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Abstract

This paper reviews the real the real business cycle models that display equilibrium indeterminacy. We first summarize the main findings in the seminal contributions by Benhabib and Farmer (1994) and Farmer and Guo (1994). We then discuss the relevant extinctions of the baseline model explored in the last two decades. We also refer to the recent development in the field after the global financial crisis of 2007-2008.

Keywords: equilibrium indeterminacy, business cycles, sunspots, extrinsic uncertainty.

JEL Classification: E21; E32; O41

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1 Introduction

If the equilibrium path of a dynamic macroeconomic model is not uniquely determined under rational expectations, which path is realized depends on a specification of expectations of agents. In this situation, non-fundamental shocks that only affect expectations of economic agents fluctuate economic activities. Therefore, in the presence of equilibrium indeterminacy, extrinsic uncertainty is a driving force of business cycles. Furthermore, if the equilibrium path of an economy is indeterminate, the long-run growth and development process of the economy would be affected by extrinsic uncertainty.[1]

Early studies on rational expectations models in the 1970s found that the rational expectations equilibrium may be multiple without imposing ad hoc restrictions.[2] Since most of the early rational expectations models lacked microfoundations, it was expected that the indeterminacy problem can be resolved, if one constructs models in which rational agents solve their dynamic optimization problems. However, as revealed by Brock (1974) and Calvo (1979), monetary dynamic models with optimizing agents easily exhibit equilibrium indeterminacy. Hence, constructing microfounded models cannot resolve the indeterminacy problem.

While the presence of equilibrium indeterminacy poses a difficult question for policy makers, it can give an alternative source of business fluctuations. This idea led to a line of research that focuses on the role of extrinsic uncertainty in macroeconomic models. Using a two-period model of general equilibrium, Cass and Shell (1983) revealed that if some agents cannot participate insurance contracts, extrinsic uncertainty has real effects even in the presence of complete financial markets. Cass and Shell (1983) called extrinsic uncertainty "sunspots."[3] Azariadis (1981) examined a two-period-lived overlapping generations model and found that extrinsic uncertainty, which is called "self-fulfilling prophecies," may generate cyclical behavior of the aggregate economy. Since then, extrinsic uncertainty has also been called "animal

[1] Cass and Shell (1983) distinguished extrinsic uncertainty from intrinsic uncertainty. The former has no effect on the fundamentals of an economy such as preferences and technologies, whereas the latter affects the fundamentals.

[2] "Multiple equilibria" and "equilibrium indeterminacy" are sometimes used as interchangeable terms. Precisely speaking, the presence of multiple equilibria in macrodynamic models is necessary but not sufficient for equilibrium indeterminacy In the literature, if a model economy involves multiple paths under rational expectations (perfect foresight in the case of deterministic environment), then the equilibrium path of the economy is called indeterminate.

[3] As is well known, Jevons (1884) claimed that solar activities could generate business cycles, because they could affect weather condition for agriculture. Hence, as opposed to Cass and Shell (1983), Jevons consided that sunspots represent intrinsic uncertainty that directly affects the agricultural production condition.

spirits," "sentiments," or "market psychology".

Although the sunspot-driven business cycles theory developed in the 1980s made an important theoretical contribution, it had little impact on the empirical research on business cycles. This is because in the two-period lived overlapping generations economy, the length of one period is about 30 years, so that fluctuations in such an environment are not suitable for describing business cycles in the conventional sense. A special issue of the *Journal of Economic Theory* published in 1994 substantially changed the situation. The articles in this issue explored equilibrium indeterminacy in infinite horizon models of growth and business cycles. Among others, Benhabib and Farmer (1994) introduced external increasing returns into an otherwise standard real business cycle model and revealed that there exists a continuum of equilibrium paths that converge to the steady state if the degree of increasing returns is sufficiently strong. Moreover, Farmer and Guo (1994) examined a calibrated version of the Benhabib and Farmer model. They found that if indeterminacy holds, the model economy exhibits empirically plausible fluctuations even in the absence of fundamental technological shocks. The Benhabib-Farmer-Guo line of research attracted a considerable attention and spawned a large body of literature in the last 20 years. The purpose of this review is to elucidate relevant issues discussed in the studies on belief-driven business cycles.[4]

This paper is organized as follows. Section 2 summarizes the main findings of the Benhabib-Farmer-Guo model. Section 3 deals with the studies responding to the critical assessment of the Benhabib-Farmer-Guo approach. Section 4 considers the models in which households' preference structure plays a key role in generating equilibrium indeterminacy. Section 5 discusses some extensions of the base model. Finally. Section 6 gives a brief review over recent studies that intend to find new directions of research.

[4] As for the development of the studies on belief-driven business cycles up to the late 1990s, Benhabib and Farmer (1999) present an extensive survey. See also Chapters7 and 8 in Farmer (2004) for a comprehensive exposition on business cycles with equilibrium indeterminacy.

2 The Benhabib-Farmer-Guo Model

2.1 Model

In Benhabib and Farmer (1994), the objective function of the representative household is a discounted sum of utilities given by

$$U = \int_0^\infty e^{-\rho t} \left(\log C - \frac{N^{1+\gamma}}{1+\gamma} \right) dt, \quad \rho > 0, \quad \gamma > 0,$$

where C is consumption and N denotes hours worked. The flow budget constraint for the household is

$$\dot{K} = rK + wN - C - \delta K, \tag{1}$$

where K is capital stock owned by the household, r is the rate of return to capital, and w is the real wage rate. In addition, δ denotes the deprecation rate of capital. Controlling consumption, C, and labor supply (hours worked), N, the household maximizes U subject to the flow budget constraint and a given initial holding of capital, K_0. In solving this problem, the representative household takes sequences of factor prices, $\{r_t, w_t\}_{t=0}^{\infty}$, as given.[5]

We set up the current value Hamiltonian function in such a way that

$$\mathcal{H} = \log C - \frac{N^{1+\gamma}}{1+\gamma} + q(rk + wN - C - \delta K),$$

where q denotes the price of capital measured in utility. Then, the optimization conditions for the above problem are

$$\max_C \mathcal{H} \Longrightarrow 1/C = q, \tag{2}$$

$$\max_N \mathcal{H} \Longrightarrow N^\gamma = qw, \tag{3}$$

$$\dot{q} = q\left(\rho + \delta - r\right), \tag{4}$$

together with the budget constraint (1) and the transversality condition such that

$$\lim_{t\to\infty} e^{-\rho t} q_t k_t = 0. \tag{5}$$

[5] Besides the seminal works by Kydland and Prescott (1982) and Long and Plosser (1983), the real business cycle approach was popularized by Hansen (1985) and King et al. (1988a). The Benhabib-Farmer-Guo model extends their formulations.

From (2) and (3), we obtain

$$CN^{\gamma} = w. \tag{6}$$

This condition means that the marginal rate of substitution between labor supply and consumption equals the real wage rate. In addition, (2) and (4) present the Euler equation of the optimal consumption:

$$\frac{\dot{C}}{C} = r - \rho - \delta. \tag{7}$$

As for the production side, it is assumed that there is a continuum of identical firms with a unit mass. The production function of the representative firm is

$$Y = XK^{a}N^{1-a}, \quad 0 < \alpha < 1,$$

where TFP of the private technology, X_t, is given by

$$X = A\bar{K}^{\alpha-a}\bar{N}^{\beta-(1-a)}, \quad A > 0, \quad \alpha > a, \quad \beta > 1 - a.$$

Here, $\bar{K}$ and $\bar{N}$ respectively denote aggregate levels of capital and labor in the economy at large. Since the mass of firms is normalized to one, Y, K and N also represent their aggregate values. Hence, the consistency conditions require that

$$\bar{K} = K, \quad \bar{N} = N \quad \text{for all } t \geq 0,$$

implying that the social production function internalizing external effects is

$$Y = AK^{\alpha}N^{\alpha}, \quad \alpha + \beta > 1. \tag{8}$$

As a result, the social technology exhibits increasing returns to scale, and A stands for TFP of the social technology.

When maximizing its profits, an individual firm takes the external effects as given. Hence, the competitive rate of rate of return and the real wage rate are respectively expressed as

$$r = a\frac{Y}{K} = aAK^{\alpha-1}N^{\beta}, \tag{9}$$

$$w = (1-a)\frac{Y}{N} = (1-a)\,AK^{\alpha}N^{\beta-1}. \tag{10}$$

Finally, the equilibrium condition of the final goods is given by

$$Y = C + \dot{K} + \delta K. \tag{11}$$

Note that the competitive equilibrium of this model can be defined by solving the following pseudo planning problem. In this problem, the planner controls C and N to maximize U subject to

$$\dot{K} = AK^{a}N^{1-a}\bar{K}^{\alpha-a}\bar{N}^{\beta-(1-a)} - C - \delta K.$$

When solving the problem, the planner takes the sequences of $\{\bar{K}_t\}_{t=0}^{\infty}$ and $\{\bar{N}_t\}_{t=0}^{\infty}$ as given. It is easy to see that the optimization conditions of the planner's problem, together with the consistency conditions, $\bar{K} = K$ and $\bar{N} = N$, yield (6) and (7).

2.2 Dynamics

Equations (6) and (10) yield

$$CN^{\gamma} = (1-a)\,AK^{\alpha}N^{\beta-1}. \tag{12}$$

The conventional analysis of the equilibrium dynamics of this model is that using (12), we express N as a function of K and C. Then, we substitute this relation into $Y = AK^{\alpha}N^{\beta}$, $r = aAK^{\alpha-1}$, and $w = (1-\alpha)\,AK^{\alpha}N^{\beta-1}$ to derive a complete dynamic system of K and C. After confirming that the steady state values of K and C are uniquely given, we linearize the dynamic system around the steady state and check the signs of characteristic roots of the coefficient matrix. In what follows, we focus on a dynamic system of Y/K and C/K, because it is more convenient for driving the indeterminacy conditions than the conventional method mentioned above. To do this, we rewrite (6) as

$$CN^{\gamma} = (1-a)\frac{Y}{N},$$

which gives

$$N = \left[(1-a)\frac{x}{z}\right]^{\frac{1}{1+\gamma}}.$$

Here, we denote $x = Y/K$ and $z = C/K$. As a result, the aggregate output is written as

$$Y = AK^{\alpha}\left[(1-a)\frac{x}{z}\right]^{\frac{\beta}{1+\beta}}.$$

Noting that $r = aY/K = ax$, the growth rates of capital, consumption and output are respectively given by

$$\frac{\dot{K}}{K} = x - z - \delta, \tag{13}$$

$$\frac{\dot{C}}{C} = ax - \rho - \delta, \tag{14}$$

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + \frac{\beta}{1+\gamma}\left(\frac{\dot{x}}{x} - \frac{\dot{z}}{z}\right). \tag{15}$$

Using (13), (14) and (15), together with $\dot{x}/x = \dot{Y}/Y - \dot{K}/K$ and $\dot{z}/z = \dot{C}/C - \dot{K}/K$, we obtain the following dynamic system of x and z:

$$\frac{\dot{x}}{x} = \frac{1}{1-\phi}\left\{\left[\alpha - 1 + \phi(1-a)\right]x - \left[\alpha - 1 + \phi\right]z + 1 - \alpha)\delta - \phi\rho\right\}, \tag{16}$$

$$\frac{\dot{z}}{z} = (a-1)x + z - \rho, \tag{17}$$

where $\phi + \beta/(1+\gamma).\phi = \frac{\beta}{1+\gamma}$.

In the steady state, K, Y and C stay constant over time. The conditions for $\dot{K} = 0$ and $\dot{z} = 0$ respectively yield

$$x - z - \delta = 0,$$

$$(1-a)x + z - \rho = 0.$$

Thus, the steady state levels of x and z are

$$x^* = \frac{\rho+\delta}{a}, \qquad z^* = \frac{\rho + (1-a)\delta}{a}. \tag{18}$$

2.3 Indeterminacy Conditions

The coefficient matrix of (16) and (17) evaluated at the steady state is

$$J = \begin{bmatrix} x^* \frac{1}{1-\phi} \left[\alpha - 1 + \phi (1-a)\right] & x^*(\alpha - 1 + \phi) \\ z^* (a-1) & z^* \end{bmatrix}.$$

The determinant of this matrix is

$$\det J = x^* z^* \frac{1}{1-\phi} a (\alpha - 1).$$

Since $0 < \alpha < 1$, if $\phi < 1$ so that

$$1 + \gamma > \beta, \tag{19}$$

then the steady state is a saddle point. Hence, there is a linear relation between x and z on the stable saddle path, which is written as $z_t = \bar{m} x_t$, where $\bar{m}$ is a constant. Therefore, on the stable saddle path, it holds that

$$Y_t = A K_t^{\alpha} \left[(1-a)\bar{m}\right]^{\frac{\beta}{1+\gamma}}.$$

Hence, if the initial level of K_0 is given, Y_0 is given, implying that $x_0 = Y_0/K_0$ is determine as well so that the equilibrium path of the economy is determinate.

If the equilibrium path is indeterminate, the initial level of C is indeterminate under a given level of K_0. This requires the steady state of our dynamic system to be a sink so that the coefficient matrix J has two stable roots. The necessary and sufficient conditions for the presence of two stable roots are $\det J > 0$ and trace $J < 0$. The trace of J is given by

$$\begin{aligned} \text{trace } J &= x^* \left[\frac{1}{1-\phi}(\alpha - 1) + \phi (1-a)\right] + z^* \\ &= \frac{\rho + \delta}{a} \left[\frac{1}{1-\phi}(\alpha - 1) + \phi (1-a)\right] + \frac{\rho + (1-a)\delta}{a}. \end{aligned}$$

Consequently, the necessary and sufficient conditions for indeterminacy in terms of model parameters are the following:

$$1 + \gamma < \beta, \tag{20}$$

$$\frac{\rho+\delta}{a}\left[\frac{1+\gamma}{1+\gamma-\beta}(\alpha-1)+\frac{\beta(1-a)}{1+\gamma}\right]+\frac{\rho+(1-a)\delta}{a}<0. \tag{21}$$

The first condition requires that the external effect associated with aggregate labor is high enough. The second condition shows that even if $\beta > 1+\gamma$, indeterminacy may not hold. For example, suppose that the aggregate capital does not yield external effects so that $\alpha = a$. Then if $\beta > 1+\gamma$, the left hand side of the second inequality condition becomes

$$\frac{(\rho+\delta)(1-a)}{a}\left[\frac{\beta}{1+\gamma}-\frac{1+\gamma}{1+\gamma-\beta}\right]+\frac{\rho+(1-a)\delta}{a}>0,$$

which violates (21). Therefore, the presence of equilibrium indeterminacy requires that the aggregate capital should exhibit external effects as well.

2.4 Calibration

Farmer and Guo (1994 and 1995) examine a stochastic, discrete time version of the Benhabib-Farmer model. Using the pseudo-planning formulation, we assume that the planner solves the following problem:

$$\max E_0 \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left[\log C_t-\frac{N_t^{1+\gamma}}{1+\gamma}\right]$$

subject to

$$K_{t+1}=(1-\delta)K_t+A_tK_t^aN_t^{1-a}\bar{K}_t^{\alpha-a}\bar{N}_t^{\beta-(1-a)}-C_t, \tag{22}$$

$$\log A_{t+1}=\theta\log A_t+(1-\theta)\log A^*+\varepsilon_{t+1},\quad 0<\theta<1. \tag{23}$$

Here, equation (23) means that the TFP follows a first-order stochastic difference equation in which ε_{t+1} is a white noise (an exogenous disturbance hitting the TFP in period $t+1$) and A^* is the steady state level of TFP in the deterministic world. When solving the optimization problem, the planner takes the sequences of external effects, $\left\{\bar{K}_t,\bar{N}_t\right\}_{t=0}^{\infty}$, as given.

Using the consistency conditions, $\bar{K}_t = K_t$ and $\bar{N}_t = N_t$, the optimal choice conditions for C_t and N_t give

$$C_tN_t^{\gamma}=(1-a)\frac{Y}{N_t}=A_tK_t^{\alpha}N_t^{\beta-1},$$

which leads to

$$N_t=\left[\frac{1-\alpha}{C_t}A_tK_t^{\alpha}\right]^{\frac{1}{1-\gamma-\beta}}. \tag{24}$$

The Euler equation of the optimal consumption is

$$\frac{1}{C_t} = \frac{1}{1+\rho} E_t \frac{1}{C_{t+1}} \left(a \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right).$$

Substituting (24) into (22) and the Euler equation, we obtain the following:

$$K_{t+1} = A_t K_t^{\alpha} \left[\frac{1-\alpha}{C_t} A_t K_t^{\alpha} \right]^{\frac{\beta}{1-\gamma-\beta}} + (1-\delta) K_t - C_t, \tag{25}$$

$$\frac{1}{C_t} = \frac{1}{1+\rho} E_t \frac{1}{C_{t+1}} \left(K_{t+1}^{\alpha} \left[\frac{1-\alpha}{C_t} A_{t+1} K_{t+1}^{\alpha} \right]^{\frac{\beta-1}{1-\gamma-\beta}} + 1 - \delta \right). \tag{26}$$

Now, let us define $x_t = \log(X_t/X^*)$ $(X_t = K_t, C_t, A_t)$. Then, log-linearizing (25), (26) and (23) at the steady state yields

$$\begin{bmatrix} k_{t+1} \\ E_t c_{t+1} \\ a_{t+1} \end{bmatrix} = J \begin{bmatrix} k_t \\ c_t \\ a_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{t+1} \end{bmatrix}, \tag{27}$$

where

$$J = \begin{bmatrix} \psi_{kk} & \psi_{kc} & \psi_{ka} \\ \psi_{ck} & \psi_{cc} & \psi_{ca} \\ 0 & 0 & \theta \end{bmatrix},$$

and ψ_{ij} $(i, j = c, k)$ denotes coefficient evaluated at the deterministic steady state.

When determinacy holds, the optimal consumption is uniquely related to k_t and a_t on the stable saddle path. Thus, the policy function of c_t approximated around the steady state is expressed as

$$c_t = \lambda_k k_t + \lambda_a a_t, \tag{28}$$

where λ_k and λ_a are undetermined coefficients. Using (28), $E_t c_{t+1}$ in the left hand side of

(27) is written as

$$\begin{aligned} E_t c_{t+1} &= \lambda_k E_t k_{t+1} + \lambda_a E_t a_{t+1} \\ &= \lambda_k(\psi_{kk} k_t + \psi_{kc} c_t + \psi_{ka} a_t) + \lambda_a \theta a_t \\ &= (\psi_{kk}\lambda_k + \psi_{kc}\lambda_k^2) k_t + (\lambda_k \psi_{ka} + \psi_{kc}\lambda_k\lambda_a + \lambda_a\theta)\, a_t. \end{aligned}$$

The second equation in (27) gives

$$E_t c_{t+1} = (\psi_{kk} + \lambda_k k_{kc})\, k_t + (\lambda_a \psi_{kc} + \theta)\, a_t.$$

Hence, we see that the following is established for any k_t and a_t:

$$\begin{aligned} & (\psi_{kk}\lambda_k + \psi_{kc}\lambda_k^2) k_t + (\lambda_k \psi_{ka} + \psi_{kc}\lambda_k\lambda_a + \lambda_a\theta)\, a_t \\ = \; & (\psi_{kk} + \lambda_k k_{kc})\, k_t + (\lambda_a \psi_{kc} + \theta)\, a. \end{aligned}$$

Comparing the coefficients of k_t and a_t in the above identity, we obtain

$$\begin{aligned} \psi_{kk}\lambda_k + \psi_{kc}\lambda_k^2 &= \psi_{kk} + \lambda_k k_{kc}, \\ \lambda_k \psi_{ka} + \psi_{kc}\lambda_k\lambda_a + \lambda_a\theta &= \lambda_a \psi_{kc} + \theta. \end{aligned}$$

Solving these two equations with respect to λ_k and λ_a, we can express λ_k and λ_a in terms of ψ_{ij} $(i, j = k, c, a)$ [6]. Consequently, the dynamic system is summarized by the following stochastic difference equations of k_t and a_t:

$$\begin{aligned} k_{t+1} &= (\psi_{kk} + \psi_{kc}\lambda_k) k_t + (\psi_{ka} + \lambda_a \psi_{ka})\, a_t, \\ a_{t+1} &= \theta a_t + \varepsilon_{t+1}. \end{aligned}$$

Under a given sequence of stochastic disturbance $\{\varepsilon_t\}_{t=0}^{\infty}$, we can conduct numerical simulations and impulse response analysis based on the above set of stochastic difference equations.

On the other hand, if the steady state is a sink and indeterminacy holds, we cannot relate c_t to k_t and a_t in the way as (28). In such a case, we exploit the fact that the rational

[6] There are two solutions of λ_k. We select one that corresponds to the stable saddle path.

expectations hypothesis means that $c_{t+1} - E_t c_{t+1} = \zeta_{t+1}$, where ξ_t is white noise. Therefore, if equilibrium indeterminacy holds, the dynamic system is expressed as

$$\begin{bmatrix} k_{t+1} \\ c_{t+1} \\ a_{t+1} \end{bmatrix} = J \begin{bmatrix} k_t \\ c_t \\ a_t \end{bmatrix} + \begin{bmatrix} 0 \\ \xi_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}. \tag{29}$$

Here, the expectations error, ξ_{t+1}, represents a sunspot shock. In particular, if there is no fundamental shock so that $a_t = 0$ for all $t \geq 0$, then the dynamic system becomes

$$\begin{aligned} k_{t+1} &= \psi_{kk} k_t + \psi_{kc} c_t, \\ c_{t+1} &= \psi_{ck} k_t + \psi_{cc} c_t + \xi_{t+1}. \end{aligned}$$

Without stochastic disturbances, the deterministic steady state $(k^*, c^*) = (0, 0)$ is a sink. In this system, the driving force of business fluctuations is the sunspot shock, ξ_t, alone. Guo and Farmer (1994) confirm that even in this simple case, the calibrated model performs reasonably well as compared with the standard RBC dynamic system summarized by (27).[7]

In fact, Kamihigashi (1997) reveals that the canonical RBC model and the corresponding sunspot model may show observationally equivalent time series data of macroeconomic variables. Kamihigashi's theoretical contribution suggests that as far as performances of calibrated models are concerned, it is difficult to evaluate whether sunspot models can substitute the canonical RBC model. Consequently, the evaluation of the sunspot models focuses on whether or not the indeterminacy conditions shown by (20) and (21) are empirically plausible. We discuss this point in the next section.[8]

[7] Following Farmer and Guo (1994, 1995), several authors conduct quantitative evaluations of sunspot-driven business cycles. Among others, Schmitt-Grohé (1997 and 2000) and Thomas (2004) present detailed evaluations of calibrated models.

[8] As for the theoretical connection between sunspot-driven and the fundamental shock driven business cycle models, Kamihigashi (1996) and Christiano and Harrison (1999) show insightful results.004) present detailed evaluations of calibrated models.

2.5 Intuition

where H_t is the human wealth defined by

$$H_t = \left(\int_t^{\infty} \exp\left(-\int_t^s (r_v - \delta)\, dv \right) w_s N_s ds \right). \tag{30}$$

We re-express (6) as

$$CN^{\gamma} = w = \bar{\Omega} A K^{\alpha} N^{1-a}, \tag{31}$$

where $\bar{\Omega} = \bar{K}^{\alpha-a}\bar{N}^{\beta-(1-a)}$ represents the external effects associated with aggregate capital and labor. Since the marginal utility of consumption is $1/C$, if we fix C, the left hand side of (31) represents the Frisch labor supply curve. On the other hand, the right hand side is the labor demand curve of firms under a given level of external effect, $\bar{\Omega}_t$.

Suppose that a positive sunspot shock makes the households anticipate that their future wage income increases. This enhances the expected present value of human wealth given by (30). Hence, from (**??**) such an income effect will raise the current consumption. In the standard RBC model without production externalities ($\bar{\Omega}_t = 1$), a rise in C_t shifts the Frisch labor supply curve upward, so that the equilibrium level of hours worked decreases. Under a given level of K_t, the decline in N_t depresses Y_t, implying that the investment in period t is lowered. This lowers capital accumulation, thereby declining future income, which contradicts the initial expectation that the future income will rise. This outcome demonstrates that a sunspot shock will not affect the equilibrium of the economy. By contrast, in the presence of production externalities, if firms increase their labor inputs, strategic complementarity among the firms' decisions raises the labor wedge, $\bar{\Omega} = \bar{K}^{\alpha-a}\bar{N}^{\beta-(1-a)}$. Note that if $\beta > 1+\gamma$, a rise in the aggregate hours, N, increases labor wedge, and thus the individual labor demand curve shifts up. If such a shift dominates the shifts of the labor supply curve, then the equilibrium level of hours worked may increase: see Figure 1.

Figure 1

To see the above result more clearly, note that since the number of firms is normalized to unity, the social labor demand curve that internalizes external effects is given by $w = AK^{\alpha}N$. Therefore, as shown by Figure 2 (a), in the standard case of $\beta < 1 + \gamma$, the social labor

demand curve is still downward sloping. On the other hand, if $\beta > 1 + \gamma$, then the social labor demand curve that internalizes externalities is upward sloping and it is stepper than the Frisch labor supply curve: see Figure 2 (b). Suppose that a positive, non-fundamental shock hits the economy and, thus, households anticipate that their real wage will increase. This increases the anticipated value of the human wealth, so that the households raise their current consumption C_t. As a result, the Frisch labor supply curve shifts upward. In the standard case in which $1 + \gamma > \beta$, the equilibrium employment and the current output will decline. Consequently, a higher C_t with a lower Y_t discourages investment of the households. Therefore, the future capital will be lowered, which contradicts the initial anticipation of higher levels of future wages. In contrast, if $\beta > 1 + \gamma$, an upward shift of the Frisch labor supply curve raises the equilibrium levels of employment and output. If such a rise in output is large enough to enhance the current investment despite the increase in C_t, the future capital stock becomes larger and the real wage will actually rise: the initial change in expectations caused by the sunspot shock will be self-fulfilled.

Figures 2 (a), 2 (b)

Using a discrete-time counterpart of the model discussed so far, Wen (2001) presents a more precise argument. In our continuous-time setting, his discussion is as follows. Substituting $w_s = C_s N_s^{\gamma}$ into (30) gives

$$H_t = \left(\int_t^{\infty} \exp\left(- \int_t^s (r_v - \delta)\, dv \right) C_s N_s^{1+\gamma} ds \right).$$

Substituting (**??**) again into the above, we obtain

$$H_t = C_t \int_t^{\infty} e^{-\rho s} N_s^{1+\gamma} ds,$$

which means that the current level of optimal consumption is written as

$$C_t = \frac{\rho K_t}{1 - \rho \int_t^{\infty} e^{-\rho s} N_s^{1+\gamma} ds}. \tag{32}$$

From $C_t = w_t N_t^{-\gamma}$, (32) gives

$$1 - \rho \int_t^\infty e^{-\rho s} N_s^{1+\gamma} ds = \frac{\rho K_t}{w_t N_t^{-\gamma}}. \tag{33}$$

Note that the term$-\int_t^\infty e^{-\rho t} N_s^{\gamma+1} ds$ represents a (subjectively) discounted sum of the disutility of labor (divided by $1+\gamma$).

Suppose that at the outset ($t = 0$), the economy stays at the steady state so that $K_0 = K^*$ and $N_0 = N^*$. Now, let us raise $N_0 > N^*$ and keep $K_t = K^*$. To establish (33), it should hold that

$$1 - \rho \int_0^\infty e^{-\rho t} N_s^{1+\gamma} dt = \frac{\rho K^*}{w_0 N_0^{-\gamma}}. \tag{34}$$

Since the system is stable, N_t must converge to N^*. According to Wen (2001), it can be shown that a higher $N_0\,(> N^*)$ yields a larger $\int_t^\infty e^{-\rho t} N_s^{1+\gamma} ds$, and thus the left hand side of (34) decreases with N_0. Moreover, if $1+\gamma > \beta$, the right hand side of (34) increases with N_0. This means that N_0 cannot diverge from N^* if K_t remains equal to K^*. In other words, the steady state is the only equilibrium under $K_t = K^*$. On the other hand, if $1+\gamma < \beta$, then the right hand side of (34) decreases with N_0, implying that N_0 can diverge from N^* even though K_t remains equal to K^*. Consequently, in the case of $\beta > 1+\gamma$, they may exist multiple paths around the steady state equilibrium.

3 Indeterminacy under Mild Increasing Returns

We have seen that the necessary condition for indeterminacy in the baseline RBC model is $\beta > 1+\gamma$. In our specification of the utility function, $1/\gamma$ represents the (Frisch) elasticity of labor supply with respect to real wage. The conventional estimated range of $1/\gamma$ is from 1.0 to 2.0, meaning that the minimum level of γ is 0.5. Thus, if β exceeds 1.5, the degree of social returns to scale, $\alpha+\beta$, is higher than 1.8 even if the external effects of capital is relatively small. (Remember that the presence of capital externality, i.e. $\alpha > a$, is necessary to generate indeterminacy). If we follow the indivisible labor supply hypothesis given by Hansen (1985) and Rogerson (1988) (so the instantaneous utility function is $u(C,N) = \log C - BN, \quad B > 0$), then $\alpha+\beta$ should be at least higher than 1.4. Since the foregoing studies on returns to scale of aggregate production functions such as Basu and

Fernald (1997) suggested that the aggregate production technology is close to constant or mild increasing returns, the indeterminacy conditions for the baseline RBC model with production externalities are empirically implausible. Researchers of the mainstream RBC theory criticized this point and claimed that the sunspot-driven business cycles are convincing; see, for example, Aiyagari (1995). Such a criticism turned the researchers' attention to the models that exhibit indeterminacy under empirically plausible external effects in production. In what follows, we refer to two ideas.

3.1 Endogenous Capital Utilization

Wen (1998) shows that if capital utilization is associated with convex costs, then the degree of external increasing returns to generate indeterminacy can be reduced. If capital stock is not fully used, the production function of an individual firm is given by

$$Y_t = A\left(s_t K_t\right)^a N_t^{1-\alpha}\left(\bar{s}\bar{K}\right)^{\alpha-a}\bar{N}^{\beta-(1-a)}, \tag{35}$$

where s denotes the capital utilization rate. According to the standard assumption on capital utilization, we assume that the depreciation rate of capital is an increasing, convex function of s. Here, we specify the depreciation function in such a way that

$$\delta = \frac{\delta_0}{1+\theta}s^{1+\theta} = \delta\left(s\right), \qquad \theta > 0.$$

The optimal rate of capital utilization maximizes the net output, $Y - \delta\left(s\right)K$, which is given by

$$s = \left(\frac{a}{\delta_0}Y\right)^{\frac{1}{1+\theta}}. \tag{36}$$

The social production function is obtained by setting $\bar{K} = K$, $\bar{N} = N$, and $\bar{s} = s$, so that the aggregate output, Y, fulfills

$$Y = A\left(sK\right)^{\alpha}N^{\beta} = A\left(\frac{a}{\delta_0}Y\right)^{\frac{\alpha}{1+\theta}}\left(K\right)^{\alpha}N^{\beta}.$$

If we ignore endogenous capital utilization, then $A\left(\frac{a}{\delta_0}Y\right)^{\frac{\alpha}{1+\theta}}$ is considered as the TFP of the social production function. In fact, the reduced form of the social production function is

written as

$$Y = A^{\frac{1+\theta}{1+\theta-a}} \left(\frac{a}{\delta_0}\right)^{\frac{a}{!+\theta-a}} K^{\alpha\frac{1+\theta}{1+\theta-\alpha}} N^{\beta\frac{1+\theta}{1+\theta-\alpha}}, \tag{37}$$

implying that the aggregate return to scale of the reduced form of the social production function is higher than $\alpha + \beta$.

Again, we focus on condition (6) . In this case, the condition, $CN^{\gamma} = w$, expressed as

$$CN^{\gamma} = A^{\frac{1+\theta}{1+\theta-a}} \left(\frac{a}{\delta_0}\right)^{\frac{a}{!+\theta-a}} K^{\alpha\frac{1+\theta}{1+\theta-\alpha}} N^{\beta\frac{1+\theta}{1+\theta-\alpha}-1}.$$

Therefore, the log-linearized labor demand curve is steeper than the log-linearized Frisch labor supply curve, if the following condition is satisfied:

$$1 + \gamma < \frac{1+\theta}{1+\theta-\alpha}\beta.$$

In the case of indivisible labor $(\gamma = 0)$, if $\theta = 0.5$ and $a = 0.35$, then the above condition is satisfies if $\beta > 0.766$. Thus, if $\alpha = 0.4\,(> a = 0.35)$, the return to scale of the social production function $(= \alpha + \beta)$ is higher than 1.2. Therefore, indeterminacy would hold under more plausible parameter values than those in the model with full utilization of capital.

3.2 Two-Sector Economy

Another popular approach that reduces the degree of external effect for generating indeterminacy is to assume that consumption and investment goods are produced by the use of different technologies. Following Benhabib and Farmer (1996), we consider a two-sector economy where one sector produces investment goods and the other sector produces pure consumption goods. The production function of each sector is

$$Y_i = A_i \bar{X}_i K_i^a N_i^{1-a}, \quad 0 < a < 1, \quad i = 1, 2. \tag{38}$$

In the above, we assume that sector 1 produces investment goods, while sector 2 produces pure consumption goods. In (38), $\bar{X}_i$ represents sector-specific externality that is specified as

$$X_i = \bar{Y}_i^{\theta}, \quad 0 < \theta < 1, \quad i = 1, 2,$$

where $\bar{Y}_i$ is the total output of sector i. Note that the production function of each sector has the same form.

The competitive factor prices are given by

$$r = a\frac{Y_1}{K_1} = pa\frac{Y_2}{K_2}, \tag{39}$$

$$w = (1-a)\frac{Y_1}{N_1} = p(1-a)\frac{Y_2}{N_2}. \tag{40}$$

Equations (39) and (40) mean that both sectors hold the same factor intensity as shown by

$$\frac{w}{r} = \frac{1-a}{a}\left(\frac{K_i}{N_i}\right), \quad i = 1, 2,$$

which leads to

$$\frac{K_2}{K_1} = \frac{N_2}{N_1}.$$

Letting $K_2/K_1 = N_2/N_1 = \lambda$ and using the full-employment conditions of capital and labor,

$$K = K_1 + K_2, \quad N = N_1 + N_2,$$

we obtain $K_1 = \lambda K$,. $K_2 = (1-\lambda)K$, $N_1 = \lambda N$, and $N_2 = (1-\lambda)N$. As a consequence, the production function of each sector is expressed as

$$Y_1 = \lambda \bar{X}_1 K^a N^{1-a}, \qquad Y_2 = (1-\lambda)\bar{X}_2 K^a N^{1-a}. \tag{41}$$

Thus, the relative price, p, is expressed as

$$p = \frac{Y_1/K_1}{Y_2/K_2} = \frac{\bar{X}_1}{\bar{X}_2} = \left(\frac{I}{C}\right)^{\theta},$$

meaning that the aggregate income (in terms of the investment good) is given by

$$Y = Y_1 + pY_2 = X_1 K^{\alpha} N^{1-a}.$$

Using $X_1 = Y_1 = I$ and $Y = I + pC$, we obtain

$$I^{1-\theta} + C^{1-\theta} = K^{\alpha} N^{1-a}. \tag{42}$$

This equation expresses the social production possibility frontier between consumption and investment goods.

Again, the household maximizes

$$U = \int_0^{\infty} e^{-\rho t} \left(\log C - \frac{N^{1+\gamma}}{1+\gamma} \right) dt$$

subject to

$$\dot{K} = rK + wN - pC - \delta K.$$

The optimal choice of C and N gives

$$1/C = pq, \tag{43}$$

$$CN^{\gamma} = w/p. \tag{44}$$

The implicit price, q, follows

$$\dot{q} = q\left(\rho + \delta - r\right). \tag{45}$$

Finally, the market equilibrium conditions for both goods are the following:

$$Y_1 = I = \dot{K} + \delta K, \quad Y_2 = C.$$

From (6) and (40), the familiar labor supply and demand relation is now given by

$$CN^{\gamma} = \frac{w}{p} = (1-a)\, C^{\theta} K^{a} N^{-a}. \tag{46}$$

Notice that due to the external effect, the labor demand depends on the level of consumption. Therefore, when C rises, not only the Frisch labor supply curve but also the labor demand

curve shift upward. Condition (46) yields

$$N = (1-a)^{\frac{1}{a+\gamma}} C^{\frac{\theta-1}{a+\gamma}} K^{\frac{a}{a+\gamma}}. \tag{47}$$

To derive a complete dynamic system of K and C, we first rewrite (42) in the following manner:

$$I = \left(K^a N^{1-a} - C^{1-\theta}\right)^{\frac{1}{1-\theta}}.$$

Substituting (47) into the above, we relate I to K and C as follows:

$$I = I(K, C). \tag{48}$$

Thus, capital stock follows

$$\dot{K} = I(K, C) - \delta K. \tag{49}$$

In addition, conditions (43) and (45) give

$$\begin{aligned} \frac{\dot{C}}{C} &= aK^{a-1}N^{1-a} - \rho - \delta - \theta\left(\frac{\dot{C}}{C} - \frac{\dot{I}}{I}\right) \\ &= aK^{a-1}N^{1-a} - \rho - \delta - \theta\left(1 - \frac{I_C C}{I}\right)\frac{\dot{C}}{C} - I_K\dot{K}. \end{aligned}$$

By use of (48), we obtain the following dynamic equation of C:

$$\frac{\dot{C}}{C} = \frac{1}{1+\theta[1-\varepsilon(C,K)]}\left[\left(K^{a-1}N^{1-a} - \rho - \delta\right) - \theta I_K\dot{K}\right], \tag{50}$$

where $\varepsilon(.) = I_K(K,C)\,C/I(K,C)$. To sum up, the dynamic behavior of the two-sector economy is described by (49) and (50).

It is easy to see that if $\theta = 0$, the dynamic system reduces to the baseline one-sector RBC model without external effects. Complexity of the two-sector model with social increasing returns stems from the fact that the social production possibility frontier is nonlinear: it is convex to the origin in (C, I) space, so that the relative price also depends on the scale parameter θ. Analyzing numerical examples of a discrete-time version of this model, Benhabib and Farmer (1996) reveal that when $a = 0.3$ and $\gamma = 0$, the steady state of the above dynamic

system is a sink even though θ is sufficiently low at 0.06, so that the aggregate return to scale, $1+\theta$, is about 1.06.[9]

4 Preference Structure

We have focused on the role of production technologies in discussing equilibrium indeterminacy. In thissection, we turn our attention to the role of preference structure in the indeterminacy issue.

4.1 Nonseprable Utility

As discussed in Section 3, the key conditions for indeterminacy in the one-sector RBC model is that the labor demand curve slopes up and is steeper than the Frisch labor supply curve. This condition ensures that a rise in consumption caused by a positive sunspot shock raises the equilibrium level of hours worked, which supports self-fulfilling expectations. One may conjecture that even though the labor demand curve has a negative slope, the same outcome can arise if the Frisch labor supply curve slopes down and is steeper than the labor demand curve. As Figure 3 shows, in this situation, an upward shift of the Frisch labor supply curve increases the equilibrium level of hours worked.

Figure 3

Obviously, the additive separable utility used so far cannot bring about the situation such as in Figure 2.3. To see when the Frisch labor curve has a negative slope, consider the following general utility function:

$$u = u(C, N).$$

Following the standard setting, we assume that $u_C > 0,\ u_N < 0,\ u_{CC} < 0$, and $u_{NN} < 0$. The

[9] Benhnabib and Farmer (1996) are the first to reveal that in a two-sector model, indeterminacy may arise under a weak degree of external increasing returns. Moreover, Benhabib and Nishimra (1998) demonstrate that equilibrium indeterminacy emerges in a two-sector model with social constant returns, so that the presence of increasing returns is not necessary for indeterminacy. For further studies on indeterminacy in two-sector models, see Harrison (2001, 2003), Guo and Harrison (2001b), and Drugeon and Venditti (2001). In addition to Guo and Harrison (2010), Dufourt et al. (2015) give a detailed analysis of a two-sector RBC model without income effect.

household's optimal choice conditions for C and N are

$$u_C(C,N) = q, \tag{51}$$

$$u_N(C,N) = -wq, \tag{52}$$

where q is the utility price of capital. The Frisch labor supply function is derived under a fixed level of marginal utility. Keeping q constant, the first-order conditions (51) and (52) in the above give

$$\begin{aligned} u_{CC}\frac{dC}{dw} + u_{CN}\frac{dN}{dw} &= 0, \\ u_{NC}\frac{\delta C}{dw} + u_{NN}\frac{dN}{dw} &= -q. \end{aligned}$$

Hence, we find that the slope of the Frisch labor supply is given by

$$\frac{dN}{dw} = \frac{u_{CC}\dfrac{u_N}{w}}{u_{CC}u_{NN} - (u_{CN})^2}. \tag{53}$$

As a result, if the utility function fails to satisfy strict concavity with respect to C and N (so the denominator in the right hand side of (53) is negative, then the Frisch labor supply curve has a negative slope.

Bennett and Farmer (2000) assume that the instantaneous utility function is

$$u(C,L) = \frac{\left(C\exp\left(-\dfrac{N}{1+\gamma}\right)\right)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \quad \gamma > 0.$$

Hence, if $\sigma = 1$, we have $u(C,N) = \log C - N^{1+\gamma}/(1+\gamma)$. Bennett and Farmer (2000) find that the necessary condition for local indeterminacy is

$$\beta - 1 > \frac{\sigma - 1}{\sigma}N^{*^{1+\gamma}} + \gamma,$$

where N^* denotes the steady state level of hours worked. The left hand side of the above inequality represents the slope of log-linearized labor demand curve, while the right hand side expresses the slope of Frisch labor supply curve linearly approximated at the steady state.

Thus, when the external effects of labor are small enough to hold $\beta < 1$, the presence of equilibrium indeterminacy requires that the Frisch labor supply curve has a negative slope. This is possible if $\sigma < 1$, so that the necessary degree of returns to scale that generates indeterminacy can be small in the case of non-separable utility. However, as pointed out by Hintermaier (2003), this outcome holds only if the instantaneous utility function violates the usual concavity assumption.

4.2 The Role of Income Effect

Regarding the intuitive implication of indeterminacy, we have discussed the case in which a positive sunspot shock raises the expected permanent income of the households, which generates an upward shift of the Frisch labor supply curve through an increase in the current consumption. To examine the role of the income effect for generating indeterminacy, it is useful to use the Greenwood-Hercowitz-Huffman (GHH) preferences under which the income (wealth) effect does not exist. Following Greenwood et al. (1988), suppose that the utility function is given by

$$u(C, N) = v(C - \Lambda(N)), \tag{54}$$

where $v(.)$ is a monotonically increasing and strictly concave function, and $\Lambda(N)$ is a monotonically increasing and strictly convex function. It is easy to see that the optimal choice of household with respect to C and N yields

$$\Lambda'(N) = w,$$

implying that the income effect that changes the current level of consumption will not affect labor supply. As a result, the optimal level of hours worked depends on the real wage alone, and it monotonically increases with the real wage rate.

In our baseline model, the GHH preference can be set as

$$u(C, N) = \log\left(C - \frac{N^{1+\gamma}}{1+\gamma}\right). \tag{55}$$

Given this specification, the household's optimization conditions (51) and (52) are respectively

given by

$$\frac{1}{C-\frac{N^{1+\gamma}}{1+\gamma}}=q, \tag{56}$$

$$N^{\gamma}\left(\frac{1}{C-\frac{N^{1+\gamma}}{1+\gamma}}\right)=wq. \tag{57}$$

These conditions yield

$$N^{\gamma}=w. \tag{58}$$

Hence, N is related to Y as $N^{1+\gamma:}=(1-a)\,Y$. This means that from (56) we obtain

$$C=\frac{1}{q}+\frac{(1-\alpha)\,Y}{1+\gamma}. \tag{59}$$

In view of (58), we find

$$N=\left[(1-a)\,AK^{\alpha}\right]^{\frac{1}{1+\gamma-\beta}},$$

meaning that the social production function can be expressed as

$$Y=A^{\frac{2+\gamma}{1+\gamma-\beta}}\,(1-a)^{\frac{1}{1+\gamma-\beta}}\,K^{\frac{\alpha(1+\gamma)}{1+\gamma-\beta}}.$$

In sum, a complete dynamic system is given by the following:

$$\dot{K}=\left(\frac{1-\alpha}{1+\gamma}\right)A^{\frac{2+\gamma}{1+\gamma-\beta}}\,(1-a)^{\frac{1}{1+\gamma-\beta}}\,K^{\frac{\alpha(1+\gamma)}{1+\gamma-\beta}}-\frac{1}{q}-\delta K,$$

$$\dot{q}=q\left[aA^{\frac{2+\gamma}{1+\gamma-\beta}}\,(1-a)^{\frac{1}{1+\gamma-\beta}}\,K^{\frac{\alpha(1+\gamma)}{1+\gamma-\beta}-1}-\rho-\delta\right].$$

Inspecting the above dynamic system, we find that if

$$\frac{\alpha(1+\gamma)}{1+\gamma-\beta}<1, \tag{60}$$

then the steady state is uniquely given. The coefficient matrix of the linearized system is given by

$$J_g=\begin{bmatrix}\alpha\left(\frac{1+\gamma}{1+\gamma-\beta}\frac{Y^*}{K^*}\right) & \frac{1}{q^{*2}}\\ -\alpha\left(\frac{\alpha(1+\gamma)}{1+\gamma-\beta}-1\right)\frac{Y^*}{K^{*2}} & 0\end{bmatrix}.$$

Therefore, if (60) holds, the determinant of the above matrix is negative, so that the equilibrium path is locally determinate around the steady state, even if the degree of external increasing returns is high enough to fulfill $\beta > 1 + \gamma$. It is easy to confirm that the same outcome holds in a more general GHH-type utility given by (54).

To clarify the role of the income effect in the indeterminacy issue, Jaimovich (2008) presents an interesting discussion. In a discrete time setting, Jaimovichi (2008) sets up the following utility function:

$$u\left(C_t, N_t\right) = \frac{\left(C_t - \psi X_t N_t^{1+\gamma}\right)^{1-\sigma}}{1-\sigma}, \quad \psi > 0, \quad \sigma > 0, \tag{61}$$

where X_t follows

$$X_t = C_t^{\chi} X_{t-1}^{1-\chi}, \quad \leq \chi \leq 1.$$

In this formulation, the utility function becomes

$$u\left(C_t, N_t\right) = \begin{cases} \dfrac{[C_t(1-\psi N_t^{1+\gamma})]^{1-\sigma}}{1-\sigma}, & \text{if } \gamma = 1, \\ \dfrac{\left(C_t - \psi \bar{X} N_t^{1+\gamma}\right)^{1-\sigma}}{1-\sigma} & \text{if } \gamma = 0. \end{cases}$$

Namely, (61) covers both the standard non-separable utility as well as the GHH preference structure. Assuming that the social production function is $Y_t = A_t K_t^{\alpha} N_t^{\beta} \quad (\alpha + \beta > 1)$, Jaimovich (2008) seeks the parameter space of $(\alpha + \beta, \gamma)$ that gives rise to equilibrium indeterminacy.

As anticipated, indeterminacy tends to emerge as $\alpha + \beta$ becomes large. However, indeterminacy will not emerge when γ is close to zero for any degree of return to scale between 0 and 2.0. The numerical experiment also shows that indeterminacy does not hold when γ is close to one, there is a minimum level of γ for generating indeterminacy, and the value of $\alpha + \beta \in [0, 2]$ if γ is relatively large. This experiment demonstrates that some level of income effect is necessary for the presence of equilibrium indeterminacy in the one-sector RBC model.

It is to be noted that the above conclusion is valid for one-sector models alone. In fact, Guo and Harrison (2010) reveal that in two-sector models with sector-specific externalities,

indeterminacy emerges under the GHH preference if the external effect associated with the investment good sector is sufficiently large.

4.3 Consumption Externalities

If the consumption behavior of each household is affected by other households' consumption decisions, then there are consumption externalities. This idea has been used in various fields of macroeconomics such as asset pricing, optimal taxation, and long-run economic growth[10]. Some authors have studied whether the consumption external effect can be a source of indeterminacy.

One popular formulations of consumption externalities is to assume that the instantaneous utility function is given by

$$u\left(C,\bar{C},N\right)=\frac{\left(C\bar{C}^{-\theta}\right)^{1-\sigma:}}{1-\sigma}-\frac{N^{1+\gamma}}{1+\gamma},\qquad \sigma>0,\quad <\theta<1. \tag{62}$$

Since the first term on the right hand side of the above can be rewritten as

$$\frac{\left(C\bar{C}^{-\theta}\right)^{1-\sigma:}}{1-\sigma}=\frac{\left[C^{1-\theta}\left(\frac{C}{\bar{C}}\right)^{\theta}\right]}{1-\sigma},$$

the felicity of the household depends on its private consumption as well as on its relative consumption, $C/\bar{C}$. According to the terminology in the literature, if $\partial u/\partial\bar{C}$ is negative (positive), consumers have jealousy (admiration) toward other consumers' consumption. In addition, if the marginal utility of private consumption, $\partial u/\partial C$ increases (decreases) with $\bar{C}$, then consumers' preferences exhibit conformism (anti-conformis). Therefore, in the above specification, if $\sigma>1$ and $\theta>0$, then the preference structure shows jealousy and conformism, which are standard assumptions in macroeconomic studies on the role of consumption externalities. In addition, under our assumption that the mass of households is one, it holds that $\bar{C}=C$ in equilibrium.

It is easy to see that in this popular formulation, the presence of consumption externalities will not yield indeterminacy. Noting that the household takes the sequence of external effects,

[10] A sample includes Abel (1990), Gali (1994) and Turnovsky. and Monteiro (2007).

$\{\bar{C}_t\}_{t=0}^{\infty}$, in deciding its optimal saving-consumption plan, we find that under (62), the optimal condition for C gives $C^{(1-\theta)(1-\sigma)-1} = q$. Therefore, the Euler equation for consumption is given by

$$\frac{\dot{C}}{C} = \frac{1}{\sigma(1-\theta)+\theta}\left[r-\rho-\delta\right],$$

implying that the (social) intertemporal elasticity in consumption is $1/\left[\sigma(1-\theta)+\theta\right]$. Except for the level of intertemporal substitutability of consumption, the optimization conditions are the same as in the standard one-sector model, the dynamic behavior of the economy essentially the same as the case of $\theta = 0$. Therefore, without assuming external increasing returns, the economy never displays indeterminacy.

This outcome suggests that if consumption externalities yield indeterminacy, we should use a complex utility function. To see this clearly, let us consider a more general utility function such as

$$u = u\left(C, \bar{C}, N\right),$$

where $\bar{C}$ is the average consumption in the economy at large. In this case, the first-order conditions for the household are the following:

$$u_c\left(C, \bar{C}, N\right) = q, \tag{63}$$

$$u_N\left(C, \bar{C}, N\right) = -wq. \tag{64}$$

Conditions (63) and (64) yield

$$(u_{CC} + u_{C\bar{C}})\frac{dC}{dw} + u_{CN}\frac{dN}{dw} = 0,$$

$$(u_{NC} + u_{N\bar{C}})\frac{dC}{dw} + u_{NN}\frac{dN}{dw} = -q.$$

As a result, the effect of a change in the real wage on the hours worked is shown by

$$\frac{dN}{dw} = \frac{-\frac{u_{NN}}{w}(u_{CC} + u_{C\bar{C}})}{(u_{CC} + u_{C\bar{C}})u_{NN} - u_{CN}(u_{NC} + u_{N\bar{C}})}. \tag{65}$$

Although the concavity assumption regarding private consumption and labor is satisfied (so

that $u_{CC}u_{NN} - u_{cN}u_{Nc} > 0$), the sign of the right hand of the above can be negative because of the effect of consumption externalities represented by $u_{C\bar{C}}$ and $u_{N\bar{C}}$. In fact, Alonso-Carerra et al. (2009) set up the following utility function :

$$u\left(C, \bar{C}, N\right) = \frac{\left(C\bar{C}^{\psi}\right)^{1-\sigma}\left(1 - N + \mu\bar{C}^{2}\right)^{\theta(1-\sigma)}}{1-\sigma}.$$

These authors examined numerical examples that yield indeterminacy under this utility function, even in the absence of production externalities. In their examples, dN/dw in (65) is negative and the relation between the labor demand and supply functions is as shown in Figure 2.3.

If there are two types of consumption goods, it is rather easy to identify examples in which consumption externality alone generates equilibrium indeterminacy. Chen et al. (2015) examine a two-sector economy where one sector produces pure consumption goods and the other sector produces general goods that can be either consumed or invested. There is no labor-leisure choice ,and thus, the household supplies one unit of labor in each moment. The instantaneous utility function of the representative household is

$$u = u\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right),$$

where C_1 (C_2) denotes consumption of general (pure consumption) goods. In addition, $\bar{C}_i$ $(i = 1, 2)$ represents external effects generated by each good. Here, the representative household solves the following problem:

$$\max \int_0^{\infty} e^{-\rho t} u\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right) dt$$

subject to

$$\dot{K} = rK + w - C_1 - pC_2 - \delta K,$$

where p is the price of the pure consumption good in terms of the general good.

In solving the optimization problem, the household takes the sequences of external effects, $\left\{\bar{C}_{1,t},\ \bar{C}_{2,t}\right\}_{t=0}^{\infty}$, as given. Denoting q as the utility price of capital, the household's

optimization gives the following conditions:

$$u_1\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right) = q, \tag{66}$$

$$u_2\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right) = pq, \tag{67}$$

$$\dot{q} = q\left(\rho + \delta - r\right). \tag{68}$$

Conditions (66) and (67) yield

$$\frac{u_2\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right)}{u_1\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right)} = p, \tag{69}$$

which implies that the private marginal substitution of good 1 for good 2 equals the relative price.

The formulation of production the side is the standard one. There is no production externalities, and each good is produced by the well-behaved, neoclassical production function with constant returns to scale:

$$Y_i = F\left(K_1, N_i\right), \quad i = 1, 2.$$

As usual, the production technology can be expressed as

$$y_i = f_i\left(k_i\right), \quad y_i = Y_i/N_i, \quad k_i = K_i/N_i, \qquad i = 1, 2.$$

Then, the competitive factor prices satisfy

$$r = f_1'\left(k_1\right) = pf_2'\left(k_2\right),$$

$$w = f_1\left(k_1\right) - f_1'\left(k_1\right)k_1 = p\left[f_2\left(k_2\right) - f_2'\left(k_2\right)k_2\right].$$

It is well known that the capital intensity of each sector is a function of p. We also see that

$$\text{sign } k_i'\left(p\right) = \text{sign } \left[k_2\left(p\right) - k_1\left(p\right)\right], \quad i = 1, 2.$$

The market equilibrium conditions in commodity markets are

$$Y_1 = \dot{K} + \delta K + C_1, \; Y_2 = C_2, \tag{70}$$

and the full employment conditions in the factor markets are given by

$$K_1 + K_2 = K, \qquad N_1 + N_2 = 1. \tag{71}$$

Using the full-employment conditions, we see that the supply function of each good is expressed as

$$Y_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} f_1(k_1(p)) = Y^1(K, p),$$

$$Y_2 = \frac{k_1(p) - K}{k_1(p) - k_2(p)} f_2(k_2(p)) = Y^2(K, p).$$

To derive a complete dynamic system, from $Y_2 = C$ in (70) and the supply function given above, we first express C_2 as a function of K and p as follows:

$$C_2 = C^2(K, p).$$

Plugging this into (69), we obtain

$$\frac{u_2\left(C_1, C^2(K,p), C_1, C^2(K,p)\right)}{u_{12}\left(C_1, C^2(K,p), C_1, C^2(K,p)\right)} = p.$$

This equation enables us to express C_1 as a function of K and p. Finally, substituting $C_2 = C^2(K, p)$ into (66) gives

$$u_1\left(C^1(K,p), C^2(K,p), C^1(K,p), C^2(K,p)\right) = q.$$

As a result, we can relate the relative price to K and q in such a way that

$$p = p(K, q).$$

To sum up, we obtain a complete dynamic system with respect to capital and its implicit

price given by the following:

$$\dot{K} = Y^1 \left(K.p\left(K,q\right)\right) - C^1 \left(K, p\left(K,q\right)\right) - \delta K,$$

$$\dot{q} = q\left[\rho + \delta - f_1' \left(k_1 \left(p\left(K,q\right)\right)\right)\right].$$

Chen et al. (2015) specify the utility function in such a way that

$$\begin{aligned} u\left(C_1, C_2, \bar{C}_1, \bar{C}_2\right) &= \frac{\left[\gamma\left(C_1 C_1^{-\theta_1}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(C_2 \bar{C}_2^{-\theta_2}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}{1-\sigma}, \\ \sigma &> 0,\ \varepsilon > 0.\ \ 0 < \gamma < 1. \end{aligned}$$

Namely, each consumption good is associated with commodity-specific external effect, and θ_i denotes the degree of externalities of the social consumption of good i. In addition, ε denotes the elasticity of substitution between the felicity generated by the consumption of goods 1 and 2. The production function of each sector is given by

$$Y_i A_i K_i^{a_i} N_i^{1-a_i}, \quad 0 < a_i < 1, \quad i = 1, 2.$$

The Cobb-Douglas formulation means that the factor intensity of each sector is expressed as

$$k_1\left(p\right) = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}} \left(\frac{1-a_1}{1-a_{2'}}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}},$$

$$k_2\left(p\right) = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}} \left(\frac{1-a_1}{1-a_{2'}}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}}.$$

These expressions show that

$$\text{sign } k_i'\left(p\right) = \text{sign } \left(\alpha_2 - \alpha_1\right), \quad i = 1, 2.$$

Given the above specifications, Chen et al. (2015) analyze a calibrated version of the model and found the parameter spaces in which indeterminacy emerges. They revealed that the indeterminacy conditions critically depend on the magnitudes of θ_i, ε as well as on the factor-intensity raking, that is, sign $(a_1 - a_2)$. They demonstrated that the model can hold

indeterminacy in a wide range of parameter space. In particular, indeterminacy tends to emerge easily if θ_i has a negative value.[11]

5 Related Issues

5.1 News versus Sunspots

Ever since the contribution of Beaudry and Portier (2004), news shocks have been considered to be useful driving forces of business cycles. The news-shock theory is based on an old idea in dynamic macroeconomics: if a future shock is anticipated, it may affect current decisions of households and firms. Since this theory considers that changes in expectations of agents generate business fluctuations, it is often called the expectations-driven business cycle theory. Although both the news-driven and sunspot-driven business cycle theories rely on the same idea that changes in expectations bring about economic fluctuations, there are two distinctive differences between these two approaches. First, the news-shock theory assumes that equilibrium is determinate. Second, news may or may not materialize: unlike the sunspot-shock theory, expectations in the news-shock models may not be self-fulfilled.

Intuitively speaking, if a positive technological shock in the future is anticipated, the current consumption, investment, hours worked, and output all rise. However, as pointed out by Beaudry and Portier (2004), the baseline RBC model with a separable utility function cannot produce the comovement of key macroeconomic variables. In the baseline model, the labor market condition is depicted by Figure1. When a future technological innovation is anticipated by the households, their expected permanent income will rise, which brings about an upward shift of the Frisch labor supply curve. Under a given level of capital stock, such a shift lowers the current levels of hours worked, output, and investment. To resolve this comovement puzzle, subsequent studies extended the baseline RBC model by introducing additional factors such as multi-sector settings, adjustment costs of investment, habit formation, and generalized preference structure with non-separability between consumption and labor.

While most subsequent investigations mentioned above assume that there is no market

[11] See also Chen and Hsu (2007), Chen et al. (2013) and Weder (2000) for further exploration on indeterminacy generated by the presence of consumption externalities.

distortion, Eusepi (2009) proposes an alternative resolution of the comovement puzzle by introducing external increasing returns into the base model. As shown in Section 2.3, if labor externality is sufficiently large to hold $\beta > 1 + \gamma$, then the labor demand curve is positively sloped and is steeper than the Frisch labor supply curve. In this situation, a rise in the current consumption caused by a positive news shock shifts the labor supply curve upward, so that hours worked, consumption and output simultaneously increase.[12] Eusepi (2009) confirms that investment also rises, implying that a positive news shock produces positive comovement of consumption, hours, output, and investment. Based on this finding, Eusepi (2008) claims that there is a strong connection between the news-driven and the sunspot-driven business cycles at least in the baseline one-sector RBC model.

Guo et al. (2012), however, point out that while the presence of strong increasing returns brings about comovement of key macroeconomic variables, a positive news shock reduces those variables when the households receive the signal about the good news. This is because if the labor demand curve is steeper than the labor supply curve, an upward shift of the labor demand caused by a positive technological shock lowers the equilibrium levels of real wage and employment (see Figure 4). Therefore, in the presence of strong increasing returns, when a positive shock materializes, it yields a negative impact on the households' income. Since the households anticipate this fact, they reduce rather than increase their current labor supply when they receive the signal. Consequently, the current levels of hours worked, output, and investment decline as well. This contradicts the empirical fact of business booms generated by good news about future technology. Guo et al. (2012) confirm this outcome by analyzing impulse responses of the calibrated model of Eusepi (2009).

Figure 4

In sum, the foregoing research has suggested that there is no direct theoretical connection between the news-driven and sunspot-driven business cycle theories.[13]

[12] Remember that $\beta > 1 + \gamma$ is necessary but not sufficient for indeterminacy. Eusepi (2009) implicitly assumed that determinacy holds even under $\beta > 1 + \gamma$.

[13] Regarding the news-driven business cycles, Beaudry and Portier (2014) provide a detailed and updated survey of this topic.

5.2 Local versus Global Indeterminacy

Our discussion on equilibrium intermediacy so far has focused on local analysis. Some authors have revealed that even if the steady state of the Benhabib-Farmer model exhibits local saddle-point property (so that determinacy holds near the steady state), there may exist stable cycles around the steady state. For example, Coury and Wen (2009) re-examine a discrete time version of the Benhabib-Farmer model in which the production function is specified as

$$Y = AK^a N^{1-a} \left(\bar{K}^a \bar{N}^{1-a}\right)^{1+\theta}, \qquad \theta > 0.$$

In this specification, the social production function is

$$Y = AK^{a(1+\theta)} N^{(1-a)(1+\theta)}.$$

Therefore, a higher θ enhances the external effects of aggregate capital and labor simultaneously.

In the absence of stochastic disturbance, the linearized deterministic dynamic system is written as

$$\begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} = M \begin{bmatrix} k_t \\ c_t \end{bmatrix},$$

where $k_t = \log(K_t/K^*)$, $c_t = \log(C_t/C^*)$, and M denotes 2×2 coefficient matrix. Coury and Wen (2009) first confirm that there is a critical level of $\theta = \hat{\theta}$: if $\theta < \hat{\theta}$, one characteristic root of M is in the unit circle, while the absolute value of the other root is higher than one. This means that the steady state is a saddle point and local determinacy holds. If $\theta > \theta^*$, then both characteristic roots are within the unit circle, implying that the steady state is a sink, thereby local indeterminacy emerges. They also show that if $\theta = \theta^*$, the absolute value of one characteristic root equals one, while the other root has an absolute value less than one. In this critical case, a flip bifurcation arises and there may exist a stable two-period cycle around the steady state. Since the presence of a stable cycle can also hold when θ is less than but close to θ^*, even if the steady state exhibits local determinacy, the economy can be on the stable cycle rather than the stable saddle path converging to the steady state. In this sense, local determinacy does not necessarily exclude the possibility of global indeterminacy.

In a similar vein, Guo and Lansing (2002) introduce factor income taxation into the above model. The flow budget constraint under income taxation is

$$K_{t+1} = (1+\tau_k))r_t K_t + (1+\tau_w)\, w_t N_t + (1-\delta)\, K_t - C_t + T_t,$$

where τ_k is the rate of tax on capital income, τ_w the rate of tax on wages, and T_t denotes a lump-sum transfer from the government. The factor prices are determined by

$$r_t = a\frac{Y_t}{K_y}, \quad w_t = (1-a)\,\frac{Y_t}{N_t}.$$

Assuming that $\tau_k = \tau_w = \tau$, Guo and Lansing (2002) focus on the relation between τ and equilibrium indeterminacy. They show that if $\tau < 0$, the steady state is a sink even if the degree of increasing returns, $1+\theta$, is small. Then, as τ rises, there is a critical level of $\hat{\tau}$ under which a flip bifurcation emerges. After τ exceeds that critical level, the steady state becomes a saddle point. Then there is another critical value of τ at which the absolute values of both roots equal to one: at this point, a Hopf bifurcation emerges, and the steady state turns from a sink to a source (i.e., total instability of the dynamic system). Again, stable cycles exist around the steady state if τ is less than but close to this second critical level.

6 Recent Development

In this section, we refer to the recent development of macroeconomic models with equilibrium indeterminacy. The global financial crisis of 2007-2008 forced macroeconomists to rethink about their analytical frameworks. The mainstream dynamic stochastic general equilibrium (DSGE) approach was severely criticized by practitioners and policy makers because it failed to offer useful policy recommendations for the financial crisis as well as for the prolonged slumps in many countries after the crisis. In the search for new directions in macroeconomic analysis, there is renewed interest in macroeconomic models with equilibrium indeterminacy. In what follows, we pick out a notable sample of the recent studies in the field.

6.1 Demand Constraint Equilibria

In a series of studies, Roger Farmer claims that the New Keynesian models, that is, DSGE models with sticky prices, fail to capture the core idea of Keynes (1936); see Farmer (2008a, 2008b, 2010, 2012, 2013). In order to provide a microfoundation to Keynes's theory of unemployment equilibrium, he adds labor market frictions to an otherwise standard general equilibrium model of macroeconomy. Unlike the conventional macroeconomic models with search frictions in labor markets, in Farmer's model, the wage is not settled by a Nash bargaining. Instead, the wage is determined competitively in the process of random matching between workers and firms. However, in the presence of labor market frictions, the competitive wage cannot be determined by the usual labor market equilibrium condition, that is, the equality between the marginal rate of substitution of consumption for labor and the marginal productivity of labor. As a consequence, the economy involves a continuum of steady state equilibria, so that the steady state levels of real wage and unemployment of labor become indeterminate as well.

To close the model, Farmer introduces the "belief function" that relates households' expected wealth to a specific steady state equilibrium. (Since the steady state constitutes a continuum, every belief can be self-fulfilled, and, hence, rationality of expectations still holds.) In this model, the households' consumption demand depends on their beliefs (animal spirits), which determine the steady state levels of real wage and unemployment. Therefore, the main outcome of the model analysis is close to Keynes's business cycle theory in which animal spirits of entrepreneurs play a key role in determining the level of effective demand (Keynes 1936, Chapter 22).

In a different context, Benhabib et al. (2015) also examine the demand constrained equilibria of a macroeconomy. In their model, each firm produces a different consumption good in monopolistically competitive markets. Firms should produce before the demand for their products materializes. The firms determine their production plans based on the information about demand conditions, but the signal they receive contains aggregate as well as idiosyncratic noises. On the other hand, households should decide their consumption demand before their income materializes. They expect their income based on their sentiments and their decisions are sent to the firms as noisy signals.

In the foregoing studies on equilibrium indeterminacy, including Farmer's theory just mentioned above, it is demonstrated that there may exist (infinitely) many fundamental equilibria and the selection of a specific equilibrium is affected by extrinsic uncertainty. In contrast, the model of Benhabib et al. (2015) involves a unique fundamental equilibrium. However, in the presence of imperfect information, there may also exist "sentiments-driven" equilibria in addition to the fundamental equilibrium, and the selection of a particular equilibrium depends on the households' sentiments (animal spirits). The modelling strategy of Benhabib et al. (2015) is closely related to a contribution of Angeletos and La'O (2013). These authors empathize that agents hold heterogeneous expectations under imperfect information. They show that in the presence of expectations heterogeneity, correlated shocks to the agents' higher-order beliefs may yield sentiment-driven business fluctuations.

6.2 Financial Frictions and Bubbles

The general equilibrium models with financial constraints initiated by Kiyotaki and Moore (1997) and others have regained research interest after the financial crisis of 2007-2008. Some authors examine the role of financial frictions for generating equilibrium indeterminacy. Harrison and Weder (2013) introduce financial constraints into a real business cycle model with external increasing returns. In their model, the production function includes land as well as labor and capital. Firms should pay for wages before production takes place, and they borrow their payments from household by using the value of land they hold as collateral. Harrison and Weder (2013) show that if the borrowing constraint is effective, equilibrium indeterminacy may arise under an empirically plausible degree of increasing returns.

In a similar vein, Benhabib and Wang (2013) examine a model in which intermediate goods are produced in monopolistically competitive markets. The final good firms must pay for intermediate goods in advance, and they borrow the payments from the households. It is also assumed that each financial transaction is associated with a fixed borrowing cost. In this situation, markups determined by the intermediate good firms are affected by the final good firms' borrowing constraints. Since the presence of the fixed borrowing cost generates nonconvexity, it plays the same role as external increasing returns in the model of Benhabib and Farmer (1994). The authors confirm that equilibrium indeterminacy may emerge even though the final good production technology satisfies constant returns to scale.

Liu and Wang (2014) also explore the relation between financial constraints and equilibrium indeterminacy in a different setting. In their model, there is a continuum of firms, each of whom has a different productivity. Each firm borrows its advance payments for production factors under a financial constraint in which the value of the firm (the value of stocks) acts as collateral. Given this setting, the presence of borrowing constraints determines the cutoff level of firm productivity, and firms that have productivity higher than the cutoff level participate in production activities. As a consequence, the financial constraints affect the total productivity of final goods. Liu and Wang (2014) reveal that equilibrium indeterminacy arises even though the production technology is not associated with external increasing returns.

Miao and Wang (2012), on the other hand, focus on bubbles in the presence of financial constraints. Like Liu and Wang (2014), in their model, firms face financial constraints for their investment and the firm value acts as a collateral. The authors first confirm that the bubble-less equilibrium is uniquely determined. However, there may also exist a continuum of bubbly equilibria. If the economy stays on a bubbly equilibrium path, the burst of bubbles caused by changes in agents' expectations yields an abrupt downturn of economic activities. This model, therefore, provides us with a possible theoretical exposition about the huge negative impact on the real side of the US economy caused by the collapse of the housing bubbles in 2007.

6.3 Decentralized Markets

If market transactions are decentralized, the matching technology of search activities of agents is relevant in characterizing the equilibrium conditions. Early studies on labor market dynamics such as Diamond (1981), Diamond and Fudenberg (1989), and Howitt and McAfee (1994) reveal that external effects associated with matching technology easily bring about equilibrium indeterminacy. In addition, Mortensen (1999) shows that a search-matching model of labor market yields global indeterminacy if the production technology exhibits increasing returns. The source of indeterminacy in these search-matching models is similar to the non-convexity of social production technology emphasized by Benhabib and Farmer (1994). The Diamond-Mortensen-Pssarides modelling of labor market frictions has been incorporated into the DSGE models. While the majority of the these studies focus on the case of equilibrium

determinacy, some authors explore the possibility of sunspot-driven business cycles generated by labor market frictions. For example, Hashimzade and Ortigueira (2005) confirm that the presence of external effects in search activities generates sunspot-driven business cycles even in the absence of external increasing returns of production technology. Furthermore, using a model without capital and investment, Kraus and Lubick (2010) demonstrate that indeterminacy may hold even though both production and matching technologies exhibit constant returns to scale.

More recently, Kaplan and Menzio (2016) and Dong et al. (2016) present new insights on business fluctuation caused by search frictions. Kaplan and Menzio (2016) consider search frictions in commodity markets and find that equilibrium indeterminacy is generated by "shopping externalities.", In their model. there are two kinds of consumption goods: one is traded in a competitive, centralized market, while the other is traded in decentralized markets. Therefore, transactions are decentralized not only in the labor markets but also in a part of final goods markets. The key idea of Kaplan and Menzio (2016) is that unemployed workers have higher income and spend less time to search commodities with lower prices than unemployed workers. In this situation, if a positive sunspot shock make firms increase employment of labor, then the number of workers with higher income increases, which brings about larger aggregate demand and higher commodity prices. As a result, the optimistic anticipation of firms are self-fulfilled, which yields multiple equilibria even in the absence of increasing returns in production and matching technologies. Kaplan and Menzio (2016) give a detailed discussion of the global dynamics of the model economy. They find that the dynamic system involves multiple steady states and that there is a continuum of equilibrium paths converging to each steady state. Consequently, the analytical properties of their model resembles these of the growth models with multiple steady states discussed in Sections 2 and 3 in Chapter 4. The authors claim that their model is consistent with the empirical findings about the difference in shopping behaviors between employed and unemployed workers.

Dong et al. (2016) introduce search frictions into credit markets. These authors assume that there are search frictions in transactions between depositors (households) and banks as well as in transactions between borrowers (firms) and banks. Given this setting, business booms enhance search activities, which raises the matching probabilities of financial transactions. This mechanism generates increasing returns of the aggregate production technology.

As a result, although production technology of individual firm and matching technologies in credit markets satisfy constant returns, the equilibrium path of the economy becomes indeterminate.

6.4 Agent Heterogeneity

In recent times, introducing heterogeneous agents into the business cycle models is an active research topic in macroeconomics. In an economy with heterogeneous households and firms, the distribution of income and wealth among households as well as firm size distribution may affect the behavior of the aggregate economy. Hence, the determinacy/indeterminacy conditions would be affected by the pattern of distribution of these variables. Based on this idea, some authors explore the relation between agents' heterogeneity and equilibrium indeterminacy.

For example, Ghiglino and Sorger (2002) introduce two types of households into Benhabib and Farmer's (1994) model of real business cycles with external increasing returns. Both types of households are assumed to have identical preference and labor efficiency but different initial wealth holdings. Since the separable utility function used in the Benhabib and Farmer model does not hold homotheticity, the behavior of the aggregate economy is not independent of wealth distribution among the households. The authors confirm that the conditions under which indeterminacy emerges is affected by the initial distribution of wealth, so that heterogeneity matters for the equilibrium determinacy/indeterminacy conditions. In a similar vein, Ghiglino and Olzal-Duquenne (2005) and Ghiglino and Venditti (2008) treat a two-sector Ramsey model with production externalities. Again, there are two types of households. In addition to their initial wealth holdings, the households' labor efficiencies are different from each other. The authors inspect the relation between equilibrium indeterminacy and the degree of heterogeneity of households. They show that depending on parameter magnitudes concerning preference structures, a higher heterogeneity may or may not enhance the possibility of emergence of indeterminacy.

Mino and Nakamoto (2012), on the other hand, explore the effects of consumption externalities in the presence of heterogenous households. In their model, there are two groups of household, and. each household's felicity is affected by the intragroup consumption externalities as well as by the intergroup consumption externalities. Mino and Nakamoto (2012)

demonstrate that if the degree of intergroup externalities are sufficiently high, that is, each household's consumption behavior is strongly affected by the consumption of households in other group, then the presence of consumption externalities yield equilibrium indeterminacy[14].

Finally. it is worth emphasizing that heterogeneity of agents also plays a key role in models with financial and search frictions as well as in sentiment-driven business cycles models explored by Angeletos and La'O (2013) and Behanbib et al. (2005). Therefore, the recent development in macroeconomic models with equilibrium indeterminacy has shifted its main concern from the representative agent settings to the heterogeneous agent settings.

7 Conclusion

It still remains to be seen whether the recent development cited in the previous section can present effective policy recommendations for long-stagnated economies such as Japan. However, it is fair to say that the research on growth and business cycle models with equilibrium indeterminacy continues serving as an attractive alternative when the standard macroeconomic models with equilibrium determinacy fail to provide us convincing explanations for relevant macroeconomic phenomena.

In this paper, we have focused on real business cycle models with equilibrium indeterminacy. Over last two decades, the research on growth and business cycle models with equilibrium indeterminacy investigate a wide range of issues that have not been discussed in this review. A sample includes the role of stabilization policies, indeterminacy in monetary economies, indeterminacy in endogenous growth models as well as indeterminacy in open economies. Mino (2017) deals with these issues in detail.

[14] Mino and Nakamoto (2016) examine a more general model in which there is a continuum of households, each of whom has different degree of conformism. They, however, do not consider the indeterminacy issue.

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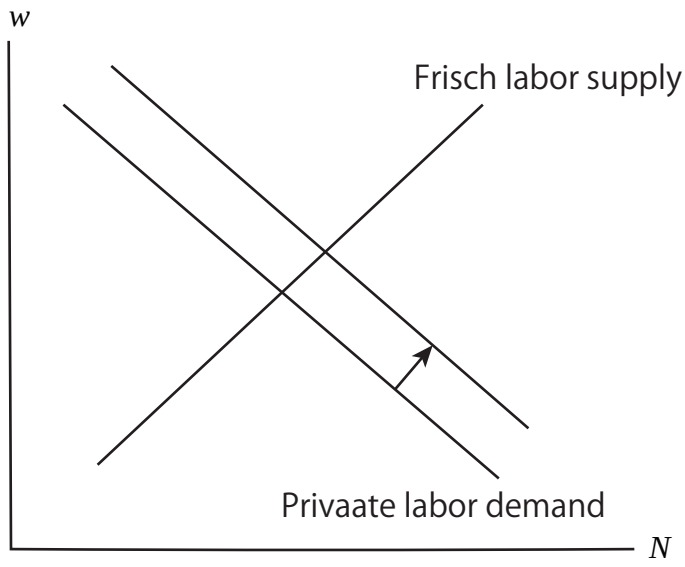

Figure 1 A shift of the private labor demand curve

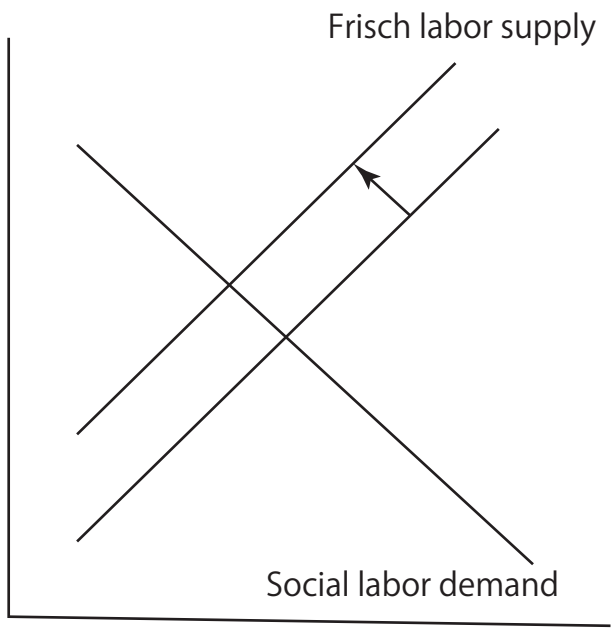

Figure 2 (a) The case of $1+\gamma > \beta$

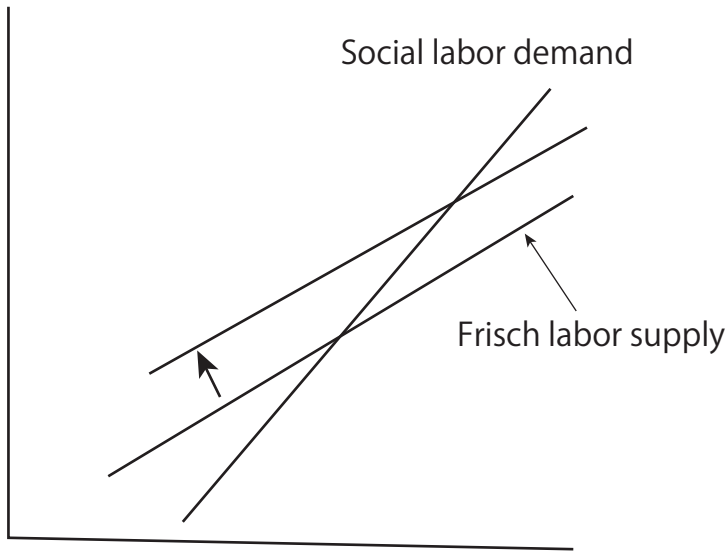

Figure 2 (b) The case of $\beta > 1+\gamma$

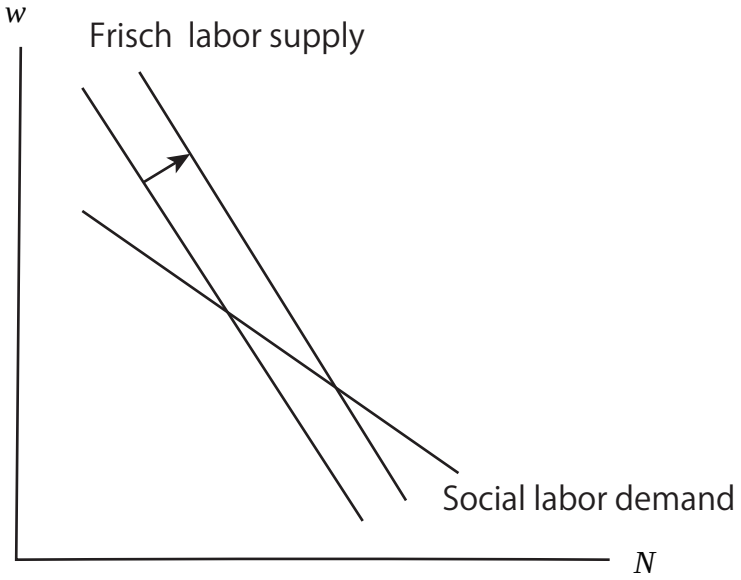

Figure 3 A shift of the Frisch labor supply curve

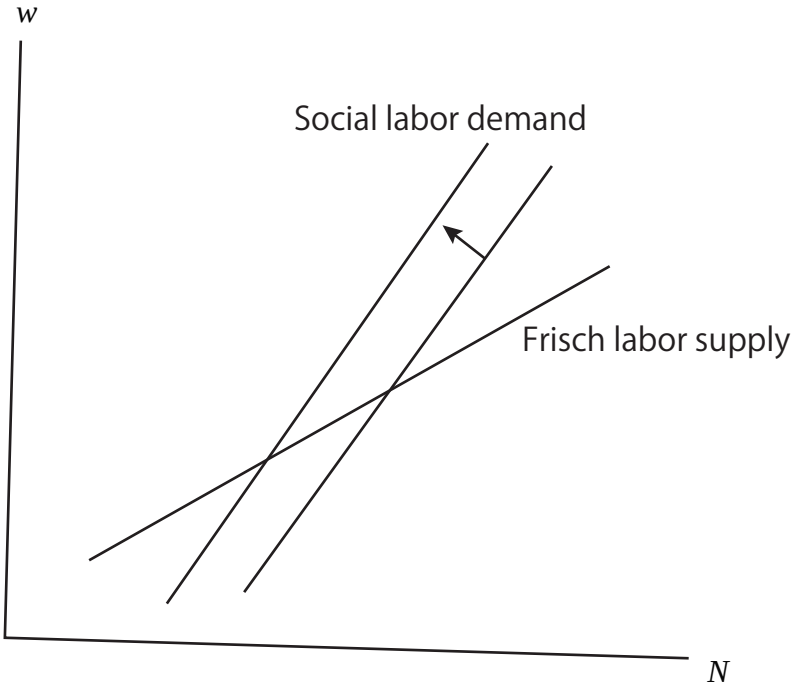

Figure 4 The effect of a positive TFP shock