（ 続紙 1 ）

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論文題目	Level set numerical approach to anisotropic mean curvature flow on obstacle （障害物上の非等方的平均曲率流のための等高面方法による数値解法）		

（論文内容の要旨）

The thesis investigates multiphase geometric evolutions with direction dependent energy densities and with global constraints on the enclosed volumes. This kind of problem has numerous applications in technology and natural science fields, while its mathematical understanding is still incomplete.

The exposition starts with a review of known results in order of increasing complexity. Thus, first, two-phase mean curvature flow model is introduced, together with its derivation and main results on well-posedness theory. This relates to the approaches to mathematically represent an evolving interface, which are described and the motivation is given for selecting the implicit level-set approach in view of possible topology changes in the investigated phenomenon. The overview then continues with results on two-phase problems with anisotropic, i.e., normal direction-dependent, energy and / or with constraint on the enclosed volume. Finally, the multiphase problem with junctions is reviewed showing that for the anisotropic case mathematical theory is available only in the explicit approach without topology changes. Next, methods to numerically approximate the multiphase anisotropic problem are reviewed with the finding that although the anisotropic problem is treated explicitly [Wang et al., 2015] and the isotropic problem is treated implicitly [Xu et al., 2017], there is no method in the literature addressing possible topology changes in the anisotropic case.

The thesis then proceeds with a detailed comparative numerical analysis of convolution kernels that are the main tool to express anisotropy in the implicit approach. The output is a list of advantages and disadvantages of four main kernels proposed in the past, which is based on quantitative evaluation of their numerical properties, such as convergence order, required computational time, ability to deal with sharp corners, etc.

The main result of the thesis is then the construction and analysis of a numerical algorithm for solving a special type of a three-phase problem, namely the obstacle problem, where one of the phases is fixed throughout time. The algorithm is based on the linearization of the corresponding level-set partial differential equation in the fashion of Bence-Merriman-Osher algorithm, where the Gaussian kernel is replaced by a suitable convolution kernel to express the anisotropy and thresholding height is modified to preserve enclosed volume. Since the algorithm is designed in such

a way that it decreases a nonlocal "heat content" approximation of the interfacial energy, which Γ-converges to the perimeter functional, the author succeeded in proving its unconditional stability with respect to time step, under the condition that the Fourier transform of the adopted convolution kernel is positive.

Finally, numerical properties of the proposed scheme are studied through a series of numerical tests, including the convergence order, behavior of contact angles, and topology changes such as splitting and merging. Since the convergence at contact point is slower than at free interface, a modification of the algorithm is proposed to improve the convergence around triple junctions. The thesis concludes with a summary of its content and topics for possible future research.

（ 続紙 2）

（論文審査の結果の要旨）

本論文は，幾何学的運動を記述する非線型偏微分方程式の数値解析手法を扱っている．具体的に，体積に関する制約のつく非等方的多相平均曲率流という数学的に解決されていない問題を取り上げ，対応する非等方的なエネルギー汎関数の非局所的な近似を線形化することで解を近似するスキームを構成し，その無条件の安定性を証明した．さらに，スキームをコンピュータコードに実装し，非等方性を表現する畳み込み核の比較解析を行うと共に，接触角の近似精度や位相変化を含む数値解析を行なった．

本博士論文では，学位申請者の Gavhale Siddharth Balu 氏は

- 先行研究がを十分に調査し，博士論文で引用しながら系統的かつ詳細に紹介すること，
- 既存結果の調査によりどこまで解っていて，どこから未解決の問題となっているかを理解し，研究の動機を明確に示すこと，
- 博士論文には，次に挙げる本質的に新しい結果を複数含めること：畳み込み核の精密な数値解析とそれによる性能比較，非等方的多相平均曲率流を近似するスキームの構築，新しいスキームの数学的な性質（とくに安定性）の厳密な証明，新しい数値スキームの実装と数値解析，
- 学位申請者による貢献度が高い（博士論文の新しい結果の部分では 50%以上である）こと，
- 信頼性のある国際ジャーナル Applications of Mathematics (Springer) に投稿し，3 名の査読者の審査を経て採録されたこと

を実現できている．さらに，公開講演の発表では

- 研究の背景を紹介し，研究成果について明確で正確な説明をしたのに加え，
- その後の質疑応答では，研究内容とその関連分野において十分な知識と理解を得ていること

が確認された．よって，本論文は博士（理学）の学位論文として価値あるものと認める．また，令和 4 年 1 月 27 日，論文内容とそれに関連した事項について試問を行った結果，合格と認めた．

要旨公開可能日：　　年　　月