Summary of thesis: Scaling laws for turbulent relative dispersion in two-dimensional energy inverse-cascade turbulence

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Transport phenomena are ubiquitous in nature. Especially, turbulence efficiently transports and promptly disperses substances such as pollutants, chemical and biological agents because of its nonlinearity of the dynamics. Turbulent relative dispersion, which is statistics of Lagrangian particle pairs, is probably the simplest statistics to characterize the nature of turbulent diffusion such as superdiffusivity and multiscaling. Here, Lagrangian particles are passively advected by turbulent flow and give no back-reaction on the flow. According to the classical phenomenology initiated by L. F. Richardson, particle pairs in turbulence completely forget quickly information on their initial conditions such as initial separations, and the mean square of the relative separations, $\langle r^2(t)\rangle$ exhibits superdiffusive scaling law, that is, so-called Richardson-Obukhov law, $\langle r^2(t)\rangle \propto t^3$. Here, r is the relative separation of a particle pair and t is the elapsed time since particle pairs started to be advected in the turbulent flow. The bracket $\langle\cdot\rangle$ is an ensemble average.

The t^3 scaling law, however, has never been clearly observed in both laboratory experiments and numerical simulations. Furthermore, the mean square of the relative separation strongly depends on the initial separation for a long time. To make matter worse, the mean square of the relative separation exhibits the t^3 power law at a special initial separation because of the initial separation dependence. Here, we cannot immediately conclude that this apparent t^3 law is consistent with the Richardson-Obukhov law because the mean squares of the relative separations at the other initial separations do not exhibit the t^3 power law.

In chapter 1, we review the phenomenology and its experimental results of the turbulent relative dispersion. Especially, we focus on two properties of turbulent relative dispersion: superdiffusivity and scaling laws. Then, we review some recent experimental results and debate on the validity of the Richardson phenomenology.

In this thesis, we study the turbulent relative dispersion in two-dimensional energy inverse-cascade turbulence in terms of the initial separation dependence in two ways: conditional sampling method and two-time Lagrangian velocity correlation function. First, in chapter 2, we develop a conditional sampling method by which the conditional mean square becomes independent of the initial separation dependence and exhibits t^3 scaling law for all initial separations. On the other hand, the conditional mean square of relative velocity of the particle pairs exhibits anomalous scaling law deviated from the prediction of Kolmogorov phenomenology, which is the standard phenomenology of Eulerian turbulence. According to these results, we conjecture that the t^3 power law exhibiting at the special initial separation is not consistent with the Richardson-Obukhov law, or the Kolmogorov phenomenology can not always apply the statistics on the turbulent relative dispersion. It is noted that the Richardson-Obukhov law is also predicted via Kolmogorov phenomenology.

Second, in chapter 3, we investigate the two-time Lagrangian velocity correlation function, which is defined as $\langle \delta \boldsymbol{v}(t_1) \cdot \delta \boldsymbol{v}(t_2) \rangle$. Here, $\delta \boldsymbol{v}(t)$ is relative velocity of a particle pair. We propose the scaling law for the two-time Lagrangian velocity correlation function by means of incomplete similarity. Then, we confirm that the proposed scaling law is consistent with the experimental data in two-dimensional energy inverse-cascade turbulence. From the scaling law for the two-time Lagrangian velocity correlation function, we demonstrate that the t^3 power law exhibiting at the special initial separation is artifact induced by finite-size effects of turbulence. Finally, we improve the Richardson-Obukhov law to be observed in experimental data and discuss validity of the Richardson-Obukhov law at infinite Reynolds number.

In chapter 4, we summarize the results of this thesis, and some concluding remarks are presented. Finally, we provide some future prospects.