Relation of Earthquake Growth and Final Size with Applications to Magnitude Determination for Early Warning

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1. Introduction

An earthquake early warning (EEW) system can forewarn strong shaking to allow people and systems to take action to protect life and property. Commonly, EEW algorithms determine the source parameters (source location, and earthquake magnitude: M) in real time from the observed seismic data while the rupture is still in progress. The current EEW systems usually determine M using a ground motion prediction equation (GMPE) in which the displacement amplitude is proportional to M at a given distance (hereafter, the GMPE approach). This study focuses on the estimation of M_W in the source parameters for EEW. A fast and accurate M_W determination enables us to issue a more accurate alert at early times.

The GMPE approach has two technical limitations. The first is M of extremely large earthquakes (typically $M_W \geq 8.3$) can be underestimated because the frequency of data recorded by seismometers can be higher than the corner frequency of the source, resulting in saturation of the observed displacement. The second technical limitation faced while using the GMPE approach is that the final M is unknown until the peak amplitude is observed, suggesting that it is necessary to wait for the rupture termination in order to determine the final size of earthquakes. This study investigates relations of rupture growth and the final size of earthquakes and proposes approaches to solve the two limitations.

2. Magnitude Estimation from the Arrival Time of the Peak Amplitude in the High-Frequency

To solve the first technical limitation indicated above, we propose an approach that utilizes the time from the onset of the S wave to the peak amplitude arrival of the S wave (T_{op}). We analyze accelerograms observed at Kyoshin Net (K-NET) stations for 223 earthquakes with $4.0 \leq M_{\mathrm{J}} \leq 9.0$. An 8 – 16 Hz recursive band-pass filter (causal type) is applied before measuring T_{op}. The relationship between M_{W} and T_{op} demonstrates that T_{op} is correlated with M_{W}. Although the coefficient (slope) of the relationship cannot be explained by the assumptions of scale-invariant stress drop and constant rupture velocity, the relationship is valuable for the estimation of M_{W}.

To increase the number of observations for larger earthquakes, we also analyze teleseismic waveforms observed at Global Seismograph Network stations with events of M_{W} 6.6 – 9.3. We find that T_{op} measured using the teleseismic data ($T_{\mathrm{op}}^{\mathrm{t}}$) has a good correlation with M_{W} complementing those using the regional records (K-NET data) and that $T_{\mathrm{op}}^{\mathrm{t}}$ is not saturated even for events exceeding M_{W} 8, suggesting that the proposed approach is applicable to M estimation even for extremely large events.

We examine the estimation performance of M for EEW analyzing the regional K-NET dataset. The analysis shows that the T_{op} measurements in the higher frequency range display less scatter and that T_{op} may be only slightly affected by distance. We search for dependencies of T_{op} on rupture directivity and source depth but cannot find any significant dependencies. It is worth examining the influence of site effects on T_{op} but this is beyond the focus of this study. The minimum root mean squares (rms) of the residual between M_{W} and M_{Top} (M estimated from T_{op}) is 0.53 magnitude unit without considering those effects when using the T_{op} measurements for 8 – 16 Hz. Note that the accuracy can probably be improved by considering those effects in the future. For best results, the proposed approach should have access to as many measurements as possible while an earthquake is underway. A retrospective application for the 2011 Tohoku earthquake (M_W 9) demonstrates that M_{Top} reaches its final value of 9.3 at 120 s after the initiation for this event which had a complex rupture process.

3. Statistical Characterization of P-Wave Growth

To overcome the second technical limitation in the GMPE approach mentioned above, we examine the statistical characteristics of the initial P-wave waveforms observed at K-NET stations for 149 earthquakes with $4.5 \leq M_W \leq 8.7$. The dataset is binned in terms of M_W by 0.1 units and R by 25 km intervals, then the average of absolute of the displacement is computed in each bin. From this analysis, we find that the P-wave absolute displacement begins in a similar way for smaller and larger earthquakes at early times, but then departs from the similarity earlier for smaller events.

We measure the departure time (T_{dp}) from the similar growth pattern by comparing a predefined parameter DPD and the departure delay which represents a systematic decline in the absolute displacement. The scaling relation between T_{dp} and M_W is determined, which demonstrates that T_{dp} is approximately 20 – 30 % of typical rupture duration. This is important for EEW because we can, in a statistical sense, find a feature that scales with the final M_W before the arrival of the peak amplitude. Note that the scaling relation is established up to M_W 7 class earthquakes in this analysis. We check the effect of filters on these results and demonstrate that filtering does not weaken our conclusion.

A controversial topic in seismology is whether the final *M* is “deterministic” at the time of initiation. A number of previous studies indicated that earthquake ruptures were not deterministic because P-wave records exhibited indistinguishable onsets for earthquakes of different size. Alternatively, other studies showed that they could find a characteristic that depends on the final *M* before the rupture is terminated. Our result show a compromise between those two viewpoints, because T_{dp} is statistically shorter than the source durations but a similar initial P-wave growth is also confirmed. As a simple kinematic rupture model cannot explain the scaling relation between M_{W} and T_{dp}, we propose a rupture growth model that is based on the initial asperity size and rupture process in and around the asperity.

4. Magnitude Determination from the Time-Dependence of P-Wave Displacement

To apply the scaling relation between T_{dp} and M_{W} for determining *M* for EEW found in the previous chapter, we propose a new technique in which the intercept of a GMPE is set as time-dependent parameter, which differs from the conventional approach where the coefficients in the GMPE approach are constant. We hold the other coefficients (including the term for the distance attenuation) constant and determine the time-dependent intercept at each time step *T* only for time after the rupture expansion has departed from the pattern of similar growth. We analyze the initial 4 s displacement for the P wave of the K-NET dataset ($4.5 \leq M_{\mathrm{W}} \leq 8.7$) to examine the effectiveness of the proposed method and obtain the time-dependent intercepts up to $T = 4$ s.

Consequently, we find that the new technique can estimate *M* without the loss of accuracy even before the arrival of the peak amplitude, whereas the conventional method that uses a constant intercept significantly underestimates *M* for large events at early *T*. We demonstrate that the proposed approach is effective up to approximately M_W 7.5. *M* estimated from the time-dependent intercept technique is only 5.1 at $T = 4$ s for the 2011 Tohoku earthquake M_W 9.0. We can apply the constant intercept method to the data after $T = 4$ s, which enables us to maintain the estimation accuracy even for the greater earthquakes. The test in this study utilizes manually picked P-wave onset times, suggesting that using automatic picking method can increase the error of the *M* estimation. Although we demonstrate that the residual of the *M* estimates is not problematic if the picking error is less than approximately 0.2 s, it is possible that larger picking errors produce bigger residuals of the *M* estimates.

5. Conclusions

We proposed an approach that uses T_{op} measurements to solve the problem of the first technical limitation of the GMPE approach (*M* can be underestimated in earthquakes with $M_W \geq 8.3$) and also proposed the time-dependent intercept technique to overcome the second limitation (the final *M* cannot be derived until the peak amplitude arrival). As this study suggests separate methodologies to solve the two problems, it is necessary to combine them for practical applications in EEW systems in the future. For example, we can include the T_{op} measurement term into a GMPE to estimate *M*. We conclude that the techniques proposed in this study can improve the performance of EEW algorithms and that our findings can provide new insights into seismology.