Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 905 (2016) 242–250

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Yang–Baxter invariance of the Nappi–Witten model

Hideki Kyono *, Kentaroh Yoshida

Department of Physics, Kyoto University, Kitashirakawa Oiwake-cho, Kyoto 606-8502, Japan

Received 23 December 2015; received in revised form 17 February 2016; accepted 17 February 2016

Available online 23 February 2016

Editor: Hubert Saleur

Abstract

We study Yang–Baxter deformations of the Nappi–Witten model with a prescription invented by Delduc, Magro and Vicedo. The deformations are specified by skew-symmetric classical r-matrices satisfying (modified) classical Yang–Baxter equations. We show that the sigma-model metric is invariant under arbitrary deformations (while the coefficient of B-field is changed) by utilizing the most general classical r-matrix. Furthermore, the coefficient of B-field is determined to be the original value from the requirement that the one-loop β-function should vanish. After all, the Nappi–Witten model is the unique conformal theory within the class of the Yang–Baxter deformations preserving the conformal invariance.

1. Introduction

The Yang–Baxter sigma-model description, which was originally proposed by Klimcik [1], is a systematic way to consider integrable deformations of 2D non-linear sigma models. According to this procedure, the deformations are specified by skew-symmetric classical r-matrices satisfying the modified classical Yang–Baxter equation (mCYBE). The original work [1] has been generalized to symmetric spaces [2] and the homogeneous CYBE [3].

Yang–Baxter deformations of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring can be studied with the mCYBE [4] and the CYBE [5]. For the former case, the metric and B-field are derived in [6] and the

* Corresponding author.
E-mail addresses: h_kyono@gauge.scphys.kyoto-u.ac.jp (H. Kyono), kyoshida@gauge.scphys.kyoto-u.ac.jp (K. Yoshida).

http://dx.doi.org/10.1016/j.nuclphysb.2016.02.017

full background has recently been studied in [7,8]. For the latter case, classical r-matrices are identified with solutions of type IIB supergravity including γ-deformations of S^5 [9,10] and gravity duals of non-commutative gauge theories [11,12], in a series of works [13–20] (for a short summary, see [21]).

Lately, Yang–Baxter deformations of 4D Minkowski spacetime have been studied [22,23]. In [22], classical r-matrices are identified with exactly-solvable string backgrounds such as Melvin backgrounds and pp-wave backgrounds. In [23], Yang–Baxter deformations of 4D Minkowski spacetime are discussed by using classical r-matrices associated with κ-deformations of the Poincaré algebra [24]. Then the resulting deformed geometries include T-duals of (A)dS_4 spaces[1] and a time-dependent pp-wave background. Furthermore, the Lax pair is presented for the general κ-deformations [23,26].

As a spin off from this progress, it would be interesting to study Yang–Baxter deformations of the Nappi–Witten model [27]. The target space of this model is given by a centrally extended 2D Poincaré group. Hence the Yang–Baxter deformed Nappi–Witten models can be regarded as toy models of the previous works [22,23], because the structure of the target space is much simpler than that of 4D Minkowski spacetime. This simplification makes it possible to study the most general Yang–Baxter deformation. As a matter of course, it is exceedingly complicated in general, hence such an analysis has not been done yet.

In this article, we investigate Yang–Baxter deformations of the Nappi–Witten model by following a prescription invented by Delduc, Magro and Vicedo [28]. We show that the sigma-model metric is invariant under the deformations (while the coefficient of B-field is changed) by utilizing the most general classical r-matrix. Furthermore, the coefficient of B-field is determined to be the original value from the requirement that the one-loop β-function should vanish. After all, the Nappi–Witten model is the unique conformal theory within the class of the Yang–Baxter deformations preserving the conformal invariance (i.e., Yang–Baxter invariance).

2. Nappi–Witten model

In this section, we shall give a concise review of the Nappi–Witten model [27].

The Nappi–Witten model is a Wess–Zumino–Witten (WZW) model whose target space is given by a centrally extend 2D Poincaré group. The associated extended Poincaré algebra $\mathfrak{g}$ is composed of two translations P_i $(i = 1, 2)$, a rotation J and the center T. The commutation relations of the generators are given by

$$[J, P_i] = \epsilon_{ij} P_j\,, \quad [P_i, P_j] = \epsilon_{ij} T\,, \quad [T, J] = [T, P_i] = 0\,, \tag{2.1}$$

where ϵ_{ij} is an anti-symmetric tensor normalized as $\epsilon_{12} = 1$. It is convenient to introduce a notation of the generators with the group index I like

$$T_I = \{P_1, P_2, J, T\} \qquad (I = 1, 2, 3, 4)\,. \tag{2.2}$$

Let us introduce a group element represented by

$$g = \exp(a_1 P_1 + a_2 P_2) \exp(u J + v T)\,. \tag{2.3}$$

By using this group element g, the left-invariant current A can be evaluated as

[1] T-dual of dS_4 can be derived as a scaling limit of η-deformed AdS_5 as well [25].

$$
\begin{aligned}
A_\alpha &\equiv g^{-1}\partial_\alpha g = A^I\, T_I \\
&= (\cos u\, \partial_\alpha a_1 + \sin u\, \partial_\alpha a_2)\; P_1 + (\cos u\, \partial_\alpha a_2 - \sin u\, \partial_\alpha a_1)\; P_2 \\
&\quad + \partial_\alpha u\, J + \left[\partial_\alpha v + \frac{1}{2}\, a_2\, \partial_\alpha a_1 - \frac{1}{2}\, a_1\, \partial_\alpha a_2\right] T\,.
\end{aligned} \tag{2.4}
$$

Here the index $\alpha = \tau,\, \sigma$ is for the world-sheet coordinates. It is also helpful to introduce the light-cone expression of A on the world-sheet like

$$
A_\pm \equiv A_\tau \pm A_\sigma\,. \tag{2.5}
$$

By using $A_\pm$, the classical action of the Nappi–Witten model is given by

$$
S[A] = \frac{1}{2}\int_\Sigma d^2\sigma\; \Omega_{IJ}\, A^I_- A^J_+ + \frac{1}{6}\int_{B_3} d^3\sigma\; \epsilon^{\hat\alpha\hat\beta\hat\gamma}\, \Omega_{KL}\, f_{IJ}{}^L A^I_{\hat\alpha} A^J_{\hat\beta} A^K_{\hat\gamma}\,. \tag{2.6}
$$

This action is basically composed of the two parts, 1) the sigma model part and 2) the Wess–Zumino–Witten (WZW) term.

The sigma model part is defined as usual on the world sheet Σ, where we assume that Σ is compact and the periodic boundary condition is imposed for the dynamical variables. A key ingredient contained in this part is the most general symmetric two-form[2]

$$
\Omega_{IJ} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & b & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{2.7}
$$

which satisfies the following condition:

$$
f_{IJ}{}^K \Omega_{LK} + f_{IL}{}^K \Omega_{JK} = 0\,. \tag{2.8}
$$

Here $f_{IJ}{}^K$ are the structure constants which determine the commutation relations

$$
[T_I, T_J] = f_{IJ}{}^K T_K\,.
$$

The WZW term in (2.6) also contains Ω_{IJ}, but, apart from this point, it is the same as the usual. The symbol B_3 denotes a 3D space which has Σ as a boundary. Hence the domain of A^I is implicitly generalized to B_3 with $\hat\alpha = \tau, \sigma$ and ξ in the WZW term,[3] where the extra direction is labeled by ξ.

A remarkable point is that the action (2.6) can be rewritten into the following form [27][4]:

$$
S = -\frac{1}{2}\int_\Sigma d^2\sigma \left[\gamma^{\alpha\beta} g_{\mu\nu}\, \partial_\alpha X^\mu \partial_\beta X^\nu - \epsilon^{\alpha\beta}\, B_{\mu\nu}\, \partial_\alpha X^\mu \partial_\beta X^\nu\right]. \tag{2.9}
$$

Here $X^\mu = \{u, v, a_1, a_2\}$ are the dynamical variables. The metric and anti-symmetric two-form on Σ are described by $\gamma^{\alpha\beta} = \mathrm{diag}(-1, 1)$ and $\epsilon^{\alpha\beta}$ normalized as $\epsilon^{\tau\sigma} = 1$. Then the space–time metric $g_{\mu\nu}$ and two-form field B are given by

[2] The overall factor of Ω_{IJ} (i.e., the level of the WZW model) is set to be 1 because it is irrelevant to the deformations we consider later.

[3] For the detail of the WZW model, for example, see [29].

[4] In this derivation, we have used the identities (12) and (13) in [27].

$$
\begin{aligned}
ds^2 &= g_{\mu\nu}\, dX^\mu dX^\nu = 2du dv + b\, du^2 + da_1^2 + da_2^2 + a_1\, da_2 du - a_2\, da_1 du\,, \\
B &= B_{\mu\nu}\, dX^\mu \wedge dX^\nu = u\, da_1 \wedge da_2\,.
\end{aligned} \tag{2.10}
$$

This is a simple 4D background.

It would be helpful to further rewrite the background (2.10). By performing the following coordinate transformation with a real constant m [30]

$$
\begin{aligned}
a_1 &\to a_1 \cos(m\,u) + a_2 \sin(m\,u)\,, \qquad a_2 \to a_1 \sin(m\,u) - a_2 \cos(m\,u)\,, \\
u &\to 2m\,u\,, \qquad v \to \frac{1}{2m}\, v - b\, m\, u\,,
\end{aligned} \tag{2.11}
$$

the background (2.10) can be rewritten as

$$
\begin{aligned}
ds^2 &= 2du dv - m^2(a_1^2 + a_2^2)\, du^2 + da_1^2 + da_2^2\,, \\
B &= -2m\, u\, da_1 \wedge da_2 + 2m^2 a_1\, u\, da_1 \wedge du + 2m^2 a_2\, u\, da_2 \wedge du\,.
\end{aligned} \tag{2.12}
$$

This is nothing but a pp-wave background. Note here that the last two terms of B-field in (2.12) contribute to the Lagrangian as the total derivatives, which can be ignored in the present setup. For this background (2.12), the world-sheet β-function vanishes at the one-loop level [27].

3. Yang–Baxter deformed Nappi–Witten model

In this section, let us consider Yang–Baxter deformations of the Nappi–Witten model.

3.1. A Yang–Baxter deformed classical action

As explained in the previous section, the Nappi–Witten model contains the WZW term. Hence it is not straightforward to study Yang–Baxter deformations of this model. Our strategy here is to follow a prescription invented by Delduc, Magro and Vicedo [28]. This is basically a two-parameter deformation. It is an easy task to extend their prescription to the Nappi–Witten model.

A deformed action we propose is the following[5]:

$$
S = \frac{1}{2} \int_\Sigma d^2\sigma\; \Omega_{IJ}\, A_-^I J_+^J + \frac{1}{2} k \int_{B_3} d^3\sigma\; \Omega_{KL}\, f_{IJ}{}^L A_\xi^I A_-^J A_+^K\,. \tag{3.1}
$$

Here the deformed current J is defined as

$$
J_\pm \equiv (1 + \omega\, \eta^2) \frac{1 \pm \tilde{A} R}{1 - \eta^2 R^2} A_\pm\,. \tag{3.2}
$$

First of all, the classical action (3.1) includes three constant parameters η, $\tilde{A}$ and k. The deformation is measured by η and $\tilde{A}$. The last parameter k is regarded as the level. When $\eta = \tilde{A} = 0$ and $k = 1$, the action (3.1) is reduced to the original Nappi–Witten model.

A key ingredient contained in J is a linear operator R: $\mathfrak{g} \to \mathfrak{g}$. In the context of Yang–Baxter deformations, it is supposed that R should be skew-symmetric and satisfy the (modified) Yang–Baxter equation [31]

[5] Here we would like to start from the WZNW model (non-conformal) rather than the WZW model (conformal). Hence, a real constant k has been put in front of the WZW term to measure the non-conformality.

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = \omega [X, Y] \qquad (X, Y \in \mathfrak{g}) . \tag{3.3}$$

The constant parameter ω can be normalized by rescaling R, hence it is enough to consider the following three cases: $\omega = \pm 1$ and 0. In particular, the case with $\omega = 0$ is the homogeneous CYBE.

3.2. The general solution of the (m)CYBE

In this subsection, we derive the general solution of the (m)CYBE.

Let us start from the most general expression of a linear R-operator:

$$R(X) = M^{IJ} \Omega_{JK} x^K T_I , \qquad M^{IJ} \equiv \sum_i \left(a_i^I b_i^J - b_i^I a_i^J \right) . \tag{3.4}$$

Here M^{IJ} is an anti-symmetric 4×4 matrix which is parametrized as

$$M^{IJ} = \begin{pmatrix} 0 & m_1 & m_2 & m_3 \\ -m_1 & 0 & m_4 & m_5 \\ -m_2 & -m_4 & 0 & m_6 \\ -m_3 & -m_5 & -m_6 & 0 \end{pmatrix} , \qquad m_i \in \mathbb{R} . \tag{3.5}$$

By using the expression (3.4) and the defining relation of Ω_{IJ} (2.8), the (m)CYBE (3.3) can be rewritten into the following form:

$$f_{LM}{}^K M^{LI} M^{MJ} + f_{LM}{}^I M^{LJ} M^{MK} + f_{LM}{}^J M^{LK} M^{MI} - \omega f_{LM}{}^K \Omega^{LI} \Omega^{MJ} = 0 . \tag{3.6}$$

Note that we define Ω^{IJ} as the inverse matrix of Ω_{IJ}. Then, by putting the expression (3.5) into (3.6), the most general solution can be determined like

$$M^{IJ} = \begin{pmatrix} 0 & \sqrt{\omega} & 0 & m_3 \\ -\sqrt{\omega} & 0 & 0 & m_5 \\ 0 & 0 & 0 & m_6 \\ -m_3 & -m_5 & -m_6 & 0 \end{pmatrix} . \tag{3.7}$$

Here the condition (3.6) has led to the following constraints:

$$m_1 = \sqrt{\omega} , \qquad m_2 = m_4 = 0 .$$

Then we have also supposed that $\omega \geq 0$ in order to preserve the reality of the background.[6] After all, m_3, m_5 and m_6 have survived as free parameters of the R-operator as well as ω.

3.3. The general deformed background

Let us consider a deformation of the Nappi–Witten model with the general solution (3.7). The resulting background is given by

$$\begin{aligned} ds^2 = {} & da_1^2 + da_2^2 + \frac{(1+\omega\eta^2)b - (m_3^2 + m_5^2)\eta^2}{1 - m_6^2 \eta^2} du^2 + 2 \frac{1+\omega\eta^2}{1 - m_6^2 \eta^2} du dv \\ & + \frac{(1+\omega\eta^2)a_2 + 2\eta^2 \{ (m_3 m_6 + m_5 \sqrt{\omega}) \cos u - (m_5 m_6 - m_3 \sqrt{\omega}) \sin u \}}{1 - m_6^2 \eta^2} da_1 du \end{aligned}$$

[6] When we consider $\omega < 0$, we need to multiply the linear R-operator by the imaginary unit i.

$$- \frac{(1+\omega\eta^2)a_1 - 2\eta^2\{(m_5m_6 - m_3\sqrt{\omega})\cos u + (m_3m_6 + m_5\sqrt{\omega})\sin u\}}{1-m_6^2\eta^2} da_2 du\,,$$

$$B = k\,u\,da_1\wedge da_2 + \frac{\tilde{A}m_6(1+\omega\eta^2)}{2(1-m_6^2\eta^2)}\left(a_2\,da_1\wedge du - a_1\,da_2\wedge du\right)\,. \tag{3.8}$$

Here we have ignored the total derivative terms that appeared in the B-field part. This background (3.8) can be simplified by performing a coordinate transformation

$$\begin{aligned}
a_1 &\to a_1\cos(m\,u) + a_2\sin(m\,u) + C_1\cos(C_3\,u) + C_2\sin(C_3\,u)\,,\\
a_2 &\to -a_2\cos(m\,u) + a_1\sin(m\,u) - C_2\cos(C_3\,u) + C_1\sin(C_3\,u)\,,\\
u &\to C_3\,u\,,\\
v &\to \frac{1}{2m}v - \frac{1}{2}b\,C_3\,u - \frac{1}{2}\Big[C_2\cos\left(\frac{C_3-C_4}{2}u\right) - C_1\sin\left(\frac{C_3-C_4}{2}u\right)\Big]a_1\\
&\quad + \frac{1}{2}\Big[C_1\cos\left(\frac{C_3-C_4}{2}u\right) + C_2\sin\left(\frac{C_3-C_4}{2}u\right)\Big]a_2\,,
\end{aligned} \tag{3.9}$$

where we have introduced the following quantities:

$$\begin{aligned}
C_1 &\equiv \frac{m_5m_6 - m_3\sqrt{\omega}}{m_6^2+\omega}\,, \qquad C_2 \equiv \frac{m_3m_6 + m_5\sqrt{\omega}}{m_6^2+\omega}\,,\\
C_3 &\equiv 2m\frac{1-m_6^2\eta^2}{1+\omega\eta^2}\,, \qquad C_4 \equiv 2m\frac{m_6^2+\omega}{1+\omega\eta^2}\eta^2\,.
\end{aligned} \tag{3.10}$$

After performing the transformation (3.9), the resulting background is given by the following pp-wave background equipped with a B-field:

$$\begin{aligned}
ds^2 &= 2du\,dv - m^2(a_1^2+a_2^2)du^2 + da_1^2 + da_2^2\,,\\
B &= -k\,C_3\,u\,da_1\wedge da_2 - m\tilde{A}m_6a_2\,da_1\wedge du + m\tilde{A}m_6a_1\,da_2\wedge du\,.
\end{aligned} \tag{3.11}$$

Here we have ignored the total derivative terms again.

Note that the B-field in (3.11) can be rewritten as (up to total derivative terms)

$$B = -(k\,C_3 - 2m\,\tilde{A}\,m_6)\,u\,da_1\wedge da_2\,. \tag{3.12}$$

Comparing (3.12) with the B-field in (2.12), one can find that only the difference is the coefficient of B-field. From the viewpoint of the original Nappi–Witten model, the coefficient of the WZW term has been changed and the resulting theory should be regarded as a WZNW model. According to this observation, it is obvious that the deformed model is exactly solvable.[7]

3.4. Yang–Baxter deformations and conformal invariance

Finally, let us show that the original Nappi–Witten model is the unique conformal theory within the class of the Yang–Baxter deformations preserving the conformal invariance.

Due to the requirement of the vanishing β-function at the one-loop level, the two B-fields should be identical as follows:

[7] In the case of [28], it is necessary to impose a condition for $\tilde{A}$ so as to preserve the integrability (cf., [32]). However, such an extra condition is not needed in the present case.

$$2m \quad \text{(the original)} \quad = \quad k\,C_3 - 2m\,\tilde{A}\,m_6 \quad \text{(the deformed)}\,. \tag{3.13}$$

This condition indicates two interesting results. The first one is the Yang–Baxter invariance of the Nappi–Witten model. If we start from the case with $k = 1$, then the original system is invariant under the Yang–Baxter deformations preserving the conformal invariance, which are specified by the parameters satisfying the condition

$$1 = \frac{1 + \omega\,\eta^2}{1 - m_6^2\,\eta^2}(1 + m_6\tilde{A})\,.$$

In other words, the Yang–Baxter invariance follows from the conformal invariance.

The second is that the Yang–Baxter deformation may map a non-conformal theory to the conformal Nappi–Witten model. Suppose that we start from the case with $k \neq 1$. Then, by performing a Yang–Baxter deformation with parameters satisfying the condition

$$k = \frac{1 + \omega\,\eta^2}{1 - m_6^2\,\eta^2}(1 + m_6\tilde{A})\,, \tag{3.14}$$

the resulting system becomes the Nappi–Witten model. In other words, the coefficient of B-field can be set to the conformal fixed point by an appropriate Yang–Baxter deformation.

4. Conclusion and discussion

In this article, we have studied Yang–Baxter deformations of the Nappi–Witten model. By considering the most general classical r-matrix, we have shown the invariance of the sigma-model metric under arbitrary deformations, up to two-form B-fields. That is, the effect coming from the deformations is reflected only as the coefficient of B-field. Then, the coefficient of B-field has been determined to be the original value from the requirement that the one-loop β-function should vanish. After all, it has been shown that the Nappi–Witten model is the unique conformal theory within the class of the Yang–Baxter deformations preserving the conformal invariance (i.e., Yang–Baxter invariance).

There are many future directions. It would be interesting to consider a supersymmetric extension of our analysis by following [33]. The number of the remaining supersymmetries should depend on Yang–Baxter deformations because the coefficient of B-field is changed. It is also nice to investigate higher-dimensional cases (e.g., the maximally supersymmetric pp-wave background [34]). As another direction, one may consider non-relativistic backgrounds such as Schrödinger spacetimes [35] and Lifshitz spacetimes [36]. Although there is a problem of the degenerate Killing form similarly, it can be resolved by adopting the most general symmetric two-form [37] as in the Nappi–Witten model. It would be straightforward to apply the techniques presented in [37] to Yang–Baxter deformations by following our present analysis.

It should be remarked that the most interesting indication of this work is the universal aspect of the dual gauge-theory side. According to our work, pp-wave backgrounds would have a kind of rigidity against Yang–Baxter deformations. This result may indicate that the ground state and lower-lying excited states of the spin chain associated with the $\mathcal{N} = 4$ super-Yang–Mills theory are *invariant*. It is quite significant to extract such a universal characteristic after classifying various examples. This is the standard strategy in theoretical physics and would be much more important than identifying the associated dual gauge theory for each of the deformations.

We hope that our result could shed light on a universal aspect of Yang–Baxter deformations from the viewpoint of the invariant pp-wave geometry.

Acknowledgements

We are very grateful to Martin Heinze and Jun-ichi Sakamoto for useful discussions. The work of K.Y. is supported by Supporting Program for Interaction-based Initiative Team Studies (SPIRITS) from Kyoto University and by the JSPS Grant-in-Aid for Scientific Research (C) No. 15K05051. This work is also supported in part by the JSPS Japan–Russia Research Cooperative Program and the JSPS Japan–Hungary Research Cooperative Program.

References

[1] C. Klimcik, Yang–Baxter sigma models and dS/AdS T duality, J. High Energy Phys. 0212 (2002) 051, arXiv:hep-th/0210095;
C. Klimcik, On integrability of the Yang–Baxter sigma-model, J. Math. Phys. 50 (2009) 043508, arXiv:0802.3518 [hep-th];
C. Klimcik, Integrability of the bi-Yang–Baxter sigma model, Lett. Math. Phys. 104 (2014) 1095, arXiv:1402.2105 [math-ph].

[2] F. Delduc, M. Magro, B. Vicedo, On classical q-deformations of integrable sigma-models, J. High Energy Phys. 1311 (2013) 192, arXiv:1308.3581 [hep-th].

[3] T. Matsumoto, K. Yoshida, Yang–Baxter sigma models based on the CYBE, Nucl. Phys. B 893 (2015) 287, arXiv:1501.03665 [hep-th].

[4] F. Delduc, M. Magro, B. Vicedo, An integrable deformation of the $AdS_5 \times S^5$ superstring action, Phys. Rev. Lett. 112 (2014) 051601, arXiv:1309.5850 [hep-th];
F. Delduc, M. Magro, B. Vicedo, Derivation of the action and symmetries of the q-deformed $AdS_5 \times S^5$ superstring, J. High Energy Phys. 1410 (2014) 132, arXiv:1406.6286 [hep-th].

[5] I. Kawaguchi, T. Matsumoto, K. Yoshida, Jordanian deformations of the $AdS_5 \times S^5$ superstring, J. High Energy Phys. 1404 (2014) 153, arXiv:1401.4855 [hep-th].

[6] G. Arutyunov, R. Borsato, S. Frolov, S-matrix for strings on η-deformed $AdS_5 \times S^5$, J. High Energy Phys. 1404 (2014) 002, arXiv:1312.3542 [hep-th].

[7] G. Arutyunov, R. Borsato, S. Frolov, Puzzles of η-deformed $AdS_5 \times S^5$, J. High Energy Phys. 1512 (2015) 049, arXiv:1507.04239 [hep-th].

[8] B. Hoare, A.A. Tseytlin, Type IIB supergravity solution for the T-dual of the η-deformed $AdS_5 \times S^5$ superstring, J. High Energy Phys. 1510 (2015) 060, arXiv:1508.01150 [hep-th].

[9] O. Lunin, J.M. Maldacena, Deforming field theories with $U(1) \times U(1)$ global symmetry and their gravity duals, J. High Energy Phys. 0505 (2005) 033, arXiv:hep-th/0502086.

[10] S. Frolov, Lax pair for strings in Lunin–Maldacena background, J. High Energy Phys. 0505 (2005) 069, arXiv:hep-th/0503201.

[11] A. Hashimoto, N. Itzhaki, Noncommutative Yang–Mills and the AdS/CFT correspondence, Phys. Lett. B 465 (1999) 142, arXiv:hep-th/9907166.

[12] J.M. Maldacena, J.G. Russo, Large N limit of noncommutative gauge theories, J. High Energy Phys. 9909 (1999) 025, arXiv:hep-th/9908134.

[13] T. Matsumoto, K. Yoshida, Lunin–Maldacena backgrounds from the classical Yang–Baxter equation – towards the gravity/CYBE correspondence, J. High Energy Phys. 1406 (2014) 135, arXiv:1404.1838 [hep-th].

[14] T. Matsumoto, K. Yoshida, Integrability of classical strings dual for noncommutative gauge theories, J. High Energy Phys. 1406 (2014) 163, arXiv:1404.3657 [hep-th].

[15] T. Matsumoto, K. Yoshida, Schrödinger geometries arising from Yang–Baxter deformations, J. High Energy Phys. 1504 (2015) 180, arXiv:1502.00740 [hep-th].

[16] I. Kawaguchi, T. Matsumoto, K. Yoshida, A Jordanian deformation of AdS space in type IIB supergravity, J. High Energy Phys. 1406 (2014) 146, arXiv:1402.6147 [hep-th].

[17] T. Matsumoto, K. Yoshida, Yang–Baxter deformations and string dualities, J. High Energy Phys. 1503 (2015) 137, arXiv:1412.3658 [hep-th].

[18] S.J. van Tongeren, On classical Yang–Baxter based deformations of the $AdS_5 \times S^5$ superstring, J. High Energy Phys. 1506 (2015) 048, arXiv:1504.05516 [hep-th];
S.J. van Tongeren, Yang–Baxter deformations, AdS/CFT, and twist-noncommutative gauge theory, Nucl. Phys. B 904 (2016) 148, arXiv:1506.01023 [hep-th].

[19] T. Kameyama, H. Kyono, J. Sakamoto, K. Yoshida, Lax pairs on Yang–Baxter deformed backgrounds, J. High Energy Phys. 1511 (2015) 043, arXiv:1509.00173 [hep-th].
[20] P.M. Crichigno, T. Matsumoto, K. Yoshida, Deformations of $T^{1,1}$ as Yang–Baxter sigma models, J. High Energy Phys. 1412 (2014) 085, arXiv:1406.2249 [hep-th];
P.M. Crichigno, T. Matsumoto, K. Yoshida, Towards the gravity/CYBE correspondence beyond integrability – Yang–Baxter deformations of $T^{1,1}$, J. Phys. Conf. Ser. 670 (1) (2016) 012019, arXiv:1510.00835 [hep-th].
[21] T. Matsumoto, K. Yoshida, Integrable deformations of the $AdS_5 \times S^5$ superstring and the classical Yang–Baxter equation – towards the gravity/CYBE correspondence –, J. Phys. Conf. Ser. 563 (1) (2014) 012020, arXiv:1410.0575 [hep-th];
T. Matsumoto, K. Yoshida, Towards the gravity/CYBE correspondence – the current status –, J. Phys. Conf. Ser. 670 (1) (2016) 012033.
[22] T. Matsumoto, D. Orlando, S. Reffert, J. Sakamoto, K. Yoshida, Yang–Baxter deformations of Minkowski spacetime, J. High Energy Phys. 1510 (2015) 185, arXiv:1505.04553 [hep-th].
[23] A. Borowiec, H. Kyono, J. Lukierski, J. Sakamoto, K. Yoshida, Yang–Baxter sigma models and Lax pairs arising from κ-Poincaré r-matrices, arXiv:1510.03083 [hep-th].
[24] J. Lukierski, H. Ruegg, A. Nowicki, V.N. Tolstoy, Q deformation of Poincaré algebra, Phys. Lett. B 264 (1991) 331;
J. Lukierski, A. Nowicki, Quantum deformations of $D = 4$ Poincaré and Weyl algebra from Q deformed $D = 4$ conformal algebra, Phys. Lett. B 279 (1992) 299.
[25] A. Pachol, S.J. van Tongeren, Quantum deformations of the flat space superstring, Phys. Rev. D 93 (2) (2016) 026008, arXiv:1510.02389 [hep-th].
[26] H. Kyono, J. Sakamoto, K. Yoshida, Lax pairs for deformed Minkowski spacetimes, J. High Energy Phys. 1601 (2016) 143, arXiv:1512.00208 [hep-th].
[27] C.R. Nappi, E. Witten, A WZW model based on a nonsemisimple group, Phys. Rev. Lett. 71 (1993) 3751, arXiv:hep-th/9310112.
[28] F. Delduc, M. Magro, B. Vicedo, Integrable double deformation of the principal chiral model, Nucl. Phys. B 891 (2015) 312, arXiv:1410.8066 [hep-th].
[29] E. Abdalla, M.C. Abdalla, K. Rothe, Non-perturbative Methods in Two-Dimensional Quantum Field Theory, World Scientific, 1991.
[30] G.W. Gibbons, C.N. Pope, Kohn's theorem, Larmor's equivalence principle and the Newton–Hooke group, Ann. Phys. 326 (2011) 1760, arXiv:1010.2455 [hep-th].
[31] M.A. Semenov-Tyan-Shanskii, What is a classical r-matrix?, Funct. Anal. Appl. 17 (1983) 259. For a nice review, see: Integrable systems and factorization problems, arXiv:nlin/0209057.
[32] I. Kawaguchi, D. Orlando, K. Yoshida, Yangian symmetry in deformed WZNW models on squashed spheres, Phys. Lett. B 701 (2011) 475, arXiv:1104.0738 [hep-th];
I. Kawaguchi, K. Yoshida, A deformation of quantum affine algebra in squashed WZNW models, J. Math. Phys. 55 (2014) 062302, arXiv:1311.4696 [hep-th].
[33] E. Kiritsis, C. Kounnas, D. Lust, Superstring gravitational wave backgrounds with space–time supersymmetry, Phys. Lett. B 331 (1994) 321, arXiv:hep-th/9404114.
[34] M. Blau, J.M. Figueroa-O'Farrill, C. Hull, G. Papadopoulos, A new maximally supersymmetric background of IIB superstring theory, J. High Energy Phys. 0201 (2002) 047, arXiv:hep-th/0110242.
[35] D.T. Son, Toward an AdS/cold atoms correspondence: a geometric realization of the Schrödinger symmetry, Phys. Rev. D 78 (2008) 046003, arXiv:0804.3972 [hep-th];
K. Balasubramanian, J. McGreevy, Gravity duals for non-relativistic CFTs, Phys. Rev. Lett. 101 (2008) 061601, arXiv:0804.4053 [hep-th].
[36] S. Kachru, X. Liu, M. Mulligan, Gravity duals of Lifshitz-like fixed points, Phys. Rev. D 78 (2008) 106005, arXiv:0808.1725 [hep-th].
[37] S. Schafer-Nameki, M. Yamazaki, K. Yoshida, Coset construction for duals of non-relativistic CFTs, J. High Energy Phys. 0905 (2009) 038, arXiv:0903.4245 [hep-th].