Quantum Entanglement of Local Operators

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Abstract

In this thesis we study dynamical properties of (Rényi) entanglement entropies for locally excited states generated by acting local operators on ground states in conformal field theories (CFTs). We define the excesses of (Rényi) entanglement entropies $\Delta S_A^{(n)}$ by subtracting those for the ground states from those for locally excited states. Here we choose a half of the total space as the subsystem A. We find that their excesses are given by the correlation functions of local operators in the replica trick [1, 2, 3]. Therefore we compute $\Delta S_A^{(n)}$ as their correlation functions in Euclidean space. After computing them, we perform a specific Wick rotation to real time and investigate the time evolutions of $\Delta S_A^{(n)}$ in various CFTs. We find that their time evolutions can be interpreted in terms of quasi-particles. The pairs of quasi-particles, which are called entangled pairs, are generated by acting local operators and propagate at the speed of light. Quantum entanglement between quasi-particles of each pair can contribute to (Rényi) entanglement entropies. When both quasi-particles of each pair are included in the region A (or the complement of A), $\Delta S_A^{(n)}$ keep to vanish. When only one quasi-particle of each pair is included in the region A and another one is included in the region B, $\Delta S_A^{(n)}$ increase. Their time evolutions obey causality and they eventually approach some constants in free massless field theories and 2 dimensional rational CFTs [1, 2, 4]. The final values of $\Delta S_A^{(n)}$

depend on the details of local operators. If we act operators of the form $(\partial^i \phi)^k$ on the ground state in the free massless scalar field theories, the final values of $\Delta S_A^{(n)}$ are given by (Rényi) entanglement entropies for a finite dimensional Hilbert space whose reduced density matrices are given by the binomial distribution [2]. Here indexes i and k are power of derivative and that of $\partial\phi$ respectively. The final values of $\Delta S_A^{(n)}$ do not depend on the spacetime dimensions. Their final values can be acquired under the entangled pair interpretation. They can be obtained by using the replica trick. We find that the results obtained under the entangled pair interpretation are consistent with those obtained by using the replica trick. We call the final values of $\Delta S_A^{(n)}$ as the (Rényi) entanglement entropies of local operators. We find a sum rule for (Rényi) entanglement entropies of local operators. If we act various local operators on the ground state, the final value of $\Delta S_A^{(n)}$ for this locally excited state is given by the sum of (Rényi) entanglement entropies of each local operator. We also study the excesses of second (Rényi) entanglement entropy for the locally excited state generated by acting a composite operator $\phi\partial\phi$, which is constructed of two species operators ϕ and $\partial\phi$, on the ground state. For this operator, the final values of (Rényi) entanglement entropy depend on the spacetime dimensions. In large N free field theories, the excesses of $\Delta S_A^{(n)}$ for $n \geq 2$ are given by $\mathcal{O}(1)$ quantities [3]. On the other hand, the von Neumann limit ΔS_A is $\mathcal{O}(\log N)$. This behavior resembles confinement / deconfinement phase transition if we regard $1/n$ as an effective temperature. We find that after taking the large N (the large central charge c) limit, we are not able to take the von Neumann limit ($n \to 1$ limit) since the final values of $\Delta S_A^{(n)}$ diverge. Therefore we have to take the von Neumann limit (the $n \to 1$ limit) before taking the large N or c limit when we study ΔS_A in large N theories. In large N (large c) interacting CFTs in any dimensions, the late time values of $\Delta S_A^{(n)}$ keep to increases logarithmically with t. The coefficient of $\log t$ for $n \geq 2$ is proportional to conformal dimensions of local operators. On the other hand a holographic result in AdS_3/CFT_2 shows that S_A $(n = 1)$ is proportional to the central charge c. In this sense its coefficient is enhanced for $n = 1$. We also find that we need information beyond the large N or c limit in order to know whether they approach some constants or diverge at late time. If they approach some constants, there can be non-perturbative N effects (non-perturbative c effects).

References

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