The study on quantum field theories from numerical approaches

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In the last several decades, the understanding of the theoretical particle physics is drastically improved. It has a significant impact on other branches of physics such as cosmology and the condensed matter physics. For example, since the correspondence between gauge theories and gravity theories is proposed, the non-perturbative study of the gauge theories in strong coupling is performed by the researchers all over the world. A famous prediction to an experiment from the correspondence is the shear viscosity of quark gluon plasma (QGP) in RHIC. The shear viscosity obtained from the AdS/CFT correspondence is close to the one given by the fluid mechanical simulation with the parameters fitted to the experiment. Similarly, in the study of the spectra in quantum chromodynamics (QCD), much improvements are also accomplished. The pion mass, which was once near 1 GeV, is reduced to physical mass in the lattice QCD studies. Moreover, large volume lattices such as a lattice with 96^4 grids are applied. Thanks to these improvements and the finding of the framework to obtain scattering phase shift , the studies of resonant states such as ρ, a_0 and even σ meson are conducted .

An underlying progress which enables such a breakthrough is the improvement in the computational resources. They enable us to investigate phenomenological parameters in realistic setup, and more importantly to forecast unknown physics. There is no decade when the infrastructure for the large scale computation is improved so fast in the whole history, and probably the new decade will be also the age of the largest improvement. For example, supercomputers today have a high performance of the order of ten peta FLOPS such as RIKEN K computer, while the performance of a supercomputer available in a decade ago was at most hundreds tera FLOPS. It means the computational capability of a supercomputer get to more than ten times larger in this ten years. On the other hand, unsolved problems in the theoretical particle physics are getting more and more difficult. This is partly because non-perturbative approaches are vital for the study of the quantum field theories in strong coupling, since perturbative approaches, which served for the study of quantum electrodynamics (QED), cannot be applied. In such problems, we have to face non-linear equations, the path-integrals with complicated weights or sophisticated algebraic calculations. These are exactly the field where numerical computations can be applied. Therefore, it is important to utilize numerical analysis for the non-perturbative study of quantum field theories.

In this thesis, we consider the studies on the scattering phase shift from the lattice QCD and chaotic solutions in superstring theory and M-theory with the knowledge of non-linear dynamics.

This thesis is organized as follows. In part I, we consider the method to obtain the phase shift in the channel which has resonant states from the combination of the potential method proposed by HAL QCD Collaboration (HAL QCD method) and Laplacian-Heaviside (LapH) smearing. We discuss the systematics in the method coming from the LapH smearing, and reveal that higher order terms in the derivative expansion is necessary for the study of high energy region. Next, the scattering phase shift of $\pi\pi$ scattering in $I = 1$ channel is considered. From the scattering phase shift in this channel, the sign of ρ resonance is obtained. Moreover, the pole is directly searched in complex energy plane, and found in some smearing parameters. Although the pole positions still deviate from the result given by finite volume method, we demonstrate the usability of HAL QCD method for conventional resonant states for the first time.

Part II is devoted to the study of chaotic solutions in superstring theory and M-theory. According to the AdS/CFT correspondence, there are operators in gauge theory side which are dual to the chaotic solutions in the classical string. Therefore, chaotic solutions in classical string are interesting probes for the study of Super Yang-Mills (SYM) theory in strong coupling beyond the supersymmetry and integrability. With the knowledge of the non-linear dynamics, we first show there are chaotic solutions in the classical string in a near Penrose limit of the $\mathrm{AdS}_5 \times \mathrm{T}^{1,1}$, and discuss the indication given by the existence. Since classical strings on $\mathrm{AdS}_5 \times \mathrm{T}^{1,1}$ background has chaotic solutions, but they disappear in the Penrose limit, it is revealed from our result that the chaotic solutions will remain until a near Penrose limit. It means that there are some almost BPS operators in the dual gauge theory. This fact will be useful for the study of the state/operator mapping. Likewise, we show the existence of chaotic solutions in BMN matrix mode. Since BMN matrix model represents supermembrane in the pp-wave background, our result means supermembrane also has chaotic solutions. In this way, we show that chaotic solutions prevail in the objects considered in superstring theory and M-theory.

Part III is devoted to the summary on whole this thesis.