数理解析研究所講究録 第2123巻 2019年 170-176

A C*-algebraic approach to quantum measurement

名古屋大学大学院情報科学研究科

岡村 和弥 *

I Introduction

A C*-algebraic approach to quantum measurement theory is proposed in the paper. Here, we treat processes of measurements in the Schrödinger picture, that enables us to totally describe dynamical changes induced by measurements in general quantum systems including those with infinite degrees of freedom.

C*-algebraic quantum theory can make the best use of noncommutative probability theory [12] and the duality theorem for C*-algebras proven by Takesaki [21] and Bichteler [2]. The existence of unitarily inequivalent *-representations of a given C*-algebra is not inevitable usually. This fact is, however, the merit of the theory rather than its difficulty: We can naturally introduce macroscopic classical levels in quantum systems described by C*-algebras. The quasi-equivalence of *-representations of C*-algebras then takes the place of the unitary equivalence.

The theory of operator algebras has been greatly contributed to quantum measurement theory from early days. In 1962, Nakamura and Umegaki [13] used the notion of conditional expectation [24, 23] to characterize the class of measurements for discrete observables called "the von Neumann-Lüders projection postulate". The importance of operator algebraic methods remains unchanged and is increasingly recognized now.

In Section II, we introduce preliminaries on algebraic quantum theory and sector theory. An equivalence relation of *-representations of C*-algebras, called the quasi-equivalence, is essential for the definition of sectors. In Section III, we describe measurements in the Schrödinger picture by completely positive (CP) instruments defined on central subspaces of duals of C*-algebras. Mathematical analysis for CP instruments defined on von Neumann algebras is efficiently used.

II Algebraic Quantum Theory and Sector Theory

To begin with, we give an axiomatic system of algebraic quantum theory.

Axiom 1 (Observables and states [17]). *All the statistical aspects of a physical system are registered in a C*-probablity space $(\mathcal{X}, \omega)$. Observables are described by self-adjoint elements of $\mathcal{X}$. On the other hand, ω statistically correponds to a physical situation (or an experimental setting).*

*okamura@math.cm.is.nagoya-u.ac.jp

Axiom 2 (Sector as event [17]). *For a state ω and a Borel set Δ of $E_{\mathcal{X}}$, $\mu_\omega(\Delta)$ gives the probability that a sector belongs to Δ under the situation described by ω. When available observables are restricted, the coarse-grained probability is given by $\mu_{\omega,\mathcal{B}}(\Delta)$ for some subalgebra $\mathcal{B}$ of $\mathcal{Z}_\omega(\mathcal{X})$.*

From now on, we shall introduce the mathematical notions appeared in the above axioms. We assume that C*-algebras are unital herein. The pair $(\mathcal{X}, \omega)$ of a C*-algebra $\mathcal{X}$ and a state ω on $\mathcal{X}$ is called a C*-probablity space. Axiom 1 states that every quantum system is described in the language of noncommutative probability theory (See [12] for an introduction to noncommutative probability theory).

Here we call a *-representation of a C*-algebra $\mathcal{X}$ a representation of $\mathcal{X}$ for simplicity. $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ denotes the GNS representation of a positive linear functional ω on $\mathcal{X}$. $\mathbb{B}(\mathcal{H})$ denotes the set of bounded linear operators on a Hilbert space $\mathcal{H}$. For any subset S of $\mathbb{B}(\mathcal{H})$, we define the commutant S' of S by $S' = \{A \in \mathbb{B}(\mathcal{H}) \mid \forall B \in S, AB = BA\}$ and the double commutant S'' of S by $S'' = (S')'$.

Definition 1 (Factor states). *A *-representation $(\pi, \mathcal{H})$ of $\mathcal{X}$ is called a factor *-representation of $\mathcal{X}$ if the center $\mathcal{Z}_\pi(\mathcal{X}) = \pi(\mathcal{X})'' \cap \pi(\mathcal{X})'$ of $\pi(\mathcal{X})''$ is trivial, i.e., $\mathcal{Z}_\pi(\mathcal{X}) = \mathbb{C}1$. A state ω on a C*-algebra $\mathcal{X}$ is called a factor state on $\mathcal{X}$ if $(\pi_\omega, \mathcal{H})$ is a factor representation of $\mathcal{X}$.*

By the definition, we can understand that each factor state corresponds to a physical situation whose values of order parameters are definite. Here we classify representations and states by the quasi-equivalence and the disjointness of them defined as follows.

Definition 2 (Quasi-equivalence and disjointness [3]).
(1) *Let $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ be *-representations of $\mathcal{X}$.*
(i) *$(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ are said to be quasi-equivalent if $\pi_1(\mathcal{X})''$ is *-isomorphic to $\pi_2(\mathcal{X})''$.*
(ii) *$(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ are said to be disjoint if there is no non-zero $V : \mathcal{H}_1 \to \mathcal{H}_2$ such that $\pi_2(X)V = V\pi_1(X)$ for all $X \in \mathcal{X}$.*
(2) *Let ω_1 and ω_2 be positive linear functionals on $\mathcal{X}$. ω_1 and ω_2 are said to be quasi-equivalent (or disjoint, respectively) if $(\pi_{\omega_1}, \mathcal{H}_{\omega_1})$ and $(\pi_{\omega_2}, \mathcal{H}_{\omega_2})$ are quasi-equivalent (or disjoint, respectively).*

Let $\mathcal{X}$ be a C*-algebra and $(\pi, \mathcal{H})$ a representation of $\mathcal{X}$. Let $\mathcal{M}$ be a von Neumann algebra on a Hilbert space $\mathcal{K}$. $\mathcal{M}_*$ denotes the set of ultraweakly continuous linear functionals on $\mathcal{M}$ and $\mathcal{S}_n(\mathcal{M})$ denotes that of normal states on $\mathcal{M}$. We define the subset $V(\pi)$ of $\mathcal{X}^*$ by

$$V(\pi) = \{\varphi \in \mathcal{X}^* \mid \exists \rho \in \pi(\mathcal{X})''_*, \forall X \in \mathcal{X}, \varphi(X) = \rho(\pi(X))\} \tag{1}$$

and the subset $\mathcal{S}_\pi(\mathcal{X})$ of $\mathcal{S}(\mathcal{X})$ by

$$\mathcal{S}_\pi(\mathcal{X}) = \{\varphi \in \mathcal{S}(\mathcal{X}) \mid \exists \rho \in \mathcal{S}_n(\pi(\mathcal{X})''), \forall X \in \mathcal{X}, \varphi(X) = \rho(\pi(X))\}. \tag{2}$$

Let C be a central projection of $\mathcal{X}^{**}$, i.e., $C \in \mathcal{Z}(\mathcal{X}'')$. A subspace $\mathcal{L}$ of $\mathcal{X}^*$ is called a central subspace of $\mathcal{X}^*$ if it has the form

$$\mathcal{L} = C\mathcal{X}^*. \tag{3}$$

Proposition 3 (See [22, Chapter III] for example)**.** *Let $\mathcal{X}$ be a C*-algebra and $(\pi, \mathcal{H})$ a representation of $\mathcal{X}$.*
There exists a central projection $C(\pi)$ of $\mathcal{X}^{**}$ such that

$$V(\pi) = C(\pi)\mathcal{X}^* = \{C(\pi)\varphi \mid \varphi \in \mathcal{X}^*\} = \{\varphi \in \mathcal{X}^* \mid C(\pi)\varphi = \varphi\}. \tag{4}$$

Proposition 4 (See [22, Chapter III] for example)**.** *Let $\mathcal{M}$ be a von Neumann algebra on a Hilbert space $\mathcal{H}$. There exists a central projection C of $\mathcal{M}^{**}$ such that $\mathcal{M}_* = C\mathcal{M}^*$.*

By the above two propositions, we see that $V(\pi)$ and $\mathcal{M}_*$ are typical central subspaces. Both the quasi-equivalence and the disjointness has equivalent conditions as follows:

Proposition 5 ([3], [22, Chapter III, Proposition 2.12])**.**
Let $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ be *-representations of $\mathcal{X}$. The following conditions are equivalent:
(i) $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ are quasi-equivalent. $\quad$ *(ii) $V(\pi_1) = V(\pi_2)$.*
(iii) $\mathcal{S}_{\pi_1}(\mathcal{X}) = \mathcal{S}_{\pi_2}(\mathcal{X})$. $\quad$ *(iv) $C(\pi_1) = C(\pi_2)$.*

Proposition 6 ([3])**.**
Let $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ be *-representations of $\mathcal{X}$. The following conditions are equivalent:
(i) $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ are disjoint. $\quad$ *(ii) $V(\pi_1) \cap V(\pi_2) = \{0\}$.*
(iii) $\mathcal{S}_{\pi_1}(\mathcal{X}) \cap \mathcal{S}_{\pi_2}(\mathcal{X}) = \emptyset$. $\quad$ *(iv) $C(\pi_1)C(\pi_2) = 0$.*

For any pair of factor states, the following theorem of alternatives holds, which enhances the importance of the quasi-equivalence of factor states.

Theorem 7. *Two factor states ω_1 and ω_2 on $\mathcal{X}$ are either quasi-equivalent or disjoint.*

This theorem follows from the proposition below.

Proposition 8 (Dixmier [5, Corollary 5.3.6])**.** *Two factor representations of $\mathcal{X}$ are either quasi-equivalent or else disjoint.*

We shall define the concept of sector, which is introduced by Ojima in order to present the extension of the superselection theory by Doplicher, Haag and Roberts [6, 7] and Doplicher and Roberts [8, 9, 10] into broken symmetry in a unified way (See also [15]).

Definition 9 (Sector [16])**.** *A quasi-equivalent class of a factor state is called a sector of $\mathcal{X}$. $\widehat{\mathcal{X}}$ denotes the set of sectors of $\mathcal{X}$.*

A sector corresponds to a "pure phase" as a generalization of thermodynamic (pure) phase. We can understand that two different factor states in the same sector of course describe different physical situations but they share the same value of order parameters. In other words, the concept of sector is a higher object than that of states and should be regarded as a generalization of the definition of thermodynamic pure phases in thermodynamics into the context of quantum theory and (quantum or noncommutative) probability theory. Geometric objects living in macroscopic classical levels are described via the sector space $\widehat{\mathcal{X}}$ of the system. To describe "mixed phases", we shall use the following theorem:

Theorem 10 ([3, Theorem 4.1.25 and Proposition 4.2.9]). *Let $\mathcal{X}$ be a C^*-algebra and ω a state on $\mathcal{X}$. There is a one-to-one correspondence between the two sets below:*
(1) The set of barycentric measures μ of ω such that $\int_\Delta \rho \, d\mu(\rho)$ and $\int_{E_\mathcal{X}\setminus\Delta} \rho \, d\mu(\rho)$ are disjoint for any $\Delta \in \mathcal{B}(E_\mathcal{X})$.
(2) The set of von Neumann subalgebras $\mathcal{B}$ of $\mathcal{Z}_\omega(\mathcal{X})$.
The above $\mathcal{B}$ is *-isomorphic to the image of the map $\kappa_\mu : L^\infty(E_\mathcal{X}, \mu) \ni f \mapsto \kappa_\mu(f) \in \mathcal{Z}_\omega(\mathcal{X})$ defined by

$$\langle \Omega_\omega | \kappa_\mu(f)\pi_\omega(X)\Omega_\omega\rangle = \int f(\rho)\rho(X) \, d\mu(\rho) \tag{5}$$

for all $X \in \mathcal{X}$ and $f \in L^\infty(E_\mathcal{X}, \mu)$.

The measure corresponding to the center $\mathcal{Z}_\omega(\mathcal{X})$ is called the central measure of ω and denoted by μ_ω. Furthermore, $\mu_{\omega,\mathcal{B}}$ denotes the barycentric measure of ω corresponding to a von Neumann subalgebra $\mathcal{B}$ of $\mathcal{Z}_\omega(\mathcal{X})$. Central measures of states have the following good property for our purpose.

Theorem 11 ([3, Theorem 4.2.10]). *For every state ω on $\mathcal{X}$, the central measure μ_ω of ω is pseudo-supported on the set $F_\mathcal{X}$ of factor states on $\mathcal{X}$. If $\mathcal{X}$ is separable, then μ_ω is supported by $F_\mathcal{X}$.*

The above theorem states that every state can be always decomposed into mutually disjoint factor states by its central measure. This fact allows us to interpret a general state as a probabilistic mixture of representatives of "sectors as elementary events". Therefore, we adopted Axiom 2.

III CP instruments defined on C*-algebras

Due to previous investigations [4, 19, 18], it is valid that we adopt the description of processes of quantum measurement in the Schrödinger picture by the concept of **completely positive (CP) instrument** when the observable algebra of the quantum system under consideration is a von Neumann algebra. It is known that there exists its operational characterization (see [18] for instance).

In order to define CP instruments on C*-algebras, we have to take transitions among sectors into account. For the purpose, we use central subspaces of the dual space of a given C*-algebra. The results presented here are simple extensions of those for the case of CP instruments defined on von Neumann algebras.

Let $\mathcal{M}$ and $\mathcal{N}$ be W^*-algebras. $P(\mathcal{M}_*, \mathcal{N}_*)$ denotes the set of positive linear maps of $\mathcal{M}_*$ into $\mathcal{N}_*$. Also, $\langle\cdot,\cdot\rangle$ denotes the pairing of $\mathcal{M}_*$ and $\mathcal{M}$.

Definition 12. *Let $\mathcal{X}$ be a C*-algebra and $(S, \mathcal{F})$ a measurable space. $\mathcal{I}$ is called an instrument for $(\mathcal{X}, S)$ if it satisfies the following three conditions:*
(1) $\mathcal{I}$ is a map of $\mathcal{F}$ into $P(C_{\rm in}\mathcal{X}^*, C_{\rm out}\mathcal{X}^*)$ for some non-zero σ-finite central projections $C_{\rm in}, C_{\rm out}$ of $\mathcal{X}^{**}$.

(2) $\langle \mathcal{I}(\Delta)\rho, 1\rangle = \langle \rho, 1\rangle$ *for all* $\rho \in C_{\rm in}\mathcal{X}^*$.
(3) *For every* $\rho \in C_{\rm in}\mathcal{X}^*$, $M \in (C_{\rm out}\mathcal{X}^*)^*$ *and mutually disjoint sequence* $\{\Delta_j\}_{j\in\mathbb{N}}$ *of* $\mathcal{F}$,

$$\langle \mathcal{I}(\cup_j \Delta_j)\rho, M\rangle = \sum_{j=1}^{\infty} \langle \mathcal{I}(\Delta_j)\rho, M\rangle. \tag{6}$$

An instrument $\mathcal{I}$ *for* $(\mathcal{X}, S)$ *is said to be completely positive (CP) if* $\mathcal{I}(\Delta)$ *is completely positive for all* $\Delta \in \mathcal{F}$.

When we emphasize that an instrument $\mathcal{I}$ for $(\mathcal{X}, S)$ is a map of $\mathcal{F}$ into $P(C_{\rm in}\mathcal{X}^*, C_{\rm out}\mathcal{X}^*)$, we say that $\mathcal{I}$ is an instrument for $(\mathcal{X}, C_{\rm in}, C_{\rm out}, S)$.

Let $\mathcal{M}$ be a von Neumann algebra on a Hilbert space $\mathcal{H}$. As seen in Section II, the predual $\mathcal{M}_*$ of $\mathcal{M}$ is a central subspace of $\mathcal{M}^*$. Thus the von Neumann algebraic definition of instruments is a special case of the above definition.

For every CP instrument $\mathcal{I}$ for $(\mathcal{X}, C_{\rm in}, C_{\rm out}, S)$ and normal state φ on $(C_{\rm in}\mathcal{X}^*)^*$, we define the probability measure $\|\mathcal{I}\varphi\|$ on $(S, \mathcal{F})$ by $\|\mathcal{I}\varphi\|(\Delta) = \|\mathcal{I}(\Delta)\varphi\|$ for all $\Delta \in \mathcal{F}$.

For every instrument $\mathcal{I}$ for $(\mathcal{X}, S)$, the dual map $\mathcal{I}^* : (C_{\rm out}\mathcal{X}^*)^* \times \mathcal{F} \to (C_{\rm in}\mathcal{X}^*)^*$ of $\mathcal{I}$ is defined by

$$\langle \mathcal{I}(\Delta)\rho, M\rangle = \langle \rho, \mathcal{I}^*(M, \Delta)\rangle \tag{7}$$

for all $\rho \in C_{\rm in}\mathcal{X}^*$, $M \in (C_{\rm out}\mathcal{X}^*)^*$ and $\Delta \in \mathcal{F}$. For every map $\mathcal{J} : (C_{\rm out}\mathcal{X}^*)^* \times \mathcal{F} \to (C_{\rm in}\mathcal{X}^*)^*$ satisfying the following three conditions, there uniquely exists an instrument $(\mathcal{X}, S)$ such that $\mathcal{J} = \mathcal{I}^*$:
(1) For every $\Delta \in \mathcal{F}$, the map $(C_{\rm out}\mathcal{X}^*)^* \ni M \mapsto \mathcal{J}(M, \Delta) \in (C_{\rm in}\mathcal{X}^*)^*$ is normal, positive and linear.
(2) $\mathcal{J}(1, S) = 1$.
(3) For every $\rho \in C_{\rm in}\mathcal{X}^*$, $M \in (C_{\rm out}\mathcal{X}^*)^*$ and mutually disjoint sequence $\{\Delta_j\}_{j\in\mathbb{N}}$ of $\mathcal{F}$,

$$\langle \rho, \mathcal{J}(M, \cup_j \Delta_j)\rangle = \sum_{j=1}^{\infty} \langle \rho, \mathcal{J}(M, \Delta_j)\rangle. \tag{8}$$

From now on, $\mathcal{I}$ denotes the dual map $\mathcal{I}^*$ of an instrument $\mathcal{I}$ for $(\mathcal{X}, S)$.

Let $\mathcal{X}, \mathcal{Y}$ be C*-algebras and $\mathcal{M}, \mathcal{N}$ von Neumann algebras. $\mathcal{X} \otimes_{\rm min} \mathcal{Y}$ denotes the injective tensor product of $\mathcal{X}$ and $\mathcal{Y}$, and $\mathcal{M} \overline{\otimes} \mathcal{N}$ does the W^*-tensor product of $\mathcal{M}$ and $\mathcal{N}$. For every CP instrument $\mathcal{I}$ for $(\mathcal{X}, S)$, there exists a unital (binormal) CP map $\Psi_{\mathcal{I}} : (C_{\rm out}\mathcal{X}^*)^* \otimes_{\rm min} L^\infty(S, \mathcal{I}) \to (C_{\rm in}\mathcal{X}^*)^*$ such that

$$\Psi_{\mathcal{I}}(M \otimes [\chi_\Delta]) = \mathcal{I}(M, \Delta) \tag{9}$$

for all $M \in (C_{\rm out}\mathcal{X}^*)^*$ and $\Delta \in \mathcal{F}$.

Here we define the normal extension property, family of posterior states and measuring process. All of them have played the role of deepening physics and mathematics of instruments defined von Neumann algebras [18]. We will see in Theorem 16 that their importance is not different for the case of instruments defined on C*-algebras.

Definition 13 (The normal extension property)**.** *Let $\mathcal{I}$ be a CP instrument for $(\mathcal{X}, S)$. $\mathcal{I}$ is said to have the normal extension property (NEP) if there exists a unital normal CP map $\widetilde{\Psi_{\mathcal{I}}}$: $(C_{\mathrm{out}}\mathcal{X}^*)^* \overline{\otimes} L^\infty(S, \mathcal{I}) \to (C_{\mathrm{in}}\mathcal{X}^*)^*$ such that*

$$\widetilde{\Psi_{\mathcal{I}}}|_{(C_{\mathrm{out}}\mathcal{X}^*)^*\otimes_{\min} L^\infty(S,\mathcal{I})} = \Psi_{\mathcal{I}}. \tag{10}$$

Definition 14 (Family of posterior states)**.** *Let $\mathcal{I}$ be an instrument for $(\mathcal{X}, S)$ and φ a normal state on $(C_{\mathrm{in}}\mathcal{X}^*)^*$. A family $\{\varphi_s\}_{s\in S}$ of normal states on $(C_{\mathrm{out}}\mathcal{X}^*)^*$ is called a family of posterior states with respect to $(\mathcal{I}, \varphi)$ if it satisfies the following two conditions:*
(1) The function $S \ni s \mapsto \varphi_s \in C_{\mathrm{out}}\mathcal{X}^*$ is weakly* $\|\mathcal{I}\varphi\|$-measurable.
(2) For all $M \in (C_{\mathrm{out}}\mathcal{X}^*)^*$ and $\Delta \in \mathcal{F}$,

$$\langle \mathcal{I}(\Delta)\varphi, M\rangle = \int_\Delta \langle \varphi_s, M\rangle \, d\|\mathcal{I}\varphi\|(s). \tag{11}$$

Definition 15 (Measuring process)**.** *A 4-tuple $\mathbb{M} = (\mathcal{K}, \sigma, E, U)$ of a Hilbert space $\mathcal{K}$, a normal state σ on $\mathbb{B}(\mathcal{K})$, a spectral measure $E : \mathcal{F} \to \mathbb{B}(\mathcal{K})$ and a unitary operator U on $\mathcal{H} \otimes \mathcal{K}$, is called a measuring process for $(\mathcal{X}, S)$ if it satisfies $\{\mathcal{I}_{\mathbb{M}}(M, \Delta) \mid M \in (C_{\mathrm{in}}\mathcal{X}^*)^*, \Delta \in \mathcal{F}\} \subset \mathcal{M}$, where C_{in} is a non-zero σ-finite central projection of $\mathcal{X}^{**}$, $\mathcal{H}$ is a Hilbert space on which faithfully represents elements of $(C_{\mathrm{in}}\mathcal{X}^*)^*$ as bounded operators and $\mathcal{I}_{\mathbb{M}} : \mathbb{B}(\mathcal{H}) \times \mathcal{F} \to \mathbb{B}(\mathcal{H})$ is defined by*[1]

$$\mathcal{I}_{\mathbb{M}}(X, \Delta) = (\mathrm{id} \otimes \sigma)[U^*(X \otimes E(\Delta))U] \tag{12}$$

for all $X \in \mathbb{B}(\mathcal{H})$ and $\Delta \in \mathcal{F}$.

The following is the main theorem of this section.

Theorem 16. *Let $\mathcal{X}$ be a C*-algebra and $(S, \mathcal{F})$ a measurable space. For an instrument $\mathcal{I}$ for $(\mathcal{X}, S)$, the following conditions are equivalent:*
(1) $\mathcal{I}$ has the NEP.
(2) For every normal state φ on $(C_1\mathcal{X}^*)^*$, there exists a strongly measurable family $\{\varphi_s\}_{s\in S}$ of posterior states with respect to $(\mathcal{I}, \varphi)$.
If $C_{\mathrm{in}} = C_{\mathrm{out}}$, the above conditions are equivalent to the condition below.
(3) There exists a measuring processes $\mathbb{M}$ for $(\mathcal{X}, S)$ such that $\mathcal{I} = \mathcal{I}_{\mathbb{M}}$.

Proof. We can prove the theorem in the same way as [18, Theorems 3.4 and 5.5]. □

参考文献

[1] W. Arveson, *An invitation to C*-algebras*, (Springer, New York, 2012).

[2] K. Bichteler, A generalization to the non-separable case of Takesaki's duality theorem for C*-algebras, Invent. Math. **9** (1969), 89–98.

[3] O. Bratteli & D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics* Vol.1 (2nd printing of 2nd ed.), (Springer, 2002).

[1]Let $\mathcal{M}$ and $\mathcal{N}$ be von Neumann algebras. For every $\sigma \in \mathcal{N}_*$, we define $\mathrm{id} \otimes \sigma : \mathcal{M} \overline{\otimes} \mathcal{N}$ by $\langle \rho \otimes \sigma, X\rangle = \langle \rho, (\mathrm{id} \otimes \sigma)(X)\rangle$ for all $\rho \in \mathcal{M}_*$ and $X \in \mathcal{M} \overline{\otimes} \mathcal{N}$.

[4] E.B. Davies & J.T. Lewis, An operational approach to quantum probability, Commun. Math. Phys. **17** (1970), 239–260.

[5] J. Dixmier, *C^*-Algebras*. (North-Holland, Amsterdam, 1977).

[6] S. Doplicher, R. Haag & J.E. Roberts, Fields, observables and gauge transformations I & II, Comm. Math. Phys. **13** (1969), 1–23; *ibid.* **15** (1969), 173–200.

[7] S. Doplicher, R. Haag & J.E. Roberts, Local observables and particle statistics, I & II, Comm. Math. Phys. **23** (1971), 199–230; *ibid.* **35** (1974), 49–85.

[8] S. Doplicher & J.E. Roberts, Endomorphism of C^*-algebras, cross products and duality for compact groups, Ann. of Math. **130** (1989), 75–119.

[9] S. Doplicher & J.E. Roberts, A new duality theory for compact groups, Invent. Math. **98** (1989), 157–218.

[10] S. Doplicher & J.E. Roberts, Why there is a field algebra with a compact gauge group describing the superselection structure in particle physics, Comm. Math. Phys. **131** (1990), 51–107.

[11] G.G. Emch, *Algebraic methods in statistical mechanics and quantum field theory*, (Wiley-Interscience, New York, 1972).

[12] A. Hora & N. Obata, *Quantum probability and spectral analysis of graphs*, Theoretical and Mathematical Physics, (Springer, Berlin, 2007).

[13] M. Nakamura & H.Umegaki, On von Neumann's theory of measurements in quantum statistics, Math. Japon. **7** (1962), 151–157.

[14] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, (Springer, Berlin, 1932); *Mathematical Foundations of Quantum Mechanics*, (Princeton UP, Princeton, 1955).

[15] I. Ojima, A unified scheme for generalized sectors based on selection criteria –Order parameters of symmetries and of thermality and physical meanings of adjunctions–, Open Sys. Inform. Dyn. **10** (2003), 235–279.

[16] I. Ojima, "Micro-Macro Duality in Quantum Physics,"pp.143-161 in Proc. Intern. Conf. on Stochastic Analysis, Classical and Quantum, (World Scientific, 2005), arXiv:math-ph/0502038.

[17] I. Ojima, K. Okamura & H. Saigo, Derivation of Born Rule from Algebraic and Statistical Axioms, Open Sys. Inform. Dyn. **21** (2014), 1450005.

[18] K. Okamura & M. Ozawa, Measurement theory in local quantum physics, J. Math. Phys. **57** (2016), 015209.

[19] M. Ozawa, Quantum measuring processes of continuous obsevables, J. Math. Phys. **25** (1984), 79–87.

[20] M. Ozawa, Uncertainty relations for noise and disturbance in generalized quantum measurements, Ann. Phys. (N.Y.) **331** (2004), 350–416.

[21] M. Takesaki, A duality in the representation theory of C^*-algebras, Ann. of Math. (2) **85** (1967), 370-382.

[22] M. Takesaki, *Theory of Operator Algebras I*, (Springer, Berlin, 1979).

[23] M. Takesaki, *Theory of Operator Algebras II*, (Springer, Berlin, 2003).

[24] H. Umegaki, Conditional expectation in an operator algebra, Tohoku Math. J. **6**(2) (1954), 177–181.