Power packet transferability via symbol propagation matrix

Shinya Nawata*, Atsuto Maki† and Takashi Hikihara*

Abstract

A power packet is a unit of electric power composed of a power pulse and an information tag. In Shannon's information theory, messages are represented by symbol sequences in a digitized manner. Referring to this formulation, we define symbols in power packetization as a minimum unit of power transferred by a tagged pulse. Here, power is digitized and quantized. In this paper, we consider packetized power in networks for a finite duration, giving symbols and their energies to the networks. A network structure is defined using a graph whose nodes represent routers, sources and destinations. First, we introduce the concept of a symbol propagation matrix (SPM) in which symbols are transferred at links during unit times. Packetized power is described as a network flow in a spatio-temporal structure. Then, we study the problem of selecting an SPM in terms of transferability, that is, the possibility to represent given energies at sources and destinations during the finite duration. To select an SPM, we consider a network flow problem of packetized power. The problem is formulated as an M-convex submodular flow problem which is a solvable generalization of the minimum cost flow problem. Finally, through examples, we verify that this formulation provides reasonable packetized power.

Keywords

power packet, router, network flow problem, electrical energy network

1 Introduction

Electric power has been considered as a continuous flow based on circuit theory, in which power flow is governed by Kirchhoff Laws and Tellegen's theorem [1]. The circuit theory can be generalized to represent various nonlinear complex systems in the system topology with energy dissipation and energy storage as a network thermodynamics [2]. Here, energy flow is handled in a continuous manner under the conservation of energy. On the other hand, it is shown in Shannon's information theory [3] that "all technical communications are essentially digital; more precisely, that all technical communications are equivalent to the generation, transmission and reception, of random binary digits" [4]. Communication networks have been developed in a digitized manner by using packet switching, which breaks messages into smaller pieces named "packets", for dynamic assignment of network resources [5]. If we handle electric power in a digitized manner, power distribution will be changed completely different from the conventional. In this paper, we consider electrical energy networks in which power is digitized and quantized through power packetization [6–15].

The concept of power packet was proposed in the 1990s to manage complicated power flows in power systems caused by various power transactions after deregulation [16]. In the proposal, electric energy routers, which include energy storage devices, were installed into the electrical energy networks.

*Department of Electrical Engineering, Kyoto University, Katsura, Nishikyo, Kyoto 615-8510, Japan

†School of Electrical Engineering and Computer Science, Royal Institute of Technology (KTH), Teknikringen 14, 100 44 Stockholm, Sweden

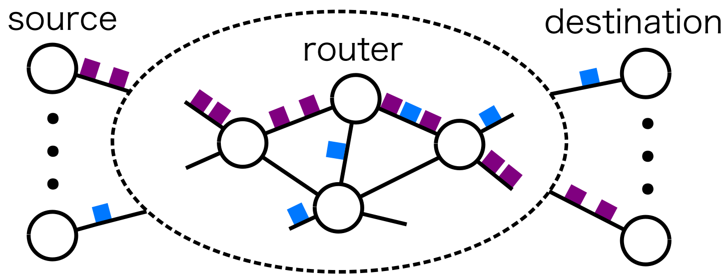


Figure 1: A schematic of power packet dispatching network.

The router manipulates its own storage device according to the flow control data transferred with the power packet so as to compensate for the difference between the generation schedule and the demand schedule. Referring to this work, power packet transactions were proposed for an electric power distribution system [17]. The concept of power packet also appeared in [18]. He *et al.* [19] proposed an electric power architecture, rooted in lessons learned from the Internet and microgrids, to produce a grid network designed for distributed renewable energy, prevalent energy storage and stable autonomous systems. "Energy packet networks" were also proposed to provide energy on demand to Cloud Computing servers [20]. There is a proposal for a controllable-delivery power grid in which electrical power is delivered through discrete power levels directly to customers [21]. In the physical layer, a universal power router is designed and evaluated for residential applications [22]. On the other hand, in most of these proposals, electric energy and information are separately transferred or the physical design is not mentioned. It has been difficult to realize the practical hardware to deal with electric power in the same way as information because of different physical scales: electric power has been high-power and low-frequency electricity, while information has been processed with low-power and high-frequency electricity. For ensuring consistency between the physical layer and the logical layer, the synchrony between energy and information is crucial for managing power.

Recently, wide-bandgap power semiconductors, such as silicon carbide (SiC) and gallium nitride (GaN), have shown material properties enabling power device operation at potentially higher temperatures, voltages and switching speeds than the current Si technology [23, 24]. High-speed gate drive circuits have been developed to achieve high-frequency switching over 1 MHz [25, 26]. This work enables us to handle high-power and high-frequency electricity, and moreover, to digitize power with power packetization [6, 8, 9]. In the developed system, an information tag is directly and physically attached to each packet with its voltage waveform. In this way, energy and information are integrated at an individual packet level. A schematic of the power packet dispatching network is shown in figure 1. The system consists of network lines and routers which stores and forwards power packets according to the tag's information. In the network, the information tag identifies different kinds of power due to different sources, destinations, voltages, control commands and so on. When we send the power packets using time-division multiplexing, it becomes possible to distinguish the power at each line by using the information tag as an index. The router has multiple storage units to identify the different kinds of power flow. Here, power is given by the density of power packets between routers [10, 14, 15].

Now, we consider a packet-centric framework of energy transfer. In Shannon's information theory, messages are represented by symbol sequences in a digitized manner [3]. Referring to this formulation, we define symbols in power packetization as a minimum unit of power transferred by a power pulse with an information tag [12, 13]. Because a symbol is a minimum unit of power, we ignore the variety of ways in which a symbol can be transferred during a unit time in the physical layer. Then, the energy of each symbol is uniquely determined as a real number.[1,2] Here, power packetization is a simultaneous representation of messages and energy with symbol sequences [12, 13]. In information theory, the representation of messages is treated as a coding problem, in which the goal is to minimize the cost such as the length of codewords. In power packetization, however, it is important to represent the given energy during a finite duration as the total amount of energy of symbols. Thus, energy representation is a problem unique to power packetization. In [12,13], this problem was considered with a set of symbol sequences which represent a given energy as the total amount.

In this paper, we consider the problem of energy representation, which was introduced in [12], in networks for a finite duration and formulate the optimization problem of power packetization. Here, the set of symbols and their energies are given to the networks. Then, packetized power is spatially and temporally transferred as symbols in a digitized and quantized manner: a symbol is transferred at a link during a unit time; at each node, energy is represented with symbols sent to and received from neighbouring nodes during a finite duration.

To mathematically represent the transmission of packetized power, we refer to the work about detecting bipedal motion using point trajectories in video sequences [27, 28]. In this work, to obtain discriminative point trajectories from image sequences over a sufficiently long time period under both image noise and occlusion, probabilistic trajectories are designed by prioritizing the concept of temporal connectedness. They are extracted from directed acyclic graphs whose edges represent temporal point correspondences and are weighted with their matching probability in terms of appearance and location.

Here, we introduce *symbol propagation matrix* (SPM), a new concept to represent packetized power, considering the transfer of a symbol as a spatio-temporal correspondence. In power packetization, unlike the probabilistic trajectories [27, 28], each symbol has its energy and packetized power is represented as a network flow. The temporal connectedness is important in power packetization to transfer power in networks with low "strain", i.e. the spatial difference of power, which is equal to the temporal change of energy stored in each router. Then, we consider the problem of selecting an SPM in terms of transferability, that is, the possibility to represent given energies at sources and destinations during the finite duration. To select an SPM, we consider a network flow problem of packetized power, weighting supplied energy from sources and supplied energy to destinations (V1), transferred energy at each link during each unit time (V2) and change of stored energy in each router (V3). The problem is formulated as an M-convex submodular flow problem which is a solvable generalization of the minimum cost flow problem [29].

Finally, through examples, we verify that the formulation provides reasonable transmission of packetized power. Here, while we consider packetized power with a network flow problem, Gelenbe and colleagues have theoretically investigated *energy packet networks* with queueing theory [30–32]. Although these approaches are fundamentally different from our problem, it is worth studying common aspects which can be done by analysing a specific system similar to the models and the examples considered in [32]. Thus, we also discuss our formulation referring to the *energy packet network* model.

2 Packetized power in networks

Here, we introduce the concept of an SPM as a representation of packetized power transferred by symbols in a digitized and quantized manner. Via SPM, packetized power is represented as a network flow in a spatio-temporal structure.

[1]With the application of a power factor correction circuit (PFC), any load can be treated as resistive. Thus, power is discussed in real numbers without loss of generality.

[2]In this setting, power pulses with the same energy can be represented by a single symbol. The properties that symbols do not specify are treated by indexing the symbols. The index becomes important in terms of redundancy of the system.

2.1 Symbol propagation matrices

The set of symbols Σ_{T} and their energies $\mathcal{E} : \Sigma_{\mathrm{T}} \to \mathbb{R}_{>0}$ are given to the network. The symbols have a partition $\{\Sigma_m\}_{m=0}^{M-1}$, whose cell represents a distinct power flow.[3] For each distinct power flow, energy is represented as a summation of the symbol's energy. Here, symbols of the same cell can be exchanged under the conservation of energy.

The network structure is given as a directed graph $G = (\tilde{V}, A)$, where $\tilde{V}$ is a disjoint union of the set of routers V, sources V_{S} and destinations V_{D}, and A is the set of links. Here, sources and destinations represent the external system of the network. The incidence relation is a couple of functions $\partial^+ : A \to \tilde{V}$ and $\partial^- : A \to \tilde{V}$. Another representation of the incidence relation is introduced as a couple of functions $\delta^+ : \tilde{V} \ni v \mapsto \{a \mid \partial^+ a = v\} \in 2^A$ and $\delta^- : \tilde{V} \ni v \mapsto \{a \mid \partial^- a = v\} \in 2^A$ [29, 33]. As for the link, power is kept between nodes. The directions of links are assigned with power directions.

Next, we set that the network is synchronized and symbols are transferred during the same unit times $\overline{T} = \{\overline{t_0}, \overline{t_1}, \cdots, \overline{t_{N-1}}\}$, where N is a positive integer, $\overline{t_n} := [t_n, t_{n+1})$, and $t_0 < t_1 < \cdots < t_N$. Here, energy is transferred by N unit times. Although various power pulses can transfer the energy of the same symbol during a unit time, we ignore the variety and focus on the integrated value of power during the unit time.

Now, we focus on a single cell Σ_m. At each link $a \in A$ during each unit time $\bar{t} \in \overline{T}$, there are three cases of transfer of symbols Σ_m: Case 1: a single symbol $\sigma \in \Sigma_m$ is transferred from node $\partial^+ a$ to node $\partial^- a$; Case 2: a single symbol $\sigma \in \Sigma_m$ is transferred from node $\partial^- a$ to node $\partial^+ a$; Case 3: no symbol is transferred.[4] Therefore, packetized power is given by a map, which we name SPM:

$$SPM_m : \overline{T} \times A \to \Sigma_m \times \{\mathrm{f}, \mathrm{b}\} \cup \{\sigma_\emptyset\}, \tag{1}$$

where $\sigma_\emptyset$ is an element which does not belong to Σ_{T}. $SPM_m(\bar{t}, a) = (\sigma, \mathrm{f})$ and $SPM_m(\bar{t}, a) = (\sigma, \mathrm{b})$ denote that σ is transferred from node $\partial^+ a$ to node $\partial^- a$ and from node $\partial^- a$ to node $\partial^+ a$, respectively. $SPM_m(\bar{t}, a) = \sigma_\emptyset$ denotes that no symbol is transferred.

2.2 Packetized power as a network flow

Here, we introduce packetized power as a network flow [29, 33]. First, we define a graph with spatio-temporal structure induced by the network structure G and unit times $\overline{T}$ as

$$\hat{G} = (\overline{T} \times \tilde{V}, \overline{T} \times A), \tag{2}$$

whose incidence relation is defined by

$$\hat{\partial}^+ : \overline{T} \times A \ni (\bar{t}, a) \mapsto (\bar{t}, \partial^+ a) \in \overline{T} \times \tilde{V} \tag{3}$$

and

$$\hat{\partial}^- : \overline{T} \times A \ni (\bar{t}, a) \mapsto (\bar{t}, \partial^- a) \in \overline{T} \times \tilde{V}. \tag{4}$$

Then, packetized power is introduced as a network flow on $\hat{G}$, i.e.

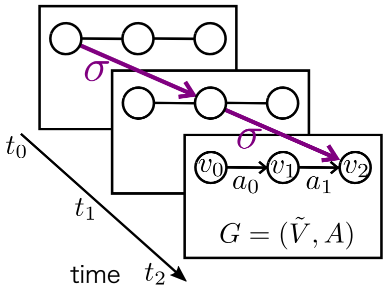
$$u : \overline{T} \times A \to \mathbb{R}, \tag{5}$$

and its boundary $\partial u : \overline{T} \times \tilde{V} \to \mathbb{R}$ is defined by

$$\partial u(\bar{t}, v) = \sum_{a \in \delta^+ v} u(\bar{t}, a) - \sum_{a \in \delta^- v} u(\bar{t}, a) \quad (\bar{t} \in \overline{T}, v \in \tilde{V}). \tag{6}$$

[3] In power packetization, energy is transferred with time division multiplexing (TDM) at links and stored in the corresponding storage unit in routers. Thus, power flow can be distinguished by the information tags of power packets.

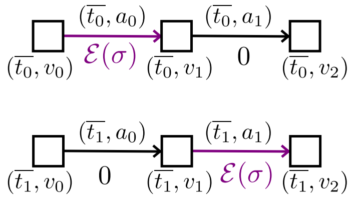
[4] Here, more than one symbol is not transferred during unit time because we do not care about the various ways that power may be transferred. For example, if twice the amount of energy of a symbol $\sigma_1 \in \Sigma_m$ is transferred, then we consider a symbol $\sigma_2 \in \Sigma_m$ with an energy $\mathcal{E}(\sigma_2) = 2\mathcal{E}(\sigma_1)$



(a)

	a_0	a_1
$\overline{t_0}$	(σ, f)	$\sigma_\emptyset$
$\overline{t_1}$	$\sigma_\emptyset$	(σ, f)

(b)



(c)

Figure 2: Schematic of a symbol propagation matrix and packetized power. We set a directed graph $G = (\tilde{V}, A)$, where $\tilde{V} = \{v_0, v_1, v_2\}$ and $A = \{a_0, a_1\}$, and unit times $\overline{T} = \{\overline{t_0}, \overline{t_1}\}$. (a) A schematic diagram of symbol propagation. A symbol $\sigma \in \Sigma_m$ is transferred at link a_0 during unit time $\overline{t_0}$ and at link a_1 during unit time $\overline{t_1}$. (b) A symbol propagation matrix shown as a table. (c) Packetized power as a network flow on $\hat{G} = (\overline{T} \times \tilde{V}, \overline{T} \times A)$.

Each link of $\hat{G}$, i.e. $(\bar{t}, a) \in \overline{T} \times A$, represents a spatio-temporal correspondence at which each symbol is transferred. Each cell Σ_m represents packetized power $u_m : \overline{T} \times A \to \mathbb{R}$ and u_m is given by SPM_m as

$$u_m(\bar{t}, a) = \begin{cases} \mathcal{E}(\sigma) & (SPM_m(\bar{t}, a) = (\sigma, \mathrm{f})), \\ -\mathcal{E}(\sigma) & (SPM_m(\bar{t}, a) = (\sigma, \mathrm{b})), \\ 0 & (SPM_m(\bar{t}, a) = \sigma_\emptyset). \end{cases} \tag{7}$$

Here, we assume the conservation of energy; more precisely, we assume that all energy exchanges in each node $v \in \tilde{V}$ are represented with symbols transferred through the adjacent links[5] $a \in \delta^+ v \cup \delta^- v$. Then, $-\partial u_m(\bar{t}, v)$ is equal to the increment of stored energy corresponding to Σ_m in the node $v \in \tilde{V}$ during the unit time $\bar{t} \in \overline{T}$. Here, packetized power is represented as a network flow, in which energy is transferred with symbols in the digitized and quantized form.

2.3 Example of symbol propagation matrix

Here, we illustrate the definitions mentioned above. Figure 2 shows a schematic of an SPM and packetized power. In this example, we set a directed graph $G = (\tilde{V}, A)$, where $\tilde{V} = \{v_0, v_1, v_2\}$ and $A = \{a_0, a_1\}$, and unit times $\overline{T} = \{\overline{t_0}, \overline{t_1}\}$. As schematically shown in figure 2(a), we consider a transmission of symbols Σ_m, in which a symbol $\sigma \in \Sigma_m$ is transferred at link a_0 during unit time $\overline{t_0}$ and at link a_1 during unit time $\overline{t_1}$. Then, we have the SPM shown in figure 2(b).

In this example, the graph $\hat{G} = (\overline{T} \times \tilde{V}, \overline{T} \times A)$ with spatio-temporal structure is defined as shown in figure 2(c). Then, packetized power u_m, which is represented by the cell Σ_m, is introduced as a network flow on this graph. Here, we have $u_m(\overline{t_0}, a_0) = \mathcal{E}(\sigma)$, $u_m(\overline{t_0}, a_1) = 0$, $u_m(\overline{t_1}, a_0) = 0$ and $u_m(\overline{t_1}, a_1) = \mathcal{E}(\sigma)$. The boundary ∂u_m is equal to the change of stored energy corresponding to Σ_m. For example, we have $-\partial u_m(\overline{t_0}, v_1) = \mathcal{E}(\sigma)$ as the "strain", i.e. as the spatial difference of the packetized power u_m, and this value is equal to the increment of the stored energy in v_1 during the unit time $\overline{t_0} = [t_0, t_1)$.

[5]In practical systems, energy can be dissipated at links and in nodes, and symbols may not be able to keep constant energy between nodes. These can become noise of power packetization, which is one of the future works of this paper.

3 Power packet transferability

To meet supply and demand, i.e. to represent given energies at sources and destinations with symbols, it is necessary to select a symbol at each link $a \in A$ during each unit time $\bar{t} \in \overline{T}$. Here, different choices lead to different transferability, that is the possibility to represent the given energies by transmission of symbols. In this section, we develop a framework to select an SPM in terms of transferability as a network flow problem of packetized power. To make the problem solvable, we focus on a single power flow represented by cell $\Sigma \in \{\Sigma_m\}_{m=0}^{M-1}$. Besides, we set that $\mathcal{E}(\Sigma)$ to be successive integers $\{1, 2, \cdots\}$, that is, we consider integer flow $u : \overline{T} \times A \to \mathbb{Z}$. Because power packet is a unit of power, it is natural to consider integer flow.

3.1 Features of packetized power contributing to transferability

In networks, packetized power appears as follows:

V1: supplied energy from sources and supplied energy to destinations,

V2: transferred energy at each link during each unit time, and

V3: change of stored energy in each router.

In terms of transferability, V1 needs to satisfy the given energies at sources and destinations, while V2 and V3 need to be small. V2 should be small to effectively use limited density of power packets between different kinds of power flows. In addition, by minimizing the summation of transferred energy, we can obtain the network flow in which energy is supplied to each destination from sources placed near the destination. As for V3, change of stored energy in routers should be suppressed to keep symbol's energy controllable with density modulation of power packets between routers.

Thus, we select the features of packetized power $u : \overline{T} \times A \to \mathbb{Z}$ contributing to transferability as V1, V2 and V3. V2 is a value of the network flow $u(\bar{t}, a)$ ($(\bar{t}, a) \in \overline{T} \times A$), while V1 and V3 are calculated from the values of the boundary $\partial u(\bar{t}, v)$ ($(\bar{t}, v) \in \overline{T} \times \tilde{V}$). V1 and V3 include time integral, such as total supplied energy from a source $s \in V_{\mathrm{S}}$, i.e. $\sum_{n=0}^{N-1} \partial u(\overline{t_n}, s)$.

We introduce a cost function of the network flow problem as the summation of costs on these values. Because our features include not only values of network flow but also values of boundary and their time integrals, it is impossible to formulate the problem as the conventional minimum cost flow problem on the spatio-temporal graph $\hat{G}(\overline{T} \times \hat{V}, \overline{T} \times A)$. Thus, in the next subsection, we provide the formulation using an M-convex submodular flow problem [29] which is the generalization of the minimum cost flow problem.

3.2 Formulation as M-convex submodular flow problem

Now, we formulate the network flow problem to provide transferable packetized power. In the spatio-temporal structure, the M-convex submodular flow problem is to find a packetized power $u : \overline{T} \times A \to \mathbb{Z}$ which minimizes the total cost $\Gamma(u)$; more precisely, the problem is described by the graph $G(\overline{T} \times \tilde{V}, \overline{T} \times A)$, univariate discrete convex functions[6] $f_{(\bar{t},a)} : \mathbb{Z} \to \mathbb{R} \cup \{+\infty\}$ ($(\bar{t}, a) \in \overline{T} \times A$), and an M-convex function $f : \mathbb{Z}^{\overline{T} \times \tilde{V}} \to \mathbb{R} \cup \{+\infty\}$ as in [29]

$$\text{Minimize} \qquad \Gamma(u) = \sum_{(\bar{t},a) \in \overline{T} \times A} f_{(\bar{t},a)}(u(\bar{t}, a)) + f(\partial u) \tag{9}$$

[6]A function $\phi : \mathbb{Z} \to \mathbb{R} \cup \{+\infty\}$ is called a univariate discrete convex function if we have

$$\phi(x-1) + \phi(x+1) \geq 2\phi(x) \text{ for all } x \in \mathbb{Z} \tag{8}$$

and $\operatorname{dom} \phi \neq \emptyset$ [29]. Note that, if a function $\phi : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$ is convex, ϕ satisfies (8).

$$\text{subject to} \quad u(\bar{t},a) \in \operatorname{dom} f_{(\bar{t},a)} \quad ((\bar{t},a) \in \overline{T} \times A), \tag{10}$$

$$\partial u \in \operatorname{dom} f, \tag{11}$$

$$u(\bar{t},a) \in \mathbb{Z} \quad ((\bar{t},a) \in \overline{T} \times A). \tag{12}$$

Here, (10) denotes capacity constraints of links, because $f_{(\bar{t},a)}$ is convex, and hence $\operatorname{dom} f_{(\bar{t},a)}$ is an interval. Therefore, we can set an upper capacity $\hat{c} : \overline{T} \times A \to \mathbb{Z} \cup \{+\infty\}$ and a lower capacity $\check{c} : \overline{T} \times A \to \mathbb{Z} \cup \{-\infty\}$, where it is assumed that $\hat{c}(\bar{t},a) \geq \check{c}(\bar{t},a)$ for each $(\bar{t},a) \in \overline{T} \times A$, by defining cost functions $f_{(\bar{t},a)}$ whose effective domain is equal to the interval $[\check{c}(\bar{t},a), \hat{c}(\bar{t},a)]$ $((\bar{t},a) \in \overline{T} \times A)$.

Then, we introduce the cost function of the boundary as the summation of the costs on V1 and V3. To this end, we prove in Sect. 3.3 that, for a laminar family[7] $\mathcal{T}$ of subsets of $\overline{T} \times \tilde{V}$ and univariate discrete convex functions $f_X : \mathbb{Z} \to \mathbb{R} \cup \{+\infty\}$ indexed by $X \in \mathcal{T}$, the function defined by

$$f(\Delta u) = \begin{cases} \sum_{X \in \mathcal{T}} f_X(\Delta u(X)) & (\Delta u(\overline{T} \times \tilde{V}) = 0), \\ +\infty & (\text{otherwise}) \end{cases} \tag{14}$$

is an M-convex function. Here, we use the notation $\Delta u(X) = \sum_{(\bar{t},v) \in X} \Delta u(\bar{t},v)$ for $\Delta u \in \mathbb{Z}^{\overline{T} \times \tilde{V}}$ and $X \subset \overline{T} \times \tilde{V}$. In (14), we can set costs on V1 and V3 by setting a laminar family $\mathcal{T}$ and univariate discrete convex functions $\{f_X\}_{X \in \mathcal{T}}$. For example, we can treat total supplied energy $\sum_{n=0}^{N-1} \partial u(\overline{t_n}, s)$ at source $s \in V_{\mathrm{S}}$ by including $\overline{T} \times \{s\}$ in $\mathcal{T}$. In the following, we set that the laminar family $\mathcal{T}$ is a disjoint union of a laminar family $\mathcal{T}_{\mathrm{S,D}}$ of subsets of $\overline{T} \times (V_{\mathrm{S}} \cup V_{\mathrm{D}})$ and laminar families $\mathcal{T}_v$ of subsets of $\overline{T} \times \{v\}$ $(v \in V)$.

To sum up, we introduce the cost function Γ of packetized power u as

$$\Gamma(u) = \sum_{X \in \mathcal{T}_{\mathrm{S,D}}} f_X(\partial u(X)) + \sum_{(\bar{t},a) \in \overline{T} \times A} f_{(\bar{t},a)}(u(\bar{t},a)) + \sum_{v \in V} \sum_{X \in \mathcal{T}_v} f_X(\partial u(X)). \tag{15}$$

The first, the second and the third term in the right-hand side of (15) correspond to the costs of V1, V2 and V3.

3.3 Proof of the M-convexity of the function f in (14)

In general, a laminar convex function has M$^\natural$-convexity[8] [29]. The following corollary shows that, if a laminar convex function is restricted to the hyper-plane, the function has M-convexity. From this corollary, we can confirm that the function f in (14) satisfies M-convexity. This is proved referring to Note 9.31. in [29] with a slight modification.

Corollary 3.1. *For a finite set V, a laminar family $\mathcal{T}$ of subsets of V, univariate discrete convex functions f_X $(X \in \mathcal{T})$, and an integer $r \in \mathbb{Z}$, a function $f : \mathbb{Z}^V \to \mathbb{R} \cup \{+\infty\}$ is defined by*

$$f(x) = \begin{cases} \sum_{X \in \mathcal{T}} f_X(x(X)) & (x(V) = r), \\ +\infty & (\textit{otherwise}). \end{cases} \tag{16}$$

Then, f has M-convexity.

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[7]By a laminar family, we mean a non-empty family $\mathcal{T}$ such that [29]

$$X, Y \in \mathcal{T} \Rightarrow X \cap Y = \emptyset \text{ or } X \subset Y \text{ or } X \supset Y. \tag{13}$$

[8]M$^\natural$-convex functions are variants of, and essentially equivalent to, M-convex functions. "M$^\natural$-convex" should be read "M-natural-convex" [29].

Proof of corollary 3.1. Without loss of generality, we assume that $\emptyset \in \mathcal{T}$, $V \in \mathcal{T}$, and every singleton set belong to $\mathcal{T}$. We represent $\mathcal{T}$ by a directed tree $G = (U, A; S, T)$ with root u_0, where $U = \{u_X \mid X \in \mathcal{T}\} \cup \{u_0\}$, $A = \{a_X \mid X \in \mathcal{T}\}$, $S = \{u_0\}$, $T = \{u_{\{v\}} \mid v \in V\}$, and $\partial^- a_X = u_X$ and $\partial^+ a_X = u_{\hat{X}}$ for $X \in \mathcal{T}$, where $\hat{X}$ denotes the smallest member of $\mathcal{T}$ that properly contains X (and $\hat{V} = 0$ by convention). We associate the given function f_X with arc a_X for $X \in \mathcal{T}$. We define an M-convex function $f' : \mathbb{Z}^S \to \mathbb{R} \cup \{+\infty\}$ by

$$f'(y) = \begin{cases} 0 & (y = r), \\ +\infty & (\text{otherwise}). \end{cases} \tag{17}$$

Then, f is the result of flow type transformation:

$$\begin{aligned} f(x) = \inf_{\xi, y} \Bigg\{ f'(y) + \sum_{X \in \mathcal{T}} f_X(\xi(a_X)) \;\Bigg|\; & \partial\xi = (y, -x, \mathbf{0}), \\ & \xi \in \mathbb{Z}^A, (y, -x, \mathbf{0}) \in \mathbb{Z}^S \times \mathbb{Z}^T \times \mathbb{Z}^{U \setminus (S \cup T)} \Bigg\} \quad (x \in \mathbb{Z}^T). \end{aligned} \tag{18}$$

Therefore, from Theorem 9.27. in [29], f has M-convexity. □

4 Examples

In this section, we verify that the formulation in Sect. 3 provides reasonable packetized power. First, we illustrate some properties of the problem, comparing three cost functions on a mesh graph. Then, we consider the formulation referring to the *energy packet network* model [32].

4.1 Illustration with three cost functions

Here, the power packet transferability is discussed through the following M-convex submodular flow problem:

$$\begin{aligned} \text{Minimize} \quad & \mathit{\Gamma}(u) = \sum_{V' \in \mathcal{T}_{V_\mathrm{S} \cup V_\mathrm{D}}} f_{\overline{T} \times V'}(\partial u(\overline{T} \times V')) + \sum_{(\bar{t}, a) \in \overline{T} \times A} \beta(a) \left| u(\bar{t}, a) \right| & (19) \\ \text{subject to} \quad & \check{c}(a) \le u(\bar{t}, a) \le \hat{c}(a) \quad ((\bar{t}, a) \in \overline{T} \times A), & (20) \\ & \partial u(\bar{t}, v) = 0 \quad ((\bar{t}, v) \in \overline{T} \times V), & (21) \\ & u(\bar{t}, a) \in \mathbb{Z} \quad ((\bar{t}, a) \in \overline{T} \times A), & (22) \end{aligned}$$

where $\mathcal{T}_{V_\mathrm{S} \cup V_\mathrm{D}}$ is a laminar family of $V_\mathrm{S} \cup V_\mathrm{D}$. Note that $\{\overline{T} \times V' \mid V' \in \mathcal{T}_{V_\mathrm{S} \cup V_\mathrm{D}}\}$ is a laminar family of $\overline{T} \times (V_\mathrm{S} \cup V_\mathrm{D})$. This problem is a special case in which we set the following:

- Total supplied energy is the only concern at sources and destinations. More precisely, $\mathcal{T}_{\mathrm{S,D}}$ is set to be $\{\overline{T} \times V' \mid V' \in \mathcal{T}_{V_\mathrm{S} \cup V_\mathrm{D}}\}$.
- Each cost function $f_{(\bar{t}, a)}$ is defined as an absolute value function with effective domain $[\check{c}(a), \hat{c}(a)]$ $((\bar{t}, a) \in \overline{T} \times A)$. The coefficients $\beta(a)$ $(a \in A)$ are non-negative real numbers. Here, $f_{(\bar{t}, a)}$ does not depend on unit times $\overline{T}$.
- Boundary of flow is set to be zero at routers V in (21) in order to transfer power without strain. Note that, because of this constraint, we have $\partial u(\{\bar{t}\} \times V_\mathrm{S}) = -\partial u(\{\bar{t}\} \times V_\mathrm{D})$ $(\bar{t} \in \overline{T})$ for a feasible flow u.

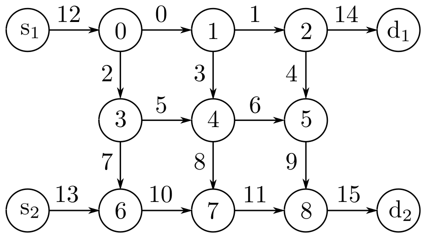

Figure 3: Network structure given as a mesh graph. For links between routers $a \in \{0, 1, \cdots, 11\}$, we set $\beta(a) = 1$, $\check{c}(a) = -1$, and $\hat{c}(a) = 1$, which imply $u(\bar{t}, a) \in \{-1, 0, 1\}$ $(\bar{t} \in \overline{T})$. For the other links $a \in \{12, 13, 14, 15\}$, we set $\beta(a) = 0$, $\check{c}(a) = -\infty$ and $\hat{c}(a) = +\infty$, which imply $u(\bar{t}, a) \in \mathbb{Z}$ $(\bar{t} \in \overline{T})$.

Table 1: Optimal flows u in E1 and their costs with various settings of given energy U_1.

N	$U_1(\mathrm{s}_1)$, $U_1(\mathrm{s}_2)$	$U_1(\mathrm{d}_1)$, $U_1(\mathrm{d}_2)$	cost $\Gamma_1(u)$	an optimal flow
1	0, 0	0, 0	0	α_0
1	1, 0	−1, 0	2	α_1
1	1, 1	−1, −1	4	α_2
1	1, 1	0, −2	6	α_3
1	2, 1	−1, −2	8	α_4
1	2, 1	−2, −1	8	α_5

In the following, we investigate the aforementioned problem with a mesh graph as an example. The network structure is shown in figure 3, where $V = \{0, \cdots, 8\}$, $V_\mathrm{S} = \{\mathrm{s}_1, \mathrm{s}_2\}$, $V_\mathrm{D} = \{\mathrm{d}_1, \mathrm{d}_2\}$ and $A = \{0, 1, \cdots, 15\}$. For links between routers $a \in \{0, 1, \cdots, 11\}$, we set $\beta(a) = 1$, $\check{c}(a) = -1$, and $\hat{c}(a) = 1$. This capacity constraint implies $u(\bar{t}, a) \in \{-1, 0, 1\}$ $(\bar{t} \in \overline{T}, a \in \{0, 1, \cdots, 11\})$. For the other links $a \in \{12, 13, 14, 15\}$, we set $\beta(a) = 0$, $\check{c}(a) = -\infty$, and $\hat{c}(a) = +\infty$, and hence we have $u(\bar{t}, a) \in \mathbb{Z}$ $(\bar{t} \in \overline{T})$.

We consider three examples of the problem, in which energy is given at the sources and the destinations in different ways: in the first example (E1), energy is given at each source and each destination by $U_1 : V_\mathrm{S} \cup V_\mathrm{D} \to \mathbb{Z}$; in the second example (E2), energy is given at each destination by $U_2 : V_\mathrm{D} \to \mathbb{Z}_{\geq 0}$; in the third example (E3), the total supplied energy is given by $U \in \mathbb{Z}_{\geq 0}$. In E1, U_1 is set to satisfy $\sum_{s \in V_\mathrm{S}} U_1(s) = -\sum_{d \in V_\mathrm{D}} U_1(d)$ because power is transferred without strain in this example. In E1, E2, and E3, the objective functions are defined respectively as

$$\Gamma_1(u) = \sum_{v \in V_\mathrm{S} \cup V_\mathrm{D}} 1000 \left| \partial u(\overline{T} \times \{v\}) - U_1(v) \right| + \sum_{(\bar{t}, a) \in \overline{T} \times A} \beta(a) \left| u(\bar{t}, a) \right|, \tag{23}$$

$$\Gamma_2(u) = \sum_{v \in V_\mathrm{D}} 1000 \left| \partial u(\overline{T} \times \{v\}) + U_2(v) \right| + \sum_{(\bar{t}, a) \in \overline{T} \times A} \beta(a) \left| u(\bar{t}, a) \right| \tag{24}$$

and

$$\Gamma_3(u) = 1000 \left| \partial u(\overline{T} \times V_\mathrm{D}) + U \right| + \sum_{(\bar{t}, a) \in \overline{T} \times A} \beta(a) \left| u(\bar{t}, a) \right|. \tag{25}$$

In these functions, the coefficients of the first terms are set to be 1000. This value is large enough to give priority to the representation of the given energy over minimization of energy transferred through links.

Now, we discuss these examples, solving them using the cycle-cancelling algorithm [29]. First, with Γ_1 in E1, we set $N = 1$ and show an optimal flow for each setting of U_1 in figure 4. The six optimal

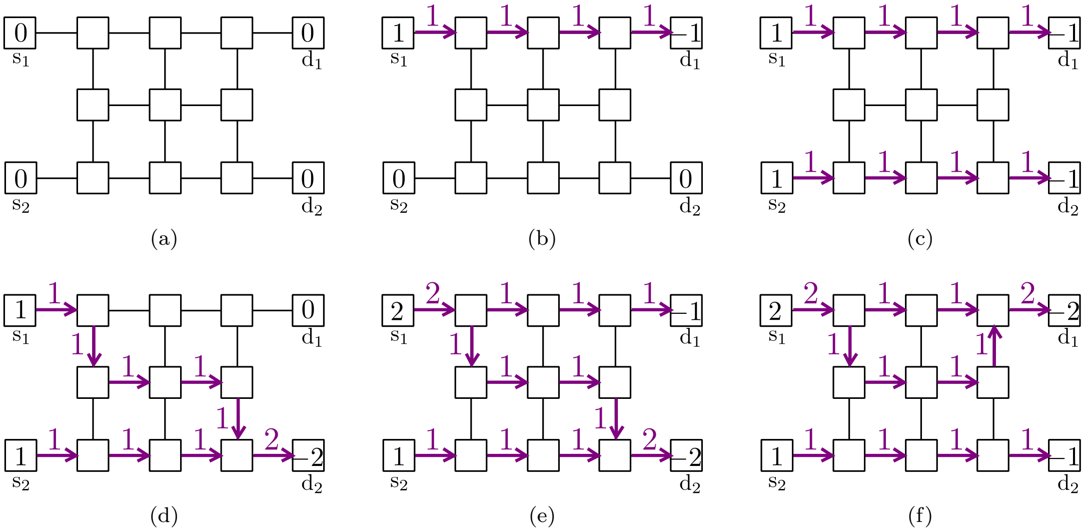

Figure 4: Optimal flows with $\mathit{\Gamma}_1$ and $N = 1$. The energies given to the sources and the destinations are shown in the corresponding nodes. (a) an optimal flow α_0, (b) an optimal flow α_1, (c) an optimal flow α_2, (d) an optimal flow α_3, (e) an optimal flow α_4 and (f) an optimal flow α_5.

Table 2: Optimal flows u in E2 and their costs with various settings of N and given energy U_2. Here, optimal flows are denoted by the sequence of $\{\alpha_i\}_{i=0}^{5}$ in time order.

N	$U_2(\mathrm{d}_1)$, $U_2(\mathrm{d}_2)$	cost $\mathit{\Gamma}_2(u)$	an optimal flow
2	3, 3	16	$\alpha_4\alpha_5$
3	3, 3	12	$\alpha_2\alpha_2\alpha_2$
5	3, 3	12	$\alpha_2\alpha_2\alpha_2\alpha_0\alpha_0$
2	5, 2	1016	$\alpha_5\alpha_5$
3	5, 2	18	$\alpha_5\alpha_5\alpha_1$
5	5, 2	14	$\alpha_2\alpha_2\alpha_1\alpha_1\alpha_1$

Table 3: Optimal flows u in E3 and their costs with various settings of N and given energy U. Here, optimal flows are denoted by the sequence of $\{\alpha_i\}_{i=0}^{5}$ in time order. The settings of N and U correspond to the settings in table 2 through the relationship $\sum_{d \in V_{\mathrm{D}}} U_2(d) = U$.

N	U	cost $\mathit{\Gamma}_3(u)$	an optimal flow
2	6	16	$\alpha_4\alpha_4$
3	6	12	$\alpha_2\alpha_2\alpha_2$
5	6	12	$\alpha_2\alpha_2\alpha_2\alpha_0\alpha_0$
2	7	1016	$\alpha_4\alpha_4$
3	7	16	$\alpha_4\alpha_2\alpha_2$
5	7	14	$\alpha_2\alpha_2\alpha_2\alpha_1\alpha_0$

flows are named as $\{\alpha_i\}_{i=0}^5$. The optimal flows u are listed in table 1 with their settings and costs $\mathit{\Gamma}_1(u)$. Next, with $\mathit{\Gamma}_2$ in E2 and with $\mathit{\Gamma}_3$ in E3, we list optimal flows as shown in tables 2 and 3, respectively. Here, each optimal flow is denoted by a sequence of $\{\alpha_i\}_{i=0}^5$ in time order. Note that the settings of N and U in table 3 correspond to the settings in table 2 through the relationship $\sum_{d\in V_{\mathrm{D}}} U_2(d) = U$. In the optimal flows in which costs exceed 1000, the given energy is not represented.

From figure 4 and table 1, the following properties are confirmed at $N = 1$:

- By minimizing the cost of V1, given energy U_1 is represented at each source and each destination.
- By minimizing the cost of V2, energy is transferred with the smallest number of links. Note that, in flow α_2 in figure 4c, the destinations d_1 and d_2 are supplied from the nearest sources, i.e. s_1 for d_1 and s_2 for d_2.
- By imposing the constraint on V3 as shown in (21), energy is transferred without strain, i.e. without change of stored energy.

Tables 2 and 3 imply the following properties:

- The cost of the optimal flow decreases as the number of time step N increases up to a certain point. From N exceeding the point, the cost takes a constant value and α_0, in which symbols are not transferred, is added to the optimal flow.
- The cost of the optimal flow decreases when the distribution of supplied energy is not specified and the total supplied energy is given. For example, the cost becomes $\mathit{\Gamma}_2(u) = 18$ when we set $N = 3$, $U_2(\mathrm{d}_1) = 5$, and $U_2(\mathrm{d}_2) = 2$ in E2, while the cost becomes $\mathit{\Gamma}_3(u) = 16$ when we set $N = 3$ and $U = 7$, which is equal to $\sum_{d\in V_{\mathrm{D}}} U_2(d) = 5 + 2$, in E3.

In other words, a more transferable SPM can be selected if symbols are transferred with less temporal and spatial restriction. These properties show that power can be packetized and be controllable while preserving reasonable properties of power.

4.2 Example referring to the energy packet network model

Here, we discuss our formulation referring to the *energy packet network (EPN)* model [32]. Although EPNs are considered with queueing theory while we consider packetized power with a network flow problem, it is worth studying common aspects of the two different approaches.

First, we clarify the features of the EPN model which we focus on in this example system[9] [32]:

- Power flow is represented by a random flow of arriving "energy packets (EPs)", which is a discrete unit of energy.
- An "energy storage unit" is modelled as a queue of EPs. EPs enter each energy storage unit at a specific rate.
- A "workstation" is modelled as a queue of jobs. Jobs enter each workstation at a specific rate. A job requires an EP to accomplish the work.
- One of the proposed cost functions considers "the needs of the consumers" with the weighted probability that more than a given number of jobs exist in the workstation, and "the desire to maintain some reserve energy" with the weighted probability that a given number of EPs does not exist in the energy storage unit.

[9] The EPN model also includes the other features, such as movement of energies between different energy storage units, movement of jobs between workstations, losses of EPs at energy storage units and losses of jobs at workstations. However, we do not consider these features for simplicity.

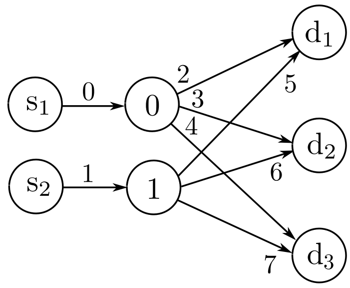

Figure 5: Network structure for comparing our formulation with the EPN model [32]. This structure appears in an example system discussed in [32].

The EPN model was illustrated with a numerical example which has three destinations and is fed by two sources [32]. The three destinations require different amounts of energy and have different priorities. At each source, there is a maximum total rate at which the source can transmit EPs. The numerical results show the following properties:

- When "the needs of the consumers" are emphasized, each energy storage unit transmits EPs at the maximum total rate.
- When "the desire to maintain some reserve energy" is emphasized, each energy storage unit does not transmit EPs at the maximum total rate.

Now, we consider an M-convex submodular flow problem for analysing a system similar to the above-mentioned EPN model and its numerical example. The problem is investigated with the graph shown in figure 5, where $V_{\mathrm{S}} = \{\mathrm{s}_1, \mathrm{s}_2\}$, $V_{\mathrm{D}} = \{\mathrm{d}_1, \mathrm{d}_2, \mathrm{d}_3\}$, $V = \{0, 1\}$ and $A = \{0, 1, \cdots, 7\}$. In this graph, the energy storage units are treated as sources and the workstations are treated as destinations. Because a job requires an EP to accomplish the work, jobs are treated as negative energy.[10] Node $v \in V$ is introduced as a bifurcation of the lines and does not store energy. Here, a significant difference exists in the representation of sources and destinations: In the EPN model, energy supplied from sources is treated as the arrival rate of EPs and energy required from destinations is treated as the arrival rate of jobs; In our formulation, energies are given to sources and destinations for a finite duration and are represented during the duration as the total amount of energy of symbols.

As a physical quantity corresponding to the queues in the EPN model, we consider stored energy in the sources and destinations at each time $t \in \{t_0, t_1, \cdots, t_N\}$. As stored energy at time t_0, we give energy $U_s \in \mathbb{Z}$ to each source $s \in V_{\mathrm{S}}$ and energy $U_d \in \mathbb{Z}$ to each destination $d \in V_{\mathrm{D}}$. Here, U_s is supposed to be non-negative ($s \in V_{\mathrm{S}}$), while U_d is supposed to be non-positive because U_d represents required energy ($d \in V_{\mathrm{D}}$). Because $-\partial u(\overline{t}, v)$ is equal to the increment of stored energy in node $v \in V_{\mathrm{S}} \cup V_{\mathrm{D}}$ during unit time $\overline{t} \in \overline{T}$, the stored energy in node v at time t_{n+1} is equal to $U_v + \sum_{n'=0}^{n}(-\partial u(\overline{t_{n'}}, v))$ ($n \in \{0, 1, \cdots, N-1\}$, $v \in V_{\mathrm{S}} \cup V_{\mathrm{D}}$).

Then, we set the following problem:

$$\text{Minimize} \qquad \mathit{\Gamma}_4(u) = \sum_{s \in V_{\mathrm{S}}} \gamma(s) \sum_{n=0}^{N-1} f_s(U_s + \sum_{n'=0}^{n}(-\partial u(\overline{t_{n'}}, s))) + \sum_{d \in V_{\mathrm{D}}} \gamma(d) \sum_{n=0}^{N-1} f_d(U_d + \sum_{n'=0}^{n}(-\partial u(\overline{t_{n'}}, d))) \tag{26}$$

[10] The requests for energy by jobs can be considered as an up-stream dispatching of required power to sources [14, 34].

$$\begin{aligned}
\text{subject to}\quad & 0 \le u(\bar{t}, a) \le \hat{c}(a) \quad ((\bar{t}, a) \in \overline{T} \times \{0, 1\}), && (27)\\
& u(\bar{t}, a) \ge 0 \quad ((\bar{t}, a) \in \overline{T} \times \{2, 3, \cdots, 7\}), && (28)\\
& \partial u(\bar{t}, v) = 0 \quad ((\bar{t}, v) \in \overline{T} \times \{0, 1\}), && (29)\\
& u(\bar{t}, a) \in \mathbb{Z} \quad ((\bar{t}, a) \in \overline{T} \times A), && (30)
\end{aligned}$$

where $f_s : \mathbb{Z} \to \mathbb{R} \cup \{+\infty\}$ and $f_d : \mathbb{Z} \to \mathbb{R} \cup \{+\infty\}$ are univariate discrete convex functions ($s \in V_{\mathrm{S}}$, $d \in V_{\mathrm{D}}$). The coefficients $\gamma(v)$ ($v \in V_{\mathrm{S}} \cup V_{\mathrm{D}}$) are non-negative real numbers. In (27), $\hat{c}(0)$ and $\hat{c}(1)$ represent the upper capacities of energy which can be transmitted during each unit time from source s_1 and from source s_2 respectively. Noting that $\sum_{n'=0}^{n}(-\partial u(\overline{t_{n'}}, v))$ is equal to $-\partial u(\{\overline{t_0}, \cdots, \overline{t_n}\} \times \{v\})$ ($n \in \{0, 1, \cdots, N-1\}$, $v \in V_{\mathrm{S}} \cup V_{\mathrm{D}}$) and $\{\{\overline{t_0}, \cdots, \overline{t_n}\} \times \{v\} \mid n \in \{0, 1, \cdots, N-1\}, v \in V_{\mathrm{S}} \cup V_{\mathrm{D}}\}$ is a laminar family, we can confirm that $\varGamma_4$ is a special case of the cost function in (15), and hence the problem is an M-convex submodular flow problem.

Referring to the cost function in [32], we define a concrete form of f_s and f_d so that the first term of (26) represents "the desire to maintain some reserve energy" and the second term of (26) represents "the needs of the consumers":

$$f_s(U) = \begin{cases} +\infty & U < 0 \\ K_s - U & 0 \le U < K_s \\ 0 & U \ge K_s \end{cases}, \tag{31}$$

$$f_d(U) = \begin{cases} -K_d - U & U < -K_d \\ 0 & -K_d \le U \le 0 \\ +\infty & U > 0 \end{cases}, \tag{32}$$

where K_s and K_d are non-negative integers ($s \in V_{\mathrm{S}}$, $d \in V_{\mathrm{D}}$). Note that, if f_s were defined as $f_s(U) = 1$ for $0 \le U < K_s$, the first term of (26) would be a good approximation of the weighted percentage of the period during which K_s energy was not stored in source s. Similarly, if f_d were defined as $f_d(U) = 1$ for $U < -K_d$, the second term of (26) would be a good approximation of the weighted percentage of the period during which more than K_d energy was required in destination d. To define univariate discrete convex functions, f_s is defined to be larger if less energy is stored in (31), and f_d is defined to be larger if more energy is required in (32).

Here, we set the parameters as shown in table 4. In a similar manner to the setting of the numerical example in [32], each destination $d \in V_{\mathrm{D}}$ requires a different amount of energy U_d and has different priorities $\gamma(d)$. As for sources, the total amount of supplied energy, i.e. $U_{\mathrm{s}_1} + U_{\mathrm{s}_2}$, is set to be less than the total amount of required energy, i.e. $-(U_{\mathrm{d}_1} + U_{\mathrm{d}_2} + U_{\mathrm{d}_3})$. The number of time step N and the upper capacities $\hat{c}(0)$ and $\hat{c}(1)$ are set to be large enough for each source $s \in V_{\mathrm{S}}$ to transmit all given energy U_s during the duration $[0, t_N)$. K_{s_1} and K_{s_2} are set to be 1, so that the cost of each source arises only at the time when the stored energy becomes zero. On the other hand, K_{d_1}, K_{d_2} and K_{d_3} are set to be 0, so that the cost of each destination at each time is proportional to the required energy.

Table 4: Parameters used in the example.

parameters	values	parameters	values
U_{d_1}	-10	U_{s_1}	4
U_{d_2}	-4	U_{s_2}	8
U_{d_3}	-1	N	4
$\gamma(\mathrm{d}_1)$	2	$\hat{c}(0)$, $\hat{c}(1)$	2
$\gamma(\mathrm{d}_2)$	1	K_{s_1}, K_{s_2}	1
$\gamma(\mathrm{d}_3)$	10	K_{d_1}, K_{d_2}, K_{d_3}	0

Table 5: An optimal flow and its cost for different settings of $\gamma(\mathrm{s}_1)$ and $\gamma(\mathrm{s}_2)$. For each optimal flow, the total supplied energy from source $s \in V_\mathrm{S}$, i.e. the value of $\sum_{n=0}^{N-1} \partial u(\overline{t_n}, s)$, and the total supplied energy to destination $d \in V_\mathrm{D}$, i.e. the value of $\sum_{n=0}^{N-1} (-\partial u(\overline{t_n}, d))$, are shown.

$\gamma(\mathrm{s}_1)$, $\gamma(\mathrm{s}_2)$	an optimal flow	cost $\mathit{\Gamma}_4(u)$	s_1	s_2	d_1	d_2	d_3
0.1	$\alpha'_0\alpha'_1\alpha'_3\alpha'_5$	37.4	4	8	10	1	1
1	$\alpha'_0\alpha'_1\alpha'_3\alpha'_4$	41	4	7	10	0	1
10	$\alpha'_0\alpha'_2\alpha'_3\alpha'_4$	44	3	7	9	0	1
100	$\alpha'_0\alpha'_2\alpha'_3\alpha'_4$	44	3	7	9	0	1

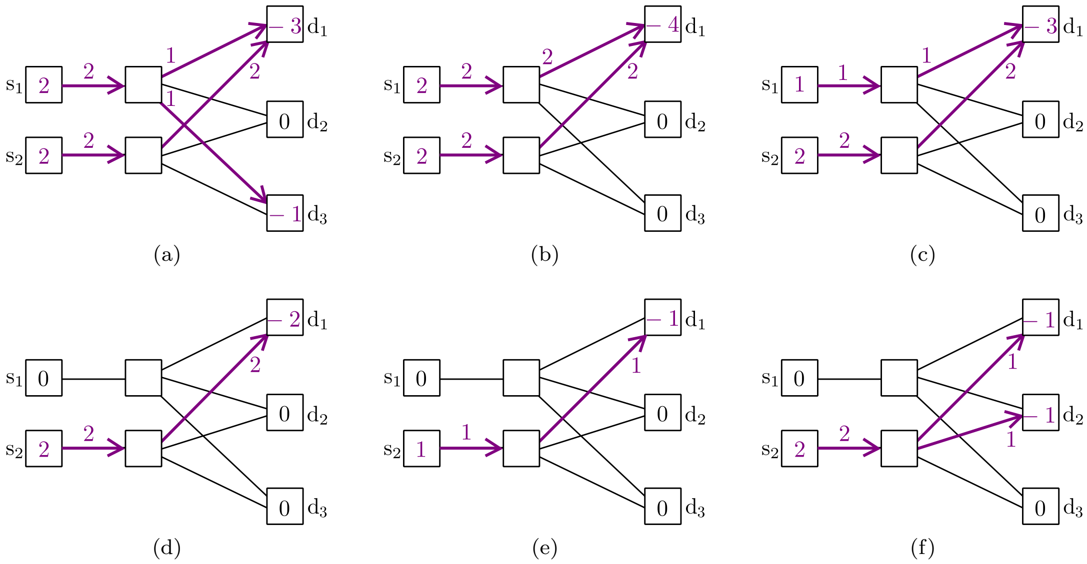

Figure 6: Definition of flows $\{\alpha'_i\}_{i=0}^{5}$. Decrements of stored energy are shown in the corresponding nodes. (a) α'_0, (b) α'_1, (c) α'_2, (d) α'_3, (e) α'_4 and (f) α'_5.

Now, we discuss the example, solving the problem using the cycle-canceling algorithm [29]. The optimal flows are shown in table 5 for different settings of $\gamma(\mathrm{s}_1)$ and $\gamma(\mathrm{s}_2)$. Note that "the needs of the consumers" are emphasized if $\gamma(\mathrm{s}_1)$ and $\gamma(\mathrm{s}_2)$ become small, while "the desire to maintain some reserve energy" is emphasized if $\gamma(\mathrm{s}_1)$ and $\gamma(\mathrm{s}_2)$ become large. Here, six flows are named as $\{\alpha'_i\}_{i=0}^{5}$ in figure 6, and each optimal flow is denoted by a sequence of $\{\alpha'_i\}_{i=0}^{5}$ in time order. For each optimal flow, we calculate the total supplied energy from source $s \in V_\mathrm{S}$, i.e. the value of $\sum_{n=0}^{N-1} \partial u(\overline{t_n}, s)$, and the total supplied energy to destination $d \in V_\mathrm{D}$, i.e. the value of $\sum_{n=0}^{N-1} (-\partial u(\overline{t_n}, d))$.

From table 5, we find the following properties:

- When $\gamma(\mathrm{s}_1) = \gamma(\mathrm{s}_2) = 0.1$, s_1 and s_2 transmit all given energies. As a result, d_3 and d_1, which have the highest priority and the second highest priority respectively, receive the whole required energy.

- When $\gamma(\mathrm{s}_1) = \gamma(\mathrm{s}_2) = 100$, s_1 and s_2 maintain reserve energy, i.e. in both the sources, not less than $K_{\mathrm{s}_1} = K_{\mathrm{s}_2} = 1$ energy is stored at each time. As a result, d_2, which has the lowest priority, does not receive any energy and d_1 does not receive the whole required energy.

These properties agree with the results of [32] which show that more energy is transmitted from sources when "the needs of the consumers" are emphasized, while less energy is transmitted from sources when "the desire to maintain some reserve energy" is emphasized. Because similar optimal flows are obtained by the two fundamentally different approaches, it is confirmed that our formulation captures fundamental properties of packetized power.

5 Conclusion

In this paper, we have established the packet-centric framework of electrical energy networks, defining symbols in power packetization as a minimum unit of electric power transferred by a power pulse with an information tag. Here, packetized power is spatially and temporally transferred as symbols in a digitized and quantized manner. At each node, the energy is represented as the total amount of energy of symbols which are sent to and received from neighbouring nodes during a finite duration.

To mathematically represent such transmission of packetized power, we introduced the SPM, in which a symbol is transferred at a link during a unit time. Via SPM, packetized power is described as a network flow in a spatio-temporal structure. Then, we considered a network flow problem for selecting an SPM in terms of transferability, that is, the possibility to represent given energies at sources and destinations during the finite duration. In networks, packetized power appears as supplied energy from sources and supplied energy to destinations (V1), transferred energy at each link during each unit time (V2), and change of stored energy in each router (V3). Setting a laminar family of subsets of nodes in spatio-temporal structure for the costs of V1 and V3, we can formulate this problem as an M-convex submodular flow problem which is a solvable generalization of the minimum cost flow problem. Unlike conventional minimum cost flow problems, here, we weighted not only values of network flow (V2) but also values of boundary of network flow and their time integrals (V1 and V3). Finally, the formulation was discussed through examples and it is shown that power can be packetized and be controllable while preserving reasonable properties of power.

The established packet-centric framework is completely different from the circuit theory, in which power is handled in a continuous manner and is governed by Kirchhoff Laws and Tellegen's theorem [1]. Here, the concept of a power packet is introduced as a unit of electric power, so that power is digitized and quantized. The results of this paper suggest a mathematical framework which integrates energy and information in electrical energy networks.

Data accessibility

This work does not have any experimental data. All computational results were obtained with the cycle-cancelling algorithm [29].

Authors' contributions

The concept of SPM was conceived by S.N. and A.M. The network flow problem was formulated and numerically simulated by S.N. T.H. designed the power packet network and initiated the study. The paper was drafted by S.N. and carefully revised by all the authors. All authors gave final approval for publication.

Competing interests

We declare we have no competing interests.

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