# Parametric Instabilities of the Yang-Mills Field and Far-from-Equilibrium Dynamics of Overpopulated Bosons

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High energy collision experiments performed at the relativistic heavy-ion collider (RHIC) and the large hadron collider (LHC) have found strong evidence that heavy-ion collisions create high temperature and energy density matter which consists of deconfined quarks and gluons. Understanding the properties of the matter known as the quark gluon plasma (QGP) is important to reveal fundamental properties of QCD. One of the remarkable findings of the collision experiments is that QGP is well described by hydrodynamical models with small viscosity. It means that QGP behaves as a nearly perfect fluid and thus achieves the local thermal equilibrium. The thermalization process of heavy-ions collisions is not well understood, and there are several theoretical puzzles to be resolved. In this thesis, we study the two issues listed below.

The first issue is referred to as the early thermalization problem. While the hydrodynamical models require that QGP should be created within $0.6 - 1.0$ fm/$c$ after the collision of nuclei, several theoretical studies suggests that the thermalization time is much larger than the phenomenological estimation. One promising scenario to resolve this puzzle is that instabilities in the early stage dynamics. In the early stage of the thermalization process of heavy-ion collisions, gluons approximately behave as classical fields due to the parametrically large occupancy of the gluon $n \sim 1/\alpha_s$. The classical configuration known as glasma is characterized by coherent color-electromagnetic fields in the longitudinal direction. It is well known that the flux tube structure of the color electromagnetic fields lead various instabilities, such as an analog of the Weibel instability and the Nielsen-Olesen instability. The recent study suggests that the color-magnetic field exhibits another type of instability known as the parametric resonance. However, the detailed features of the parametric resonance of gluon fields was not completely investigated.

The second issue is the theoretical understanding of the evolution of overpopulated systems. It has been also pointed out that the large occupancy of the initial gluons $n \sim 1/\alpha_s$ leads to the formation of Bose-Einstein condensate (BEC) when elastic scattering processes dominates. However, the formation of BEC is still controversial for heavy-ion collisions where the gauge coupling constant is large $g = \sqrt{4\pi\alpha_s} \sim 2$. One reliable method to attack this problem is the two-particle irreducible (2PI) functional techniques combined with the $1/N$-expansion.

In this thesis, we study the two issues proposed in the contest of the thermalization puzzle of heavy-ion collisions: the nature of instabilities of the classical Yang-Mills equation in the early stage dynamics and the time evolution of the overpopulated bosonic system.

**Parametric instabilities of the classical Yang-Mills field**

We study the nature of instabilities of the classical Yang-Mills equation under the time-dependent homogeneous color magnetic field in non-expanding and expanding geometries. In Chapter 2, we consider a homogeneous SU(2) Yang-Mills theory in a non-expanding geometry. The feature of the background gauge field is that it does not have spatial dependence, but depend on time. We find that some eigenmodes of the equation of motion (EOM) of fluctuation obeys the Lame equation. In particular, one of the dispersion relation is given by $\omega^2 s = p_z^2 - B_0 \mathrm{cn}^2(t)$. We point out that the negative sign of the second term originates from the spin-magnetic field interaction, and as a consequence, it can lead to a strong instability. We perform the analytic and numerical stability analyses for the Lame equation, and reproduce the well-known band structure in momentum space caused by the parametric instability. We also work out the linear analysis of the full CYM equation on the basis of the Floquet theory. We find that there is a broad instability band which have the maximum growth rate around zero momentum region in the $(p_z, p_T)$-plane. In particular, we show that the instability of the fluctuation with $p = 0$ is most unstable, and its nature can be understood in terms of the instability band of the Lames equation. Besides, we discuss that similarities and differences of the parametric and Nielsen-Olesen instabilities. We have point out that the parametric instability is caused by but by bunch of unstable modes with a band structure extending to continuous transverse momenta instead of the instability of the lowest Landau level which plays an important role for the emergence of the Nielsen-Olesen instability.

In Chapter 3, we perform instability analyses in an expanding geometry. We introduce a conformal time in order to map the expanding problem approximately into the nonexpanding problem. and show asymptotic forms of EOM of fluctuations around the boost-invariant magnetic field. We find that the EOM in the expanding geometry contains the effective momentum which is a function of the initial momentum and the conformal time. These features suggest that the expanding system under the time-dependent background field can show parametric instability, but the instability emerges in a different way from that in the nonexpanding geometry. We confirm that the parametric instability occurs even in an expanding geometry by numerical simulations. Due to the time dependence of the effective momenta, the way of growth of unstable modes is qualitatively different between finite $p_\eta$ and finite $p_T$ modes. Since the longitudinal effective momentum decreases monotonically, unstable fluctuations with finite $p_\eta$ grow exponentially after the transition from the oscillating behavior in the earliest stage. The remarkable point is that not only low momentum modes but also high momentum modes show parametric instability. On the other hand, since the transverse effective momentum is not monotonic in time, exponential growth in finite $p_T$ modes is limited in a particular temporal interval. In this regime, effective momenta are small and located within the instability band, then fluctuations can grow exponentially. In accordance with the band structure found in the nonexpanding geometry, only the modes with small $p_T$ show significant instability. The physical time scale of these instability have been estimated as $\tau \simeq 0.26(0.53)$ fm/$c$ at LHC (RHIC) energy, and this could be a relevant time scale for heavy-ion collisions.

**Time evolution of the overpopulated bosonic system**

We investigate the far-from-equilibrium dynamics of the strongly interacting O($N$)-symmetric scalar fields on the basis of the 2PI formalism and the resummation of diagrams to the next-to-leading order of the $1/N$-expansion. We perform the numerical simulations for underpopulated, overpopulated and strongly overpopulated initial conditions. In the underpopulated case, we find that the low momentum region of the particle distribution can be described by Bose-Einstein distribution already at early times. The chemical potential is always negative and eventually approaches the equilibrium value $\mu = 0$. In the overpopulated and strongly overpopulated cases, we find qualitative difference as compared with the underpopulated case. The most interesting behavior is that the chemical potential quickly turns positive and approaches to the quasi-particle mass. This phenomenon coincides with the rapid growth of the occupancy of the zero momentum mode $f(p = 0)$. While these findings are reminiscent of the dynamical formation of Bose-Einstein condensation in particle-number-conserving situations, we do not find that the divergence of $f(p = 0)$ due to the strong decaying process $3 \to 1$. Our result suggests that the formation of BEC is hindered in the present full-quantum approach.