Abstract

Studies on Approximation Algorithms for Bin-Packing and Train Delivery Problems

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The classical one-dimensional bin packing and the vehicle routing problem have long served as a proving ground for new approaches and techniques to the analysis of approximation algorithms. The two problems are among the most studied NP-hard problems. They are studied by a lot of research fields, including artificial intelligence, logic design, scheduling, without saying computational complexity.

Bin packing was one of the first combinatorial optimization problems for which the idea of worst-case performance guarantees was investigated. The idea of proving lower bounds on the performance of online algorithm was also first developed in the domain of bin packing problem.

In this thesis, we study *online bin packing with* $(1, 1)$ *and* $(2, R)$ *bins problem* (OBP1R), the *black and white bin packing problem* ($B\&W$) and *train delivery problem* (TDP). OBP1R and $B\&W$ are variants of online bin packing problem, while the TDP problem can be viewed as a variant of one dimensional vehicle routing problem with unsplittable demands. TDP problem is also correlated with bin packing and scheduling problems. We give several new algorithms and analyses for them.

In chapter 2, we first consider a generalization of the bin packing problem, the *online bin packing with* $(1, 1)$ *and* $(2, R)$ *bins problem* (OBP1R). The OBP1R problem is a variant of online bin packing problem, in OBP1R there are two types of bins: $(1, 1)$ and $(2, R)$, i.e., unit size bin with cost 1 and size 2 bin with cost $R > 1$, the objective is to minimize the total cost occurred when all the items are packed into the two types of bins. The problem is related to the generalized cost variable sized bin packing problem, which is a generalization of the variable sized bin packing problem. In the Generalized Cost Variable Sized Bin Packing Problem (GCVS), there will be infinite supply of r types of bins whose sizes are $0 < b_r < ... < b_1 = 1$

respectively, each bin of type i is associated with cost $c_i > 0$, the goal is to find a feasible solution with a minimal cost. There is an APTAS (Asymptotic Polynomial Time Approximation Scheme), for this problem. However, the APTAS does not give us too much idea about how the different costs affect the packing of items. In this thesis we consider the most basic case that only two different types of bins are used, and we study the online case, i.e., items are given one by one, once items are packed, we cannot repack them. The OBP1R problem is also related to the parametric bin packing problem, in which all the items are with sizes in $(0, 1/r]$, where $r > 0$ is an integer, the goal is to pack all the items in a minimum number of unit size bins. In the problem, the value of R has a critical effect on the solution of the problem. When $R > 3$, the offline version of OBP1R problem is equivalent to the classical bin packing problem. We focus on the case $R \leq 3$, and propose online algorithms and obtain lower bounds for the problem. We show our upper bounds and lower bounds by graph, they are almost tight, and for the part that the problem is equivalent to a parametric bin packing problem, we can get a better lower bound than the previous result of this problem.

In chapter 3, we study the *black and white bin packing* problem ($B\&W$), which is a variation of the classical bin packing problem. In the $B\&W$ problem, we have infinite supply of unit size bins, input are items of size in $(0, 1]$ and color either black or white, each item should not pack onto an item with the same color, and the total size of items in a bin should not exceed 1, the object is to minimize the total number of bins used. The previous online upper bound for this problem is provided by Dosa, they designed a Pseudo Algorithm for the problem, the upper bound is 3. They also proved lower bound for the problem which is 2. If the items have more than two colors, this problem is called colorful bin packing problem, in this case the best algorithm has upper bound 4, and the lower bound for colorful bin packing is 2.5. We find that the upper bound of Dosa′s algorithm is still 3 even all the input items have sizes in $(0, 1/2]$, the algorithm do not work better for smaller items. It has been conjectured that there be a "better than 3" competitive algorithm for $B\&W$. In this thesis we settle this conjecture in the affirmative for the case when sizes in $(0, 1/2]$.

In chapter 4, we study the train delivery problem, which is a generalization of

the bin packing problem, and is also equivalent to one dimensional version of the vehicle routing problem with unsplittable demands. In the problem, there are trains with capacity W and a set of products, which are called customers, each customer is associated with (s, p), where s is the size and p is the position. A set of products with a total size at most W can be shipped together, however the cost of the shipment is dominated by the largest position value of products in the train. We are asked to ship all the products and minimize the total cost occurred. If all the positions are equal to one, the problem is degenerated to one dimensional bin packing problem. Based on the techniques of scheduling problem, we give an APTAS for the general problem with time complexity $O(n^{O(\epsilon^{-4})})$, which is better than the previous one $O(n^{(\frac{1}{\epsilon})^{O(\frac{1}{\epsilon})}})$, where n is the number of input elements.