

Letter

How close can we approach the event horizon of the Kerr black hole from the detection of gravitational quasinormal modes?

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 Using the Wentzel–Kramers–Brillouin method, we show that the peak location (r_{peak}) of the potential, which determines the quasinormal mode frequency of the Kerr black hole, obeys an accurate empirical relation as a function of the specific angular momentum a and the gravitational mass M . If the quasinormal mode with $a/M \sim 1$ is observed by gravitational wave detectors, we can confirm the black-hole space-time around the event horizon, $r_{\text{peak}} = r_+ + O(\sqrt{1-q})$, where r_+ is the event horizon radius. However, if the quasinormal mode is different from that of general relativity, we are forced to seek the true theory of gravity and/or face the existence of the naked singularity.

Subject Index E01, E02, E31, E38

1. *Introduction* Coalescing binary black holes (BHs) form a BH in numerical relativity simulations [1–3]. The BH radiates characteristic gravitational waves (GWs) with quasinormal mode (QNM) frequencies that dominate in the final phase of the merger of two BHs. QNM will be detected by the second-generation GW detectors such as Advanced LIGO (aLIGO) [4], Advanced Virgo (AdV) [5], and KAGRA [6,7]. QNM is also one of the targets for space-based GW detectors such as eLISA [8] and DECIGO [9]. However, to confirm QNM GWs, we need a sufficiently high signal-to-noise ratio (see, e.g., Refs. [10,11]).

To calculate GWs from the Kerr BH [12], we need to solve the Teukolsky equation [13]. The radial Teukolsky equation for gravitational perturbations in the Kerr space-time is expressed as

$$\Delta^2 \frac{d}{dr} \frac{1}{\Delta} \frac{dR}{dr} - VR = -T, \tag{1}$$

where T is the source and

$$V = -\frac{K^2}{\Delta} - \frac{2iK\Delta'}{\Delta} + 4iK' + \lambda, \tag{2}$$

with

$$K = (r^2 + a^2)\omega - am, \tag{3}$$

where $\Delta = r^2 - 2Mr + a^2$ with M and a being the mass and the spin parameter, respectively. In this paper, we use the geometric unit system, where $G = c = 1$. Here, we consider the Kerr metric in the

Boyer–Lindquist coordinates as

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mar \sin^2\theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Ma^2r}{\Sigma} \sin^2\theta\right) \sin^2\theta d\phi^2, \quad (4)$$

where $\Sigma = r^2 + a^2 \cos^2\theta$. The constants m and λ in the Teukolsky equation come from the spin-weighted spheroidal function $Z_{\ell m}^{a\omega}(\theta, \phi)$. A prime is the derivative with respect to r . When we consider GWs emitted by a test particle falling into the Kerr BH, the source term T diverges as $\propto r^{7/2}$ and the potential V is the long-range one, which motivated Sasaki and Nakamura [14–16] to consider the change of the variables and the potential. Using two functions $\alpha(r)$ and $\beta(r)$ for the moment, let us define various variables as

$$X = \frac{\sqrt{r^2 + a^2}}{\Delta} \left(\alpha R + \frac{\beta}{\Delta} R'\right), \quad (5)$$

$$\gamma = \alpha \left(\alpha + \frac{\beta'}{\Delta}\right) - \frac{\beta}{\Delta} \left(\alpha' + \frac{\beta}{\Delta^2} V\right), \quad (6)$$

$$F = \frac{\Delta}{r^2 + a^2} \frac{\gamma'}{\gamma}, \quad (7)$$

$$U_0 = V + \frac{\Delta^2}{\beta} \left[\left(2\alpha + \frac{\beta'}{\Delta}\right)' - \frac{\gamma'}{\gamma} \left(\alpha + \frac{\beta'}{\Delta}\right)\right], \quad (8)$$

$$G = -\frac{\Delta'}{r^2 + a^2} + \frac{r\Delta}{(r^2 + a^2)^2}, \quad (9)$$

$$U = \frac{\Delta U_0}{(r^2 + a^2)^2} + G^2 + \frac{dG}{dr^*} - \frac{\Delta G}{r^2 + a^2} \frac{\gamma'}{\gamma}. \quad (10)$$

Then, we have a new wave equation for X from the Teukolsky equation as

$$\frac{d^2 X}{dr^{*2}} - F \frac{dX}{dr^*} - UX = 0, \quad (11)$$

where $dr^*/dr = (r^2 + a^2)/\Delta$.

We define α and β by

$$\alpha = A - \frac{iK}{\Delta} B, \quad (12)$$

$$\beta = \Delta B, \quad (13)$$

where

$$A = 3iK' + \lambda + \Delta P, \quad (14)$$

$$B = -2iK + \Delta' + \Delta Q, \quad (15)$$

with

$$P = \frac{r^2 + a^2}{gh} \left(\left(\frac{g}{r^2 + a^2} \right)' h \right)', \quad (16)$$

$$Q = \frac{(r^2 + a^2)^2}{g^2 h} \left(\frac{g^2 h}{(r^2 + a^2)^2} \right)'. \quad (17)$$

Here, g and h are free functions under the restrictions to guarantee the convergent source term and the short-range potential that is given by

$$g = \text{const}, \quad h = \text{const}, \quad (18)$$

for $r^* \rightarrow -\infty$, and

$$g = \text{const} + O(r^{-2}), \quad h = \text{const} + O(r^{-2}), \quad (19)$$

for $r^* \rightarrow +\infty$. In Refs. [14,15], h and g are adopted as

$$h = 1, \quad g = \frac{r^2 + a^2}{r^2}. \quad (20)$$

Defining a new variable Y by $X = \sqrt{\gamma} Y$, we have

$$\frac{d^2 Y}{dr^{*2}} + (\omega^2 - V_{\text{SN}}) Y = 0, \quad (21)$$

where

$$V_{\text{SN}} = \omega^2 + U - \left[\frac{1}{2} \frac{d}{dr^*} \left(\frac{1}{\gamma} \frac{d\gamma}{dr^*} \right) - \frac{1}{4\gamma^2} \left(\frac{d\gamma}{dr^*} \right)^2 \right]. \quad (22)$$

In our previous paper [17], it was shown that this V_{SN} has double peaks for $q = a/M > 0.8$ to refuse the approach to determine the complex frequency of QNMs from the peak location r_{peak} , which is real-valued, of the absolute value of V_{SN} . Our new g is defined by

$$g = \frac{r(r-a)}{(r+a)^2}. \quad (23)$$

The above new g seems to violate the necessary dependence for $r \rightarrow \infty$. However, Eq. (19) indicates only the sufficient condition, not the necessary one. In reality, our new V_{NNT}^1 is confirmed to be short-ranged, i.e., V_{NNT} is in proportion to $1/r^2$ for $r \rightarrow \infty$ and becomes the Regge–Wheeler potential [18] for $a = 0$ as V_{SN} .

After adopting the new potential V_{NNT} , what we will do is to ask which part of the Kerr metric determines the QNMs for given a $0.8 < q < 1$. Conversely, if the QNM GWs are observed, to say which part of the Kerr BH is confirmed, is the main theme of this paper.

The essential technique to determine the QNMs by using the Wentzel–Kramers–Brillouin (WKB) method² was proposed by Schutz and Will [20] for the Schwarzschild BH using the Regge–Wheeler potential V_{RW} [18,21]. They approximated V_{RW} near its peak radius at $r_0 \approx 3.28M$ as

$$V_{\text{RW}}(r^*) = V_{\text{RW}}(r_0^*) + \frac{1}{2} \frac{d^2 V_{\text{RW}}}{dr^{*2}} \Big|_{r^*=r_0^*} (r^* - r_0^*)^2, \quad (24)$$

where $r_0^* = r_0 + 2M \ln(r_0/2M - 1)$. The QNM frequencies are expressed as

$$(\omega_r + i\omega_i)^2 = V_{\text{RW}}(r_0^*) - i \left(n + \frac{1}{2} \right) \sqrt{-2 \frac{d^2 V_{\text{RW}}}{dr^{*2}} \Big|_{r^*=r_0^*}}, \quad (25)$$

with $n = 0, 1, 2, \dots$. As for the accuracy of the fundamental $n = 0$ QNM frequency with $\ell = 2$, the errors of the real ($\omega_r = \text{Re}(\omega)$) and the imaginary ($\omega_i = \text{Im}(\omega)$) parts are 7% and 0.7%, respectively,

¹ To distinguish the original V_{SN} from the new SN equation, we use V_{NNT} from now on.

² See the interesting review [19] for various methods of QNM calculations.

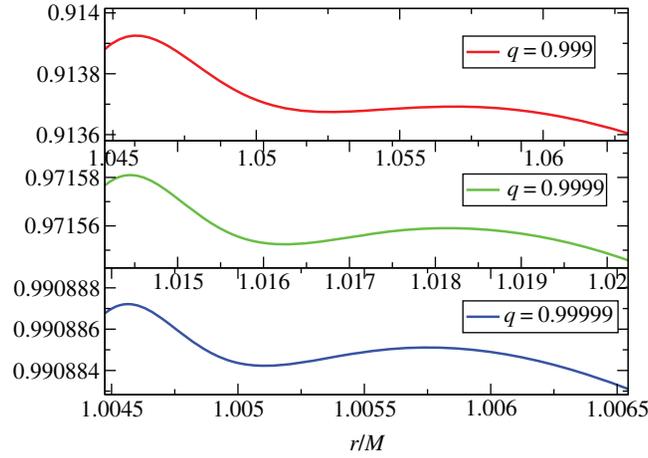


Fig. 1. $|V_{\text{NNT}}|$ with $(\ell = 2, m = 2)$ for $q = 0.999$ (top), 0.9999 (middle), and 0.99999 (bottom) with respect to r/M . We will calculate the QNM frequencies by using the strong peak for each q .

compared with the numerical results of Chandrasekhar and Detweiler [22]. This suggests that, for the fundamental QNM of the $a = 0$ case, the space-time of a Schwarzschild BH around $r \approx 3.28M$ is confirmed through the detection of the QNM GWs. The word “around” has two meanings, that the GW cannot be localized due to the equivalence principle and that the imaginary part of the QNMs is determined by the curvature of the potential, which reflects the space-time structure of the Schwarzschild BH around $r \approx 3.28M$.

In our previous papers [17,23], we succeeded in performing similar procedures up to $q = 0.98$ for the Kerr BH with the Detweiler potential [24] (see also Ref. [25]). However, above $q = 0.98$, we could not derive consistent QNMs with the method using r_{peak} of the absolute value of the potential.

In Fig. 1, we show that the new Sasaki–Nakamura potential V_{NNT} has only a single strong peak to allow us to identify r_{peak} up to $q \rightarrow 1$ successfully. Here, the QNM frequencies have been calculated accurately by Leaver’s method [26]. It is noted that, when we consider an additional function $a(M - a)/r^2$ for Eq. (23), the peak location r_{peak} changes by only 0.4% and 0.06% for $q = 0.98$ and 0.9999 , respectively, which shows that the results do not depend significantly on the choice of g .

2. Result First, we calculate the peak location of V_{NNT} for various spin parameters q , and show various curves in Fig. 2. The black dots are obtained by finding the maximum of the absolute value of the new Sasaki–Nakamura potential $|V_{\text{NNT}}|$ with $(\ell = 2, m = 2)$, where the contribution from the imaginary part is small. In practice, $|\text{Im}(V_{\text{NNT}})/\text{Re}(V_{\text{NNT}})|$ is 9% ($q = 0.98$), 7% ($q = 0.99$), 2% ($q = 0.999$), 0.7% ($q = 0.9999$), and 0.2% ($q = 0.99999$).

The blue curve in Fig. 2 is the fitting curve with

$$r_{\text{fit}}/M = 1 + 1.4803 (-\ln q)^{0.503113}, \quad (26)$$

which is derived from 13 data between $q = 0.98$ and $q = 0.9999$, yielding a correlation coefficient of 0.999995 and a chance probability of 7.4×10^{-29} . The red and green dotted curves show the event horizon radius,

$$\begin{aligned} r_+/M &= 1 + \sqrt{1 - q^2} \\ &= 1 + \sqrt{2} (1 - q)^{1/2} + O\left((1 - q)^{3/2}\right), \end{aligned} \quad (27)$$

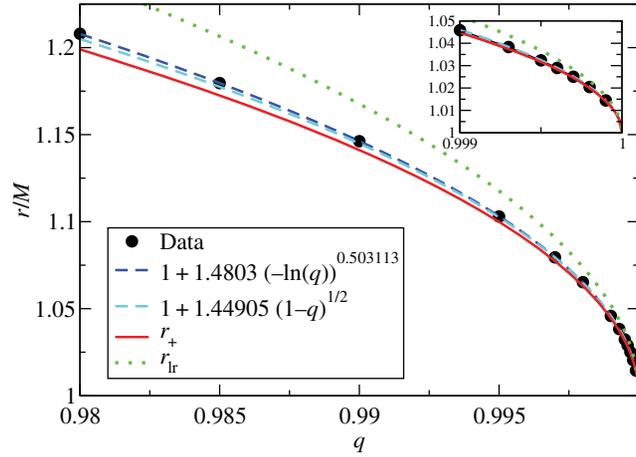


Fig. 2. Black dots denote the locations of the maximum of the absolute value of $|V_{\text{NNT}}|$ with $(\ell = 2, m = 2)$ for various spin parameters $q = a/M$. The blue and light-blue dashed curves are the fitting curve and the curve derived analytically. The red curve is the event horizon r_+ , and the inner light ring radius r_{lr} is represented by the green dotted curve.

and the inner light ring radius [27],

$$\begin{aligned} r_{\text{lr}}/M &= 2 + 2 \cos \left[\frac{2}{3} \cos^{-1}(-q) \right] \\ &= 1 + \frac{2\sqrt{2}}{\sqrt{3}} (1 - q)^{1/2} + O(1 - q). \end{aligned} \tag{28}$$

The latter radius is evaluated in the equatorial ($\theta = \pi/2$) plane. It is noted that there are various studies on the relation between the QNMs and the orbital frequency of the light ring orbit (see the useful lecture note in Ref. [28]).

The light-blue dashed curve in Fig. 2 is obtained as follows. Here, we introduce a fitting curve to evaluate λ by using ${}_s A_{\ell m}$, which is a constant defined in Eq. (25) of Ref. [29], as

$$-{}_2 A_{22} = 0.545\,652 + (6.024\,97 + 1.385\,91\,i)(-\ln q)^{1/2}, \tag{29}$$

where λ is related to ${}_s A_{\ell m}$ as $\lambda = {}_s A_{\ell m} - 2am\omega + a^2\omega^2$. Also, the $(n = 0)$ QNM frequency with $(\ell = 2, m = 2)$ is described by Ref. [30] as

$$M\omega = \frac{Mq}{r_+} - \frac{i}{4} \frac{r_+ - M}{r_+}. \tag{30}$$

Then, using $q = 1 - \epsilon$ and expanding V_{NNT} with respect to ϵ , we estimate r_{peak} analytically. Instead of finding the peak location of $|V_{\text{NNT}}|$, we derive the location of $dV_{\text{NNT}}/dr^* = 0$. The location has a $O(\sqrt{\epsilon})$ term that is consistent with r_{fit} because $(-\ln(1 - \epsilon))^{1/2} = \sqrt{\epsilon} + O(\epsilon^{3/2})$, and the result is

$$r_{\text{ana}}/M = 1 + 1.449\,05 (1 - q)^{1/2}. \tag{31}$$

Here, we have ignored the tiny imaginary contribution of $-0.020\,157\,i$. It should be noted that a different choice of g makes a difference of $O((1 - q)^{1/2})$ in the estimation of the peak location. We will discuss the details of this in future work.

To confirm our analysis, we present the QNM frequencies via the WKB approximation in Fig. 3. The errors in the real and imaginary parts of the QNM frequencies are plotted in Fig. 4.

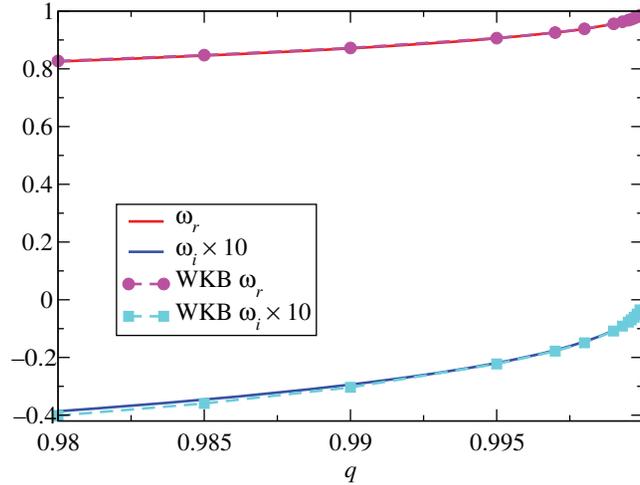


Fig. 3. The real and imaginary parts of the fundamental ($n = 0$) QNM frequencies with $V_{\text{NNT}} (\ell = 2, m = 2)$ evaluated for various spin parameters $q = a/M$. The exact frequencies ω_r and ω_i are from Refs. [31,32], and Leaver's method [26].

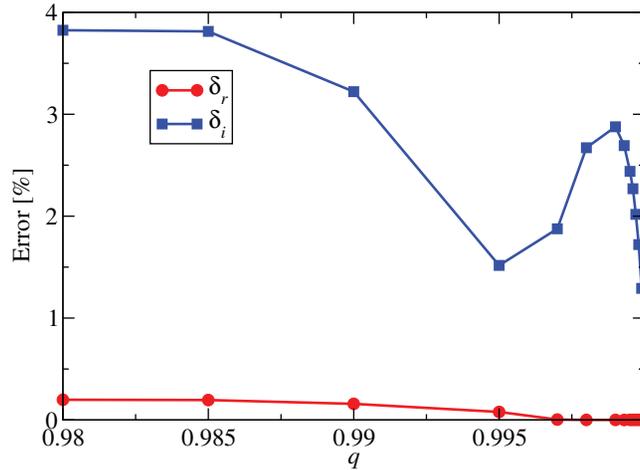


Fig. 4. Absolute value of relative errors for the real and imaginary parts of the QNM frequencies with $V_{\text{NNT}} (\ell = 2, m = 2)$, $\delta_r = |(\text{WKB } \omega_r)/\omega_r - 1|$, and $\delta_i = |(\text{WKB } \omega_i)/\omega_i - 1|$ between the exact value and that of the WKB approximation in Fig. 3.

3. *Discussion* There is an ergoregion in the Kerr BH where the timelike Killing vector turns out to be spacelike. The boundary of the ergoregion is given by

$$r_{\text{ergo}}(\theta)/M = 1 + \sqrt{1 - q^2 \cos^2 \theta}, \quad (32)$$

which is called the ergosphere. Note here that $r_{\text{ergo}}(0) = r_+$ and $r_{\text{ergo}}(\pi/2) = 2M$. The physical origin of the Penrose mechanism [33] and the Blandford–Znajek mechanism [34] is the ergoregion, which enables the extraction of the rotational energy of the Kerr BH. There are many papers using these mechanisms, but confirmation of the existence of the ergoregion has not been made observationally. The detection of QNM GWs, e.g., for $q = 0.9999$, can confirm its existence. Let us define the covering solid angle $4\pi C$ of the ergoregion for a given sphere of radius r_{peak} . Then, C is given by $C = \cos \theta_m$ with $r_{\text{peak}}/M = 1 + \sqrt{1 - q^2 \cos^2 \theta_m}$. Then if, e.g., the QNM with $q = 0.9999$ is observed by GW detectors, we can confirm the space-time around $r = 1.01445M$ covering

99.9996% of the ergoregion. Here, the horizon radius for $q = 0.9999$ is $1.01414M$. Therefore, the space-time at only 1.0003 times the event horizon can be confirmed. If the QNM is confirmed to be different from that of general relativity by the detection of GWs, a very serious problem is raised since the Kerr BH is the unique solution of the stationary vacuum solution of the Einstein equation under the assumption of cosmic censorship [35–37]. The Einstein equation and/or the cosmic censorship would be wrong. In the former case, the true theory of gravity should be determined to be compatible with the data on QNMs. In the latter case, we face the existence of the naked singularity, which requires a new physics law possibly related to quantum gravity.

As for the detection rate, the population Monte Carlo simulation by Kinugawa et al. [38–40] showed that, for Pop III binary BHs, 0.43% have the final $q > 0.98$ in their standard model, in which they adopted various parameters and functions for Pop I stars like the sun, except for the initial mass function. Since the Pop III star, which is the first star in our universe without metal that has an atomic number greater than carbon has not been observed, these parameters are highly unknown. This suggests that the percentage of BHs with $q > 0.98$ could be either larger or smaller than 0.43%. Massive BHs from Pop I and Pop II BH binaries are also expected [41], although the spin parameter data are not available from that paper at present. This suggests that the second-generation detectors might detect QNM GWs of such a BH with $q > 0.98$. Third-generation detectors, such as the Einstein Telescope (ET) [42], will increase the detection number by ~ 1000 times. Another possibility is QNM GWs from very massive BHs of mass $\sim 10^4 M_\odot$ and $\sim 10^7 M_\odot$ for DECIGO [9] and eLISA [8], respectively.

In conclusion, the present and future GW detectors with the frequency $\sim 10^{-3}$ Hz to 100 Hz should observe the very strong gravity almost at the event horizon radius to clarify the true theory of gravity.

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