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Beyond Urban Form: How Masahisa Fujita Shapes Us

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Beyond Urban Form: How Masahisa Fujita Shapes Us

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Abstract

Literature from several phases of the career of Masahisa Fujita is surveyed chronologically, with a view toward future contributions in these areas. First we address the economic structure of the interior of a city with mobile consumers, adding production. Next we provide a critical discussion of the New Economic Geography, in particular distinguishing between recent approaches employing two regions and more than two regions, both in theory and in application to data. Finally, we discuss knowledge creation in groups and briefly touch on his current work in artificial intelligence.

JEL Codes: D83, O31, R12, R13, R14

Keywords: New urban economics, New economic geography, Knowledge creation, Knowledge diversity, Robot economist

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1 Introduction

Over the years, much has been written about both the work of Masahisa Fujita, henceforth Masa, and its impact. Here, we wish to take the time and space not just to thank him, which is rather superficial, but to give an integrative and critical view of the past, present and future of some Masa-relevant literatures from our rather special vantage points, those of a long time collaborator and a student. We do not wish to put words in his mouth, so to speak, nor to imply that our vantage points are more important than those of others. Rather, we think that our view of intellectual progress over the course of Masa's career is different from that of the many others who have contributed to urban economics, economic geography, and related disciplines. Naturally, a researcher's milieu and context have much to do with how a person's research proceeds. Clearly Masa's give and take with others working in parallel or at cross purposes with him, in other words interactions in person or in print, or coauthor and mentor relationships, have produced the research path we have experienced and will experience. There is a great deal of path dependence and path co-dependence in the development of new ideas.

Naturally, many of Masa's innovations come from tensions in the literature. Theory, stylized facts, and data all play a role. Tensions can arise either within or between any of these. But what is unique about Masa is his systematic, exhaustive organization and categorization of the literature prior to addressing any of the tensions. This may involve working out versions of models and results in his notebooks, possibly results never published, in order to make sure that all the logical possibilities are known and available to him. Or it may involve cataloging the previous results in a literature. This can be seen, for example, in the book *Urban Economic Theory*. It makes the referee process easy in many cases, as that process simply involves giving page numbers in the book.

We shall return to this big point in the Epilogue.

Our survey of Masa's work is neither comprehensive nor random. We select work, from each phase of his career thus far, that provides a way forward. It is generally not useful to survey deceased or zombie literatures with no evident future.

The remainder of this article is organized as follows. Section 2 discusses

material related to the early part of Masa's career, namely classical urban economics, up to around 1990. Section 3 discusses material related to the New Economic Geography (NEG), from around 1990 until around 2003. Section 4 discusses material from around 2003 until around 2011. It is about knowledge creation and transfer in groups. Section 5 provides a brief discussion of work from around 2011 into the future, related to artificial intelligence.

2 Episode 1: Inside a City

The literature from the early part of Masa's career, summarized in his encyclopedic work *Urban Economic Theory*, is to a good degree settled.[1] So this section will take the form of a pictorial tribute, presenting some new material and ideas informally. It may tax your intuition some, but will not be technical. Your homework is to write a paper or two.

To begin, we detail Alonso (1964a)'s famous model of a city, as further developed by Berliant and Fujita (1992) and as described pictorially in Berliant and LaFountain (2006). It is the analog of an Edgeworth box exchange economy in the urban context. We shall cover this older literature briefly. The city is linear, so there is one unit of land available at each distance from the central business district (or CBD), where the latter is located at 0, so the supply of land is the interval $[0, l)$, where l is the exogenous extent of the city. The total amount of composite consumption commodity available in the economy is $C > 0$. There are two consumers called A and B. The use of two consumers is essential for the diagrams employed, much as they are in an Edgeworth box economy, but the definitions and results for our model extend easily to an arbitrary but finite number of consumers. When dealing with positive issues such as equilibrium, it is necessary to add individual endowments and an absentee landlord who is endowed with all of the land and likes only composite commodity, but we shall focus on normative issues here.

Each consumer will be allocated some composite good, c^A, $c^B \geq 0$ and an interval of land: $[x^A, x^A + s^A)$, $[x^B, x^B + s^B)$ where x^A and x^B are the

[1] One area of ongoing dispute, that we shall suppress here due to constraints on time, space, and energy, are the differences between models with a continuum of agents and those with a finite number of agents.

driveway locations of consumers A and B, namely the closest point in their respective parcels to the central business district, whereas s^A and s^B are the interval lengths for the respective consumers, namely the sizes of their parcels. Marginal commuting cost per unit of distance is $t > 0$, so the total commuting cost in terms of consumption good for consumers A and B is $t \cdot x^A$ and $t \cdot x^B$.

For this basic model of exchange, an allocation is represented by a vector $\{c^A, c^B, x^A, x^B, s^A, s^B\}$ of non-negative numbers. Such an allocation is called *feasible* if and only if $C = t \cdot x^A + t \cdot x^B + c^A + c^B$, $[x^A, x^A + s^A) \cup [x^B, x^B + s^B) = [0, l)$, and $[x^A, x^A + s^A) \cap [x^B, x^B + s^B) = \emptyset$. Using our notation, an example of the locational component of a feasible allocation can be found in Figure 1.

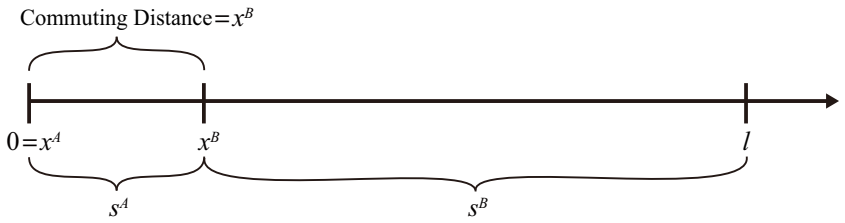

Figure 1: A Feasible Allocation

The consumers are identical, and have smooth, strictly convex, monotonic preferences over bundles consisting of land and composite consumption commodity represented by a utility function. Given this structure, Pareto optimum is defined in the usual way: A feasible allocation is efficient if there is no other feasible allocation that gives some consumer higher utility and leaves all consumers with utility at least as high. In the previously cited literature, a number of basic results were established. At an efficient allocation, consumers ordered from the central business district outward are also ordered by weakly increasing land consumption and, if land is a normal good, weakly increasing utility levels. Equilibrium, that we have not defined for reasons of brevity, was shown to exist. The welfare theorems and core[2] were examined.

Our exchange model can be illustrated in a modified Edgeworth box given in Figure 2.

[2]See Berliant and ten Raa (2007).

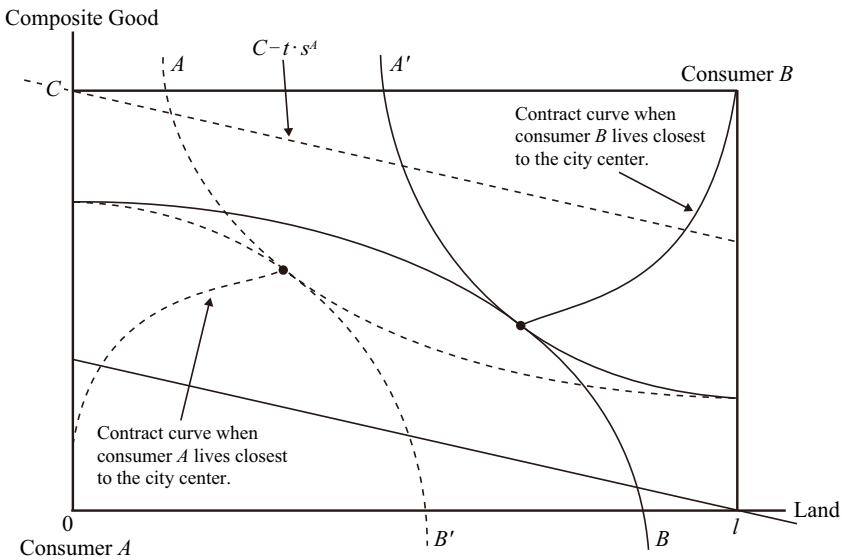

Figure 2: The Modified Edgeworth Box

In this figure, the vertical axis represents composite good consumption, whereas the horizontal axis represents land consumption. A modification of the standard Edgeworth box is required to account for the composite commodity used for commuting cost, turning the figure into a trapezoid.[3] For intuition, let us focus on the case where consumer A lives closer to the city center. Then its parcel is smaller than that of consumer B. Thus, the allocation lies in the lower left part of the box. To represent feasible allocations, the upper part of the box must be truncated by the line $C - t \cdot s^A = C - t \cdot x^B$ to account for the commuting cost in terms of composite good used by B; the commuting cost of consumer A is 0. Moreover, in this case, although consumer A's indifference curves are unchanged, consumer B's indifference curves must be shifted to account for the change in the origin from which consumer B's consumption is measured, due to the truncation. In particular, the upper right corner of the box is no longer feasible, due to commuting cost. The linear shift is given by $C - t \cdot s^A$, and is represented by indifference curve B'. As usual, under some regularity conditions, the set of Pareto optima are given by the set of tangencies, or the contract curve. However, in our diagrams, these are tangencies between the indifference curves of consumer A and the *shifted* indifference

[3]In contrast, the modification of the Edgeworth box used by public finance to account for a pure public good turns it into the Kolm triangle.

curves of consumer B, labelled B'. This leads to a first order condition for efficiency different from that using the standard Edgeworth box, namely the marginal rate of substitution (i.e. the marginal willingness to pay for land in terms of composite good) of consumer A is equal to the marginal rate of substitution of consumer B plus the marginal commuting cost t. In fact, when interpreted in terms of equilibrium by substituting land price for marginal rate of substitution everywhere, this condition becomes the analog of the classical Muth-Mills condition for our model. The piece of the contract curve in the lower left of the box applies as long as both the land consumption and utility level of consumer A are lower than those of B at the efficient allocation. Now move up along the contract curve from the lower left corner. The utility condition generally binds first. When utility levels are equal, it is time for the two consumers to switch positions, and there is a discontinuity in the contract curve.

The entire diagram is symmetric around the line from the upper left corner of the box to the lower right corner, so we represent the case where consumer B lives closer to the city center accordingly in Figure 2.

Everything stated above can be found in the cited papers. Next, we detail our innovation. We shall add production to this model.[4] We shall assume that composite good is produced by a single producer at the city center using land. It would be easy to add labor as a factor for the firm with inelastic labor supply on the part of consumers, but we shall refrain from doing so in order to keep the diagrams simple.

Please refer to Figure 3. As before, land is represented on the horizontal axis whereas composite consumption good is represented on the vertical axis. The production possibilities frontier, derived from a production function, is the outermost curve. This diagram is a hybrid of the classical Edgeworth box for a production economy and our modified Edgeworth box for our urban economy.

[4] A related paper with a totally different focus is Berliant and ten Raa (2003).

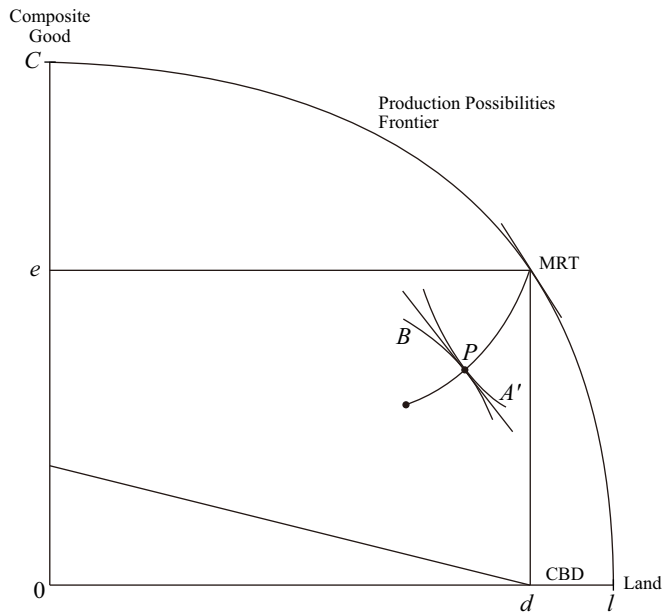

Figure 3: The Modified Edgeworth Box with Production: Consumer B Innermost

In this case, the central business district (CBD), where the firm is exogenously located, lies rightmost on the horizontal axis. It uses land $[d, l)$ for production of e units of composite good. Consumer B is adjacent to the producer, and consumer A is farther away. With this production plan, we have drawn the exchange economy with the upper right corner of the modified box at (d, e). Then, inside, we have drawn the contract curve of the exchange economy for this configuration of agents. As usual, the slope of the production possibilities frontier at (d, e) is the marginal rate of transformation (MRT). The point P represents a Pareto optimum. It occurs exactly when a first order condition is met: The marginal rate of transformation is equal to the marginal rate of substitution of B, which in turn is equal to the marginal rate of substitution of A plus t, in other words the slope of the indifference curve A'. To find Pareto optima, we trace out this first order condition for all production plans (d, e). But wait, there's more!

The case where the CBD is located to the left of the agents and consumer A is closer to the CBD than consumer B must also be considered. The diagram for this case is in Figure 4.

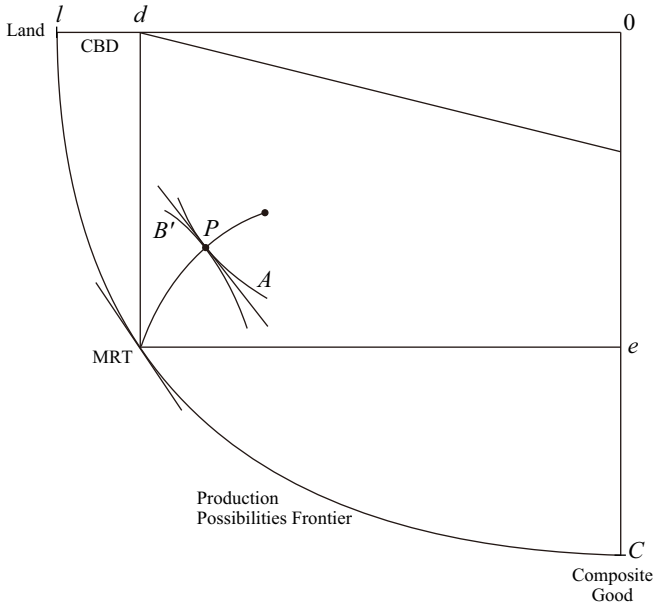

Figure 4: The Modified Edgeworth Box with Production: Consumer A Innermost

It is identical to Figure 3 except that the diagram is rotated 180 degrees and the two consumers A and B exchange roles. We will not bore you with repetition of the arguments for the case where consumer A is closer to the CBD.

We could continue the development of the model to address equilibrium by using our diagram that is a hybrid of the Edgeworth box with production and our Edgeworth box for a two consumer exchange economy modified for urban economics. It is natural and classical to set prices of land at various locations to marginal rates of substitution or the marginal rate of transformation. More consumers and producers could be added, but we shall stop here.

Before concluding this episode, we note two items of particular historical importance. First, in the course of systematically elaborating models of city entrails, the important work of Ogawa and Fujita (1980) and Fujita and Ogawa (1982) was conceived. The models allowed not just consumers but also firms to be mobile, subject to a spatial externality. These papers were about 20 years ahead of their time. Second, and also in the course of this line of research, the foundations of Episode 2: The New Economic Geography were developed. This can be seen most clearly in section 3 of Fujita (1986),

a precursor of Fujita (1991) as well as the work of Abdel-Rahman and Fujita (1990, 1993). Here the famous Starrett Spatial Impossibility Theorem is used to motivate why standard, classical general equilibrium models cannot generate cities endogenously, and proposals for modifying them are made. One notable passage is quoted here (p. 124): "We could generate many interesting problems by appropriately fusing different models in the above three categories, A, B and C. It would be wise, however, to thoroughly study each *pure* category first."

3 Episode 2: The New Economic Geography

Masa's contribution to the development of the new economic geography (NEG) has been well documented in his two seminal books, Fujita, Krugman, and Venables (1999) and Fujita and Thisse (2013). Whereas the topics covered by NEG today have become far more diverse than when Masa initiated the field with Paul Krugman, we focus on his original motivation and how it led to the birth of NEG. We also discuss recent developments that exceed Masa's original aspirations.

Before the Dawn

In the late 1980s, like other fields of economics, increasing returns and imperfect competition started to play a major role in urban economics.[5] Neoclassical assumptions of constant returns and perfect competition have been awkward especially in the context of urban economics, since the very reason for the presence of cities — agglomeration of economic activities — cannot be justified in the absence of scale economies and positive externalities of some sort.[6]

Masa was one of the central players in this literature, and wrote a few important papers with Hideaki Ogawa explaining the endogenous formation of the CBD on a continuous location space within a city. City formation is explained in terms of a Marshallian externality that is meant to represent positive spillovers among firms (Fujita and Ogawa, 1982; Ogawa and Fujita,

[5]Initial such attempts were much earlier; see, for example, Beckmann (1976); Solow and Vickrey (1971).

[6]The first nature advantage such as being a natural port may have triggered the birth of a city there. But, that by itself cannot fully account for the presence of very large cities like New York and Tokyo.

1980, 1989). While there were several competing attempts,[7] Masa's work was distinctive in that it involved *the formation of multiple business districts*.

In the beginning of the 1990s, Masa was seeking a formal model capable of explaining the spatial distribution of cities — *inter-city spatial structure* — in addition to the formation of business districts and land use within each city — *intra-city spatial structure*. Although many models of city formation with micro-founded agglomeration economies were developed in the 1990s and early 2000s,[8] they have little to say about the spatial relations among the cities. The reason is simple: the spatial pattern of cities was a hard question to tackle formally given the available modeling techniques at that time. They were typically based on either the classical single-city model (Alonso, 1964b; Muth, 1969; Mills, 1972) or the systems-of-cities model by Henderson (1974), both of which abstract from inter-city space.

Multiple cities emerge because economies of agglomeration are eventually dominated in large cities by diseconomies, and the spacing of cities is determined by the tension between these two forces, depending on the specific mechanism underlying the increasing returns and externalities. Thus, it was necessary to develop microfoundations for both economies and diseconomies of agglomeration to address the spatial distribution of cities.

Masa's first such attempt was made in Fujita (1988) where he replaced inter-firm externalities from his previous models with pecuniary externalities based on product differentiation and plant-level scale economies. There, mono- and poly-centric internal city structures were shown to emerge in equilibrium as in Fujita and Ogawa (1982), but this time under micro-founded pecuniary externalities. It was, however, still not possible to go beyond a single city model in this formulation.[9]

In the meantime, Paul Krugman was stimulated by the fact that national borders became less and less important in the course of increasing economic integration in the 1980's and 1990's such as EU, NAFTA and MERCOSUR. The observation that around 80% or more of the value traded among countries is indeed accounted for by cities led him to develop his first models of the NEG: Krugman (1991, 1993). The coincidence of aims and this opportune timing resulted in the seminal collaboration between the two in Fujita

[7] See Fujita and Smith (1990) for a survey of this literature.
[8] E.g., Helsley and Strange (1990, 2002); Kim (1990, 1991).
[9] See Fujita and Thisse (2013, §7.2) for an overview of these models.

and Krugman (1995).

The Birth of NEG

The breakthrough came right after Fujita (1988), when Masa wrote Fujita (1993), his first NEG model, as an extension of the model developed earlier by Krugman (1991, 1993). Rather than trying to deal with both intra- and inter-city spaces simultaneously, he focused on inter-city space while *abstracting from intra-city space*. Such a setup is indeed standard in the classical central place theory of Christaller (1933) and Lösch (1940). What Masa did is to utilize an old idea in the context of modern urban economic modeling. In particular, although city formation in both Fujita (1988) and Fujita (1993) is based on a monopolistically competitive sector, unlike the former model, the latter model involves neither land consumption nor land input for production in cities, and hence, each city is formed at a point in location space not occupying any land. This is in contrast with the systems-of-cities model of Henderson (1974), which preserves intra-city space while abstracting from inter-city space.

To be fair, this "new" idea from the past was almost present in Krugman (1993), where multiple industrial (and population) agglomerations emerge spontaneously in a homogeneous (discrete) many-region space. The key dispersion force underlying the formation of multiple agglomerations in this model was the presence of immobile consumers distributed exogenously over the regions.

In Fujita (1993), since the only immobile factor is land and all consumers are mobile,[10] the dispersion of consumers is an endogenous outcome from the presence of a land-intensive sector. Hence, it was the first general equilibrium model of endogenous agglomeration on a continuous location space in which all households and firms are fully mobile. Here, households are homogeneous, and each household consists of a single worker. Based on this setup, in Fujita (1993), Masa laid out his road map for NEG development in the 1990s, to include the models explored in Fujita and Krugman (1995); Fujita and Mori (1997); Fujita, Krugman, and Mori (1999).

This series of models started with the simplest setup in which there are

[10]More precisely, there are landlords attached each parcel of land, and they spend their entire income from renting their land for consumption. Alternatively, public land ownership can be assumed without altering the results.

two types of consumption goods: a single set of horizontally (and symmetrically) differentiated "manufactured" goods and a single homogeneous "agricultural" good with the utility function given by

$$U = M^{\mu} A^{1-\mu} \tag{1}$$

$$M \equiv \left[\int_0^n m(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where M is the composite of manufactured goods *à la* Dixit and Stiglitz (1977) with elasticity of substitution $\sigma > 1$, where A is the consumption of agricultural good, and where $\mu \in (0, 1)$. Each differentiated variety of manufactured good is produced using only labor under plant-level scale economies, whereas the agricultural good is produced under constant returns technology using labor and land. In this context, the well-known circular causation of agglomeration takes place through the interaction among love for variety, transportation costs, and plant-level scale economies associated with consumption and production of manufactured goods.

Fujita and Krugman (1995) investigated the single-city equilibrium of this model, and developed the concept of *the market potential function* which is a micro-founded generalization of the classical market potential introduced by Harris (1954). The location space is the one dimensional line, $\mathbf{X} \equiv (-\infty, \infty)$, over which homogeneous land is distributed uniformly. There is a given population of mobile workers, and each of them lives and works at the same location. The location at which manufacturing production takes place is called a *city*. Naturally, the region surrounding each city will be specialized in agricultural production. Output from city and country is exchanged.[11] Let the city location be the origin, $r = 0$, of $\mathbf{X}$. Then the agricultural hinterland of this city will extend symmetrically around the origin, say $[-f, f]$ for some $f > 0$ which is determined as an increasing function of the total worker population.[12] For simplicity, let us assume that transportation is costly only for manufactured goods.

The market potential is the ratio of the *hypothetical* zero-profit output to the equilibrium (zero-profit) output of a manufacturing firm if it unilaterally

[11] It can be interpreted as the formalization of the spatial economy described by Cronon (1991) in explaining the emergence of Chicago in the nineteenth century.

[12] See also Fujita and Hamaguchi (2001) for a version based on product variety in intermediate goods.

deviated from the city to a given location $r \in \mathbf{X}$. Such a computation is a simple matter, since each monopolistically competitive firm is small, and hence the equilibrium configuration remains exactly the same after the unilateral deviation of this firm. The market potential, $\Omega(r)$, is strictly positive at each location $r \in \mathbf{X}$. If the deviation is strictly more (less) profitable, then $\Omega(r) > 1$ (< 1), and $\Omega(r) = 1$ if the hypothetical profit is zero at r, i.e., the same as the equilibrium profit.

Figure 5 depicts the typical shape of the market potential function in equilibrium.[13] Since the spatial pattern is symmetric with respect to the city location, $r = 0$, the figure only shows the right half of the location space, $r \geq 0$. The market potential function has an S-shape as indicated by the solid curve in the figure, where $\Omega(0) = 1$ and $\Omega(r) \leq 1$ for $r \neq 0$ in equilibrium so that there is no incentive for manufacturing firms to deviate from the city. The market potential in the single-city equilibrium can be decomposed into two parts: one accounting for the potential profit from the market in the city, depicted by the solid thin curve, and the other from the market in the agricultural hinterland, depicted by the dashed curve. Since there is a mass of consumers at the city, there is substantial loss by moving away from the city, which results in the sharp kink of the solid-thin curve at the city location. Though the average distance to the consumers in the agricultural hinterland is minimized at $r = 0$, the potential profit from selling to the agricultural hinterland is not maximized at $r = 0$, since the competition with other manufacturing firms is toughest there. Instead, a somewhat remote location around $r = 1.0$ offers a larger profit. In this remote market, the deviating firm can enjoy more local monopoly power by having a larger market share as it can sell at a lower price there than its competitors in the city. As a result, the potential profit from the market in the agricultural hinterland becomes hump-shaped as indicated by the dashed curve.

These two curves add up to the S-shaped market potential function. In particular, the agglomeration of manufacturing firms in the city casts an *agglomeration shadow* in the vicinity of the city, within which the distance from the city is not large enough to mitigate the competition in the local market with firms in the city. The concept of agglomeration shadow, first

[13] As long as the total population of mobile workers is sufficiently small, equilibrium with a single city exists and is unique up to a translation of the single city location.

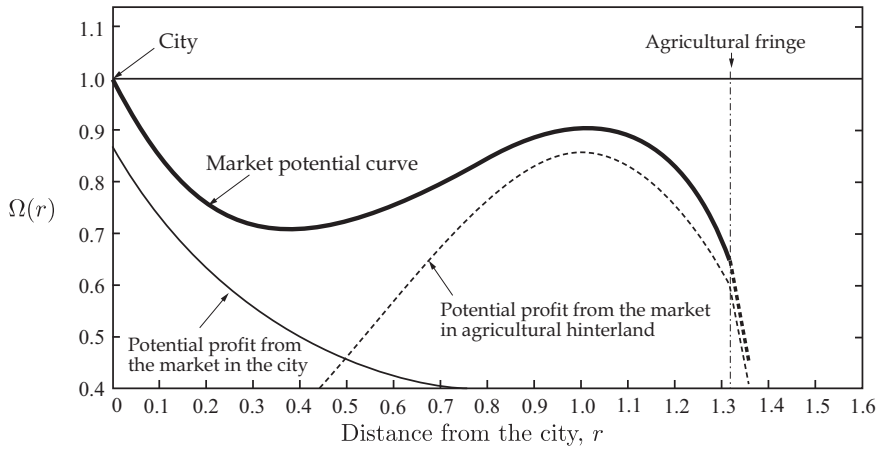

Figure 5: The market potential curve of a single-city equilibrium

introduced by Arthur (1994), has thus been formalized in the context of a general equilibrium model by Fujita and Krugman (1995). The size of the agglomeration shadow is larger for industries producing more differentiated goods (i.e., with a smaller value of σ) and/or industries that are less sensitive to transport costs.

Figure 6 shows the response of the market potential function to an exogenous change in the total mass of mobile workers, $N > 0$.[14] As the population size of the economy increases, the agricultural hinterland expands, which in turn makes the local market in a larger portion of agricultural hinterland less competitive. Consequently, at some critical population size, $\widetilde{N}$, the market potential reaches 1 at some remote location, $\widetilde{r} \gg 0$, at which firms are indifferent between the city and location $\widetilde{r}$. A further increase in the population will make location $\widetilde{r}$ more profitable than the city, and hence, a new city will emerge at $\widetilde{r}$.[15]

This critical distance $\widetilde{r}$ between the two adjacent cities depends on the nature of the differentiated products, and hence is specific to each industry. Namely, agglomerations form more densely for industries producing less differentiated goods and/or those that are more sensitive to transport costs. Fujita and Mori (1997) studied the evolution of the city system under

[14]Each market potential curve is plotted only over the agricultural hinterland. Note that the agricultural fringe, f, increases for a larger value of N. This figure is based on Fujita and Krugman (1995, Fig.4).

[15]Here, it is assumed that the elasticity of substitution, σ, is sufficiently large, so that the market potential exceeds one at some location $r \neq 0$ for sufficiently large N.

increasing population size in a long, narrow location space.[16]

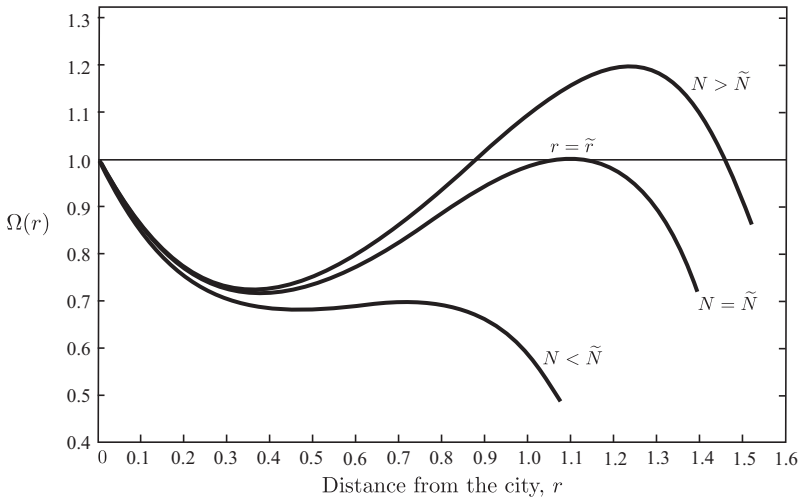

Figure 6: Formation of a new city and the market potential

The finding of industry-specific spacing of agglomerations had a far reaching implication in understanding the prevailing large diversity in the size and industrial composition of cities in reality. This direction of research was pursued by Fujita, Krugman, and Mori (1999) in which the model by Fujita and Krugman (1995) was extended to include multiple groups of differentiated goods with the utility function given by

$$U = A^{\mu^A} \prod_{h=1}^{H} \left(M^h\right)^{\mu^h}, \quad \mu^A + \sum_h \mu^h = 1 \tag{3}$$

where M^h is the CES composite of differentiated goods as in eq. (2) except that the value of the elasticity of substitution, σ^h, differs across commodity groups, $h = 1, 2, \ldots, H$. As suggested above, the size of the agglomeration shadow cast around an agglomeration differs across industries, so that the number and spacing of cities are also different among industries. For a given population size of the economy, a larger number of cities supply fewer differentiated goods and/or the goods that are more sensitive to transport costs. In this extended model, an interesting phenomenon takes place: The agglomerations of different industries exhibit a particular spatial coor-

[16]Fujita and Mori (1996) studied the interaction between agglomeration economies (second nature advantage) and the first nature advantage of natural ports.

dination. Namely, *the agglomeration of an individual industry takes place at a roughly common spatial cycle, and the cycles synchronize across industries*.

Figure 7 illustrates this coordination expressed in terms of the market potential curves of three groups of differentiated goods. The values of the elasticity of substitution differs among these groups as follows: $1 < \sigma^1 < \sigma^2 < \sigma^3$.[17] Thus, industry 1 produces goods that are highly differentiated, whereas industry 3 produces goods that are least differentiated. In Fujita, Krugman, and Mori (1999), the evolution of the city system is studied under a gradual exogenous increase in the total population size of mobile workers, N. When N is small, a unique single-city equilibrium exists so that all differentiated products are supplied from this single city.[18] Like Fujita and Krugman (1995), let the location of this city be $r = 0$. As N increases, new cities are formed in the agricultural hinterland. Since the firms producing less differentiated goods have an incentive to deviate from the city earlier, the first new cities to be formed are by the agglomerations of industry 3.

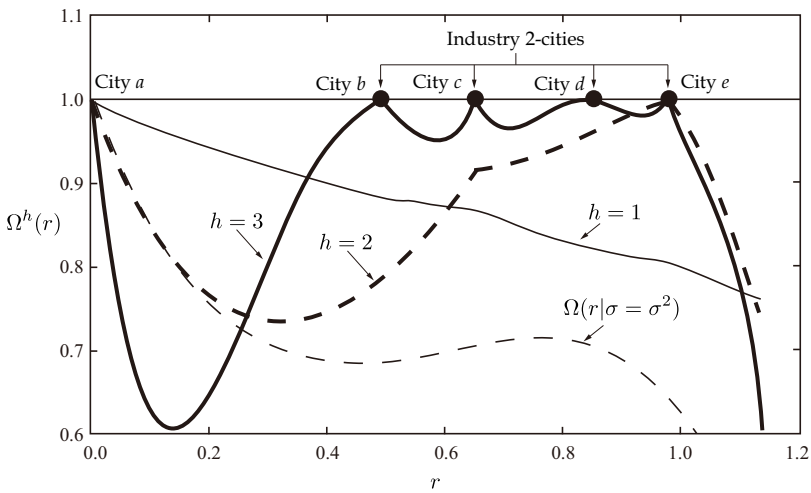

Figure 7: Spatial coordination of industrial agglomerations

The figure depicts an equilibrium in which nine cities exist in equilibrium, where city a at $r = 0$ supplies the products of all three industries, whereas four cities (b, c, d, e) on either side of the central city produce only

[17]This figure is based on Fujita, Krugman, and Mori (1999, Fig.8).

[18]The market potential curves for all goods are smaller than one for all locations $r \in \mathbf{X}$ except for the city at $r = 0$.

the least differentiated goods of industry 3. The market potential value for industry 3 equals one for all nine cities, whereas the market potential for the other two industries (1 and 2) is one only at city a: $\Omega^h(r) < 1$ for all $r \neq 0$ for $h = 1$ and 2. For comparison, the thin dashed curve indicates the market potential function for industry 2 under the single-city equilibrium in the Fujita and Krugman (1995) model of a single industry (as in Figure 5 with $\sigma = \sigma^2$).

Notice that the market potential functions are not S-shape any more when multiple groups of differentiated goods are involved. In particular, the market potential curve exhibits kinks at the city locations, reflecting the presence of a mass of consumers in these locations. The comparison between the thick-dashed market potential curve for industry 2 under the presence of the four cities b, c, d and e, and thin-dashed curve in the absence of these four cities indicates that the mass of consumers in these four rural cities pull up the market potential of industry 2. In fact, the market potential for industry 2 is about to reach one at city e; at this point the spatial cycle of agglomeration for industry 2 and that for industry 3 are going to synchronize. Industry 2 will start agglomerating in city e given a further increase in N. The market potential function for industry 1 producing more differentiated goods is, not surprisingly, less influenced by the spatial distribution of consumers.[19]

In this way, the resulting industrial composition of cities in this economy naturally exhibits *hierarchical structure* such that cities having more differentiated goods also provide all the less differentiated goods. The difference in the range of products supplied in cities translates into diversity in city population, reminiscent of Christaller (1933)'s *hierarchy principle*.[20] This hierarchy formation further implies a certain spatial fractal structure in the spatial distribution of cities, *a spacing-out property*, that larger cities are formed farther apart from one another than smaller cities, and that a larger city is surrounded by a larger number of smaller cities.[21]

Although their demonstration of the spatial coordination of industrial

[19]Here we skip the details of the adjustment dynamics. For these details, see Ikeda, Murota, Akamatsu, and Takayama (2016), that adopts essentially the same dynamics.

[20]These results were confirmed formally by Tabuchi and Thisse (2011) using the version of the NEG model proposed by Pflüger (2004), and by Hsu (2012) using an alternative modeling approach based on spatial competition.

[21]Hsu, Mori, and Smith (2014) show evidence of the spacing-out property using the US data.

agglomeration is limited to only three industries, the results obtained are suggestive that the large diversity in the actual city sizes may accrue from the large diversity in the degree of product differentiation and/or transport costs. In the international trade literature, it is common to assume the presence of only a single group of differentiated products, and the elasticity of substitution is estimated to be around 3 to 5 (Head and Mayer, 2015). But, if the elasticities of substitution of individual product groups are estimated separately, they appear to be quite different.

Figure 8 shows the distribution of substitution elasticities of $13,930$ products according to the harmonized tariff schedule (HTS) classification estimated from the data on US imports between 1990 and 2001 by Broda and Weinstein (2006). The cumulative share of the US import value at $\sigma^1 = 1.25$, $\sigma^2 = 4$ and $\sigma^3 = 10$ considered in the numerical example by Fujita, Krugman, and Mori (1999) are 1.26%, 50.2% and 77.5%, respectively. The range of implied markup, $1/(\sigma - 1)$, is found to be as large as [0.00023,33.8] with mean 0.784. The result by Broda and Weinstein (2006) indicates that actual product diversity is far wider and finer than that in the simple economy of this exercise, which at the same time suggests that spatial coordination among these diverse industries may be the primary source of diversity in the size and industrial composition of cities in reality.[22]

[22]See Mori, Nishikimi, and Smith (2008); Mori and Smith (2009, 2011) for evidence supporting this statement using Japanese data.

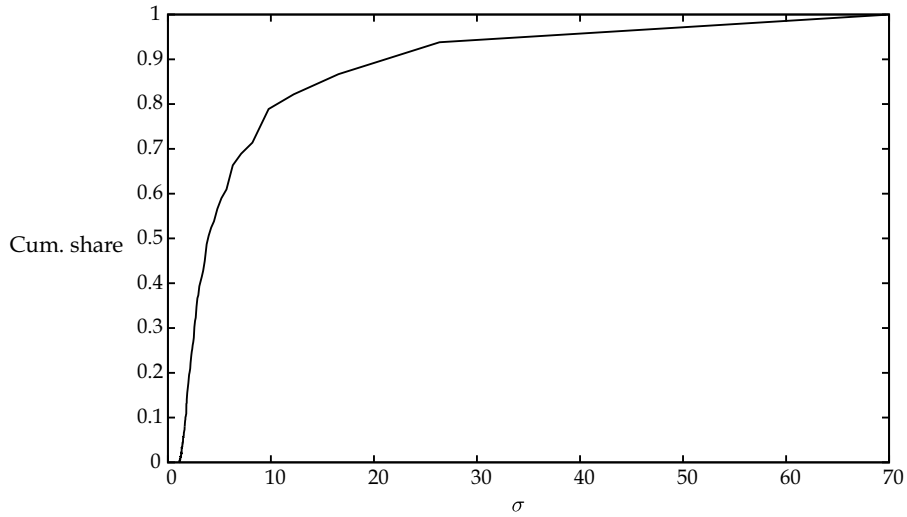

Figure 8: Estimated cumulative distribution of the elasticity of substitution elasticities of the imported products in the US

Twists and Turns

Even today, the NEG remains one of the few general equilibrium frameworks that can explicitly address the spatial distribution of endogenous agglomerations. But, the highly non-linear nature of the model almost prohibits formal analysis under a location space with more than two regions. As a result, rather heuristic numerical analyses dominated the literature in the 1990s (especially in the context of many-region and continuous location models). This lack of rigor promoted the retreat of NEG modeling during the 2000's from the many-region setup to the minimum two-region setup used at the very beginning of the development by Krugman (1991), and this situation persisted for a decade.[23]

It is worth mentioning two directions of major progress in the 2000s. First is the generalization of the dispersion force. Aside from the exogenous spatial dispersion of consumers considered in the original NEG models, *urban costs* were added by Helpman (1998); Tabuchi (1998); Murata and Thisse (2005) and *randomness in location preference* by Tabuchi and Thisse (2002); Murata (2003).[24] These alternative dispersion forces were shown

[23]See Baldwin, Forslid, Martin, Ottaviano, and Robert-Nicoud (2003) for an extensive survey of alternative specifications of NEG under the two-region setup.

[24]Another important dispersion force not discussed here is the price-competition effect introduced by Ottaviano, Tabuchi, and Thisse (2002) and Behrens and Murata (2007).

to influence location patterns differently from the one used in the original formulation by Krugman (1991, 1993). In particular, *dispersion takes place when transport costs for differentiated goods are low, rather than high*, unlike in the original model.

Second is the refinement of the equilibrium concept by introducing forward-looking behavior (e.g., Ottaviano, 2001; Oyama, 2009a,b). In particular, Oyama (2009b) has shown that in the context of a two-region economy *à la* Krugman (1991), the multiplicity of stable equilibria is an artifact of myopic behavior and symmetry of the two regions, and that the unique full-agglomeration equilibrium is a robust outcome in the two-region economy under forward-looking behavior and asymmetry of the two regions.[25]

But all these formal results have only limited implications for the actual economy, as they strongly depend on the two-region setup. It is generally not possible to draw reasonably precise implications in the many-region context from the results of two-region models. Taking Oyama (2009b)'s result on equilibrium refinement just discussed, for instance, "full-agglomeration" in a two-region economy does not necessarily mean agglomeration in a single region in the context of a many-region economy.

This ambiguity comes essentially from the abstraction of the spatial scale of agglomeration and dispersion in two-region models, as pointed out by Mori and Smith (2015); Akamatsu, Mori, and Takayama (2015). Although dispersion due to the exogenous spread of consumers and that due to urban costs look exactly the same in the two-region setup, they are often qualitatively different in a many-region economy. On the one hand, it has already been suggested by Krugman (1993); Fujita and Mori (1997), and formally proved by Akamatsu, Takayama, and Ikeda (2012), that the spread of immobile consumers results in a larger number of smaller agglomerations dispersed over the location space, i.e., the dispersion in this case takes place *at a global scale* over the entire (active) location space. Ikeda, Murota, Akamatsu, and Takayama (2016) show that this result essentially persists in a long-narrow economy *à la* Fujita and Krugman (1995).

On the other hand, dispersion due to urban costs is associated with the spatial expansion of individual agglomeration, i.e., it takes place *at a local scale* (see, e.g., Ikeda, Murota, Akamatsu, and Takayama, 2016). As an extreme case, Akamatsu, Mori, and Takayama (2015) show that in a many-

[25]See also the discussion in Fujita and Thisse (2013, §8.2.3.2).

region extension of the model of Helpman (1998), the distribution of mobile workers is at most *unimodal*, and lower transport costs (for differentiated goods) simply makes this distribution flatter.[26]

These differences in the spatial scale of dispersion can be identified only in a many-region setup. Thus, the strong results under the two-region setup should be interpreted with caution. Just having many regions is still not enough. In particular, there is no distinction between global and local scales if transport cost between any pair of locations is the same, as in Tabuchi, Thisse, and Zeng (2005); Tabuchi (2014).

There was in fact a revival of many-region models (with asymmetric inter-regional distances) in the late 2000's, when, like other fields of economics, *evidence-based approaches* became fashionable in spatial economics in response to the increasing availability of micro and geographically disaggregated data. But, this revival happened without any associated technical advance in the study of many-region models over the heuristic numerical approach used in the late 1990's. Its consequence can be seen most notably in the counterfactual exercise of Redding and Sturm (2008) and a similar attempt by Behrens, Mion, Murata, and Süedelum (2014) using many-region extensions of the two-region NEG models of Helpman (1998) and Behrens and Murata (2007), respectively.

A common feature of these two studies is that they introduce an *unobserved city-specific amenity* in their model to make up *any* gap between the actual and calibrated city sizes. The unobserved amenities are thus nothing but the residuals after fitting their model to the actual city size distribution. Using this approach, Redding and Sturm (2008) and Behrens, Mion, Murata, and Süedelum (2014) calibrated their models to fit the pre-war German city size distribution in 1939 and the US city size distribution in 2007, respectively.

If the log of the actual city size is regressed on the log of the "estimated" unobserved amenity (i.e., the residuals after fitting their models), one can immediately find that most of the variation in city size is explained by these residuals, as indicated in Figure 9.[27]

[26] In Helpman (1998), land is a necessary input for the production of differentiated goods, and thus every location hosts mobile agents whose labor produces differentiated goods.

[27] The dashed lines indicate the fitted OLS model.

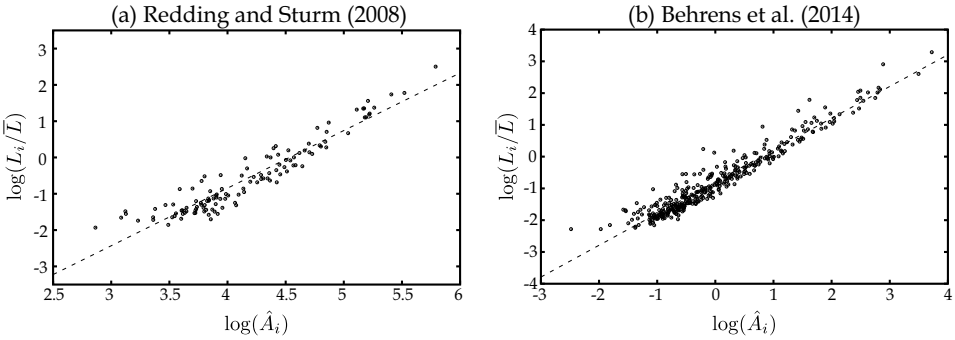

Figure 9: The relationship between city sizes and the estimated unobserved amenities

In the case of Redding and Sturm (2008), this regression yields the following result:

$$\log(L_i/\overline{L}) = \underset{(0.210)}{-7.191} + \underset{(0.050)}{1.587} \log(\hat{A}_i)\,, \text{adj. } R^2 = 0.896\,, \tag{4}$$

where L_i and $\overline{L}$ are the population size of city i and the average city size in 1939 in Germany, and $\hat{A}_i$ is the estimated unobserved amenity in city i. The numbers in the parentheses are the standard errors. In the case of Behrens, Mion, Murata, and Süedelum (2014), the analogous regression yields

$$\log(L_i/\overline{L}) = \underset{(0.015)}{-0.790} + \underset{(0.014)}{1.00} \log(\hat{A}_i)\,, \text{adj. } R^2 = 0.933\,, \tag{5}$$

where the notation is the same as above except that the regression uses data for US cities in 2007.[28] It is clear that these models have little explanatory power, since distribution of city size is driven primarily by the residuals.

The reason for the poor performance of these models is rather simple. Recall that a many-region extension of Helpman (1998) can generate at most a unimodal agglomeration in the absence of exogenous location-specific advantage. It follows that the setup in Redding and Sturm (2008) was, from the start, inappropriate to endogenously generate multi-modal agglomeration patterns in reality. What is worse still, their calibration was conducted in the parameter range in which the only possible location pattern is complete dispersion, i.e., no endogenous agglomeration (even a unimodal one) can occur (see the detailed discussion in Akamatsu, Mori, and Takayama,

[28]The data for these regressions are available from Redding and Sturm (2008, Online Appendix) and Behrens, Mion, Murata, and Süedelum (2014, Table 4), respectively.

2015).[29,30]

As for Behrens, Mion, Murata, and Süedelum (2014), their model can generate multiple agglomerations endogenously just like Krugman (1993). Allowing for the presence of city-specific exogenous (observed) heterogeneity, they could account for a part of the actual city size diversity. But, the regression result (5) indicates that their model still could replicate only 6.7% of the variation in the actual city sizes in the US. A major source of the misfit may be that their model allows for only a single group of differentiated products. It has already been suggested repeatedly in the early results of Krugman (1993); Fujita and Mori (1997) as well as in the more recent and formal results of Akamatsu, Takayama, and Ikeda (2012); Ikeda, Murota, Akamatsu, and Takayama (2016) that NEG models with a single group of differentiated products can yield little diversity in city size at stable equilibria. Indeed, this was the reason that led Masa to develop a multiple industry version of the NEG model in Fujita (1993); Fujita, Krugman, and Mori (1999).

The Way Forward

In the 2010's, researchers from engineering who recently entered the field made significant contributions to this literature by developing systematic analytical and numerical approaches for studying complex spatial models.

A breakthrough was made by Akamatsu, Takayama, and Ikeda (2012) in the analysis of a many-region economy by applying a discrete Fourier transformation to the NEG model with the racetrack location space *à la* Krugman (1993). The formal comparative static analysis for essentially the entire range of parameter values of the model has become possible, and the corresponding evolutionary paths of stable equilibria can be obtained. In particular, it was shown formally that a gradual decrease in transport costs in the model of Krugman (1993) leads to a *spatial period doubling bifurcation* of industrial agglomerations.

[29]When the location space is asymmetric, as in Germany or any real country, the location with the best accessibility gets a first nature advantage, and attracts a larger population. One interpretation is that this exogenous advantage is responsible for the 10.4% of the city size variation explained by the model in (4).

[30]The many-region extension of Helpman (1998) is adopted and the parameter range for complete dispersion is also assumed in other counterfactual analyses by Michaels, Rauch, and Redding (2013); Monte, Redding, and Rossi-Hansberg (2015).

Though this approach cannot be used in the case of the long narrow economy of Fujita and Krugman (1995), the advances in numerical methods over the last two decades made it possible to conduct systematic simulations for a high dimensional space of endogenous variables under symmetry as well as certain asymmetries of location space. Most notably, as demonstrated by Ikeda, Murota, an Tatsuhito Kono, and Takayama (2014); Ikeda, Murota, and Takayama (2014); Ikeda, Murota, Akamatsu, and Takayama (2016), it is now possible to formally predict the bifurcation path of stable equilibria in many-region models by utilizing a combination of *group-theoretic* and *computational bifurcation theory* (see, e.g., Ikeda, Akamatsu, and Kono, 2012). Furthermore, an advance in numerical optimization methods, *the merit function approach* of Fukushima (1992), adopts *projection dynamics* (instead of the standard *replicator dynamics*) as the migration mechanism. This advance made it possible to conduct a large scale Monte Carlo simulation for highly disaggregated models (with respect to both geography and industry), e.g., under the industrial structure estimated by Broda and Weinstein (2006) above. An initial such attempt is Akamatsu, Mori, and Takayama (2016). Thus, although once almost dismissed, Masa's initial aspirations for construction and application of the NEG framework are now being succeeded by the next generation of models and researchers.

Finally, a relatively unexplored but important direction of research is the dynamics of NEG. The myopic dynamics in the NEG have been essentially a comparative statics exercise. Let agents be forward looking, i.e., not myopic as in the most of the NEG models. Consider a version of the model with a finite number of firms (or product varieties) and a continuum of consumers. Parameters change period to period but exogenously. So this is literally not a repeated game, but close. The firms are the players in the game. The question is: Can a variant of the Folk Theorem be proved in this context? If so, then most everything is an equilibrium. And the attempts to sort out the difficult dynamics in these models are doomed, because there are versions of the Folk Theorem for refinements. These attempts may be similar to the attempts to sort out dynamics in repeated games many years ago, which led to the Folk Theorem.

4 Episode 3: Knowledge Creation

Masa's work on innovation and knowledge creation derives directly from elaboration of the research and development sector in NEG models. In variations of these models, this sector produces ideas for new differentiated products, and sells patents to the production sector. He was a discussant at the RSAI meetings of an early version of Berliant, Reed III, and Wang (2006), and saw connections. The basic model we work with here can be found in Berliant and Fujita (2008), where details and extensions are provided. We shall be concise and intuitive here. Please write a paper or two to complete this material.

The novel question of interest below is how researchers sort into groups according to their knowledge productivity. In particular, if researchers have exogenous, heterogeneous technologies for knowledge creation partnerships, where some are better in partnerships with ideas in common and others are better in partnerships with larger knowledge differential between the agents, how will they sort and what size research groups will they form in equilibrium?

Ideas are differentiated horizontally, but are treated as symmetric. For an agent i, the number of ideas in their knowledge base at time t is denoted by $n_i(t)$, an integer. For agents i and j, the number of ideas that they share, or have in common, at time t is denoted by $n^c_{ij}(t)$. The number of ideas that i has but j doesn't have at time t is denoted by n^d_{ij}. Lastly, the number of ideas in the knowledge base of agents i and j combined at time t is denoted by $n^{ij}(t)$. These identities follow:

$$n_i(t) = n^d_{ij}(t) + n^c_{ij}(t)$$
$$n^{ij}(t) = n^d_{ij}(t) + n^c_{ij}(t) + n^d_{ji}(t)$$

Knowledge creation is governed by the following equations. If agent i is working alone at time t, we denote this by $\delta_{ii}(t) = 1$ and the rate of knowledge creation for this person is given by:

$$a_{ii}(t) = \alpha \cdot n_i(t)$$

where $\alpha > 0$, so knowledge creation is proportional to their current knowledge stock. These ideas are not shared with anyone. If agents i and j ($i \neq j$)

are working together at time t, we denote this by $\delta_{ij}(t) = 1$, and the rate of knowledge creation for this pair is given by:

$$a_{ij}(t) = \beta \cdot \left(n^c_{ij}(t)\right)^{\theta} \cdot \left(n^d_{ij}(t) \cdot n^d_{ji}(t)\right)^{\frac{1-\theta}{2}} \tag{6}$$

where $\beta > 0$ and $0 < \theta < 1$ are parameters. These new ideas become common to the two creators as they are produced. The basic idea behind this knowledge production function is that for joint knowledge creation, both knowledge in common and differential knowledge contribute to collaborative productivity. Best is a balance. The parameter β represents overall joint productivity, whereas the key parameter θ is more important for our purposes here, as it expresses the productivity weight on knowledge in common relative to differential or exclusive knowledge in partnerships.

Next, we define normalized variables to make our analysis easier. For simplicity, we drop the time argument (always implicitly present).

$$m^c_{ij} = \frac{n^c_{ij}}{n^{ij}} = \frac{n^c_{ji}}{n^{ij}}, m^d_{ij} = \frac{n^d_{ij}}{n^{ij}}, m^d_{ji} = \frac{n^d_{ji}}{n^{ij}}$$

Then we have the following:

$$1 = m^c_{ij} + m^d_{ij} + m^d_{ji}$$
$$n_i = \left(1 - m^d_{ji}\right) \cdot n^{ij}$$

Dividing both sides of (6) by n_i, we obtain the percent knowledge growth rate relationship for the partnership between i and j:

$$\frac{a_{ij}}{n_i} = \frac{\beta \cdot \left(m^c_{ij}\right)^{\theta} \cdot \left(m^d_{ij} \cdot m^d_{ji}\right)^{\frac{1-\theta}{2}}}{1 - m^d_{ji}}$$

Under symmetry across agents working together, we next obtain the knowledge creation growth rate curve, $g(m)$. For every symmetric partnership with percent differential knowledge $m^d = m$ and percent knowledge in common $m^c = 1 - 2m$, the percent knowledge growth is:

$$\frac{a_{ij}}{n_i} = g(m) = \beta \cdot \frac{(1-2m)^{\theta} \cdot (m)^{1-\theta}}{1-m} \tag{7}$$

In Figure 10, we graph this function for parameter values $\beta = 1$ and $\theta = \frac{1}{3}$. Its peak or bliss point, called B, is at knowledge differential level $m^B = .4$.

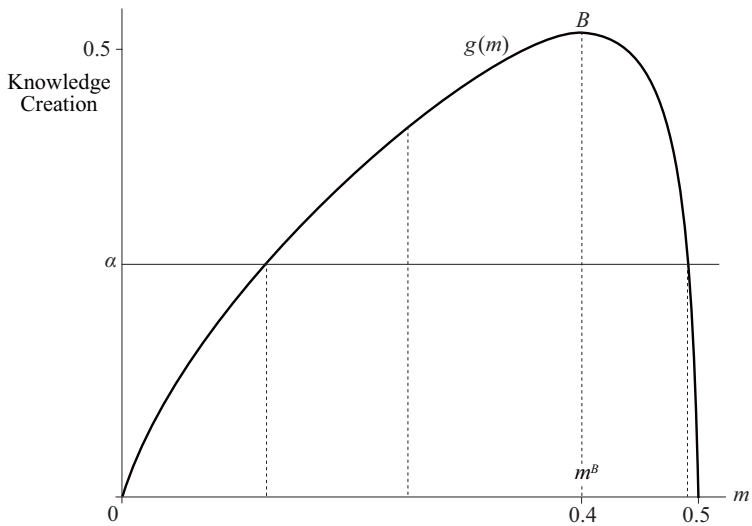

Figure 10: The $g(m)$ Curve and the Bliss Point B

Notice that if the productivity of the pair of agents working together falls below α, then the partnership dissolves and the agents work independently.

We shall not discuss dynamics here, as they are discussed extensively elsewhere. It is important to remark, however, that once the bliss point is attained, maintaining it requires agents to change partners rapidly within a well-defined group of agents. Such a pattern prevents buildup of too much knowledge in common and a reduction in m. For example, when $\theta = \frac{1}{3}$, this research group size is 4. More generally, this group size is $1 + \frac{1}{\theta}$.

The next step in our development is to understand how the bliss point depends on θ. By taking the derivative of (7) and setting it to 0, we find that:

$$m^B(\theta) = \frac{1-\theta}{2-\theta} \tag{8}$$

In Figure 11, we graph bliss point knowledge differential m^B as a function of θ, the relative importance of knowledge in common, that is exogenous.

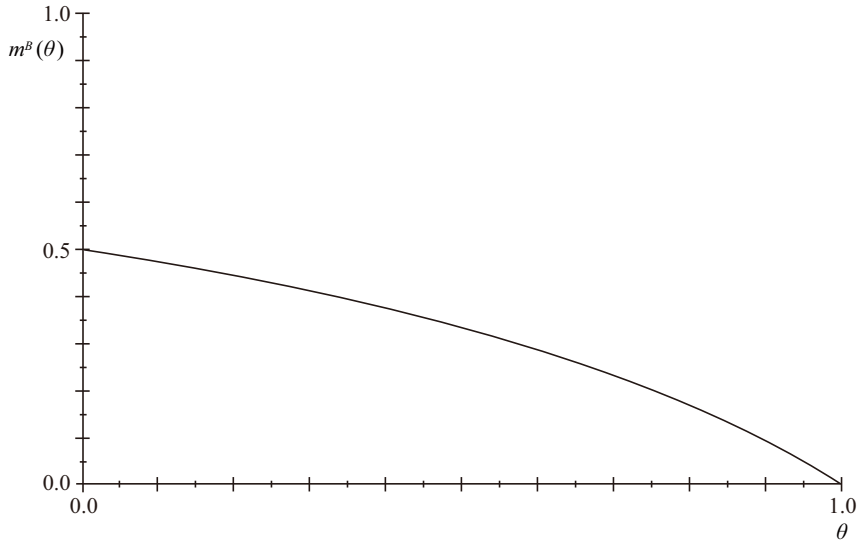

Figure 11: Knowledge Differential at the Bliss Point B as a Function of θ

Finally, we compose (7) with (8) to obtain bliss point productivity as a function of θ:

$$g(m^B(\theta)) = \beta \cdot \theta^{\theta} \cdot (1-\theta)^{1-\theta} \tag{9}$$

This function is graphed in Figure 12.

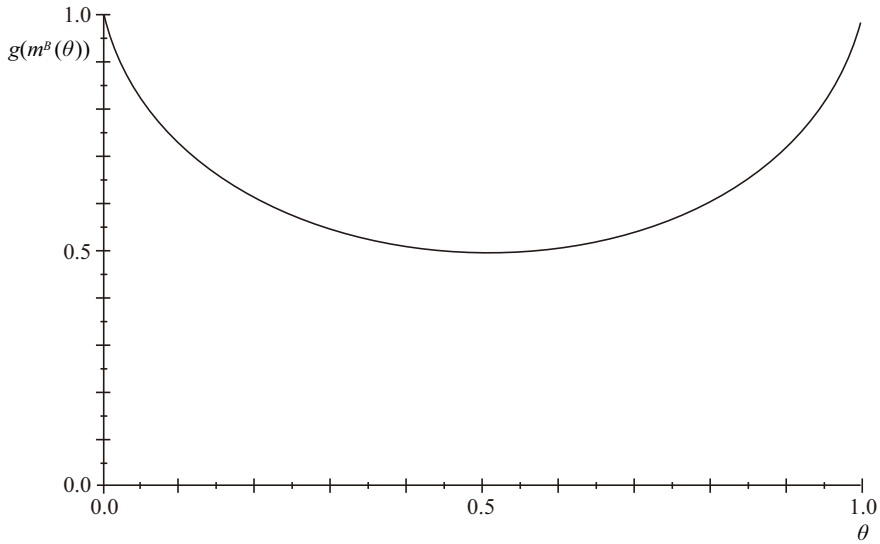

Figure 12: Knowledge Productivity as a Function of θ

Notice that it is a convex function.

Finally, we commence our extension. Suppose that agents are heterogeneous, in that θ, the relative importance of knowledge in common in knowledge creation, differs. For agent i, we take $\theta_i \in (0, 1)$ as exogenous. There could be an exogenously given distribution over θ, but for our purposes we do not need to give notation. When two agents i and j work together, we assume that the knowledge creation function given in equation (6) holds for the average of the two values of θ, namely $\theta \equiv \frac{\theta_i+\theta_j}{2}$.

If the function given in equation (9) were concave, then with this structure, we would expect agents with dissimilar values of θ to work together in equilibrium, forming research groups of approximately the same size. However, the function is *convex*,[31] so the prediction is that to maximize the rate of knowledge creation,[32] agents with similar values of θ will match. This is called *assortative matching* in the literature. In sum, the prediction is that we will have a large variety of research group sizes, each group having members who are relatively homogeneous in θ.

5 Epilogue: Inside a Brain

One among many items that Masa has taught us is how, ideally, research is to be done. This is not just about scientific method, but something much more specific. It is about how to organize an academic literature in one's mind, what steps are needed to complete a literature, and how to deal with tensions in a literature to produce new ideas as a next step. It is both methodical and methodological. And it is precisely this structure and specific set of steps that we are trying to model in current work, first as a theory and then as a practical algorithm, in an attempt to construct a robot economist.

Next we describe the static structure of knowledge as a brief introduction to this work. At the outset, there are four finite sets: Assumptions, Models, Implications, and Observations. The order of these spaces matters, as there are maps between subsets of each space, which are given the natural set lattice structure, and the next. The map from Assumptions to Models simply indicates which subset of assumptions comprise a particular model.

[31] Of course, in theory the function could have been neither convex nor concave.

[32] We have shown in Berliant and Fujita (2011) that this is equivalent to maximizing researcher income or utility.

The map from a subset of Models to Implications tells us what conclusions are consistent with all of the models in the subset. Similarly, the map from a subset of Implications to Observations tells us what observations are consistent with all of the elements of the subset of Implications. Finally, the map from a subset of Observations back to Assumptions reveals which assumptions are consistent with the observations. This last map is special, since it involves reverse engineering back through the other spaces and maps, more precisely taking the inverse image. Fixed points of the composition of these maps represent a complete paper, and in terms of mathematics the fixed points form a complemented lattice that is called a literature.

This theory is both positive, in the sense that it describes how Masa does research, and normative, in the sense that his students and coauthors tend to jump around when doing research, much to Masa's dismay.[33] According to Masa, we should not be performing these acrobatics. Thus, the theory does not describe the how a random economist does research; rather, it is a model of Masa's brain: Berliant and Fujita (2015).

[33]This is one easy way to provoke him. Soothing him requires the music of AKB 48.

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