Summary of "A bifurcation phenomenon of Stokes curves around a double turning point, and influence of virtual turning points upon the transition probabilities for three-level systems"

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Non-adiabatic transition problems of three-level systems have been studied by many researchers ([CH], [BE], [JP] etc.). From the viewpoint of the exact WKB analysis, Aoki, Kawai and Takei ([AKT2]) studied the following equation:

$$i\frac{d}{dt}\Psi = \eta H(t,\eta)\Psi,$$

where $\eta > 0$ is a large parameter, $\Psi = \Psi(t,\eta)$ is a 3-vector and $H(t,\eta)$ is a 3×3 matrix of the form

$$\begin{aligned} H(t,\eta) &= H_0(t) + \eta^{-1/2}H_{1/2} \\ &= \begin{pmatrix} \rho_1(t) & 0 & 0 \\ 0 & \rho_2(t) & 0 \\ 0 & 0 & \rho_3(t) \end{pmatrix} + \eta^{-1/2}\begin{pmatrix} 0 & c_{12} & c_{13} \\ \overline{c_{12}} & 0 & c_{23} \\ \overline{c_{13}} & \overline{c_{23}} & 0 \end{pmatrix}. \end{aligned}$$

Here $\rho_1(t)$, $\rho_2(t)$, $\rho_3(t)$ are polynomials and c_{12}, c_{13}, c_{23} are complex constants. In [AKT2], it is shown that if $\rho_j(t)$' s are real polynomials and if the Stokes geometry is not degenerate, new Stokes curves (NSCs, for short)[1] are inert (namely the Stokes coefficients are zero) near the real axis. This means that virtual turning points (VTPs)[1] and new Stokes curves have no effect on the connection problem of solutions (or transition probabilities). Shudo ([Sh]) and the present author ([Sa]) considered the situations where the Stokes geometry is degenerate (by resolving the degeneration with small complex perturbation), and showed that virtual turning points and new Stokes curves are not negligible for the computation of transition probabilities in general situations.

In the present paper, we show the importance of VTPs and NSCs by presenting some concrete examples where their contributions to transition probabilities are greater than those of ordinary turning points (OTPs) and ordinary Stokes curves (OSCs). In the first example we compare two different transitions (e.g., transitions $3 \to 2$ and $3 \to 1$) one of which is caused by an OTP and the other by a VTP, and show that the transition probability for the latter one is bigger (§3.1). In the second example we take one transition to which an OTP and a VTP both contribute, and show that the contribution by the VTP is greater (§3.2). These results show that VTPs and NSCs are indispensable to the calculation of transition probabilities for three-level systems, and vital even for their approximate calculation.

[1]For multi-level systems, new Stokes curves are needed ([BNR]). [AKT1] introduced the notion of virtual turning points so that new Stokes curves can be interpreted as Stokes curves emanating from virtual turning points.

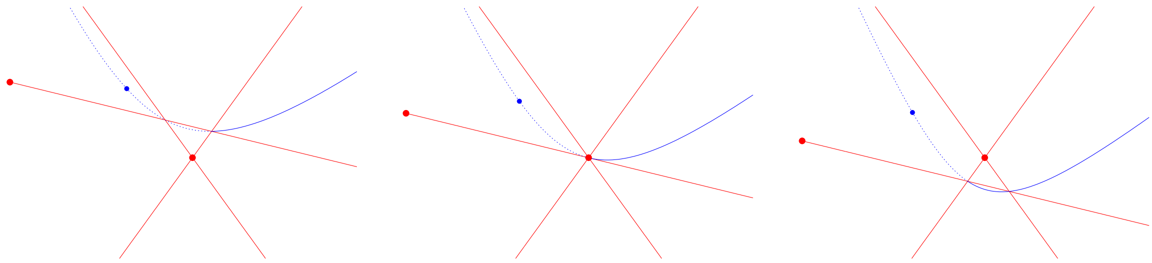

Figure 1: Bifurcation around a double turning point

To this problem, a bifurcation phenomenon of Stokes curves around a double turning point (cf. Figure 1) is closely related, and we analyze this phenomenon as well (§2). Specifically, we clarify the relation of Stokes coefficients before and after the bifurcation (Theorem 1). In the above calculation of transition probabilities, it may occur that the expressions of transition probabilities change in an apparently discontinuous manner when the perturbation parameter varies due to the bifurcation phenomenon (§1). By applying Theorem 1, we can verify that such apparent discontinuous changes are superfluous ones (§3).

In the last section (§4), we further present some future problems related to another kind of degeneration of Stokes geometry, that is, the existence of Stokes segments. Stokes segments are one of the central issues of the exact WKB theory.

References

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