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# Twisted Fourier-Mukai number of a $K3$ surface

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In my poster, I exhibited a counting formula for the twisted Fourier-Mukai (FM) partners of a projective $K3$ surface. Let $S$ be a projective $K3$ surface over $\mathbb{C}$. A twisted $K3$ surface $(S', \alpha')$ is called a *twisted FM partner* of $S$ if there is an exact equivalence $D^b(S) \simeq D^b(S', \alpha')$ between their derived categories. Let $\mathrm{FM}^d(S)$ be the set of isomorphism classes of twisted FM partners $(S', \alpha')$ of $S$ with $\mathrm{ord}(\alpha') = d$. I calculated the number $\#\mathrm{FM}^d(S)$ from severel lattice-theoretic informations about the lattice $H^2(S, \mathbb{Z})$ equipped with a natural Hodge structure. The number $\sum_d \#\mathrm{FM}^d(S)$ has the following meanings.

- The number of certain geometric origins of the category $D^b(S)$.
- The number of isomorphism classes of 2-dimensional compact moduli spaces of stable sheaves on $S$, considered with natural obstruction classes.
- The number of the 0-dimensional cusps of the Kahler moduli of $S$.

Now the formula is stated as follows.

**Theorem 0.1.** *Let $\varepsilon(d) = 1$ or $2$ according to $d \leq 2$ or $\geq 3$. For a projective $K3$ surface $S$ the following formula holds.*

$$\#\mathrm{FM}^d(S) = \sum_x \left\{ \sum_M \#\Big( O_{Hodge}(T_x, \alpha_x) \backslash O(D_M)/O(M) \Big) + \varepsilon(d) \sum_{M'} \#\Big( O_{Hodge}(T_x, \alpha_x) \backslash O(D_{M'})/O(M') \Big) \right\}.$$

*Here $x$ runs over the set $O_{Hodge}(T(S)) \backslash I^d(D_{NS(S)})$ and the lattices $M$, $M'$ run over the sets $\mathcal{G}_1(M_x)$, $\mathcal{G}_2(M_x)$ respectively.*

This formula is simplified if $S$ satisfies either of the following conditions : (1) The Neron-Severi lattice $NS(S)$ contains the hyperbolic plane $U$. (2) $NS(S)$ is 2-elementary. (3) The rank of $NS(S)$ equals to 1.

As an application of the formula, I gave a set of explicit Mukai vectors for a projective $K3$ surface of Picard number 1 such that the set of the corresponding moduli spaces of stable sheaves, considered with natural obstruction classes, coincides with the set $\mathrm{FM}^d(S)$.

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