can increase the population ratio of poor to rich people, per capita GDP may decrease even with the decrease in the poor's fertility rate until the start of education investment. Further, an increase in the wage rate could be more effective in helping the poor to escape poverty than health investment if the survival probability depends on the income level. When people accumulate their human capital, an increase in the survival probability has ambiguous effects on their fertility rate and human capital level at the stable steady state.<sup>2</sup>

The remainder of this paper is organized as follows. The next section places the study in the context of the existing literature. Section 3 describes our model. We investigate how declines in the child mortality rate affect the fertility and education decisions of rich and poor people. We also see macroeconomics. In Section 4, we examine the panel estimation results. We conclude in Section 5 with some policy implications.

## 2. Related literature

Zeira (1998) and Moav (2002) examined the evolution of income distributions by assuming credit constraints and convex educational expenditure. They showed polarization of income distributions with persistent poverty. We extend their models by considering fertility under uncertainty about the number of surviving children. Because we examine whether an increase in the survival probability of a child can help poor people start their education investment, we can evaluate policies designed to decrease child mortality rates. Furthermore, we examine how the difference in

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<sup>2</sup> Education investment per child may not increase because of an increase in the number of surviving children. However, an increase in the wage rate can decrease the fertility rate but increase the human capital level.

the numbers of surviving children between rich and poor people affects economic development.<sup>3</sup>

Kalemli-Ozcan (2008) developed a model in which parents decide on their fertility rate before the uncertainty is realized, but they choose to invest only in the human capital of their surviving children. By extending his model, this paper examines how an increase in the survival probability of a child helps poor people to escape poverty by starting education investment. We allow the decision of no educational expenditure under uncertainty about child mortality.<sup>4</sup>

Some empirical controversy surrounds the effects of declining mortality.<sup>5</sup> Using panel estimation, we found positive effects for the child mortality rate on the fertility rate and the number of surviving children but a negative effect on education investment. We also found a negative effect for the number of surviving children on per capita GDP.

### 3. Model

#### 3.1 Fertility and education decisions under uncertainty

In an overlapping-generations economy, individuals can live for two periods if they survive. We consider child mortality before children work as child labor or go to school. When parents make their children work as child labor, they obtain their own labor income and child labor income. They determine the education level of their surviving children after the uncertainty about the number of surviving children is

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<sup>3</sup> See also Galor and Moav (2002), Kremer and Chen (2002), de la Croix and Doepke (2003), Doepke (2004), and Nakamura and Seoka (2014) that examined economic development with differential fertility.

<sup>4</sup> Cigno (1998), Kalemli-Ozcan (2002), Strulik (2004a,b), Soares (2005), Galor (2005), and Azarnert (2006) examined the effect of child mortality decline. Hazan and Berdugo (2002), Doepke (2004), Strulik (2004a,b), Moav (2005), and Load and Rangazas (2006) explored development with child labor.

<sup>5</sup> See Load and Rangazas (2006), Acemoglu and Johnson (2007), Weil (2007), Lorentzen, McMillan, and Wacziarg (2008), Angeles (2010), Cervellati and Sunde (2011), Galor (2012), and Nakamura and Mihara (2014).

realized, only if they decide that their children go to school, but not work as child labor. Rich people, but not poor people, are assumed to be educated initially.

We first describe the utility function of parents, which depends on their consumption level, the number of surviving children, and the education level of their children by allowing zero education investment:

$$U_{it} \equiv \gamma \ln c_{it} + \eta \ln N_{it} + (1 - \gamma - \eta) \ln(o + e_{it}N_{it}), \quad (1)$$

$$s.t. \quad (1 - zn_{it})h_{it}w + (1 - x_{it})\theta wN_{it} = c_{it} + x_{it}e_{it}N_{it}, \quad (2)$$

where  $i = r, p$ .  $r$  and  $p$  respectively represent rich and poor people. We assume  $0 < \gamma < 1$ ,  $0 < \eta < 1$ ,  $0 < 1 - \gamma - \eta < 1$ , and  $0 < o$ .  $U_{it}$  is the utility level,  $c_{it}$  is the consumption level,  $N_{it}$  is the number of surviving children,  $e_{it}$  is the education level,  $n_{it}$  is the fertility rate,  $h_{it}$  is the human capital level of a parent, and  $w$  is the wage rate.<sup>6</sup>  $x_{it}$  represents the decision with respect to work or education of children and takes a value of zero or unity.

We assume that the amount of educational expenditure increases the parents' utility level because it increases the human capital level of their children.<sup>7</sup> We describe the human capital level as follows:

$$h_{it+1} = (1 + e_{it})^\delta, \quad (3)$$

where  $i = r, p$ . We assume  $0 < \delta < 1$ .

The human capital level is still positive even without education investment. The decision  $x_{it} = 0$  implies that  $e_{it} = 0$ . Parents choose the education level with the decision  $x_{it} = 1$ .

Because child mortality rates would be highly correlated with the income level *ex post*, we assume that the survival probability of a child depends positively on the

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<sup>6</sup> We consider a linear production technology with perfect substitutability of labor; that is, the wage rate is equal to the productivity of labor.

<sup>7</sup> If we assume that the parents' utility level depends on the human capital level of their children, then the consumption level and total amount of educational expenditure would depend on the number of surviving children. The analysis would be then complicated.

income level:

$$q_{it} = q(h_{it}w), \quad (4)$$

where we assume  $0 < q_{it} < 1$ ,  $\frac{\partial q_{it}}{\partial h_{it}w} > 0$  and  $\frac{\partial^2 q_{it}}{\partial (h_{it}w)^2} < 0$ .

Thus, a low level of human capital implies a low survival probability of a child. The number of surviving children is distributed as a binomial distribution:

$$\Phi(N_{it}; n_{it}, q_{it}) \equiv \binom{n_{it}}{N_{it}} q_{it}^{N_{it}} (1 - q_{it})^{n_{it} - N_{it}}. \quad (5)$$

We now examine the utility maximization problem. When parents decide their consumption level and fertility rate, they face uncertainty about the number of surviving children. They decide the education level of their children after the uncertainty is resolved. Thus, we use backward induction to solve the utility maximization problem. We first examine that parents can afford education for their children. Given that uncertainty is resolved, individuals maximize their utility level by education investment:

$$\max_{e_{it}} U(e_{it} | n_{it}, N_{it}, x_{it} = 1), \quad (6)$$

where the utility level depends on the level of education in which the utility level is conditional on the fertility rate, the number of surviving children, and the decision of education investment:

$$U(e_{it} | n_{it}, N_{it}, x_{it} = 1) \equiv \gamma \ln[(1 - zn_{it})wh_{it} - e_{it}N_{it}] + \eta \ln N_{it} + (1 - \gamma - \eta) \ln(o + e_{it}N_{it}).$$

The education level is represented as follows:

$$e_{it} = \frac{(1 - \gamma - \eta)(1 - zn_{it})wh_{it} - \gamma o}{(1 - \eta)N_{it}}. \quad (7)$$

Whereas the expenditure of education is convex with respect to the income level, it is divided equally among the surviving children. The ratio of educational expenditure to income increases with an increase in the income level.

Income is allocated to consumption and educational expenditure, and thus we have the following consumption level:

$$c_{it} = \frac{\gamma(1 - zn_{it})wh_{it} + \gamma o}{1 - \eta}. \quad (8)$$

Both the consumption level and the total amount of educational expenditure depends on the fertility rate, but not on the number of surviving children. That is, a decision of the fertility rate implies the allocation between consumption and educational expenditure.

The education level in (7) and the consumption level in (8) imply the following utility level:

$$U_{it}|_{x_{it}=1} = (1 - \eta) \ln[(1 - zn_{it})wh_{it} + o] + \eta \ln N_{it} + D_e, \quad (9)$$

where  $D_e \equiv \ln \frac{\gamma^\gamma (1 - \gamma - \eta)^{1 - \gamma - \eta}}{(1 - \eta)^{1 - \eta}}$ .

By considering uncertainty about the number of surviving children, we can represent the expected utility as:

$$E(U_{it}|_{x_{it}=1}) = \sum_{N_{it}=0}^{n_{it}} U(n_{it}, N_{it}|_{x_{it}=1}) \Phi(N_{it}; n_{it}, q_{it}). \quad (10)$$

By use of the delta method which is a Taylor expansion of the second order around the mean, the expected utility in (10) can be approximated as follows:

$$E(U_{it}|_{x_{it}=1}) = (1 - \eta) \ln[(1 - zn_{it})wh_{it} + o] + \eta \ln n_{it} q_{it} + D_e - \eta \frac{1 - q_{it}}{2q_{it}n_{it}}. \quad (11)$$

The expected utility level increases with the mean in the number of surviving children but decreases with the variance. The last term represents the variance which implies the risk about the child mortality. An increase in the fertility rate or the survival probability decreases the variance.

We obtain the fertility rate by maximizing the expected utility in (11). Given the parental human capital level, the fertility rate satisfies:

$$F(n_{it}, h_{it}) \equiv -(1 - \eta) \frac{wh_{it}z}{(1 - zn_{it})wh_{it} + o} + \eta \frac{1}{n_{it}} + \eta \frac{1 - q_{it}}{2q_{it}n_{it}^2} = 0. \quad (12)$$



We assume  $n_{it} > 1$  to avoid a decrease in the population.

The fertility rate decision is affected crucially by the variance in the number of surviving children.<sup>8</sup> An increase in the income level decreases the fertility rate because of an increase in the opportunity cost of child rearing:

$$\frac{\partial n_{it}|_{x_{it}=1}}{\partial h_{it}w} = -\frac{\partial F(n_{it}, h_{it})/\partial h_{it}w}{\partial F(n_{it}, h_{it})/\partial n_{it}} < 0.$$

We next consider that parents make their children work as child labor; that is, they do not afford education for their children. The expected utility level is represented as follows:

$$E(U_{it}|_{x_{it}=0}) = \sum_{N_{it}=0}^{n_{it}} U(n_{it}, N_{it}|_{x_{it}=0})\Phi(N_{it}; n_{it}, q_{it}), \quad (13)$$

where  $U_{it}|_{x_{it}=0} = \gamma \ln[(1 - zn_{it})h_{it} + \theta n_{it}q_{it}] + \eta \ln N_{it} + (1 - \gamma - \eta) \ln o$ .

Using the delta method, the expected utility can be approximated as:

$$E(U_{it}|_{x_{it}=0}) = \gamma \ln[(1 - zn_{it})h_{it} + \theta n_{it}q_{it}] + \eta \ln n_{it}q_{it} + D_{ne} - \eta \frac{(1 - q_{it})}{2q_{it}n_{it}} - \gamma \frac{n_{it}q_{it}(1 - q_{it})\theta^2}{2[(1 - zn_{it})h_{it} + \theta n_{it}q_{it}]^2}, \quad (14)$$

where  $D_{ne} \equiv \ln w^\gamma o^{1-\gamma-\eta}$ .

The number of surviving children affects the utility level not only directly, but also indirectly through the income level because of child labor. The last term in (14) represents the variance in the number of surviving children via the income level. We obtain  $n_{it}$  by maximizing (14). The fertility rate satisfies:

$$G(n_{it}, h_{it}) = -\gamma \frac{zh_{it} - \theta q_{it}}{(1 - zn_{it})h_{it} + \theta n_{it}q_{it}} + \eta \frac{1}{n_{it}} + \eta \frac{(1 - q_{it})}{2q_{it}n_{it}^2} - \gamma \frac{q_{it}(1 - q_{it})\theta^2}{2[(1 - zn_{it})h_{it} + \theta n_{it}q_{it}]^2} \left[ 1 + \frac{2n_{it}(zh_{it} - \theta q_{it}n_{it})}{(1 - zn_{it})h_{it} + \theta n_{it}q_{it}} \right] = 0. \quad (15)$$

The variance in the number of surviving children affects the fertility rate with the productivity of child labor.

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<sup>8</sup> In the appendix, we consider the perfect foresight about the number of surviving children which yields the different result.

We assume that the opportunity cost of child rearing exceeds the income of child labor:

$$zh_{it} > \theta q_{it}. \quad (A1)$$

Parents cannot afford education for their children—that is, they choose  $x_{it} = 0$  when the marginal benefit of education cannot outweigh the marginal cost of education—then we have:

$$B(h_{it}w) \equiv (1 - \gamma - \eta)wh_{it}(1 - zn_{it}|_{x_{it}=1}) - \gamma o < 0. \quad (16)$$

Furthermore, even when  $B(h_{it}w) > 0$  holds, when the expected utility with education investment cannot outweigh the expected utility without education investment—then we have:

$$IC(h_{it}w) \equiv E(U_{it}|_{x_{it}=1}) - E(U_{it}|_{x_{it}=0}) < 0, \quad (17)$$

parents make their children work as child labor. Note that  $E(U_{it}|_{x_{it}=1})$  can be defined when  $B(h_{it}w) > 0$ .

### 3.2 How do child mortality declines affect the fertility and education decisions of poor and rich people?

In this subsection, we first examine the effect of an increase in the survival probability of a poor's child on their start of education investment. We consider the threshold in the wage rate at which the marginal benefit of education investment is equal to its marginal cost:

$$B(\hat{w}) = (1 - \gamma - \eta)\hat{w}(1 - zn_p|_{x_{pt}=0}) - \gamma o = 0, \quad (18)$$

where  $n_p|_{x_{pt}=0}$  is given by  $G(n_p, h_p)$  with  $q_p = q(\hat{w})$ .

When  $w < \hat{w}$  holds, we have  $B(w) < 0$ ; that is, it is impossible for poor people to afford education for their children because of that  $h_{p0} = 1$ . We consider an exogenous health investment on poor people, which can increase the survival probability of their

children. The survival probability of a poor's child is represented by  $q_p = q(w : v)$  in which  $v$  is health investment and we assume  $\frac{\partial q_p}{\partial v} > 0$ .<sup>9</sup>

The effect of an increase in the survival probability on the fertility rate is represented as  $\frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} = -\frac{\partial G(n_p, h_p)/\partial q_p}{\partial G(n_p, h_p)/\partial n_p}$ . Given the fertility rate, the increase in the survival probability can increase both the mean and variance in the expected utility through the income of child labor. When we have a low  $z - \theta q_p$  implied by a low cost of child rearing, a high productivity of child labor, and a high survival probability, given  $q_p \geq 1/2$  which implies a decline in the variance represented by  $n_p q_p (1 - q_p)$ , the increase in the survival probability can increase the fertility rate,  $\frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} > 0$  that can increase the income of child labor.

The effect of an increase in the productivity of child labor on the poor's fertility rate is represented as  $\frac{\partial n_p|_{x_{pt}=0}}{\partial \theta} = -\frac{\partial G(n_p, h_p)/\partial \theta}{\partial G(n_p, h_p)/\partial n_p}$ . When we have a low  $z - \theta q_p$ , the increase in the productivity of child labor can increase the fertility rate,  $\frac{\partial n_p|_{x_{pt}=0}}{\partial \theta} > 0$  because of the increases in the income of child labor.

**Proposition 1:** (*Effect of an increase in the survival probability on the fertility and education decisions of poor people in the presence of child labor*). Suppose that Assumption (A1) holds. An increase in the survival probability of a child may increase the threshold in the wage rate for the start of education investment because of the presence of child labor. Thus, it may not help poor people to start education investment.

**Proof:** The effect of an increase in the survival probability on  $\hat{w}$  is represented as  $\frac{\partial \hat{w}}{\partial q_p} = -\frac{\partial B(\hat{w})/\partial q_p}{\partial B(\hat{w})/\partial \hat{w}}$  in which

$$\frac{\partial B(w)}{\partial \hat{w}} = 1 - \gamma - \eta \quad \text{and} \quad \frac{\partial B(w)}{\partial q_p} = (1 - \gamma - \eta)wz \frac{\partial n_p|_{x_{pt}=0}}{\partial q_p}.$$

When we have a low level in  $z - \theta q_p$ ,  $\frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} > 0$  can hold. We then obtain  $\frac{\partial \hat{w}}{\partial q_p} > 0$ . ||

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<sup>9</sup> Increases in the numbers of skilled maternity nurses, improved distribution of maternity health records, and the administration of particle nutrients to pregnant mothers help to reduce maternal and child mortality rates. Vaccines, the administration of nutrients to children, and mosquito nets can decrease the child mortality rate.

An increase in the survival probability has an ambiguous effect on the incentive-compatibility condition for educational expenditure. Given a positive education investment, we have  $\frac{\partial n_p|x_{pt}=1}{\partial q_p} < 0$  and  $\frac{\partial q_p n_p|x_{pt}=1}{\partial q_p} > 0$ , whereas it increases the expected utility. Thus, the effect of the increase in the survival probability on the incentive-compatibility condition depends on the increases in the expected utility with and without education investment.

Let us assume that child labor is prohibited. The expected utility can be approximated as follows:

$$E(U_{pt}|x_{pt}=0) = \gamma \ln(1 - zn_p)w + \eta \ln n_p q_p + (1 - \gamma - \eta) \ln o - \frac{\eta(1 - q_p)}{2q_p} \frac{1}{n_p}. \quad (19)$$

By maximizing the expected utility, the fertility rate is given by:

$$G(n_p, h_p) = -\gamma \frac{z}{1 - zn_p} + \eta \frac{1}{n_p} + \eta \frac{1 - q_p}{2q_p} \frac{1}{n_p^2} = 0. \quad (20)$$

From (18), we have  $E(U_{pt}|x_{pt}=0) = E(U_{pt}|x_{pt}=1)$  at  $w = \hat{w}$  because of no child labor. Thus, when  $w > \hat{w}$ , we can have the incentive-compatibility condition,  $E(U_{pt}|x_{pt}=1) > E(U_{pt}|x_{pt}=0)$ .

An increase in the survival probability of a child always decreases the fertility rate because of no child labor; that is, we have  $\frac{\partial n_p|x_{pt}=0}{\partial q_p} = -\frac{\partial G(n_p, h_p)/\partial q_p}{\partial G(n_p, h_p)/\partial n_p} < 0$ . Furthermore, the increase in the survival probability increases the number of surviving children; that is, we have  $\frac{\partial q_p n_p|x_{pt}=0}{\partial q_p} > 0$ , whereas it implies an increase in the expected utility level.

**Lemma 1:** (*Comparison in the effects of survival probability on the fertility rates between with education investment and without education investment under the prohibition of child labor*). (a) *The fertility rate with education investment is lower than that with no education investment:  $n_p|x_{pt}=0 > n_p|x_{pt}=1$ .* (b) *An increase in the survival probability can decrease more rapidly the fertility rate without education investment than with education investment:  $-\frac{\partial n_p|x_{pt}=0}{\partial q_p} > -\frac{\partial n_p|x_{pt}=1}{\partial q_p}$ .* (c) *The elasticity of the fertility rate with respect to the survival probability can be higher with education investment than without education investment:  $-\frac{\partial \ln n_p|x_{pt}=1}{\partial q_p} > -\frac{\partial \ln n_p|x_{pt}=0}{\partial q_p}$ .*

**Proof:** (a) Let us temporarily assume that:

$$\frac{\gamma}{1 - zn_p|_{x_{pt}=0}} > \frac{(1 - \eta)w}{(1 - zn_p|_{x_{pt}=1})w + o}.$$

This implies  $n_p|_{x_{pt}=1} > n_p|_{x_{pt}=0}$ . Furthermore, we have:

$$(1 - \eta - \gamma)w(1 - zn_p|_{x_{pt}=0}) - \gamma o < \gamma w(n_p|_{x_{pt}=0} - n_p|_{x_{pt}=1}).$$

Although this implies  $(1 - \eta - \gamma)w(1 - zn_p|_{x_{pt}=0}) - \gamma o < 0$ , this contradicts  $B(w) > 0$  which implies the decision,  $x_{pt} = 1$ .

Thus, the following inequality holds:

$$\frac{\gamma z}{1 - zn_p|_{x_{pt}=0}} < \frac{(1 - \eta)wz}{(1 - zn_p|_{x_{pt}=1})w + o}, \quad (21)$$

which implies  $n_p|_{x_{pt}=1} < n_p|_{x_{pt}=0}$ .

(b) By use of (12) and (20), the effects of an increase in the survival probability on the fertility rates without education investment and with education investment, respectively, are represented as follows:

$$\begin{aligned} \frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} &= \frac{-\frac{\eta}{2q_p^2}}{\gamma^{-1}[\eta + \frac{\eta(1-q_p)}{2q_p n_p|_{x_{pt}=0}}]^2 + \eta + \frac{\eta(1-q_p)}{q_p n_p|_{x_{pt}=0}}}, \\ \frac{\partial n_p|_{x_{pt}=1}}{\partial q_p} &= \frac{-\frac{\eta}{2q_p^2}}{(1 - \eta)^{-1}[\eta + \frac{\eta(1-q_p)}{2q_p n_p|_{x_{pt}=1}}]^2 + \eta + \frac{\eta(1-q_p)}{q_p n_p|_{x_{pt}=1}}}. \end{aligned}$$

We have  $-\frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} > -\frac{\partial n_p|_{x_{pt}=1}}{\partial q_p}$  because  $n_p|_{x_{pt}=0} > n_p|_{x_{pt}=1}$  and  $1 - \eta - \gamma > 0$ .

(c) We obtain:

$$-\frac{\frac{\partial n_p|_{x_{pt}=1}}{\partial q_p}}{n_p|_{x_{pt}=1}} > -\frac{\frac{\partial n_p|_{x_{pt}=0}}{\partial q_p}}{n_p|_{x_{pt}=0}}, \quad (22)$$

because  $n_p|_{x_{pt}=0} > n_p|_{x_{pt}=1}$ ,  $1 - \eta - \gamma > 0$ , and  $\frac{\partial[\eta + \frac{\eta(1-q_p)}{2q_p n_p}]^2 n_p}{\partial n_p} > 0$  that holds with  $q_p \geq 1/3$ .  $\parallel$

We have  $\frac{\partial IC(w)}{\partial q_p} > 0$ ; that is, the incentive-compatible condition for education investment can hold more easily with an increase in the survival probability because  $n_p|_{x_{pt}=1} < n_p|_{x_{pt}=0}$ . We now examine how increases in health investment and labor productivity are effective for the start of education investment.

**Proposition 2:** (Effect of an increase in the survival probability on the fertility and education decisions of poor people under the prohibition of child labor). (a) An increase in the survival probability decreases the threshold in the wage rate through a decrease in the fertility rate. Thus, it can help poor people to start education investment. (b) Suppose that  $\frac{\partial q_p}{\partial w} = \frac{\partial q_p}{\partial v}$ . An increase in the wage rate helps poor people to start education investment more strongly than health investment does.

**Proof:** (a) We have  $\frac{\partial \hat{w}}{\partial q_p} = -\frac{\partial B(\hat{w})/\partial q_p}{\partial B(\hat{w})/\partial \hat{w}} < 0$  because

$$\frac{\partial B(w)}{\partial q_p} = (1 - \gamma - \eta)wz \frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} < 0.$$

(b) The effects of the wage rate and health investment on  $B(w)$  are, respectively,:

$$\frac{\partial B(w)}{\partial w} = (1 - \gamma - \eta) + (1 - \gamma - \eta)wz \frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} \frac{\partial q_p}{\partial w},$$

$$\text{and } \frac{\partial B(w)}{\partial v} = (1 - \gamma - \eta)wz \frac{\partial n_p|_{x_{pt}=0}}{\partial q_p} \frac{\partial q_p}{\partial v}.$$

Thus, when  $\frac{\partial q_p}{\partial w} = \frac{\partial q_p}{\partial v}$ , we obtain  $\frac{\partial B(w)}{\partial w} > \frac{\partial B(w)}{\partial v}$ .  $\parallel$

An increase in the wage rate can have both a direct effect through the income level and an indirect effect through the fertility rate. However, health investment has only an indirect effect through the fertility rate. Thus, the increase in the wage rate can help poor people to escape poverty more strongly than health investment can.

We next examine how an increase in the survival probability of a child affects the fertility and education decisions of rich people. Given the human capital of parents, an increase in the survival probability decreases their fertility rate and increases their number of surviving children; that is, we have  $\frac{\partial n_{rt}|_{x_{rt}=1}}{\partial q_{rt}} < 0$  and  $\frac{\partial q_{rt}n_{rt}|_{x_{rt}=1}}{\partial q_{rt}} > 0$ . Thus, their expected utility level can increase because of health investment:  $\frac{\partial E(U_{rt}|_{x_{rt}=1})}{\partial q_{rt}} > 0$ .

Using the law of large numbers, the average level of education investment can be represented as:

$$e_{rt} = \frac{(1 - \gamma - \eta)(1 - zn_{rt})wh_{rt} - \gamma o}{(1 - \eta)n_{rt}q_{rt}}. \quad (23)$$

An increase in the survival probability of a child can increase education investment:  $\frac{\partial e_{rt}}{\partial q_{rt}} > 0$  when the following inequality holds:

$$\frac{(1 - \gamma - \eta)wzh_{rt}\left(-\frac{\partial n_{rt}}{\partial q_{rt}}\right)}{(1 - \gamma - \eta)h_{rt}w(1 - zn_{rt}) - \gamma o} > \frac{\partial q_{rt}n_{rt}}{\partial q_{rt}}.$$

Because an increase in the survival probability decreases the fertility rate, an increase in the income level can increase educational expenditure. However, given the total amount of educational expenditure, an increase in the number of surviving children decreases the amount of educational expenditure per child. When the former effect outweighs the latter effect, education investment per child can increase because of an increase in the survival probability.

The dynamics of the human capital level can be represented as follows:

$$h_{rt+1} = \left[ \frac{(1 - \gamma - \eta)(1 - zn_{rt})wh_{rt} - \gamma o + (1 - \eta)n_{rt}q_{rt}}{(1 - \eta)n_{rt}q_{rt}} \right]^\delta \equiv h(h_{rt}, n_{rt}). \quad (24)$$

Note that  $\frac{\partial n_{rt}}{\partial h_{rt}} < 0$ .

We assume concavity in (24):

$$\frac{\partial h_{rt+1}}{\partial h_{rt}} > 0 \quad \text{and} \quad \frac{\partial^2 h_{rt+1}}{\partial h_{rt}^2} < 0. \quad (A2)$$

Assuming the existence of steady states, Assumption (A2) implies the unstable steady state represented by  $h^{**}$  and the stable steady state represented by  $h^*$ ; that is, we have  $\frac{\partial h_{rt+1}}{\partial h_{rt}}|_{h_r=h^{**}} > 1$  and  $\frac{\partial h_{rt+1}}{\partial h_{rt}}|_{h_r=h^*} < 1$ . We also obtain  $n^{**}$  and  $n^*$  which respectively, correspond with  $h^{**}$  and  $h^*$  ( $n^{**} > n^*$ ). As shown in Figure 1, given  $h_{r0} > h^{**}$ , the human capital level monotonically converges to  $h^*$ .

Let us examine the effects of an increase in the survival probability on the fertility rates and human capital levels at the steady states. We consider (12) and (24) at the steady states in which  $F(n_r, h_r) = 0$  and  $H(n_r, h_r) \equiv h - h(h_r, n_r) = 0$ . Thus, the comparative statics are represented as follows:

$$\begin{pmatrix} \frac{\partial n_r}{\partial v} \\ \frac{\partial h_r}{\partial v} \end{pmatrix} = -\frac{1}{\Omega(n_r, h_r)} \begin{pmatrix} \frac{\partial H(n_r, h_r)}{\partial h_r} \frac{\partial F(n_r, h_r)}{\partial v} - \frac{\partial F(n_r, h_r)}{\partial h_r} \frac{\partial H(n_r, h_r)}{\partial v} \\ -\frac{\partial H(n_r, h_r)}{\partial n_r} \frac{\partial F(n_r, h_r)}{\partial v} + \frac{\partial F(n_r, h_r)}{\partial n_r} \frac{\partial H(n_r, h_r)}{\partial v} \end{pmatrix}, \quad (25)$$

where  $\Omega(n_r, h_r) \equiv \frac{\partial F(n_r, h_r)}{\partial n_r} \frac{\partial H(n_r, h_r)}{\partial h_r} - \frac{\partial F(n_r, h_r)}{\partial h_r} \frac{\partial H(n_r, h_r)}{\partial n_r}$ . We have  $\frac{\partial F(n_r, h_r)}{\partial v} < 0$ ,  $\frac{\partial H(n_r, h_r)}{\partial v} > 0$ ,  $\frac{\partial H(n_r, h_r)}{\partial n_r} > 0$ ,  $\frac{\partial F(n_r, h_r)}{\partial h_r} < 0$ , and  $\frac{\partial F(n_r, h_r)}{\partial n_r} < 0$ .

Under Assumption (A2), we have  $\Omega(n_r, h_r) > 0$  at  $h_r = h^{**}$ . We assume that at  $h_r = h^*$ ,

$$\Omega(n_r, h_r) < 0. \quad (A3)$$

This assumption easily holds when we have a strong concavity in (24) at the stable steady state. We consider the different signs of  $\Omega(n_r, h_r)$  between the stable and unstable steady states.

We obtain ambiguous effects of an increase in the survival probability on the steady-state human capital levels at the stable steady state, even under Assumptions (A2) and (A3). Although the total amount of educational expenditure increases as a result of the increase in the survival probability, the effect on educational expenditure per a child is ambiguous because of an increase in the number of surviving children. Thus, the effects of an increase in the survival probability on the steady-state fertility rates are also ambiguous. However, under Assumptions (A2) and (A3), an increase in the wage rate decreases  $n^*$  but increases  $n^{**}$ , whereas an increase in the wage rate increases  $h^*$  but decreases  $h^{**}$ .<sup>10</sup>

**Proposition 3:** *(Effect of an increase in the survival probability on the fertility and education decisions of rich people). Suppose that Assumptions (A2) and (A3) hold. (a) Given the human capital level, health investment decreases the fertility rate. An increase in the survival probability has ambiguous effects on the fertility rate and human capital levels at the stable steady state. (b) An increase in the wage rate decreases the fertility rate but increases the human capital level at the stable steady state.*

### 3.3 Macroeconomics

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<sup>10</sup> An increase in the survival probability can decrease the fertility rate at the unstable steady state, although the effect on the human capital level is ambiguous. Additionally, an increase in the wage rate decreases the human capital level at the unstable steady state, although the effect on the fertility rate is ambiguous.



The total population includes the numbers of rich and poor people:

$$L_t = L_{rt} + L_{pt},$$

where  $L_t$  is the population,  $L_{rt}$  is the number of rich people who survive in the second period, and  $L_{pt}$  is the number of poor people who survive in the second period. That is, we have  $L_{rt} = N_{rt}L_{rt-1}$  and  $L_{pt} = N_{pt}L_{pt-1}$ .

We define the ratio of rich people to the total population as  $\lambda_t \equiv \frac{L_{rt}}{L_t}$ . Thus, the ratio of poor people to the total population is represented as  $1 - \lambda_t = \frac{L_{pt}}{L_t}$ .

When child labor is unavailable, per capita GDP is represented as follows:

$$y_t = w[\lambda_t h_{rt} + (1 - \lambda_t)]. \quad (26)$$

Per capita GDP depends on the human capital level of rich people and the population ratio of rich to poor people.

The dynamics of the ratio of rich people to poor people depend on the ratio of the number of poor to rich surviving children:

$$\frac{1 - \lambda_t}{\lambda_t} = g(h_{rt}) \frac{1 - \lambda_{t-1}}{\lambda_{t-1}}, \quad (27)$$

where  $g(h_{rt}) \equiv \frac{n_p q_p}{n_r q_r}$ .

While the fertility rate of rich people decreases with their accumulation of human capital, their fertility rate is always lower than that of poor people. However, the survival probability of rich children always exceeds that of poor children. We assume that the number of rich's surviving children falls below that of poor's surviving children:

$$g(h_{rt}) > 1. \quad (A4)$$

The population ratio of poor to rich people in the current period depends positively on the ratio of the number of poor to rich surviving children and the ratio of poor to rich people in the previous period. Under Assumption (A4), the population ratio of poor to rich people continues to diverge if poor people cannot start their education investment (see Figure 2). Thus, per capita GDP depends on the relative

growth rates of the human capital level of rich people and the population ratio of poor to rich people.

The population growth rate is represented as:

$$\frac{L_t}{L_{t-1}} = n_{rt}q_{rt}\lambda_{t-1} + n_pq_p(1 - \lambda_{t-1}). \quad (28)$$

Under Assumption (A4), the population growth rate continues to increase as long as poor people remain in poverty.

**Proposition 4:** *(An increase in the survival probability of poor's children and macroeconomics). Under Assumption (A4), the ratio of poor people to the total population increases when  $w < \hat{w}$ . An increase in the survival probability of poor's children may decrease per capita GDP even with a decrease in their fertility rate until they can accumulate their human capital.*

**Proof:** We have  $\frac{\partial q_p n_p |_{x_{pt}=0}}{\partial q_p} > 0$ . This implies  $\frac{\partial g(h_{rt})}{\partial q_p} > 0$ . ||

Furthermore, even if poor people start education investment, if its level does not exceed the low steady-state level,  $h^{**}$ , they still cannot accumulate their human capital.

#### 4. Panel estimation

In this section, by applying a panel estimation with fixed effects, we examine how mortality and health investment affect economic development. We examine both child and adult mortality to see their different effects. We first explore the effect of child and adult mortality rates on the fertility rate:

$$\ln n_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_m \ln mt_{it} + \beta_{am} \ln amt_{it} + \epsilon_{it}, \quad (29)$$

where  $i = 1, 2, \dots, U$  ( $U = 24$  for the low-income subsample and  $U = 70$  for the full sample) and  $t = 1990, 1995, 2000, 2005$ .  $n_{it+1}$  is the fertility rate,  $h_{it}$  is the average years of total schooling,  $mt_{it+1}$  is the child mortality rate between birth and five years of age,  $amt_{it}$  is the adult (male) mortality rate, which is the percentage of adults dying between the ages of 15 and 60, and  $\epsilon_{it}$  is an error term.<sup>11</sup>

<sup>11</sup> The data were derived from the World Bank, whereas educational data were from Barro and

Column [i] in Table 2 provides the results. In the low-income and full samples, the human capital level is negative and significant. This implies a trade-off between the number of children and their human capital level. The child mortality rate has a positive effect on the fertility rate in both samples, whereas the adult mortality rate has a negative effect.<sup>12</sup> Thus, in developing economies, people have a high fertility rate because of a high child mortality rate. A decline in the child mortality rate would decrease the fertility rate.

We explore the number of surviving children but not the fertility rate:

$$\ln qn_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_m \ln mt_{it} + \beta_{am} \ln amt_{it} + \epsilon_{it}, \quad (30)$$

where  $qn_{it+1} \equiv \frac{1000 - mt_{it+1}}{1000} n_{it+1}$  which is the number of surviving children.

Column [ii] in Table 2 provides the results. In the low-income and full samples, the human capital level has a negative effect on the number of surviving children. The child mortality rate is positive and significant in both samples. Thus, a decline in the child mortality rate can help to decrease the number of surviving children. The adult mortality rate is negative and significant in the low-income sample but not in the full sample.

We next examine the level of education investment:

$$\ln e_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_m \ln mt_{it} + \beta_{am} \ln amt_{it} + \epsilon_{it}, \quad (31)$$

where  $e_{it}$  is the gross secondary enrollment rate.

Column [iii] in Table 2 details the results. In the low-income and full samples, the human capital level in period  $t$  has a positive effect on the level of education investment in period  $t + 1$ . Furthermore, the child mortality rate has a negative

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Lee (2010). These data were chosen because of data availability. We chose a low-income subsample in which per capita GDP in 2010 was less than 3000 USD. We obtained the same results with the adult (female) mortality rate.

<sup>12</sup> The alternative hypotheses were  $\beta_m \neq 0$  and  $\beta_{am} \neq 0$ . The Hausman test has a  $\chi^2$  distribution in which the degrees of freedom are equal to the number of explanatory variables. Rejection implies the existence of correlations between the individual effects and explanatory variables.

effect in the low-income subsample but not in the full sample. Thus, a decline in the child mortality rate can help to increase education investment in developing economies. It is impossible to confirm a negative effect of the adult mortality rate in the two samples. The effect of a decline in the adult mortality rate on education investment may be weak.

We explore the effect of the number of surviving children on per capita GDP:

$$\ln y_{it+1} = \beta_i + \beta_s s_{it} + \beta_h \ln h_{it} + \beta_{qn} \ln qn_{it} + \epsilon_{it}, \quad (32)$$

where  $y_{it+1}$  is per capita GDP and  $s_{it}$  is per capita domestic savings.<sup>13</sup>

Column [iv] in Table 2 shows the results. In the low-income and full samples, the savings are positive but not significant. This might be because of negative savings in some economies. The human capital level is positive and significant in both samples. We obtained a negative effect of the number of surviving children on per capita GDP in the full sample but not in the low-income subsample. Although a decline in the child mortality rate is important for economic development, the number of surviving children may not decrease sufficiently in the less-developed economies. Finally, we use the neonatal mortality rate, which is the percentage of children dying in the first month, rather than the child mortality rate. As shown in column [v] in Table 2, by using the neonatal mortality rate, we obtain a negative effect of the number of surviving children on per capita GDP in both samples.

Next, we examine the effect of health investment on economic development. We first consider the effect of health investment on the child mortality rate:

$$\ln mt_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_{vu} \ln vpu_{it} + \beta_{vr} \ln vpr_{it} + \epsilon_{it}, \quad (33)$$

where  $i = 1, 2, \dots, U$  ( $U = 24$  for the low-income subsample and  $U = 70$  for the full sample which are the same with the estimation of (29)–(32)) and  $t = 1995, 2000, 2005$  because of the availability of data on health expenditure.  $vpu_{it}$  is per capita public health expenditure, and  $vpr_{it}$  is per capita private health expenditure.

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<sup>13</sup> We did not take the logarithm of savings because some observations are negative.

Column [i] in Table 3 shows the results. In the low-income and full samples, the human capital level has a negative effect on the child mortality rate. It may be appropriate to assume that the survival probability of a child depends on the parents' income level. Public health investment is negative and significant in both samples. Private health investment is also negative and significant in both samples. Thus, both public and private health investment may help to decrease the mortality rate.

We see the effect of health investment on the fertility rate:

$$\ln n_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_{vu} \ln vpu_{it} + \beta_{vr} \ln vpr_{it} + \epsilon_{it}. \quad (34)$$

Column [ii] in Table 3 shows the results. In the low-income and full samples, the human capital level has a negative effect on the fertility rate. Public health investment is negative and significant in the full sample but not in the low-income subsample. Private health investment is negative but not significant in both samples. Thus, public health investment may have a weak negative effect on the fertility rate.

We explore the effect of health investment on the number of surviving children:

$$\ln qn_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_{vu} \ln vpu_{it} + \beta_{vr} \ln vpr_{it} + \epsilon_{it}. \quad (35)$$

Column [iii] in Table 3 provides the results. The human capital level is negative and significant in the full sample but not in the low-income subsample. Public health investment is negative and significant in the full sample but not in the low-income subsample. Private health investment is negative but not significant in both samples. Thus, we have the same result for the fertility rate and the number of surviving children.

We examine the effect of health investment on education:

$$\ln e_{it+1} = \beta_i + \beta_h \ln h_{it} + \beta_{vu} \ln vpu_{it} + \beta_{vr} \ln vpr_{it} + \epsilon_{it}. \quad (36)$$

Column [iv] in Table 3 provides the results. In the low-income and full samples, the human capital level has a positive effect on education investment. Public health

investment is significant in the low-income sample. Private health investment is not significant in both samples.

We finally explore the effect of the number of surviving children on per capita GDP with health investment:

$$\ln y_{it+1} = \beta_i + \beta_s s_{it} + \beta_h \ln h_{it} + \beta_{qn} \ln qn_{it} + \beta_{vu} \ln vpu_{it} + \beta_{vr} \ln vpr_{it} + \epsilon_{it}. \quad (37)$$

Column [v] in Table 3 details the results. In the low-income and full samples, the savings are positive and significant. The human capital level is positive but not significant in both samples. The effect of the number of surviving children is negative and significant in the full sample but not in the low-income subsample. The number of surviving children may not decrease sufficiently in the less-developed economies. Public and private health investment both have negative and significant impacts on per capita GDP in both samples.

## 5. Concluding remarks

This study examined the escape from poverty by poor people under uncertainty about the number of surviving children. Under the prohibition of child labor, poor people can start education investment with a sufficient decline in the child mortality rate because of a decrease in their fertility rate. The population ratio of poor to rich people increases with an increase in the number of poor people's surviving children because of an increase in the survival probability of poor people. Thus, per capita GDP may decrease until poor people start education investment.

The government should continue decreasing the child mortality rate of poor people, even though it temporarily increases their number of surviving children. Furthermore, the government should prohibit child labor because its existence could prevent an escape from poverty by poor people. If it takes time to eliminate child labor, this could further disturb development because of an increasing population ratio of poor to rich people.

## Appendix.

In this appendix, we assume that parents know the number of surviving children with perfect foresight. The utility maximization problem can be rewritten as follows:

$$\max_{c_{it}, n_{it}, e_{it}, x_{it}} \gamma \ln c_{it} + \eta \ln q_{it} n_{it} + (1 - \gamma - \eta) \ln(o + q_{it} n_{it} e_{it}), \quad (B1)$$

$$s.t. \quad (1 - z n_{it}) w h_{it} + (1 - x_{it}) \theta w q_{it} n_{it} = c_{it} + x_{it} q_{it} n_{it} e_{it}, \quad (B2)$$

where  $i = r, p$ .

The marginal benefit of education investment by rich people outweighs its marginal cost when the following inequality holds:

$$(1 - \gamma - \eta) w h_{rt} - (\eta + \gamma) o > 0. \quad (B3)$$

Furthermore, when the incentive-compatibility condition for education investment holds, we have the following first-order conditions:

$$n_{rt} = \eta \frac{\eta(w h_{rt} + o)}{z w h_{rt}}, \quad (B4)$$

$$e_{rt} = \frac{(1 - \gamma - \eta) w h_{rt} - (\gamma + \eta) o}{q_{rt} n_{rt}}. \quad (B5)$$

There is no effect of the survival probability on the fertility rate. An increase in the survival probability decreases education investment:  $\frac{\partial e_{rt}}{\partial q_{rt}} < 0$ . Thus, a declining child mortality rate leads to a decline in education if child mortality occurs before schooling starts.<sup>14</sup>

When the marginal benefit of education investment by poor people falls below its marginal cost, it is impossible for them to afford education for their children. The fertility rate is represented as:

$$n_p = \frac{\eta}{(\gamma + \eta)(z - \theta q_p)}. \quad (B6)$$

An increase in the survival probability increases the fertility rate because of the existence of child labor:  $\frac{\partial n_p}{\partial q_p} > 0$ . Under the prohibition of child labor, an increase in the survival probability does not affect the fertility rate, but increases the number of surviving children. Thus, it is difficult for poor people to start education investment, regardless of the existence of child labor.

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<sup>14</sup> See also Azarnert (2006) and Ozcan (2008).

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Table 1. Economic growth, mortality, fertility, education, and poverty

<i>sub – Saharan countries</i>	1960s	1970s	1980s	1990s	2000s
<i>per capita GDP growth</i>	2.44	0.93	−1.01	−0.45	2.09
<i>child mortality rate</i>	254.61	216.36	185.33	167.65	127.68
<i>adult mortality ratio</i>	465.01	419.58	390.81	407.64	395.28
<i>fertility rate</i>	6.67	6.78	6.58	6.02	5.49
<i>secondary school enrollment</i>		13.76	21.94	24.09	33.25
<i>poverty headcount ratio</i>			55.30	58.00	52.93
<i>East Asian and Pacific countries</i>	1960s	1970s	1980s	1990s	2000s
<i>per capita GDP growth</i>	3.80	4.55	5.78	7.02	8.20
<i>child mortality rate</i>		91.98	64.47	50.67	30.08
<i>adult mortality ratio</i>	363.00	228.63	202.53	184.26	149.76
<i>fertility rate</i>	5.93	4.14	3.02	2.10	1.84
<i>secondary school enrollment</i>		43.81	36.30	51.73	69.32
<i>poverty headcount ratio</i>			63.01	40.76	22.19

*Note:* The data are from the World Bank. We calculate the averages between ten years wherever possible. Child mortality rate is the ratio of dying between birth and five years of age per 1000 live births. Adult mortality rate is the ratio of dying between the age of 15 and 60 years per 100 thousands live births. Secondary school enrollment is the gross rate. Poverty headcount ratio is represented at 1.25 dollar a day (% of population).

Table 2. Panel estimation, which considers mortality

$U = 24$	$[i] \ln n_{it+1}$	$[ii] \ln qn_{it+1}$	$[iii] \ln e_{it+1}$		$[iv] \ln y_{it+1}$	$[v] \ln y_{it+1}$
$\ln h_{it}$	-0.339 (-2.36*)	-0.376 (-2.67**)	0.748 (5.62**)	$s_{it}$	0.00354 (0.95)	0.00545 (1.51)
$\ln mt_{it}$	0.139 (2.47*)	0.176 (3.20**)	-0.172 (-3.31*)	$\ln h_{it}$	1.024 (4.72**)	0.762 (3.25**)
$\ln amt_{it}$	-0.300 (-2.19*)	-0.616 (-4.58**)	0.0807 (0.64)	$\ln qn_{it}$	-0.196 (-0.92)	-0.459 (-2.35**)
$\bar{R}^2, H$	0.968, 4.83	0.969, 7.60	0.921, 8.37*		0.915, 6.13	0.921, 8.37*
$U = 70$	$[i] \ln n_{it+1}$	$[ii] \ln vn_{it+1}$	$[iii] \ln e_{it+1}$		$[iv] \ln y_{it+1}$	$[v] \ln y_{it+1}$
$\ln h_{it}$	-0.364 (-4.14**)	-0.384 (-4.20**)	1.010 (11.87**)	$s_{it}$	0.00148 (0.80)	0.00174 (0.96)
$\ln mt_{it}$	0.139 (4.04**)	0.197 (5.52**)	-0.0304 (-0.92)	$\ln h_{it}$	0.759 (5.57**)	0.627 (4.50**)
$\ln amt_{it}$	-0.156 (-2.46*)	-0.0642 (-0.97)	-0.018 (-0.84)	$\ln qn_{it}$	-0.662 (-5.78**)	-0.740 (-6.73**)
$\bar{R}^2, H$	0.964, 21.38**	0.963, 26.94**	0.977, 2.60		0.987, 18.43**	0.987, 15.71**

*Note:* The numbers in the parentheses are the  $t$ -values. \*, \*\* represent significance at the 5% and 1% levels, respectively.  $H$  is the Hausman test. We use child mortality rate in  $[iv]$  and neonatal mortality rate in  $[v]$ .

Table 3. Panel estimation, which considers health investment

$U = 24$	$[i] \ln m_{it+1}$	$[ii] \ln n_{it+1}$	$[iii] \ln qn_{it+1}$	$[iv] \ln e_{it+1}$		$[v] \ln y_{it+1}$
$\ln h_{it}$	-1.377 (-4.80**)	-0.479 (-2.70**)	-0.502 (-2.96**)	0.828 (4.76**)	$s_{it}$	0.00256 (0.88)
$\ln vpu_{it}$	-0.347 (-4.17**)	-0.00997 (0.19)	0.0946 (1.92)	0.115 (2.29*)	$\ln h_{it}$	0.460 (1.67)
$\ln vpr_{it}$	-0.191 (-5.98**)	-0.0252 (-1.45)	-0.0272 (-2.94**)	0.0111 (0.57)	$\ln vn_{it}$	-0.0237 (-0.11)
					$\ln vpu_{it}$	0.382 (5.32**)
					$\ln vpr_{it}$	0.102 (3.77**)
$\bar{R}^2, H$	0.965, 10.95*	0.976, 8.31*	0.979, 11.34**	0.981, 5.82		0.967, 10.88
$U = 70$	$[i] \ln m_{it+1}$	$[ii] \ln n_{it+1}$	$[iii] \ln vn_{it+1}$	$[iv] \ln e_{it+1}$		$[v] \ln y_{it+1}$
$\ln h_{it}$	-1.285 (-7.04**)	-0.306 (-2.72**)	-0.417 (-3.77**)	1.112 (10.63**)	$s_{it}$	0.00552 (3.35**)
$\ln vpu_{it}$	-0.249 (-5.13**)	-0.0645 (-2.16*)	-0.0701 (-2.39**)	0.0191 (0.69)	$\ln h_{it}$	0.228 (1.38)
$\ln vpr_{it}$	-0.192 (-6.42**)	-0.0143 (-0.77)	-0.0273 (-1.51)	0.00173 (0.10)	$\ln vn_{it}$	-0.297 (-2.43**)
					$\ln vpu_{it}$	0.280 (6.58**)
					$\ln vpr_{it}$	0.127 (4.89**)
$\bar{R}^2, H$	0.986, 21.13**	0.972, 15.08**	0.983, 1.37	0.983, 3.50		0.994, 99.09**

*Note:* The numbers in the parentheses are the  $t$ -values. \*, \*\* represent significance at the 5% and 1% levels, respectively.  $H$  is the Hausman test.

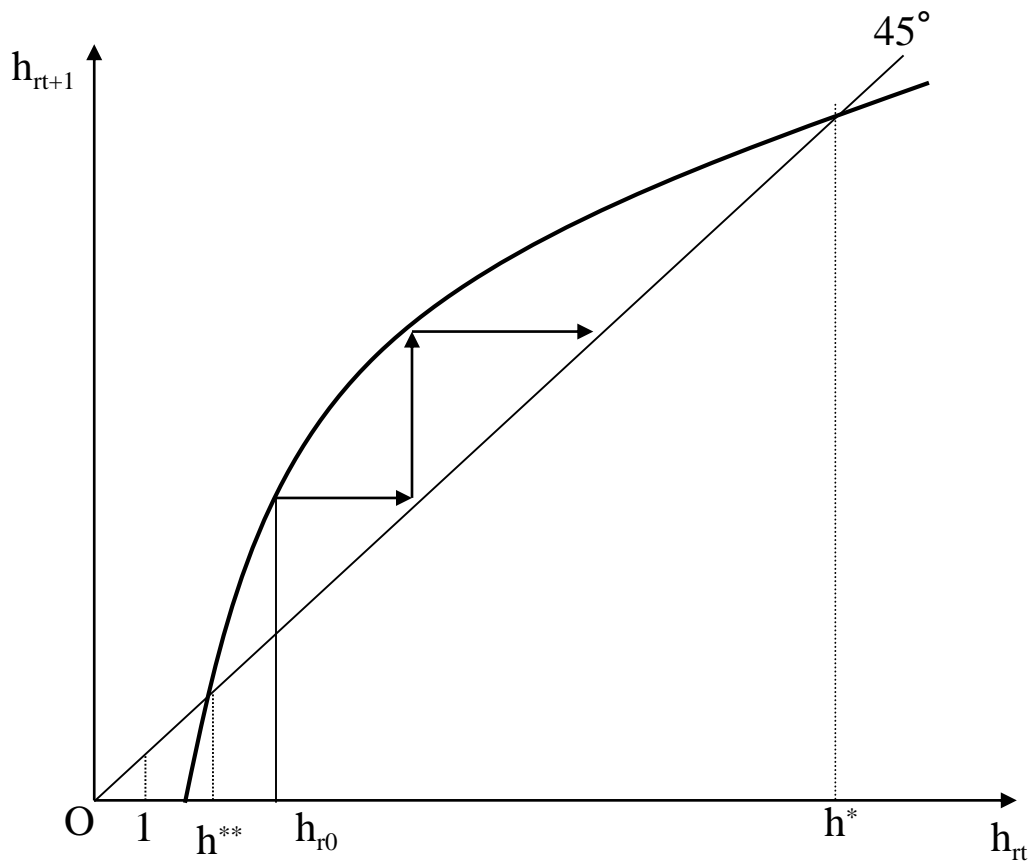


Figure 1. Dynamics of human capital level for rich people

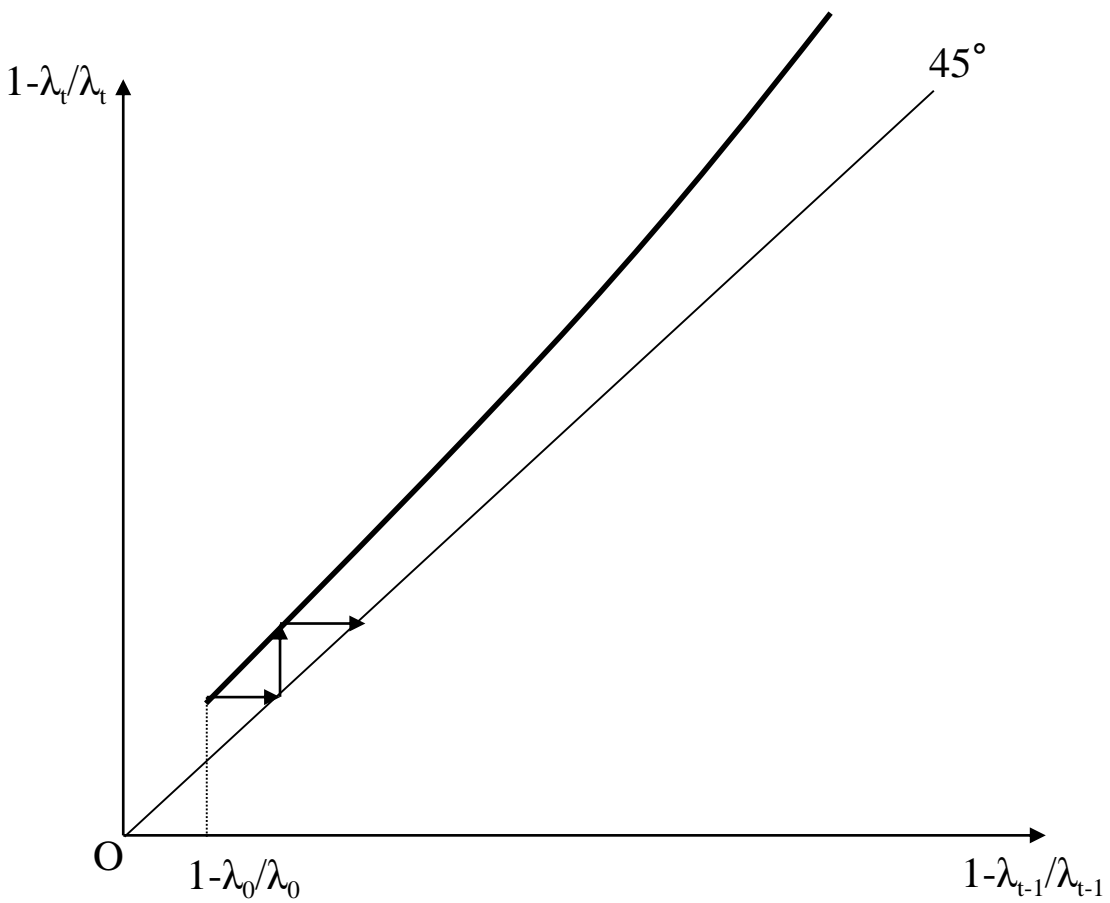


Figure 2. Dynamics of population ratio of poor to rich people