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Energy Budget in Stratified Turbulence

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In stratified turbulence in atmosphere and oceans, often observed is the coexistence of weakly-nonlinear internal-wave turbulence, eddy turbulence and vertically-sheared horizontal flows. The internal-wave turbulence appears at the wavenumbers where the periods of the internal waves, τ_{wave}, are smaller than the eddy-turnover time, τ_{eddy}. On the other hand, the eddy turbulence appears at the wavenumbers larger than the buoyancy (Ozmidov) wavenumber, at which the eddy-turnover time is comparable with the Brunt-Väisälä period, τ_{BV}.

The energy is given to the internal gravity waves mainly from winds and tidal currents. The resonant triad interactions such as the induced diffusion, the elastic scattering and the parametric subharmonic instability play a dominant role in the energy transfer among internal waves according to the weak turbulence theory [2]. The internal waves break to the large-wavenumber eddies. The energy is further transferred to the large wavenumbers through the interactions among eddies, and is finally dissipated to the heat.

The energy transfer among the waves and that among the eddies are relatively well understood through the weak turbulence theory and Kolmogorov's theory, respectively, but the energy transfer between the waves and the eddies is still under investigation. A conjecture that gives the energy transfer is proposed [4]. The conjecture referred to as *critical balance* tells us that the energy is transferred along the wavenumbers at which the periods of the internal waves are comparable with the eddy-turnover time. The critical balance was partially supported in the MHD turbulence [3]. In this work, the energy transfers in the stratified turbulence where the weakly-nonlinear internal-wave turbulence, eddy turbulence and vertically-sheared horizontal flows coexist are numerically investigated.

The governing equation of a stably-stratified flow in the z direction with a constant Brunt-Väisälä frequency N under the Boussinesq approximation is given as

$$\frac{\partial}{\partial t}\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + b\boldsymbol{e}_z + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \tag{1a}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{1b}$$

$$\frac{\partial}{\partial t}b + (\boldsymbol{u}\cdot\nabla)b = -N^2\boldsymbol{u}\cdot\boldsymbol{e}_z + \kappa\nabla^2 b. \tag{1c}$$

Here, $\boldsymbol{u}$ and b are velocity and buoyancy variable, respectively. In the direct numerical simulation, where the periodic boundary condition is employed, the pseudo-spectral method with the aliasing removal is used for the calculation of the nonlinear term. The maximal wavenumber is $k_{\mathrm{max}} = 2^{11}\sqrt{2}/3 \approx 970$. The external force that excites the small wavenumbers, $7/2 \le |\boldsymbol{k}| \le 9/2$, is three-dimensional and two-component, and given by colored noise with the correlation time $1/N$ [1]. The Prandtl number is $Pr = 1$, that is, $\kappa = \nu$, and the Brunt-Väisälä is set to $N = 10$.

The kinetic energy of waves is defined as $K_{\mathrm{wave}\boldsymbol{k}} = |u_{\mathrm{wave}\boldsymbol{k}}|^2/2 = k^2|u_{z\boldsymbol{k}}|^2/(2k_\perp^2)$ according to the Craya-Herring decomposition. The ratio of the wave kinetic energy to the total kinetic energy $K_{\mathrm{wave}\boldsymbol{k}}/K_{\boldsymbol{k}} = K_{\mathrm{wave}\boldsymbol{k}}/(K_{\mathrm{eddy}\boldsymbol{k}} + K_{\mathrm{wave}\boldsymbol{k}})$ is drawn in Fig. 1. The wave kinetic energy is much larger than the eddies at the wavenumbers $\tau_{\mathrm{wave}} < \tau_{\mathrm{eddy}}/3$. Similarly, the wave kinetic energy is also comparable with the potential energy, and the polarization anisotropy is large at such wavenumbers. These facts demonstrate that the internal-wave turbulence is dominant at the wavenumbers $\tau_{\mathrm{wave}} < \tau_{\mathrm{eddy}}/3$, and the buffer region appears at the wavenumbers $\tau_{\mathrm{eddy}}/3 < \tau_{\mathrm{wave}} < \tau_{\mathrm{BV}}$.

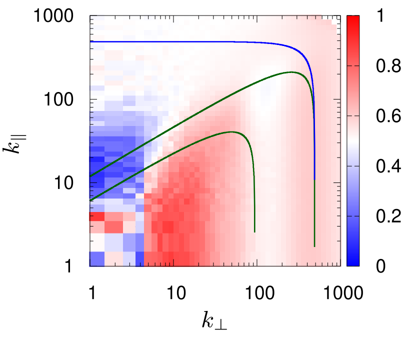

Figure 1: The wave kinetic energy normalized by the total kinetic energy. The green curves shows the critical wavenumbers at which the ratios of the wave period to the eddy-turnover time are 1/3 and 1. The blue curve indicates indicates the Ozmidov wavenumber.

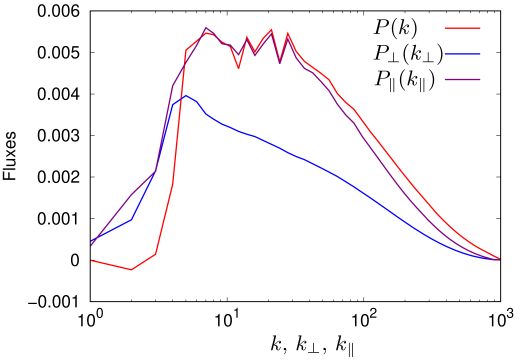

Figure 2: Isotropic, horizontal and vertical energy fluxes.

The energy transfer from/to a wavenumber via nonlinear interactions among wavenumbers is defined as $T_{\boldsymbol{k}} = \boldsymbol{u}_{\boldsymbol{k}}^* \cdot \boldsymbol{N}_{\boldsymbol{k}}/2 + \text{c.c.}$, where $\boldsymbol{N}_{\boldsymbol{k}}$ is the nonlinear term, which is derived from the advection and pressure terms, expressed in the Fourier space from in Eq. (1). Because it is a triad interaction that governs the nonlinear energy transfer, it is difficult to obtain one-to-one transfer or energy-flux *vector*. We first examine the energy flux as a scalar function here. The energy flux used in researches of the isotropic turbulence is defined as

$$P(k) = -\int_{|\boldsymbol{k}'|<k} T_{\boldsymbol{k}'} d\boldsymbol{k}', \tag{2}$$

which is called a spherical energy flux in this proceeding. Similarly, the horizontal energy flux and the vertical energy flux can be respectively defined as

$$P_\perp(k_\perp) = -\int_{|\boldsymbol{k}'_\perp|<k_\perp} T_{\boldsymbol{k}'} d\boldsymbol{k}', \qquad P_\parallel(k_\parallel) = -\int_{|k'_\parallel|<k_\parallel} T_{\boldsymbol{k}'} d\boldsymbol{k}'. \tag{3}$$

These energy fluxed are drawn in Fig. 2. In the inertial subrange, i.e., $k, k_\perp, k_\parallel = O(10)$, the vertical energy flux is almost the same as the spherical energy flux, and the horizontal energy flux is roughly half of those. It suggests that the energy is transferred to the large vertical wavenumbers, and the energy transfer to the large horizontal wavenumbers is small. It also implies that the energy transfer along the critical wavenumbers at which the periods of the internal waves are comparable with the eddy-turnover time is not so large.

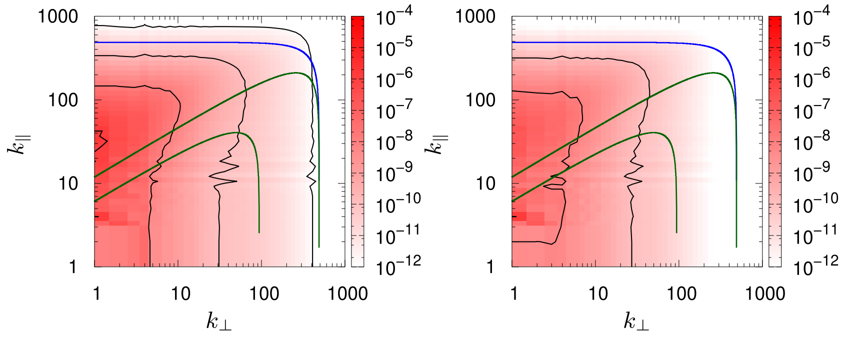

Figure 3: Kinetic energy dissipation rate (left) and diffusion rate (right).

The small energy transfer along the critical wavenumbers appears to be inconsistent with the critical balance. To observe this inconsistency, the kinetic energy dissipation rate and the diffusion rate are drawn in Fig. 3. Both of the kinetic energy dissipation rate and the diffusion rate are large at the small-horizontal large-vertical wavenumbers. Therefore, the energy cannot be transferred along the critical wavenumbers, since the energy is dissipated before it. We have never validated nor invalidated the critical balance, because the computational domain is not sufficiently large in the present numerical study. To conclude the energy transfer in the buffer region at the wavenumbers $\tau_{\text{eddy}}/3 < \tau_{\text{wave}} < \tau_{\text{BV}}$, much larger numerical simulations are required.

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[1] Y. Kimura and J. R. Herring, J. Fluid Mech., **698**, 19–50 (2012).
[2] C. H. McComas, J. Phys. Oceanogr., **7**, 836–845 (1977).
[3] R. Meyrand, S. Galtier, and K. H. Kiyani, Phys. Rev. Lett., **116**, 105002 (2016).
[4] S. V. Nazarenko and A. A. Schekochihin, J. Fluid Mech., **677**, 134–153 (2011).
[5] P. Sagaut and C. Cambon, Homogeneous Turbulence Dynamics, Cambridge University Press (2008).