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Letter

Saddle point inflation in string-inspired theory

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The observed value of the Higgs mass indicates the possibility that there is no supersymmetry below the Planck scale and that the Higgs can play the role of the inflaton. We examine the general structure of saddle point inflation in string-inspired theory without supersymmetry. We point out that the string scale is fixed to be around the GUT scale $\sim 10^{16}$ GeV in order to realize successful inflation. We find that the inflaton can be naturally identified with the Higgs field.

Subject Index B53, B72

1. *Introduction* The particle recently observed by the ATLAS [1] and CMS [2] experiments at the Large Hadron Collider (LHC) is consistent with the Standard Model (SM) Higgs with a mass around 125 GeV. Up to now, no significant deviation has been observed from the SM, nor has there been any hint of new physics. Once the Higgs mass is determined, we have fixed all the parameters in the SM and can extrapolate it up to its ultraviolet (UV) cutoff scale. In particular, the quadratically divergent bare Higgs mass is found to be suppressed when the UV cutoff is around the Planck scale [3–6]; see also Ref. [7,8]. Furthermore, the quartic Higgs coupling becomes tiny at the same time; see, e.g., Refs. [3–6,9–13]. This opens up the possibilities of identifying the Higgs field as the inflaton [14–24], and of the absence of supersymmetry below the Planck scale. Although non-supersymmetric vacua are ubiquitous in string theory [25–33], their phenomenology has not been well studied. It becomes important to explore the phenomenology starting from non-supersymmetric theory.

In this letter, we consider the saddle point inflation scenario starting in string-inspired theory without supersymmetry. The potential is generated perturbatively, in contrast to the supersymmetric case where the potential comes only non-perturbatively. Then, we calculate the cosmological parameters by assuming that the potential is tuned in such a way that the first n derivatives vanish at some point. The predicted cosmological parameters are consistent with the recent Planck 2015 result [34]. Furthermore, we can estimate the order of the string scale from the height of the potential that is given roughly by the string scale to the fourth multiplied by the rather small ten-dimensional one-loop factor.

To realize the saddle point, some degree of fine-tuning is needed. This fine-tuning can be achieved by some principles which are beyond the ordinary local field theory, e.g. the multiple point criticality principle [35,36] and the maximum entropy principle [37–41].

This letter is organized as follows. In the next section, we consider the potential that has a saddle point where the first n derivatives vanish. Then we calculate the cosmological parameters of the model. In Sect. 3, we estimate the string scale in the case of the non-supersymmetric heterotic-like string model. In Sect. 4, we summarize our results.

2. *Saddle point inflation and observables* We start with a general potential V as a function of an inflaton field φ. We will discuss the possibility of identifying it as the SM Higgs in the next section. We expand the potential around the saddle point φ_c as $\varphi = \varphi_c + \delta\varphi$:

$$V = \sum_{n=0}^{\infty} \frac{V_c^{(n)}}{n!} \delta\varphi^n = V_c + V_c' \, \delta\varphi + \frac{V_c''}{2} \, \delta\varphi^2 + \frac{V_c'''}{3!} \, \delta\varphi^3 + \cdots . \tag{1}$$

We assume that the first n (≥ 2) derivatives vanish at φ_c:

$$V_c' = V_c'' = \cdots = V_c^{(\mathsf{n})} = 0. \tag{2}$$

Here, we also assume $V_c^{(\mathsf{n}+1)} > 0$ (<0) for even (odd) n so that φ rolls down from φ_c towards 0. This is because we are going to identify φ as the Higgs field.

The slow roll parameters around the saddle point are obtained as:[1]

$$\epsilon := \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 = \frac{M_P^2 \left(V_c^{(\mathsf{n}+1)} \right)^2}{2 \, (\mathsf{n}!)^2 \, V_c{}^2} \, \delta\varphi^{2\mathsf{n}} + O\left(\delta\varphi^{2\mathsf{n}+1} \right), \tag{3}$$

$$\eta := M_P^2 \frac{V''}{V} = \frac{M_P^2 \, V_c^{(\mathsf{n}+1)}}{(\mathsf{n}-1)! \, V_c} \, \delta\varphi^{\mathsf{n}-1} + O\left(\delta\varphi^{\mathsf{n}} \right), \tag{4}$$

$$\zeta^2 := M_P^4 \frac{V''' V'}{V^2} = \frac{M_P^4 \left(V_c^{(\mathsf{n}+1)} \right)^2}{(\mathsf{n}-2)! \, \mathsf{n}! \, V_c^2} \, \delta\varphi^{2\mathsf{n}-2} + O\left(\delta\varphi^{2\mathsf{n}-1} \right). \tag{5}$$

We see that $\epsilon \ll |\eta| \, , \zeta^2$ for $\delta\varphi \ll M_P$. The inflation ends when ϵ becomes of order unity, and we define its end point by $\epsilon \, (\delta\varphi_{\rm end}) = 1$ to get

$$(\delta\varphi_{\rm end})^{\mathsf{n}} \simeq \frac{\sqrt{2} \, \mathsf{n}! \, V_c}{M_P \, V_c^{(\mathsf{n}+1)}}. \tag{6}$$

The e-folding number N from a given stage of the inflation $\varphi = \varphi_c + \delta\varphi$ to its end $\varphi_{\rm end} = \varphi_c + \delta\varphi_{\rm end}$ is:[2]

$$\begin{aligned} N = \int_{\varphi_{\rm end}}^{\varphi} \frac{{\rm d}\varphi}{M_P^2} \, \frac{V}{V'} &= \frac{\mathsf{n}!}{(\mathsf{n}-1)} \, \frac{V_c}{M_P^2 \, V_c^{(\mathsf{n}+1)}} \left[\frac{1}{(\delta\varphi_{\rm end})^{\mathsf{n}-1}} - \frac{1}{(\delta\varphi)^{\mathsf{n}-1}} \right] \\ &\simeq \frac{\mathsf{n}!}{(\mathsf{n}-1)} \, \frac{V_c}{M_P^2 \left| V_c^{(\mathsf{n}+1)} \right| \, |\delta\varphi|^{\mathsf{n}-1}}, \end{aligned} \tag{7}$$

[1] If φ_c is the only theoretical mass scale in the model we will consider in the next section, we have $V_c^{(n+2)} / V_c^{(n+1)} \sim \varphi_c^{-1}$. Therefore, the condition for the validity of neglecting the higher-order terms is

$$\frac{V_c^{(n+2)} \delta\varphi^{n+2}}{(n+2)!} \Big/ \frac{V_c^{(n+1)} \delta\varphi^{n+1}}{(n+1)!} \sim \frac{\delta\varphi}{\varphi_c} \ll 1.$$

[2] The problem with the initial condition can be avoided by considering the eternal inflation scenario at the saddle point [24].

where we have assumed $|\delta\varphi_{\rm end}| \gg |\delta\varphi|$ in the last step. From Eqs. (3), (4), (5), and (7), we obtain

$$\epsilon = \frac{1}{2M_P^2}\left[\frac{\mathsf{n}!}{(\mathsf{n}-1)^{\mathsf{n}}}\frac{1}{N^{\mathsf{n}}}\frac{V_c}{M_P^2\left|V_c^{(\mathsf{n}+1)}\right|}\right]^{\frac{2}{\mathsf{n}-1}}, \quad \eta = -\frac{\mathsf{n}}{(\mathsf{n}-1)\,N}, \quad \zeta^2 = \frac{\mathsf{n}}{(\mathsf{n}-1)\,N^2}. \tag{8}$$

The cosmological observables, namely the scalar perturbation A_s, spectral index n_s, tensor-to-scalar ratio r, and running index $\mathrm{d}n_s/\mathrm{d}\ln k$,[3]

$$A_s = \frac{V}{24\pi^2\epsilon M_P^4}, \tag{9}$$

$$n_s = 1 - 6\epsilon + 2\eta \simeq 1 - \frac{2\mathsf{n}}{(\mathsf{n}-1)\,N}, \tag{10}$$

$$r = 16\epsilon, \tag{11}$$

$$\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\zeta^2 \simeq 2\frac{\mathsf{n}}{(\mathsf{n}-1)\,N^2}, \tag{12}$$

are constrained by the Planck 2015 data [34],

$$A_s \simeq 2.2\times 10^{-9}, \quad 0.954 < n_s < 0.980, \quad r < 0.168, \quad -0.03 < \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} < 0.007, \tag{13}$$

at the 95% CL.[4] The e-folding number

$$N_* = 62 - \ln\left(\frac{10^{16}\,\mathrm{GeV}}{V_{\rm end}^{1/4}}\right) \simeq 64 + \frac{1}{4}\ln\epsilon \tag{14}$$

corresponds to the stage of inflation observed by the Planck experiment. We note that this model gives a concave potential, $\eta < 0$, which is favored by the recent Planck data.[5]

3. *Saddle point inflation in string-inspired theory* In this section, we consider the saddle point inflation in the non-supersymmetric heterotic-like string model. Here we assume that the tree-level potential of the inflaton is absent. This is realized if the inflaton comes from the extra component of the gauge field/metric, for example. Then the dominant contribution to the potential is the one-loop correction, which is suppressed compared to the string scale by the loop factor:

$$\int\frac{d^dk}{(2\pi)^d} = \frac{S_{d-1}}{2\,(2\pi)^d}\int dk^2\left(k^2\right)^{\frac{d}{2}-1} \sim \frac{S_{d-1}}{2\,(2\pi)^d}M_s^d. \tag{15}$$

For $d = 10$, we obtain the following numerical value:

$$C_{\rm loop} \equiv \frac{S_{d-1}}{2\,(2\pi)^d} = \frac{2\pi^5}{\Gamma\,(5)}\,\frac{1}{2\,(2\pi)^{10}} \simeq 1.3\times 10^{-7}. \tag{16}$$

In fact, the ten-dimensional cosmological constant of $SO\,(16)\times SO\,(16)$ heterotic string theory [42–44] is calculated as

$$\Lambda_{SO(16)\times SO(16)} \simeq 3.9\times 10^{-6}M_s^{10}. \tag{17}$$

[3] It appears that these quantities change their values discretely with n. This is because n is the number of fine-tunings. However, if we take the next-order term into account, we can explicitly check that the limit of $V_c^{(\mathsf{n}+1)} \to 0$ continuously connects the case n to $\mathsf{n}+1$. Thus we can have fractional n effectively.

[4] To give the most conservative bound, here we employ the constraint from the Planck TT+lowP data.

[5] We thank the referee for pointing this out.

Because we assume that the tree potential of the inflaton vanishes, the effective action below the string scale becomes

$$\begin{aligned} S &= \frac{M_s^8}{g_s^2}\int d^{10}x\sqrt{g}A(\chi)\mathcal{R} + \frac{M_s^8}{g_s^2}\int d^{10}x\sqrt{g}B(\chi)(\partial\chi)^2 + C_{\rm loop}M_s^{10}\int d^{10}x\sqrt{g}V(\chi) + \cdots \\ &= \frac{M_s^8}{g_s^2}V_6\int d^4x\sqrt{g}A(\chi)\mathcal{R} + \frac{M_s^8}{g_s^2}V_6\int d^4x\sqrt{g}B(\chi)(\partial\chi)^2 + C_{\rm loop}M_s^{10}V_6\int d^4\sqrt{g}V(\chi) + \cdots . \end{aligned} \tag{18}$$

Here, χ is the dimensionless inflaton field, g_s is the string coupling, $V(\chi)$ is the one-loop potential, and V_6 is the compactification volume. Because M_s is the only mass scale of the theory, $A(\chi)$, $B(\chi)$, and $V(\chi)$ should be functions of order one,

$$A(\chi) = a_0 + a_2 g_s^2\chi^2 + \cdots, \quad B(\chi) = b_0 + b_2 g_s^2\chi^2 + \cdots, \quad V(\chi) = v_0 + v_2 g_s^2\chi^2 + \cdots, \tag{19}$$

with the a_is, b_is, and v_is being order one constants. Next let us move to the Einstein frame. Namely, we redefine the metric in such a way that $A(\chi)$ becomes 1. In the Einstein frame, we have

$$S = M_P^2\int d^4x\sqrt{g}\mathcal{R} + M_P^2\int d^4x\sqrt{g}C(\chi)(\partial\chi)^2 + C_{\rm loop}g_s^2M_P^2M_s^2\int d^4x\sqrt{g}U(\chi). \tag{20}$$

Here,

$$M_P^2 = \frac{M_s^2}{g_s^2}\left(M_s^6V_6\right), \quad C(\chi) = c_0 + c_2 g_s^2\chi^2 + \cdots, \quad U = u_0 + u_2 g_s^2\chi^2 + \cdots, \tag{21}$$

where the c_is and u_is are order one constants. In terms of the dimensionless canonical field ϕ, the action becomes

$$S = M_P^2\int d^4x\sqrt{g}\,\mathcal{R} + M_P^2\int d^4x\sqrt{g}\,(\partial\phi)^2 + C_{\rm loop}g_s^2M_P^2M_s^2\int d^4x\sqrt{g}\,W(\phi), \tag{22}$$

where $W(\phi)$ is a function of order one.

The argument so far is quite general. In the following, we assume that the potential has a saddle point where the first n derivatives vanish, as in Sect. 2. This may happen by some mechanism beyond the ordinary local field theory such as the multiple point criticality principle [35,36] and the maximum entropy principle [37–41]. Here, we take

$$W(\phi) = W_0\left(1 - \left(1 - \frac{\phi}{\phi_c}\right)^{\mathsf{n}+1}\right) \tag{23}$$

as a simple possibility. We expect that ϕ_c is an order one quantity. In terms of the canonical field $\varphi = M_P\phi$, the potential $V(\varphi)$ becomes

$$V(\varphi) = C_{\rm loop}g_s^2M_P^2M_s^2 \times W\left(\frac{\varphi}{M_P}\right). \tag{24}$$

Then, from Eq. (8), we get

$$\epsilon = \frac{1}{2M_P^2}\left[\frac{\mathsf{n}!}{(\mathsf{n}-1)^{\mathsf{n}}}\frac{1}{N^{\mathsf{n}}}\frac{V_c}{M_P^2\left|V_c^{(\mathsf{n}+1)}\right|}\right]^{\frac{2}{\mathsf{n}-1}} = \frac{1}{2}\left[\frac{\mathsf{n}!}{(\mathsf{n}-1)^{\mathsf{n}}}\frac{1}{N^{\mathsf{n}}}\frac{\phi_c^{\mathsf{n}+1}}{(\mathsf{n}+1)!}\right]^{\frac{2}{\mathsf{n}-1}}. \tag{25}$$

Furthermore, Eq. (25) and the COBE normalization, Eq. (9), fix the value of $V_c = C_{\rm loop}g_s^2M_P^2M_s^2W_0$, from which we can obtain the string scale. In Table 1, we present the predictions of the cosmological

Table 1. The predictions of cosmological parameters and the string scale for $N = 60$, $C_{\rm loop} = 10^{-7}$.

n	n_s	ϵ	V_c/M_P^4	$g_s M_s \sqrt{W_0}/M_P$
2	0.933...	$4.3 \times 10^{-9}\phi_c^6$	$2.2 \times 10^{-15}\phi_c^6$	1.5×10^{-4}
3	0.95	$7.2 \times 10^{-8}\phi_c^4$	$3.8 \times 10^{-14}\phi_c^4$	6.1×10^{-4}
4	0.955...	$1.7 \times 10^{-7}\phi_c^{10/3}$	$8.6 \times 10^{-14}\phi_c^{10/3}$	9.3×10^{-4}
5	0.95833...	$2.3 \times 10^{-7}\phi_c^3$	$1.2 \times 10^{-13}\phi_c^3$	1.1×10^{-3}
6	0.96	$2.6 \times 10^{-7}\phi_c^{14/5}$	$1.4 \times 10^{-13}\phi_c^{14/5}$	1.2×10^{-3}

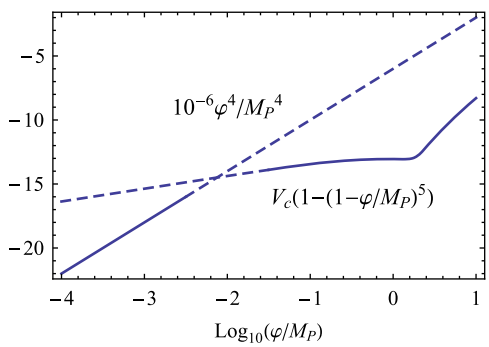

Fig. 1. Matching between Eq. (24) and Eq. (26).

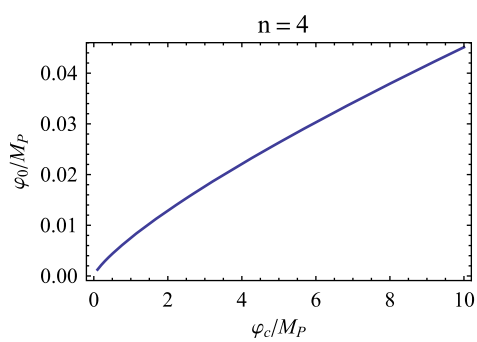

Fig. 2. φ_0 as a function of φ_c. φ_0 is the value of φ for which $V_{\rm SM}$ equals V.

parameters taking $C_{\rm loop} = 10^{-7}$, $N = 60$. From this table, we can see that $\mathsf{n} \geq 4$ is favored by the current observation, Eq. (13). The tensor to scalar ratio is very small compared to the current limit provided that ϕ_c is of order one. As we vary n from 2 to 6, $g_s M_s$ takes values from 4×10^{14} GeV to 3×10^{15} GeV for $W_0 = 1$. If g_s is $\mathcal{O}(0.1)$, the result indicates that M_s is around the GUT scale, $\sim 10^{16}$ GeV.

Finally, let us discuss the possibility of identifying the inflaton as the SM Higgs. Recent analysis shows that the Higgs potential for large values of the Higgs field h is roughly given by

$$V_{\rm SM} \sim 10^{-6} h^4 \tag{26}$$

in the SM [3–6,9–13] and its simple extensions [45–53] when the top mass is around 171–172 GeV. We examine whether $V_{\rm SM}$ can be connected to the potential V in Eq. (24) under the assumption that φ is identified as h. In Fig. 1, Eqs. (24) and (26) are plotted. Here we take $\mathsf{n} = 4$, $\phi_c = 1$, and $W_0 = 1$ as an example. One can see that the two lines cross at around $\varphi \simeq 10^{16}$ GeV, which we call φ_0. We interpret this as an indication that the potential is given by the SM at lower energies, and becomes

stringy, Eq. (24), above the string scale $\sim 10^{16}$ GeV. We also show φ_0 as a function of $\varphi_c = \phi_c M_P$ in Fig. 2. φ_0 takes vaues of the order of 10^{16} GeV for $\varphi_c = \mathcal{O}(M_P)$.

4. *Summary* We have examined the possibility of saddle point inflation in the context of non-supersymmetric string theory, which is ubiquitous and becomes more realistic in light of the recent LHC result. Contrary to supersymmetric theory, the potential is generated perturbatively. We have assumed that the potential of the inflaton is identically zero at the tree level, and it is radiatively generated by the loop effect. We have estimated the string scale that realizes a successful inflation assuming that the potential is tuned so that it has a saddle point where the first n derivatives vanish. Interestingly, the string scale becomes around the GUT scale, $\sim 10^{16}$ GeV, if the string coupling is $\mathcal{O}(0.1)$. Furthermore, we have found that it is reasonable to identify the inflaton as the Higgs field. It is interesting that, in addition to the LHC results, the scale of the inflation supports non-supersymmetric string theory.

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