Initial value problem to a shallow water model with a floating solid body

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1 Introduction

We consider the two-dimensional motion of the water over a flat bottom together with the motion of a floating solid body on the water surface under the assumption that there are only two contact points where the water, the air, and the body meet. Let t be the time, x the horizontal spatial coordinate, and z the vertical spatial coordinate. The horizontal coordinates of these contact points at time t are denoted by $x_-(t)$ and $x_+(t)$, which satisfy $x_-(t) < x_+(t)$. Let $\mathcal{I}(t)$ and $\mathcal{E}(t)$ be the projections on the horizontal line of the parts where the water surface contacts with the floating body and the air, respectively, that is,

$$\begin{cases} \mathcal{I}(t) = (x_-(t), x_+(t)), \\ \mathcal{E}(t) = \mathcal{E}_-(t) \cup \mathcal{E}_+(t), \quad \mathcal{E}_-(t) = (-\infty, x_-(t)), \quad \mathcal{E}_-(t) = (x_+(t), \infty). \end{cases}$$

The corresponding water regions to $\mathcal{I}(t)$ and $\mathcal{E}(t)$ will be called the interior and the exterior regions, respectively. We consider the case where overhanging waves do not occur and suppose that the surface elevation of the water in the exterior region is denoted by $Z_{\mathrm{e}}(t,x)$ and that the underside of the floating body is parameterized by $Z_{\mathrm{i}}(t,x)$. See Figure 1. Let h_0 be the mean depth of the water, so that the water depth in the interior

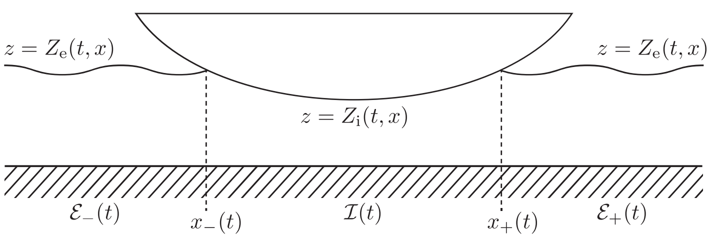

Figure 1: Waves interacting with a floating body

and exterior regions are given by $H_{\mathrm{i}}(t,x) = h_0 + Z_{\mathrm{i}}(t,x)$ and $H_{\mathrm{e}}(t,x) = h_0 + Z_{\mathrm{e}}(t,x)$, respectively. We denote by $\overline{V}(t,x)$ the vertically averaged horizontal velocity of the water and put $Q = H\overline{V}$, which is the horizontal flux of the water. The restrictions of Q to the interior and the exterior regions will be denoted by $Q_{\rm i}$ and $Q_{\rm e}$, respectively. Let $\underline{P}_{\rm i}(t,x)$ be the pressure of the water at the underside of the floating body. This pressure is an important unknown quantity and should be determined together with the motion of the water. In the case where the floating body moves freely, the body interacts with the water through the force exerted by this pressure. The shallow water model proposed by D. Lannes [2] was derived from the full water wave equations by using the assumption that $\partial_x\big(\int_{-h_0}^{Z(t,x)} V(t,x,z)^2 {\rm d}z\big) \simeq \partial_x\big(H(t,x)\overline{V}(t,x)^2\big)$, where $V(t,x,z)$ denotes the horizontal component of the velocity field in the water, and that the pressure $P(t,x,z)$ can be approximated by the hydrostatic pressure, that is,

$$P(t,x,z) \simeq \begin{cases} P_{\rm atm} - \rho {\tt g}(z - Z_{\rm e}(t,x)) & \text{in} \quad \mathcal{E}(t), \\ \underline{P}_{\rm i}(t,x) - \rho {\tt g}(z - Z_{\rm i}(t,x)) & \text{in} \quad \mathcal{I}(t), \end{cases}$$

where ρ is the density of the water, ${\tt g}$ the gravitational constant, and $P_{\rm atm}$ a constant atmospheric pressure. Then, the shallow water model for the water has the form

$$\begin{cases} \partial_t H_{\rm e} + \partial_x Q_{\rm e} = 0 & \text{in} \quad \mathcal{E}(t), \\ \partial_t Q_{\rm e} + \partial_x\left(\dfrac{Q_{\rm e}^2}{H_{\rm e}} + \dfrac{1}{2}{\tt g}H_{\rm e}^2\right) = 0 & \text{in} \quad \mathcal{E}(t), \end{cases} \tag{1}$$

in the exterior region, while under the floating body we have

$$\begin{cases} \partial_t H_{\rm i} + \partial_x Q_{\rm i} = 0 & \text{in} \quad \mathcal{I}(t), \\ \partial_t Q_{\rm i} + \partial_x\left(\dfrac{Q_{\rm i}^2}{H_{\rm i}} + \dfrac{1}{2}{\tt g}H_{\rm i}^2\right) = -\dfrac{1}{\rho}H_{\rm i}\partial_x\underline{P}_{\rm i} & \text{in} \quad \mathcal{I}(t), \end{cases} \tag{2}$$

with matching conditions

$$H_{\rm e} = H_{\rm i}, \quad Q_{\rm e} = Q_{\rm i}, \quad \underline{P}_{\rm i} = P_{\rm atm} \quad \text{on} \quad \Gamma(t), \tag{3}$$

where $\Gamma(t) = \partial\mathcal{I}(t) = \partial\mathcal{E}(t)$ denotes the contact points, which together with $(Z_{\rm e}, Q_{\rm e})$ and $(Z_{\rm i}, Q_{\rm i}, \underline{P}_{\rm i})$ are unknown quantities in our problem. We note that the equations in (1) are well-known nonlinear shallow water equations. As for the motion of the floating body, we consider three cases: the floating body is fixed; the motion of the body is prescribed; and the body moves freely according to Newton's laws, and that we also need to prescribe equations of the motion of the floating body according to these three cases.

1.1 The case of a fixed floating body

In the case where the body is fixed, we impose the condition

$$Z_{\rm i} = Z_{\rm lid} \quad \text{on} \quad \mathcal{I}(t), \tag{4}$$

where $Z_{\rm lid} = Z_{\rm lid}(x)$ is a given function defined on an open interval $I_{\rm f}$ and represents the shape of the underside of the floating body.

1.2 The case of a floating body with a prescribed motion

Since the floating body is allowed only to a solid motion, its motion is completely determined by $(x_G(t), z_G(t))$ the coordinates of the center of mass and $\theta(t)$ the rotational angle of the body. Without loss of generality, we have $\theta|_{t=0} = 0$. Suppose that the underside of the floating body is initially parameterized by $Z_{\rm lid}(x)$ on an open interval $I_{\rm f}$, so that $Z_{\rm i}|_{t=0} = Z_{\rm lid}$ in $\mathcal{I}(0)$. Consider a point of the underside of the body and denote the coordinates of the point at $t = 0$ by (X, Z). Let the coordinates of the point at time t be (x, z). Then, it holds that

$$Z = Z_{\rm lid}(X), \qquad z = Z_{\rm i}(t, x),$$

and that

$$\begin{pmatrix} x - x_G(t) \\ z - z_G(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & -\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) \end{pmatrix} \begin{pmatrix} X - x_G(0) \\ Z - z_G(0) \end{pmatrix}.$$

Therefore, we obtain

$$\begin{aligned} &(Z_{\rm i}(t,x) - z_G(t))\cos\theta(t) - (x - x_G(t))\sin\theta(t) + z_G(0) \\ &= Z_{\rm lid}\big((x - x_G(t))\cos\theta(t) + (Z_{\rm i}(t,x) - z_G(t))\sin\theta(t) + x_G(0)\big). \end{aligned} \tag{5}$$

This is the equation for the motion of the body and gives an expression of $Z_{\rm i}$ implicitly in terms of x_G, z_G, θ, and $Z_{\rm lid}$. Here we note that x_G, z_G, and θ are also given functions since we suppose that the motion of the floating body is prescribed in this case.

1.3 The case of a freely floating body

Finally, we consider the case where the floating body moves freely according to Newton's laws under the action of the gravitational force and the pressure from the air and from the water. Let $\mathfrak{m}$ and $\mathfrak{i}_0$ be the mass and the inertia coefficient of the body. Then, Newton's laws for the conservation of linear and angular momentum have the form

$$\begin{cases} \mathfrak{m}\partial_t \mathbf{U}_G(t) = -\mathfrak{m}\mathbf{g}\mathbf{e}_z - \displaystyle\int_{\partial\mathcal{C}(t)} P\boldsymbol{n}\,\mathrm{d}S, \\ \mathfrak{i}_0 \partial_t \omega(t) = \displaystyle\int_{\partial\mathcal{C}(t)} P\mathbf{r}_G^{\perp}\cdot\boldsymbol{n}\,\mathrm{d}S, \end{cases}$$

where $\mathbf{U}_G = (\partial_t x_G, \partial_t z_G)^{\rm T}$ and $\omega = \partial_t\theta$ are the velocity of the center of mass and the angular velocity of the body, respectively, $\mathcal{C}(t)$ is the domain occupied by the floating body at time t, P the pressure of the air and the water on the surface of the body, $\boldsymbol{n}$ the unit outward normal to $\partial\mathcal{C}(t)$, and $\mathbf{r}_G$ a position vector relative to the center of mass. Let $\partial_{\rm W}\mathcal{C}(t)$ and $\partial_{\rm A}\mathcal{C}(t)$ be portions of $\partial\mathcal{C}(t)$ in contact with the water and the air respectively.

Then, we see that

$$\begin{aligned}\int_{\partial\mathcal{C}(t)} P\boldsymbol{n}\mathrm{d}S &= \int_{\partial_{\mathrm{W}}\mathcal{C}(t)} \underline{P}_{\mathrm{i}}\boldsymbol{n}\mathrm{d}S + \int_{\partial_{\mathrm{A}}\mathcal{C}(t)} P_{\mathrm{atm}}\boldsymbol{n}\mathrm{d}S \\ &= \int_{\partial_{\mathrm{W}}\mathcal{C}(t)} (\underline{P}_{\mathrm{i}} - P_{\mathrm{atm}})\boldsymbol{n}\mathrm{d}S \\ &= -\int_{\mathcal{I}(t)} (\underline{P}_{\mathrm{i}}(t,x) - P_{\mathrm{atm}})N_{\mathrm{i}}(t,x)\mathrm{d}x,\end{aligned}$$

where

$$N_{\mathrm{i}}(t,x) = \begin{pmatrix} -\partial_x Z_{\mathrm{i}}(t,x) \\ 1 \end{pmatrix}.$$

In the derivation of the above equalities, we used the identity

$$\int_{\partial\mathcal{C}(t)} P_{\mathrm{atm}}\boldsymbol{n}\mathrm{d}S = \iint_{\mathcal{C}(t)} \nabla P_{\mathrm{atm}}\mathrm{d}x\mathrm{d}z = 0.$$

Similarly, we have

$$\int_{\partial\mathcal{C}(t)} P\mathbf{r}_G^{\perp}\cdot\boldsymbol{n}\mathrm{d}S = -\int_{\mathcal{I}(t)} (\underline{P}_{\mathrm{i}}(t,x) - P_{\mathrm{atm}})\mathbf{r}_G(t,x)^{\perp}\cdot N_{\mathrm{i}}(t,x)\mathrm{d}x,$$

where

$$\mathbf{r}_G(t,x) = \begin{pmatrix} x - x_G(t) \\ Z_{\mathrm{i}}(t,x) - z_G(t) \end{pmatrix}.$$

Therefore, Newton's laws for the conservation of linear and angular momentum are written in the form

$$\begin{cases} \mathfrak{m}\partial_t\mathbf{U}_G(t) = -\mathfrak{m}\mathbf{g}\mathbf{e}_z + \displaystyle\int_{\mathcal{I}(t)} (\underline{P}_{\mathrm{i}}(t,x) - P_{\mathrm{atm}})N_{\mathrm{i}}(t,x)\mathrm{d}x, \\ \mathfrak{i}_0\partial_t\omega(t) = -\displaystyle\int_{\mathcal{I}(t)} (\underline{P}_{\mathrm{i}}(t,x) - P_{\mathrm{atm}})\mathbf{r}_G(t,x)^{\perp}\cdot N_{\mathrm{i}}(t,x)\mathrm{d}x, \end{cases} \tag{6}$$

which together with (5) constitute the equations of motion for the floating body.

In this communication we report that the initial value problem to these wave-structure interactions are well-posed locally in time. The result was obtained through a joint research with David Lannes at University of Bordeaux.

2 Reformulation of the problem

We proceed to consider the initial value problem to the shallow water models with a floating solid body (1)–(3) together with (4), (5), or/and (6) according to the motion of the floating body. By using the equations of the floating body, we can reduce the shallow water equations (2) in the interior region $\mathcal{I}(t)$ to simple ordinary differential equations as follows.

2.1 The case of a fixed floating body

It follows from (4) that $H_{\rm i}(t,x)=h_0+Z_{\rm lid}(x)$ does not depend on t, so that the continuity equation in (2) yields $\partial_x Q_{\rm i}=0$. This means that $Q_{\rm i}$ does not depend on x, so that we can write $Q_{\rm i}(t,x)=q_{\rm i}(t)$. Plugging this into the momentum equation in (2) we have

$$\partial_t q_{\rm i}+\partial_x\biggl(\frac{q_{\rm i}^2}{H_{\rm i}}+\frac{1}{2}\mathtt{g}H_{\rm i}^2\biggr)=-\frac{1}{\rho}H_{\rm i}\partial_x\underline{P}_{\rm i},$$

which is equivalent to

$$\frac{\partial_t q_{\rm i}}{H_{\rm i}}+\partial_x\biggl(\frac{1}{2}\frac{q_{\rm i}^2}{H_{\rm i}^2}+\mathtt{g}H_{\rm i}\biggr)=-\frac{1}{\rho}\partial_x\underline{P}_{\rm i}.$$

Therefore, $\underline{P}_{\rm i}$ satisfies a simple boundary value problem

$$\begin{cases}\partial_x\underline{P}_{\rm i}=-\rho\biggl(\dfrac{\partial_t q_{\rm i}}{H_{\rm i}}+\partial_x\biggl(\dfrac{1}{2}\dfrac{q_{\rm i}^2}{H_{\rm i}^2}+\mathtt{g}H_{\rm i}\biggr)\biggr) & \text{in}\quad \mathcal{I}(t),\\ \underline{P}_{\rm i}=P_{\rm atm} & \text{on}\quad \Gamma(t).\end{cases}\tag{7}$$

Integrating the first equation in (7) and using the boundary condition, we obtain

$$\partial_t q_{\rm i}\int_{\mathcal{I}(t)}\frac{1}{H_{\rm i}}{\rm d}x+\biggl[\!\!\biggl[\frac{1}{2}\frac{q_{\rm i}^2}{H_{\rm i}^2}+\mathtt{g}H_{\rm i}\biggr]\!\!\biggr]=0,\tag{8}$$

where $[\![F]\!]=F(t,x_-(t))-F(t,x_+(t))$ for a function $F=F(t,x)$. This is a solvability condition of the boundary value problem (7) for $\underline{P}_{\rm i}$. Conversely, once $q_{\rm i}$ and $x_\pm$ are given so that (8) holds, we can resolve (7) for the pressure $\underline{P}_{\rm i}$ explicitly as

$$\begin{aligned}\underline{P}_{\rm i}(t,x)=P_{\rm atm}-\rho\biggl\{&\partial_t q_{\rm i}(t)\int_{x_-(t)}^{x}\frac{{\rm d}x'}{H_{\rm i}(x')}\\ &+\frac{1}{2}q_{\rm i}(t)^2\biggl(\frac{1}{H_{\rm i}(x)^2}-\frac{1}{H_{\rm i}(x_-(t))^2}\biggr)+\mathtt{g}\bigl(H_{\rm i}(x)-H_{\rm i}(x_-(t))\bigr)\biggr\}.\end{aligned}$$

Therefore, the equations in the interior region (2) are reduced to a scalar ordinary differential equation (8).

To summarize, the problem is reduced to the nonlinear shallow water equations in the exterior region

$$\begin{cases}\partial_t H_{\rm e}+\partial_x Q_{\rm e}=0 & \text{in}\quad \mathcal{E}(t),\\ \partial_t Q_{\rm e}+\partial_x\biggl(\dfrac{Q_{\rm e}^2}{H_{\rm e}}+\dfrac{1}{2}\mathtt{g}H_{\rm e}^2\biggr)=0 & \text{in}\quad \mathcal{E}(t),\end{cases}\tag{9}$$

the matching condition on the contact points

$$H_{\rm e}=H_{\rm i},\quad Q_{\rm e}=q_{\rm i}\quad\text{on}\quad\Gamma(t),\tag{10}$$

and the ordinary differential equation

$$\partial_t q_{\rm i}=-\frac{1}{\int_{\mathcal{I}(t)}\frac{1}{H_{\rm i}}{\rm d}x}\biggl[\!\!\biggl[\frac{1}{2}\frac{q_{\rm i}^2}{H_{\rm i}^2}+\mathtt{g}H_{\rm i}\biggr]\!\!\biggr],\tag{11}$$

where $H_{\rm i}$ is given by $H_{\rm i}(x)=h_0+Z_{\rm lid}(x)$. In this case, the initial conditions are given by

$$(Z_{\rm e},Q_{\rm e})|_{t=0}=(Z_{\rm e}^{\rm in},Q_{\rm e}^{\rm in})\quad\text{in}\quad\mathcal{E}(0),\quad x_\pm|_{t=0}=\underline{x}_\pm^{\rm in},\quad q_{\rm i}|_{t=0}=q_{\rm i}^{\rm in}.\tag{12}$$

2.2 The case of a floating body with a prescribed motion

We remind that (5) determines $Z_{\rm i}$ in terms of $\mathbf{G}=(x_G,z_G,\theta)$ and $Z_{\rm lid}$. More precisely, by the implicit function theorem we see that there exists a function ψ determined by $Z_{\rm lid}$ such that

$$Z_{\rm i}(t,x)=\psi(x-x_G(t),\theta(t))+z_G(t). \tag{13}$$

Differentiating (5) with respect to t and x, we see that

$$\partial_t Z_{\rm i}=(\mathbf{U}_G-\omega\mathbf{r}_G^{\perp})\cdot N_{\rm i}=-\partial_x\biggl(\begin{pmatrix}\mathbf{U}_G\\ \omega\end{pmatrix}\cdot\mathbf{T}(\mathbf{r}_G)\biggr),$$

where $\mathbf{T}(\mathbf{r}_G)$ is defined by

$$\mathbf{T}(\mathbf{r}_G)=\begin{pmatrix}-\mathbf{r}_G^{\perp}\\ \frac{1}{2}|\mathbf{r}_G|^2\end{pmatrix}.$$

This together with the continuity equation in (2) yields that there exists a function $\overline{q}_{\rm i}(t)$ of t such that

$$Q_{\rm i}(t,x)=\begin{pmatrix}\mathbf{U}_G(t)\\ \omega(t)\end{pmatrix}\cdot\mathbf{T}(\mathbf{r}_G(t,x))+\overline{q}_{\rm i}(t). \tag{14}$$

Plugging this into the momentum equation in (2), we obtain an equation for $\underline{P}_{\rm i}$. As a solvability condition of the boundary value problem to the equation for $\underline{P}_{\rm i}$ with the boundary condition $\underline{P}_{\rm i}=P_{\rm atm}$ on $\Gamma(t)$, we obtain an ordinary differential equation in the form

$$\partial_t\overline{q}_{\rm i}=F(\overline{q}_{\rm i},\mathbf{G},\partial_t\mathbf{G},\partial_t^2\mathbf{G},x_-,x_+)$$

with some function F. For the explicit form of this function F, we refer to T. Iguchi and D. Lannes [1] and D. Lannes [2].

To summarize, the problem is reduced to the nonlinear shallow water equations in the exterior region

$$\begin{cases}\partial_t H_{\rm e}+\partial_x Q_{\rm e}=0 & \text{in} \quad \mathcal{E}(t),\\ \partial_t Q_{\rm e}+\partial_x\biggl(\dfrac{Q_{\rm e}^2}{H_{\rm e}}+\dfrac{1}{2}\mathrm{g}H_{\rm e}^2\biggr)=0 & \text{in} \quad \mathcal{E}(t),\end{cases} \tag{15}$$

the matching condition on the contact points

$$H_{\rm e}=H_{\rm i},\quad Q_{\rm e}=Q_{\rm i}\quad\text{on}\quad\Gamma(t), \tag{16}$$

and the ordinary differential equation

$$\partial_t\overline{q}_{\rm i}=F(\overline{q}_{\rm i},\mathbf{G},\partial_t\mathbf{G},\partial_t^2\mathbf{G},x_-,x_+), \tag{17}$$

where $H_{\rm i}=h_0+Z_{\rm i}$ and $Q_{\rm i}$ are given by (13) and (14), respectively, and $\mathbf{G}=(x_G,z_G,\theta)$ are given functions of t. In this case, the initial conditions are also given by

$$(Z_{\rm e},Q_{\rm e})|_{t=0}=(Z_{\rm e}^{\rm in},Q_{\rm e}^{\rm in})\quad\text{in}\quad\mathcal{E}(0),\quad x_\pm|_{t=0}=\underline{x}_\pm^{\rm in},\quad\overline{q}_{\rm i}|_{t=0}=\overline{q}_{\rm i}^{\rm in}. \tag{18}$$

2.3 The case of a freely floating body

We note that the calculations in Section 2.2 are valid although $\mathbf{G} = (x_G, z_G, \theta)$ are unknown functions in this case and are governed by Newton's laws (6). Under the solvability condition (17) we can solve the pressure $\underline{P}_{\mathrm{i}}$. Plugging the expression of the pressure $\underline{P}_{\mathrm{i}}$ into Newton's laws (6), after some calculations we have a system of ordinary differential equations of the form

$$\partial_t \mathbf{W} = \boldsymbol{F}(\mathbf{W}, x_-, x_+)$$

with some function $\boldsymbol{F}$, where $\mathbf{W} = (\mathbf{G}, \partial_t \mathbf{G}, \overline{q}_{\mathrm{i}})$ are functions of t. Therefore, the problem is reduced to the nonlinear shallow water equations in the exterior region

$$\begin{cases} \partial_t H_{\mathrm{e}} + \partial_x Q_{\mathrm{e}} = 0 & \text{in} \quad \mathcal{E}(t), \\ \partial_t Q_{\mathrm{e}} + \partial_x \left(\dfrac{Q_{\mathrm{e}}^2}{H_{\mathrm{e}}} + \dfrac{1}{2} \mathtt{g} H_{\mathrm{e}}^2 \right) = 0 & \text{in} \quad \mathcal{E}(t), \end{cases} \tag{19}$$

the matching condition on the contact points

$$H_{\mathrm{e}} = H_{\mathrm{i}}, \quad Q_{\mathrm{e}} = Q_{\mathrm{i}} \quad \text{on} \quad \Gamma(t), \tag{20}$$

and the ordinary differential equations

$$\partial_t \mathbf{W} = \boldsymbol{F}(\mathbf{W}, x_-, x_+), \tag{21}$$

where $H_{\mathrm{i}} = h_0 + Z_{\mathrm{i}}$ and Q_{i} are given by (13) and (14), respectively. In this case, the initial conditions are given by

$$\begin{cases} (Z_{\mathrm{e}}, Q_{\mathrm{e}})|_{t=0} = (Z_{\mathrm{e}}^{\mathrm{in}}, Q_{\mathrm{e}}^{\mathrm{in}}) \quad \text{in} \quad \mathcal{E}(0), \qquad x_\pm|_{t=0} = \underline{x}_\pm^{\mathrm{in}}, \\ \overline{q}_{\mathrm{i}}|_{t=0} = \overline{q}_{\mathrm{i}}^{\mathrm{in}}, \qquad (x_G, z_G, \theta, \mathbf{U}_G, \omega)|_{t=0} = (x_G^{\mathrm{in}}, z_G^{\mathrm{in}}, 0, \mathbf{U}_G^{\mathrm{in}}, \omega^{\mathrm{in}}). \end{cases} \tag{22}$$

Now, our initial value problem to the shallow water model with a floating solid body on the water surface was reduced to (9)–(12) in the case of a fixed floating body, to (15)–(18) in the case of a floating body with a prescribed motion, and to (19)–(22) in the case of a freely floating body. However, all of the reduced problems have the same structure: a free boundary problem to the nonlinear shallow water equations with a Dirichlet type boundary condition on the free boundary coupled with a system of ordinary differential equations. These considerations motive us to analyze a new type of free boundary problem to a quasilinear hyperbolic system of equations.

3 Free boundary problem to 2×2 quasilinear hyperbolic system

Motivated by the reduction in Section 2, we consider a free boundary problem to a 2×2 quasilinear hyperbolic system of partial differential equations in a moving domain $(\underline{x}(t), \infty)$:

$$\partial_t U + A(U) \partial_x U = 0 \quad \text{in} \quad (\underline{x}(t), \infty) \tag{23}$$

with a Dirichlet type boundary condition

$$U = U_{\mathrm{i}} \quad \text{on} \quad x = \underline{x}(t), \tag{24}$$

where $U_{\mathrm{i}} = U_{\mathrm{i}}(t,x)$ is a given $\mathbf{R}^2$-valued function, whereas $\underline{x}(t)$ is an unknown function. As for the coefficient matrix $A(U)$ we assume the following.

Assumption 1 *Let $\mathcal{U}$ be an open set in $\mathbf{R}^2$, which represents a phase space of U.*

i. *$A \in C^\infty(\mathcal{U})$.*
ii. *For each $U \in \mathcal{U}$, the matrix $A(U)$ has eigenvalues $\pm\lambda_\pm(U)$ satisfying $\lambda_\pm(U) > 0$.*

In the case of nonlinear shallow water equations

$$\begin{cases} \partial_t H + \partial_x Q = 0, \\ \partial_t Q + \partial_x \left(\dfrac{Q^2}{H} + \dfrac{1}{2} \mathtt{g} H^2 \right) = 0, \end{cases}$$

the coefficient matrix $A(U)$ with $U = (Z, Q)^{\mathrm{T}}$ are given by

$$A(U) = \begin{pmatrix} 0 & 1 \\ \mathtt{g}H - (\frac{Q}{H})^2 & 2\frac{Q}{H} \end{pmatrix}$$

with $H = h_0 + Z$, so that the eigenvalues $\pm\lambda_\pm(U)$ are given by $\lambda_\pm(U) = \sqrt{\mathtt{g}H} \pm \frac{Q}{H}$. Therefore, the second condition in Assumption 1 corresponds to the subsonic condition in the gas dynamics.

3.1 Equation for the contact point

In the free boundary problem (23)–(24), we do not have any explicit equation for the contact point $\underline{x}(t)$ such as the kinematic boundary condition in the standard free boundary problems in the fluid dynamics. In our problem, the equation of the contact point $\underline{x}(t)$ is a part of the boundary condition (24). In fact, differentiating the boundary condition $U(t, \underline{x}(t)) = U_{\mathrm{i}}(t, \underline{x}(t))$ with respect to t, we have

$$\partial_t U + (\partial_t \underline{x}) \partial_x U = \partial_t U_{\mathrm{i}} + (\partial_t \underline{x}) \partial_x U_{\mathrm{i}} \quad \text{on} \quad x = \underline{x}(t),$$

which implies

$$\partial_t \underline{x} = - \frac{(\partial_t U - \partial_t U_{\mathrm{i}}) \cdot (\partial_x U - \partial_x U_{\mathrm{i}})}{|\partial_x U - \partial_x U_{\mathrm{i}}|^2} \bigg|_{x=\underline{x}(t)}.$$

In view of this, a discontinuity of the spatial derivative $\partial_x U$ on the free boundary is crucial to the free boundary problem (23)–(24) whereas U itself is continuous. Note that we have also

$$(\partial_t U - \partial_t U_{\mathrm{i}}) \cdot (\partial_x U - \partial_x U_{\mathrm{i}})^\perp = 0 \quad \text{on} \quad x = \underline{x}(t),$$

which can be viewed as a boundary condition to the hyperbolic system (23), so that the nonlinearity of the problem (23)–(24) is very high, especially, on the boundary. We also note that only one boundary condition is allowed under the subsonic condition in Assumption 1, that is, a part of the condition in (24) is used as a standard boundary condition and another part is used to determine the contact point $\underline{x}(t)$.

In the case of the shallow water model with a floating solid body on the water surface, this discontinuity condition is equivalent to a transversality condition for the water surface to the underside of the floating body. More precisely, we have the following proposition.

Proposition 1 *Suppose that* $U_\mathrm{e}=(Z_\mathrm{e},Q_\mathrm{e})^\mathrm{T}$, $U_\mathrm{i}=(Z_\mathrm{i},Q_\mathrm{i})^\mathrm{T}$, $\underline{P}_\mathrm{i}$, *and* $x_\pm$ *satisfy* (1)–(3). *Then, the condition* $\partial_x U_\mathrm{e}-\partial_x U_\mathrm{i}\neq 0$ *on* $\Gamma(t)$ *is equivalent to* $\partial_x Z_\mathrm{e}-\partial_x Z_\mathrm{i}\neq 0$ *on* $\Gamma(t)$.

3.2 Coordinate transformation

To analyze the free boundary problem (23)–(24) mathematically rigorously, we need to transform this problem on a moving domain $(\underline{x}(t),\infty)$ into another one cast on a fix domain, say, $\mathbf{R}_+$. This is done through a diffeomorphism $\varphi(t,\cdot)$ that maps $\mathbf{R}_+$ onto $(\underline{x}(t),\infty)$ for each time t, which should satisfy $\varphi(t,0)=\underline{x}(t)$. Several choices are possible for φ and we choose it appropriately. Here, we should emphasize that this diffeomorphism $\varphi(t,\cdot)$ is also unknown quantity. Once we have the contact point $\underline{x}(t)$, $\varphi(t,\cdot)$ is determined. We put $u=U\circ\varphi$ and introduce the notations

$$\partial_x^\varphi u=(\partial_x U)\circ\varphi,\qquad \partial_t^\varphi u=(\partial_t U)\circ\varphi.$$

Then, we have the relation

$$\partial_x^\varphi u=\frac{1}{\partial_x\varphi}\partial_x u,\quad \partial_t^\varphi u=\partial_t u-\frac{\partial_t\varphi}{\partial_x\varphi}\partial_x u=\partial_t u-(\partial_t\varphi)\partial_x^\varphi u, \tag{25}$$

and the free boundary problem (23)–(24) can be recast as a problem on a fix domain

$$\begin{cases}\partial_t^\varphi u+A(u)\partial_x^\varphi u=0 & \text{in}\quad (0,T)\times\mathbf{R}_+,\\ u=u_\mathrm{i} & \text{on}\quad (0,T)\times\{x=0\},\end{cases} \tag{26}$$

where $u_\mathrm{i}(t)=U_\mathrm{i}(t,\underline{x}(t))$ contains an unknown contact point $\underline{x}(t)$. The first equation in (26) is written in the standard form

$$\partial_t u+\mathcal{A}(u,\partial\varphi)\partial_x u=0,$$

where

$$\mathcal{A}(u,\partial\varphi)=\frac{1}{\partial_x\varphi}(A(u)-(\partial_t\varphi)\mathrm{Id}).$$

The eigenvalues of $\mathcal{A}(u,\partial\varphi)$ are given by $\pm\frac{1}{\partial_x\varphi}(\lambda_\pm(u)\mp\partial_t\varphi)$ and we will consider the solution $(u,\underline{x})$ of (26) satisfying $\lambda_\pm(u(t,x))\mp\partial_t\varphi(t,x)>0$. Under Assuption 1 and an appropriate choise of the diffeomorphism $\varphi(t,\cdot)$, this condition would be equivalent to $\lambda_\pm(u(t,0))\mp\partial_t\underline{x}(t)>0$. Without loss of generality, we can assume that $\underline{x}(0)=0$, so that we impose the initial conditions of the form

$$u|_{t=0}=u^\mathrm{in}\quad\text{in}\quad\mathbf{R}_+,\qquad \underline{x}(0)=0. \tag{27}$$

3.3 Linearized equations and good unknowns

We linearize (26) around $(u,\underline{x})$ and denote the variation in the linearization by $(\delta u,\delta\underline{x})$. It is important to introduce good unknowns $(\dot{u},\dot{\underline{x}})$ by

$$\dot{\underline{x}}:=\delta\underline{x},\qquad \dot{u}:=\delta u-(\delta\varphi)\partial_x^\varphi u.$$

In terms of these unknowns the linearized equations have the form

$$\begin{cases} \partial_t^\varphi \dot{u} + A(u)\partial_x^\varphi \dot{u} = \dot{f} & \text{in} \quad (0,T)\times \mathbf{R}_+, \\ \dot{u} + \underline{\dot{x}}(\partial_x^\varphi u - \partial_x^\varphi u_{\mathrm{i}}) = \dot{g} & \text{on} \quad (0,T)\times \{x=0\}, \end{cases} \tag{28}$$

where $\dot{f}$ and $\dot{g}$ are given functions. We note that thanks of the introduction of the good unknowns, the structure of the equations, that is, the principal part of the first equation in (28) does not change under this linearization. We decompose the boundary condition into the direction $\partial_x^\varphi u - \partial_x^\varphi u_{\mathrm{i}}$ and its perpendicular one to obtain

$$\begin{cases} \partial_t^\varphi \dot{u} + A(u)\partial_x^\varphi \dot{u} = \dot{f} & \text{in} \quad (0,T)\times \mathbf{R}_+, \\ \nu \cdot \dot{u} = \dot{g}_1 & \text{on} \quad (0,T)\times \{x=0\}, \end{cases} \tag{29}$$

and

$$\underline{\dot{x}} = \frac{\nu^\perp \cdot \dot{u}}{|\partial_x^\varphi u - \partial_x^\varphi u_{\mathrm{i}}|^2} + \dot{g}_2 \quad \text{on} \quad x=0, \tag{30}$$

where $\nu = (\partial_x^\varphi u - \partial_x^\varphi u_{\mathrm{i}})^\perp$. Here, we note that the first equation in (29) can be written as

$$\partial_t \dot{u} + \mathcal{A}(u, \partial\varphi)\partial_x \dot{u} = \dot{f},$$

where we are assuming that $\mathcal{A}(u,\partial\varphi)$ has positive and negative eigenvalues. Therefore, the equations for $\dot{u}$ and that for $\dot{x}$ are now decoupled, and that it is sufficient to analyze the initial and boundary value problem to (29).

3.4 Energy estimate and Kreiss–Lopatinskiĭ condition

In view of (29), we need to consider the classical initial and boundary value problem to a linear hyperbolic 2×2 system of equations

$$\begin{cases} \partial_t u + A(t,x)\partial_x u = f(t,x) & \text{in} \quad (0,T)\times \mathbf{R}_+, \\ \nu(t)\cdot u = g(t) & \text{on} \quad (0,T)\times\{x=0\}, \\ u = u^{\mathrm{in}}(x) & \text{on} \quad \{t=0\}\times \mathbf{R}_+, \end{cases} \tag{31}$$

where u, u^{in}, f, and ν are $\mathbf{R}^2$-valued functions and g is a real-valued function, while A takes their values in the space of 2×2 real-valued matrices. We also make the following assumption on the hyperbolicity of the system.

Assumption 2 *There exists $c_0 > 0$ such that the following assertions hold.*

- **i.** $A \in W^{1,\infty}((0,T)\times \mathbf{R}_+)$, $\nu \in C([0,T])$.
- **ii.** *For any $(t,x) \in (0,T)\times \mathbf{R}_+$, the matrix $A(t,x)$ has eigenvalues $\pm\lambda_+(t,x)$ satisfying $\lambda_\pm(t,x) \geq c_0$.*

Under the condition **ii** in Assumption 2, the system in (31) is particularly strictly hyperbolic, so that one can easily construct a symmetrizer $S(t,x)$, that is, a positive matrix with the property that $S(t,x)A(t,x)$ is symmetric. Then, by the standard calculations we have

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbf{R}_+} S(t,x)u(t,x)\cdot u(t,x)\mathrm{d}x - S(t,0)A(t,0)u(t,0)\cdot u(t,0)\\&= \int_{\mathbf{R}_+}\{(\partial_t S(t,x)+\partial_x(S(t,x)A(t,x)))u(t,x)+2S(t,x)f(t,x)\}\cdot u(t,x)\mathrm{d}x.\end{aligned}$$

In order to obtain a useful energy estimate, we have to control the boundary term $S(t,0)A(t,0)u(t,0)\cdot u(t,0)$. The next proposition relates the uniform Kreiss–Lopatinskiĭ condition with a control of this boundary term, and particularly, the condition yields the maximal dissipativity on the boundary.

Proposition 2 *Suppose that the conditions in Assumption 2, $|\nu(t)|\geq c_0$, and $|A(t,x)|\leq 1/c_0$ hold for some $c_0>0$. Then, the following four statements are all equivalent.*

- **i.** *There exist a symmetrizer $S\in W^{1,\infty}((0,T)\times\mathbf{R}_+)$ and positive constants α_0 and β_0 such that $\alpha_0\mathrm{Id}\leq S(t,x)\leq\beta_0\mathrm{Id}$ and that for any $v\in\mathbf{R}^2$ satisfying $\nu(t)\cdot v=0$ we have*
$$v^{\mathrm{T}}S(t,0)A(t,0)v\leq 0.$$
- **ii.** *(The maximal dissipativity.) There exist a symmetrizer $S\in W^{1,\infty}((0,T)\times\mathbf{R}_+)$ and positive constants α_0, β_0, α_1, and β_1 such that $\alpha_0\mathrm{Id}\leq S(t,x)\leq\beta_0\mathrm{Id}$ and that for any $v\in\mathbf{R}^2$ we have*
$$v^{\mathrm{T}}S(t,0)A(t,0)v\leq -\alpha_1|v|^2+\beta_1|\nu(t)\cdot v|^2.$$
- **iii.** *There exists a positive constant α_0 such that*
$$|\pi_-(t,0)\nu(t)^\perp|\geq\alpha_0,$$
where $\pi_\pm(t,x)$ is the eigenprojector associated to the eigenvalue $\pm\lambda_\pm(t,x)$ of $A(t,x)$.
- **iv.** *(The uniform Kreiss–Lopatinskiĭ condition.) There exists a positive constant α_0 such that*
$$|\nu(t)\cdot\mathbf{e}_+(t,0)|\geq\alpha_0,$$
where $\mathbf{e}_\pm(t,x)$ is the unit eigenvector associated to the eigenvalue $\pm\lambda_\pm(t,x)$ of $A(t,x)$.

Thanks of this proposition, if we impose the uniform Kreiss–Lopatinskiĭ condition, then we can show the well-posedness of the initial and boundary value problem (31). Now, we turn to consider the free boundary problem to the 2×2 quasilinear hyperbolic system (26). In view of the linearized problem (29), the corresponding Kreiss–Lopatinskiĭ condition would be

$$|\nu\cdot\mathbf{e}_+(u)|>0\quad\text{on}\quad x=0, \tag{32}$$

where $\nu=(\partial_x^\varphi u-\partial_x^\varphi u_{\mathrm{i}})^\perp$ and $\mathbf{e}_\pm(u)$ is the unit eigenvector associated to the eigenvalue $\pm\lambda_\pm(u)$ of $A(u)$. The following proposition helps us to check this Kreiss–Lopatinskiĭ condition.

Proposition 3 *Suppose that* u *together with* $\underline{x}$ *is a regular solution to* (26) *satisfying* $(\partial_x^\varphi u - \partial_x^\varphi u_\mathrm{i})|_{x=0} \neq 0$ *and* $\lambda_\pm(u(t,0)) \mp \partial_t \underline{x}(t) > 0$. *Then, there exists a unique unit vector* $\mu = \mu(t)$ *up to the sign such that*

$$\mu \cdot (\partial_t^\varphi u_\mathrm{i} + A(u_\mathrm{i})\partial_x^\varphi u_\mathrm{i})|_{x=0} = 0.$$

Moreover, we have the following identity on $x = 0$*:*

$$|\nu \cdot \mathbf{e}_+(u)| = \frac{(\lambda_+(u) - \partial_t \underline{x})|\partial_x^\varphi u - \partial_x^\varphi u_\mathrm{i}|}{|((\partial_t \underline{x})\mathrm{Id} - A(u))^\mathrm{T} \mu|} |\mu \cdot \mathbf{e}_+(u)|.$$

Therefore, under our restriction $(\partial_x^\varphi u - \partial_x^\varphi u_\mathrm{i})|_{x=0} \neq 0$ and $\lambda_\pm(u(t,0)) \mp \partial_t \underline{x}(t) > 0$, the Kreiss–Lopatinskiĭ condition (32) is equivalent to

$$|\mu \cdot \mathbf{e}_+(u)| > 0 \quad \text{on} \quad x = 0. \tag{33}$$

In the case of the shallow water model (1)–(3) with a floating solid body on the water surface, the eigenvector $\mathbf{e}_+(u)$ and μ are given by

$$\mathbf{e}_+(u) = \frac{1}{\sqrt{1 + \lambda_+(u)^2}} \begin{pmatrix} 1 \\ \lambda_+(u) \end{pmatrix}, \quad \mu = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

so that the Kreiss–Lopatinskiĭ condition is automatically satisfied.

4 Local well-posedness

We now fix the diffeomorphism $\varphi(t,\cdot) : \mathbf{R}_+ \to (\underline{x}(t), \infty)$ by

$$\varphi(t,x) = x + \psi\Big(\frac{x}{\varepsilon}\Big)\underline{x}(t), \tag{34}$$

where $\psi \in C_0^\infty(\mathbf{R})$ is a cut-off function such that $\psi(x) = 1$ for $|x| \leq 1$ and $= 0$ for $|x| \geq 2$, and $\varepsilon > 0$ is chosen to be sufficiently small. As for the local well-posedness of the initial value problem to (26), we have the following theorem.

Theorem 1 *Let* $m \geq 2$ *be an integer. Suppose that Assumption 1 is satisfied. If* $u^\mathrm{in} \in H^m(\mathbf{R}_+)$ *takes its values in a compact and convex set* $\mathcal{K}_0 \subset \mathcal{U}$ *and if the data* u^in *and* $U_\mathrm{i} \in W^{m,\infty}((0,T) \times (-\delta,\delta))$ *satisfy*

- **i.** $\lambda_\pm(u^\mathrm{in}|_{x=0}) \mp \underline{x}_1^\mathrm{in} > 0$,
- **ii.** $(\partial_x u^\mathrm{in})|_{x=0} - (\partial_x U_\mathrm{i})|_{t=x=0} \neq 0$,
- **iii.** $((\partial_x u^\mathrm{in})|_{x=0} - (\partial_x U_\mathrm{i})|_{t=x=0})^\perp \cdot \mathbf{e}_+(u^\mathrm{in}|_{x=0}) \neq 0$,

where $\underline{x}_1^\mathrm{in} = (\partial_t \underline{x})|_{t=0}$ *is determined by*

$$\underline{x}_1^\mathrm{in} = \frac{(A(u^\mathrm{in}|_{x=0})(\partial_x u^\mathrm{in})|_{x=0} + (\partial_t U_\mathrm{i})_{t=x=0}) \cdot ((\partial_x u^\mathrm{in})|_{x=0} - (\partial_x U_\mathrm{i})_{t=x=0})}{|(\partial_x u^\mathrm{in})|_{x=0} - (\partial_x U_\mathrm{i})_{t=x=0}|^2},$$

and the compatibility conditions up to order $m-1$*, then there exist* $T_1 \in (0,T]$ *and a unique solution* $(u,\underline{x})$ *to* (26) *in the time interval* $[0,T_1]$ *with the diffeomorphism* φ *defined by* (34) *and the solution satisfies*

$$\begin{cases} \partial_t^j u \in C([0,T_1];H^{m-j}(\mathbf{R}_+)) & \textit{for} \quad 0 \le j \le m-1, \\ (\partial_x^k u)|_{x=0} \in H^{m-k}(0,T_1) & \textit{for} \quad 0 \le k \le m, \\ \underline{x} \in H^m(0,T). \end{cases}$$

We turn to consider the local well-posedness of the initial value problem to the shallow water model with a floating solid body on the water surface. For simplicity, we restrict our consideration to the case of a freely floating body, so that we consider the initial value problem (19)–(22). The other cases can be treated in the same way. As before, we need to use a coordinate transformation to reduce the equations on the unknown region $\mathcal{E}(t)$ to those on a fixed region $\underline{\mathcal{E}}$. Let $\underline{x}_-^{\rm in}$ and $\underline{x}_+^{\rm in}$ be the initial contact points at time $t=0$ such that $\underline{x}_-^{\rm in} < \underline{x}_+^{\rm in}$ and put $\underline{\mathcal{E}}_- = (-\infty, \underline{x}_-^{\rm in})$, $\underline{\mathcal{E}}_+ = (\underline{x}_+^{\rm in}, \infty)$, and $\underline{\mathcal{E}} = \underline{\mathcal{E}}_- \cup \underline{\mathcal{E}}_+$. We use a diffeomorphism $\varphi(t,\cdot) : \underline{\mathcal{E}} \to \mathcal{E}(t)$ and put $\zeta_{\rm e} = Z_{\rm e} \circ \varphi$, $h_{\rm e} = H_{\rm e} \circ \varphi$, $q_{\rm e} = Q_{\rm e} \circ \varphi$, $\zeta_{\rm i} = Z_{\rm i} \circ \varphi$, and $q_{\rm i} = Q_{\rm i} \circ \varphi$. Such a diffeomorphism φ can be constructed as in (34), that is,

$$\varphi(t,x) = \begin{cases} x + \psi\left(\dfrac{x-\underline{x}_-^{\rm in}}{\varepsilon}\right)(x_-(t) - \underline{x}_-^{\rm in}) & \text{for} \quad x \in \underline{\mathcal{E}}_-, \\ x + \psi\left(\dfrac{x-\underline{x}_+^{\rm in}}{\varepsilon}\right)(x_+(t) - \underline{x}_+^{\rm in}) & \text{for} \quad x \in \underline{\mathcal{E}}_+, \end{cases} \tag{35}$$

where $\psi \in C_0^\infty(\mathbf{R})$ is a cut-off function such that $\psi(x) = 1$ for $|x| \le 1$ and $= 0$ for $|x| \ge 2$, and $\varepsilon > 0$ is chosen to be sufficiently small. As before, we will use the notation ∂_x^φ and ∂_t^φ which were defined by (25). Now, the problem under consideration is reduced to

$$\begin{cases} \partial_t^\varphi h_{\rm e} + \partial_x^\varphi q_{\rm e} = 0 & \text{in} \quad \underline{\mathcal{E}}, \\ \partial_t^\varphi q_{\rm e} + \partial_x^\varphi \left(\dfrac{q_{\rm e}^2}{h_{\rm e}} + \dfrac{1}{2}\mathtt{g} h_{\rm e}^2 \right) = 0 & \text{in} \quad \underline{\mathcal{E}}, \end{cases} \tag{36}$$

with the matching condition

$$h_{\rm e} = h_{\rm i}, \quad q_{\rm e} = q_{\rm i} \quad \text{on} \quad \partial\underline{\mathcal{E}}, \tag{37}$$

the ordinary differential equations

$$\partial_t \mathbf{W} = \boldsymbol{F}(\mathbf{W}, x_-, x_+), \tag{38}$$

and the initial conditions

$$\begin{cases} (\zeta_{\rm e}, q_{\rm e})|_{t=0} = (\zeta_{\rm e}^{\rm in}, q_{\rm e}^{\rm in}) \quad \text{in} \quad \mathcal{E}(0), & x_\pm|_{t=0} = \underline{x}_\pm^{\rm in}, \\ \overline{q}_{\rm i}|_{t=0} = \overline{q}_{\rm i}^{\rm in}, & (x_G, z_G, \theta, \mathbf{U}_G, \omega)|_{t=0} = (x_G^{\rm in}, z_G^{\rm in}, 0, \mathbf{U}_G^{\rm in}, \omega^{\rm in}). \end{cases} \tag{39}$$

Let us calculate $\underline{x}_{\pm,1}^{\rm in} = (\partial_t x_\pm)|_{t=0}$ in terms of the initial data. Differentiating the boundary condition $Z_{\rm e}(t, x_\pm(t)) = Z_{\rm i}(t, x_\pm(t))$ with respect to t and using the equation $\partial_t Z_{\rm e} + \partial_x Q_{\rm e} = 0$, we obtain $(\partial_x Z_{\rm e} - \partial_x Z_{\rm i})\partial_t x_\pm = (\partial_x Q_{\rm e} + \partial_t Z_{\rm i})$ on $\partial\underline{\mathcal{E}}_\pm$, so that

$$\underline{x}_{\pm,1}^{\rm in} = \left.\frac{Z_{{\rm i},1}^{\rm in} + \partial_x q_{\rm e}^{\rm in}}{\partial_x \zeta_{\rm e}^{\rm in} - \partial_x Z_{\rm lid}}\right|_{x=\underline{x}_\pm^{\rm in}}, \tag{40}$$

where $Z_{\mathrm{i},1}^{\mathrm{in}} = (\partial_t Z_{\mathrm{i}})|_{t=0}$ is given by

$$Z_{\mathrm{i},1}^{\mathrm{in}}(x) = \left(\mathbf{U}_G^{\mathrm{in}} + \omega^{\mathrm{in}} \begin{pmatrix} Z_{\mathrm{lid}}(x) - z_G^{\mathrm{in}} \\ -(x - x_G^{\mathrm{in}}) \end{pmatrix} \right) \cdot \begin{pmatrix} -\partial_x Z_{\mathrm{lid}}(x) \\ 1 \end{pmatrix}.$$

The following theorem asserts the local well-posedness of the initial value problem (36)–(39) to the shallow water model with a freely floating solid body on the water surface.

Theorem 2 *Let* $m \geq 2$ *be an integer and* I_{f} *an open interval. If the data* $(\zeta_{\mathrm{e}}^{\mathrm{in}}, q_{\mathrm{e}}^{\mathrm{in}}) \in H^m(\underline{\mathcal{E}})$, $\underline{x}_{\pm}^{\mathrm{in}} \in I_{\mathrm{f}}$, $(\overline{q}_{\mathrm{i}}^{\mathrm{in}}, x_G^{\mathrm{in}}, z_G^{\mathrm{in}}, \mathbf{U}_G^{\mathrm{in}}, \omega^{\mathrm{in}}) \in \mathbf{R}^6$, *and* $Z_{\mathrm{lid}} \in W^{m,\infty}(I_{\mathrm{f}})$ *satisfy*

- **i.** $\underline{x}_- < \underline{x}_+$,
- **ii.** $\inf_{x \in I_{\mathrm{f}}} (h_0 + Z_{\mathrm{lid}}(x)) > 0, \quad \inf_{x \in \underline{\mathcal{E}}} (h_0 + \zeta_{\mathrm{e}}^{\mathrm{in}}(x)) > 0$,
- **iii.** $\inf_{x \in \underline{\mathcal{E}}} \left(\sqrt{\mathsf{g}(h_0 + \zeta_{\mathrm{e}}^{\mathrm{in}}(x))} - \frac{|q_{\mathrm{e}}^{\mathrm{in}}(x)|}{h_0 + \zeta_{\mathrm{e}}^{\mathrm{in}}(x)} \right) > 0$,
- **iv.** $\left. \left(\sqrt{\mathsf{g}(h_0 + \zeta_{\mathrm{e}}^{\mathrm{in}})} - \left| \frac{q_{\mathrm{e}}^{\mathrm{in}}}{h_0 + \zeta_{\mathrm{e}}^{\mathrm{in}}} - \underline{x}_{\pm,1}^{\mathrm{in}} \right| \right) \right|_{\partial \underline{\mathcal{E}}} > 0$,
- **v.** $(\partial_x Z_{\mathrm{lid}} - \partial_x \zeta_{\mathrm{e}}^{\mathrm{in}})|_{\partial \underline{\mathcal{E}}} \neq 0$,

where $\underline{x}_{\pm,1}^{\mathrm{in}}$ *is defined by* (40), *and the compatibility conditions up to order* $m-1$, *then there exist* $T > 0$ *and a unique solution* $(\zeta_{\mathrm{e}}, q_{\mathrm{e}}, x_{\pm}, \overline{q}_{\mathrm{i}}, x_G, z_G, \theta)$ *in the time interval* $[0,T]$ *to* (36)–(39) *with the diffeomorphism* φ *given by* (35) *and the solution satisfies*

$$\begin{cases} \partial_t^j \zeta_{\mathrm{e}}, \partial_t^j q_{\mathrm{e}} \in C([0,T]; H^{m-j}(\underline{\mathcal{E}})) \quad \textit{for} \quad 0 \leq j \leq m-1, \\ (\partial_x^k \zeta_{\mathrm{e}})|_{\partial \underline{\mathcal{E}}}, (\partial_x^k q_{\mathrm{e}})|_{\partial \underline{\mathcal{E}}} \in H^{m-k}(0,T) \quad \textit{for} \quad 0 \leq k \leq m, \\ x_{\pm} \in H^m(0,T), \quad \overline{q}_{\mathrm{i}} \in H^{m+1}(0,T), \quad x_G, z_G, \theta \in H^{m+2}(0,T). \end{cases}$$

The details in this communication will be published in T. Iguchi and D. Lannes [1].

References

[1] T. Iguchi and D. Lannes, Hyperbolic free boundary problems and applications to wave-structure interactions, arXiv:1806.07704.

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