

# The Gribov problem beyond Landau gauge Yang-Mills theory

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Quantum chromodynamics (QCD) is the fundamental gauge theory of the strong interaction based on quarks and gluons. In low energy QCD, a variety of nonperturbative phenomena such as chiral symmetry breaking and color confinement appear.

Color confinement, which means that color non-singlet objects such as quarks and gluons are confined, plays an important role for the formation of hadrons. Although color confinement has been shown numerically using lattice simulations by calculating the Wilson loop, the confinement mechanism is still an open problem because of mathematical difficulties in the non-Abelian gauge theory.

In Landau gauge, scenarios for the confinement mechanism has been proposed by Kugo and Ojima, and Gribov and Zwanziger. According to the scenarios, confinement is related to the (deep) infrared behavior of the gluon propagator and the ghost propagator in Landau gauge. Gribov and Zwanziger suggested that, because of the effects of the Gribov region, which eliminates the remaining gauge degrees of freedom, the functional form of the gluon propagator is largely changed, and it approaches zero as the momentum decreases. This behavior shows the violation of the Kallen-Lehmann representation, and thus the gluon does not appear as a physical particle. From this point of view, the gluon propagator in Landau gauge Yang-Mills theory has been studied using lattice QCD calculations and analytical methods.

In this thesis, we study the confinement mechanism beyond Landau gauge Yang-Mills theory from the perspective of Gribov-Zwanziger scenario: maximally Abelian gauge (MAG) Yang-Mills theory and the Landau gauge Higgs model.

First, we construct the Gribov-Zwanziger action in  $SU(2)$  MAG, which eliminates the remnant gauge degrees of freedom from the QCD action by generalizing Zwanziger's work in Landau gauge. In MAG, it has been discussed from the perspective of the dual-superconductor picture, where the Abelian degree of freedom leads to the confinement because of monopole condensation. However, the Gribov-Zwanziger scenario in MAG has not yet been discussed enough. To investigate the scenario in MAG, we restrict the functional integral region to the Gribov region in MAG, give the nonlocal action, and localize it by introducing new fields. Then we obtain the final expression by performing the shift of the new scalar fields. Using this action, the diagonal gluon propagator in MAG at tree level behaves like the propagator of the Gribov-Zwanziger action in Landau gauge, while the off-diagonal gluon propagator does not show the influence.

Next, to investigate the Gribov-Zwanziger scenario in MAG, we study the diagonal and off-diagonal gluon propagators in MAG using  $SU(2)$  lattice simulations in two dimensions. The reason why we study two-dimensions is that in two-dimensional Landau gauge, the behavior of the propagator is well described by the propagator of the Gribov-Zwanziger action in accordance with the scenario. We calculate the gluon propagators and their dressing functions. In the infrared region, the transverse component in the diagonal propagator is suppressed with decreasing lattice spacing, while the transverse component in

the off-diagonal propagator hardly shows the dependence of the lattice spacing. Furthermore, the dressing function of the diagonal propagator shows a maximum on a fine lattice, while the transverse off-diagonal propagator shows a monotonically increasing behavior. These behavior supports that the violation of the Kallen-Lehmann representation is visible for the diagonal gluon, consistent with the Gribov-Zwanziger scenario in MAG. Thus, the scenario is also valid for the case of MAG in two dimensions.

Finally we investigate the gluon propagator and the phase structure of the SU(2) Higgs model in two dimensions using lattice simulations. The lattice result show that the gluon propagator in Landau gauge would vanish at zero momentum in the infinite-volume limit. The behavior supports that the Gribov-Zwanziger scenario is also valid for the Higgs model in two dimensions. Furthermore this result is consistent with an analytical study which shows that in two dimensions, for the Landau gauge inside the Gribov region, the gluon propagator vanishes at zero momentum whether the action includes other fields or not.

To clarify the phase structure, we also calculate gauge-invariant quantities, in particular the static potential from the Wilson loop, the  $W$  propagator, and the plaquette expectation value. Our results suggest that a confinement-like region and a Higgs-like region appear even in two dimensions. In the confinement-like region, the static potential rises linearly, with string breaking at large distances, while in the Higgs-like region, it is of Yukawa type, consistent with the Higgs mechanism. The correlation length obtained from the  $W$  propagator has a finite maximum between these regions. The plaquette expectation value shows a smooth cross-over consistent with the Fradkin-Shenker-Osterwalder-Seiler theorem. From these results, we suggest that there is no phase transition in two dimensions.