HADRONIC CONTRIBUTIONS TO $(g-2)_\mu$ AND $\alpha_{\rm QED}(M_Z^2)$*

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We review recent developments in the evaluations of the muon anomalous magnetic moment, putting some emphasis on the leading-order hadronic contribution to it. We also discuss related quantities such as the QED gauge coupling at the Z-pole $\alpha_{\rm QED}(M_Z^2)$ and the running QED gauge coupling $\alpha_{\rm QED}(q^2)$ as a function of q^2.

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1. Introduction

The lepton anomalous magnetic moment a_ℓ has been an important quantity since the very early stages of particle physics: one of the successes of the Dirac theory is that the Dirac equation can predict that the lepton gyromagnetic ratio g_ℓ must be 2. Also, one of the early triumphs of QED is that Schwinger was able to predict that the lowest-order QED correction to a_ℓ is $\alpha/(2\pi)$.

Not only historically but also today the anomalous magnetic moment a_μ of the muon is important since it can probe/constrain New Physics beyond the Standard Model (SM) (see *e.g.*, Refs. [1–4] for reviews). At the moment, it is measured to the precision of 0.5 ppm [5][1]

$$a_\mu({\rm exp}) = (11659208.9 \pm 6.3) \times 10^{-10}\,. \tag{1}$$

The SM prediction for this quantity is also known to a similar precision. The number quoted in Ref. [8] is

$$a_\mu({\rm SM}) = (11659182.8 \pm 4.9) \times 10^{-10}\,. \tag{2}$$

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[1] The value in Eq. (1) is the one quoted in Ref. [6] which takes into account the new value of the muon-to-proton magnetic moment ratio [7] on top of the original experimental result [5].

By comparing Eqs. (1) and (2), we obtain, for the difference between experiment and theory,

$$\delta a_\mu \equiv a_\mu(\exp) - a_\mu(\mathrm{SM}) = (26.1 \pm 8.0) \times 10^{-10}\,, \tag{3}$$

which means a 3.3σ discrepancy. Similar results are reported by other groups as well [9, 10]. A recent paper [11] claims a discrepancy of more than 4σ based on an appropriately broken Hidden Local Symmetry model.

There are two planned experiments to measure the muon $g-2$ at J-PARC [12] and Fermilab [13], which both aim to reduce the experimental uncertainty by a factor of about 4. If the mean values of the experimental and theoretical values do not change very much from Eqs. (1) and (2), respectively, then these experiments will establish the discrepancy of more than 5σ, if successful, which makes studies in this field extremely important.

2. Standard Model prediction for $(g-2)_\mu$

The SM prediction for a_μ comes from three contributions: the QED, electroweak (EW), and hadronic contributions

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})\,. \tag{4}$$

The QED and EW contributions are perturbatively calculable quantities, and hence they are known precisely enough. In Ref. [8], the value

$$a_\mu(\mathrm{QED}) = (11658471.808 \pm 0.015) \times 10^{-10} \tag{5}$$

is used as the value of the QED contribution [14, 15]. The value of the EW contribution calculated in Ref. [16] is

$$a_\mu(\mathrm{EW}) = (15.4 \pm 0.2) \times 10^{-10}\,. \tag{6}$$

The uncertainties in $a_\mu(\mathrm{QED})$ and $a_\mu(\mathrm{EW})$ are completely negligible as compared to the current uncertainties in the experiment and the SM prediction, see Eqs. (1) and (2).

The hadronic contributions can conveniently be written as the sum of the leading-order (LO) hadronic, next-to-leading-order (NLO) hadronic and light-by-light (l-by-l) scattering terms

$$a_\mu(\mathrm{hadronic}) = a_\mu(\mathrm{had,LO}) + a_\mu(\mathrm{had,NLO}) + a_\mu(\mathrm{had,l\text{-}by\text{-}l})\,. \tag{7}$$

The LO and NLO hadronic contributions can be calculated with the help of dispersion relations by using experimental data of the reaction $e^+e^- \to$ hadrons as input. For these contributions, Ref. [8] gives

$$a_\mu(\mathrm{had,LO}) = (694.9 \pm 4.3) \times 10^{-10}\,, \tag{8}$$

and

$$a_\mu(\text{had}, \text{NLO}) = (-9.8 \pm 0.1) \times 10^{-10}\,, \tag{9}$$

respectively. To evaluate the l-by-l contribution, we have to rely on a model of hadrons. Some representative values are

$$a_\mu(\text{had}, \text{l-by-l}) = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{Ref. [17]}\,, \\ (11.6 \pm 3.9) \times 10^{-10} & \text{Ref. [2]}\,. \end{cases} \tag{10}$$

We should keep in mind that even though the uncertainty in this contribution is smaller than that of the LO hadronic contribution, it will become more important to reduce the uncertainty in this contribution once the LO hadronic contribution is evaluated more precisely.

By adding the numbers in Eqs. (5), (6), (8), (9) and (10) (the one from Ref. [17]), the authors of Ref. [8] obtain Eq. (2) as the total SM prediction for the muon $g-2$.

By comparing Eqs. (5), (6), (8), (9) and (10), we see that the largest error comes from the hadronic LO contribution. It is, therefore, very important to evaluate this contribution as precisely as possible in order to calculate the SM prediction precisely.

2.1. Recent updates in Standard Model prediction for $(g-2)_\mu$

During the past few years, there have been updates in some of the numbers above.

One of the updates is in the QED contribution. Recently, it has been calculated up to and including the 5-loop order [18]. The authors of Ref. [18] give the revised value,

$$a_\mu(\text{QED}) = (11\ 658\ 471.8951 \pm 0.0080) \times 10^{-10}\,, \tag{11}$$

where the uncertainty is dominated by that of the input value of the QED gauge coupling α. The EW contribution is also updated [19], where the improvement mainly comes from the Higgs boson mass which has become known only recently. The new value reads

$$a_\mu(\text{EW}) = (15.36 \pm 0.10) \times 10^{-10}\,. \tag{12}$$

The knowledge of the Higgs boson mass makes the uncertainty smaller compared to the previous evaluations.

There are some updates also in the hadronic contributions. First, the NNLO hadronic contribution is calculated [20], which reads

$$a_\mu(\text{had, NNLO}) = (1.24 \pm 0.01) \times 10^{-10}\,. \tag{13}$$

Although the mean value of this contribution is not very large, it is not negligible compared to the uncertainty of the hadronic LO contribution or that of the experimental value of the muon $g-2$. It follows that it is necessary to take this term into account for future evaluations of $a_\mu(\mathrm{SM})$. Another update in this sector is the NLO corrections to the l-by-l contribution [21]. Reference [21] evaluates this contribution to be

$$a_\mu(\text{had, l-by-l NLO}) = (0.3 \pm 0.2) \times 10^{-10}\,, \tag{14}$$

which is actually negligible, but it is always good to confirm that higher order terms are really negligible.

None of the updates mentioned in this subsection changes the value of the total SM prediction very much, and hence the $\gtrsim 3\,\sigma$ discrepancy is still there.

3. Leading-order hadronic contribution to $(g-2)_\mu$

In the previous section, we have seen that it is the LO hadronic contribution which gives the dominant error in the SM prediction for a_μ. It is, therefore, extremely important to evaluate this contribution as precisely as possible.

The LO hadronic contribution is given as an integral of the total cross section σ_{had} of $e^+e^- \to$ hadrons with a known weight function $\hat{K}(s)$,

$$a_\mu(\text{had, LO}) = \frac{m_\mu^2}{12\pi^3} \int_{m_\pi^2}^{\infty} \frac{ds}{s}\ \hat{K}(s)\ \sigma^0_{\mathrm{had}}(s)\,, \tag{15}$$

where σ^0_{had} is the undressed cross section (*i.e.*, the cross section which does not include vacuum polarization corrections). The function $\hat{K}(s)$ is a monotonically increasing function with $\hat{K}(m_\pi^2) = 0.40$, $\hat{K}(4m_\pi^2) = 0.63$, and $\hat{K}(s) \to 1$ for $s \to \infty$. The lower end of the integral must be $s = m_\pi^2$ in order to take into account a small contribution from $e^+e^- \to \pi^0\gamma$. Since the weight factor in the integral put emphasis on lower energies, the $\pi^+\pi^-$ channel gives the most important contribution. In fact, about 73% of the total value of $a_\mu(\text{had, LO})$ comes from the $\pi^+\pi^-$ channel.

In Table I, we show the breakdown of the contributions to $a_\mu(\mathrm{had,LO})$ with respect to the energy region. As we can see from the table, the most important contribution comes from the lowest energy region which includes the ρ and ω mesons. Reference [8] uses the sum of the exclusive measurement of the hadronic cross sections below 2 GeV (except the threshold region where they use chiral perturbation theory (ChPT) since sometimes no or only poor-quality data are available there), above which the authors use the inclusive measurement of the hadronic R-ratio $R_{\rm had}(s)$ up to 11.09 GeV. They add the contributions from the narrow resonances separately, and above 11.09 GeV they use perturbative QCD (pQCD).

TABLE I

Breakdown of the contributions to the LO hadronic contribution to a_μ with respect to the energy region. The numbers are given in units of 10^{-10}, and are taken from Ref. [8].

Source	Contribution to $a_\mu(\text{had}, \text{LO})$
$2m_\pi$ — 0.32 GeV (ChPT, 2π)	2.36 ± 0.05
$3m_\pi$ — 0.66 GeV (ChPT, 3π)	0.01 ± 0.00
m_π — 0.60 GeV (ChPT, $\pi^0\gamma$)	0.13 ± 0.01
m_η — 0.69 GeV (ChPT, $\eta\gamma$)	0.00 ± 0.00
$\phi \to$ unaccounted modes	0.04 ± 0.04
0.32–1.43 GeV	606.50 ± 3.35
1.43–2 GeV	34.61 ± 1.11
2–11.09 GeV	41.19 ± 0.82
$J/\psi + \psi'$	7.80 ± 0.16
$\Upsilon(1S - 6S)$	0.10 ± 0.00
11.09–∞ (pQCD)	2.11 ± 0.00
Sum	694.86 ± 3.64

In Table II, we show the contributions from important channels to the dispersion integral Eq. (15), together with the comparison between Refs. [8] and [9]. From this table, we can easily see which channel is important and which is not. By far the most important is the $\pi^+\pi^-$ channel, whose mean value and uncertainty dominate over those of the others.

The recent data for the $\pi^+\pi^-$ channel include BaBar [22, 23], KLOE [24, 25][2], CMD-2 [27, 28], SND [29]. Out of these four experiments, BaBar and KLOE take the pion form factor data by using the initial state radiation (ISR) method [30], while CMD-2 and SND use direct scan. It is known that there is a tension between the BaBar data and the KLOE data (see, *e.g.*, Fig. 4 of Ref. [8]). It is strongly desired to resolve this tension in order to more firmly establish the discrepancy in the muon $g-2$.

Very recently, new data of the pion form factor appeared from the BESIII Collaboration [31]. The data 'interpolate' between the BaBar and the KLOE data, but when integrated over the data with the weight function, the contribution to $a_\mu(\text{had}, \text{LO})$ from the BESIII data alone is closer to that from the KLOE data alone than to that obtained from the BaBar data alone (see Fig. 7 of Ref. [31]).

[2] Note that after publication of Ref. [8], new data of the pion form factor appeared from KLOE [26].

TABLE II

Contributions from important channels to the dispersion integral Eq. (15) from the energy region of $\sqrt{s} < 1.8$ GeV. The numbers in the second column are taken from Ref. [8], while the numbers in the third column are taken from Ref. [9]. The last column is the difference between the two evaluations.

Channel	HLMNT [8]	Davier *et al.* [9]	Diff.
$\pi^+\pi^-$	505.65 ± 3.09	507.80 ± 2.84	-2.15
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	46.00 ± 1.48	1.38
K^+K^-	22.09 ± 0.46	21.63 ± 0.73	0.46
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	18.01 ± 1.24	0.61
$2\pi^+2\pi^-$	13.50 ± 0.44	13.35 ± 0.53	0.15
$K^0_{\rm S}K^0_{\rm L}$	13.32 ± 0.16	12.96 ± 0.39	0.36
$\pi^0\gamma$	4.54 ± 0.14	4.42 ± 0.19	0.12
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sum	634.28 ± 3.53	633.93 ± 3.61	0.35

4. QED gauge coupling at the Z-pole and other important byproducts

An important byproduct of the dispersive analysis discussed above is the QED gauge coupling at the Z-pole, $\alpha_{\rm QED}(M_Z^2)$ (below, we suppress the subscripts "QED"). This quantity is extremely important since it is usually used as one of the input parameters to define the EW sector of the SM *e.g.* when performing EW precision studies.

The QED coupling at the Z-pole can be written in terms of the contributions from the leptons $\Delta\alpha_{\rm lep}(M_Z^2)$, 5-flavor quarks $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, and the top quark $\Delta\alpha_{\rm top}(M_Z^2)$

$$\alpha\left(M_Z^2\right) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}\left(M_Z^2\right) - \Delta\alpha^{(5)}_{\rm had}\left(M_Z^2\right) - \Delta\alpha_{\rm top}(M_Z^2)}\,. \tag{16}$$

The leptonic and top-quark contributions are perturbatively calculable, and are known precisely enough in literature. The leptonic contribution obtained in Ref. [32] is

$$\Delta\alpha_{\rm lep}\left(M_Z^2\right) = 0.03149769\,, \tag{17}$$

and the top-quark contribution is [33]

$$\Delta\alpha_{\rm top}\left(M_Z^2\right) = -0.0000728(14)\,. \tag{18}$$

Note that in this number the value $m_t = (172.0 \pm 1.6)$ GeV is used [34].

The 5-flavor hadronic contribution can be written as the dispersive integral below,

$$\Delta\alpha_{\rm had}^{(5)}\left(M_Z^2\right) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{P}\int\frac{R_{\rm had}(s')ds'}{s'\left(s'-M_Z^2\right)}\,, \tag{19}$$

where 'P' stands for the principal value of the integral, and $R_{\rm had}(s)$ is the hadronic R-ratio. By using the hadronic data, Ref. [8] obtains

$$\Delta\alpha_{\rm had}^{(5)}\left(M_Z^2\right) = (276.3\pm 1.4)\times 10^{-4} \tag{20}$$

and

$$\alpha\left(M_Z^2\right)^{-1} = 128.944\pm 0.019\,. \tag{21}$$

Another important quantity which can be calculated as a byproduct of the dispersive analysis is the running QED coupling, $\alpha(q^2)$, as a function of q^2. By using q^2 in place of M_Z^2 in Eqs. (16) and (19), we can obtain the QED coupling at q^2 and the hadronic contribution to it, respectively. The authors of Ref. [8] have made the Fortran subroutine which returns the value of the running QED coupling $\alpha(q^2)$ at both space-like and time-like q^2, which is available from the authors upon request. This routine is included in some of Monte Carlo event generators for e^+e^- colliders such as `BabaYaga@NLO` [35] and `PHOKHARA` [36] (see *e.g.*, Ref. [37] for a review).

It is also possible to calculate another important related quantity, the LO hadronic contribution to the *electron* $g-2$. By using the same data sets as in Ref. [8], the authors of Ref. [38] give the results

$$a_e(\mathrm{had,LO}) = (18.66\pm 0.11)\times 10^{-13}\,, \tag{22}$$

$$a_e(\mathrm{had,NLO}) = (-2.234\pm 0.001)\times 10^{-13}\,. \tag{23}$$

See Ref. [38] for further details and background information.

5. Summary and discussion

In this paper, we have discussed that the largest uncertainty in the SM prediction for the muon $g-2$ comes from the LO hadronic contribution. Since there is more than 3σ discrepancy between the experimental and theoretical values of the muon $g-2$, which could be a hint of New Physics beyond the SM, it is extremely important to evaluate the LO hadronic contribution as precisely as possible. This has been done by using new precise data of $e^+e^-\to$ hadrons, in particular the pion form factor data, as input. Recent pion form factor data include those from BaBar, CMD-2, KLOE, and

SND (in alphabetical order), and we have discussed that there is a tension between the KLOE and BaBar data. Reference [8] obtains the results of Eqs. (8) and (9) for the LO and NLO hadronic contributions, respectively. By combining these results with other contributions such as the QED and EW contributions, we arrive at the full SM prediction, Eq. (2), which means a 3.3σ discrepancy from the experimental result, Eq. (1).

Some comments on the near-future prospects for improving this contribution are in order. From Table II, it is clear that it is the most important to reduce the error from the $\pi^+\pi^-$ channel. The most efficient way to do this is to have good experimental data. In the near future, new pion form factor data are expected to appear from the CMD-3 and SND experiments at VEPP-2000. According to the estimate given in Ref. [39], the new data from CMD-3 and SND, together with the very recent data from BESIII could reduce the error in the LO hadronic contribution by about 40%. Another important progress in the theoretical predictions may come from lattice QCD. The authors of Ref. [39] say that "it should be possible to compete with the e^+e^- determination of a_μ(HVP) by the end of the decade".

If the planned experiments at J-PARC and Fermilab are carried out, they may establish a signal of physics beyond the SM. It is most exciting for a particle physicist to imagine such a situation. To achieve this goal, not only experimentalists but also theorists should do their best to sharpen the theoretical prediction for a_μ.

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