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"Incomplete Contract and Verifiability"

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Abstract

While the theory of incomplete contracts has contributed greatly to our understanding many topics such as the nature and financial structure of the firm, its rigorous foundation has been debated. Maskin and Tirole (1999) show that the usual "observable but not verifiable" assumption is not sufficient for the incomplete contract to be optimal, provided that parties can commit themselves not to renegotiate. We show that the assumption is not necessary, either. In sequential bargaining where parties can write a contract contingent on (ex post) verifiable variables, an equilibrium contract turns out to be a null contract (the ex post Nash bargaining solution). A key to our result is endogenous revealing of private information during contract negotiations. The possibility of renegotiations is irrelevant.

JEL classification: C72; C78; D82
Keywords: incomplete contract; ex post Nash bargaining solution; information revealing; sequential bargaining; verifiability

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1 Introduction

The theory of incomplete contracts is now an important field of economic theory, and has contributed significantly to the investigation of the nature of the firm and organizational governances.[1] "This approach has been useful for understanding topics such as the meaning of ownership and the nature and financial structure of the firm" (Hart and Moore 1999). Nevertheless, since its emergence the theory has been criticized for the lack of rigorous foundations. The criticisms are made as follows. While almost all economists agree that real contracts are incomplete due to so-called "transaction costs" often described as unforeseen contingencies, costs of writing contracts and cost of enforcing contracts, there is no well-accepted formal model of incomplete contracts involving transaction costs. Putting it simply, there is no clear definition of incomplete contracts. Few attempts were made to explain the restrictions to a simple class of incomplete contracting in the optimal contracting paradigm.

Maskin and Tirole (1999) show that transaction costs, specifically the indescribability of states of nature, are irrelevant to optimal contracting, provided that agents can forecast their possible future payoffs. In other words, they argue that the usual "observable but not verifiable" assumption is not sufficient for the incomplete contract to be optimal. Maskin and Tirole's criticism is rebutted by Hart and Moore (1999) on the ground that agents are assumed to be able to commit themselves not to renegotiate the contract. Segal (1999) show that as the environment (the number of future trading opportunities) becomes complex, the optimal contract approximately converges to the null contract (the incomplete contract in the extreme form) in a hold-up model with contract renegotiation. Che and Hausch (1998) show a similar result under cooperative investments. From a methodological point of view, one of the lessons we can learn from the debate is that the inability to commit not to renegotiate is crucial to the justification of incomplete contracts, rather than

[1] Tirole (1999) provides an excellent review on the topic.

transaction costs.

The aim of this note is to discuss some implications of a non-cooperative sequential bargaining game with incomplete information for the debate regarding incomplete contracts. For clarity of exposition, the discussion is based on a simple example of negotiations under uncertainty. For a general treatment, see Okada (2016).

We consider the following situation. Two agents negotiate for contracts contingent on ex post verifiable variables at the interim stage when they possess private information. Unlike the standard principal-agent model, the bargaining protocol is not the ultimatum one, but the Rubinstein (1982)-type sequential bargaining model with time discounting. A proposer is randomly selected. In the limit that the common discount factor for future payoffs goes to one, it is shown that a stationary sequential equilibrium of the bargaining game necessarily converges to the ex post Nash bargaining solution under an equilibrium refinement based on self-selection. The ex post Nash bargaining solution is the Nash bargaining solution that is defined ex post, that is, after all contractable variables become verifiable. Equivalently, it is the null contract ex ante. While agents can make an (ex ante) contract contingent on verifiable variables, the equilibrium describes that agents contract trading ex post. Agents' bargaining power is determined by their probability of making proposals. This result implies that the usual "observable but not verifiable" assumption is not necessary for the incomplete contract to be justified as an equilibrium one. Even when all contractable variables are ex post verifiable, agents make the null contract in equilibrium. Since agents can negotiate about ex post efficient contracts, our result is irrelevant to whether or not agents can commit themselves not to renegotiate contracts.

The key property to our result is endogenous revealing of private information during contract negotiations. Agents' private information may affect their preferences over contracts. To reach a preferable agreement, agents may want to reveal or conceal their types. Private information may leak through

actions in negotiations. Our equilibrium refinement is introduced to capture the possibility of endogenous information revealing. It assumes that, if a responder is offered an unexpected contract, then he infers that a true type of a proposer must be among those who have incentives to propose such a contract, and that he updates his prior belief about the proposer's type based on the revealed information. Our refinement is similar to the notion of a perfect sequential equilibrium of Grossman and Perry (1986) in the literature of signalling games.

The remainder of the note is organized as follows. Section 2 presents a model of contract bargaining with incomplete information. Section 3 obtains the result. Section 4 discusses some implications of the result related to the theory of incomplete contracts.

2 The Model

For clarity of exposition, our analysis is restricted to a simple example of resource allocations with two possible states. A general formulation is given in Okada (2016). Consider two agents 1 and 2 with two private types, $T_1 = \{t_1, t_1'\}$ and $T_2 = \{t_2, t_2'\}$, respectively. They know only their own types at the time of negotiations. The prior belief π of players is given by the uniform distribution on $T = T_1 \times T_2$. The feasible set $U(t)$ of the bargaining problem depends on a type profile $t \in T$ of players, and it is given by

$$\begin{aligned} U(t_1, t_2) = U(t_1', t_2') &= \{(x_1, x_2) \in R_+^2 |\ x_1 + 2x_2 \leq 1\} \\ U(t_1', t_2) = U(t_1, t_2') &= \{(x_1, x_2) \in R_+^2 |\ 2x_1 + x_2 \leq 1\}. \end{aligned}$$

The four possible feasible sets are illustrated in Figure 1. In Figure 1, U_{12} denotes the feasible set $U(t_1, t_2)$ where agent 1 is of type t_1 and agent 2 is of

type t_2. Other notations of feasible sets can be interpreted similarly.[2]

In the model, two agents negotiate for a division of a fixed amount of money at the interim stage when they possess private information. The value of money to agents depends on a state (type profile) ex post verifiable. If negotiations fail, then the agents receive zero payoffs, regardless of their types. An important assumption in our analysis is that the agents' types become publicly known and verifiable ex post. The agents can negotiate for contracts of allocations, not for single allocations. According to the complete contract theory, a contract is an allocation plan contingent on ex post verifiable variables, that is, profiles of agents' types. For every type profile $t = (t_1, t_2)$, a contract x assigns a payoff allocation $x(t) \in U(t)$.

Real world examples of negotiations with incomplete information and verifiable types are abundant. Usually, many contracts include clauses that adjust contractual details depending on uncertain events in future. Below are some examples.

1. A domestic firm and a foreign firm negotiate for trading contracts with clauses that trading prices may be adjusted by exchange rates.

2. A manager and workers negotiate for wage contracts contingent on tax reform in future, for example, whether consumption tax or corporate tax is increased.

3. An automobile company and a part producing maker negotiate for contracts that may adjust trading prices, depending on car selling prices and production costs of parts.

[2]The example is briefly discussed in Okada (2016). We employ the type formulation according to the traditional approach of Harsanyi (1968). Equivalently, the model can be given in a state-space formulation where there are four states $\omega^1 = (t_1, t_2)$, $\omega^2 = (t_1, t_2')$, $\omega^3 = (t_1', t_2)$ and $\omega^4 = (t_1', t_2')$. Two states ω^1 and ω^4 have the same feasible set, and two states ω^2 and ω^3 do so. Agents have only imperfect information on a true state. For example, when the true state is ω^1, agent 1 knows only that it is either ω^1 or ω^2. Their common prior is given by the uniform distribution on the state space.

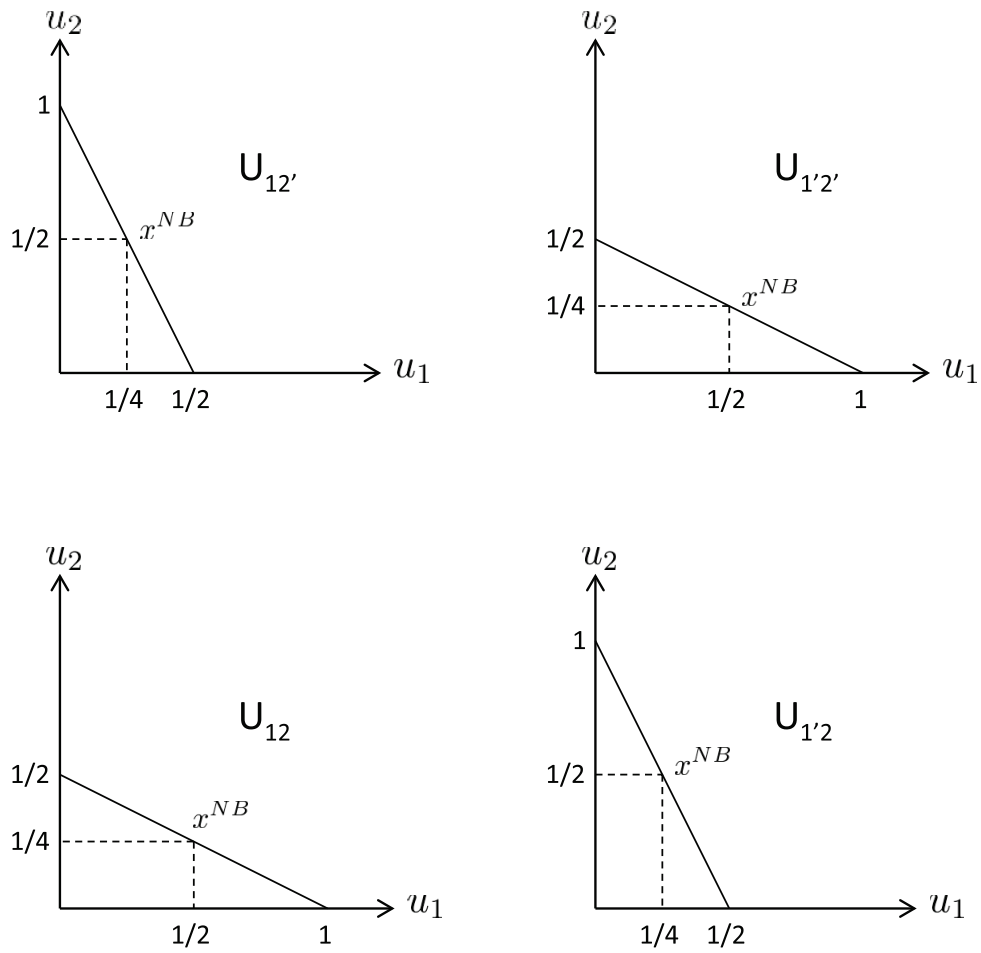

Figure 1 Two-person Bayesian bargaining problem

The null contract is a special (extremely incomplete) contract where two agents agree to a payoff allocation ex post, that is, after their private types become publicly known and verifiable. In the following, we consider the ex post Nash bargaining solution where agents agree to the Nash bargaining solution, given their types. For simplicity of exposition, we assume that agents' bargaining powers are identical. Concretely, the ex post Nash bargaining solution x^{NB} is given by

$$x^{NB}(t_1, t_2) = x^{NB}(t_1', t_2') = (\frac{1}{2}, \frac{1}{4}),\ x^{NB}(t_1', t_2) = x^{NB}(t_1, t_2') = (\frac{1}{4}, \frac{1}{2}). \quad (1)$$

For the general case that agents have different bargaining powers, see Okada (2016). In the following, the ex post Nash bargaining solution is simply referred to as the null contract.

Since agents can negotiate for contracts at the interim stage when they possess private information, their suitable preferences for contracts should be measured by the interim expected utility, that is, the conditional expected utility given a private type. For the null contract, every agent's interim expected utility for his every type is given by

$$\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4} = \frac{3}{8}.$$

It is easy to show that the null contract is not efficient in terms of the interim expected utility. Consider another example of a contract, denoted by x^E, satisfying

$$x^E(t_1, t_2) = x^E(t_1', t_2') = (1, 0),\ x^E(t_1', t_2) = x^E(t_1, t_2') = (0, 1). \quad (2)$$

For every agent, the interim expected utility of x^E for his every type is given by 1/2 which is larger than 3/8.

We shall show that, when agents are sufficiently patient[3], two agents agree

[3]Equivalently, the stopping probability of negotiations after rejection is very small. A

to the null contract in equilibrium, and moreover that the null contract is a unique equilibrium agreement under a version of refinement of self-selection employed in the literature of signalling games.

We consider a Rubinstein-type (1982) sequential bargaining game for contracts. After agents' types are realized and revealed to them privately, an agent is selected with equal probability as a proposer and proposes a contract to the other agent. If the opponent accepts it, then the proposed contract is agreed, and it will be implemented at the ex post stage where players' types become publicly known. Otherwise, negotiations may continue in the next round under the same rule. When negotiation does not stop, agents receive zero payoffs. In negotiations, agents know perfectly a history of the game except the opponent's type. It is assumed that agents discount future payoffs by the common discount factor $\delta < 1$

A (pure) *strategy* σ_i for agent i in the sequential bargaining game is defined in a usual manner. It is a function that assigns a choice to each of his possible moves, depending on the information he receives. A *belief* system for agents is a function μ that assigns every agent i to his every information set a belief about the other agent's type, a probability distribution on T_j. Let $\sigma = (\sigma_1, \sigma_2)$ be a strategy profile for agents. In what follows, We employ a weakly sequential equilibrium for the bargaining game. Roughly, a pair (σ, μ) of a strategy profile and a belief system is a weakly sequential equilibrium if the strategy of every agent is a best response to the other's strategy in the part of the game that follows each of his information sets under the belief system μ, where μ is consistent with the strategy profile σ by Bayes' rule on equilibrium path. A weakly sequential equilibrium allows an arbitrary belief off equilibrium play (Osborne 2004). In what follows, we simply call it an equilibrium. Our refinement concept introduces a self-selection property to agents' belief off-equilibrium path, taking the viewpoint that an unexpected proposal may

standard principal-agent model employs the ultimatum rule that negotiations stop with probability one when an agent rejects a proposal from a principal.

reveal a proposer's incentive to screen himself.

3 Result

Proposition 1. Let $\delta < 1$ be the discount factor for future payoffs. For every δ, there exists an equilibrium (σ, μ) of the bargaining game such that for every $i = 1, 2$, agent i proposes the same contract $x^{i,\delta}$, independent of his type, and it is accepted with probability one. The equilibrium contract $x^{i,\delta}$ converges to the null contract (the ex post Nash bargaining solution) in the limit that δ goes to one, regardless of a proposer.

Proof. First we construct an equilibrium contract $x^{1,\delta}$ proposed by agent 1 for every δ. Let x^{NB} be the null contract in (1). Recall that the Pareto frontiers of the feasible sets $U(t_1, t_2)$ and $U(t_1', t_2')$ are given by $x_1 + 2x_2 = 1$ where $x_1, x_2 \geq 0$, and those of $U(t_1', t_2)$ and $U(t_1, t_2')$ by $2x_1 + x_2 = 1$ where $x_1, x_2 \geq 0$.

Let $i = 1$. For a type profile $t = (t_1, t_2), (t_1', t_2')$, consider the following equations

$$y + 2\delta\ x_2^{NB}(t) = 1 \text{ and } x_2^{NB}(t) = \frac{1}{4}.$$

This solves $y = \frac{2-\delta}{2}$. Define

$$x^{1,\delta}(t_1, t_2) = x^{1,\delta}(t_1', t_2') = (\frac{2-\delta}{2}, \frac{\delta}{4}). \tag{3}$$

Similarly, for a type profile $t = (t_1', t_2), (t_1, t_2')$, consider the following equations

$$2y + \delta\ x_2^{NB}(t) = 1 \text{ and } x_2^{NB}(t) = \frac{1}{2}.$$

This solves $y = \frac{2-\delta}{4}$. Define

$$x^{1,\delta}(t_1', t_2) = x^{1,\delta}(t_1, t_2') = (\frac{2-\delta}{4}, \frac{\delta}{2}). \tag{4}$$

The equilibrium contract $x^{1,\delta}$ are defined by (3) and (4).

By permutation, the equilibrium contract proposed by agent 2 is given by

$$x^{2,\delta}(t_1, t_2) = x^{2,\delta}(t_1', t_2') = (\frac{\delta}{2}, \frac{2-\delta}{4}), \tag{5}$$

and

$$x^{2,\delta}(t_1', t_2) = x^{2,\delta}(t_1, t_2') = (\frac{\delta}{4}, \frac{2-\delta}{2}). \tag{6}$$

It is clear that the equilibrium contracts for both agents converge to the null contract in the limit that δ goes to one.

Let $i, j = 1, 2$ $(i \neq j)$. An equilibrium strategy $\sigma = (\sigma_1, \sigma_2)$ is such that (i) agent i proposes $x^{i,\delta}$ and j accepts it, independent of their types and a history of play, and (ii) every type t_j of agent j responds optimally to a proposal by agent i under the belief constructed below. The tie-breaking rule is employed so that agent j accepts it when he is indifferent to a response. A belief μ is given as follows:

(a) when every type t_j of j responds to the equilibrium contract $x^{i,\delta}$ in the first round, he has the same belief as the prior one, that is, the uniform distribution on types t_i of agent i,

(b) when type t_j responds to any contract $y \neq x^{i,\delta}$ in the first round, he has the posterior belief that a true type of agent i must be in the set $T_i^+ = \{t_i' \in T_i | y_i(t_i', t_j) > x_i^{i,\delta}(t_i', t_j)\}$ where T_i^+ is a non-empty set,[4] and

(c) after the first round, the same rules as (a) and (b) are applied to a responder's belief where his prior belief is possibly updated according to a game play in previous rounds.

Property (a) simply means that the belief of agents are consistent with their equilibrium strategies according to Bayes' rule. Property (b) defines every agent' belief off equilibrium path such that he believes that a true type of

[4] If T_i^+ is an empty set, the responder's posterior belief receiving an unexpected contract can be arbitrary.

the opponent must be among those who are better off by a non-equilibrium proposal than the equilibrium contract, given his own true type. Recall that the notion of a weakly sequential equilibrium makes no restriction about agents' belief off equilibrium path.

First, we show that the strategy $\sigma = (\sigma_1, \sigma_2)$ defined above prescribes an optimal proposal for every type t_i of every agent i. Without loss of generality, let $i = 1$ and suppose that type t_1 deviates from σ_1 and proposes another contract y satisfying that there exists some t_2 such that $y_1(t_1, t_2) > x^{1,\delta}(t_1, t_2) = \frac{2-\delta}{2}$. By (b), type t_2 believes that the true type of agent 1 must be in the set $T_1^+ = \{t_1' \in T_1 | y_1(t_1', t_2) > x_1^{1,\delta}(t_1', t_2)\}$. Since the payoff vector $x_1^{1,\delta}(t_1', t_2)$ is Pareto efficient in $U(t_1', t_2)$, it holds that $y_2(t_1', t_2) < x_2^{1,\delta}(t_1', t_2)$ for every $t_1' \in T_1^+$. Thus, type t_2 optimally rejects y. Type t_1 never obtains a payoff higher than $x_1^{1,\delta}(t_1, t_2)$ for any possible type t_2 by proposing y. Thus, it is optimal for t_1 to propose $x^{1,\delta}$. Since the arguments above do not depend on an initial belief of proposer 1, it can be applied not only to the first round but also to other rounds in which the proposer's belief may be updated by a history of play.

Second, we show that it is optimal for every type t_i of every agent i to accept the equilibrium contract $x^{j,\delta}$. Without loss of generality, consider $i = 1$ and $t_i = t_1$. If he accepts $x^{2,\delta}$, he receives

$$\frac{1}{2}\{\frac{\delta}{2} + \frac{\delta}{4}\} = \frac{3}{8}\delta.$$

If type t_1 of agent 1 rejects $x^{2,\delta}$, he receives the same discounted payoff $\frac{3}{8}\delta$. Recall that $\frac{3}{8}$ is the conditional expected payoff on equilibrium path. Therefore, it is optimal for type t_1 of every agent 1 to accept the equilibrium contract $x^{2,\delta}$. By definition, the equilibrium strategy prescribes the optimal response for every type t_1 of every agent 1 off equilibrium path. Q.E.D.

The proposition is a special case of a general result proved by Okada (2016, Theorem 4.1). The intuition for it can be explained as follows. For simplicity,

suppose that agents are sufficiently patient. Suppose that type t_1 of agent 1 proposes the interim efficient contract x^E in (2). Receiving this unexpected proposal, type t_2 of agent 2 believes that a true type of agent 1 must be of type t_1, given his own type, knowing that only type t_1 has the incentive to do so. Under this belief, type t_2 receives zero payoff worse than the null contract, and he rejects it. A similar argument can be applied to type t_2'. Thus, even if the interim efficient contract x^E is proposed by type t_1 of agent 1, it is rejected by all types of agent 2. The same arguments can be applied to all contracts except the null contract.

The equilibrium constructed in Proposition 1 has the following three properties. The first property is that agents' behavior is independent of past actions.

Definition 1. An equilibrium (σ, μ) is *stationary* if every proposer i's behavior in every round depends only on his type t_i, and (ii) every responder's behavior depends only on his type and a proposal.

The second property is that of self-selection. It is formally defined as follows.

Definition 2.(Okada, 2016) Let (σ, μ) be a stationary equilibrium in which every agent $i = 1, 2$ proposes a contract x_i (independent of his type). An equilibrium (σ, μ) is said to satisfy *self-selection* if, when every type $t_j \in T_j$ of responder $j(\neq i)$ receives a proposal y_i from player i satisfying that the set

$$T_i^+ = \{t_i \in T_i | y_i(t) > x_i(t) \text{ for } t = (t_i, t_j)\}$$

is non-empty, the belief system μ assigns to type t_j of responder j a posterior belief of which support is equal to T_i^+. If T_i^+ is an empty set, then no restriction on the belief system is imposed.

The self-selection is a version of equilibrium refinement employed in the literature of signaling games. See Grossman and Perry (1986) for a related concept. It means that if a responder receives an unexpected proposal, then he believes that a true type of a proposer must be among those who have the incentive to make such a proposal, given his own type.

The last one is that every type of an agent responds to an equilibrium proposal and a non-equilibrium proposal in the same way if they prescribe the same payoff allocations, given his own type, in every contingency for the other agent's type. Every type of an agent makes the same responses to two contracts if *he knows* that they are identical. Every agent type's response to a proposal is independent of the allocations it assigns to his other (irrelevant) types.

Let M be the set of all feasible contracts. Also, for every strategy profile σ, let $M(\sigma)$ be the set of all contracts proposed in σ. For each $t_i \in T_i$, $T(t_i)$ denotes the cylinder set $\{t_i\} \times T_j (j \neq i)$.

Definition 3.(Okada, 2016) A stationary equilibrium (σ, μ) is said to satisfy *independence of irrelevant types* (IIT) if, for every $i = 1, 2$, $t_i \in T_i$, $x \in M(\sigma)$, and $y \in M$,

$$x = y \text{ on } \{t_i\} \times T_j \text{ implies } \sigma_i(t_i, x) = \sigma_i(t_i, y),$$

where $\sigma_i(t_i, x)$ and $\sigma_i(t_i, y)$ are the responses of agent i to x and y, respectively, prescribed by σ_i when his type is t_i.

The reason that the equilibrium constructed in Proposition 1 satisfies IIT can be explained as follows. We shall show that, regardless of his belief, it is optimal for every agent $i = 1, 2$ to accept any non-equilibrium proposal as long as it offers the same payoffs to him as the equilibrium proposal, given his true type. To do this, it is sufficient for us to show the same thing, given *every* type profile $t = (t_1, t_2)$. With no loss of generality, let $i = 1$. For a type profile

$t = (t_1, t_2)$, agent 1's equilibrium proposal $x^{1,\delta}$ satisfies by (3)

$$x^{1,\delta}(t_1, t_2) = (\frac{2-\delta}{2}, \frac{\delta}{4})$$

and agent 2's equilibrium proposal $x^{2,\delta}$ does by (5)

$$x^{2,\delta}(t_1, t_2) = (\frac{\delta}{2}, \frac{2-\delta}{4}).$$

In equilibrium, those contracts are accepted and thus the expected payoffs of the agents are equal to the Nash bargaining solution $(\frac{1}{2}, \frac{1}{4})$ in the feasible set $U(t_1, t_2)$. If agent 1 deviates from the equilibrium and rejects agent 2's proposal, then negotiations continue in the next round. Since the equilibrium is stationary, the discounted expected payoff of agent 1 is $\frac{\delta}{2}$. This means that agent 1 is indifferent to a response to agent 2's non-equilibrium proposal y as long as it offers the same payoff $\frac{\delta}{2}$ to him as the equilibrium proposal $x^{2,\delta}$. The payoffs proposed to his other types are irrelevant to his response. Since the equilibrium employs the tie-breaking rule that agent 1 accepts y when he is indifferent to a response, the equilibrium in Proposition 1 satisfies IIT.

In the last part of the paper, we shall show that an equilibrium satisfying the three properties above necessarily results in the null contract. We need two general lemmas proved by Okada (2016).

Lemma 1. If a stationary equilibrium (σ, μ) satisfies IIT, then every agent's proposal is accepted in the initial round with probability one.

The intuition for the lemma can be explained as follows. Let $v(t)$ be the expected equilibrium payoffs for agents, given a type profile t. Recall that agents are randomly selected as a proposer. Since the equilibrium is stationary, $v(t)$ is independent of a history in negotiations. Suppose that some type t_i^* of agent i, say $i = 1$, makes an unacceptable proposal x in the initial round. Then, the type set of agent 2 is divided into two subsets T_2^a and T_2^r, where T_2^a

and T_2^r are the type subsets of agent 2 who accept and reject x, respectively. Type t_1^* of agent 1 can construct and propose a new contract y such that (i) both agents are strictly better off in y than in $\delta v(t)$ for any type profile t in $T_1 \times T_2^r$, and (ii) x and y are identical on $T_1 \times T_2^a$. Property (i) implies that all rejection types T_2^r of agent 2 accept y, regardless of their beliefs about agent 1's type. A contract y satisfying property (i) can be constructed since the continuation payoff vector $\delta v(t)$ is in the interior of the feasible set $U(t)$ for all $t \in T$. Property (ii) means that all acceptance types in T_2^a know that x and y prescribe the same outcomes. Thus, IIT implies that they respond to x and y in the same manner, that is, they accept y. Since all types of agent 2 accept y, type t_1^* of agent 1 is better off if he proposes the non-equilibrium contract y. This is a contradiction.

Lemma 2. (Okada 2016) For any stationary equilibrium (σ, μ) satisfying IIT, there exists some stationary equilibrium (σ', μ') of Γ that satisfies IIT and the following properties:

(i) (σ, μ) and (σ', μ') are *outcome-equivalent*; that is, both equilibria generate the same outcomes for every type profile $t \in T$.

(ii) In (σ', μ'), all types of every agent $i = 1, 2$ propose the same contract $x_i^* \in M$. The other agent accepts it, independent of his type.

This lemma was first shown by Myerson (1983) in a principal-agent framework, and was called the *principle of inscrutability.* The lemma implies that we can restrict our analysis to a pooling equilibrium where all types of a proposer choose the same contracts. The lemma holds in a general set-up of a sequential bargaining game with respect to a non-stationary sequential equilibrium without IIT. The intuition for the result is as follows. When different types of the proposer propose different contracts, they can compose a single contract that assigns payoff allocations over the type space according to original different

contracts. This kind of operation can be done since a contract is a function from the type space of agents to the feasible set of payoff allocations.

With help of the two lemmas, we shall prove the necessity of the null-contract under three assumptions. The result is proved in a general framework by Okada (2016). In the following, we present an elementary proof of it for the simple model in this note.

Proposition 2. For every discount factor δ, every agent $i = 1, 2$ proposes a contract x_i^δ, independent of his type, in a stationary equilibrium (σ, μ) satisfying IIT and self-selection if and only if x_1^δ and x_2^δ are given by (3) and (4), and (5) and (6), respectively. These contracts converge to the null contract (the ex post Nash bargaining solution) as δ goes to one.

Proof. It suffices us to prove the "only-if" part. Let (σ, μ) be any stationary equilibrium satisfying IIT and self-selection. For every type profile $t \in T$ and every agent $i = 1, 2$, let $v_i(t)$ be the expected payoff for agent i, and let $x^{i,\delta}(t)$ be a payoff allocation prescribed by a contract $x^{i,\delta}$ proposed by agent i. It follows from Lemma 2 that agent i proposes the same contract $x^{i,\delta}$, independent of his type, and is accepted with probability one. The proof is done in the three steps.

Step 1. We show that for every $i, j = 1, 2 (i \neq j)$ and every type profile $t \in T$, it holds that $x_j^{i,\delta}(t) = \delta v_j(t)$.

Since every type of agent i proposes the same contract in equilibrium, responder j never receives additional information, and so he does not update the prior belief π (the uniform distribution). Let t_j be any type of agent j. Since type t_j accepts $x^{i,\delta}$ by Lemma 1, it must hold that

$$\sum_{t_i \in T_i} \frac{1}{2} x_j^{i,\delta}(t) \geq \sum_{t_i \in T_i} \frac{1}{2} \delta v_j(t). \tag{7}$$

Suppose that $x_j^{i,\delta}(s) > \delta v_j(s)$ for some $s \in T$ with $s_j = t_j$. Then, there

exists a payoff allocation $a \in U(s)$ such that $a_i > x_i^{i,\delta}(s)$ and $x_j^{i,\delta}(s) > a_j > \delta v_j(s)$ by making a slight "payoff transfer" between i and j at $x^{i,\delta}(s)$. Consider the contract y^i that assigns the payoff allocation a to s and coincides with $x^{i,\delta}$ for all other type profiles. Then y^i satisfies

$$y_i^i(s) > x_i^{i,\delta}(s) \tag{8}$$

$$y_i^i(t) = x_i^{i,\delta}(t) \text{ for every } t \neq s, \tag{9}$$

$$y_j^i(s) > \delta v_j(s). \tag{10}$$

Since (σ, μ) satisfies self-selection, it follows from (8) and (9) that type s_j of agent j believes that the true type of agent i must be s_i, if type s_i of agent i proposes y_i. By (10), type s_j optimally accepts y^i. For all other types of j, y^i prescribes the same allocations as $x^{i,\delta}$. Thus, IIT requires that they should respond to y^i in the same way as to $x^{i,\delta}$. That is, they accept the proposal. Since all types of j accept y^i, (8) implies that type s_i of agent i is better off by proposing y^i in (σ, μ) than $x^{i,\delta}$. This is a contradiction. Therefore, $x_j^{i,\delta}(t) \leq \delta v_j(t)$ for every $s \in T$ with $s_j = t_j$. Then, It follows from (7) that $x_j^{i,\delta}(t) = \delta v_j(t)$ for every $s \in T$ with $s_j = t_j$. Since t_j is arbitrary, Step 1 holds.

Step 2. We show that for every agent $i = 1, 2$ and every type profile $t \in T$, it holds that $x^{i,\delta}(t)$ is Pareto efficient in $U(t)$.

Suppose that $x^{i,\delta}(s)$ is not Pareto efficient in $U(s)$ for some $s \in T$. Then there exists some $a \in U(s)$ such that $a_i > x^{i,\delta}(s)$ and $a_j > x_j^{i,\delta}(s) = \delta v_j(s)$. The last equality comes from Step 1. Similarly to the proof of Step 1, consider the contract y^i that assigns the payoff allocation a to s and coincides with $x^{i,\delta}$ for all other type profiles. Then, y^i satisfies $y_j^i(s) = a_j > \delta v_j(s)$ and (8) and (9). By the same arguments as in Step 1, if type s_i of agent i proposes y^i, then all types of agent j accept it, and thus type s_i is better off than in (σ, μ). This is a contradiction.

Step 3. We show that for every type profile $t \in T$, the expected equilibrium

payoff $v(t)$ is equal to the ex post Nash bargaining solution in (1), and that the contracts $x^{i,\delta}$ proposed by agent $i = 1, 2$ are given by (3)-(6).

It follows from Step 1 that for every type profile $t \in T$, $x^{1,\delta}(t) = (x_1^{1,\delta}(t), \delta v_2(t))$, and $x^{2,\delta}(t) = (\delta v_1(t), x_2^{2,\delta}(t))$, and also that $v(t)$ is Pareto efficient in $U(t)$. Since agents 1 and 2 are selected as proposers with equal probability, $v(t)$ is the mid-point between $x^{1,\delta}(t)$ and $x^{2,\delta}(t)$. Thus,

$$2v_1(t) = x_1^{1,\delta}(t) + \delta v_1(t), \quad 2v_2(t) = x_2^{2,\delta}(t) + \delta v_2(t). \tag{11}$$

Then, it holds that

$$\frac{x_2^{2,\delta}(t) - \delta v_2(t)}{x_1^{1,\delta}(t) - \delta v_1(t)} = \frac{v_2(t)}{v_1(t)}. \tag{12}$$

The left-hand side of (12) is equal to the absolute value of the slope of the Pareto frontier line of $U(t)$. (12) is the well-known formula which shows that $v(t)$ is equal to the Nash bargaining solution of the feasible set $U(t)$ with the disagreement point $(0, 0)$. Thus, $v(t)$ is equal to the ex post Nash bargaining solution in (1). Also, it follows from (11) that the contracts $x^{i,\delta}$ proposed by agents $i = 1, 2$ satisfy (3)-(6). The last part of the proposition holds from (11). Q.E.D.

4 Discussion

We have illustrated a simple example of bilateral negotiations with incomplete information where two agents have their own private information on contractable (ex post verifiable) variables at the time of negotiations. We have shown that an equilibrium contract becomes necessarily the null contract (the ex post Nash bargaining solution) in the limit that the agents are sufficiently patient, even when all contractable variables are verifiable. This implies that the usual "observable but unverifiable" assumption is not a necessary condition for the incomplete contract to appear in equilibrium. We shall discuss the result critically, clarifying some assumptions underlining it.

First, the result never means that the null contract prevails in every case of contract negotiations with incomplete information. A critical assumption in the model is that if the agents exchange their private information truthfully, then they can obtain perfect information on the uncertain events. Putting it formally, the common refinement of the agents' information partitions on the state space is the most refined partition where every information set consists of a single state. This is certainly a special case. In a general case that there remains uncertain events even after informational exchange, the agents may agree to some interim efficient contract. For example, suppose that agents do not know neither his own type nor the other's. In such a case, the agents may agree to the interim (and ex ante, also) efficient contract x^E in (2) where the most productive agent exploits the total resource.

Second, the result holds under two assumptions on agents' belief off equilibrium play. The equilibrium refinement of self-selection means that if a responder receives an unexpected proposal, he believes that a true type of a proposer must be among those who have the incentive to do so, given his type. Although similar notions of self-selection are often employed in the literature of signalling games, there is no well-accepted consistency of belief off equilibrium play in the equilibrium refinement theory. Empirical investigations are needed for us to understand how actual agents update their belief. Note that we employ a much weaker condition than Definition 2 in the proof of Proposition 2. From a viewpoint of a responder, a non-equilibrium contract makes only the true type of a proposer better off than an equilibrium contract, and assigns the same payoffs to all other types of the proposer, regardless of his type, as the equilibrium contract. Proposed such an unexpected contract, the responder can easily infer a true type of the proposer. IIT condition means that every type of an agent responds to an equilibrium proposal and a non-equilibrium proposal in the same way if they prescribe the same payoff allocations, given his own type, and that his response is independent of offers to his other types. Generally, this condition restricts a responder's belief in the way that such a

response is optimal. We, however, remark that there is no restriction on belief in the equilibrium (constructed in Proposition 1) supporting the ex post Nash bargaining solution since every type of a responder is indifferent to a response.

Finally, the possibility of renegotiations is one of key issues in the debate on the theory of incomplete contracts. It has been shown that whether agents can commit themselves not to renegotiate makes a crucial difference in the structure of the optimal contract. Assuming dynamic programming rationality, Maskin and Tirole (1999) show that, even when transaction costs (unforeseen contingencies) exist, the outcome of the optimal complete contracting can be implemented by designing an appropriate mechanism which requires agents to report possible future payoffs, provided that the agents can commit themselves not to renegotiate. Segal (1999) and Hart and Moore (1999) show that the optimal contract approximately converges to the null contract as the environment becomes complex, if renegotiations are possible. In contrast to these works, the possibility of renegotiations is not relevant to our result since agents propose ex post efficient contracts.

In sum, this note shows that, due to information revealing during negotiations, the null contract can be in equilibrium even when all contractable variables are verifiable.

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