代数幾何学シンポジウム記録
2008年度　pp.129-129

Non-symplectic automorphisms of prime order on $K3$ surfaces

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We study non-symplectic automorphisms of prime order on algebraic $K3$ surface X which act trivially on the Néron-Severi lattice S_X i.e. $\varphi^*\omega_X = \zeta\omega_X$.
(ω_X: nowhere vanishing holomorphic 2-form, ζ: primitive p-th root of unity.).

Main Theorem

Let r be the Picard number of X. We assume that $S_X^*/S_X \simeq (\mathbb{Z}/p\mathbb{Z})^s$.

(1) If $22 - r - (p-1)s \notin 2(p-1)\mathbb{Z}_{\geq 0} \Rightarrow \not\exists\, \varphi$.

(2) If $22 - r - (p-1)s \in 2(p-1)\mathbb{Z}_{\geq 0} \Rightarrow \exists\, \varphi$.

$$X^\varphi = \begin{cases} \phi & \text{if } S_X = U(2) \oplus E_8(2) \\ C^{(1)} \amalg C^{(1)} & \text{if } S_X = U \oplus E_8(2) \\ \{P_1\} \amalg \cdots \amalg \{P_M\} & \text{if } p \neq 2,\ b_1(X^\varphi) = 0 \text{ and } \chi(X^\varphi) = M \\ \{P_1\} \amalg \cdots \amalg \{P_M\} \amalg C^{(g)} \amalg E_1 \amalg \cdots \amalg E_{N-1} & \text{otherwise} \end{cases}$$

- P_j is an isolated point.
- $C^{(g)}$ is a non-singular curve with genus g.
- E_k is a non-rational curve.

$$\chi(X^\varphi) = \frac{pr - (24-2p)}{p-1}, \qquad b_1(X^\varphi) = \frac{22 - r - (p-1)s}{p-1},$$

$$g = \frac{22 - r - (p-1)s}{2(p-1)},$$

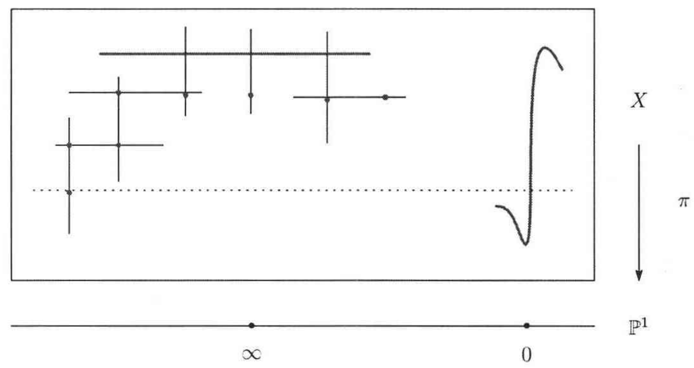
$$M = \begin{cases} 0 & p = 2 \\ \dfrac{(p-2)r + 22}{p-1} & p = 17, 19 \\ \dfrac{(p-2)r - 2}{p-1} & \text{otherwise} \end{cases}, \qquad N = \begin{cases} \dfrac{r - s + 2}{2} & p = 2 \\ \dfrac{2p + r - (p-1)s - 24}{2(p-1)} & p = 17, 19 \\ \dfrac{2p + r - (p-1)s}{2(p-1)} & \text{otherwise} \end{cases}.$$

Example

We give affine equations of elliptic $K3$ surfaces $X : y^2 = x^3 + x + t^7$. Then $S_X = U \oplus E_8$.

The elliptic fibration π has a singular fiber of type II^* over $t = \infty$ and 14 singular fibers of type I_1 over $t^{14} = -4/27$.

We put $\varphi(x, y, t) = (x, y, \zeta t)$. Then φ is a non-symplectic automorphisms of order 7.



Now $p = 7$, $r = 10$ and $s = 0 \rightsquigarrow X^\varphi = C^{(1)} \amalg \mathbb{P}^1 \amalg \{pt\} \times 8$.

Remark

We assume $22 - r - (p-1)s \in 2(p-1)\mathbb{Z}_{\geq 0}$. For each (r, s), there exist a $K3$ surface which has a non-symplectic automorphism of order p acting trivially on S_X.

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