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Constraint on ghost-free bigravity from gravitational Cherenkov radiation

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We investigate gravitational Cherenkov radiation in a healthy branch of background solutions in the ghost-free bigravity model. In this model, because of the modification of dispersion relations, each polarization mode can possess subluminal phase velocities, and the gravitational Cherenkov radiation could be potentially emitted from a relativistic particle. In the present paper, we derive conditions for the process of the gravitational Cherenkov radiation to occur and estimate the energy emission rate for each polarization mode. We found that the gravitational Cherenkov radiation emitted even from an ultrahigh energy cosmic ray is sufficiently suppressed for the graviton's effective mass less than 100 eV, and the bigravity model with dark matter coupled to the hidden metric is therefore consistent with observations of high energy cosmic rays.

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I. INTRODUCTION

The LIGO detection of gravitational wave signal from a pair of merging black holes finally proved the propagation of gravitational waves [1], and it was reported that the Einstein theory of gravity is consistent with gravitational wave observations with high accuracy [2] in addition to solar-system tests [3]. On the other hand, theoretical and observational evidences imply that the Universe is undergoing a phase of accelerated expansion at the present epoch, and one has to introduce an energy component with negative pressure, dubbed as dark energy, to describe our Universe [4,5]. Recently, modifications of Einstein's gravity have attracted considerable attention as a substitute of dark energy and have been investigated in many literatures (see for reviews e.g. [6,7]).

One of the simplest modifications of general relativity is to introduce a graviton mass to general relativity. This hypothetical massive graviton has been first introduced by Fierz and Pauli (FP) in the context of linear theory, where its special structure of the mass term prevents an additional degree of freedom from appearing in a flat background space time [8]. One would naively expect that this linearized theory of massive gravity in the massless limit reduces to general relativity. However, one gets an order-one modification of the propagator in the massless limit, known as the van Dam-Veltman-Zakharov (vDVZ) discontinuity [9,10] (see the recent developments in [11,12]). A solution of this problem by taking into account the nonlinear effect was proposed by Vainshtein [13], which is responsible for screening a scalar degree of freedom in massive graviton. Although nonlinearities are essential to solve the van Dam-Veltman-Zakharov discontinuity, an additional 6th degree of freedom, called Boulware-Deser (BD) ghost, generally appears in such a theory [14]. However, it has been recently shown that the serious problem in the FP theory can be avoided by carefully choosing the potential, which consists of an infinite series of interaction terms determined in such a way that it eliminates BD ghost at all orders in perturbation theory [15]. This infinite series of interactions can be expressed in a compact form [16], referred to as the dRGT mass terms, and the absence of BD ghost in nonperturbative description has been shown in [17]. These mass terms added to general relativity can successfully mimic the cosmological constant in open Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime [18,19], though this solution involves serious instabilities in scalar and vector modes [20–22].

An extension of the dRGT theory is a bimetric theory of gravity, which can be straightforwardly constructed without reintroducing BD ghost by promoting the reference metric of the dRGT theory to a dynamical variable [23,24]. In this theory, referred to as the ghost-free bigravity, the physical degrees of freedom can be decomposed into five from the massive spin-2 field and two from the massless spin-2 field. Although similar type of FLRW solutions in the dRGT theory suffer from the catastrophic instabilities stated above, a new healthy branch of solutions (in the absence of matter field which couples to the second metric [25] and in the presence of two matter fields each of which couples to either the first or second metric [26]) can be obtained in a large fraction of the model parameter space (see [27–33] about other cosmological solutions). Although the bare graviton mass in this type of healthy solution is chosen to be larger than the Hubble parameter, one can evade the stringent constraints from solar-system tests by tuning the parameters in such a way that the Vainshtein radius is sufficiently large [25]. Furthermore, in this background, because of the modified dispersion relations, the phase and group velocities for all polarization modes of graviton deviate from the speed of light.

If a phase velocity of graviton is slower than the speed of light, a relativistic particle emits gravitational Cherenkov radiation (GCR), analogous to the electromagnetic Cherenkov radiation [34–36]. Interestingly, this GCR process can put a tight constraint on the phase velocity of graviton from the condition that the damping from GCR is not significant for ultrahigh energy cosmic rays, and it is confirmed to be useful in concrete examples of modified gravity [37,38], such as the new Ether-Einstein gravity [39,40] and the most general second order scalar-tensor theory [41–43]. For example, in the latter theory the phase velocity c_T is constrained as $c - c_T < 2\times 10^{-15}c$ [37], and most of parameter space in which at least one phase velocity is subluminal is not allowed because of significant energy loss of high energy cosmic rays. Furthermore, the authors in [44] investigated the constraints on modified gravity theories with Lorentz-violating modified dispersion relations [45], $\omega^2 = k^2c_s^2 + m^2 + Ak^\alpha$, where c_s and m are the sound speed and the mass of graviton, and α and A are model parameters. Although the constraint on the graviton mass is not stringent in this model, the authors found that α and A can be tightly constrained by observations of high energy cosmic rays. Constraint on more general modified dispersion relations including spatial anisotropies was investigated in Ref. [46]. The ghost-free bigravity model could be also constrained by the same process of GCR, and if so, the model parameters should be chosen to be consistent with observations. To this end, in the present paper we estimate the emission rate of GCR from a relativistic particle and derive constraints on the ghost-free bigravity model from observations of high energy cosmic rays.

The rest of the present paper is organized as follows. In Sec. II we briefly review the ghost-free bigravity theory and its FLRW cosmology. Then, in Sec. III we derive the emission rate of the gravitational Cherenkov radiation of the tensor and the vector modes. In Sec. IV we discuss consistency with observations of high energy cosmic rays. Section V is devoted to conclusion.

Throughout the paper, we use units in which the speed of light and the Planck constant are unity, $c = \hbar = 1$, and we follow the metric signature convention $(-,+,+,+)$.

II. FLRW BACKGROUNDS

In this section we briefly review the ghost-free bigravity model and spatially homogeneous and isotropic cosmological solutions, investigated in detail in [26]. The action for the ghost-free bigravity is written as

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}R[g] + \frac{\kappa M_g^2}{2}\int d^4x\sqrt{-f}R[f] + m^2M_g^2\int d^4x\sqrt{-g}\sum_{i=0}^{4}\alpha_i\mathcal{L}_i + S_m[g] + S_m[f], \quad (2.1)$$

where $g_{\mu\nu}$ and $f_{\mu\nu}$ are, respectively, the physical and the hidden metrics, M_g is the 4-dimensional bare Planck mass for the physical metric $g_{\mu\nu}$, κ represents the ratio of the squared bare Planck masses for $g_{\mu\nu}$ and $f_{\mu\nu}$, and α_i are dimensionless model parameters. $S_m[g]$ ($S_m[f]$) is the action of a matter field that couples only to $g_{\mu\nu}(f_{\mu\nu})$, which is referred to as g-matter (f-matter). The interaction Lagrangian $\mathcal{L}_i$ (dRGT mass terms) is given by

$$\mathcal{L}_0 = 1, \quad \mathcal{L}_1 = [\mathcal{K}],$$
$$\mathcal{L}_2 = \frac{1}{2}([\mathcal{K}]^2 - [\mathcal{K}^2]), \quad \mathcal{L}_3 = \frac{1}{6}([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]), \quad (2.2)$$

$$\mathcal{L}_4 = \frac{1}{24}([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]), \quad (2.3)$$

where we introduce

$$\mathcal{K}^\mu{}_\nu = \delta^\mu{}_\nu - (\sqrt{g^{-1}f})^\mu{}_\nu, \quad (2.4)$$

and $[\mathcal{K}^n] = \mathrm{Tr}(\mathcal{K}^n)$. We consider the cosmological background described by the following flat FLRW metrics:

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2\delta_{ij}dx^i dx^j, \quad (2.5)$$

$$f_{\mu\nu}dx^\mu dx^\nu = -n^2dt^2 + \alpha^2\delta_{ij}dx^i dx^j, \quad (2.6)$$

where we set the lapse function for the physical metric to unity, $n = n(t)$ is the lapse function for the hidden metric, and $a = a(t)$ and $\alpha = \alpha(t)$ are the scale factors for the respective metrics. The background equations are given by

$$3H^2 = m^2\hat{\rho}_{m,g} + \frac{\rho_g}{M_g^2}, \quad (2.7)$$

$$3H_f^2 = \frac{m^2}{\kappa}\hat{\rho}_{m,f} + \frac{\rho_f}{\kappa M_g^2}, \quad (2.8)$$

$$2\dot{H} = m^2\xi J(\tilde{c} - 1) - \frac{\rho_g + P_g}{M_g^2}, \quad (2.9)$$

$$2\frac{\dot{H}_f}{n} = -\frac{m^2}{\kappa\xi^2\tilde{c}}\xi J(\tilde{c} - 1) - \frac{\rho_f + P_f}{\kappa M_g^2}, \quad (2.10)$$

and the energy conservation laws for g-matter and f-matter. Here, we defined $H \equiv \dot{a}/a$, $H_f \equiv \dot{\alpha}/(n\alpha)$, an overdot as the differentiation with respect to t, and ρ_g, P_g, ρ_f and P_f as the energy density of g-matter, the pressure of g-matter, the energy density of f-matter and the pressure of f-matter, respectively. Also, we introduced

$$\hat{\rho}_{m,g} \equiv U(\xi) - \frac{\xi}{4}U'(\xi), \tag{2.11}$$

$$\hat{\rho}_{m,f} \equiv \frac{1}{4\xi^3}U'(\xi), \tag{2.12}$$

$$J(\xi) \equiv \frac{1}{3}\left[U(\xi) - \frac{\xi}{4}U'(\xi)\right]', \tag{2.13}$$

where $'$ is the differentiation with respect to ξ and

$$\xi \equiv \frac{\alpha}{a}, \qquad \tilde{c} \equiv \frac{na}{\alpha}, \tag{2.14}$$

$$U(\xi) \equiv -\alpha_0 + 4(\xi-1)\alpha_1 - 6(\xi-1)^2\alpha_2 + 4(\xi-1)^3\alpha_3 - (\xi-1)^4\alpha_4. \tag{2.15}$$

A constraint is given by the divergence of the equation of motion for $g_{\mu\nu}$ (or equivalently by the divergence of the equation of motion for $f_{\mu\nu}$) as

$$J(H - \xi H_f) = 0. \tag{2.16}$$

In this paper we focus on the healthy branch of solutions with $H = \xi H_f$, equivalently $\tilde{c}\alpha\dot{a} - a\dot{\alpha} = 0$ [25,26]. Then, from Eqs. (2.7) and (2.8), we obtain

$$\hat{\rho}_{m,g}(\xi) - \frac{\xi^2}{\kappa}\hat{\rho}_{m,f}(\xi) = -\frac{\rho_g}{m^2M_g^2} + \frac{\xi^2\rho_f}{\kappa m^2 M_g^2}. \tag{2.17}$$

For convenience, we define $\Gamma(\xi)$ and the time-dependent effective graviton mass $\mu(\xi)$,

$$\Gamma(\xi) \equiv \xi J(\xi) + \frac{(\tilde{c}-1)\xi^2}{2}J'(\xi), \tag{2.18}$$

$$\mu^2(\xi) \equiv \frac{1+\kappa\xi^2}{\kappa\xi^2}m^2\Gamma(\xi). \tag{2.19}$$

As is seen in the next section, μ corresponds to the effective mass in the long wave length limit.

Since we are interested in the late-time cosmology, we take the low energy limit, $\rho_g/(\mu^2M_g^2) \ll 1$ and $\xi^2\rho_f/(\kappa\mu^2M_g^2) \ll 1$. In this limit, from Eq. (2.17), we find that ξ converges to a constant ξ_c determined by

$$\hat{\rho}_{m,g}(\xi_c) - \frac{\xi_c^2}{\kappa}\hat{\rho}_{m,f}(\xi_c) = 0. \tag{2.20}$$

Expanding Eq. (2.17) around ξ_c, ξ is given by

$$\left(\frac{3m^2(1+\kappa\xi_c^2)J(\xi_c)}{\kappa\xi_c} - 2\Lambda\right)\frac{\xi-\xi_c}{\xi_c} \approx -\frac{\rho_g}{M_g^2} + \frac{\xi_c^2\rho_f}{\kappa M_g^2}, \tag{2.21}$$

as a function of ρ_g and ρ_f, where Λ is defined as

$$\Lambda \equiv m^2\hat{\rho}_{m,g}(\xi_c). \tag{2.22}$$

At least in the low energy limit, the ξ parameter is monotonic, which can be seen from Eq. (2.21), since ρ_g and ρ_f are also monotonic. Then, the modified Friedmann equation for the physical metric $g_{\mu\nu}$ can be written as

$$3H^2 \simeq \frac{\rho_g + \tilde{\kappa}^{-1}\rho_f}{\tilde{M}_g^2} + \Lambda \quad \text{for } \left|\frac{\Lambda}{\mu^2}\right| \ll 1, \tag{2.23}$$

where $\tilde{M}_g^2 = (1+\kappa\xi_c^2)M_g^2$ and $\tilde{\kappa} = 1/\xi_c^4$, and Λ turns out to be the effective cosmological constant.

The equation that determines the evolution of $\tilde{c}$ can be derived from the equation of motion for $f_{\mu\nu}$ as

$$\tilde{c} = 1 + \frac{1}{2WM_g^2}\left[\rho_g + P_g - \frac{\tilde{c}\xi^2}{\kappa}(\rho_f + P_f)\right], \tag{2.24}$$

where we define

$$W \equiv \frac{(1+\kappa\xi^2)J}{2\kappa\xi}m^2 - H^2 = \frac{1}{2}\left(\mu^2 - \frac{\tilde{c}-1}{2}\frac{(1+\kappa\xi^2)J'}{\kappa}m^2 - 2H^2\right). \tag{2.25}$$

In the low energy limit, we obtain

$$\tilde{c} \simeq 1 + \frac{1}{M_g^2(\mu^2 - 2H^2)}\left[\rho_g + P_g - \frac{\xi^2}{\kappa}(\rho_f + P_f)\right]. \tag{2.26}$$

Assuming that $W > 0$, which is required to avoid the Higuchi ghost [26], a matter field that satisfies $\rho_g + P_g < \xi^2(\rho_f + P_f)/\kappa$ implies $\tilde{c} < 1$ and vice versa. We will show that the tensor modes of graviton possess subluminal phase velocity when $\tilde{c} < 1$ in Sec. III. It is naively expected that cosmic-ray observations will prohibit the dominance of f-matter because it leads to the gravitational Cherenkov radiation. Also, we investigate the gravitational Cherenkov radiation of the vector modes of graviton. The vector modes can possess subluminal phase velocity for any $\tilde{c}$, which will be seen in Sec. IV. Therefore, even when g-matter dominates, the allowed parameter region of the ghost-free bigravity can be potentially considerably restricted.

III. GRAVITATIONAL CHERENKOV RADIATION OF TENSOR MODES

In this section we investigate the gravitational Cherenkov radiation of the tensor modes in bigravity model. The tensor perturbations for the respective metrics can be introduced as small deviations from the background metrics (2.6), $\delta g_{ij} = a^2(h_+\varepsilon^+_{ij} + h_\times \varepsilon^\times_{ij})$ and $\delta f_{ij} = \alpha^2(\tilde{h}_+\varepsilon^+_{ij} + \tilde{h}_\times\varepsilon^\times_{ij})$, where ε^+_{ij} and $\varepsilon^\times_{ij}$ denote the polarization tensors for plus and cross modes. We normalize the polarization tensors as $\varepsilon^{\mu\nu(\lambda)}\varepsilon^{(\lambda')}_{\mu\nu} = \delta_{\lambda\lambda'}$. Hereafter we omit the index $+/\times$ since the equations of motion are identical for both polarizations. The quadratic action for the tensor modes is given by [26]

$$S_{\rm T} = \frac{M_g^2}{8}\int d^4x\left[\dot{h}^2 - (\partial_\ell h)^2 - m^2\Gamma(h-\tilde{h})^2 + \frac{\kappa\xi_c^2}{\tilde{c}}(\dot{\tilde{h}}^2 - \tilde{c}^2(\partial_\ell\tilde{h})^2)\right]. \tag{3.1}$$

Here, we assumed that the leading effect of nonflat background is due to the deviation of $\tilde{c}$ from unity and neglected the other cosmic expansion effects.[1] Then, the equations of motion are given by

$$\ddot{h} - \triangle h + m^2\Gamma(h-\tilde{h}) = 0, \tag{3.2}$$

$$\ddot{\tilde{h}} - \tilde{c}^2\triangle\tilde{h} + \frac{\tilde{c}m^2\Gamma}{\kappa\xi_c^2}(\tilde{h}-h) = 0. \tag{3.3}$$

One can find eigenfrequencies $\omega_{1,2}$ from the above equations of motion as

$$\frac{\omega_{1,2}^2}{k^2} = 1 + \frac{1-\tilde{c}}{x}\left[1 - x \mp \sqrt{(1-x)^2 + \frac{4\kappa\xi_c^2}{1+\kappa\xi_c^2}x}\right] + \mathcal{O}((1-\tilde{c})^2), \tag{3.4}$$

where the upper (lower) sign is for ω_1 (ω_2), k is the wave number, i.e., $\triangle = -k^2$, and x is defined as[2]

$$x = \frac{2k^2(1-\tilde{c})}{\mu^2}. \tag{3.5}$$

Here, the expression inside the square root in Eq. (3.4) is always positive, meaning $\omega_1^2 \neq \omega_2^2$, and we define $\omega_{1,2}$ so that the mode labeled with 1 becomes massless while ω_2^2 reduces to μ^2 in the long wave length limit $k \to 0$. When $\tilde{c} > 1$, x becomes negative and both modes always have superluminal phase velocities, i.e., $\omega_{1,2} > k$. On the other hand, when $\tilde{c} < 1$, the phase velocity of the mode labeled with 1 becomes subluminal while that labeled with 2 is superluminal, for any x. In order to study the gravitational Cherenkov radiation, we investigate the case with $\tilde{c} < 1$, in which the mode labeled with 1(2) corresponds to $\tilde{h}$ (h) in the high energy limit $k \to \infty$.[3] The orthogonalized action is given by

$$S_{\rm T} = \frac{M_g^2}{8}\int dt d^3k \sum_{A=1,2}(|\dot{h}_A|^2 - \omega_A^2|h_A|^2), \tag{3.6}$$

where the eigenfunctions h_1 and h_2 are given by

$$h_1 = \cos\theta_g h + \sin\theta_g\frac{\sqrt{\kappa}\xi_c}{\sqrt{\tilde{c}}}\tilde{h}, \tag{3.7}$$

$$h_2 = -\sin\theta_g h + \cos\theta_g\frac{\sqrt{\kappa}\xi_c}{\sqrt{\tilde{c}}}\tilde{h}, \tag{3.8}$$

with the mixing angle,

$$\theta_g = \frac{1}{2}\cot^{-1}\left(-\frac{1+\kappa\xi_c^2}{2\sqrt{\kappa}\xi_c\sqrt{\tilde{c}}}x + \frac{\tilde{c}-\kappa\xi_c^2}{2\sqrt{\kappa}\xi_c\sqrt{\tilde{c}}}\right), \tag{3.9}$$

defined as a continuous function of x with $0 < \theta_g < \pi/2$.

Now, we are ready to quantize the tensor modes, and the field operators can be expanded as

$$h_{A\mu\nu} = \sqrt{\frac{4}{M_g^2}}\sum_\lambda\int\frac{d^3k}{(2\pi)^{3/2}}[\varepsilon^{(\lambda)}_{\mu\nu}\hat{a}^{(\lambda)}_{A\mathbf{k}}u_{Ak}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + \varepsilon^{(\lambda)}_{\mu\nu}\hat{a}^{(\lambda)\dagger}_{A\mathbf{k}}u^*_{Ak}(t)e^{-i\mathbf{k}\cdot\mathbf{x}}], \tag{3.10}$$

where $\boldsymbol{A} = 1, 2$, and $\hat{a}^{(\lambda)\dagger}_{A\mathbf{k}}$ and $\hat{a}^{(\lambda)}_{A\mathbf{k}}$ are the creation and annihilation operators, which satisfy the commutation relation $[\hat{a}^{(\lambda)}_{A\mathbf{k}}, \hat{a}^{(\lambda')\dagger}_{A'\mathbf{k}'}] = \delta_{AA'}\delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}')$, and the mode function

$$u_{Ak}(t) = \frac{e^{-i\omega_A(k)t}}{\sqrt{2\omega_A(k)}}, \tag{3.11}$$

satisfies

$$\left(\frac{d^2}{dt^2} + \omega_A^2(k)\right)u_{Ak}(t) = 0, \tag{3.12}$$

and $\dot{u}^*_{Ak}(t)u_{Ak}(t) - \dot{u}_{Ak}(t)u^*_{Ak}(t) = i$.

[1] One might think the other cosmic expansion effects become important at $k < H_0$. However, our results will not change as long as $H_0 < \mu$. Otherwise, the estimation of Eq. (3.34) could be altered.

[2] Here the sign of x is different from the one in Ref. [26].

[3] In the case where g-matter is dominant and hence $\tilde{c} > 1$, on the contrary, the mode labeled with 1(2) reaches h ($\tilde{h}$) when $k \to \infty$.

We are interested in the GCR from a high energy particle, e.g., a high energy proton. For simplicity, we consider a complex scalar field with the action

$$S_m = \int d^4x\sqrt{-g}[-g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - M^2\psi^*\psi], \qquad (3.13)$$

instead of a Dirac fermion. Neglecting the cosmic expansion and the coupling to the metric perturbation, the free part of ψ can be quantized as

$$\hat{\psi}(t,\mathbf{x}) = \int \frac{d^3p}{(2\pi)^{3/2}}[\hat{b}_{\mathbf{p}}\psi_p(t)e^{i\mathbf{p}\cdot\mathbf{x}} + \hat{c}^\dagger_{\mathbf{p}}\psi^*_p(t)e^{-i\mathbf{p}\cdot\mathbf{x}}], \qquad (3.14)$$

where $\hat{b}_{\mathbf{p}}$ and $\hat{c}^\dagger_{\mathbf{p}}$ are the annihilation and creation operators of the particle and antiparticle, respectively, which satisfy the commutation relations $[\hat{b}_{\mathbf{p}}, \hat{b}^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p}-\mathbf{p}')$, $[\hat{c}_{\mathbf{p}}, \hat{c}^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p}-\mathbf{p}')$, and the mode function

$$\psi_p(t) = \frac{1}{\sqrt{2\Omega_p}}e^{-i\Omega_p t}, \qquad (3.15)$$

obeys

$$\left(\frac{d^2}{dt^2} + p^2 + M^2\right)\psi_p(t) = 0, \qquad (3.16)$$

with $\Omega_p = \sqrt{p^2+M^2}$. The interaction part of the action (3.13) is given by

$$S_I = -\int dtd^3x h^{ij}\partial_i\psi\partial_j\psi^*, \qquad (3.17)$$

and the interaction Hamiltonian is

$$H_I = \int d^3x h^{ij}\partial_i\psi\partial_j\psi^*. \qquad (3.18)$$

(Strictly speaking, all time derivatives must be replaced by means of the conjugate momenta in the Hamiltonian.)

In order to evaluate the total energy of the gravitational Cherenkov radiation, we adopt the method developed in [47,48]. (Note that the gravitational Cherenkov radiation can be also derived classically as in the case of the electromagnetic Cherenkov radiation [36].) Based on the in-in formalism [49], at the lowest order of the expectation value of the number operator of graviton is given by

$$\langle\hat{a}^{\dagger(\lambda)}_{\mathbf{Ak}}\hat{a}^{(\lambda)}_{\mathbf{Ak}}\rangle = i^2\int_{t_{\rm in}}^t dt_2\int_{t_{\rm in}}^{t_2}dt_1\langle{\rm in}|[H_I(t_1), [H_I(t_2), \hat{a}^{\dagger(\lambda)}_{\mathbf{Ak}}\hat{a}^{(\lambda)}_{\mathbf{Ak}}]]|{\rm in}\rangle \qquad (3.19)$$

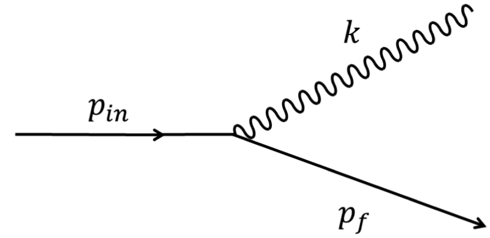

FIG. 1. Feynman diagram for the process.

for the initial state with one scalar particle with the momentum, $\mathbf{p}_{\rm in}$, i.e., $|{\rm in}\rangle = \hat{b}^\dagger_{\mathbf{p}_{\rm in}}|0\rangle$. This gives the transition probability of the process in which one graviton with the momentum $\mathbf{k}$ is emitted from a scalar particle with the initial momentum $\mathbf{p}_{\rm in}$ as shown in Fig. 1. Equation (3.19) can be rewritten as [50]

$$\langle\hat{a}^{\dagger(\lambda)}_{\mathbf{Ak}}\hat{a}^{(\lambda)}_{\mathbf{Ak}}\rangle = 2\Re\int_{t_{\rm in}}^t dt_2\int_{t_{\rm in}}^{t_2}dt_1 \times \langle{\rm in}|H_I(t_1)\hat{a}^{\dagger(\lambda)}_{\mathbf{Ak}}\hat{a}^{(\lambda)}_{\mathbf{Ak}}H_I(t_2)|{\rm in}\rangle. \qquad (3.20)$$

Hereafter, we omit the tensor mode labeled with 2, whose phase velocity is always superluminal. Then, the total radiation energy emitted from the scalar particle into the tensor mode labeled with 1 can be estimated as $E = \sum_\lambda\sum_{\mathbf{k}}\omega_k\langle\hat{a}^{\dagger(\lambda)}_{\mathbf{Ak}}\hat{a}^{(\lambda)}_{\mathbf{Ak}}\rangle$, which leads to

$$E_{\rm T} = \int\frac{d^3k}{(2\pi)^3}\omega_1\left|\int_{t_{\rm in}}^t dt_1\sqrt{\frac{4}{M_g^2}}u_{1k}(t_1)\psi_{p_{\rm f}}(t_1)\psi^*_{p_{\rm in}} \times (t_1)\varepsilon^{(\lambda)}_{ij}p^i_{\rm in}p^j_{\rm f}\right|^2\cos^2\theta_g, \qquad (3.21)$$

where $\mathbf{p}_{\rm f} + \mathbf{k} = \mathbf{p}_{\rm in}$ ($p^i_{\rm f} + k^i = p^i_{\rm in}$). With the aid of the relation $\sum_\lambda|\varepsilon^{(\lambda)}_{ij}p^i_{\rm in}p^j_{\rm f}|^2 = p^4_{\rm in}\sin^4\theta/2$, we have

$$E_{\rm T} = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\omega_1 p^4_{\rm in}\sin^4\theta\left|\int_{t_{\rm in}}^t dt_1\sqrt{\frac{4}{M_g^2}}u_{1k}(t_1) \times \psi_{p_{\rm f}}(t_1)\psi^*_{p_{\rm in}}(t_1)\right|^2\cos^2\theta_g. \qquad (3.22)$$

After plugging the mode functions into (3.22), the total radiation energy (3.22) reduces to

$$E_{\rm T} \simeq \frac{1}{4M_g^2}\int\frac{d^3k}{(2\pi)^3}\frac{p^4_{\rm in}\sin^4\theta}{\Omega_{\rm f}\Omega_{\rm in}}2\pi T\delta(\Omega_{\rm in}-\Omega_{\rm f}-\omega_1)\cos^2\theta_g, \qquad (3.23)$$

where $\Omega_{\rm in} = \sqrt{\mathbf{p}^2_{\rm in}+M^2}$ and $\Omega_{\rm f} = \sqrt{(\mathbf{p}_{\rm in}-\mathbf{k})^2+M^2}$, and we used

$$\left|\int_{t_{\rm in}}^{t} dt_1 \exp\left[i(\Omega_{\rm in}-\Omega_{\rm f}-\omega_1)(t_1-t_{\rm in})\right]\right|^2 \simeq 2\pi T\delta(\Omega_{\rm in}-\Omega_{\rm f}-\omega_1), \tag{3.24}$$

assuming the long time duration of the integration, where $T = t - t_{\rm in}$. Then, we have the expression in the relativistic limit of the scalar particle, $p_{\rm in} \gg M$,

$$\frac{dE_T}{dt} = \frac{p_{\rm in}^3}{4M_g^2}\int_0^\infty \frac{dkk^2}{2\pi}\int_{-1}^{1} d(\cos\theta)\frac{\sin^4\theta}{\Omega_{\rm f}} \times \cos^2\theta_g \delta(\Omega_{\rm in}-\Omega_{\rm f}-\omega_1). \tag{3.25}$$

Now, we consider the delta function, which can be written as

$$\delta(\Omega_{\rm in}-\Omega_{\rm f}-\omega_1) = 2\Omega_{\rm f}\delta(\Omega_{\rm f}^2-(\Omega_{\rm in}-\omega_1)^2)\Theta(\Omega_{\rm in}-\omega_1), \tag{3.26}$$

where $\Theta(y)$ is the Heaviside function. Using the identity (3.26), we may write

$$\delta(\Omega_{\rm in}-\Omega_{\rm f}-\omega_1) = \frac{\Omega_{\rm f}}{p_{\rm in}k}\delta\left(\cos\theta - \frac{k}{2p_{\rm in}}\left(1-\frac{\omega_1^2}{k^2}\right) - \sqrt{1+\frac{M^2}{p_{\rm in}^2}\frac{\omega_1}{k}}\right)\Theta(\Omega_{\rm in}-\omega_1). \tag{3.27}$$

Integration over θ in Eq. (3.25) makes a nontrivial contribution when

$$\cos\theta = \frac{k}{2p_{\rm in}}\left(1-\frac{\omega_1^2}{k^2}\right) + \sqrt{1+\frac{M^2}{p_{\rm in}^2}\frac{\omega_1}{k}} \le 1, \tag{3.28}$$

which is a necessary condition for GCR to arise. Assuming $M/p_{\rm in} \ll 1$ and $1-\omega_1^2/k^2 \ll 1$, the above condition can be rewritten as

$$\cos\theta \approx 1 + \frac{M^2}{2p_{\rm in}^2} - \frac{p_{\rm in}-k}{2p_{\rm in}}\left(1-\frac{\omega_1^2}{k^2}\right) \le 1. \tag{3.29}$$

From the presence of $\Theta(\Omega_{\rm in}-\omega_1)$, the possible range of k is restricted to $k \simeq \omega_1 \lesssim p_{\rm in}$, and hence we reconfirm that $\omega_1^2 < k^2$ is a necessary condition for GCR. Then, the condition (3.29) leads to

$$1-\frac{\omega_1^2}{k^2} \ge \frac{M^2}{p_{\rm in}(p_{\rm in}-k)}. \tag{3.30}$$

As we will see soon, the left-hand side of the above inequality is approximated as $1-\omega_1^2/k^2 \sim \mathcal{O}(1-\tilde{c})$ for any k. Thus, the condition for emitting GCR is simply given by $1-\tilde{c} \gtrsim M^2/p_{\rm in}^2 \sim 10^{-22}$ for a ultrahigh energy cosmic-ray proton with $p_{\rm in} \sim 10^{11}$ GeV and $M \sim 1$ GeV. Although $M^2/p_{\rm in}^2$ term could be important when $1-\tilde{c} \sim M^2/p_{\rm in}^2$, the effect of mass merely reduces the GCR efficientcy. We will find later that the constraint is always weak even if we neglect the proton mass to discuss the constraints from the tensor GCR. Therefore we can safely ignore $M^2/p_{\rm in}^2$ term in this context. Then, the condition (3.30) is understood as the one that the effective refractive index exceeds unity, $n_A = k/\omega_A > 1$. Thus, GCR is emitted only through the mode labeled with 1 when $1-\tilde{c} > 0$.

We integrate Eq. (3.25) by adopting the small angle approximation $\theta \ll 1$, and we have[4]

$$\frac{dE_{\rm T}}{dt} = \frac{1}{8\pi M_g^2}\int_0^{p_{\rm in}} dkk\left((p_{\rm in}-k)\left(1-\frac{\omega_1^2}{k^2}\right)\right)^2\cos^2\theta_g. \tag{3.31}$$

Because of the complex k dependence in $1-\omega_1^2/k^2$ and $\cos\theta_g$, one cannot simply integrate Eq. (3.31). To approximately estimate dE_T/dt, we consider the limiting cases with $|x| \ll 1$ and $|x| \gg 1$. In both limits, $1-\omega_1^2/k^2$ is estimated from Eq. (3.4) as

$$1-\frac{\omega_1^2}{k^2} \simeq \begin{cases} \frac{2\kappa\xi_c^2}{1+\kappa\xi_c^2}(1-\tilde{c}) + \mathcal{O}(x), & (x \ll 1), \\ 2(1-\tilde{c}) + \mathcal{O}(x^{-1}), & (x \gg 1). \end{cases} \tag{3.32}$$

Also, we estimate $\cos^2\theta_g$ from Eq. (3.9) as

$$\cos^2\theta_g \simeq \begin{cases} \frac{1}{1+\kappa\xi_c^2} + \mathcal{O}(x), & (x \ll 1), \\ \frac{\kappa\xi_c^2}{(1+\kappa\xi_c^2)^2}x^{-2} + \mathcal{O}(x^{-3}), & (x \gg 1). \end{cases} \tag{3.33}$$

Then, we can now estimate dE_T/dt using the approximate expressions (3.32) and (3.33). Denoting the wave number at $x = 1$ as $k_D \equiv \mu_c/\sqrt{2(1-\tilde{c})}$, we discuss two cases: $k_D < p_{\rm in}$ and $k_D > p_{\rm in}$, one by one. For the case with $k_D < p_{\rm in}$, we can estimate dE_T/dt by dividing the interval of the integral into two at $x = 1$, and we get[5]

[4]Contrary to the vector case, which will be seen in the next section, the condition (3.28) does not impose the lower limit of the integration. This is because the subluminal mode labeled with 1 corresponds to the massless mode in the limit $k \to 0$.

[5]Since the dominant contribution to the integral (3.34) lies at $x \sim 1$, the expression (3.34) does not smoothly connect with (3.35) at $k_D \sim p_{\rm in}$. However, it is sufficient to understand the dependence of $dE_{\rm T}/dt$ on μ and $1-\tilde{c}$ for our present purpose.

$$\frac{dE_{\rm T}}{dt} \simeq \frac{1}{8\pi M_g^2}\int_0^{k_D} dk \left[k\left((p_{\rm in}-k)\left(1-\frac{\omega_1^2}{k^2}\right)\right)^2\cos^2\theta_g\right]_{|x|\ll 1} + \frac{1}{8\pi M_g^2}\int_{k_D}^{p_{\rm in}} dk \left[k\left((p_{\rm in}-k)\left(1-\frac{\omega_1^2}{k^2}\right)\right)^2\cos^2\theta_g\right]_{|x|\gg 1} \simeq \frac{1}{8\pi M_g^2}\frac{\kappa\xi_c^2(1+2\kappa\xi_c^2)}{(1+\kappa\xi_c^2)^3}p_{\rm in}^2\mu^2(1-\tilde{c}). \tag{3.34}$$

If $k_D \geq p_{\rm in}$, we only need to consider $x \ll 1$ region, and then dE_T/dt can be approximated as

$$\frac{dE_{\rm T}}{dt} \simeq \frac{1}{8\pi M_g^2}\int_0^{p_{\rm in}} dk \left[k\left((p_{\rm in}-k)\left(1-\frac{\omega_1^2}{k^2}\right)\right)^2\cos^2\theta_g\right]_{|x|\ll 1} \simeq \frac{1}{8\pi M_g^2}\frac{\kappa^2\xi_c^4}{3(1+\kappa\xi_c^2)^3}p_{\rm in}^4(1-\tilde{c})^2. \tag{3.35}$$

IV. GRAVITATIONAL CHERENKOV RADIATION FROM VECTOR MODES

In this section we investigate the gravitational Cherenkov radiation of the vector modes of graviton in the ghost-free bigravity. We introduce vector perturbations around the cosmological background as $\delta g_{0i} = aB_i$, $\delta f_{0i} = n\alpha b_i$, $\delta g_{ij} = a^2\partial_{(i}E_{j)}$, and $\delta f_{ij} = \alpha^2\partial_{(i}S_{j)}$, where B_i, b_i, E_i and S_i are transverse vectors. Following the discussion in Ref. [26], the effective action for the vector modes is written in terms of one dynamical vector variable for each polarization, while the other vectors are constrained or left unspecified corresponding to gauge degrees of freedom. The quadratic action for the vector mode expanded in vector harmonics is given as

$$S_{\rm V} = \frac{M_-^2}{8}\int dt d^3k a^3 A\left[\dot{\mathcal{E}}^i\dot{\mathcal{E}}_i^* - \left\{\frac{k^2}{a^2}c_{\rm V}^2 + m_{\rm V}^2\right\}\mathcal{E}^i\mathcal{E}_i^*\right], \tag{4.1}$$

where

$$\mathcal{E}_i \equiv \sqrt{\frac{1+\kappa\xi^2}{\kappa\xi^2}}k(E_i - S_i), \tag{4.2}$$

$$A \equiv \frac{\kappa\xi^2}{1+\kappa\xi^2}\left[\frac{(\tilde{c}+1)\Gamma k^2}{2a^2\mu^2\xi J} + \frac{\tilde{c}+\kappa\xi^2}{1+\kappa\xi^2}\right]^{-1}, \tag{4.3}$$

$$M_-^2 \equiv \frac{\kappa\xi^2}{1+\kappa\xi^2}M_g^2, \tag{4.4}$$

$$c_{\rm V}^2 \equiv \frac{(\tilde{c}+1)\Gamma}{2\xi J}, \tag{4.5}$$

$$m_{\rm V}^2 \equiv \frac{\tilde{c}+\kappa\xi^2}{1+\kappa\xi^2}\mu^2. \tag{4.6}$$

B_i and b_i are nondynamical degrees of freedom and written in terms of E_i and S_i by means of the constraints as

$$B_i \equiv a\left[\frac{\dot{E}_i}{2} - \frac{A}{2}(\dot{E}_i - \dot{S}_i)\right], \tag{4.7}$$

$$b_i \equiv a\left[\frac{\dot{S}_i}{2\tilde{c}} - \frac{A}{2\kappa\xi_c^2}(\dot{E}_i - \dot{S}_i)\right]. \tag{4.8}$$

In the low-energy limit where $\xi \simeq \xi_c$, $c_{\rm V}^2$ is written as

$$1 - c_{\rm V}^2 \simeq \frac{1-\tilde{c}}{2}(1+\mathcal{C}), \tag{4.9}$$

where we define $\mathcal{C} \equiv \xi_c J'(\xi)/J(\xi)$. Then, subluminal phase velocity can be achieved when $1+\mathcal{C} > 0$ for f-matter dominant case $(1-\tilde{c} > 0)$ or $1+\mathcal{C} < 0$ for g-matter dominant case $(1-\tilde{c} < 0)$. Imposing the positivity of the effective mass squared, $\mu^2 > 0$ and $J > 0$, and the absence of gradient instability $c_{\rm V}^2 \geq 0$, the conditions that $c_{\rm V}$ is subluminal are given by

$$\left(\mathcal{C} < -1 \cap 1 < \tilde{c} < \frac{-2+\mathcal{C}}{\mathcal{C}}\right) \cup \left(\mathcal{C} > -1 \cap \frac{-1+\mathcal{C}}{1+\mathcal{C}} < \tilde{c} < 1\right). \tag{4.10}$$

According to [25], the Vainshtein radius is given by

$$r_{\rm V} = \mathcal{O}\left(\left(\frac{|\mathcal{C}|r_g}{\mu^2}\right)^{1/3}\right), \tag{4.11}$$

where r_g is the gravitational radius of the star. Therefore, $|\mathcal{C}|$ need to be sufficiently large for the Vainshtein mechanism to work. One can find such parameter spaces in the region (4.10) for small $|1-\tilde{c}|$, and the smallness of $|1-\tilde{c}|$ is also consistent with the background equation (2.26). Therefore, $c_{\rm V}^2$ can be significantly subluminal both in the g-matter dominant and f-matter dominant cases.

Then, we obtain the quantized vector gravitational perturbation:

$$\hat{\mathcal{E}}_i = \frac{1}{a}\sum_\lambda\int\frac{d^3k}{(2\pi)^{3/2}}\sqrt{\frac{2}{AM_-^2}}[\varepsilon_i^{(\lambda)}\hat{a}_{\rm V\mathbf{k}}^{(\lambda)}u_{{\rm V}k}(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \varepsilon_i^{(\lambda)}\hat{a}_{\rm V\mathbf{k}}^{(\lambda)\dagger}u_{{\rm V}k}^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}], \tag{4.12}$$

where $\varepsilon_i^{(\lambda)}$ is the polarization vector, which is normalized as $\varepsilon^{\mu(\lambda)}\varepsilon_\mu^{(\lambda')} = \delta_{\lambda\lambda'}$, $\hat{a}_{\rm V\mathbf{k}}^{(\lambda)\dagger}$ and $\hat{a}_{\rm V\mathbf{k}}^{(\lambda)}$ are the creation and annihilation operators, which satisfy the commutation relation $[\hat{a}_{\rm V\mathbf{k}}^{(\lambda)}, \hat{a}_{\rm V\mathbf{k}'}^{(\lambda')\dagger}] = \delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}')$. Neglecting the effect of cosmic expansion and considering $a \simeq 1$, the mode function

$$u_{\mathrm{V}k}(t) = \frac{e^{-i\omega_{\mathrm{V}}(k)t}}{\sqrt{2\omega_{\mathrm{V}}(k)}} \tag{4.13}$$

satisfies

$$\left(\frac{d^2}{dt^2} + \omega_{\mathrm{V}}^2(k)\right) u_{\mathrm{V}k}(t) = 0, \tag{4.14}$$

and $\dot{u}^*_{\mathrm{V}k}(t)u_{\mathrm{V}k}(t) - \dot{u}_{\mathrm{V}k}(t)u^*_{\mathrm{V}k}(t) = i$ with

$$\omega_{\mathrm{V}}^2(k) = c_{\mathrm{V}}^2 k^2 + m_{\mathrm{V}}^2. \tag{4.15}$$

The coupling between the vector graviton and the complex scalar field ψ is given as

$$I_{\mathrm{int}} = -\int dt d^3x h^{\mu\nu}\left[\partial_\mu\psi\partial_\nu\psi^* - \frac{1}{2}\eta_{\mu\nu}(\partial^\lambda\psi\partial_\lambda\psi^* + 2M^2\psi\psi^*)\right]. \tag{4.16}$$

Since the whole action is invariant under a coordinate transformation, we impose a convenient gauge fixing condition

$$S_i = \frac{A-1}{A}E_i, \tag{4.17}$$

so that h_{0i} components vanish. In this gauge, E_i is written in terms of $\mathcal{E}_i$ as

$$E_i = \sqrt{\frac{\kappa\xi_c^2}{1+\kappa\xi_c^2}}\frac{A}{k}\mathcal{E}_i, \tag{4.18}$$

and the Hamiltonian for the interaction between the graviton and scalar field becomes

$$H_{\mathrm{int}} = \int d^3x \partial_{(i}E_{j)}\partial^i\psi\partial^j\psi^*. \tag{4.19}$$

As in Sec. III, we calculate the gravitational radiation energy emitted from the process shown in Fig. 1 as

$$\begin{aligned} E_{\mathrm{V}} &= \sum_\lambda \int \frac{d^3k}{(2\pi)^3}\omega_{\mathrm{V}}\left|\int_{t_{\mathrm{in}}}^t dt_1 \frac{\sqrt{2A}}{M_g}u_{\mathrm{V}k}(t_1)\psi_{p_{\mathrm{f}}}(t_1)\psi^*_{p_{\mathrm{in}}} \times (t_1)\hat{k}_{(i}\varepsilon^{(\lambda)}_{j)}p^i_{\mathrm{in}}p^j_{\mathrm{f}}\right|^2 \\ &= \int\frac{d^3k}{(2\pi)^3}\omega_{\mathrm{V}}\frac{A}{4M_g^2}p_{\mathrm{in}}^2\sin^2\theta(2p_{\mathrm{in}}\cos\theta - k)^2 \times\left|\int_{t_{\mathrm{in}}}^t dt_1 u_{\mathrm{V}k}(t_1)\psi_{p_{\mathrm{f}}}(t_1)\psi^*_{p_{\mathrm{in}}}(t_1)\right|^2, \end{aligned} \tag{4.20}$$

where we define a unit vector parallel to $\mathbf{k}$, $\hat{k}_i \equiv k_i/|\mathbf{k}|$, and θ as the angle between p_{in} and $\mathbf{k}$, and use $\mathbf{p}_{\mathrm{in}} = \mathbf{k} + \mathbf{p}_{\mathrm{f}}$. Assuming the long time duration of the time integration (3.24), E_{V} is estimated as

$$E_{\mathrm{V}} = \frac{t-t_{\mathrm{in}}}{32\pi^2M_g^2}\int d^3kA\sin^2\theta\frac{p_{\mathrm{in}}^2(2p_{\mathrm{in}}\cos\theta-k)^2}{\Omega_{\mathrm{in}}\Omega_{\mathrm{f}}} \times\delta(\Omega_{\mathrm{in}}-\Omega_{\mathrm{f}}-\omega_{\mathrm{V}}). \tag{4.21}$$

Since the delta function in the above equation is the same expression as in the tensor case, we get the same condition (3.28) by replacing ω_1 to ω_{V}. Assuming $M/p_{\mathrm{in}} \ll 1$, the condition can be rewritten by solving the quadratic inequality as

$$1 - \frac{\omega_{\mathrm{V}}}{k} \geq \frac{M^2}{2p_{\mathrm{in}}(p_{\mathrm{in}}-k)}. \tag{4.22}$$

The condition for the vector GCR emission is therefore given by $1 - c_{\mathrm{V}} \gtrsim M^2/p_{\mathrm{in}}^2$. For the same reason as in the tensor case, we can safely ignore M^2/p_{in}^2 in the present paper. Then, Eq. (4.22) determines the lower limit of the integration,

$$k_{\mathrm{min,V}} \equiv \frac{m_{\mathrm{V}}}{\sqrt{1-c_{\mathrm{V}}^2}}, \tag{4.23}$$

and thus Eq. (4.3) can be written as

$$A = \frac{\kappa\xi_c^2}{1+\kappa\xi_c^2}\frac{\mu^2}{k^2}\left[1-(1-c_{\mathrm{V}}^2)\left(1-\frac{k^2_{\mathrm{min,V}}}{k^2}\right)\right]^{-1}. \tag{4.24}$$

Then, we find the contribution from $k \simeq k_{\mathrm{min,V}}$ in Eq. (4.21) is not dominant. Assuming $k \gg k_{\mathrm{min,V}}$, we finally obtain an approximation to the energy emission rate of the vector GCR,

$$\begin{aligned} \frac{dE_{\mathrm{V}}}{dt} &\simeq \frac{1}{4\pi M_g^2}\frac{\kappa\xi_c^2}{1+\kappa\xi_c^2}p_{\mathrm{in}}^2\mu^2(1-c_{\mathrm{V}}^2) \times\int_{k_{\mathrm{min,V}}}^{p_{\mathrm{in}}} dk\frac{1}{k}\left(1-\frac{k}{p_{\mathrm{in}}}c\mathrm{V}-\frac{k^2}{4p_{\mathrm{in}}^2}(1-c_{\mathrm{V}}^2)\right) \times\left(1-\frac{k}{2p_{\mathrm{in}}}c_{\mathrm{V}}\right)^2 \\ &\simeq \frac{1}{4\pi M_g^2}\frac{\kappa\xi_c^2}{1+\kappa\xi_c^2}p_{\mathrm{in}}^2\mu^2(1-c_{\mathrm{V}}^2)\ln\left(\frac{p_{\mathrm{in}}}{k_{\mathrm{min,V}}}\right). \end{aligned} \tag{4.25}$$

Here, we used $m_{\mathrm{V}}^2 \simeq \mu^2$ at low energies, and we only kept the leading contribution for $p_{\mathrm{in}} \gg k_{\mathrm{min,V}}$ in the last line.

V. CONSTRAINTS FROM HIGH ENERGY COSMIC RAYS

In this section we derive the condition that the damping due to GCR is not significant for an ultrahigh energy cosmic ray with initial energy p_{in} during time t, i.e., the

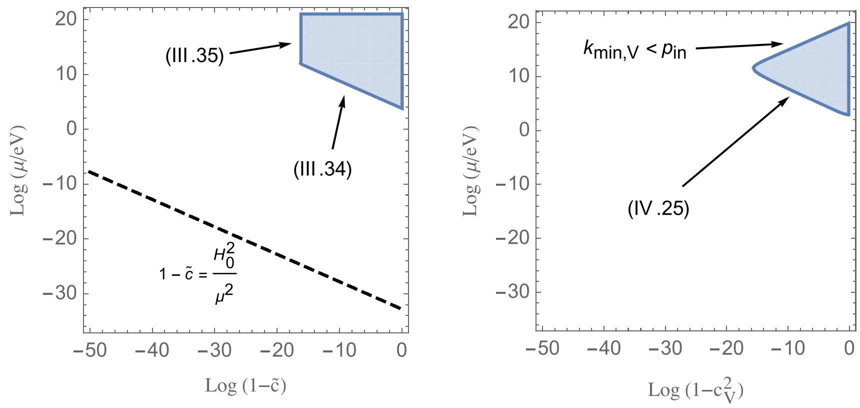

FIG. 2. Left: The excluded region in the $\mu-(1-\tilde{c})$ plane obtained by the constraint from the tensor GCR. The black dashed line shows the line $(1-\tilde{c})=H_0^2/\mu^2$, and $\kappa\xi_c^2=1$. Right: The excluded region in the $\mu-(1-c_{\rm V}^2)$ plane obtained by the constraint from the vector GCR for $\kappa\xi_c^2=1$.

condition that $dE_{\rm total}/dt < p_{\rm in}/t$ is satisfied, where $E_{\rm total}=E_{\rm T}+E_{\rm V}+E_{\rm S}$. Because of the complexity of the scalar perturbation, we only focus on the vector and tensor GCR discussed in Sec. III. We assume that the origins of high energy cosmic rays are located at a cosmological distance, $ct \gtrsim 1$ Mpc, and the initial momentum of the high energy cosmic rays of our concern is $p_{\rm in}\sim 10^{11}$ GeV.

Let us first examine the cosmological solution introduced in Sec. II. The deviation of $\tilde{c}$ from unity is related to the effective graviton mass μ through Eq. (2.26) as $1-\tilde{c}\sim H_0^2/\mu^2$. Then, the condition for the tensor GCR to occur can be simply given by $\mu \lesssim 10^{-31}$ GeV. In this case we always have $k_D < p_{\rm in}$, and the energy emission rate of the tensor GCR is therefore given by Eq. (3.34). Assuming $\kappa\xi_c^2\sim\mathcal{O}(1)$, we have

$$\frac{dE_{\rm T}}{dt}\sim\frac{p_{\rm in}^2H_0^2}{M_g^2}\ll\frac{p_{\rm in}}{t}. \tag{5.1}$$

Since the energy loss of a high energy cosmic ray due to the tensor GCR is extremely small, there is no conflict with observations, as we have anticipated earlier.

Let us next consider the vector GCR. Assuming $\kappa\xi_c^2\sim\mathcal{O}(1)$ and $\ln(p_{\rm in}/k_{\rm min,V})\sim\mathcal{O}(1)$, we have

$$\frac{dE_{\rm V}}{dt}\sim\frac{p_{\rm in}^2\mu^2}{M_g^2}(1-c_{\rm V}^2). \tag{5.2}$$

The constraint on the effective graviton mass is now given by

$$\mu\lesssim 100(1-c_{\rm V}^2)^{-1/2}\ \text{eV}, \tag{5.3}$$

which will allow the whole range of the graviton mass of our interest. Therefore the ghost-free background solutions introduced in Sec. II are consistent with observations of high energy cosmic rays.

Owing to the relation $1-\tilde{c}\sim H_0^2/\mu^2$, the tensor GCR emission is suppressed by H_0^2/M_g^2. Relaxing this relation, we now consider the constraint on $\tilde{c}$ and μ assuming as if they could be independently determined. The shaded region in the left panel of Fig. 2 shows the excluded region in the $\mu-(1-\tilde{c})$ plane obtained by the constraint from the tensor GCR. The lower and the left boundaries are, respectively, determined by the estimates of the emission rate (3.34) and (3.35). One can see that the cosmological solution in Sec. II, which lies at $1-\tilde{c}\sim H_0^2/\mu^2$ (black dashed line), is far from the excluded region. Even if we independently treat $1-\tilde{c}$ and μ, the tightest constraint on the effective graviton mass is $\mu\lesssim 100$ eV. In the right panel of Fig. 2, we present the excluded region in the $\mu-(1-c_{\rm V}^2)$ plane obtained by the constraint from the vector GCR. The lower and the upper boundaries are, respectively, determined by the emission rate (4.25) and the condition $k_{\rm min,V}<p_{\rm in}$. Also in the vector case the tightest constraint on the effective graviton mass is $\mu\lesssim 100$ eV similarly to the tensor case. Hence, this model is consistent with observations of ultrahigh energy cosmic rays.

VI. CONCLUSION

In this paper, we studied the consistency of the ghost-free bigravity model with observations of ultrahigh energy cosmic rays. The GCR can be emitted from a relativistic particle when a phase velocity of graviton is slower than the speed of light. If such a process is possible, a high energy cosmic ray reduces its energy during its propagation to the Earth, and a subluminal phase velocity of graviton could be strongly constrained. In the ghost-free bigravity model that we considered in this paper [26], the light speed in the hidden metric becomes subluminal or superluminal and then the graviton can possess a subluminal phase velocity. We confirmed that a relativistic particle emits the GCR in this model and derived the conditions for such a process to occur. The energy emission rate of the GCR of the tensor mode and the vector mode was estimated, and it turned out to be suppressed as far as the effective graviton mass is sufficiently small to satisfy $\mu \lesssim 100$ eV, which will cover most of the parameter region that is interesting when we consider gravity modification relevant at a late epoch.

Although we did not derive the emission rate of the scalar GCR due to the complexity of the dispersion relations, we think it natural to assume the emission rate of the scalar GCR is also suppressed for the following reason. In Ref. [51] the coupling between the scalar mode of a simple FP massive graviton and a real conformal scalar field in de Sitter background was computed, and the coupling squared, which is proportional to the transition amplitude, was reported to be suppressed by the factor $\mu^2(\mu^2 - 2H^2)/\tilde{M}_g^2 k^2$. (We use $\tilde{M}_g^2$ instead of M_g^2 since the only option here is the effective gravitational constant in the context of the FP massive graviton.) Although this computation was done not in the context of bigravity without taking into account the coupling between g- and f-matters and graviton, the factor mentioned above is, in a naive sense, the quantity to be compared with the vector mode counterpart

$$\frac{\kappa\xi_c^2}{1+\kappa\xi_c^2}\frac{k^2 A}{M_-^2} \approx \frac{\kappa\xi_c^2}{1+\kappa\xi_c^2}\frac{\mu^2}{c_{\rm V}^2 M_g^2}, \tag{6.1}$$

in the present setup. Neglecting the factor related to $\kappa\xi_c^2$ and the deviation of $c_{\rm V}^2$ from unity, we find that the coupling between the scalar mode of massive graviton and the incident high energy particle is as suppressed as in the case of the vector mode. On the other hand, the propagation speed of the scalar mode of graviton in bigravity has been calculated in Ref. [25], and the obtained expression is similar to the vector case at low energies [Eq. (88) in [25]].

When we consider nonconformal field, we need to keep the trace part of the metric perturbation, which was neglected in the computation in Ref. [51]. This neglected contribution gives a coupling to the trace part of the energy momentum tensor, which is absent as long as we consider conformally invariant matter fields. As is expected from the presence of the vDVZ discontinuity, the trace part of the energy momentum tensor will couple to the scalar mode of massive graviton without any suppression even in the massless limit. However, such a nonconformal component of the matter energy momentum tensor will be suppressed by the degree of violation of the conformal invariance, i.e., by the ratio of mass to momentum squared, $M^2/p_{\rm in}^2$, in the case of a Dirac fermion, instead of the suppression by $\mu^2/p_{\rm in}^2$. As a result, the transition amplitude should have a suppression factor proportional to $(M^2/p_{\rm in}^2)^2$. Then, based on the dimensional argument, the GCR emission rate would be, at most, given by $dE/dt \approx M^4/M_g^2$. When ultrahigh energy cosmic-ray protons are concerned, the fraction of the energy that is lost by the GCR after traveling a cosmological distance is much less than unity. Therefore, we expect that the scalar GCR will be harmless and conclude that the ghost-free bigravity with a sufficiently small mass is consistent with the observations of high energy cosmic rays, although confirmation by an explicit computation for scalar mode is needed to obtain a conclusive answer.

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