Neoclassical transport and flow analysis in Heliotron J plasmas

Kenji Nishioka

Abstract

In this thesis, the neoclassical flow in Heliotron J (H-J) plasmas is studied. Since plasma flows are related to the suppression of the turbulence transport, they play important roles on improvement of plasma confinement properties. The bootstrap (BS) current, which is produced by neoclassical particle flow parallel to the magnetic field line, changes the equilibrium magnetic configuration. In previous researches, however, the multi-ion species effect on these quantities is not investigated in non-axisymmetric plasmas. In addition, when neutral beam (NB) injections are used for heating plasmas, parallel flow is driven by external momentum sources. Recently, plasma flows are investigated experimentally in H-J. Estimations of plasma flow velocities based on the neoclassical theory, however, has not been performed yet. In order to clarify the physical mechanism of plasma flows, the moment approach to solve the neoclassical transport matrix is applied for H-J plasmas.

For estimation of plasma flow, the monoenergetic viscosity coefficients obtained from solution of the drift kinetic equation (DKE) is required. For plasmas with complex magnetic geometry such as H-J, the numerical method is widely applied to solve the DKE. But the accuracy of the numerical solution degrades in collisional, collisionless, and large electric field limits. In order to resolve the difficulty, the method using the analytical solutions together with the numerical solutions is shown.

The neoclassical parallel ion flows in NB heated plasmas in H-J are estimated. In such plasmas, the parallel momentum balance and the resulting neoclassical transport matrix is changed due to external parallel momentum sources from NB. The method to include the effect of external momentum source into the neoclassical transport analysis consistently is proposed. In addition, this method is applied to the analysis of the measured parallel ion flows. It is found that the experimentally measured C^{6+} impurity flow velocities do not contradict clearly with the neoclassical estimations and the dependence of parallel flow velocities on the magnetic field ripples are similar in both results. Previous experimental researches show the bumpy field dependence of the BS current in H-J plasmas. In order to estimate effects of the ambipolar electric field and multi-ion species, the moment method is applied. This study shows that the effect of ambipolar electric field on the geometric factor of the BS current is not negligible. In addition, finite multi-ion species effect on the BS current is predicted.

1. Introduction

Nuclear fusion is one of the powerful energy sources to resolve the energy problems, because resources of the fusion are easily available and plentiful all over the world and fusion reaction does not emit CO_2 at all. For realization of nuclear fusion power generation, confinement of high temperature plasmas is a crucial factor.

For improvement of plasma confinement, analysis of plasma transport is one of the important tasks. Plasma transports are comprised of classical, neoclassical, and turbulent transport. The radial transport is dominated by the turbulent transport which is generated by fluctuations of magnetic fields, plasma temperatures, and so on. However, neoclassical transport, which is determined by the non-uniformity of the magnetic field strength in torus plasmas and Coulomb collisions, has roles on determination of several parameters such as parallel plasma flows and the bootstrap (BS) current. Parallel plasma flows yields the suppression of turbulent transport. The BS current, which is produced by parallel flow of charged particles, changes the plasma confinement property through the magnetic geometry. Especially in non-axisymmetric plasmas, it is considered that the contribution of the neoclassical transport is more important than that in axisymmetric plasmas. Radial transport in non-axisymmetric collisionless plasmas can be extremely large compared to that in axisymmetric plasmas due to trapped particles by helical ripples. Moreover, neoclassical transport theory contributes to the determination of the radial electric field. Since radial transport produced by non-axisymmetric components breaks the charge neutrality without radial electric field, the radial electric field which satisfies the ambipolar condition is determined by the neoclassical theory in non-axisymmetric plasmas. These demands require us to estimate neoclassical transport and flow.

Here, previous studies of parallel transport in Heliotron J (H-J) are shown. Dependence of BS currents on bumpy field component is clarified experimentally [1]. Such dependency is confirmed by the numerical estimation using the BSC code [2]. In addition to this, recently, measurement of parallel ion flow in neutral beam (NB) heated plasmas is carried out [3]. This study shows that when tangential NB is injected, the parallel ion flow is driven. It is also found that NB-driven flow can be suppressed by changing the magnetic field configuration. However, the physical mechanism of plasma flows has not been clarified yet. Therefore, the parallel flow analysis within the neoclassical transport theory is carried out in this research.

In the neoclassical transport analysis of this study, several effects on the neoclassical transport theory are newly taken into account. First, when two or more ion species exist in plasmas, neoclassical flow is affected by the ion-ion friction. Moreover, since the neoclassical parallel flow is deeply related to neoclassical radial transport, effects of ambipolar electric field should be considered. Furthermore, when tangential momentum is injected by NB systems, the parallel momentum balance equation has to be re-defined. Such external parallel momentum sources changes not only parallel flow, but also radial transport by beam-driven particle fluxes. Including such effects into the neoclassical transport theory enables us to clarify the physical mechanism of plasma flows with more accuracy.

This thesis is organized as follows. In Section 2, neoclassical transport equations are introduced. In this thesis, the moment method proposed by Sugama-Nishimura [4,5] is applied to the neoclassical transport calculation in H-J plasmas. Section 3 shows the method to obtain the monoenergetic viscosity coefficients which are required to carry out the neoclassical transport analysis. The method to obtaining these coefficients in arbitrary collision frequencies and radial electric fields by applying the analytical methods together with numerical methods is proposed. Based on the obtained monoenergetic coefficients by this method, estimations in Sections 4 and 5 are carried out. Neoclassical parallel flow estimation in NB-heated plasmas is shown in Section 4. In order to estimate plasma flows in the NB-heated plasmas, the method to include external momentum input terms into conventional Sugama-Nishimura's method is proposed. This method is also applied to analyze the experimental results. The BS current investigations in H-J plasmas are carried out in Section 5. In previous researches on the BS current, the effects of multi-ion species and radial electric fields can be only partly treated. Consistent estimation of such effects on the BS current is carried out in this section. Finally, summary is shown in Section 6.

2. Neoclassical transport analysis

In this thesis, one of the moment approaches to estimate the neoclassical transport, the Sugama-Nishimura's moment method is applied. Neoclassical transport is produced by the deviation of particle distribution function f_{a1} from the local Maxwellian distribution function $f_{a\mathrm{M}}$. This deviation term is obtained from the drift kinetic equation (DKE). This DKE is given by [4,5]

$$(V_{\|} + V_E) f_{a1} - C_a^L(f_{a1}) = -\mathbf{v}_{da} \cdot \nabla f_{aM} + \frac{e_a}{T_a} v_{\|} B \frac{\langle BE_{\|} \rangle}{\langle B^2 \rangle} f_{aM} , \tag{1}$$

where $V_{\|}$ is the parallel orbit propagator, V_E is the $\mathbf{E} \times \mathbf{B}$ drift operator, C_a^{L} is the linearized collision operator, and $\mathbf{v}_{\mathrm{d}a}$ is the radial drift velocity. The moment approach [6,7] approximates the f_{a1} by the orthogonal polynomial expansion. Here, the Legendre polynomial and the Laguerre polynomials are applied in the expansion of the f_{a1} in the $\xi = v_{\|}/v$ space and the energy space, respectively. In this case, the expansion coefficients are obtained by taking the velocity moment of the distribution function. The $l = 1$ component, which denotes the parallel flow, is related to the break of parallel momentum conservation in the numerical method which uses the approximation in collision for simplicity, where l is the order of Legendre polynomials. The Sugama-Nishimura's moment method [4,5] is the method to obtain the $l = 1$ and the $l = 2$ components are obtained by the analytical method and the numerical method, respectively, where the $l = 2$ components are related to radial flux and parallel viscosity. The parallel momentum balance equation [8] is obtained by integrating the DKE for $l = 2$ components, as follows:

$$\left[-\begin{bmatrix} \mathbf{M}_a & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{M}_N \end{bmatrix} + \langle B^2 \rangle \begin{bmatrix} \mathbf{\Lambda}_{aa} & \cdots & \mathbf{\Lambda}_{aN} \\ \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_{Na} & \cdots & \mathbf{\Lambda}_{NN} \end{bmatrix} \right] \cdot \begin{bmatrix} \mathbf{U}_a \\ \vdots \\ \mathbf{U}_N \end{bmatrix} = \begin{bmatrix} \mathbf{N}_a & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{N}_N \end{bmatrix} \cdot \begin{bmatrix} \mathbf{X}_a \\ \vdots \\ \mathbf{X}_N \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_a \\ \vdots \\ \mathbf{Z}_N \end{bmatrix} \langle BE_{\|} \rangle , \tag{2}$$

where $\mathbf{X}_a$ is the thermodynamic forces, $\mathbf{Z}_a$ is the momentum driven by parallel electric field, $\mathbf{U}_a$ is the parallel flow, and $\mathbf{M}_a$, $\mathbf{N}_a$, and $\mathbf{\Lambda}_{ab}$ are the matrices related to the parallel viscosity, the viscosity due to thermodynamic forces, and the friction coefficients, respectively. The left hand side term indicates the damping terms due to the parallel viscosity and the friction. On the other hand, the right hand side term denotes the driving terms due to the thermodynamic forces and parallel electric fields. In this relation, components of $\mathbf{M}_a$, $\mathbf{N}_a$, and $\mathbf{\Lambda}_{ab}$ matrices are expressed by the moment expansions.

It should be noted that parallel plasma flows calculated only by Eq. (2) are not self-consistent because the matrices in Eq. (2) depend on the radial electric field. The radial electric field E_r which satisfies $\sum_a e_a \Gamma_a \left(E_r\right) = 0$ is determined by the neoclassical radial particle flux Γ_a in non-axisymmetric plasmas. Since matrices $\mathbf{M}_a$ and $\mathbf{N}_a$ the thermodynamic forces $\mathbf{X}_a$ depend on the E_r, it is necessary for consistent estimation of neoclassical parallel flows to calculate the radial particle fluxes and determine the ambipolar electric field. The neoclassical radial flux is obtained from the Flux-flow relation

$$\begin{bmatrix} \Gamma_a \\ -q_a / T_a \end{bmatrix} = \begin{bmatrix} \mathbf{N}_a^{\mathrm{Tr}} & \mathbf{L}_a \end{bmatrix} \cdot \begin{bmatrix} \mathbf{U}_a \\ \mathbf{X}_a \end{bmatrix}, \tag{3}$$

where q_a is the radial heat flux and $\mathbf{L}_a$ is the matrix related to the radial diffusion. Matrices $\mathbf{L}_a$, $\mathbf{M}_a$, and $\mathbf{N}_a$ are obtained from the solutions of the DKE. By solving Eqs. (2) and (3) simultaneously under the ambipolar condition, neoclassical transport calculation can be carried out consistently.

3. Monoenergetic viscosity coefficients of arbitrary collision frequencies and radial electric fields

In this section, the method to obtain the matrices $\mathbf{L}_a$, $\mathbf{M}_a$, and $\mathbf{N}_a$, which are used in Eqs. (2) and (3) is shown. These matrices are obtained by the energy integral, as follows:

$$\begin{aligned} \left[\mathrm{L}_{aij}, \mathrm{M}_{aij}, \mathrm{N}_{aij}\right] = n_a \frac{2}{\sqrt{\pi}} \int_0^\infty dK \sqrt{K} e^{-K} L_i^{(3/2)}\left(K\right) L_j^{(3/2)}\left(K\right) \\ \times \left[L^* L'\left(K\right), M^* M'\left(K\right), N^* N'\left(K\right)\right] \end{aligned}, \tag{4}$$

where $L_i^{(3/2)}\left(K\right)$ is the i-th order Sonine polynomials, $K = \left(v/v_{ta}\right)^2$, L^*, M^*, N^* are the non-energy dependent term in the integrand, and $L'\left(K\right), M'\left(K\right), N'\left(K\right)$ are the energy dependent term in the integrand. Here, L^*, M^*, N^* (so-called "monoenergetic viscosity coefficients") can be expressed by the solution of the DKE. In order to calculate the matrices accurately, wide range of monoenergetic coefficients in collision frequencies and radial electric fields are required.

In previous calculations, the DKES code [9,10] is used for obtaining the coefficients. The DKES calculation is consistent with the analytical estimation in H-J [11]. However, this numerical result has large error bars when radial electric field becomes large. Furthermore, several assumptions seem to degrade the accuracy of the DKES calculation in collisionless, collisional, and large electric field limit. We use the analytical solution including these effects in the region where the accuracy of DKES solution degrades.

This research shows that continuous monoenergetic coefficients can be obtained by combination of the DKES and the analytical solutions in arbitrary collision frequencies and radial electric fields. In addition to this, the large parallel viscosity is estimated in the configuration with large magnetic ripple. Parallel particle flows are expected to be suppressed by this large viscosity.

4. Plasma flow analysis in neutral beam heated plasma

In this section, analyses of parallel ion flows in NB heated plasmas are carried out. Since parallel plasma flows are deeply related to the suppression of turbulent transport, estimation of them contributes to understanding the radial transport of plasmas. Neoclassical parallel plasma flows can be estimated by solving Eqs. (2) and (3). But when plasmas are heated by tangential NB systems, external parallel momentum is injected to plasmas, as well as particles and energies. When there is external momentum input, the DKE is rewritten by

$$(V_{\|}+V_E) f_{a1} - \left\{ C_a^L(f_{a1}, f_{bM}) + C_a^L(f_{aM}, f_{b1}) \right\} = -\mathbf{v}_{da}\cdot\nabla f_{aM} + \frac{e_a}{T_a} v_{\|} B \frac{\langle BE_{\|}\rangle}{\langle B^2\rangle} f_{aM} - C_a^L(f_{aM}, f_{\mathrm{f}}), \tag{5}$$

where the last term in RHS denotes the interaction between background particles and fast ions. Parallel momentum balance equation is obtained by integrating Eq. (5), as follows:

$$
\left[-\begin{bmatrix} \mathbf{M}_a & 0 & \cdots & 0 \\ 0 & \mathbf{M}_b & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{M}_N \end{bmatrix} + \langle B^2 \rangle \begin{bmatrix} \mathbf{\Lambda}_{aa} & \mathbf{\Lambda}_{ab} & \cdots & \mathbf{\Lambda}_{aN} \\ \mathbf{\Lambda}_{ba} & \mathbf{\Lambda}_{bb} & \cdots & \mathbf{\Lambda}_{bN} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_{Na} & \mathbf{\Lambda}_{Nb} & \cdots & \mathbf{\Lambda}_{NN} \end{bmatrix} \right] \cdot \begin{bmatrix} \mathbf{U}_a \\ \mathbf{U}_b \\ \vdots \\ \mathbf{U}_N \end{bmatrix}
$$

$$
= \begin{bmatrix} \mathbf{N}_a & 0 & \cdots & 0 \\ 0 & \mathbf{N}_b & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{N}_N \end{bmatrix} \cdot \begin{bmatrix} \mathbf{X}_a \\ \mathbf{X}_b \\ \vdots \\ \mathbf{X}_N \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_a \\ \mathbf{Z}_b \\ \vdots \\ \mathbf{Z}_N \end{bmatrix} \langle BE_{\|} \rangle - \begin{bmatrix} \mathbf{C}_a \\ \mathbf{C}_b \\ \vdots \\ \mathbf{C}_N \end{bmatrix} \langle BF_{\mathrm{f}\|1} \rangle . \qquad (6)
$$

The last term is comprised of the momentum transfer coefficient $\mathbf{C}_a$ and total external momentum input $\langle BF_{\mathrm{f}\|1} \rangle$. Parallel plasma flows including the beam driven component can be obtained from Eq. (6). In addition to this, by inserting the obtained parallel flow into Eq. (3), change in ambipolar condition due to beam driven fluxes also can be taken into account.

Numerical estimations of the C^{6+} impurity flow velocity in D^{+}-C^{6+} plasmas in H-J is carried out. When considering the measurement errors, assumption in plasma profiles and inherent defect of the treatment of fast ions, experimentally measured C^{6+} impurity flow velocity does not contradict with the neoclassical estimations. Thus, we cannot conclude that there is a clear evidence of existence of the anomalous component. Also, both experimentally measured and numerically estimated parallel ion flows are suppressed across the whole plasma region by the magnetic ripple. Furthermore, estimation of the bulk ion flow velocity, which is difficult to be measured directly can be numerically predicted. This investigation shows that there is small difference in flow velocities between of bulk ions and of C^{6+} impurities in collisional plasmas.

5. Bootstrap current analysis

In this section, the effects of the ambipolar electric fields and multi-ion species on the BS currents are numerically predicted. For stabilization of plasma confinements, estimation of the BS current is one of the important works because the magnetic configuration is changed by the BS current. Previous experiments show that the BS current can be controlled by changing the bumpy field component of the magnetic field [1]. This bumpy field dependence is also confirmed by the numerical estimation using the BSC code [2]. However, effects of the multi-ion species and radial electric field have not been investigated. In addition to this, the BSC code can take into account such effects partly, but it cannot estimate the isotope effect such as H^+-D^+ effect and change in geometric factor of the BS current due to ambipolar electric field. In this research, the moment method is applied to estimate these effects in H-J consistently.

The BS currents are studied for the H-J equilibrium magnetic configuration. In high electron temperature plasmas, existence of multiple roots and only electron roots is predicted. In $E_r = 0$ cases, the BS current is increased with the electron temperature because fraction of the trapped particle increases. On the other hand, when taking into account the effect of ambipolar electric field, strong reduction and resulting reversal of the direction in the BS current on electron roots is predicted in high temperature plasmas. In such situations, changes in geometric factor of the BS current $G^{(\mathrm{BS})}$ itself due to ambipolar electric field should be taken into account. Effect of the multi-ion species is also performed. This study shows the reduction in the BS current with increase in Z_{eff} by the friction with high-Z impurities. In addition to this, study of H^+-D^+ effects on electron roots in the high-Z plasmas shows that the change in ion flow velocity and resulting negative BS current cannot be ignored. This study also shows the finite difference of flow velocity between bulk ions and impurities in collisionless plasmas.

6. Summary

In this thesis, neoclassical parallel flow velocity, and the BS current in the H-J plasmas are estimated by the Sugama-Nishimura's moment method for the purpose of clarification of the physical model of parallel plasma flows. In order to carry out the neoclassical transport analysis, the monoenergetic viscosity coefficients in wide-range of collision frequency and radial electric field are needed. However, monoenergetic coefficients which are obtained by numerical method are inappropriate for the large electric field, collisionless, and collisional limit. Combination of the analytical solutions with the numerical solutions by the DKES enables us to obtain the monoenergetic coefficients in arbitrary collision frequencies and radial electric fields. These coefficients contribute to the following neoclassical flow analyses.

Effect of external momentum on the neoclassical parallel flow is studied. In order to estimate the effects consistently, the parallel momentum balance equation with external momentum source terms is proposed. This method is applied to the numerical analysis of the parallel ion flow velocity in NB-heated H-J plasmas. Measured C^{6+} flow velocity does not contradict with the neoclassical estimation within the accuracy of the measurement and treatment of the fast ions. This suggests that we cannot conclude that there is a clear evidence of existence of the anomalous viscosity. This advanced momentum approach contributes to clarification of the physical model for parallel plasma flows.

Effects of ambipolar electric field and multi-ion species effect on the BS current are also estimated. This study predicts that when the strong electric field (such electron roots) exists, effects of radial electric field on the geometric factor itself should be taken into account. Furthermore, change in the BS current not by only high-Z impurity effect, but also by H^{+}-D^{+} isotope effect is predicted. In addition to this, finite difference of ion flow velocities between bulk ions and impurity ions is expected in collisionless plasmas.

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