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On the Non-Existence of a Zero-Tax Steady State with Incomplete Asset Markets*

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Abstract

Previous analyses suggest that a government can finance its expenditure by only using its asset income without taxes in the long run. We show that uninsured idiosyncratic earnings risk may overturn this result. In an Aiyagari-type model, we theoretically show that increasing government assets eventually decreases the interest rate below zero, suggesting an upper bound on government asset income. Hence, when government expenditure exceeds a threshold, there exists no zero-tax steady-state equilibrium, and the zero-tax policy is infeasible. Quantitatively, a government can raise small revenues without taxes. Increasing government assets may also generate rational asset price bubbles.

Keywords: Government assets. Equilibrium existence. Zero taxes. Bubbles. Incomplete markets. Heterogeneous agents.

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1 Introduction

Can a government raise all required revenues without taxes in the long run by accumulating a sufficient amount of assets? According to the conventional view, such a zero-tax policy is feasible and sometimes optimal. For example, Aiyagari, Marcet, Sargent, and Seppälä (2002) analyze an incomplete market version of the Lucas and Stokey (1983) economy without capital. They show that under some specifications, tax revenues almost surely converge to zero in the Ramsey outcome, regardless of the maximum government spending in the long run. Farhi (2010) reaches a similar conclusion for an economy with capital when capital income taxes are predetermined. Aiyagari, Marcet, Sargent, and Seppälä (2002) further argue that under a more general setting, it is impossible to rule out a long-run zero-tax policy as the Ramsey outcome. Thus, existing analyses suggest the feasibility of a long-run zero-tax policy.[1]

In the present study, we argue that a zero-tax policy is indeed feasible in the long run on the assumption that no uninsured idiosyncratic earnings risk exists. Using an Aiyagari (1994)-style heterogeneous agent, incomplete market model, we show that the presence of this risk makes a zero-tax policy infeasible at the steady state when government expenditure exceeds a certain threshold.

The model used here is based on the Aiyagari (1994) model, which is a neoclassical growth model with uninsured idiosyncratic earnings risk. We augment the model with endogenous labor supply; a similar framework has been widely used for fiscal policy analysis (e.g., Aiyagari and McGrattan (1998), Flodén (2001), and Féve, Matheron, and Sahuc (2018)).[2] We first theoretically prove that there exists an upper bound on government revenues, raised only with its assets and without taxes. This is established as follows. As government assets increase, the interest rate falls and eventually becomes

[1] Often, the optimal policy is to use a capital levy, if available, accumulate enough assets to finance government expenditures, and set all distortionary taxes to zero. See, for example, Ljungqvist and Sargent (2012).

[2] Other examples are Alonso-Ortiz and Rogerson (2010) and Nakajima and Takahashi (2019).

negative.[3] Further, the interest rate in an incomplete-market economy is below the interest rate in a corresponding complete-market economy, which is invariant to the amount of government assets. These facts imply that, in an incomplete-market economy, the government revenue raised only using its assets cannot exceed the minimum amount of government assets that leads to negative interest rates times the complete-market interest rate.[4] Thus, when government spending exceeds this threshold, there exists no zero-tax steady-state equilibrium, meaning that the zero-tax policy is infeasible. The infeasibility of the zero-tax policy holds for cases in which government spending is its consumption, lump-sum transfers to households, and any combination of the two.

We then evaluate the quantitative relevance of this result using a numerical method. We find that the zero-tax policy is unrealistic because without taxes, the government can raise only a small amount of revenue compared to the government spending in practice. Specifically, the maximum government revenue without taxes is around 0.7% of GDP for the United States. The numerical analysis also finds that there exists an inverse U-shaped relationship between government assets and the income from them.

Our theoretical analysis relies on an influential work by Zhu (2018), who establishes the existence of a steady-state equilibrium in an Aiyagari (1994) model with endogenous labor supply. In the model, assets are only claims for physical capital. For the capital-labor ratio, Zhu (2018) then introduces the supply and demand functions of the interest rate. The supply function summarizes the household-sector choice of the capital-labor ratio, whereas the demand function is derived from the firm-side optimization. Zhu (2018) shows that the demand and supply curves intersect, implying the existence of an equilibrium. We introduce government assets and define the supply and demand functions for the wealth-labor ratio. Due to the presence of government assets, the demand function in our setting has properties different from those in the Zhu (2018) setting. Neverthe-

[3] Aiyagari and McGrattan (1998), Flodén (2001), and Féve, Matheron, and Sahuc (2018) numerically show that an increase in government assets reduces the interest rate. We here theoretically establish that the interest rate eventually falls below zero as government assets increase.

[4] There could be multiple equilibria with different interest rates in the incomplete-market economy.

less, we can concretely show that increasing government assets reduces eventually the interest rate below zero and that there exists no zero-tax steady-state equilibrium when government expenditure exceeds a certain level.

Our result might be helpful for policy analysis conducted in a Bewley-Huggett-Aiyagari-type heterogeneous agent model with uninsured idiosyncratic earnings risk. Since this type of model cannot be solved analytically, researchers numerically solve for equilibrium under various policies and then search for the optimal policy.[5] It is essential to consider a wide range of policies because with idiosyncratic uncertainty and heterogeneity across agents, an extreme policy can be optimal under a certain welfare criteria.[6] Thus, showing the infeasibility of some policies, which is difficult to establish numerically, can facilitate policy evaluation. We also conjecture that a result similar to ours holds in a model that includes life-cycle elements in addition to idiosyncratic risk and heterogeneity, such as the model developed by Conesa, Kitao, and Krueger (2009).[7]

Lastly, we theoretically establish that increasing government assets may generate rational asset price bubbles. As government assets increase, the interest rate decreases and eventually becomes lower than the growth rate of the economy. Consequently, there exists an equilibrium where assets with no fundamental value have a positive price. Aoki, Nakajima, and Nikolov (2014) also argue that the presence of idiosyncratic risk may generate asset price bubbles, but they analyze investment risk in an endogenous growth model and do not discuss the role of fiscal policy. We here show that bubbles can emerge under a certain fiscal policy in an exogenous growth model with uninsured idiosyncratic labor income risk. We believe that this is a new finding in the literature.

The remainder of the present paper is organized as follows. Section 2 explains the

[5] Boháček and Kejak (2018) develop a new approach to characterize the optimal policy, but the method is currently applied to a model with fixed labor supply.

[6] For example, Flodén (2001) analyzes the optimal combination of government debt and transfers. When the government cares about inequality, the optimal policy is to set debt to the minimum. In contrast, without consideration for inequality, it is optimal to set debt to the highest level.

[7] Conesa, Kitao, and Krueger (2009) numerically show that the interest rate falls as government assets increase in their model.

model. Section 3 theoretically shows that there exists no zero-tax steady-state equilibrium when government expenditure exceeds a certain level. Section 4 uses a numerical method and quantifies the maximum government revenue that can be raised without taxes. Section 5 discusses the possibility of asset price bubbles. Section 6 concludes the paper. The appendix explains a numerical method.

2 Model

We use a Bewley-Huggett-Aiyagari model with incomplete insurance against idiosyncratic earnings risk. Time is discrete and goes from zero to infinity. Both physical capital accumulation and labor supply are endogenous. The model is similar to that of Aiyagari and McGrattan (1998), whose variants are widely used for analysis on fiscal policy (e.g., Flodén (2001), Alonso-Ortiz and Rogerson (2010), Féve, Matheron, and Sahuc (2018), and Nakajima and Takahashi (2019)). The exposition here particularly follows that of Zhu (2018), which shows the existence of steady-state equilibrium for this type of model without fiscal policy and on which our theoretical analysis relies.

2.1 Firms

Perfectly competitive firms produce a single good by renting factors of production from households. The production function $Y = F(K, N)$, where Y is output, K is capital input, and N is labor input, satisfies the following assumptions.

Assumption 1 (Assumption 6 of Zhu (2018)) *F displays constant returns to scale with $F_1, F_2 > 0$, and $F_{11}, F_{22} < 0$.*[8] *Further, F satisfies the Inada conditions: $\lim_{K\to\infty} F_1(K, 1) = 0$ and $\lim_{K\to 0} F_1(K, 1) = \infty$.*

The first-order conditions for profit maximization are

[8] Here F_1 is the partial derivative of F with respect to its first argument. Other expressions are defined in a similar way.

$$r = F_1(K, N) - \delta \tag{1}$$

and

$$w = F_2(K, N), \tag{2}$$

where r is the return on capital, w is the wage rate per efficiency unit of labor, and $\delta \in (0, 1)$ is the capital depreciation rate.

2.2 Households

There is a continuum of households (measure one), each of which is endowed with one unit of time in each period. The momentary utility function $u(c, l)$, where c is consumption and l is leisure, satisfies the following assumptions.

Assumption 2 (Assumptions 1–3 and 5$'$ of Zhu (2018)) *$u : R_+ \times [0, 1] \to R$ is twice continuously differentiable. $u(c, l)$ is strictly increasing and is strictly concave in c and l. The Inada conditions hold:*

$$\lim_{c \to 0} u_1(c, l) = \infty, \forall l \in [0, 1], \lim_{l \to 0} u_2(c, l) = \infty, \forall c \geq 0. \tag{3}$$

Further, $u(c, l) \in [0, M], M > 0$ and $u_{12} \geq 0$.

Households differ in their labor productivity x, which follows a discrete Markov chain. Let $\pi(x'|x)$ be the transition probability from x to x'. Hereinafter, a prime indicates the next period value.

Assumption 3 (Assumption 4 of Zhu (2018)) *$x \in X \equiv \{x_1, x_2, ..., x_n\}$, with $0 < x_1 < x_2 < \cdots < x_n$. $\sum_{x'} \pi(x'|x) = 1$ for all $x \in X$. Further, $\pi(x'|x) > 0$ for all $x, x' \in X$.*

Households cannot fully insure against idiosyncratic risk because asset markets are incomplete. There is a risk-free asset, which earns the risk-free return r.[9] Let a be a household's wealth. There is a borrowing constraint: $a^{'} \geq 0$. At the beginning of each period, households are distinguished by $(a, x) \in [0, \infty) \times X$.

Households maximize the discounted sum of the expected utility, subject to the budget and borrowing constraints. Let $V(a, x)$ be the value function for households. The problem of households is written as the following Bellman equation:

$$V(a,x) = \max_{\{c,l,a^{'}\}} \left\{ u(c,l) + \beta E[V(a^{'}, x^{'})|x] \right\} \tag{4}$$

$$\text{subject to } (1+\tau_c)c + a^{'} \leq [1 + (1-\tau_k)r]a + (1-\tau_n)wx\,(1-l) + T$$

$$c \geq 0, a^{'} \geq 0, 1 \geq l \geq 0,$$

where $\beta \in (0,1)$ is the discount factor and E is the conditional expectation. The first constraint is the budget constraint: τ_c, τ_k, and $\tau_n \in [0,1]$ denote the consumption, capital income, and labor income tax rates, respectively, while $T \geq 0$ is lump-sum transfers from the government to households.

2.3 Government

The government finances its consumption and lump-sum transfers to households through taxes and its assets. The government budget constraint in each period is

$$G + T + B^{'} = (1+r)B + \tau_n wN + \tau_k r(K-B) + \tau_c C, \tag{5}$$

[9]In equilibrium, the total savings of the household sector must be equal to the stock of physical capital minus the household sector's borrowing from the government. See the asset-market clearing condition in Section 2.4.

where G is government consumption, B represents the assets held by the government, and C is aggregate private consumption.

2.4 Stationary Equilibrium

Let $S \equiv [0, \bar{a}] \times X$ and $s \equiv (a, x) \in S$. Let Γ be the probability measure that describes the distribution of households over idiosyncratic state s, which consists of wealth a and idiosyncratic productivity x, defined on $(S, \mathbf{B}(S))$, where $\mathbf{B}(S)$ is the Borel algebra on S.[10] We focus on stationary equilibrium in which aggregate variables and the cross-sectional distribution are unchanged over time. Given the government policy $(G, T, B, \tau_c, \tau_k, \tau_n)$, a stationary competitive equilibrium is $(w, r, V, c, a^{'}, l, Y, K, N, C, \Gamma)$ that satisfies the following conditions.

1. Households' optimization:

   $V(s)$ satisfies (4), while $c(s), a^{'}(s)$, and $l(s)$ are the associated policy functions.

2. Firms' optimization:

   The representative firm chooses K and N to satisfy (1) and (2).

3. Labor market clearing:

   $N = \int_S x\,[1 - l(s)]\, d\Gamma.$

4. Asset market clearing:

   $K^{'} - B^{'} = \int_S a^{'}(s) d\Gamma.$

5. Goods market clearing:

   $C + \delta K + G = Y$ with $C = \int_S c(s) d\Gamma.$

[10]Zhu (2018) shows that there is an upper bound on household wealth under $r < 1/\beta - 1$, which holds in equilibrium.

6. Government budget constraint:

   The government budget constraint (5) holds.

7. Stationary household distribution:

   For all $D \in \mathbf{B}(S)$,

   $\Gamma(D) = \int_S P(s, D) d\Gamma$,

   where $P(s, D)$ is the transition function, which shows the probability that a household with idiosyncratic state s moves into idiosyncratic states in $D \in \mathbf{B}(S)$ and which can be constructed from households' policy function for savings and the transition probability of x.

3 Theoretical Analysis

In this section, we theoretically establish the infeasibility of the zero-tax policy when government expenditure exceeds a certain level by showing the non-existence of stationary equilibrium under the policy. The following proposition summarizes the result.

Proposition 4 *(1) Suppose that $T = 0$. There exists a $\bar{G}$ such that there is no zero-tax steady-state equilibrium for any $G > \bar{G}$.*

(2) Suppose that $G = 0$. There exists a $\bar{T}$ such that there is no zero-tax steady-state equilibrium for any $T > \bar{T}$.

Proof. Consider (1) so that $\tau_k = \tau_n = \tau_c = T = 0$ and $G = rB$. As Theorem 3 of Zhu (2018) shows, there is a unique invariant household distribution $\Gamma(s|r)$ for each level of the interest rate $r \in (-\delta, 1/\beta - 1)$. Define $\zeta(r)$ as

$$\zeta(r) \equiv \frac{\int_S a d\Gamma(r)}{\int_S x[1 - l(s|r)] d\Gamma(r)}, \tag{6}$$

where $l(s|r)$ is the policy function for leisure, which depends on the interest rate r. The optimal leisure choice, of course, also depends on the wage rate w, but under firms' optimization, w is a function of r. Note that $\zeta(r)$ is considered to be the aggregate "supply function" for the wealth-labor ratio. As Lemma 9 of Zhu (2018) shows, $\zeta(r)$ is continuous in $r \in (-\delta, 1/\beta - 1)$ and $\lim_{r \uparrow 1/\beta - 1} \zeta(r) = +\infty$. However, $\zeta(r)$ may not be monotone.

Next, define $\xi(r)$ as

$$\xi(r) \equiv f^{-1}(r + \delta) - \frac{B}{\int_S x[1 - l(s|r)] d\Gamma(r)}, \tag{7}$$

where

$$f\left(\frac{K}{N}\right) \equiv F_1\left(\frac{K}{N}, 1\right) = F_1(K, N). \tag{8}$$

Note that $\xi(r)$ can be seen as the aggregate "demand function" for the wealth-labor ratio.

As for the properties of $\xi(r)$, we need some discussion. In the case of Zhu (2018), since there is no government asset, $B = 0$ and $\xi(r) \equiv f^{-1}(r + \delta)$. Therefore, $\xi(r)$ is continuous and monotonically decreasing in $r \in (-\delta, 1/\beta - 1)$, with $\lim_{r \uparrow 1/\beta - 1} \xi(r) = f^{-1}(1/\beta - 1 + \delta)$ and $\lim_{r \downarrow -\delta} \xi(r) = +\infty$. By contrast, when $B > 0$, the property of $\xi(r)$ depends on how $\int_S x[1 - l(s|r)] d\Gamma(r)$ changes with r. As Lemma 8 of Zhu (2018) shows, $\int_S x[1 - l(s|r)] d\Gamma(r)$ is continuous in $r \in (-\delta, 1/\beta - 1)$, which implies that $\xi(r)$ is continuous. Zhu (2018) also shows that $\int_S x[1 - l(s|r)] d\Gamma(r) > 0$ for $r \in [-\delta, 1/\beta - 1)$ and $\lim_{r \uparrow 1/\beta - 1} \int_S x[1 - l(s|r)] d\Gamma(r) = 0$. Hence, for $B > 0$, $\lim_{r \uparrow 1/\beta - 1} \xi(r) = -\infty$ and $\lim_{r \downarrow -\delta} \xi(r) = +\infty$. Note that in contrast to the case of Zhu (2018), $\xi(r)$ may not be monotone.

The equilibrium interest rate is a solution to

$$\zeta(r) = \xi(r). \tag{9}$$

Given the properties of $\zeta(r)$ and $\xi(r)$ discussed above, the aggregate supply and demand curves for the wealth-labor ratio intersect, implying the existence of an equilibrium. However, it may not be unique because $\zeta(r)$ and $\xi(r)$ may not be monotone.[11] Note that for the case considered by Zhu (2018), the non-monotonicity of $\zeta(r)$ is the only potential source of multiple equilibria.

Define $\bar{\xi}(r)$ as

$$\bar{\xi}(r) \equiv f^{-1}(r+\delta) - \frac{B}{\int_S x d\Gamma(r)}. \tag{10}$$

Note that for $r \in (-\delta, 1/\beta - 1)$ and $B \geq 0$,

$$\xi(r) \leq \bar{\xi}(r). \tag{11}$$

Further, since $\int_S x d\Gamma(r)$ is the mean of idiosyncratic productivity, it is independent of r. Therefore, $\bar{\xi}(r)$ is continuous and monotonically decreasing in r, with $\lim_{r \uparrow 1/\beta - 1} \bar{\xi}(r) = f^{-1}(1/\beta - 1 + \delta) - B/\left[\int_S x d\Gamma(r)\right]$ and $\lim_{r \downarrow -\delta} \bar{\xi}(r) = +\infty$. Given these properties of $\bar{\xi}(r)$, there exists an r that satisfies $\zeta(r) = \bar{\xi}(r)$. There could be multiple solutions because $\zeta(r)$ may not be monotone.

Let $\bar{\Omega}(B)$ be the set of solutions to $\zeta(r) = \bar{\xi}(r)$ and $\Omega(B)$ the set of solutions to $\zeta(r) = \xi(r)$. Then,

$$r^*(B) \equiv \max \Omega(B) \leq \bar{r}(B) \equiv \max \bar{\Omega}(B). \tag{12}$$

Given $T = 0$, increasing B does not affect $\zeta(r)$, but shifts $\bar{\xi}(r)$ leftward. Since $\zeta(r)$ is continuous, there exists a $\bar{B}$ such that $\bar{r}(B) < 0$ for $B > \bar{B}$. Hence, for any $B \in [0, \bar{B}]$,

$$r^*(B)B \leq \bar{r}(B)B < \left(\frac{1}{\beta} - 1\right)\bar{B} \equiv \bar{G}. \tag{13}$$

This implies that there is no zero-tax steady-state equilibrium for $G > \bar{G}$.

[11] In the numerical analysis below, however, we do not find any multiple equilibria, consistent with the results of previous numerical analysis such as Aiyagari and McGrattan (1998) and Flodén (2001).

Consider (2). In this case, $\tau_k = \tau_n = \tau_c = G = 0$ and $T = rB$. A change in B now affects T and shifts $\zeta(r)$. However, $\zeta(0)$ does not move from the $B = 0$ case because for any $B > 0$, $T = 0$ at $r = 0$. Hence, the above discussion can be applied in a straightforward way. ■

There are two remarks. First, there could be no equilibrium even when $G \leq \bar{G}$ because $G > \bar{G}$ is a sufficient condition for the non-existence of zero-tax steady-state equilibrium. The next section uses a numerical method and quantifies the maximum government spending that can be financed through its asset income alone. Second, the non-existence result obviously holds when the government uses a part of its revenue for its consumption and the rest for lump-sum transfers to households.

4 Quantitative Analysis

The previous section theoretically shows that no zero-tax steady-state equilibrium exists when government expenditure exceeds a certain level. This section uses a numerical method and analyzes the quantitative relevance of the result.

4.1 Parameter Values

The model is calibrated to the U.S. economy. We use standard parameter values (Table 1), which are mostly taken from Trabandt and Uhlig (2011). One period in the model is set to one year. The capital depreciation rate δ is 0.07. The production function is $Y = K^{\theta}N^{1-\theta}$, which satisfies Assumption 1. The capital share θ is 0.38. With respect to the government policy, the consumption tax rate τ_c is 0.05. The labor income tax rate τ_n is 0.28, while the capital income tax rate τ_k is 0.36. The share of government consumption in GDP (G/Y) is 0.18. The ratio of government assets to GDP (B/Y) is –0.63, meaning that government debt is 63% of GDP. The amount of lump-sum transfers to households is endogenously determined to satisfy the government budget constraint.[12]

Symbol	Meaning	Value
β	Discount factor	0.9740
δ	Capital depreciation rate	0.07
θ	Capital share	0.38
μ	Risk aversion	1.5
η	Consumption share	0.3426
ρ	Persistence in idiosyncratic productivity	0.94
σ	Volatility of idiosyncratic productivity shocks	0.205
τ_c	Consumption tax rate	0.05
τ_n	Labor income tax rate	0.28
τ_k	Capital income tax rate	0.36
G/Y	Government consumption-output ratio	0.18
B/Y	Government asset-output ratio	–0.63

Table 1: Parameter values.

The utility function takes the form of $u(c,l) = \left(c^{\eta} l^{1-\eta}\right)^{1-\mu} / (1-\mu)$. Following Aiyagari and McGrattan (1998), we set $\mu = 1.5$, which ensures that Assumption 2 ($u_{12} \geq 0, \forall c \geq 0, \forall l \in [0,1]$) is satisfied.

Idiosyncratic productivity x follows a 17-state Markov chain. The Markov chain is obtained by approximating an AR(1) process, $\ln x' = \rho \ln x + \varepsilon', \varepsilon' \sim N(0, \sigma^2)$, using the method of Tauchen (1986). Note that the procedure ensures that Assumption 3 is satisfied. We set $\rho = 0.94$ and $\sigma = 0.205$, following Alonso-Ortiz and Rogerson (2010). These values are in line with existing estimates.

Lastly, we choose the discount factor β and the consumption share in the utility function η by targeting the after-tax interest rate $(1-\tau_k)r$ and the total hours worked $H =$

[12] The amount of transfers is equal to 4.2% of GDP.

$\int_S [1 - l(s)]\, d\Gamma$. Following Trabandt and Uhlig (2011), we target the total hours worked of 0.25. As for the after-tax return, Trabandt and Uhlig (2011) target 4% under the growth rate of 2%. What matters for the government finance is the difference between the interest and growth rates. Since the growth rate is 0% here, we target a return of 2%. The results are $\beta = 0.9740$ and $\eta = 0.3426$.[13]

4.2 Results

We analyze how a change in government assets affects the asset income and the interest rate in a zero-tax steady-state equilibrium. We set all tax rates to zero, $\tau_k = \tau_n = \tau_c = 0$, and consider a government asset-GDP ratio of $\{0\%, 10\%, ..., 290\%, 300\%\}$, that is, $B/Y \in \{0.0, 0.1, ..., 2.9, 3.0\}$. In one case, transfers are zero (i.e., $T = 0$) and government consumption is adjusted to satisfy the government budget constraint, implying that $G = rB$. In the other case, government consumption is zero (i.e., $G = 0$) and transfers to households are adjusted, so that $T = rB$. Other parameter values are fixed to their benchmark values.

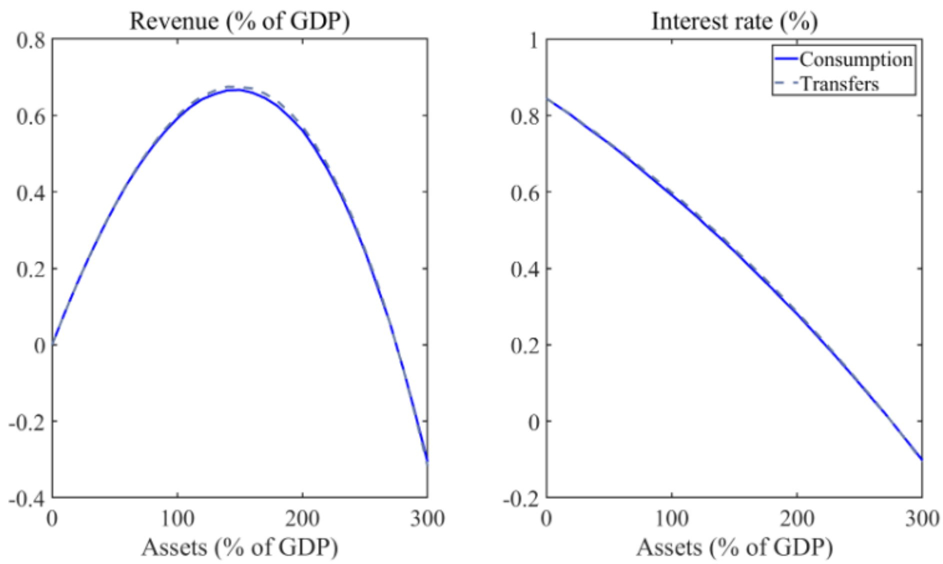

Figure 1: Effects of government assets on its asset income and the interest rate.

[13]As Aiyagari and McGrattan (1998) show, the elasticity of labor supply is computed by $[1 - \eta(1 - \mu)](1 - N)/(\mu N)$, and here it is equal to 1.341, with $N = 0.3681$.

Figure 1 shows how government assets affect government revenue (i.e., asset income) and the interest rate at a zero-tax steady-state equilibrium. As shown, the results are similar between when government consumption is adjusted and when transfers to households are adjusted. In particular, there is an inverse U-shaped relationship between government assets and government revenue. When government assets are zero, of course, asset income is also zero. As the government accumulates assets, its asset income initially increases. However, the interest rate falls monotonically with government assets, which works to decrease its asset income. Hence, there is a single peak in government asset income, which then eventually decreases below zero because the interest rate becomes negative. The maximum revenue is about 0.7% of GDP when government assets are 150% of GDP. Thus, considering the actual size of the U.S. government expenditure, we conclude that a zero-tax policy is an unrealistic option. The interest rate becomes negative when government assets are between 270% and 280% of GDP.

5 Bubbles

This section shows that rational asset price bubbles can emerge in equilibrium when government assets exceed a certain level. Here, bubbles mean that assets with no fundamental value are traded at a positive price.

Let $Z_t = Z > 0$ be the constant aggregate quantity of a bubble asset, which has no intrinsic value. Let p_t be the price of the bubble asset in t, while z_t indicates the amount of the bubble asset that each household holds at the beginning of t. At the beginning of any given period, households are then distinguished by three variables: the holding of the risk-free asset a_t, the holding of the bubble asset z_t, and idiosyncratic productivity x_t. Households' budget constraint is given by

$$c_t + a_{t+1} + p_t z_{t+1} \leq (1 + r_t) a_t + p_t z_t + w_t x_t (1 - l_t) + T_t. \tag{14}$$

We impose the same borrowing (i.e., non-negative) constraint for the risk-free and bubble assets: $a_{t+1} \geq 0$ and $z_{t+1} \geq 0$.

For simplicity, we here assume that the government does not consume, but it makes lump-sum transfers to households. With all taxes set to zero, the government budget constraint is then

$$B_{t+1} + T_t = (1 + r_t) B_t. \tag{15}$$

The market-clearing condition for the bubble asset is

$$Z = \int_{S_z} z_{t+1}(a_t, z_t, x_t) d\Gamma_{z,t}, \tag{16}$$

where $\Gamma_{z,t}(a_t, z_t, x_t)$ is the probability measure that describes the cross-sectional distribution over the risk-free asset a_t, the bubble asset z_t, and idiosyncratic productivity x_t.[14]

The next proposition shows that when government assets exceed a certain level, there exists a zero-tax steady-state equilibrium where the bubble asset is traded at a positive price.

Proposition 5 *There exists $\hat{B}$ such that for any $B > \hat{B}$, there exists a zero-tax steady-state equilibrium with bubbles in which $r_t = T_t = 0$, $B_t = B$, and $p_t = p > 0$ for all t.*

Proof. Let $\hat{B} > 0$ be the level of government assets that achieves $r = 0$ in equilibrium. The discussion in the previous section implies that such a $\hat{B}$ uniquely exists. Express the equilibrium objects at $\hat{B}$ using a hat, such as $\hat{K}$ and $\hat{a}'(a, x)$. Consider $B > \hat{B}$. We show below that there exists a zero-tax steady-state equilibrium with $B > \hat{B}$, $r = T = 0$, and $p > 0$. Since the risk-free and bubble assets face the same borrowing constraint, the two assets must earn the same return in equilibrium, which indicates that p is constant and the portfolio of households is indeterminate. Let $m \equiv a + pz$ be the wealth of each household. Then, at the beginning of any period, households are distinguished by m

[14] Let $S_z \equiv [0, \bar{a}] \times [0, \bar{z}] \times X$, where $\bar{z}$ is the upper bound on the holding of the bubble asset. Then, Γ_z is defined on $(S_z, \mathbf{B}(S_z))$, where $\mathbf{B}(S_z)$ is the Borel algebra on S_z.

and x. The policy functions, the value function, and the cross-sectional distribution have these two variables as their arguments, such as $m^{'}(m,x)$ and $\Gamma_m(m,x)$, where $\Gamma_m(m,x)$ is the probability measure that describes the cross-sectional distribution over wealth m and idiosyncratic productivity x.[15]

Note that $r = \hat{r}(=0)$ implies that $w = \hat{w}$. Recall also that $T = \hat{T}(=0)$. Consequently, for any $y \in [0, \bar{a}]$, $m^{'}(y,x) = \hat{a}^{'}(y,x)$, $l(y,x) = \hat{l}(y,x)$, $c(y,x) = \hat{c}(y,x)$, $\Gamma_m(y,x) = \hat{\Gamma}(y,x)$, and $V(y,x) = \hat{V}(y,x)$. Therefore, $K = \hat{K}$ and $N = \hat{N}$. Note that

$$\int_{S_m} m^{'}(m,x) d\Gamma_{m.} = \int_S \hat{a}^{'}(a,x) d\Gamma = \hat{K}^{'} - \hat{B}^{'}. \tag{17}$$

Suppose that

$$pZ = B - \hat{B}. \tag{18}$$

Then,

$$\begin{aligned} \int_{S_m} m^{'}(m,x) d\Gamma_m &= \hat{K}^{'} - \hat{B}^{'} \\ &= K^{'} - B^{'} + pZ^{'}, \end{aligned} \tag{19}$$

which implies that the asset market clears. Other equilibrium conditions are obviously satisfied. Hence, there exists a zero-tax steady-state equilibrium under which $r_t = T_t = 0$, $B_t = B > \hat{B}$, and $p_t = p = (B - \hat{B})/Z > 0$ for all t. ■

There are three remarks. First, a similar result holds when the government uses its revenue for its consumption.

Second, some argue that increasing government assets (and reducing government debt) has the same effect as tightening the borrowing limit. See, for example, Aiyagari and McGrattan (1998). Further, the borrowing limit is thought to reflect the condition or development of financial systems. See, for example, Guerrieri and Lorenzoni (2017).

[15] Let $S_m \equiv [0, \bar{a}] \times X$. Then, Γ_m is defined on $(S_m, \mathbf{B}(S_m))$, where $\mathbf{B}(S_m)$ is the Borel algebra on S_m.

With these interpretations, our finding indicates that weakening the financial system may generate asset price bubbles.

Third, the quantitative analysis in the previous section indicates that asset price bubbles can emerge when the government accumulates assets above 280% of GDP.

6 Conclusion

Can a government set all taxes to zero and finance its spending solely using its asset income in the long run? Previous analyses find that it can, and such a zero-tax policy sometimes emerges as the Ramsey outcome, no matter how large long-run government expenditure is. In the present study, we argue that the feasibility of a zero-tax policy found in previous studies may depend on their assumption of the absence of uninsured idiosyncratic earnings risk. In an Aiyagari (1994)-type heterogeneous agent model with such risk, we theoretically show that a zero-tax policy is infeasible at the steady state when government spending exceeds a certain threshold. The maximum revenue raised only from government asset income is quantitatively small compared to the actual level of government spending in the United States, and hence a zero-tax policy is an unrealistic option. We also show how accumulating government assets may generate rational asset price bubbles.

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Appendix: Numerical Method

We explain the numerical method used to solve for a stationary equilibrium.

1. Discretize the idiosyncratic state (a, x). Set 10 log-spaced points over [0, 60] for assets a. For idiosyncratic productivity x, set 17 evenly spaced points over $[-3\sigma/\sqrt{1-\rho^2}, 3\sigma/\sqrt{1-\rho^2}]$, and compute the transition matrix using the method of Tauchen (1986).

2. Set a guess for the after-tax interest rate $(1-\tau_k)r$ and aggregate labor input N. The wage rate w is given by (2). The transfers-to-output ratio T is given by the government budget constraint of (5), with the aggregate resource constraint, $1 = C + \delta K + G$, where the capital-to-output ratio K is computed from (1).

3. Given $(\tau_c, \tau_n, \tau_k, r, w, T)$, solve the household optimization problem and obtain the beginning-of-period value function $V(a, x)$.

   (a) Set a guess for the beginning-of-period value function $V^0(a, x)$.

(b) Solve the household problem as in (4). Use cubic spline interpolation to approximate the conditional expectation at $a^{'}$ off their grid points.

(c) If $V(a,x)$ becomes sufficiently close to $V^0(a,x)$, then proceed to the next step. Otherwise, update the value function to $V^0(a,x) = V(a,x)$ and return to (b).

4. Compute the stationary distribution of households $\Gamma(a,x)$.

(a) Choose points used to approximate the distribution. Use 250 log-spaced points over $[0,60]$ for a, and the points chosen in Step 1 for x.

(b) Using $V(a,x)$ obtained in Step 3 (c), solve the household problem this time for 250×17 pairs of (a,x), and determine their optimal asset holding $a^{'}(a,x)$, consumption $c(a,x)$, and leisure $l(a,x)$.

(c) Suppose $a_m \leq a^{'}(a,x) < a_{m+1}$, where a_m and a_{m+1} are two sequential asset points. Starting from an initial guess, keep updating the distribution until the distribution converges as follows: Households with (a,x) move to $(a_m, x^{'})$, with probability $\omega\pi(x^{'}|x)$, and to $(a_{m+1}, x^{'})$, with probability $(1-\omega)\pi(x^{'}|x)$, where $\omega = (a_{m+1} - a^{'})/(a_{m+1} - a_m)$. The result is the stationary household distribution $\Gamma(a,x)$.

5. Check whether the asset and labor markets clear: $K = \int_S a d\Gamma + B$ and $N = \int_S x\,[1 - l(a,x)]\,d\Gamma$. If the market-clearing conditions are satisfied, then stop. Otherwise, set different guesses for $(1-\tau_k)r$ and N and repeat Steps 2–5.