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"Capital Allocation and Wealth Distribution in a Global Economy with Financial Frictions"

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Capital Allocation and Wealth Distribution in a Global Economy with Financial Frictions*

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Abstract

This paper constructs a two-country model in which firms with heterogeneous production efficiency are subject to financial constraints. In our setting, the total factor productivity of the aggregate production function in each county depends on the cutoff level of production efficiency of firms. We first show that in the presence of international capital mobility, the cutoff condition is affected by the wealth distribution between the two countries. We then examine the existence and stability of the steady-state equilibrium of the world economy as well as the long-run impacts of real and financial shocks. It is shown that, compared to global financial shocks, global real shocks have larger impact on income and wealth in each country, especially if heterogeneity of production efficiency among firms is sufficiently low. The tractability of the model made it possible to analytically derive the main results.

Keywords: two-country model, financial frictions, firm heterogeneity, wealth distribution, capital mobility

JEL Classification: F21, F32, F41

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1 Introduction

Ever since the 2007-2008 global financial crisis and the subsequent worldwide recession, there has been a renewed interest in the effects of financial frictions on macroeconomic activities. This topic has been actively explored in the field of international macroeconomics: many authors have examined behaviors of open economies in the presnce of financial frictions. So far, research on this topic has analyzed two types of models: small open economy models and world economy models with two large countries. Most studies on small open economy models employed the standard neoclassical growth model with infinitely lived agents which is the basic analytical framework for studies on open economy macroeconomic models without financial frictions. Therefore, it is easy to understand the effects of financial frictions in the existing small open economy models by comparing them with the standard models without financial market imperfection. On the other hand, studies on two-country models have utilized various types of models. As mentioned below, some authors use two-period models, while others use overlapping generations framework. Infinite-horizon models have also been employed in the literature. Moreover, in general, the structure of the model economy is specified for discussing particular problems the researchers intend to address. This reflects the fact that two-country models are more complex than small open economy models, and hence, the researchers should specify a model structure to derive meaningful outcomes. Since the existing two-country models with financial frictions are not necessarily based on the prototype neoclassical growth model with infinitely-lived agents, it is often difficult to understand the effects of financial frictions by comparing them with the standard two-country models without financial frictions[1].

In this paper, we construct a simple two-country model with financial frictions whose analytical framework is close to the standard neoclassical growth model. An advantage of our model is its tractability that allows us to analytically investigate the existence and stability of the steady-state equilibrium of the world economy. Further, the long-run impacts of real and financial shocks can be easily analyzed. Since our model is based on the standard neoclassical growth model without financial frictions, it can clearly show how financial frictions affect

[1] It is to be noted that the international business cycle studies use the prototype neoclassical growth models with stochastic disturbances. However, the central concern of such studies is the quantitative evaluation of model economies rather than the qualitative analysis.

resource allocation and wealth distribution in the global economy.

Specifically, in our model economy, each country consists of workers and entrepreneurs. In the baseline model, it is assumed that workers do not save, so that their decisions do not play a substantial role in determining the behavior of the world economy (In Section 5, we consider the case wherein workers save). Each entrepreneur owns a firm, which is hit by an idiosyncratic technological shock every moment. Employment of physical capital by a firm is subject to financial constraints under which the level of physical capital employed is proportional to the net worth held by the entrepreneur who owns the firm. Combining this assumption of financial constraints with the heterogeneity in firms' productivity, there is an endogenously determined cutoff of capital efficiency: the firms whose productivity level exceeds the cutoff employ capital and produce. Otherwise, entrepreneurs act as rentiers. As a result, the total factor productivity (TFP) of the aggregate technology of each country is affected by the efficiency cutoff, and the cutoff condition in turn depends on the aggregate wealth-capital ratio in each country.

We assume that the entrepreneurs in each country can lend to or borrow from the entrepreneurs in the other country by transacting international bonds. Hence, the rate of return to capital in each country equals the real interest rate on bonds, so that both countries hold the same rate of returns at each moment. Since the rate of return to capital is affected by the cutoff condition, the equalization of the rate of returns means that the aggregate wealth-capital ratios in both countries are related to each other. Consequently, TFP of the aggregate production function in one country depends not only on the wealth holdings of the domestic entrepreneurs but also on the stock of wealth held by the foreign entrepreneurs. Namely, the aggregate productivity in each country is affected by the aggregate wealth distribution between the two countries.

Given the setting mentioned above, we first investigate the existence and stability of the steady-state equilibrium of the world economy. We analytically confirm that the existence and stability of the steady-state equilibrium are generally established. Then, we conduct steady-state analyses. We inspect how a negative financial or technological shock in one country affects the other country's economic activities. We also examine the impacts of global financial or real shocks, that is, the shocks that simultaneously hit both countries. We find that compared to financial shocks, real shocks have larger impacts on the levels of income and wealth in both countries. This is particularly true if heterogeneity of production efficiency among the firms is sufficiently low. This finding may give a simple answer in our model as to why the recent COVID-19 pandemic generated a larger scale worldwide recession than that caused by the 2007-2008 global financial crisis.

The paper also discusses some extensions of the baseline model. We consider the effects of income and consumption taxes as well as the case in which workers in both countries save. Additionally, we treat a model of endogenous growth in which income in each country continues to rise in the long-run equilibrium.

Background Literature

There is a large body of literature on open economies with financial frictions. In what follows, we restrict our attention to the studies that are closely related to our paper.

(i) Two-country Models without Financial Frictions

The two-country model without financial frictions based on the neoclassical growth model was first investigated in the models with homogeneous goods and representative household in each country. An early study is Ono and Shibata (1992) that explores the international diffusion effects of factor income taxation by use of a two-country model of this type[2]. The model has been extended to a two-good economy in which each country produces traded as well as nontradable goods: see, for example, Turnovsky (1997, Chapter 7) and Hu and Mino (2013). The neoclassical growth model with two countries has also been used by the international business cycle research: Backus et al. (1992) set up a baseline one-sector RBC model with two countries, while Baxter and Crucini (1995) analyze a two-sector model with nontradable goods[3].

(ii) Small Open Economy Models with Financial Frictions

Many studies have examined the standard one- or two-sector models of small open economies under alternative forms of borrowing constraints. Korinek and Mendoza (2014) explore a small open economy in which the debt limit of the representative household is proportional to its income. In a similar setting, Christiano et al. (2011) analyze a small-open

[2] This type of model is discussed in leading textbooks such as Turnovsky (1997, Chapters 5) and Ljungqvist and Sargent (2018, Chapter 11).

[3] Cole (1988) and Cole and Obstfeld (1991) treat two-country business cycle models with incomplete financial markets. Altug (2010, Chapter 4) presents a useful survey on international business cycle literature.

economy version of the New Keynesian model with unemployment. Schmitt-Grohé and Uribe (2020) discuss the presence of multiple equilibria in a small open economy subject to flow-based collateral constraints. On the other hand, Mendoza (2010) and Bianchi and Mendoza (2020) study the sudden stop problem using small open economy models with stock-based collateral constraints under which the upper bound of debt is proportional to the asset holdings of the representative household. Schmitt-Grohé and Uribe (2017) discuss the presence of multiple equilibria in the case of stock-based collateral constraints. Itskhoki and Moll (2019) construct a small open economy model with stock-based financial constraints and firm heterogeneity to investigate optimal policies in developing countries.

(iii) Two-country Models with Financial Frictions

As mentioned above, a variety of models have been used by the foregoing studies concerning this topic. For example, Antras and Caballero (2009) study a $2 \times 2 \times 2$ Heckscher-Ohlin model in which firms in the investment good sector are subject to financial constraints. The main concern of the authors is to examine complementarity between commodity trade and capital mobility[4]. Matsuyama (2014) constructs a world economy model in which firms have heterogeneous investment projects under financial constraints, and discusses international capital flow in a two-period setting[5]. Coeurdacier et al. (2015) explore an overlapping-generations model in which agents in each cohort work both in their young and middle ages, and retire in their old age. The authors assume that the young agents are subject to borrowing constraints and explore capital flow between two countries with different stages of financial development[6]. Wang et al. (2017) treat an infinite-horizon model with heterogeneous firms that conduct foreign direct investment under borrowing constraints. The authors show that in their setting, real capital may flow from a developed country to an underdeveloped country, while financial assets may flow in the opposite direction[7]. In addition to the theoretical investigations, many authors working on international business cycles have examined calibrated stochastic two-country models with financial frictions. A recent sample

[4]The main discussion by Antras and Caballero (2009) is based on a static model. To make the model tractable, in their dynamic analysis, the authors assume that agents are finitely lived and they consume their entire wealth at the end of their lives.

[5]Matsuyama (2014) points out that his two-period model can be extended to an infinite horizon setting. See also Matsuyama (2004).

[6]When analyzing a calibrated model, Coeurdacier et al. (2015) assume that agents live for n periods.

[7]An early theoretical study on capital flow between a developed and a less developed country is Gertler and Rogoff (1990). See also Hamada and Sakuragawa (2001), and Caballero et al. (2008).

includes Faia (2007), Devereux and Yetman (2010), Yao (2019), and Pintus et al. (2019).

According to the studies reviewed above, our model is close to the standard two-country model without financial frictions. However, our model is different from the standard setting in the sense that we introduce stock-based borrowing constraints and heterogeneous firms. Since our model is based on the prototype neoclassical growth model, it is not only different from but also much simpler than the two-country models with financial frictions cited above. Of course, we do not claim that simple models are always better than complex models: our model cannot address some of the relevant issues explored by the existing literature. However, our model is useful for understanding how the liberalization of financial transactions affects resource allocation and wealth distribution of the world economy in the presence of financial frictions and firm heterogeneity. Moreover, the simplicity of our baseline model makes it rather easy to extend it into various directions to discuss specific topics in international macroeconomics[8].

The rest of the paper proceeds as follows. Section 2 summarizes the key outcomes obtained in a prototype two-country model with homogeneous firms in the absence of financial frictions. Section 3 constructs a baseline model with financial frictions. Section 4 discusses the existence and stability of the steady-state equilibrium of the world economy and characterizes the long-run impacts of financial versus real shocks. Section 5 presents some extensions of the baseline model. Section 6 concludes.

2 A Two-Country Model without Financial Frictions

Before discussing the model with financial frictions, we briefly summarize the behavior of a baseline two-country model without financial frictions. This would be useful in clarifying the effects of financial frictions on the behavior of the global economy.

2.1 International Capital Mobility

The baseline analytical setting of this study is a one-good, two-country model with international capital mobility. There are two countries in the world, home and foreign. The stock of physical capital in the home country is denoted by K_t, and that in the foreign country is

[8] Some extensions are discussed in Section 5.

denoted by K_t^*. In this paper, the foreign variables are denoted by attaching an asterisk to the corresponding variables of the home country. When discussing international capital mobility, there are two alternative implications[9]. The first is to assume that domestic households can directly own the physical capital installed in the foreign county. In this case, we may write

$$K_t = K_{h,t} + K_{f,t}, \tag{1a}$$

$$K_t^* = K_{h,t}^* + K_{f,t}^*, \tag{1b}$$

where $K_{h,t}$ and $K_{f,t}$ denote the home country's capital stock owned by the domestic and foreign households, respectively. Similarly, $K_{h,t}^*$ and $K_{f,t}^*$ denote the foreign country's capital stock owned by the home country's households and foreign households, repectively. The aggregate wealth in each country is thus defined as

$$\begin{aligned} X_t &= K_{h,t} + K_{h,t}^* \\ X_t^* &= K_{f,t} + K_{f,t}^*. \end{aligned}$$

In this setting, the net asset position of the home country, $\hat{B}_t$, is given by

$$\begin{aligned} \hat{B}_t &= K_{h,t}^* - K_{f,t}, \\ \hat{B}_t^* &= K_{f,t} - K_{h,t}^* = -\hat{B}_t. \end{aligned}$$

As a result, the levels of the net worth of each country, denoted by X_t and X_t^*, are written as

$$\begin{aligned} X_t &= K_t + \hat{B}_t, \\ X_t^* &= K_t^* - \hat{B}_t. \end{aligned}$$

Thus, the equilibrium condition for the world capital market is expressed as

$$X_t + X_t^* = K_t + K_t^*. \tag{2}$$

[9] The following discussion follows Chapter 11 in Ljungqvist and Sargent (2018). See also Chapter 6 in Turnovsky (1997).

Alternatively, we may assume that there is a global financial market in which international bonds (IOUs) are traded. If physical capital is also internationally traded and if bonds and capital are perfectly substitute for each other, then the households' portfolio choice between real and financial assets becomes indeterminate. To avoid such an indeterminacy problem, the small-open economy models used in international macroeconomics often assume that real investment is associate with adjustment costs. In this paper, we simply assume that households in each country do not purchase the physical capital installed in the other country, that is, foreign direct investment is not allowed. Instead, the households conduct financial investment by purchasing IOUs in the global financial market. Letting B_t be the net asset (in terms of final goods) held by households in the home country, if $B_t > 0$ (resp. $B_t < 0$), households in the home country lend to (resp. borrow from) the foreign households. Hence, the net worth of each country is defined as

$$X_t = K_t + B_t, \tag{3a}$$

$$X_t^* = K_t^* - B_t. \tag{3b}$$

Again, the equilibrium condition of the global capital market is given by (2). In this paper, we adopt the second implication of international capital mobility.

2.2 Consumption and Production

In each country, there is a continuum of identical households with a unit mass. The households in the home country solve the following optimization problem:

$$\max \int_0^\infty e^{-\rho t} \log C_t dt,$$

subjective to

$$\dot{X}_t = r_t X_t + w_t - C_t, \quad X_0 = \text{given},$$

where C_t is consumption, X_t net worth holding, r_t real interest rate, w_t real wage, and $\rho\,(>0)$ denotes a time discount rate[10]. According to the second implication of international

[10] The logarithmic utility function is assumed for analytical simplicity.

capital market mentioned above, the flow budget constraint of the household is expressed as

$$\dot{K}_t + \dot{B}_t = r_t \left(K_t + B_t\right) + w_t - C_t.$$

Note that in the competitive equilibrium of the capital market, the domestic physical capital and foreign bonds yield the same rate of return[11]. Since we have assumed that the mass of the household is unity, K_t, X_t and C_t denote their aggregate values as well.

The optimal consumption follows the Euler equation,

$$\dot{C}_t = C_t \left(r_t - \rho\right), \tag{4}$$

and B_t is assumed to satisfy the non-Ponzi game scheme:

$$\lim_{t\to\infty} \exp\left(-\int_t^\infty r_s\right) B_t ds = 0,$$

The production function of the home country is

$$Y_t = AK_t^\alpha N_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{5}$$

The competitive factor prices satisfy the following:

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha}, \quad w_t = \left(1-\alpha\right) AK_t^\alpha N_t^{-\alpha}. \tag{6}$$

We assume that the utility and production functions take the same forms as those of the home country except for the level of TFP. Hence, the Euler equation and the factor prices held in the foreign country are respectively given by

$$\dot{C}_t^* = C_t^* \left(r_t^* - \rho\right), \tag{7}$$

[11] If there is no international bond market and the domestic household directly owns the capital stock in the foreign country, from $(1a)$ the flow budget constraint for the households is written as

$$\dot{K}_{h,t} + \dot{K}_{h,t}^* = r_t \left(K_{h,t} + K_{h,t}^*\right) + w_t - C_t,$$

which leads to

$$\dot{K}_t + \frac{d}{dt}\hat{B}_t = r_t \left(X_t + \hat{B}_t\right) + w_t - C_t.$$

$$r_t^* = \alpha A^* K_t^{*\alpha-1} N_t^{*1-\alpha}, \quad w_t^* = (1-\alpha) A^* K_t^{*\alpha} N_t^{*-\alpha}. \tag{8}$$

Here, each variable with an asterisk indicates the corresponding foreign variable.

2.3 The Steady-State Distribution of Wealth

Following the standard setting, we assume that the households of the home country freely lend to or borrow from the foreign households, and physical capital and financial assets are perfect substitutes for each other. Then it holds that

$$\alpha A K_t^{\alpha-1} N_t^{1-\alpha} = r_t = r_t^* = \alpha A^* K_t^{*\alpha-1} N_t^{*1-\alpha}. \tag{9}$$

Since the market equilibrium conditions in the labor market in each country gives $N_t = N_t^* = 1$, (9) gives

$$\frac{K_t^*}{K_t} = \left(\frac{A^*}{A}\right)^{\frac{1}{1-\alpha}}. \tag{10}$$

Hence, at every moment, the allocation of capital stocks between the home and foreign countries depends on the relative TFP, and it stays constant over time. Denoting the aggregate capital stock in the world by $K_t^w = K_t + K_t^*$, the above relation gives

$$K_t = \frac{K_t^w}{1 + (A/A^*)^{\frac{1}{1-\alpha}}}, \quad K_t^* = \frac{(A/A^*)^{\frac{1}{1-\alpha}} K_t^w}{1 + (A/A^*)^{\frac{1}{1-\alpha}}}. \tag{11}$$

Hence, the output of each country and the rate of return to capital are respectively expressed as

$$Y_t = \chi (K_t^w)^\alpha, \quad Y_t^* = \chi^* (K_t^w)^\alpha, \quad r_t = \alpha\chi (K_t^w)^{\alpha-1}, \tag{12}$$

where

$$\chi = A\left(\frac{1}{1 + (A/A^*)^{\frac{1}{1-\alpha}}}\right)^\alpha, \quad \chi^* = A^*\left(\frac{(A/A^*)^{\frac{1}{1-\alpha}}}{1 + (A/A^*)^{\frac{1}{1-\alpha}}}\right)^\alpha.$$

The market equilibrium condition for the world goods market is

$$Y_t + Y_t^* = C_t + C_t^* + \dot{K}_t + \dot{K}_t^* + \delta (K_t + K_t^*). \tag{13}$$

Denoting the aggregate consumption in the world by $C_t^w = C_t + C_t^*$, we obtain a complete dynamic system of the global economy:

$$\dot{K}_t^w = (\chi + \chi^*)\left(K_t^w\right)^\alpha - C_t^w, \tag{14}$$

$$\dot{C}_t^w = C_t^w\left[\alpha\chi\left(K_t^w\right)^{\alpha-1} - \rho\right]. \tag{15}$$

Since our setting allows us to describe the aggregate dynamics of the world economy as a closed economy model and the dynamic system is essentially the same as the Ramsey model with two types of agents, the dynamic system consisting of (14) and (15) has unique steady-state levels of $\bar{K}^w$ and $\bar{C}^w$ that satisfy

$$\bar{K}^w = \left(\frac{\rho}{\alpha\chi}\right)^{\frac{1}{\alpha-1}}, \quad \bar{C}^w = (\chi + \chi^*)\left(\frac{\rho}{\alpha\chi}\right)^{\frac{\alpha}{\alpha-1}}.$$

In addition, the steady state equilibrium exhibits saddle-point stability. On the stable saddle path, we obtain a unique relation between C_t^w and K_t^w, which is written as

$$C_t^w = \Psi\left(K_t^w\right), \quad \Psi'\left(K_t^w\right) > 0.$$

Note that (4) and (7) mean that $\dot{C}_t/C_t = \dot{C}_t^*/C_t$ for all $t \geq 0,$ and thus it holds that

$$C_t^* = \bar{\lambda} C_t, \tag{16}$$

where λ is a positive constant. Considering the non-Ponzi-game conditions, the intertemporal budget constraint for the household in each country is respectively given by

$$\begin{aligned} X_0 + \int_0^\infty \exp\left(-\int_0^t r_s ds\right) w_t dt &= \int_0^\infty \exp\left(-\int_0^t r_s ds\right) C_t dt, \\ X_0^* + \int_0^\infty \exp\left(-\int_0^t r_s ds\right) w_t^* dt &= \int_0^\infty \exp\left(-\int_0^t r_s ds\right) C_t^* dt. \end{aligned}$$

Thus, using (16), we obtain

$$\bar{\lambda} = \frac{X_0^* + \int_0^\infty \exp\left(-\int_0^t r_s ds\right) w_t^* dt}{X_0 + \int_0^\infty \exp\left(-\int_0^t r_s ds\right) w_t dt}. \tag{17}$$

Since on the stable saddle path r_s and w_s are monotonic functions of K_t^w, the value of $\bar{\lambda}$ is uniquely determined under given levels of initial wealth holdings, X_0 and X_0^*. Therefore, the steady-state levels of consumption and wealth of each country are specified as

$$C = \frac{1}{1+\bar{\lambda}}C^w \quad C^* = \frac{\bar{\lambda}}{1+\lambda}C^w, \quad X = \frac{C-w}{r}, \quad X^* = \frac{C^*-w}{r},$$

where variables without time subscript denote their steady-state values. The above equations show that the steady-state distribution of consumption and wealth between the home and foreign countries depend not only on the fundamentals that characterize the stable saddle path but also on the initial levels of net worth in both countries. Since K_t^w andC_t^w monotonically converge to their steady-state values, the initial difference between X_0 and X_0^* tends to be preserved during the transition. Consequently, wealth and income distributions between the two countries mainly depend on the initial distribution of wealth[12].

3 The Baseline Model with Financial Frictions

We now introduce financial frictions and firm heterogeneity into the standard two-country model discussed so far. Following Kiyotaki and Moore (1997), Moll (2014), Liu and Wang (2014), and Itskhoki and Moll (2019), we assume that each country consists of homogeneous workers and heterogeneous entrepreneurs. Specifically, our baseline model is a two-country version of Moll (2014) and Itskhoki and Moll (2019) whose analytical setting is based on the standard neoclassical growth model[13]. In discussing behaviors of workers and entrepreneurs, we focus on the home country. The behaviors of the agents in the foreign country are in Section 3.4.

3.1 Workers

There is a continuum of identical workers with a unit mass. The behavior of workers is simple. They are myopic and do not save. The representative worker solves the following

[12] See Chapter 6 in Turnovsky (1997) for the detailed discussion on the prototype two-country model with capital accumulation and its application to fiscal policy in the global economy.

[13] Moll (2014) examines a closed economy model, while Itskhoki and Moll (2019) treat a small open economy model. See also Giméntz and Neto (2016).

optimization problem at every each moment of time:

$$\max_{C_t^w, N_t} U_{w,t} = \max_{C_t^w, N_t} \log C_t^w - \frac{N^{1+\gamma}}{1+\gamma},$$

subject to $C_t^w = w_t N_t$, where C_t^w denotes consumption of the representative worker. The optimal choice yields

$$C_t^w = w_t, \quad N_t = 1. \tag{18}$$

In Section 5, we discuss the case in which workers in both countries save[14].

3.2 Entrepreneurs

Production Decision

Entrepreneurs also constitute a continuum with a unit measure. Each entrepreneur owns a firm. The production technology of a firm is

$$y_t = A\left(z k_t\right)^{\alpha} n_t^{1-\alpha}, \qquad A > 0, \quad 0 < \alpha < 1, \tag{19}$$

where y_t, k_t and n_t denote theoutput, capital, and labor of a firm, respectively. It is assumed that the efficiency of capital denoted by z is heterogeneous among firms. The above specification shows that each entrepreneur has the same form of production function except for the level of z. We assume that evry moment, an entrepreneur draws capital efficiency z from a stationary Pareto distribution whose cumulative distribution function is given by

$$F\left(z\right) = 1 - z^{-\phi}, \qquad \phi > 1, \quad z \geq 1. \tag{20}$$

[14]If the workers save, their optimization problem is

$$\max \int_0^{\infty} r^{e-\eta t}\left(\log C_t^w - \frac{N^{1+\gamma}}{1+\gamma}\right) dt$$

subject to

$$\dot{S}_t = r_t S_t + w_t N_t - C_{w,t},$$

where $\eta\,(>0)$ is the time discount rate of workers and S_t denotes workers' asset holding. As point out by Moll (2014) (Footnote 19), if the time discount rate is sufficiently high so that $\eta > r$ holds in the steady state and if the workers cannot borrow ($S_t \geq 0$ for all $t \geq 0$), then the workers optimal choice yields $C_{w,t} = w_t N_t$ in the long run. In this case, he hand-to-mouth behavior of workers reflects their optimal decision at lease in the long run.

Here, the shape parameter, ϕ, expresses the degree of heterogeneity in production efficiency: a lower value of ϕ means a higher level of heterogeneity in production technology among firms. We may interpret z as an idiosyncratic technological shock that hits each firm every moment. In what follows, according to Liu and Wang (2013) and Itskhoki and Moll (2019), we assume that z is identically and independently distributed *(iid)* over time as well as across agents. As a result, owing to the law of large numbers, the share of capital held by the firms who draw a particular level of z is stationary and deterministic.

We also assume that the entrepreneur's debt is subject to financial constraints. Denoting the net debt of an entrepreneur as d_t, the borrowing constraint is given by

$$d_t \leq \lambda k_t, \quad 0 \leq \lambda \leq 1.$$

This implies that the level of debt is subject to financial constraints in which capital stock acts as a collateral. In other words, a part of real capital is financed by borrowing. Then, letting the net worth held by an entrepreneur be $x_t = k_t - d_t$, the borrowing constraint is written as

$$k_t \leq \theta x_t, \quad \theta = \frac{1}{1-\lambda} \geq 1. \tag{21}$$

Namely, the leverage ratio, k_t/x_t, should be less than θ. Thus, if $\theta = 1$ $(\lambda = 0)$, the entrepreneur's investment must be self-financed, while there is no financial friction if $\theta = +\infty$ $(\lambda = 1)$.

We first formulate the entrepreneur's employment policy of labor and capital as a static optimization problem. As a producer, each firm employs labor and rents capital from the households. Thus, ignoring capital depreciation for notational simplicity, the pfofit of the firm is given by $\pi_t = y_t - w_t n_t - r_t k_t$. The firm maximizes π_t by choosing n_t and k_t under the constraints of production technology (19) and financial constraint (21). The first-order condition for the choice of labor input is

$$(1-\alpha) A \left(\frac{z k_t}{n_t}\right)^{\alpha} = w_t. \tag{22}$$

From (22), the profit of the firm is expressed as

$$\pi_t = \alpha A z k_t \left[\frac{w_t}{(1-\alpha) A}\right]^{-\frac{1-\alpha}{\alpha}} - r_t k_t.$$

Each entrepreneur selects k_t to maximize π_t under the constraints of $0 \le k_t \le \theta x_t$. The optimal choice of k_t is given by

$$k_t = \begin{cases} 0, & \text{for } z < \underline{z_t}, \\ \theta x_t \ , & \text{for } z \ge \underline{z_t}. \end{cases} \tag{23}$$

where $\underline{z_t}$ denotes the cutoff level of capital efficiency at period t, which is given by the zero-profit condition, $\pi_t = 0$. Thus, $\underline{z_t}$ is given by

$$\underline{z_t} = \frac{r_t}{\alpha A}\left[\frac{w_t}{(1-\alpha)A}\right]^{\frac{1-\alpha}{\alpha}}. \tag{24}$$

Since heterogeneity among firms is characterized by production efficiengy, z, and level of wealth of the firm owner, x_t, the profit function of the firm indexed by $(x_t,\ z_t)$ is expressed as

$$\pi(x_t,\ z) = \begin{cases} 0, & \text{if } z < \underline{z_t}, \\ \left\{\alpha A z\left[\frac{w_t}{(1-\alpha)A}\right]^{-\frac{1-\alpha}{\alpha}} - r_t\right\}\theta x_t = \hat{\pi}(z, w_y, r_t)\,\theta x_t, & \text{if } z \ge \underline{z_t}, \end{cases} \tag{25}$$

where

$$\hat{\pi}(z, w_t, r_t) = z\alpha A\left[\frac{w_t}{(1-\alpha)A}\right]^{\frac{\alpha-1}{\alpha}} - r_t.$$

Consumption and Savings

As a consumer, an etrepreneur maximizes a discounted, expected sum of utilities

$$U_{e,t} = E_0\int_0^{\infty} e^{-\rho t}\log c_{e,t}dt, \qquad \rho > 0,$$

subject to the flow budget constraint:

$$\dot{x}_t = r x_t + \pi(x_t,\ z) - c_{e,t}. \tag{26}$$

where $c_{e,t}$ is consumption of an entrepreneur and $\pi(x_t, z)$ is given by (25). The optimal consumption of the entrepreneur also follows the trasversality condition: $\lim e^{-\rho t}x_t/c_t = 0$.

To derive the optimal consumption of the entrepreneur, define the stationary value function, $v_t(x_t, z) = \max E_0 \int_t^{\infty} e^{\rho r_e(t-s)} \log c_{e,s} ds$. Then the Bellman equation is given by[15]

$$\rho_e v_t(x_t, z) = \max_{c_{e,,t}} \left\{ \log c_{e,t} + \frac{1}{dt} E_t dv_t(x_t, , z) \right\},$$

where x_t changes according to

$$dx_t = [[r_t + \hat{\pi}(z.r_t, w_t)\theta] x_t - c_{e,t}] \, dt.$$

As shown by Itskhoki and Moll (2019), as long as z is *iid* over time, the optimal consumption of an active entrepreneur is given by $c_{e,t} = \rho x_t$[16]. As a result, the optimal consumption function of each entrepreneur is given by

$$c_{e,t} = \rho x_t. \tag{27}$$

3.3 Aggregation

Production Function

As mentioned before, in our formulation, each entrepreneur is characterized by asset holdings, x, and production efficiency, z, Since we have assumed that z is *iid* and independently distributed across agents, the distributions of z and x are independent of each other[17].

[15] Note that the value function is not stationary because it involves r_t and w_t. When $dt \to 0$, this equation becomes the Hamilton-Jacobi equation, that is, the continuous-time counterpart of the Bellman equation.

[16] Supposet that the value function takes a form of $v_t(x_t, z) = M \log x_t + \mu \chi_t(z)$,where M and μ are undetermined constants. Such a specification yields $E_t dv_t(x_t, z) = M(dx_t/x_t) + \mu E_t d\chi_t(z)$.Then, using the flow budget constraint, the Bellman equation is written as

$$\begin{aligned} & \rho_e M \chi_t(z) + \rho_e M \log x_t \\ = \ & \max_{c_{i,t}} \left\{ \log c_{e,t} + \frac{M}{x_t}[r_t + \hat{\pi}(z, r_t, w_t,)\theta x_{,t} - c_{e,t}] + \mu \frac{1}{dt} E_t d\chi_t(z) \right\} \end{aligned}$$

Based on the guess and verify approach, they find that $\mu = \rho_e$ and, hence, the first-order condition, $1/c_{i,t} = \mu/a_{it}$, leads to $c_{e,,t} = \rho_e x_t$.

[17] If z is persistent over time, we should deal with the dynamic behavior of the joint distribution of wealth holdings among the entrepreneur and idiosyncratic shocks. Moll (2014) assumes that z follows a diffusion process. In this case, there is no stationary joint distribution function of (x, z) but the wealth share defined by$(1/K_t) \int_x x \, d\Gamma_t(x, z)$ will be stationary in the long run, where $\Gamma_t(.)$ denotes the joint distribution function at time t. (Note that in his closed economy model, it holds that $X_t = K_t$.) As shown by Moll (2014), wealth distribution among entrepreneurs and transition dynamics are sensitive to the specification of the stochastic process of z. However, since productivity shocks are idiosyncratic, the steady-state values of the aggregate

Therefore, the aggregate level of capital, hours worked and output are respectively written as

$$K_t = \int_x \int_{z \geq \underline{z}} k_t(x,z)\, dG_t(x)\, dF(z), \qquad N_t = \int_x \int_{z \geq \underline{z}} n_t(x,z)\, dG_t(x)\, dF(z), \tag{28}$$
$$Y_t = \int_x \int_{z \geq \underline{z}} y_t(x,z)\, dG_t(x)\, dF(z).$$

Here, $G_t(x)$ denotes the cumulative distribution function of x at time t, so that the entrepreneurs' aggregate wealth is

$$X_t = \int_x x dG_t(x).$$

Notice that the firms with $z \geq \underline{z}_t$ employ capital and are subject to financial constraint as long as $\underline{z}_t > 1$,. Thus, in view of (20) and $k_t(x,z) = \theta x_t$, the aggregate capital is expressed as

$$K_t = \int_x \int_{z \geq \underline{z}} k_t(x,z)\, dG_t(x)\, dF(z) = \theta X_t \int_{z \geq \underline{z}} dF(z) = \theta X_t \underline{z}_t^{-\phi}. \tag{29}$$

This means that $\underline{z}_t$ is related to aggregate capital and wealth in such a way that

$$\underline{z}_t = \left(\frac{\theta X_t}{K_t} \right)^{\frac{1}{\phi}}. \tag{30}$$

Using (22), an individual output is written as

$$y_t = Azk_t \left[\frac{w_t}{(1-\alpha)A} \right]^{\frac{\alpha-1}{\alpha}}.$$

Thus, the aggregate output is given by

$$\begin{aligned} Y_t &= \int_{x_t} \int_{z \geq \underline{z}} Azk_t(x,z) \left[\frac{w_t}{(1-\alpha)A} \right]^{\frac{\alpha-1}{\alpha}} dG_t(x)\, dF(z) \\ &= A \left[\frac{w_t}{(1-\alpha)A} \right]^{\frac{\alpha-1}{\alpha}} \theta \int_x x dG_t(x) \int_{z \geq \underline{z}_t} z dF(z) \\ &= A \left[\frac{w_t}{(1-\alpha)A} \right]^{\frac{\alpha-1}{\alpha}} \theta X_t \frac{\phi}{\phi - 1} \underline{z}_t^{1-\phi}. \end{aligned} \tag{31}$$

variables are the same as in the case of *iid* shocks. Therefore, as long as we focus on the aggregate wealth distribution between the two countries rather than the wealth distribution among the entrepreneurs in each country, our assumption of *iid* shocks is still effective in characterizing the steady-state equilibrium.

Considering that the optimization condition (22) is expressed as $w_t n_t = (1-\alpha) y_t$, we see that aggregating both sides of this relation gives

$$w_t = (1-\alpha) \frac{Y_t}{N_t}. \tag{32}$$

Then, substituting (32) into (31) and solving it with respect to Y, we obtain

$$Y_t = A \left(\frac{\phi}{\phi - 1} \underline{z} \right)^{\alpha} K_t^{\alpha} N_t^{1-\alpha}. \tag{33}$$

Since the average productivity of the firms whose production efficiency is higher than $\underline{z}$ is given by $\int_{z \geq \underline{z}} z dF(z) = \frac{\phi}{\phi - 1} \underline{z}_t$, the above expression implies that the TFP of the aggregate technology depends positively on the average productivity of the active firms.

Finally, substituting (30) into (33) yields

$$Y = A \left(\frac{\phi}{\phi - 1} \right)^{\alpha} \theta^{\frac{\alpha}{\phi}} \left(\frac{X_t}{K_t} \right)^{\frac{\alpha}{\phi}} K_t^{\alpha} N_t^{1-\alpha}. \tag{34}$$

Therefore, under given levels of K_t and N_t, TFP of the home country becomes higher as the aggregate net worth of the entrepreneurs becomes large. It is to be noted that if there is no firm heterogeneity so that $\phi = \infty$, then (34) becomes

$$Y_t = A K_t N_t^{1-\alpha},$$

which gives the aggregate production function with homogeneous firms. In the presence of financial frictions and firm heterogeneity, the term $A \left(\frac{\phi}{\phi - 1} \right)^{\alpha} \theta^{\frac{\alpha}{\phi}} \left(\frac{X_t}{K_t} \right)^{\frac{\alpha}{\phi}}$ expresses the efficient wedge of aggregate technology. Given the parameter values of α, ϕ, and θ, the efficiency wedge becomes higher, as the asset share of the entrepreneurs, X_t / K_t, increases. Additionally, under a given level of X_t / K_t, the efficiency wedge is higher, either if the degree of firm heterogeneity, is larger (ϕ is smaller) or if the financial constraint is weaker (θ is larger).

Rate of Return and Profit Income

Note that in addition to (30), $\underline{z}$ also satisfies (24). Using (24) and (33), we find

$$Y_t = \left(\frac{\phi r_t}{(\phi-1)\alpha A} \left[\frac{w_t}{(1-\alpha)A} \right]^{\frac{1-\alpha}{\alpha}} \right)^{\alpha} K_t^{\alpha} N_t^{1-\alpha}.$$

Substituting (32) into the above equation and solving it with respect to Y, we obtain the following relation:

$$r_t = \frac{\phi-1}{\phi} \alpha \frac{Y_t}{K_t}. \tag{35}$$

If firms are homogeneous, the competitive net rate of return to capital is given by $r_t = \alpha Y_t/K_t$. Therefore, $\frac{\phi-1}{\phi}\,(<1)$ represents an efficiency wedge[18].

Since national income consists of factor incomes of labor and capital as well as the excess profits, it holds that $Y_t = w_t N_t + r_t K_t + \Pi_t$. As a result, from (32) and (35) the aggregate excess profit is given by

$$\Pi_t = \frac{\alpha}{\phi} Y_t. \tag{36}$$

3.4 Foreign Firms and Households

The preference and production structures in the foreign country are the same as those in the home country. It is assumed that $\alpha = \alpha^*$, $\phi = \phi^*$, but it may hold that $A \neq A^*$, $\theta \neq \theta^*$, $\rho \neq \rho^*$. Similar to the home country, the aggregate production function and factor prices in the foreign country are respectively given as

$$\underline{z}_t^* = \left(\frac{\theta^* X_t^*}{K_t^*} \right)^{\frac{1}{\phi}}, \tag{37a}$$

$$Y_t^* = A^* \left(\frac{\phi}{\phi-1} \underline{z}_t^* \right)^{\alpha} K_t^{*\alpha} N_t^{*1-\alpha}, \tag{37b}$$

$$r_t^* = \alpha \left(\frac{\phi-1}{\phi} \right) \frac{Y_t^*}{K_t^*}, \tag{37c}$$

$$w_t^* = (1-\alpha) \frac{Y_t^*}{N_t^*}. \tag{37d}$$

[18] Since $(\phi-1)/\phi$ increases with ϕ, a higher heterogeneity among firms, that is, a smaller value of ϕ, indicates a higher degree of distortion.

The optimal consumption and labor supply of the foreign workers are expressedd as

$$C_t^w = w_t^*, \quad N_t^* = 1. \tag{37e}$$

Finally, the optimal consumption of each foreign entrepreneur is

$$c_t^e = \rho^* x_t^*. \tag{37f}$$

4 The Behavior of the Global Economy

In this section, we analyze the baseline model constructed in the previous section.

4.1 Allocation of Physical Capital

In our setting, the households in the home country can freely borrow from or lend to the foreign households under a common interest rate on the financial asset (IOUs). As a result of competitive portfolio choice, the rate of return to physical capital in each country equals the international interest rate on IOUs. Hence, it holds that $r_t = r_t^*$, (35), and $(37c)$ lead to

$$\frac{Y_t}{K_t} = \frac{Y_t^*}{K_t^*}. \tag{38}$$

We have assumed that labor cannot cross the border, and thus, the labor market equilibrium conditions in both countries are $N_t = 1$ and $N_t^* = 1$. As a result, (38) gives

$$\frac{K_t^*}{K_t} = \left(\frac{A^*}{A}\right)^{\frac{\phi}{(1-\alpha)\phi+\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}} \left(\frac{X_t^*}{X_t}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}}. \tag{39}$$

In view of (39), we find:

Proposition 1 *At every moment, allocation of capital between the home and foreign countries, K_t^*/K_t increases with the relative TFP, A^*/A, the relative degree of financial development, θ^*/θ, as well as the relative levels of wealth holdings, X_t^*/X_t.*

If $\phi = +\infty$ so that the financial constraints are not effective, (39) reduces to the case of perfect capital mobility given by (10), in which the capital allocation between the home and foreign countries depends on the relative TFPs alone. Note that, in contrast to the baseline model without financial frictions, the global allocation of capital is not stationary as long as the relative wealth changes during the transition process.

$$K_t^* = \Gamma \left(\frac{X_t^*}{X_t} \right)^{\psi} K_t, \tag{40}$$

where

$$\Gamma \equiv \left(\frac{A^*}{A} \right)^{\frac{\phi}{(1-\alpha)\phi+\alpha}} \left(\frac{\theta^*}{\theta} \right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}} > 0 \ \ , \ \psi \equiv \frac{\alpha}{(1-\alpha)\phi+\alpha} \in (0, \alpha). \tag{41}$$

From the equilibrium condition of the world financial market, $K_t + K_t^* = X_t + X_t^*$, and (40), we obtain

$$K_t = \Lambda \left(\frac{X_t^*}{X_t} \right) X_t, \quad K_t^* = \Lambda^* \left(\frac{X_t^*}{X_t} \right) X_t, \tag{42}$$

where

$$\Lambda \left(\frac{X_t^*}{X_t} \right) = \frac{1}{1 + \Gamma \left(\frac{X_t^*}{X_t} \right)^{\psi}} \left(1 + \frac{X_t^*}{X_t} \right),$$

$$\Lambda^* \left(\frac{X_t^*}{X_t} \right) = \frac{\Gamma \left(\frac{X_t^*}{X_t} \right)^{\psi-1}}{1 + \Gamma \left(\frac{X_t^*}{X_t} \right)^{\psi}} \left(1 + \frac{X_t^*}{X_t} \right).$$

We see that $\Lambda' \left(X_t^*/X_t \right) > 0$ unless X_r^*/X_t is sufficiently small, while $\Lambda^{*\prime} \left(X_t^*/X_t \right) < 0,$,unless we X_r^*/X_t is sufficiently large. Appendix 1 discusses the properties of $\Lambda\left(.\right)$ and $\Lambda^*\left(.\right)$ functions in detail.

Using the outcomes derived so far, we find the following:

Proposition 2 *The aggregate production function of each country can be expressed as*

$$Y_t = A \left(\frac{\phi}{\phi - 1} \right)^{\alpha} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} \left[\Lambda \left(\frac{X_t^*}{X_t} \right) \right]^{\alpha\left(1-\frac{1}{\phi}\right)}, \tag{43a}$$

$$Y_t^* = A^* \left(\frac{\phi}{\phi - 1} \right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^* \left(\frac{X_t^*}{X_t} \right) \right]^{\alpha\left(1-\frac{1}{\phi}\right)}. \tag{43b}$$

Proof. From (30) and (42), we obtain

$$\underline{z}_t = \left(\frac{\theta X_t}{K_t}\right)^{\frac{\alpha}{\phi}} = \theta^{\frac{\alpha}{\phi}}\left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{-\frac{\alpha}{\phi}}, \tag{44a}$$

$$\underline{z}_t^* = \left(\frac{\theta^* X_t^*}{K_t^*}\right)^{\frac{\alpha}{\phi}} = \theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{-\frac{\alpha}{\phi}}. \tag{44b}$$

Substituting the above equations into (33) and (37b), respectively, and using (42) again, we obtain (43a) and (43b). ■

Due to the properties of $\Lambda(.)$ and $\Lambda^*(.)$ functions, a rise in X_t^*/X_t decreases the cutoff level of capital efficiency in the home country, and raises the cutoff level in the foreign country. As (33) shows, a decrease in cutoff lowers the average productivity of active firms in the home country so that TFP of the aggregate production decreases in the home country. By contrast, a rise in X_t^*/X_t increases the aggregate productivity in the foreign country[19]

$$Y_t = A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\gamma}}K_t^{\alpha}N_t^{1-\alpha}, \quad Y_t^* = A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{*\frac{\alpha}{\alpha}}K_t^{\alpha}N_t^{1-\alpha}.$$

Hence, TFP of the aggregate production function of each country is fixed over time even out of the steady-state equilibrium...

4.2 Dynamic System

Aggregating the budget constraints of entrepreneurs yields the following dynamic equation of X_t:

$$\dot{X}_t = r_t X_t + \Pi_t - C_{e,t},$$

where $C_{e,t}$ is the aggregate consumption of entrepreneurs. Combining aggregate expressions of (35) and (42), we find the above equation can be expressed as

$$\begin{aligned}\dot{X}_t &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha}\left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ &\quad+\frac{\alpha}{\phi}A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha}\left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}} - \rho X_t.\end{aligned} \tag{45a}$$

[19] Note that if both countries are in the state of financial autarky, it then holds that $X_t = K_t$ and $X_t^* = K_t^*$. for all $t \geq 0$. Then from (33), (37b), (30,) and (37a), the aggregate production function of each country are reduced to

In a similar vein, the dynamic behavior of $\dot{X}_t^*$ is described by

$$\begin{aligned}\dot{X}_t^* &= \alpha A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^* \left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ &+\frac{\alpha}{\phi} A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^* \left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}} - \rho^* X_t^*. \end{aligned} \tag{45b}$$

Equations $(45a)$ and $(45b)$ constitute a complete dynamic system with respect to X_t and X_t^*.

When analyzing $(45a)$ and $(45b)$, it is convenient the dynamic system in the following manner. Letting $X_t^*/X_t = m_t$, $(45a)$ and $(45b)$ are expressed as

$$\begin{aligned}\dot{X}_t &= d\left(m_t\right) X_t^{\alpha} - \rho X_t, \\ \dot{X}_t^* &= d^*\left(m_t\right) X_t^{*\alpha} - \rho^* X_t^*,\end{aligned}$$

where

$$d(m_t) = \alpha A \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{\frac{\alpha}{\phi}} \left[\Lambda\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} + \frac{\alpha}{\phi} A \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} \left[\Lambda\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}}, \tag{46a}$$

$$d^*(m_t) = \alpha A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{*\frac{\alpha}{\phi}} \left[\Lambda^*\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} + \frac{\alpha}{\phi} A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} \left[\Lambda^*\left(m_t\right)\right]^{\alpha-\frac{\alpha}{\phi}}. \tag{46b}$$

Hence, we have an alternative dynamic system with respect to X_t and m_t :

$$\dot{X}_t = d(m_t) X_t^{\alpha} - \rho X_t, \tag{47a}$$

$$\dot{m}_t = m_t X_t^{\alpha-1} \left[m_t^{\alpha-1} d^*(m_t) - d(m_t) + (\rho^* - \rho) X_t^{1-\alpha}\right]. \tag{47b}$$

4.3 Steady-State Equilibrium

Steady-state equilibrium holds when $\dot{m}_t = \dot{X}_t = 0$ in $(47a)$ and $(47b)$, so that X_t and X_t^* stay constant over time. In the following, we eliminate the time suffix from the time-dependent endogenous variables to express their steady-state values. Then, the steady-state values of m_t and X_t satisfy the following:

$$d(m) = \rho X^{1-\alpha}, \tag{48a}$$

$$m^{\alpha-1}d^*(m) = d(m) + (\rho^* - \rho) X^{1-\alpha}. \tag{48b}$$

Since $\Lambda(m)$ and $\Lambda^*(m)$ are increasing and decreasing functions, respectively, we may define

$$\underline{z} = \theta^{\frac{\alpha}{\phi}} [\Lambda(\bar{m})]^{-\frac{\alpha}{\phi}} = 1, \quad \underline{z}^* = \theta^{*\frac{\alpha}{\phi}} [\Lambda^*(\underline{m})]^{-\frac{\alpha}{\phi}} = 1,$$

where $\underline{z}$ and $\underline{z}^*$ are the steady-state levels of cutoff efficiency. Specifically, $\underline{m}$ and $\bar{m}$ satisfy

$$\frac{1+\bar{m}}{1+\Gamma\bar{m}^{\psi}} = \theta, \quad \frac{\Gamma\underline{m}^{\psi-1}(1+\underline{m})}{1+\Gamma\underline{m}^{\psi}} = \theta^*. \tag{49}$$

Thus, $\underline{z} > 1$ for $m < \bar{m}$, and $\underline{z}^* > 1$ for $m > \underline{m}$. This means that if $\underline{m} > \bar{m}$ and if the steady-state level of m_t satisfy $\underline{m} > m > \bar{m}$, then $\underline{z} < 1$ and $\underline{z}^* < 1$. This means that all the firms in both countries are active. Hence, all of the entrepreneurs are borrowers and there is no lender in the global financial market. To avoid this situation, we assume the following[20]:

Assumption 1 *It holds that $\bar{m} > \underline{m}$, where $\bar{m}$ and $\underline{m}$ are determined by* (49).

Then, we can show the following:

Proposition 3 *Under Assumption 1, the world economy has a unique steady-state equilibrium.*

Proof. See Appendix 2. ■

Once the steady-state level of m_t is uniquely given, X is determined by $(48b)$, so that X^* fulfills $X^* = m[d(m)/\rho]^{\frac{1}{1-\alpha}}$. Accordingly, the steady-state values of other key variables are determined in the following manner:

$$\begin{aligned}
K &= \Lambda(m) X, \quad K^* = \Lambda^*(m) mX, \\
Y_t &= A\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} [\Lambda(m)]^{\alpha\left(1-\frac{1}{\phi}\right)}, \\
Y_t^* &= A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} (mX)^{\alpha} [\Lambda^*(m)]^{\alpha\left(1-\frac{1}{\phi}\right)}, \\
\underline{z} &= [\Lambda(m)]^{-\frac{1}{\phi}}, \quad \underline{z}^* = [\Lambda^*(m)]^{-\frac{1}{\phi}}, \\
r &= r_t^* = \alpha\left(\frac{\phi-1}{\phi}\right) X^{\alpha-1} [\Lambda(m)]^{\alpha\left(1-\frac{1}{\phi}\right)-1}.
\end{aligned}$$

[20] See Figure 3 in Section 4.5 shown below.

We should also note the following result:

Corollary 1 *If entrepreneurs in both countries have an identical time preference rate, i.e. $\rho = \rho^*$, then the steady-state level of m is given by*

$$m = \left(\frac{A^*}{A}\right)^{\frac{1}{1-\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi}}, \tag{50}$$

and financial autarky holds in the steady state, that is, $X = K$ and $X^* = K^*$.

Proof. (A6) in Appendix 2 shows that if $\rho = \rho^*$, then the steady state level of m_t satisfies $\Lambda(m) = \Lambda^*(m)$. Thus, by the definitions of $\Lambda(m)$ and $\Lambda^*(m)$ functions, the steady-state level of m fulfills

$$\Gamma m^{\psi-1} = 1,$$

which means that m is given by

$$m = \Gamma^{\frac{1}{\psi-1}} = \left[\left(\frac{A^*}{A}\right)^{\frac{\phi}{(1-\alpha)\phi+\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}}\right]^{1-\frac{\alpha}{(1-\alpha)\phi+\alpha}} = \left(\frac{A^*}{A}\right)^{\frac{1}{1-\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi}}. \tag{51}$$

In addition, when $\Gamma m^{\psi-1} = 1$, Appendix 2 shows that $\Lambda(m) = \Lambda^*(m) = 1$. Thus, from (42), it holds that $X = K$ and $X^* = K^*$. ■

It is noteworthy that financial autarky is established in the steady state, regardless of the levels of A, A^*, θ, and θ^*. In the standard model without financial frictions discussed in Section 2, the steady-state value of K^*/Kdepends on the relative TFP, A^*/A. while the steady-state value of X^*/X depends on the initial value of X_o^*/X_0 as well. Therefore, financial autarky in the steady state, $K = X$ and $K^* = X^*$ does not generically hold, and the net asset position of each country is determined by the prepayment values as well as by the initial asset holdings of the representative households in each country.

4.4 Stability

As for the stability of the dynamic system consisting of $(47a)$ and $(47b)$, we can analytically show the following result:

Proposition 4 *If entrepreneurs have an identical time preference rate, that is , $\rho = \rho^*$, then* *the steady-state equilibrium of the world economy is globally stable.*

Proof. See Appendix 3. ■

The phase diagram of (47a) and (47b) under $\rho = \rho^*$ is shown in Figure 1. In this case, the $\dot{m}_t = 0$ locus is a vertical line in the (m_t, X_t) space. Appendix 3 confirms that if $\rho = \rho^*$, the linearized system of (47a) and (47b) has two stable roots. Since both X_t and $m_t \, (= X_t^*/X_t)$ are non-jump variables, as depicted by Figure 1, the world economy satisfies global stability.

[Figure 1: Phase Diagram under $\rho = \rho^*$]

If $\rho \neq \rho^*$, then the local stability of the dynamic system cannot be guaranteed analytically. Hence, we examine some numerical examples. As was expected, if the differences in the parameter values in both countries are relatively small so that the steady-state level of m_t is not far from 1.0, then the behavior of the dynamic system is similar to one when entrepreneurs' time discount rate is same in both countries[21]. Thus, we focus on the examples in which asymmetries between the home and foreign countries are relatively large, and thus the level of m is far from 1.0. Figure 2 displays the phase diagrams of four examples. All the examples set $\alpha = 1/3$, $\phi = 2$, $A = 2$, $\theta = 1.5$, and $\theta^* = 1.65$. In Examples 1 and 2, the time discount rate of the foreign entrepreneurs is slightly higher than that of the entrepreneurs in the home country ($\rho = 0.01$ and $\rho^* = 0.011$). Example 1 assumes that $A = 2$ and $A^* = 0.25$, while Example 2 sets $A = 2$ and $A^* = 6$. As a result of large divergences in TFP of firms beteen the two countries, the steady state level of m is much smaller than 1.0 in Example 1, and much higher than 1.0 in Example 2. In Example 3, we set $\rho = 0.01$ and $\rho^* = 0.02$. By contrast, $\rho = 0.02$ and $\rho^* =$0.01 in Example 4. Again, the steady-state value of m is much smaller than 1.0 in Example 3, while the opposite outcome holds in Example 4. Is to be noted that the level of m is highly sensitive to the difference between ρ and ρ^*. As the phase diagrams show, all examples establish global stability. Therefore, it is safe to conclude that under plausible parameter values, the global stability of the world economy generally holds.

[21] For example, if $\phi = 2$, $\alpha = 1/3$, $A = A^{*:} = 2$, $\theta = 1.5$, $\theta^* 1.65$, $\rho = 0.001$, and $\rho^* = 0.011$, we obtain $m = 0.83$ and the phase diagram exhibits the global stability of the dynamic system.

[Figure 2: Phase Diagrams of Numerical Examples]

4.5 Financial vs. Real Shocks

In this subsection, we consider the impacts of real and financial shocks on the steady-state equilibrium as well as on the transition process of the world economy.

Financial Shocks

Since we will examine the effects of changes in the parameter values, we express $\Lambda(m_t)$, $\Lambda^*(m_t)$, $d(m_t)$ and $d^*(m_t)$ functions as $\Lambda(m_t;\Gamma)$, $\Lambda^*(m_t;\Gamma)$, $dm_t;\Gamma, A, \theta)$, and $d^* m_t;\Gamma, A^*, \theta^*)$, respectively. We see that

$$\frac{\partial\Lambda(.)}{\partial\Gamma} < 0, \quad \frac{\partial\Lambda^*(.)}{\partial\Gamma} > 0, \quad \frac{\partial d(.)}{\partial A} > 0, \quad \frac{\partial d^*(.)}{\partial A^*} > 0, \quad \frac{\partial d(.)}{\partial\theta} > 0, \quad \frac{\partial d^*(.)}{\partial\theta^*} > 0.$$

Figure 3 depicts the steady-state conditions of the world economy. From (A6) in Appendix 2, the steady-state level of m_t satisfies

$$\frac{\Lambda^*(m;\Gamma)}{\phi-1} = \left(\frac{\rho^*}{\rho}-1\right) + \left(\frac{\rho^*}{\rho}\right)\frac{\Lambda(m;\Gamma)}{\phi-1}. \tag{52}$$

The upper panel in Figure 3 shows graphs of the left-hand side (LHS) and the right-hand side (RHS) in (52). The lower panel exhibits the determination of the steady state-level of efficiency cutoffs in both countries.

[Figure.3: Graphical Exposition of the Steady-state Equilibrium]

Now, suppose that the world economy initially stays in the steady state, and there is a permanent, negative financial shock in the foreign country,that is, a permanent decrease in θ^*. Since a lower θ^* reduces Γ, the graph of the LHS of (52) shifts downward, while the that of the RHS shifts upward. Hence, as shown by Figure 4, the steady-state level of m_t $(= X_t^*/X_t)$ decreases from m to m'. At the same time, from $(44a)$ and $(44b)$, the graph of $\underline{z}_t$ shifts downward and that of $\underline{z}_t^*$ shifts upward. The resulting new steady-state level of cutoff in each country may or may not be higher than that of its old value. We should note that from Corollary 1, if $\rho = \rho^*$, then (52) reduces to $\Lambda(m, \Gamma) = \Lambda^*(m, \Gamma)$, and the steady-state levels of cutoffs are $z = \theta^{\frac{\alpha}{\phi}}$ and $z = (\theta^*)^{\frac{\alpha}{\phi}}$, .repectively. This means that a fall in θ^* lowers both m and z^*, and the level of z remains the same. Therefore, a negative financial shock lowers TFP of the aggregate production in the foreign country alone.

[Figure 4: Effects of Decrease in θ^*]

Next, consider the effects of a worldwide negative financial shock. This may correspond to the 2007-2008 global financial crisis. For simplicity of analysis, we assume that the declining rates of θ and θ^* are the same, so that θ^*/θ does not change. In this case, the value of Γ remains the same, meaning that the steady-state level of m determined by (52) will not change either. On the other hand, the decreases in θ and θ^* shift both z and z curves downward, which leads to simultaneous declines in TFPs of both countries (see Figure 5). Moreover, since the steady-state level of wealth in each country is given by

$$X = \left[\frac{d(m; \Gamma, A, \theta)}{\rho}\right]^{\frac{1}{1-\alpha}}, \quad X = \left[\frac{d^*(m; \Gamma, A^*, \theta^*)}{\rho}\right]^{\frac{1}{1-\alpha}},$$

the simultaneous reductions in θ and θ^* decrease the steady-state levels of wealth in both countries.

[Figure 5: Effects of Simultaneous Decreases in θ and θ^*]

To sum up, we have found:

Proposition 5 *If the financial constraints become tighter in the foreign country, the steady-state level of wealth ratio, X^*/X, decreases. If there are simultaneous and proportional decreases in θ and θ^*, then X^*/X will not change but aggregate TFPs in both countries fall.*

So far, we have focused on the steady-state equilibrium of the world economy. We now briefly consider the transition process after the shocks. As an example, Figure 6 depicts the transition process in the case of $\rho = \rho^*$. In the figure, the initial steady state is E_0. If θ^* decreases permanently, the new steady state shifts to point E_1, and the world economy will move from Point E_0 to Point E_1. As figure shows, on this transition path, both X_t and m_t continue to decrease. Note that the current account of each country is given by

$$\begin{aligned} CA_t &= \dot{B}_t = Y_t - C_{e,t} - C_{w.t,} - \dot{K}_t, \\ CA_t^* &= \dot{B}_t^* = Y_t^* - C_{e,t}^* - C_{w.t,}^* - \dot{K}_t^*. \end{aligned}$$

From (42), it holds that

$$\dot{K}_t = \Lambda_m \left(m_t^*; \Gamma'\right) \dot{m}_t + \Lambda \left(m_t; \Gamma'\right) \dot{X}_t,$$

where Γ' denotes the value of Γ after a reduction in θ^*. Combining this and $\dot{K}_t + \dot{B}_t = \dot{X}_t$, we obtain

$$\dot{B}_t = \left[1 - \Lambda \left(m_t; \Gamma'\right)\right] \dot{X}_t - \Lambda_m \left(m_t; \Gamma'\right) \dot{m}_t.$$

Since $1 > \Lambda\left(m_t, \Gamma'\right), \Lambda_m\left(.\right) > 0$, $\dot{X}_t < 0$ and $m_t < 0$ on the transition path, $CA_t = \dot{B}_t > 0$ (so that $CA_t^* = \dot{B}^* = -\dot{B}_t < 0$) during the transition. Hence, after the negative financial shock in the foreign country, financial assets move from the foreign country to the home country until the world economy reaches the new steady-state equilibrium.

On the other hand, if both θ and θ^* fall, and θ^*/θ remains constant, the world economy moves from Point E_0 to Point E_2 in Figure 6. As the figure demonstrates, the reduction in X in the case of a global financial crisis is higher than that in the case of country-specific negative financial shock. In the former, m_t does not change and $\Lambda\left(m_t, \Gamma'\right) = 1$ holds even during the transition process. Namely, there is neither capital inflow nor capital outflow between the two countries after the global financial crisis,

[Figure 6: Phase Diagram after the Shocks]

Real Shocks

If a negative technological shock hits the foreign country so that A^* and Γ decrease, then its qualitative impacts on both countries are essentially the same as those of a decline in θ^*. Such a result also holds for simultaneous falls in A and A^*. However, the quantitative impacts of real shocks are larger than those of financial shocks. For example, compare the effects of simultaneous falls in θ and θ^* to those of simultaneous falls in A and A^*. Note that in the steady state, TFPs of the aggregate production function of the home and foreign countries are respectively given by

$$\begin{aligned} \text{TFP} &= A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}\left[\Lambda\left(m;\Gamma\right)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}, \\ \text{TFP}^* &= A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*\left(m;\Gamma\right)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}. \end{aligned}$$

Since we have considered the case in which simultaneous negative shocks do not change m and Γ, a 10% decrease in A and A^* gives rise to a 10% decrease of TFPs of both countries. However, 10% decreases in θ and θ^* yield $(\alpha/\phi)\times 10\%$ falls in TFPs of both countries. If $\alpha = 0.35$ and $\phi = 1.5$, then the negative real impact on aggregate productions by a global real shock is about three times as large as that caused by a global financial shock. Moreover, such a gap in magnitudes of impacts becomes larger as heterogeneity in capital efficiency among firms becomes smaller, that is, ϕ takes a larger value. This finding may explain why the recent COVID-19 pandemic resulted a larger scale worldwide recession,than that caused by the 2007-2008 global financial crisis.

Proposition 6 *A country-specific as well as a global technological shock qualitatively yields the same long-run effects as those caused financial shocks. However, magnitude of impact generated by real shocks is larger than that of financial shocks.*

Finally, we consider the effects of preference shock. Suppose that the time discount rate of foreign entrepreneurs, ρ^*, rises permanently. The steady-state effects of this shock are described in Figure 7. Due to a rise in ρ^*, the RHS graph in (52) shifts upward, which lowers the level of m. Since the relations between the efficiency cutoffs and m_t are independent of ρ and ρ^*, the graphs of $\underline{z}_t$ and $\underline{z}_t^*$ will not shift. Consequently, the steady-state level of cutoff in the foreign country decreases, while that of the home country increases. This is because the reduction in X^*/X makes the borrowing constraints on the foreign firms more strict, and hence, a part of foreign firms with lower productivity give up production. In the home country, the opposite outcome will hold. Thus, a negative preference shock in the foreign country increases (resp. decreases) the average productivity of active firms in the foreign (resp. home) country.

Proposition 7 *If the time discount rate of foreign entrepreneurs rises, then (i) X^*/X decreases, (ii) efficiency cutoff in the home country falls, and (iii) efficiency cutoff in the foreign country rises.*

[Figure 7: Effects of a Rise in ρ^*]

5 Extensions

In this section, we discuss possible extensions of our baseline model.

5.1 Tax Policy

Let us assume that the government in each country levies income and consumption taxes on entrepreneurs and the workers. In our model with hand-to-mouth workers, the workers' decision does not affect the behavior of the global economy. Thus, we focus on the effects of taxation on the entrepreneurs' income and consumption. In the presence of income and consumption taxes, the flow budget constraints for entrepreneurs in the home and foreign countries respectively become

$$\begin{aligned} \dot{x}_t &= (1-\tau_y)(r_t x_t + \pi_t) - (1+\tau_c)\rho x_t + v_t, \\ \dot{x}_t &= (1-\tau_k^*)(r_t x_t + \pi_t^*) - (1+\tau_c^*)\rho^* x_t^* + v_t^*, \end{aligned}$$

where τ_y and τ_y^* respectively denote the rate of income tax in the home and foreign countries, and τ_c and τ_c^* express the consumption tax rates. In parallel with the discussion in Section 3.2, the individual budget constraints are re-expressed as

$$\begin{aligned} dx_t &= [(1-\tau_y)[r_t + \hat{\pi}(z_t.r_t, w_t)]\theta x_t - (1+\tau_c) c_{e,t}] dt, \\ dx_t^* &= \left[(1-\tau_y^*)[r_t + \hat{\pi}(z_t.r_t, w_t)]\theta^* x_t^* - (1+\tau_c^*) c_{e,t}^*\right] dt. \end{aligned}$$

We assume that the entire tax revenue is consumed by the government, so that the government budget constraint in each country is

$$\begin{aligned} G_t &= \tau_y (r X_t + \Pi_t) + \tau_c C_{e,t}, \\ G_t^* &= \tau_y^* (r X_t^* + \Pi_t^*) + \tau_c^* C_{e,t}^*. \end{aligned}$$

Using the guess and verify method, we find that the optimal consumption of entrepreneurs in each country satisfies

$$c_{e,t} = \frac{\rho x_t}{1+\tau_c}, \qquad c_{e,t}^* = \frac{\rho^* x_t^*}{1+\tau_c^*}. \tag{53}$$

Consequently, the aggregate dynamic system of the world economy consists of the following:

$$\begin{aligned} \dot{X}_t &= (1-\tau_y)\,\alpha A \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} \left[\Lambda(m_t)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ &\quad + (1-\tau_y)\frac{\alpha}{\phi} A \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{\frac{\alpha}{\phi}} X_t^{\alpha} \left[\Lambda(m_t)\right]^{\alpha-\frac{\alpha}{\phi}} - \frac{\rho X_t}{1+\tau_c}, \end{aligned} \tag{54a}$$

$$\begin{aligned} \dot{X}_t^* &= \left(1-\tau_y^*\right)\alpha A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha-1} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^*(m_t)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ &\quad + \left(1-\tau_y^*\right)\frac{\alpha}{\phi} A^* \left(\frac{\phi}{\phi-1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Lambda^*(m_t)\right]^{\alpha-\frac{\alpha}{\phi}} - \frac{\rho^* X_t}{1+\tau_c^*}. \end{aligned} \tag{54b}$$

Following the derivation of Equatation (A6) in Appendix 2, we find that the steady-state level of m_t is determined by

$$\frac{\Lambda^*\left(m;\hat{\Gamma}\right)}{\phi-1} = \frac{\rho^*(1+\tau_c)}{\rho\left(1+\tau_c^*\right)} - 1 + \left[\frac{\rho^*(1+\tau_c)}{\rho\left(1+\tau_c^*\right)}\right] \frac{\Lambda\left(m;\hat{\Gamma}\right)}{\phi-1}, \tag{55}$$

where

$$\hat{\Gamma} \equiv \left[\frac{\left(1-\tau_y^*\right)A^*}{\left(1-\tau_y\right)A}\right]^{\frac{\phi}{(1-\alpha)\phi+\alpha}} \left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{(1-\alpha)\phi+\alpha}}.$$

From (55), it is easy to see that a rise in the rate of income tax in the foreign country exactly corresponds to the case of a negative real shock in the foreign country (a fall in A^*) stated in Proposition 5. On the other hand, a rise in the consumption tax rate in the foreign country corresponds to a positive preference shock in the foreign country, that is, a decrease in ρ^*. Hence, its impact is opposite to the result shown in Proposition 6: a higher consumption tax rate in the foreign country raises the steady state level of X^*/X, and TFP of the foreign country, and lowers TFP of the home country. Consequently, change in tax policy in one country affects not only the economic activities in that country but also those in the other country through changes in capital allocation and wealth distribution between the two countries.

5.2 Workers' Savings

So far, we have assumed that workers in both countries do not save. To consider workers' savings, let us assume the workers in the home country solve the following life-time utility-maximizing problem:

$$\max \int_0^\infty e^{-\eta t} \log C_{w,t} dt,$$

subject to

$$\dot{S}_t = r_t S_t + w_t - C_{w,t},$$

where η (> 0) is time discount rate of the workers and S_t denotes bond holdings of the workers[22]. Then the optimal consumption satisfies the Euler equation such that

$$\dot{C}_{w,t} = C_{e,t} \left(r_t - \eta \right).$$

Similarly, the Euler equation and the flow budget constraint of the foreign workers are respectively given by

$$\begin{aligned} \dot{C}^*_{w,t} &= C^*_{w,t} \left(r_t - \eta \right), \\ \dot{S}^*_t &= r_t S^*_t + w^*_t - C^*_{w,t}. \end{aligned}$$

Here, we assume that workers in both countries have an identical time discount rate, $\eta \, (> 0)$.

Let us define $S^w_t = S_t + S^*_t$ and $C^w_{w,t} = C_{w,t} + C^*_{w,t}$. Then the total consumption of the

[22] Here, for expositional simplicity, we assume that workers' labor supply is fixed and normalized to one. If labor and leisure choices are allowed, the equilibrium level of labor becomes a function of the aggregate capital and workers' consumption. Thus, the reduces form of the dynamic system still consists of five endogenous variables shown below.

workers in the world and their aggregate asset respectively follow

$$\dot{C}^{w}_{w,t} = C^{w}_{t}\left(r_t - \eta\right), \tag{56a}$$

$$\dot{S}^{w}_{t} = r_t S^{w}_{t} + (1-\alpha)\left(Y_t + Y^{*}_{t}\right) - C^{w}_{w,t}. \tag{56b}$$

Similar to the model without financial frictions, it holds that $C^{*}_{w,t} = \bar{\mu} C_{w,t}$, and a positive constant, $\bar{\mu}$, is determined by the intertemporal budget constraints of workers in both countries. This means that the steady-state level of asset holdings of workers in both countries depend on the initial distribution of wealth of the workers between the home and foreign countries.

The equilibrium condition of the world financial market is now replaced with

$$K_t + K^{*}_{t} = X_t + X^{*}_{t} + S_t + S^{*}_{t}. \tag{57}$$

Since $K^{*}_{t} = \Gamma m^{\psi}_{t} K_t$ still holds, we obtain the following equations:

$$K_t = \Omega\left(m_t, s_t\right) X_t, \qquad K^{*}_{t} = \Omega^{*}\left(m_t, s^{*}_{t}\right) X^{*}_{t},$$

where

$$\Omega\left(m_t, s_t\right) = \frac{1}{1+\Gamma m^{\psi}_{t}}\left(1 + m_t + s_t\right), \qquad s_t = \frac{S^{w}_{t}}{X_t},$$

$$\Omega^{*}\left(m_t, s^{*}_{t}\right) = \frac{\Gamma m^{\psi-1}_{t}}{1+\Gamma m^{\psi}_{t}}\left(1 + m_t + s^{*}_{t}\right), \qquad s^{*}_{t} = \frac{S^{w}_{t}}{X^{*}_{t}}.$$

In this case, the cutoff conditions are given by

$$\underline{z_t} = \theta^{\frac{\alpha}{\phi}}\left[\Omega(m_t, s_t)\right]^{-\frac{\alpha}{\phi}},$$

$$\underline{z_t}^{*} = \theta^{*\frac{\alpha}{\phi}}\left[\Omega^{*}(m_t, s^{*}_{t})\right]^{-\frac{\alpha}{\phi}}.$$

As a result, the aggregate production function in the home and foreign countries is respectively described by

$$Y_t = A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}} X^{\alpha}_{t}\left[\Omega(m_t, s_t)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}, \tag{59a}$$

$$Y_t^* = A^* \left(\frac{\phi}{\phi - 1}\right)^{\alpha} \theta^{*\frac{\alpha}{\phi}} X_t^{*\alpha} \left[\Omega^*(m_t, s_t^*)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}. \tag{59b}$$

We can show that a complete dynamic system involves five endogenous variables, m_t, X_t, s_t, s_t^*, and $C_{w,t}^w / X_t$: see Appendix 4 for the differential equations that constitute a complete dynamic system. A key difference from the baseline model discussed in the previous sections is that the distribution of wealth as well and allocation of capital in the steady state partially depend on the initial distribution of wealth holdings among the workers in both countries. Therefore, the long-run characterization of the world economy combines the standard model without financial frictions treated in Section 2 and the model examined in Sections 3 and 4.

5.3 Endogenous Growth

In this subsection, we again assume that workers in both countries do not save[23]. A simple way to allow endogenous growth of the world economy is to assume that the efficiency of labor input of an individual firm is affected by external effects generated by the aggregate capital in the sense of Romer (1986). In this case, the production function of an individual firm in each country is formulated as follows:

$$\begin{aligned} y_t &= A\left(z_t k_t\right)^{\alpha}\left(\bar{K}_t n_t\right)^{1-\alpha}, \\ y_t^* &= A^*\left(z_t^* k_t^*\right)^{\alpha}\left(\bar{K}_t^* n_t^*\right)^{1-\alpha}, \end{aligned}$$

where $\bar{K}_t$ and $\bar{K}_t^*$ denote country-specific external effects of capital. Since the mass of firms is normalized to one in each country, it holds that $\bar{K}_t = K_t$ and $\bar{K}_t = K_t^*$ for all $t \geq 0$. When deciding the optimal production plan, each firm takes the external effects as given. Under such an assumption, it is easy to show that the aggregate, social production function in each country that involves country-specific external effects is

$$Y_t = A\left(\frac{\phi}{\phi - 1}\underline{z}\right)^{\alpha} K_t, \quad Y_t^* = A^*\left(\frac{\phi}{\phi - 1}\underline{z^*}\right)^{\alpha} K_t^*.$$

[23] It is easy to analyze an endogenous growth version of the model discussed in the previous subsection.

Therefore, the production function in each country expressed in terms of X_t and X_t^* is as follows:

$$\begin{aligned} Y_t &= A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}\left[\Lambda\left(m_t\right)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}X_t, \\ Y_t^* &= A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*(m_t)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}X_t^*. \end{aligned}$$

Using the above eauations, we find that the dynamic equations of X_t and X_t^* are respectively given by

$$\begin{aligned} \frac{\dot{X}_t}{X_t} &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ &\quad +\frac{\alpha}{\phi}A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}\left[\Lambda\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}}-\rho, \end{aligned} \tag{60a}$$

$$\begin{aligned} \frac{\dot{X}_t^*}{X_t^*} &= \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}-1} \\ &\quad +\frac{\alpha}{\phi}A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^*\left(\frac{X_t^*}{X_t}\right)\right]^{\alpha-\frac{\alpha}{\phi}}-\rho^*. \end{aligned} \tag{60b}$$

As a result, the complete dynamic system of the world economy reduces to a single differential equation of m_t :

$$\frac{\dot{m}_t}{m_t} = d^*\left(m_t\right) - d\left(m_t\right) + \rho - \rho^*, \tag{61}$$

where $d\left(m_t\right)$ and $d^*\left(m_t\right)$ are defined by $(46a)$ and $(46b)$.

It is easy to confirm that there generally exists a unique steady-state level of m_t that fulfills

$$d\left(m\right) - \rho = d^*\left(m\right) - \rho^*. \tag{62}$$

When $m_t = m$ determined by (62), the world economy stays on the balanced growth path where X_t, X_t^*, K_t, K_t^*, Y_t, Y_t^*, , $C_{e,t}$ and $C_{e,t}^*$ grow at a common rate. Additionally, the balanced growth rate of the world economy is given by $g = d\left(m\right) - \rho$. Based on the outcomes shown abvovestudy, we can examine the impacts of real and financial shocks on the balanced growth path of the world economy.

6 Conclusion

In this study, we analyzed a simple two-country model with financial frictions and firm heterogeneity. The key outcome of our study is that the TFP of the aggregate production function of each country depends on the wealth distribution between the two countries. We then explored the uniqueness and stability of the steady-state equilibrium of the world economy and discussed the effects of financial and real shocks on capital allocation and wealth distribution in the global economy. Owing to the tractability of our model, most of the main results are derived analytically without relying on numerical considerations. In addition, by comparing our model with the prototype neoclassical two-country model without financial frictions, it is easy to see how the presence of financial frictions affects the behavior of the global economy.

Besides the extensions presented in Section 5, three further extensions of our model would be interesting. The first is to consider multiple commodities. Since we assumed that both countries produce homogeneous goods, we considered intertemporal trade alone in this study. As assumed by Antras and Caballero (2009), if both countries produce multiple goods, we may consider intratemporal as well as intertemporal trade simultaneously. Such an extension of our model would be a useful integration of trade theory and international macroeconomics in the presence of financial frictions and firm heterogeneity[24]. Second, we can consider alternative forms of financial constraints. Although the stock-based collateral constraints assumed in our model contribute to the tractability of the model, the flow-based financial constraints have been also used in the foregoing literature. As the form of financial constraints sometimes affects the model behavior, re-examination of our model under alternative forms of financial constraints would be interesting. Finally, we may drop the assumption that productivity shocks are *iid*. If we assume that shocks are persistent over time, we may examine the wealth distribution among the agents in each country and the distribution of aggregate wealth between the two countries under a unified framework. Although this is a technically challenging topic, it deserves further investigation.

[24] Jin (2010) presents a useful overview on the integration of trade theory and open economy macroeconomics. See also Furusawa and Yanagawa (2016) and Ohodoi (2020) who emphasize the role of commodity trade in global economy with financial frictions.

Appendices

Appemdix 1: Properties of $\Lambda\left(X_t^* X_t\right)$ and $\Lambda^*\left(X_t^*/X_t\right)$ functions

Letting $X_t^*/X_t = m_t$, $\Lambda\left(.\right)$ and $\Lambda^*\left(.\right)$ functions are given by

$$\Lambda\left(m_t\right) = \frac{1}{1+\Gamma {m_t}^{\psi}}\left(1+m_t\right), \quad \Lambda^*\left(m_t\right) = \frac{\Gamma m_t^{\psi-1}}{1+\Gamma {m_t}^{\psi}}\left(1+m_t\right).$$

We see that

$$\Lambda'\left(m\right) = \frac{1+\Gamma m^{\psi-1}\left(1-\psi\right)\left(m-\frac{\psi}{1-\psi}\right)}{\left(1+\Gamma {m_t}^{\psi}\right)^2} = \frac{1+\left(1-\psi\right)\Gamma\left[m^{\psi}-\left(\frac{\psi}{1-\psi}\right)\frac{1}{m^{1-\psi}}\right]}{\left(1+\Gamma m^{\psi}\right)^2}, \quad \text{(A1)}$$

$$\Lambda^{*\prime}\left(m\right) = \frac{\Gamma m^{\psi-2}\left[m_t\left(\psi-\Gamma m^{\psi-1}\right)-\left(1-\psi\right)\right]}{\left(1+\Gamma m^{\psi}\right)^2} = \frac{\Gamma m_t^{\psi-2}\left[\left(\psi m-\Gamma m^{\psi}\right)-\left(1-\psi\right)\right]}{\left(1+\Gamma m^{\psi}\right)^2} \quad \text{(A2)}$$

Thus we find

$$\Lambda\left(0\right)=1, \quad \Lambda^*\left(0\right)=\infty, \quad \lim_{m\to\infty}\Lambda\left(m\right)=\infty, \quad \lim_{m\to\infty}\Lambda^*\left(m\right)=1.$$

Moreover, since $0<\psi<\alpha<1$, if $m_t > \frac{\psi}{1-\psi} = \frac{\alpha}{(1-\alpha)\phi}$, hen $\Lambda'\left(m_t\right)>0$. Even if $m_t < \frac{\alpha}{(1-\alpha)\phi}$, it holds that $\Lambda'(m_t)>0$, unless m_t has a sufficiently small value. On the other hand the expresion of $\Lambda^*\left(m_t\right)$ function means that unless m_t takes an sufficiently large value so that $\psi < \Gamma m_t^{\psi-1}$, it holds that $\Lambda^{*\prime}\left(m_t\right)<0$.

Appendix 2: Proof of Proposition 2

Combining $(48a)$ and $(48b)$, we obtain

$$\frac{\rho^*}{\rho}m^{1-\alpha} = \frac{d^*\left(m\right)}{d\left(m\right)}.$$

From the definitions of $d\left(m_t\right)$ and $d^*\left(m_t\right)$ functions given by $(46a)$ and $(46b)$, the above equation is expressed as

$$\frac{\rho^*}{\rho}m^{1-\alpha} = \left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}}\left[\frac{\Lambda^*\left(m\right)}{\Lambda\left(m\right)}\right]^{\alpha-\frac{\alpha}{\phi}-1}\left[\frac{\Lambda^*\left(m\right)+\left(\phi-1\right)}{\Lambda\left(m\right)+\left(\phi-1\right)}\right]. \quad \text{(A3)}$$

Note that the steady-state expressions of $\Lambda(m_t)$ and $\Lambda^*(m_t)$ are respectively given by

$$\Lambda(m) \equiv \frac{1}{1+\Gamma m^{\psi}}(1+m), \quad \Lambda^*(m) = \frac{\Gamma m^{\psi-1}}{1+\Gamma m^{\psi}}(1+m). \tag{A4}$$

Hence, (A3) can be written as

$$\frac{\rho^*}{\rho} m^{1-\alpha} = \left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}} \left(\Gamma m^{\psi-1}\right)^{\alpha-\frac{\alpha}{\phi}-1} \left[\frac{\Lambda^*(m)+(\phi-1)}{\Lambda(m)+(\phi-1)}\right], \tag{A5}$$

Here, note that since $\psi = \frac{\alpha}{(1-\alpha)\phi+\alpha}$, it holds that $(\psi-1)\left(\alpha-\frac{\alpha}{\phi}-1\right) = 1-\alpha$. Thus m satisfies

$$\frac{\rho^*}{\rho} = \left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}} \Gamma^{\alpha-\frac{\alpha}{\phi}-1} \left[\frac{\frac{\Gamma m^{\psi-1}}{1+\Gamma m^{\psi}}(1+m)+(\phi-1)}{\frac{1}{1+\Gamma m^{\psi}}(1+m)+(\phi-1)}\right].$$

Moreover, it holds that

$$\left(\frac{A^*}{A}\right)\left(\frac{\theta^*}{\theta}\right)^{\frac{\alpha}{\phi}} \Gamma^{\alpha-\frac{\alpha}{\phi}-1} = 1,$$

meaning that (51) is reduced to

$$\frac{\Gamma m^{\psi-1}(1+m)}{(1+\Gamma m^{\psi})(\phi-1)} + 1 = \left(\frac{\rho^*}{\rho}\right)\left[\frac{1+m}{(\phi-1)(1+\Gamma m^{\psi})}+1\right].$$

We can express the above equation in the above as

$$\frac{\Lambda^*(m)}{\phi-1} = \left(\frac{\rho^*}{\rho}-1\right) + \left(\frac{\rho^*}{\rho}\right)\frac{\Lambda(m)}{\phi-1}. \tag{A6}$$

As Figure 3 in the main text shows, $\Lambda(m) \geq 1$ for all $m \leq \bar{m}$ and $\Lambda^*(m) \geq 1$ for all $m \geq \underline{m}$. Appendix 1 states that unless m takes a sufficiently large value, the left-hand-side in (A6) monotonically decreases with m. On the other hand, unless m is sufficiently small, the right-hand-side of (A6) monotonically increases with m. Hence, it is easy to see that (A6) has a unique positive solution: see the upper panel in Figure 3 in the main text.

Proof of Proposition 3

We first inspect the local stability of the symmetric steady state. Using one of the steady-state condition, $X^{\alpha} d(m) = \rho X$, we find that the coefficient matrix of the dynamic system linearized at the steady state is given by

$$J=\begin{bmatrix} -(1-\alpha)\rho & X^{\alpha}d'(m) \\ m\left[(1-\alpha)(\rho-\rho^{*})\frac{1}{X}\right] & X^{\alpha-1}m\left[(\alpha-1)m^{\alpha-2}d^{*}(m)+m^{\alpha-1}d^{*\prime}(m)-d'(m)\right] \end{bmatrix}.$$

In the above, $d'(m)$ and $d^{*\prime}(m)$ are repectively given by

$$\begin{aligned} d'(m) &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Lambda(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\Lambda'(m)\left[\frac{\alpha}{\phi-1}\left(1-\frac{1}{\phi}\right)\Lambda(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right], \\ d^{*\prime}(m) &= \alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^{*}(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\Lambda^{*\prime}(m)\left[\frac{\alpha}{\phi}\Lambda^{*}(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right]. \end{aligned}$$

From Corollary 1, if $\rho=\rho^{*}$, it holds that $\Gamma m^{\psi-1}=1$ and $\Lambda(m)=\Lambda^{*}(m)=1$. Hence, we see the following;

$$\begin{aligned} \Lambda'(m) &= \frac{1+(1-\psi)\left(m-\frac{\psi}{1-\psi}\right)}{\left(1+\Gamma m^{\psi}\right)^{2}} \\ &= \frac{(1-\psi)(1+m)}{(1+m)^{2}}=\frac{1-\psi}{1+m}>0, \\ \Lambda^{*\prime}(m_t) &= \frac{\Gamma m_t^{\psi-1}\left[m(\psi-1)-(1-\psi)\right]}{m\left(1+\Gamma m^{\psi}\right)^{2}}=\frac{\psi-1}{m(1+m)}<0. \end{aligned}$$

$$\begin{aligned} d(m_t) &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[1+\frac{1}{\phi-1}\right], \\ d^{*}(m_t) &= \alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[1+\frac{1}{\phi-1}\right], \end{aligned}$$

$$\begin{aligned} d'(m) &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Lambda(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\Lambda'(m)\left[\frac{\alpha}{\phi-1}\left(1-\frac{1}{\phi}\right)\Lambda(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right] \\ &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{1+m}\right)\left[\frac{\alpha}{\phi}-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right] \\ &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{m(1+m)}\right)(\alpha-1)<0, \end{aligned}$$

$$\begin{aligned} d^{*\prime}(m) &= \alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\Lambda^{*}(m)\right]^{\alpha-\frac{\alpha}{\phi}-2}\Lambda^{*\prime}(m)\left[\frac{\alpha}{\phi}\Lambda^{*}(m)-\left(1+\frac{\alpha}{\phi}-\alpha\right)\right] \\ &= \alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left(\frac{\psi-1}{m(1+m)}\right)(\alpha-1)>0. \end{aligned}$$

Therefore, we find that

$$\begin{aligned} (m^{-1}X^{1-\alpha})\frac{d\dot{m}_t}{dm_t}\Big|_{m_t=m,} &= (\alpha-1)\,m^{\alpha-2}d^{*}(m)+m^{\alpha-1}d^{*\prime}(m)-d^{\prime}(m) \\ &= m^{2-\alpha}(\alpha-1)\,\alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left[\frac{\phi}{\phi-1}\right] \\ &\quad +m^{\alpha-1}\alpha A^{*}\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}\left(\frac{\psi-1}{m(1+m)}\right)(\alpha-1) \\ &\quad -\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{1+m}\right)(\alpha-1) \end{aligned}$$

From (50), it holds that $m^{\alpha-1}=\left(\frac{A}{A^{*}}\right)\left(\frac{\theta}{\theta^{*}}\right)^{\frac{\alpha}{\phi}}$ and $m^{\alpha-2}=\frac{1}{m}\left(\frac{A}{A^{*}}\right)\left(\frac{\theta}{\theta^{*}}\right)^{\frac{\alpha}{\phi}}$. Substituting those relations into the above, we find:

$$\begin{aligned} (m^{-1}X^{1-\alpha})\frac{d\dot{m}_t}{dm_t}\Big|_{m_t=m,} &= \frac{1}{m}(\alpha-1)\,\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\frac{\phi}{\phi-1}\right] \\ &\quad +\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{\psi-1}{m(1+m)}\right)(\alpha-1) \\ &\quad -\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left(\frac{1-\psi}{1+m}\right)(\alpha-1) \\ &= (\alpha-1)\,\alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\frac{\phi-(1-\psi)(\phi-1)}{m(\phi-1)}\right]<0. \end{aligned}$$

If $\rho=\rho^{*}$, the eigenvalues if J are $-(1-\alpha)\,\rho$ and $X^{\alpha-1}m\left[(\alpha-1)\,m^{\alpha-2}d^{*}(m)+m^{\alpha-1}d^{*\prime}(m)-d^{\prime}(m)\right]$. Since both are negative, the stationary point is a stable node, meaming that the world economy will not exhibit cyclical behaviors around the steady state. Combining this result with the phase diagram given by Figure 1, we may conclude that the dynamic system is globally stable.

Appendix 4: A complete dynamic system for a model with workers' savings

Note that $r_t=\alpha\left(\frac{\phi-1}{\alpha}\right)\frac{Y_t}{X_t}$. Thus, using $(59a)$ and $(59b)$, the dymamic equations displayed in the main text, we obtain the following:

$$\frac{\dot{C}_t^w}{C_t^w} = A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Omega(m_t,s_t)\right]^{\alpha-\frac{\alpha}{\phi}-1}\frac{X_t}{C_t^w}-\eta,$$

$$\begin{aligned}\frac{\dot{S}_t^w}{S_t^w} &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}\left[\Omega(m_t,s_t)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}+(1-\alpha)A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha}\left[\Omega(m_t,s_t)\right]\frac{X_t}{S_t^w}\\ &+(1-\alpha)A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{*\frac{\alpha}{\phi}}X_t^{*\alpha}\left[\Omega^*(m_t,s_t^*)\right]^{\alpha\left(1-\frac{1}{\phi}\right)}\frac{X_t^*}{S_t^w}-\frac{C_{w,.t}^w}{S_t^w},\end{aligned}$$

$$\begin{aligned}\frac{\dot{X}_t}{X_t} &= \alpha A\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha-1}\left[\Omega\left(m_t,s_t\right)\right]^{\alpha-\frac{\alpha}{\phi}-1}\\ &+\frac{\alpha}{\phi}A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha}\left[\Omega\left(m_t,s_t\right)\right]^{\alpha-\frac{\alpha}{\phi}}-\rho,\end{aligned}$$

$$\begin{aligned}\frac{\dot{X}_t^*}{X_t} &= \alpha A^*\left(\frac{\phi}{\phi-1}\right)^{\alpha-1}\theta^{*\frac{\alpha}{\phi}}X_t^{*\alpha}\left[\Omega(m_t,s_t^*)\right]^{\alpha-\frac{\alpha}{\phi}-1}\\ &+\frac{\alpha}{\phi}A\left(\frac{\phi}{\phi-1}\right)^{\alpha}\theta^{\frac{\alpha}{\phi}}X_t^{\alpha}\left[\Omega^*\left(m_t,s_t^*\right)\right]^{\alpha-\frac{\alpha}{\phi}}-\rho.\end{aligned}$$

Using the four differential equations displayed above and noting that $X_t/C_t^w = \left(\frac{X_t}{S_t^w}\right)\left(\frac{S_t^w}{C_t^w}\right)$, we can obtain a complete dynamic system consisting of five differential equations with respect to five endogenous variables, $m_t\left(=X_t^*/X_t\right)$, X_t, $s_t\left(=S_t^w/X_t\right)$, $s_t^*\left(=S_t^w/X_t^*\right)$, and C_t^w/X_t.

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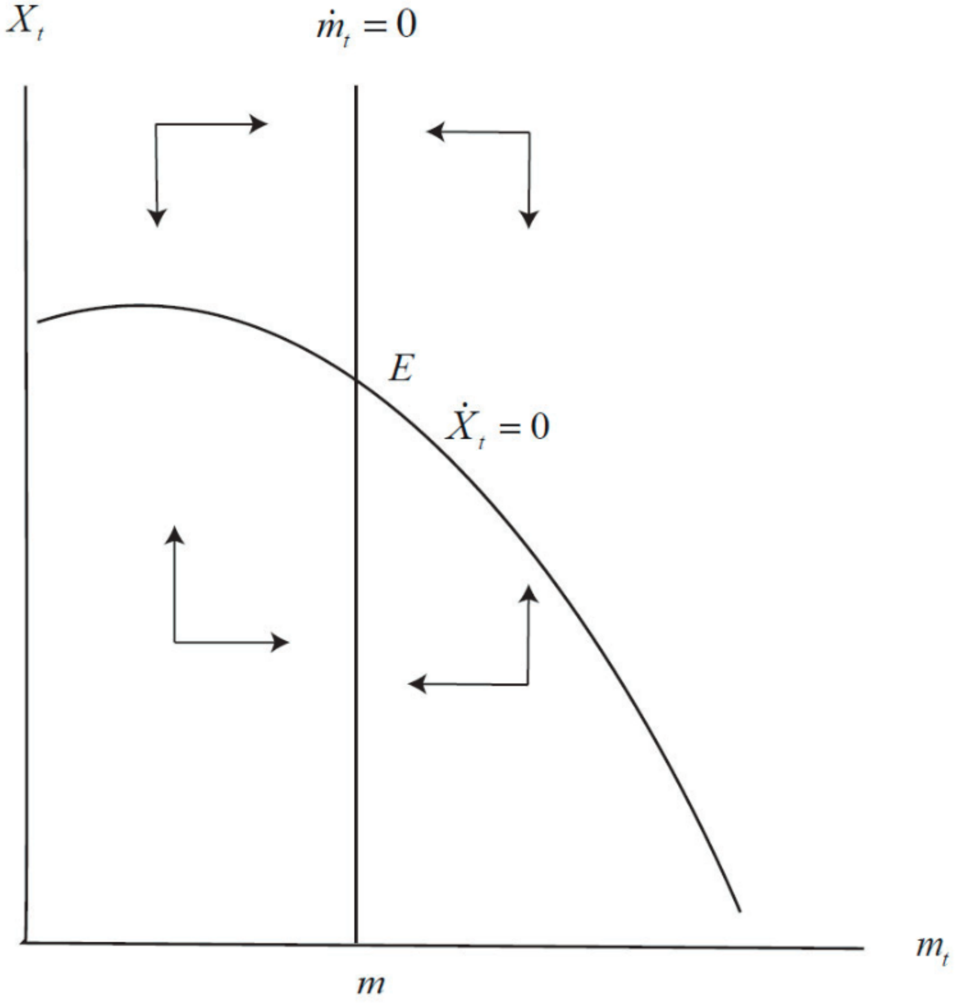
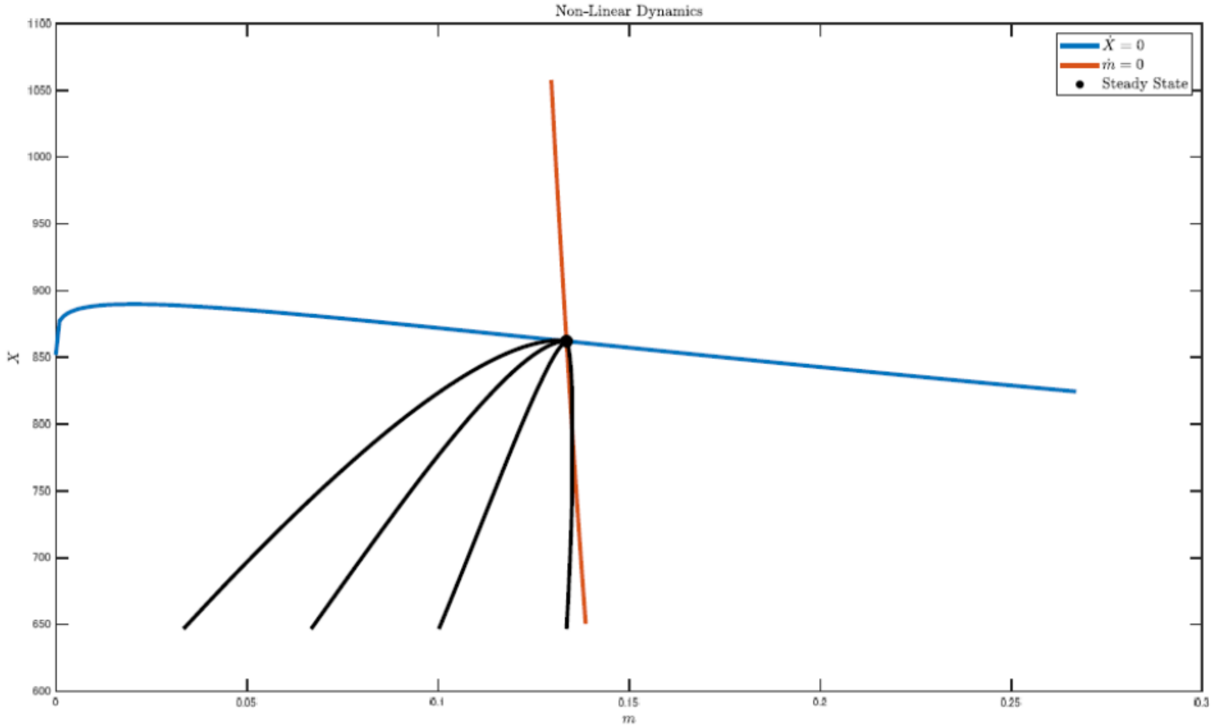
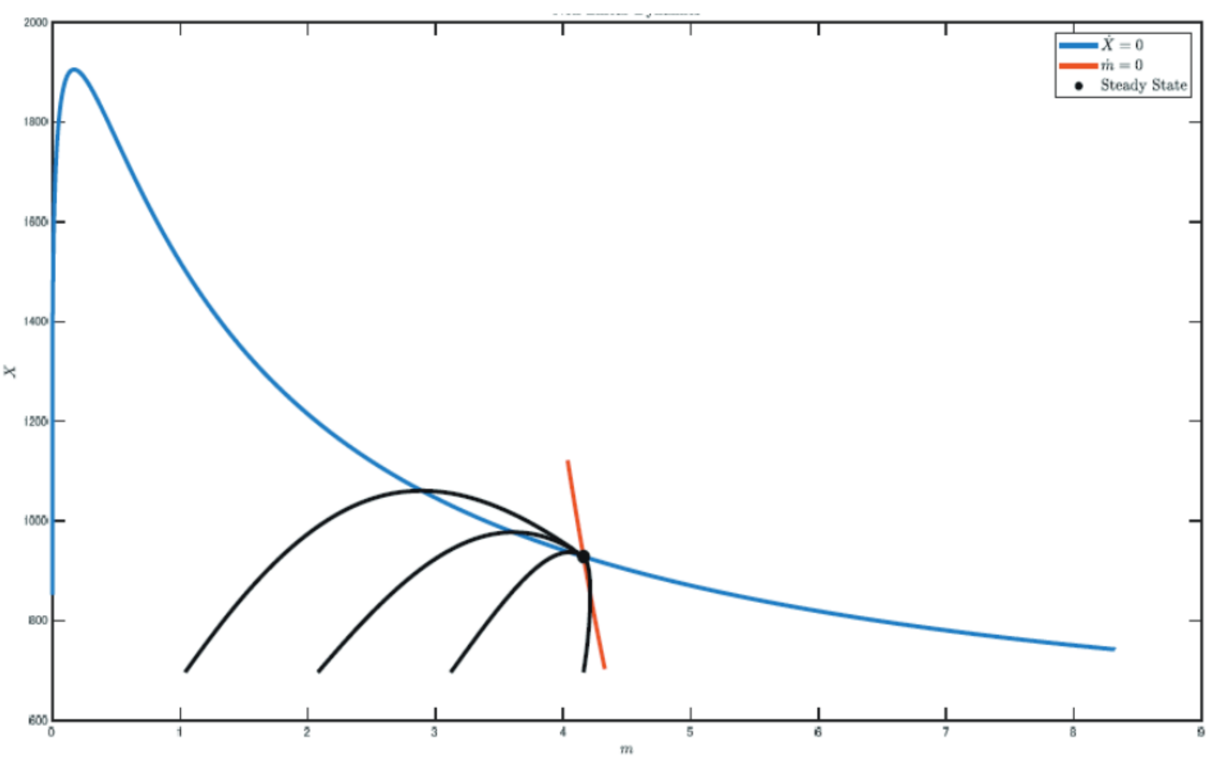


Figure 1: Phase Diagram under $\rho = \rho^*$

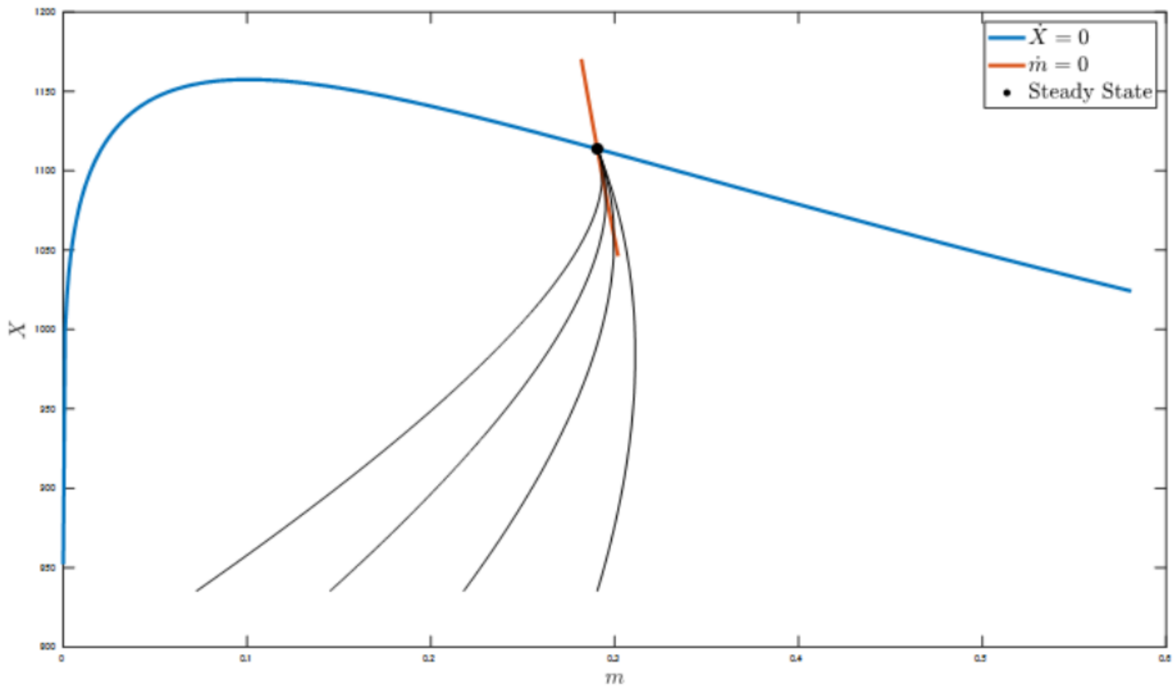


Example 1: $\rho = 0.01$, $\rho^* = 0.011$, $A=2$, $A^* = 0$`0,25, $\theta = 1.5$, $\theta^* = 1.65$, $m = 0.12$

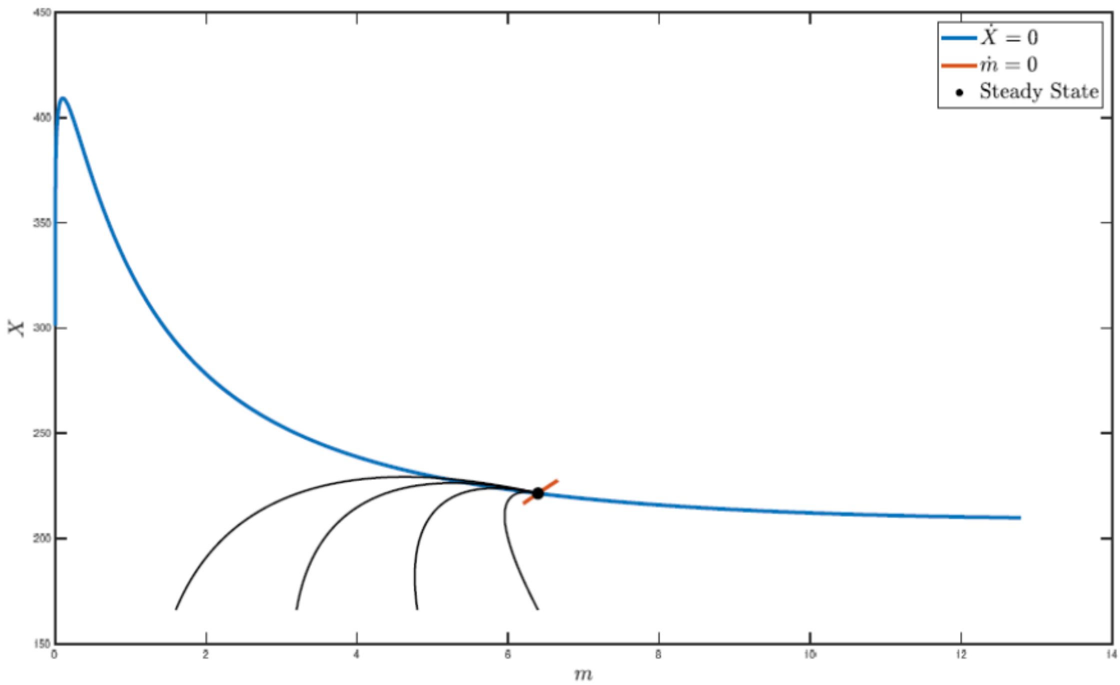


Example 2: $\rho = 0.01$, $\rho^* = 0.011$, $A=2$, $A^* = 6$ $\theta = 1.5$, $\theta^* = 1.65$, m $= 4.21$

Figure 2: Phase Diagrams of the Nuemerical Examples



Example 3: $\rho = 0.01$, $\rho^* = 0.02$, A=2, A^*=2.5 θ =1.5, θ^*= 1.65, m =0.29



Example 4: $\rho = 0.02$, $\rho^* = 0.01$, A=2, A* =2.5 θ =1.5, θ^*= 1.65, m =6.4

Figure 2(continued): Phase Diagrams of the Numerical Examples

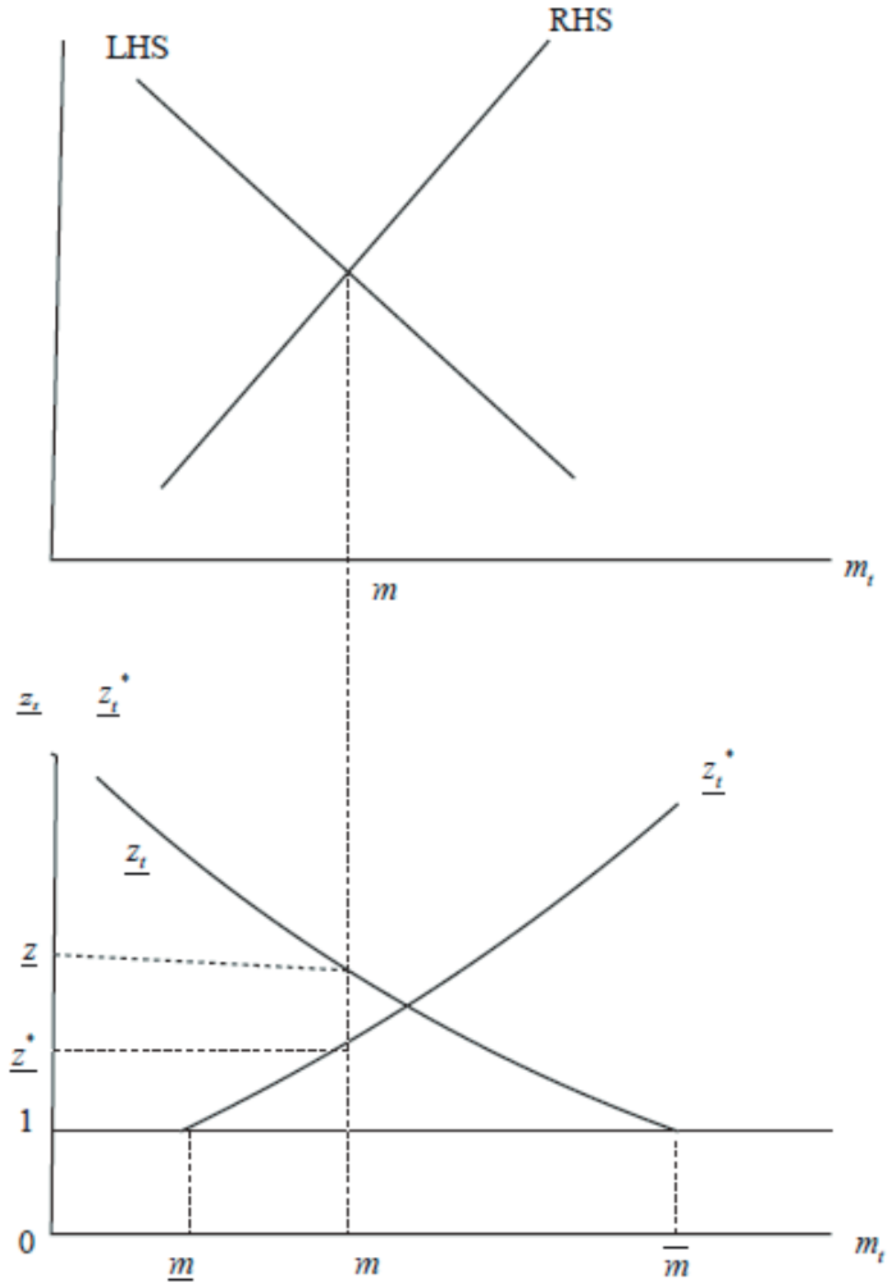


Figure 3: Graphical Exposition of the Steady-State Equilibrium

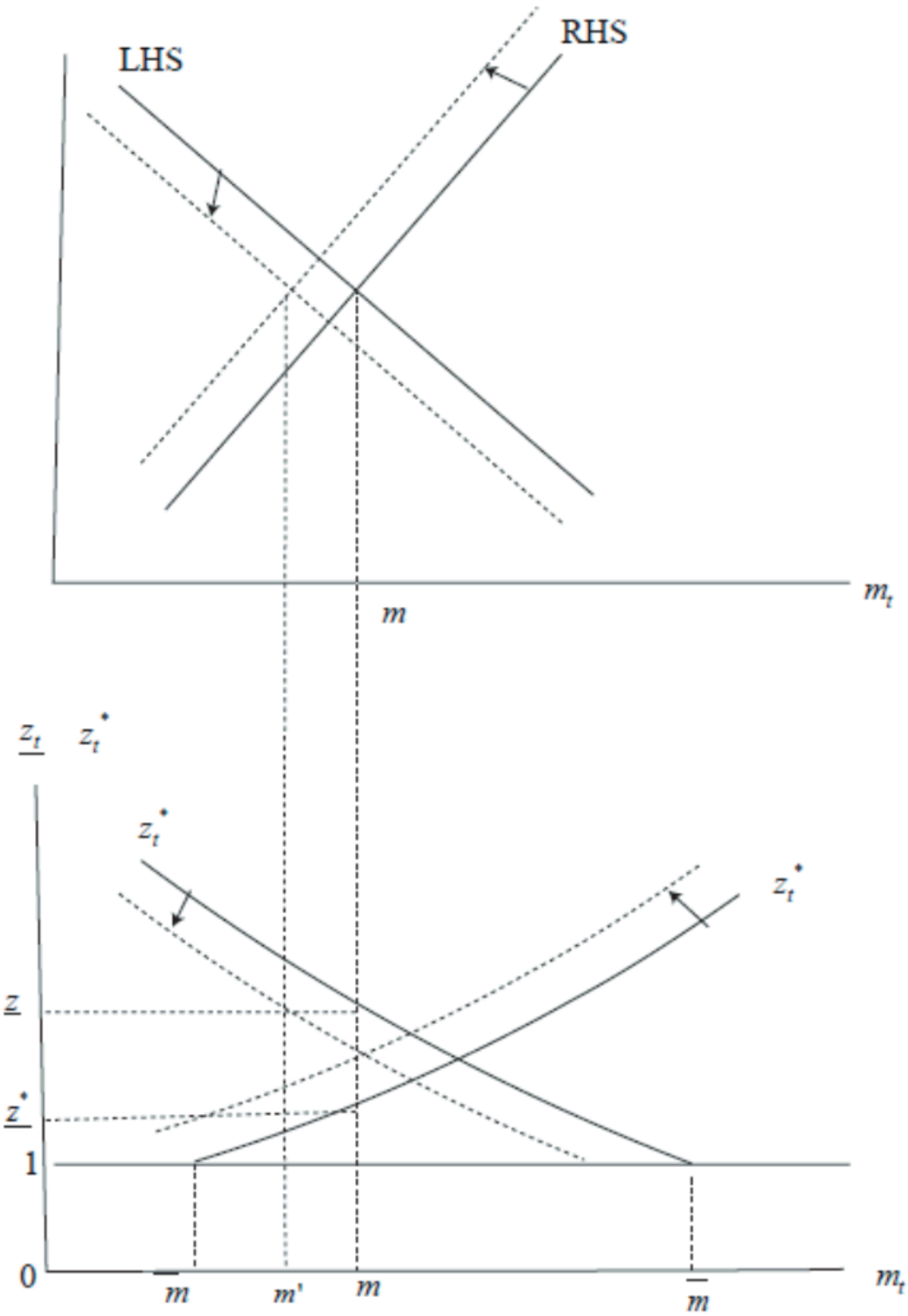


Figure 4: Effects of a Decrease in `θ*

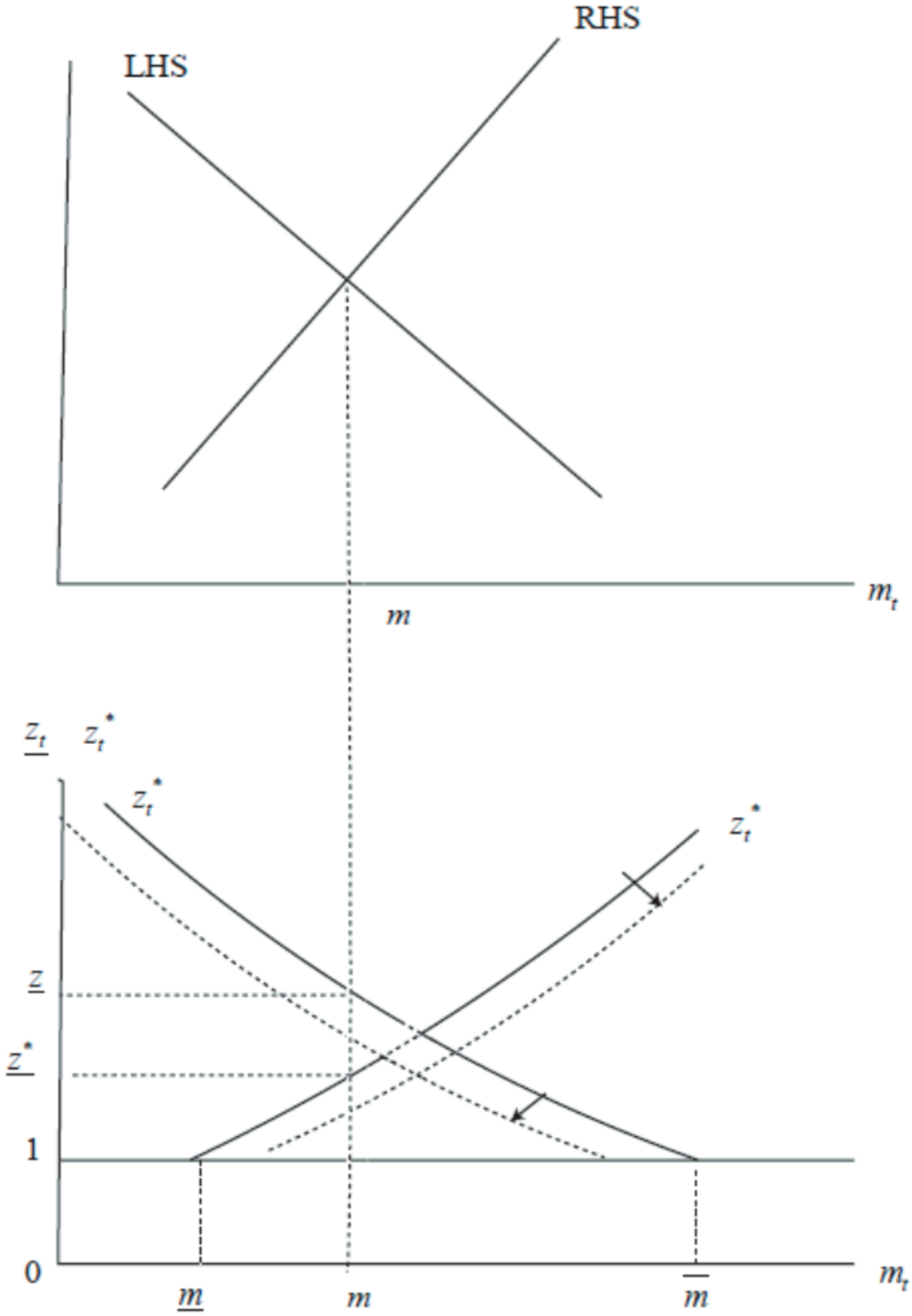


Figure 5: Effects of Simultaneous Decreases in θ and θ *

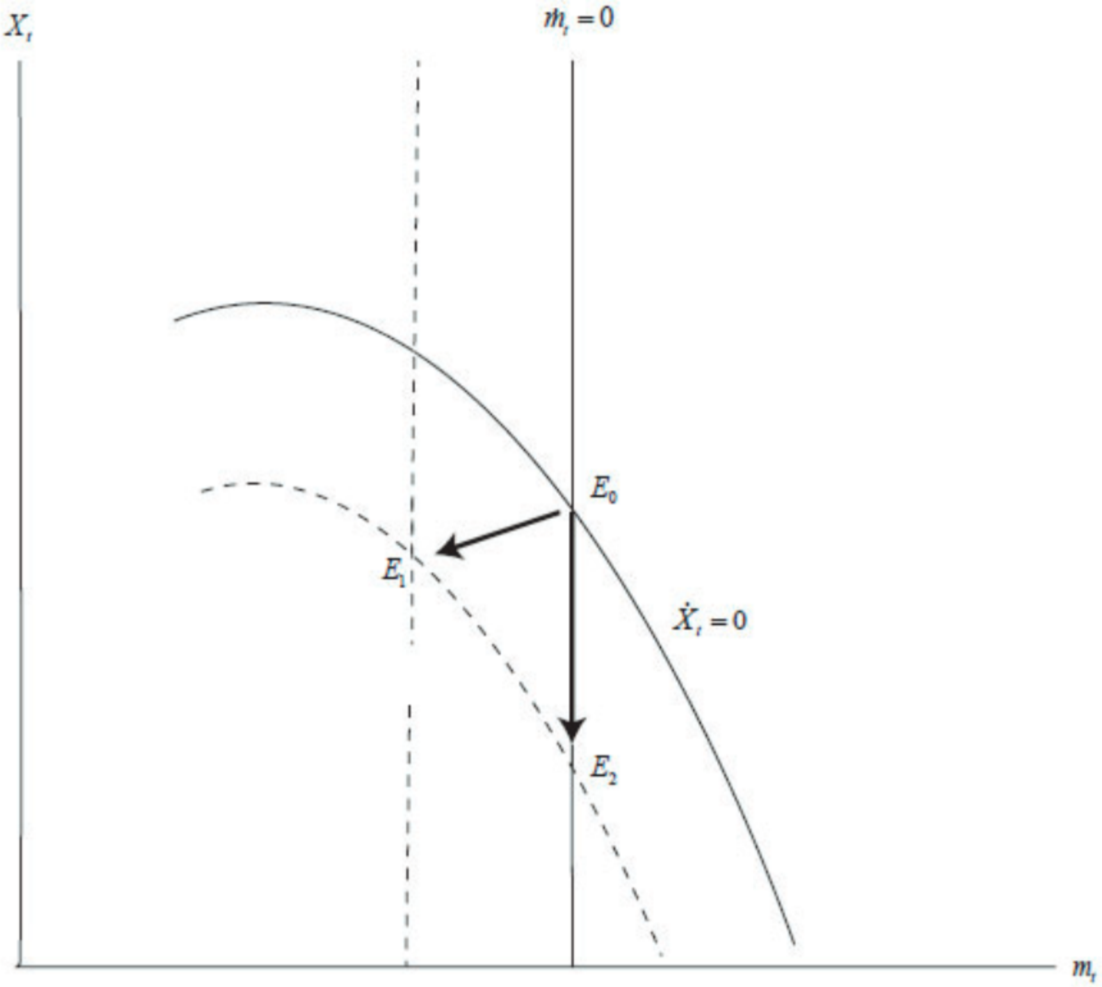
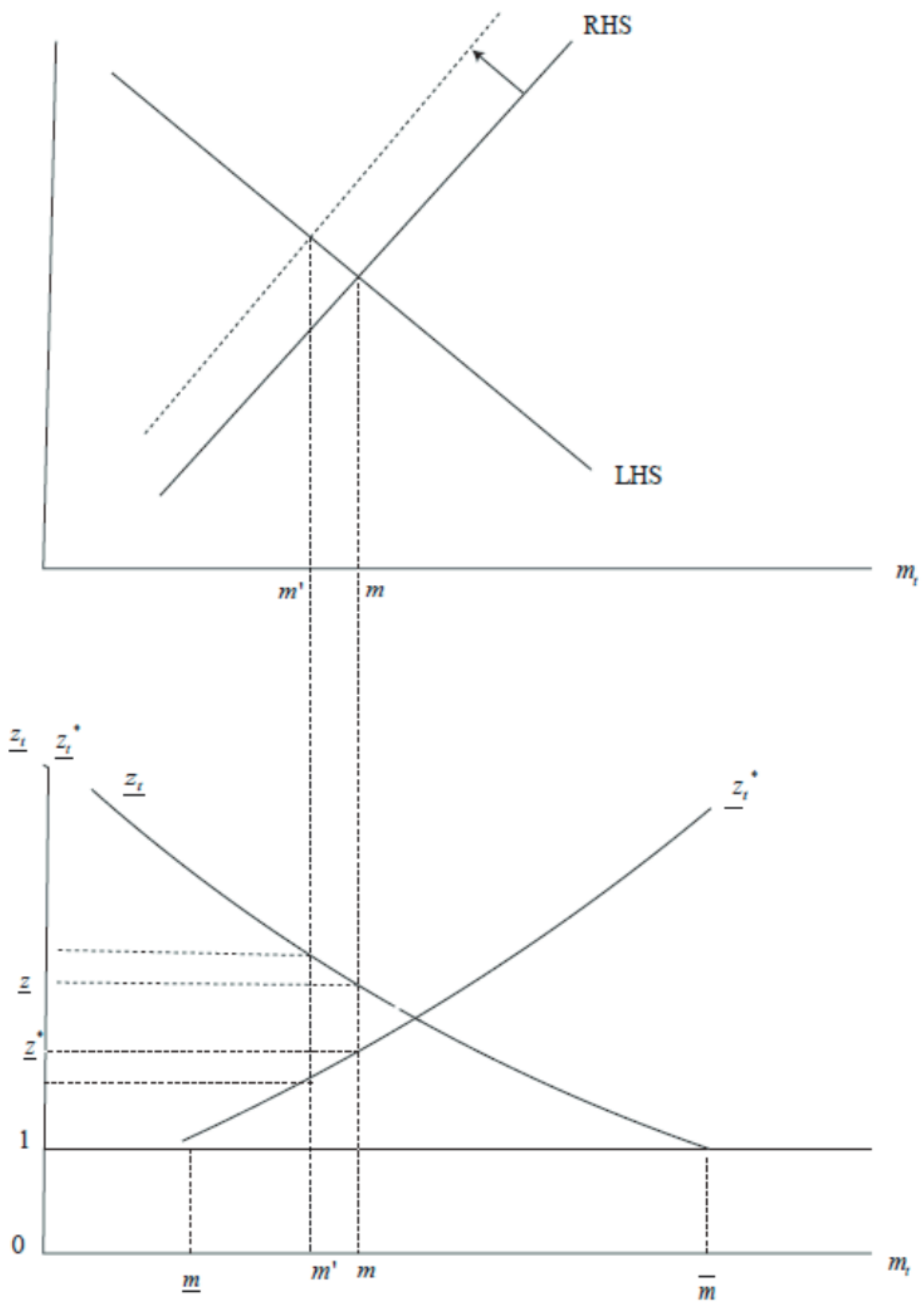


Figure 6: Phase Diagram after the Shocks



Fifure 7: Effects of a rise in ρ*