Essays on Business Cycles and Monetary Policy

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Abstract

This dissertation provides three essays on business cycles and the monetary policy.

Chapter 1, "The Impacts of Credit Standards on Aggregate Fluctuations in a Small Open Economy: The Role of Monetary Policy," constructs a small open economy model with financial frictions to generate the countercyclical movement in credit standards. Our analysis demonstrates that countercyclical fluctuations in credit standards work as an amplifier of shocks to the economy. In particular, the existence of endogenous credit standards increases output volatility by 21%. We also suggest three alternative tools for policymakers to dampen the effects of endogenous credit standards on macroeconomic volatility. First, the introduction of credit growth to the monetary policy succeeds in counteracting the fluctuation of lending, and thus decreasing the additional volatility considerably. Second, the exchange rate augmented monetary policy, if well-constructed, is considered an efficient tool to eliminate most of the additional fluctuations caused by deep habits in the banking sector. Finally, the introduction of the foreign interest augmented policy also proves successful in dampening the effect of endogenous movements in lending standards.

Chapter 2, "Modeling Inflation Dynamics: A Bayesian Comparison between GARCH and Stochastic Volatility," employs a prominent model comparison criterion, namely the Bayes factor, to compare three commonly used GARCH models with their stochastic volatility (SV) counterparts in modeling the dynamics of inflation rates. By using consumer price index (CPI) data from 18 developed countries to evaluate these models, we find that the GARCH models are generally outperformed by their stochastic volatility counterparts. Furthermore, the stochastic volatility in mean (SV-M) model is shown to be the best for all 18 countries considered. The paper also examines which model characteristics play a main role in modeling inflation rates. It turns out that inflation volatility feedback is a crucial feature that we should take into consideration when modeling inflation rates. The relevance of a leverage effect, however, is found to be rather ambiguous. Finally, the forecasting results using the log predictive score confirm these findings.

Chapter 3, “Monetary Policy in Practice: Do Central Banks Respond to Movements in Exchange Rate and Credit Growth?” with Nguyen Minh Phuong, examines the importance of exchange rate and credit growth fluctuations when designing the monetary policy in Thailand. We construct and estimate a small open economy DSGE model with banking using Bayesian methods. We consider generalized Taylor rules, in which policymakers adjust the policy rates in response to output, inflation, credit growth, and exchange rate fluctuations. The marginal likelihoods are then employed to investigate whether the Bank of Thailand responds to fluctuations in the exchange rate and credit growth. Our findings indicate that the monetary authority does target exchange rates, whereas there is no evidence in support of incorporating credit growth in the policy rules. These findings survive various robustness checks. In addition, we demonstrate that domestic shocks contribute significantly to the domestic business cycles. Although the terms of trade shock plays a minor role in the Thai business cycles, it explains the largest proportion of exchange rate fluctuations, followed by the country risk premium shock.

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Chapter 1

The Impacts of Credit Standards on Aggregate Fluctuations in a Small Open Economy: The Role of Monetary Policy

1.1 Introduction

Credit standards, such as banking spreads and collateral requirements, move in a countercyclical direction, according to numerous empirical investigations. Santos and Winton (2008), employing US data for the credit market, show that banking markup can rise up to 95 basis points in a recession. Even when credit risk is taken into consideration, Aliaga-Díaz and Olivero (2011) demonstrate that countercyclical banking spreads can be found. Similar results are observed in numerous OECD countries using both Bankscope data and International Financial Statistics (IFS) data (see Olivero, 2010). As for the empirical evidence supporting countercyclical fluctuations in collateral requirements, Asea and Blomberg (1998), using a large dataset for commercial and industrial loans issued in the US during the period 1977-1993, indicate that a remarkable increase in the probability of collateral pledge is attributed to higher aggregate unemployment. In other words, collateral requirements are empirically proven to be countercyclical. Similarly, Jimenez et al. (2006) employ data from Spain for all loans exceeding 6000 euros made between 1984 and 2002 to show that loans made during booms are less likely to be collateralized than those made during downturns.

By introducing the lending relationship, a number of models are successful in producing countercyclical banking spreads on a theoretical basis (see, e.g., Aliaga-Díaz and Olivero, 2010; Aksoy et al., 2013; Melina and Villa, 2014). Ravn (2016) extends the previous literature by al-

"The Impacts of Credit Standards on Aggregate Fluctuations in a Small Open Economy: The Role of Monetary Policy," *Economies* 9(4):203.

lowing banks to compete not only in interest rate spreads but also in collateral requirements. These papers, however, are closed economy models, and thus various foreign aspects such as exchange rates and foreign demand, which could be the source of fluctuations, are not considered. In the present paper, we attempt to fill this gap in the literature. We present a small open economy model with Calvo price setting. In our model, the endogenous fluctuation in credit standards emerges as a result of the presence of a lending relationship between lenders and borrowers. The features of our model are characterized as follows:

First, deep habits in banking have been shown to be effective at capturing characteristics of the lending relationship between borrowers and lenders.[1] Therefore, we follow Aliaga-Díaz and Olivero (2010) and assume that wholesale good entrepreneurs form deep habits in the demand for loans from banks to incorporate the lending relationship in our model. Second, empirical evidence has indicated that collateral requirements, as one measure of credit standards, fluctuate over the business cycle and that they move in a countercyclical fashion. To account for this finding, we follow Ravn (2016) and endogenize the fluctuation in collateral requirements into our model by an assumption that banks compete with each other on both interest rate spread and collateral pledge when giving loans.[2] Finally, we extend the current setting to a small open economy model by introducing the small open economy feature of Galí and Monacelli (2016). Specifically, we assume that the size of the domestic economy is relatively small compared to that of the world economy. As a result, we can neglect its impact on the world economy, and thus consider the world aggregate as exogenous. However, unlike Galí and Monacelli (2016) who assume the existence of complete international financial markets to close the open economy, we relax this assumption and allow for the incomplete asset markets. To induce stationarity, we follow Schmitt-Grohé and Uribe (2003) and employ the debt elastic interest

[1] Several studies support this finding. For example, Aliaga-Díaz and Olivero (2010) point out that banks obtain an information monopoly over the creditworthiness of customers when they monitor borrowers, which triggers costs for borrowers to switch to other banks (switching costs). Deep habits, as proposed by Ravn et al. (2006), can be indicated as a parsimonious way of incorporating switching costs into the dynamic general equilibrium model. Furthermore, the deep habits model can generate countercyclical credit standards, which is in line with empirical findings. The explanation is that an expansionary shock triggers an increase in output, and thus the demand for loans from entrepreneurs. In order to set the new bank spread, banks will consider the following trade-off: (1) Increasing the current profit by setting a higher spread; (2) Generating a higher future market share by lowering the spread to attract more borrowers. Due to the persistence of the shock, the latter effect dominates the former one. As a result, a positive shock will lead to a lower credit spread.

[2] We believe that bank competition on the amount of collateral that firms need to pledge is particularly relevant for the market of bank loans. Cerqueiro et al. (2016), using the difference-in-difference method for Swedish data, demonstrate that collateral is crucial for both borrowers and lenders and that with a high-quality collateral pledge, borrowers can experience a lower lending rate and an increase in credit availability. Furthermore, as the duration of the lending relationship increases, collateral requirements tend to relax. Berger and Udell (1995), for example, show that a long relationship with banks reduces the probability of collateral pledging for borrowers.

rate to close our model.

The paper is motivated by the following questions which are not addressed in previous deep habits-related literature: How do fluctuations in the credit standards emerging from the deep habits in the banking sector amplify macroeconomic volatility in a small open economy setting? In particular, what are the quantitative impacts of endogenous credit standards on output volatility when taking into account the open economy features? How do credit standards move in response to an increase in foreign demand? What are recommendations for policymakers in order to diminish the volatility brought about by the existence of the lending relationship? To answer these questions, we incorporate 4 shocks into our model: (1) technology shock; (2) labor supply shock; (3) monetary policy shock; (4) foreign demand shock as in Gali and Monacelli (2005), and calibrate the model based on Swedish data.[3] We find that the countercyclical movement in credit standards indeed works as a financial accelerator of these shocks to the economy. More specifically, the presence of credit standards increases output volatility by approximately 21%. This demonstrates the quantitative importance of endogenous credit standards over the business cycle that we should take into analysis. To combat the impact of endogenous fluctuations in credit standards, we introduce three alternative monetary policies. First, we show that credit growth augmented monetary policy is an effective tool in reducing the additional volatility arising from endogenous credit standards. Second, the addition of the exchange rate to the monetary policy also proves successful in counteracting the fluctuations in lending, thus eliminating most of the additional volatility. Third, we let the policy interest rate respond to changes in the foreign interest rate in order to indirectly counteract movements in lending. This policy, if well designed, can eliminate the additional volatility substantially.

There have been other studies that investigate the impacts of deep habits in the banking sector on economic fluctuation. However, to the best of our knowledge, existing studies on this field all employ the closed economy framework. Aliaga-Díaz and Olivero (2010), using the deep habits mechanism developed by Ravn et al. (2006) for the banking sector to model the switching cost of borrowers, show that the interest rate spread moves in a countercyclical pat-

[3]For this study, we choose the Swedish economy to calibrate our models for the following reasons: First, Sweden is a small open economy with a floating exchange rate regime; thus, the fluctuation of the exchange rate could play an important role in the formulation of monetary policy. Bjørnland and Halvorsen (2014) provide empirical evidence to support this hypothesis. Specifically, by employing a structural VAR model with sign and zero restrictions, they find that monetary policy responds strongly to the movements in the exchange rate in the case of Sweden. Second, empirical studies demonstrate that credit standards in Sweden are countercyclical (see Olivero, 2010). Therefore, investigating the impacts of credit standards on aggregate fluctuations in the Swedish economy is extremely important. Finally, as previously indicated, bank competition over the amount of collateral that entrepreneurs must pledge is particularly relevant for the Swedish bank loan market.

tern, as observed in the US data. Furthermore, they find that countercyclical spreads do indeed work as a financial accelerator of the productivity shock in the US economy. Melina and Villa (2014), by endogenizing the bank spread through the deep habits framework in banking, are able to replicate the negative response of the spread to an expansionary fiscal shock observed in the data. Also, their findings point out that countercyclical fluctuations in the interest rate spread generate an amplification mechanism in the transmission of the government spending shock. Ravn (2016) incorporates empirically demonstrated endogenous fluctuations in interest rate spreads and collateral requirements into macroeconomic models. He shows that the countercyclical lending standards amplify the impacts of macroeconomic shocks on the economy, with output volatility going up by 25%. Melina and Villa (2018) incorporate the financial frictions arising from deep habits into the DSGE model and apply the Bayesian technique to estimate the model. They discover that monetary policy in the United States responds to credit growth during the Great Moderation. Airaudo and Olivero (2019) use a DSGE model with financial frictions arising from the existence of the lending relationship in banking to examine the optimal monetary policy. Their analysis demonstrates that countercyclical fluctuations in lending spreads exacerbate the inflation-output trade-off when designing the optimal policy in both discretion and commitment instances. Furthermore, they show that the welfare cost of committing to suboptimal rules increases as we raise the magnitude of deep habits. Shapiro and Olivero (2020) introduce deep habits into an RBC model with endogenous labor force participation to investigate the role of labor force participation as an accelerator of financial shocks in the model. They show that the impacts of countercyclical spreads on labor market dynamics are magnified by endogenous participation.

We also contribute to the growing body of literature on open economy models commenced by Mendoza (1991). Monacelli (2005) introduces the imperfect exchange rate pass-through in the small open economy setting and finds that the monetary policy analysis in an open economy model is not isomorphic to that in a closed version under the presence of incomplete pass-through.[4] Galí and Monacelli (2016) employ the small open economy model with sticky prices and sticky wages to investigate the impacts of increased wage flexibility. They discover that higher wage flexibility leads to a welfare reduction, especially in countries with a fixed exchange rate regime. Recently, many studies have incorporated financial frictions into the small

[4]Various empirical studies support the viewpoint of Monacelli (see, e.g., Campa and Goldberg, 2005; McCarthy, 2007). Moreover, Ferrero et al. (2008), using data from Sweden, Italy, and the United Kingdom, provide evidence on how import prices react to a sharp initial depreciation of the exchange rate and conclude that the pass-through on import prices is high, but with a delay.

open framework. Céspedes et al. (2004) introduce financial frictions formulated by Bernanke et al. (1999) to investigate the relationships among balance sheets, exchange rates, and outcomes in the small open economy setting. They point out that the external financing premium determined by an entrepreneur's net worth is unaffected by the exchange rate regime, which is contrary to previous literature. In their model, they show that the flexible exchange rate regime plays a better role in containing the external shocks and is optimal in terms of welfare. Christiano et al. (2011) incorporate both financial frictions and employment frictions into the small open economy framework, and employ Bayesian methods to estimate the model for Swedish data. They find that the entrepreneur's wealth shock is crucial to explaining movements in both GDP and investment, whereas a shock to the marginal efficiency of investment only plays a limited role in variance decomposition. Their analysis also shows that in general, the impact of demand shocks is reduced while that of supply shocks is magnified once the open economy feature is introduced. Afrin (2020) investigates the impacts of financial frictions emerging from oligopolistic bank competition on Australian business cycles. She demonstrates that the oligopolistic banking sector produces a distinct shock propagation mechanism that frequently accelerates business cycles.

The remainder of the paper is organized as follows. Section 2 present the small open economy model. Section 3 show the calibration strategies. We report main results, robustness checks and policy analyses in section 4. Finally, section 5 concludes.

1.2 Model

The DSGE model presented here is fully based on the models of Ravn (2016), Galí and Monacelli (2016), Monacelli (2005), and Schmitt-Grohé and Uribe (2003). The economy is inhabited by households; entrepreneurs; domestic retailers; importers; commercial banks; and monetary authorities. Households choose consumption, deposit, and labor to maximize their utility subject to the budget constraint. Entrepreneurs, borrowing from the banks, employ capital stock and labor services to produce homogeneous goods. Domestic retailers then differentiate the goods at no cost and resell them in a monopolistically competitive market for consumption, investment, and export. Importers also operate in monopolistic competition, importing differentiated goods from the world economy and selling them in the home economy. Banks maximize the expected discounted value of profits by choosing their demand for deposits, external debt, and loan rates.

1.2.1 Households

There is a continuum of households indexed by $i \in (0,1)$. Household preferences are given by the following utility function:

$$E_0 \sum_{t=0}^{\infty} (\beta^p)^t \left[\log(C_t^{i,p} - h^p C_{t-1}^{i,p}) - Z_t \frac{(N_t^i)^{1+\psi}}{1+\psi} \right], \tag{1.1}$$

where $\beta^p \in (0,1)$, $h^p \in (0,1)$, and ψ denote the discount factor, the habit parameter in consumption, and the inverse Frisch elasticity, respectively. $C_t^{i,p}$ is a composite consumption index, and N_t^i is labor. The superscript p is used since households are assumed to be more patient than entrepreneurs. Z_t represents the disutility of a labor supply shock. The shock evolves as follows:

$$\log Z_t = \rho_z \log Z_{t-1} + (1-\rho_z) \log Z + \sigma_z \varepsilon_{z,t},$$

where $\varepsilon_{z,t}$ is an i.i.d. process with standard deviation σ_z, $Z > 0$ is the steady state value of the labor supply shock, and $\rho_z \in (0,1)$ is the persistence of the shock. Let $C_t^{i,p} = [(1-v)^{\frac{1}{\eta}} (C_{H,t}^{i,p})^{\frac{\eta-1}{\eta}} + v^{\frac{1}{\eta}} (C_{F,t}^{i,p})^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}$ be a composite index of domestic final good consumption $C_{H,t}^{i,p}$ produced by domestic retailers and imported good consumption $C_{F,t}^{i,p}$ imported by local retailers. $\eta > 0$ measures the intratemporal elasticity of substitution between domestic and imported goods. $v \in (0,1)$ denotes the share of imported goods in consumption basket of the home country.

In each period t, households face two optimization problems: an optimal allocation of goods and a utility maximization problem. First, the optimal allocation of expenditures between domestic and imported goods implies

$$C_{H,t}^{i,p} = (1-v) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t^{i,p}, \quad C_{F,t}^{i,p} = v \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t^{i,p}, \tag{1.2}$$

where $P_t = [(1-v)P_{H,t}^{1-\eta} + vP_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ denotes the consumer price index (CPI). $P_{H,t} = (\int_0^1 P_{Hj,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$ and $P_{F,t} = (\int_0^1 P_{Fj,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$ are the price indexes of domestic and imported final goods respectively, both expressed in the home currency. $\epsilon > 1$ denotes the elasticity of substitution across final goods within each category of domestic or foreign goods.[5]

[5] Note that $C_{H,t}^{i,p}$ and $C_{F,t}^{i,p}$ are in turn composites of domestic differentiated goods and imported differentiated

Second, households, taking the deposit rate, nominal wage, and the sum of profit as given, choose consumption, labor supply, and stock of deposit to maximize their utility function. The optimization can be summarized as follows:

$$\max_{C_t^{i,p}, N_t^i, M_{b,t}^i} E_0 \sum_{t=0}^{\infty} (\beta^p)^t \left[\log(C_t^{i,p} - h^p C_{t-1}^{i,p}) - Z_t \frac{(N_t^i)^{1+\psi}}{1+\psi} \right], \tag{1.3}$$

s.t.

$$P_t C_t^{i,p} + \int_0^1 M_{b,t}^i db \leq W_t N_t^i + R_{t-1}^d \int_0^1 M_{b,t-1}^i db + \Upsilon_t^i, \tag{1.4}$$

where W_t denotes the nominal wage, R_{t-1}^d is the gross interest rate on the deposit $M_{b,t-1}^i$ of household i in bank b, and Υ_t^i denotes the sum of profits gained by household i. Equation (1.4) represents the budget constraint of the household. The first-order conditions yield[6]

$$\frac{1}{C_t^p - h^p C_{t-1}^p} - \beta^p E_t \frac{h^p}{C_{t+1}^p - h^p C_t^p} = \lambda_t^p, \tag{1.5}$$

$$Z_t N_t^{\psi} = w_t \lambda_t^p, \tag{1.6}$$

$$\lambda_t^p = \beta^p R_t^d E_t \left(\frac{\lambda_{t+1}^p}{\Pi_{t+1}} \right), \tag{1.7}$$

where λ_t^p is the Lagrange multiplier associated with equation (1.4), $w_t = \frac{W_t}{P_t}$ is the real wage, and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the gross inflation rate. Equation (1.6) describes the optimal choice for labor supply, while a combination of equation (1.5) and equation (1.7) can be interpreted as an Euler equation for consumption.

1.2.2 Wholesale Good Entrepreneurs

The economy is inhabited by a continuum of entrepreneurs indexed by $e \in (0,1)$. Entrepreneur e eventually maximizes the following utility acquired from consuming both domes-

goods indexed by $j \in (0,1)$:

$$C_{H,t}^{i,p} = \left[\int_0^1 (C_{Hj,t}^{i,p})^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad C_{F,t}^{i,p} = \left[\int_0^1 (C_{Fj,t}^{i,p})^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}.$$

[6] The derivations of first-order conditions are available upon request. Following the standard literature in models with deep habits, we consider a symmetric equilibrium only.

tic and imported final goods:

$$E_0 \sum_{t=0}^{\infty} (\beta^I)^t \log(C_t^{e,I} - h^I C_{t-1}^{e,I}), \tag{1.8}$$

where $\beta^I \in (0,1)$ and $h^I \in (0,1)$ denote the discount factor and the habit parameter in consumption, respectively. Similar to household consumption, it is assumed that the consumption of entrepreneur $C_t^{e,I}$, defined as $C_t^{e,I} = [(1-v)^{\frac{1}{\eta}} (C_{H,t}^{e,I})^{\frac{\eta-1}{\eta}} + v^{\frac{1}{\eta}} (C_{F,t}^{e,I})^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}$, is a composite index of domestic and imported final goods.[7]

Following Kiyotaki and Moore (1997), Iacoviello (2005), Gerali et al. (2010), and Ravn (2016), we assume that the entrepreneur's loan from each bank is restricted by the collateral constraint as follows:

$$L_{b,t}^e \leq \frac{1}{R_{b,t}^l} \xi_{b,t} a_t^e, \tag{1.9}$$

where $L_{b,t}^e$, $R_{b,t}^l$, and $\xi_{b,t}$ denote the borrowing of entrepreneur e from bank b, the bank b's gross lending rate, and the loan to value (LTV) ratio allowed by bank b, respectively.[8] a_t^e is the expected value of the entrepreneur e's asset and is given as follows:

$$a_t^e = E_t Q_{t+1} K_t^e, \tag{1.10}$$

where Q_t denotes the price of installed capital, K_t^e is the stock of capital of entrepreneur e.

In each period t, entrepreneur e faces two main optimization problems: an optimal allocation of loans from different banks, which results in the lending relationship; and a utility maximization problem. The former can be summarized as follows:

$$\min_{L_{b,t}^e} \left[\int_0^1 \Gamma_{b,t} L_{b,t}^e db \right], \tag{1.11}$$

s.t.

$$eq.(1.9),$$

$$\left[\int_0^1 (L_{b,t}^e - h^l S_{b,t-1}^l)^{\frac{\eta_l - 1}{\eta_l}} db \right]^{\frac{\eta_l}{\eta_l - 1}} = (D_t^l)^e, \tag{1.12}$$

[7]For simplicity, we assume that the share of imported goods in the consumption basket of entrepreneurs is the same as that of households.

[8]For simplicity, it is assumed that the LTV ratio allowed by bank b is the same for all entrepreneurs.

$$S_{b,t}^{l} = \rho_l S_{b,t-1}^{l} + (1-\rho_l) L_{b,t}, \tag{1.13}$$

where L_{bt}^{e} denotes the entrepreneur e's demand for loans offered by bank b, while $(D_t^l)^e$ is the demand for loans by the firm augmented by lending relationships. The term $S_{b,t}^{l}$, defined as $S_{b,t}^{l} = \int_0^1 (S_{b,t}^{l})^e de$, indicates that habits are external, as in Ravn et al. (2006). The parameter $h^l \in (0,1)$ denotes the degree of habit in lending, η_l is the elasticity of substitution across different banks' loans, and ρ_l is the persistence of lending relationships. $\Gamma_{b,t}$ is defined as $\Gamma_{b,t} = R_{b,t}^{l} + \frac{v_b}{\xi_{b,t}}$, with the first term indicating the interest rate payments and the second one being the amount of collateral. The parameter v_b denotes the relative weight of collateral-minimization desire. Equation (1.11) demonstrates the minimization problem of the entrepreneur. Following Ravn (2016), we assume that entrepreneurs take both interest rate expenditures and collateral requirements into consideration when they choose optimal demand for loans from each bank.[9] Equation (1.12) shows that entrepreneurs form deep habits in their relationship with banks, while Equation (1.13) indicates the evolution of stock of habit. The assumption of deep habits in wholesale good entrepreneurs' demand for bank loans, as explained by Aliaga-Díaz and Olivero (2010), yields a wedge between effective borrowing and actual borrowing. The wedge displays switching costs.[10] Given a total demand for loan $(D_t^l)^e$, each entrepreneur e chooses $L_{b,t}^{e}$ to minimize both the interest rate expenditure and the amount of collateral. The solution to the problem yields the demand for bank b's loans:

$$L_{b,t}^{e} = \left(\frac{\Gamma_{b,t}}{\Gamma_t}\right)^{-\eta_l} (D_t^l)^e + h^l S_{b,t-1}^{l}, \tag{1.14}$$

where $\Gamma_t \equiv R_t^l + \frac{v_b}{\xi_t}$, with $R_t^l \equiv \left[\int_0^1 (R_{b,t}^{l})^{1-\eta_l} db\right]^{\frac{1}{1-\eta_l}}$ and $\xi_t \equiv \left[\int_0^1 \xi_{b,t}^{1-\eta_l}\right]^{\frac{1}{1-\eta_l}}$ being the aggregate lending rate and LTV ratio, respectively.

In addition to the allocation of lending expenditure, in each period t, entrepreneur e chooses consumption, capital, labor, investment, and borrowing to maximize the utility function. The

[9] There are a number of reasons for entrepreneurs to minimize their collateral pledges. One crucial reason is that they do not want to lose control of assets in the event of default. Moreover, the process of asset valuation induces some additional costs, which entrepreneurs would prefer to avoid. It is worth noting that when v_b is equal to zero, entrepreneurs are just concerned with the interest rate expenditure, as in Aliaga-Díaz and Olivero (2010).

[10] The actual borrowing, denoted by $L_{b,t}^{e}$, is the amount that entrepreneur e will pay back to bank b in the following period. The effective loan, instead, demonstrates the fund available to that entrepreneur after deducting the switching costs to pay for investments, labor services, etc.

maximization problem can be summarized as follows:

$$\max_{C_t^{e,I}, K_t^e, N_t^e, I_t^e, (D_t^l)^e} E_0 \sum_{t=0}^{\infty} (\beta^I)^t \log(C_t^{e,I} - h^I C_{t-1}^{e,I}),$$

s.t.

$$eq.(1.9), eq.(1.12),$$

$$(Y_t^e)^w = A_t (K_{t-1}^e)^\alpha (N_t^e)^{1-\alpha}, \tag{1.15}$$

$$\log(A_t) = \rho_a \log(A_{t-1}) + \sigma_a \varepsilon_{a,t}, \tag{1.16}$$

$$K_t^e = (1-\delta) K_{t-1}^e + I_t^e \left[1 - \frac{\gamma}{2}\left(\frac{I_t^e}{I_{t-1}^e} - 1\right)^2\right], \tag{1.17}$$

$$P_t C_t^{e,I} + \int_0^1 R_{b,t-1}^l L_{b,t-1}^e db \leq P_t^w (Y_t^e)^w - W_t N_t^e - P_t I_t^e + (D_t^l)^e + \Xi_t + \Psi_t. \tag{1.18}$$

The wholesale goods are produced via the technology in equation (1.15), where α is the capital share in production, $(Y_t^e)^w$ denotes wholesale goods, and N_t^e is labor. The total productivity A_t is assumed to be the same across entrepreneurs and follow the AR(1) process as in equation (1.16), with $\rho_a \in (0,1)$ being the persistence of the shock and $\varepsilon_{a,t}$ following an i.i.d. process with standard deviation σ_a. Equation (1.17) is the evolution of capital with, $\delta \in (0,1)$ being the depreciation rate of physical capital and I_t^e being entrepreneur e's investment. Following Galí and Monacelli (2016), we assume that investment is subject to an adjustment cost function.[11] Entrepreneur e's budget constraint is given by equation (1.18), where P_t^w denotes the wholesale price at which entrepreneur e sells its goods in a competitive market to domestic retailers, and $W_t N_t^e$ is the wage bill. Ξ_t and Ψ_t are two lump-sum transfers given exogenously to entrepreneurs.[12] The first-order conditions yield

$$\frac{1}{C_t^I - h^I C_{t-1}^I} - \beta^I E_t \frac{h^I}{C_{t+1}^I - h^I C_t^I} = \lambda_t^I, \tag{1.19}$$

[11]Analogous to consumption, investment is a composite index of domestic and foreign goods, i.e., $I_t^e = [(1-v)^{\frac{1}{\eta}} (I_{H,t}^e)^{\frac{\eta-1}{\eta}} + v^{\frac{1}{\eta}} (I_{F,t}^e)^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}$. $I_{H,t}^e$ and $I_{F,t}^e$ are in turn composite indexes of domestic and imported differentiated goods, respectively. Following Galí and Monacelli (2016), we assume that the share of imported goods in the investment basket is the same as that in the consumption basket for simplifying reasons.

[12]$\Xi_t \equiv h^l \int_0^1 \frac{\xi_{b,t}}{\xi_t} S_{b,t-1}^l db$ represents the difference between effective and actual borrowings, while $\Psi_t \equiv \int_0^1 (1 - \chi_{b,t-1})(R_{b,t-1}^l L_{b,t-1} - \tau \xi_{t-1} a_{t-1}) db$ indicates the wedge between effective and actual repayment of loans. $\chi_{b,t}$ is the probability of repayment and the parameter τ captures the fact that the value of the collateral is lower in liquidation, as we discuss in more detail in the banking sector. The two lump-sum transfers are to guarantee that all markets clear.

$$\beta^I E_t \frac{\lambda_{t+1}^I}{\Pi_{t+1}} R_t^l + \nu_t^I R_t^l = \lambda_t^I, \tag{1.20}$$

$$w_t = (1-\alpha)\frac{mc_t}{p_t^{h,p}} A_t K_{t-1}^{\alpha} N_t^{-\alpha}, \tag{1.21}$$

$$\begin{aligned} \lambda_t^I = {} & \lambda_t^I q_t \left[1 - \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \gamma \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] \\ & + \beta^I E_t \left[\gamma \lambda_{t+1}^I q_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right], \end{aligned} \tag{1.22}$$

$$\lambda_t^I q_t = \beta^I \alpha E_t \left(\lambda_{t+1}^I \frac{mc_{t+1}}{p_{t+1}^{h,p}} A_{t+1} K_t^{\alpha-1} N_t^{1-\alpha}\right) + \beta^I (1-\delta) E_t(\lambda_{t+1}^I q_{t+1}) + \nu_t^I \xi_t E_t(q_{t+1}\Pi_{t+1}), \tag{1.23}$$

where $q_t = \frac{Q_t}{P_t}$ is the price of installed capital measured in units of consumption goods. Note that this price must be equal to the shadow price of capital in units of consumption goods, which means that the equation $q_t = \frac{\kappa_t^I}{\lambda_t^I}$ holds at all times. λ_t^I, κ_t^I, and ν_t^I denote the Lagrange multipliers associated with the budget constraint (1.18), the law of motion for capital (1.17), and the collateral constraint (1.9), respectively. $p_t^{h,p} = \frac{P_t}{P_{H,t}}$ denotes the relative price, while $mc_t = \frac{P_t^w}{P_{H,t}}$ is the real marginal cost of domestic retailers in terms of final goods prices.

A combination of equation (1.19) and equation (1.20) yields a standard Euler equation. Equation (1.21) describes the optimal choice for labor, which equalizes the marginal product of labor with the marginal cost of labor. Equation (1.22) characterizes the optimal decision for investment, equalizing the marginal cost of investment to its marginal benefit. Finally, equation (1.23) indicates that the cost of acquiring one extra unit of capital equalizes the expected value of price plus the payoff from holding capital. The latter, in turn, integrates the marginal product of capital with the ability to pledge as collateral.

1.2.3 Retailers

There is a continuum of retailers indexed by $j \in [0,1]$. In each period t, retailer j buys the homogeneous wholesale goods from domestic entrepreneurs at the wholesale price P_t^w in a competitive market, differentiates them at no cost, and sells them in a monopolistically competitive market at the price $P_{Hj,t}$. The total domestic final good is a composite of individual retail goods:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $Y_{j,t}$ is the output of firm j and Y_t indicates the total final goods.[13] To introduce price stickiness, we allow for monopolistic competition to occur at the retail level as in Bernanke et al. (1999).[14] Specifically, we use a Calvo pricing setting with the degree of price stickiness θ_H, which means that in each period t the retailer j can re-optimize their price with a constant probability $1 - \theta_H$. Therefore, the probability that the price set at time t will still hold at time $t + s$ is θ_H^s. The problem of domestic retailer j can be written as follows:

$$\max_{\bar{P}_{H,t}} \sum_{s=0}^{\infty} E_t[\theta_H^s \Lambda_{t,t+s}^p (\bar{P}_{H,t} - P_{t+s}^w) Y_{j,t+s}], \tag{1.24}$$

s.t.

$$Y_{j,t} = \left(\frac{\bar{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} Y_t, \tag{1.25}$$

where $\Lambda_{t,t+s}^p$ denotes a relevant stochastic discount factor for retailers. Since retailers are owned by households, the discount factor is given by $\Lambda_{t,t+s}^p = (\beta^p)^s \frac{\lambda_{t+s}^p}{\lambda_t^p} \frac{1}{\Pi_{t+s}}$. Equation (1.24) indicates the discounted profits of domestic retailer j, while equation (1.25) represents the demand for retailer j's goods. The first order condition with respect to $\bar{P}_{H,t}$ yields the following:

$$\bar{P}_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\theta_H)^s \Lambda_{t,t+s}^p (P_{H,t+s})^{\epsilon+1} Y_{t+s} mc_{t+s}}{E_t \sum_{s=0}^{\infty} (\theta_H)^s \Lambda_{t,t+s}^p (P_{H,t+s})^{\epsilon} Y_{t+s}},$$

where $mc_{t+s} = \frac{P_{t+s}^w}{P_{H,t+s}}$ denotes the real marginal cost of domestic retailer j in terms of final goods price at period $t + s$; $\frac{\epsilon}{\epsilon-1} > 1$ is the markup earned by retailers. Since all retailers who can reoptimize their prices at time t choose the same price, the aggregate price index of final domestic goods evolves according to

$$P_{H,t} = [\theta_H P_{H,t-1}^{1-\epsilon} + (1 - \theta_H)(\bar{P}_{H,t})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}.$$

[13] Since we assume that retailers involve no cost at differentiating goods and that each retailer is matched to one entrepreneur randomly, the following equation must hold for all time t: $Y_t^e = Y_{j,t}$.

[14] This assumption renders our analysis of entrepreneurs' problems simpler, as pointed out by Bernanke et al. (1999).

Therefore, the inflation rate of domestic goods is

$$\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} = \left[\theta_H + (1-\theta_H)\bar{\Pi}_{H,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{1.26}$$

where $\bar{\Pi}_{H,t}$ is defined as $\bar{\Pi}_{H,t} = \frac{\bar{P}_{H,t}}{P_{H,t-1}}$.

1.2.4 Banks

The economy is inhabited by a continuum of banks indexed by b. They receive deposits ($M_{b,t}^i$) from households, borrow from foreign countries, and use these funds to lend to entrepreneurs ($L_{b,t}^e$). Following Schmitt-Grohé and Uribe (2003), we employ the debt elastic interest rate to close our open economy model and induce stationarity. Specifically, we assume that banks borrow from foreign countries ($D_{b,t}$) at an interest rate R_t^f. The rate R_t^f, in turn, rises in the aggregate level of debt and is assumed to take the following form:

$$R_t^f = R^* + \bar{p} + \varphi\left(e^{\frac{\tilde{D}_t}{P_t^*} - \frac{D}{P^*}} - 1\right),$$

where R^* denotes the (gross) world interest rate, which is assumed to be constant for simplicity, $\bar{p}$ captures the invariant component of a country-specific interest rate premium, and the remaining term is the variant component of the premium. The variable $\tilde{D}_t$ denotes the aggregate level of foreign debt, which is taken as exogenous by the bank, and P_t^* is the foreign price index.[15] The parameter φ measures the elasticity of domestic interest rate with respect to changes in the external debt.

In each period t, the individual bank b chooses foreign debt $D_{b,t}$, the total amount of loans $L_{b,t}$, its lending rate $R_{b,t}^l$, and its LTV ratio $\xi_{b,t}$ to maximize its expected discounted profits.[16] Since banks are owned by households, the discount factor $\Lambda_{t,t+s}^p$ is also given by households' marginal rate of substitution. The maximization problem of the bank can be written as follows:

$$\max_{R_{b,t}^l, L_{b,t}, \xi_{b,t}, D_{b,t}} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}^p \left\{\chi_{b,t-1+s} R_{b,t-1+s}^l L_{b,t-1+s}\right. \tag{1.27}$$
$$+[1-\chi_{b,t-1+s}]\frac{L_{b,t-1+s}}{\int_0^1 L_{b,t-1+s}db}\tau\xi_{t-1+s}a_{t-1+s} + \int_0^1 M_{b,t+s}^i di$$

[15] In the symmetric equilibrium, we have $\tilde{D}_t = D_t$.

[16] The total amount of loans $L_{b,t}$ is defined as $L_{b,t} = \int_0^1 L_{b,t}^e de$.

$$+D_{b,t+s}\mathcal{E}_{t+s} - L_{b,t+s} - R^d_{t-1+s}\int_0^1 M^i_{b,t-1+s}di - R^f_{t-1+s}D_{b,t-1+s}\mathcal{E}_{t+s}\Bigg\},$$

s.t.

$$\chi_{b,t} = \chi + \Theta(\xi_{b,t} - \xi), \tag{1.28}$$

$$L_{b,t} = \int_0^1 M^i_{b,t}di + D_{b,t}\mathcal{E}_t, \tag{1.29}$$

$$L_{b,t} = \int_0^1 \left[\left(\frac{\Gamma_{b,t}}{\Gamma_t}\right)^{-\eta_l}(D^l_t)^e + h^l S^l_{b,t-1}\right]de = \left(\frac{\Gamma_{b,t}}{\Gamma_t}\right)^{-\eta_l} D^l_t + h^l S^l_{b,t-1}, \tag{1.30}$$

where $D^l_t = \int_0^1 (D^l_t)^e de$ is the total demand for the loan composite, $\mathcal{E}$ is the exchange rate, $\chi_{b,t}$ and χ denote the probability of repayment and its steady state value, respectively. The parameter Θ determines the elasticity of repayment probability to the difference between the LTV ratio and its steady state value ξ. Equation (1.27) indicates the bank b's profit. The first term represents the payoff obtained from loans made to entrepreneurs with probability $\chi_{b,t-1}$, while the second one indicates the liquidation value of collateral retrieved from entrepreneurs with probability $(1 - \chi_{b,t-1})$.[17] Following Barro (1976), we assume that there is a drop in the value of collateral retrieved in liquidation, which is captured by the parameter $\tau \in (0, 1)$. Equation (1.28) imposes a positive relationship between the probability that loans offered by bank b are repaid and the collateral requirement of that bank.[18] As a result, each bank faces a trade-off when choosing its LTV ratio $\xi_{b,t}$: An increase in the LTV ratio raises the profitability of loans through a rise in both current and future market shares, at the cost of higher credit risk.[19] Equation (1.29) indicates bank b's balance sheet, which equates total loans made by bank b to the integration of total deposits received from all households and foreign debt denominated in domestic currency. Equation (1.30) represents the total demand for bank b's loans from all entrepreneurs.

[17]Following Ravn (2016), we assume that a fraction of bank b's loans to a specific entrepreneur relative to the total loans of that bank $\left(\frac{L^e_{b,t-1}}{\int_0^1 L^e_{b,t-1}db}\right)$ is equivalent to a fraction of that bank's total loans relative to the total loans of all banks in the economy $\left(\frac{L_{b,t-1}}{\int_0^1 L_{b,t-1}db}\right)$. Furthermore, it is assumed that a lump-sum transfer Ψ_t is made to entrepreneurs for compensation of handing over their assets in order to guarantee that no money falls out of the economy.

[18]For simplicity, we assume that entrepreneurs do not internally consider that they can not pay back the loans with some positive probability. This implies that when offering loans to entrepreneurs, the bank recognizes that a proportion of the total loans will end up not being repaid *ex-post*, while each entrepreneur simply think that they can repay the loan they obtain. The wedge between effective and actual repayment of loans arising from this assumption ends up in the lump-sum transfer earned by the entrepreneur. The more credit standards are relaxed, the larger the wedge becomes.

[19]Each bank also faces a trade-off when choosing its lending rate: while raising the lending rate $R^l_{b,t}$ leads to higher profits, it comes at the cost of losing market share as entrepreneurs switch to other banks.

The first-order conditions with respect to $D_{b,t}$,$L_{b,t}$, $R^l_{b,t}$, and $\xi_{b,t}$ after imposing a symmetric equilibrium yield

$$R^d_t = R^f_t E_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right), \tag{1.31}$$

$$\nu^l_t = E_t\Lambda^p_{t,t+1}\left[\chi_t R^l_t + (1-\chi_t)\tau\frac{\xi_t a_t}{L_t} - R^d_t + h^l(1-\rho_l)\nu^l_{t+1}\right], \tag{1.32}$$

$$E_t\Lambda^p_{t,t+1}\chi_t L_t = \eta_l \nu^l_t D^l_t \frac{\xi_t}{v_b + \xi_t R^l_t}, \tag{1.33}$$

$$\eta_l \nu^l_t D^l_t \frac{v_b}{\xi_t(v_b + \xi_t R^l_t)} = -\Theta E_t \lambda^p_{t,t+1}(R^l_t L_t - \tau\xi_t a_t), \tag{1.34}$$

where ν^l_t denotes the Lagrange multiplier associated with the constraint (1.30). Therefore, ν^l_t can be interpreted as the shadow value of lending an extra dollar to the entrepreneur. Equation (1.31) is the uncovered interest rate parity condition. Equation (1.32) expresses that the shadow value of lending an additional unit is determined by the benefit obtained from offering the extra unit minus the borrowing cost, which is the deposit rate (R^d_t). The former, in turn, incorporates the weighted probability of repayment and the benefit that the borrower will borrow more in the next period due to the habit developed for that bank. Equation (1.33) states that the marginal benefit obtained from a rise in the lending rate must be equal to the marginal cost of a higher lending rate due to the market share loss. Finally, equation (1.34) indicates that the marginal benefit gained from an increase in the LTV ratio is equal to its marginal cost.

1.2.5 Importers and Incomplete Pass-Through

The setting here, following Monacelli (2005), features an incomplete exchange rate pass-through and allows the deviation from the law of one price to be gradual and persistent. The incomplete exchange rate pass-through is induced by the price setting of the importers according to the Calvo pricing rule. Specifically, we assume that there is a continuum of monopolistically competitive retailers, indexed by $j \in (0, 1)$, who import differentiated goods from the rest of the world at a cost $\mathcal{E}_t P^*_{Fj,t}$, where $P^*_{Fj,t}$ is the price of the imported good j ($IM_{j,t}$) denominated in the foreign currency. The problem of importer j can be summarized as follows:

$$\max_{\bar{P}_{F,t}} \sum_{s=0}^{\infty} \Lambda^p_{t,t+s}\theta^s_F[\bar{P}_{F,t} - \mathcal{E}_{t+s}P^*_{Fj,t+s}]IM_{j,t+s}, \tag{1.35}$$

s.t.

$$IM_{j,t+s} = \left(\frac{\bar{P}_{F,t}}{P_{F,t+s}}\right)^{-\epsilon} IM_{t+s}, \tag{1.36}$$

where θ_F denotes the degree of price stickiness and IM_t is the aggregate imported good. Since importers are owned by households, the stochastic discount factor $\Lambda^p_{t,t+s}$ is also given by households' marginal rate of substitution. Equation (1.35) represents the discounted profits of importer j denominated in home currency, while equation (1.36) is the demand for importer j's goods. The first order condition with respect to $\bar{P}_{F,t}$ yields the following:[20]

$$\bar{P}_{F,t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\theta_F)^s \Lambda^p_{t,t+s} (P_{F,t+s})^{\epsilon+1} IM_{t+s} Lg_{t+s}}{E_t \sum_{s=0}^{\infty} (\theta_F)^s \Lambda^p_{t,t+s} (P_{F,t+s})^{\epsilon} IM_{t+s}},$$

where $Lg_{t+s} = \frac{\mathcal{E}_{t+s} P^*_{F,t+s}}{P_{F,t+s}}$ is the real marginal cost of importers in terms of the price of imported final goods at period $t+k$.[21] Since all importers who can reoptimize their prices at time t choose the same price, the aggregate price index of imported final goods evolves according to

$$P_{F,t} = [\theta_F P^{1-\epsilon}_{F,t-1} + (1-\theta_F)(\bar{P}_{F,t})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}.$$

The inflation rate of imported goods is

$$\Pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} = \left[\theta_F + (1-\theta_F)\bar{\Pi}^{1-\epsilon}_{F,t}\right]^{\frac{1}{1-\epsilon}}, \tag{1.37}$$

where $\bar{\Pi}_{F,t}$ is defined as $\bar{\Pi}_{F,t} = \frac{\bar{P}_{F,t}}{P_{F,t-1}}$.

1.2.6 Central Bank

The central bank sets its interest rate according to the following rule:[22]

$$\log\left(\frac{R^d_t}{R^d}\right) = \rho_r \log\left(\frac{R^d_{t-1}}{R^d}\right) + (1-\rho_r)\phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \varepsilon_{r,t}, \tag{1.38}$$

where R^d and Π denote the steady state value of the policy interest rate and the inflation target, respectively. ρ_r and ϕ_π denote policy parameters representing interest smoothing and the re-

[20]We impose the condition $P^*_{Fj,t} = P^*_{F,t}$, for all t because prices are assumed to be flexible in the world economy. Therefore, the marginal cost is the same for all importers as in Monacelli (2005).

[21]Note that Lg_t measures the deviation from the law of one price. When we shut down the incomplete pass-through feature, the law of one price holds, i.e., $Lg_t = 1$ for all t.

[22]This assumption is reasonable because the Sveriges Riksbank (the central bank of Sweden) has introduced the inflation target since 1993.

sponse of the policy rate to the deviation of inflation from its steady state. The monetary shock $\varepsilon_{z,t}$ is an i.i.d. process with standard deviation σ_r.

1.2.7 Demand for Exports

Following Galí and Monacelli (2016), we assume that the foreign demand for domestically produced goods is given by

$$C^*_{H,t} = v\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} Y^*_t,$$

where P^*_t denotes the price level of the world economy, $P^*_{H,t}$ is the export price index, and $C^*_{H,t}$ is the aggregate export index.[23] The world output, denoted by Y^*_t, is assumed to follow the AR(1) process:

$$\log Y^*_t = \rho_y \log Y^*_{t-1} + (1-\rho_y)\log Y^* + \sigma_y \varepsilon_{y,t}, \tag{1.39}$$

with $\rho_y \in (0,1)$ being the persistence of the shock and $\varepsilon_{y,t}$ following an i.i.d. process with standard deviation σ_y. Since the size of the open economy is small enough compared to the world economy, we can neglect its impact on the world economy. As a result, the price level of the world economy equalizes the price of foreign products, i.e., $P^*_t = P^*_{F,t}$. Furthermore, because the export price is assumed to be flexible and determined by the law of one price, we obtain the following condition $\mathcal{E}_t P^*_{H,t} = P_{H,t}$. Therefore, the foreign demand can be rewritten as follows:

$$C^*_{H,t} = v\left(p^{h,f}_t Lg_t\right)^{\eta} Y^*_t, \tag{1.40}$$

where $p^{h,f}_t$ defined as $p^{h,f}_t = \frac{P_{F,t}}{P_{H,t}}$ indicates the term of trade.

[23]The aggregate export $C^*_{H,t}$ is produced via the following technology: $C^*_{H,t} = \left[\int_0^1 (C^*_{Hj,t})^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$, where $C^*_{Hj,t}$ is the export good j.

1.2.8 Equilibrium

In the symmetric equilibrium, all markets clear.[24] Domestic goods are used for domestic consumption, investment, and exports. Therefore, the final good market clearing is as follows:

$$Y_t = C_{H,t} + I_{H,t} + C^*_{H,t}, \tag{1.41}$$

where $C_{H,t}$, defined as $C_{H,t} = C^p_{H,t} + C^I_{H,t}$, indicates the total consumption for domestic final goods. Since foreign goods are imported for both consumption and investment, the market clearing for imported goods is given by

$$IM_t = C_{F,t} + I_{F,t}, \tag{1.42}$$

where $C_{F,t}$, defined as $C_{F,t} = C^p_{F,t} + C^I_{F,t}$, represents the total consumption for foreign goods. The aggregate consumption is given by

$$C_t = C^p_t + C^I_t. \tag{1.43}$$

Furthermore, the budget constraints of households and entrepreneurs are both binding in equilibrium. A combination of these two equilibrium conditions yields the evolution of foreign debt:

$$R^f_{t-1} D_{t-1} \mathcal{E}_t = D_t \mathcal{E}_t + P_{H,t} C^*_{H,t} - \mathcal{E}_t P^*_{F,t} IM_t \Delta_{F,t}. \tag{1.44}$$

where $\Delta_{F,t}$ denotes the price dispersion for imported goods and is given by the following expression:

$$\Delta_{F,t} = (1-\theta_F)\left(\frac{\Pi_{F,t}}{\bar{\Pi}_{F,t}}\right)^{\epsilon} + \theta_F (\Pi_{F,t})^{\epsilon} \Delta_{F,t-1}. \tag{1.45}$$

Similarly, the price dispersion for domestically produced goods ($\Delta_{H,t}$) is given as follows:

$$\Delta_{H,t} = (1-\theta_H)\left(\frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}}\right)^{\epsilon} + \theta_H (\Pi_{H,t})^{\epsilon} \Delta_{H,t-1}. \tag{1.46}$$

[24] In fact, we consider a semi-symmetric equilibrium as in Airaudo and Olivero (2019). On one hand, we assume that all households in the consumption sector, all wholesale goods entrepreneurs in the producing sector, and all banks in the financial sector do behave identically. On the other hand, we assume that price is sticky in the retail sector. Specifically, there exists a faction of $1 - \theta_H$ of retailers that can reoptimize their prices, whereas a fraction of θ_H cannot. As a result, the pricing is different among domestic retailers. A similar assumption is applied to the imported sector. A full list of equilibrium conditions is available upon request. The model is linearized and solved using DYNARE (Adjemian et al., 2011).

1.3 Calibration

The model is calibrated for the Swedish economy. There are *31* structural parameters that need to be calibrated:

- *18* structural parameters: σ, δ, ψ, η, υ, ϵ, β, α, θ_H, θ_F, θ_w, r^*, d,φ, h^l, ρ_l, η_l, and π that appear in the steady-state system; and
- *13* structural parameters γ, ϕ_π, ϕ_e, ϕ_l, ρ_r, ρ_a, ρ_y, ρ_ϵ^r, ρ_z, σ_a^2, σ_y^2, σ_r^2, and σ_z^2 that do not appear in the steady-state system.

The time unit in the model corresponds to one quarter. The full set of calibrated values is displayed in Table 1.1. The intertemporal discount factor β^p for households is calibrated to yield an annual interest rate of 2.25% as in Christiano et al. (2011). The discount factor for entrepreneurs β^I is set at 0.95, which is commonly assumed in the literature.[25] The steady state of labor supply shock Z is chosen so that 25% of households' time is devoted to working. Accordingly, Z is set at 2.63. The habit parameter for consumption is calibrated so that the volatility of aggregate consumption relative to aggregate output is consistent with the Swedish data. The calibrating result of 0.6 is in line with the estimates of Christiano et al. (2011).

Based on parameter values commonly used in much of the related business-cycle literature, the capital share α is set at 0.32 and the depreciation rate δ is set equal to 0.025, which implies an annual depreciation of 10%. Our baseline setting for the capital adjustment cost parameter γ is 2.58 following the estimation of Christiano et al. (2011). The Calvo pricing parameters for both domestic final goods and imported final goods θ_H and θ_F are set equal to 0.8, implying the average duration of changing prices of five quarters, which is in line with the estimated values of Christiano et al. (2011). Following Ravn (2016), we set a value of 0.05 for the relative weight of collateral-minimization desire υ_b.[26]

As for the deep habits parameters, we resort to the value estimated by Aliaga-Díaz and Olivero (2010) and set the deep habits formation equal to 0.72. The persistence of the lending relationship ρ_l is set equal to 0.85 following Ravn et al. (2006) and Aliaga-Díaz and Olivero (2010). Following Ravn (2016), we set the elasticity of substitution in the banking sector η_l

[25] Note that a significant difference between discount factors is to guarantee that in the steady state, the collateral constraint is binding, i.e., $\nu_t^I > 0$. Interested readers are referred to Gerali et al. (2010) for a more detailed discussion.

[26] This is also in line with Booth and Booth (2006), who find that the concern about collateral minimization is of limited importance to firms. Therefore, we assign a small value of 0.05, and later we report a robustness check for this parameter.

Table 1.1: Calibrated parameters

Parameter	**Value**	**Description**
Preference parameters		
β^p	0.9994	Discount factor for households
β^I	0.95	Discount factor for entrepreneurs
Z	2.63	Steady state of labor supply shock
h^p	0.6	Consumption habits for housholds
h^I	0.6	Consumption habits for entrepreneurs
ϵ	6	Elasticity of substitution across goods
Production parameters		
α	0.32	Capital share in production
δ	0.025	Depreciation rate of capital
γ	2.58	Capital adjustment cost
θ_H	0.8	Calvo index for domestic price
θ_F	0.8	Calvo index for imported price
υ_b	0.05	Relative weight of collateral optimization
Banking parameters		
h^l	0.72	Deep habits formation
ρ_l	0.85	Persistence of deep habits stock
η_l	230	Elasticity of substitution for banks
χ	0.989	Steady state probability of repayment
Θ	-1.5	Elasticity of repayment probability
τ	0.9425	Recovery rate of assets
Openness parameters		
υ	0.3759	Degree of openness
η	2	Trade elasticity of substitution
R^*	1.01	World interest rate
$\bar{p}$	-0.0094	Constant component of country premium
φ	0.18	Debt elasticity interest rate
d	0.01	Foreign debt parameter
Y^*	1.244	Steady state of foreign shock
Policy parameters		
Π	1.005	Steady state gross inflation target
ρ_r	0.819	Interest rate smoothing
ϕ_π	1.909	Response to the deviation of inflation
Shock parameters		
ρ_y	0.9579	Persistence of foreign shock
σ_y	0.0031	SD of foreign shock
ρ_a	0.95	Persistence of technology shock
σ_a	0.0015	SD of technology shock
ρ_z	0.95	Persistence of labor supply shock
σ_z	0.0015	SD of labor suppy shock
σ_r	0.00075	SD of monetary shock

at 230. The recovery rate of assets τ is set to target the LTV ratio of 0.75 in the steady state, following Liu et al. (2013). The resulting value of 0.9425 is in line with the calibration of Ravn (2016). To calibrate the value for the steady state of repayment probability χ, we use data for non-performing loans to gross loans in Sweden collected from the Federal Reserve Economic Data of the Federal Reserve Bank of St. Louis over the period 1998–2016. The average value for this ratio is 1.1%. Accordingly, we set $\chi = 0.989$. The elasticity of credit risk Θ is set at -1.5 as in Ravn (2016).

As for the openness-related parameters, the share of foreign goods in the total consumption basket v is calibrated to match the observed average real import/real GDP ratio in Sweden. Accordingly, v is set at 0.3759. The elasticity of substitution between domestic and foreign goods η is set equal to 2.[27] We set the elasticity of substitution across varieties of goods ϵ at 6 to target a gross mark-up of 1.2, as is common practice in the literature. The world interest rate is set at 1.01 (4% per year). Our baseline setting for the debt elastic interest rate parameter φ is 0.18, which is the median value estimated by Uribe and Schmitt-Grohé (2017) for various countries . We also set the parameter d at 0.01 as in Uribe and Schmitt-Grohé (2017). Following Gali and Monacelli (2005), we employ real U.S. GDP as a proxy for world output and apply the AR(1) process to calibrate the parameters of the foreign output shock.[28] We obtain the following result:

$$\log(Y_t^*) = \underset{(0.0064)}{0.004} + \underset{(0.0274)}{0.9579} \log(Y_{t-1}^*) + \epsilon_{y,t}, \sigma_y = 0.0031.$$

The persistence ρ_y and the steady state of the foreign demand shock $\bar{Y}^*$ are thus equal to 0.9579 and 1.244, respectively. As for the monetary policy parameters, we set the interest rate smoothing parameter ρ_r at 0.819 and the response to the deviation of inflation from its steady state ϕ_π at 1.909, which follows the estimation of Christiano et al. (2011). We resort to the estimation of Smets and Wouters (2007) and set the persistence of technology shock ρ_a at 0.95. The persistence of labor supply shock ρ_z is set at 0.95, which is roughly in line with Liu et al. (2013). As for the standard deviation of those shocks, we draw on the relative size calibrated by Ravn (2016), who finds that the standard deviation of the technology shock is an order of magnitude the same as that of the labor supply shock. Since they do not include the monetary

[27] Since there is no consensus on the trade elasticity of substitution in the literature, we also check the sensitivity of our results for different values of this parameter.

[28] We use the quarterly data over the period 1993Q1-2018Q3 for the real U.S. GDP. The series is then detrended by the log quadratic method as in Uribe and Schmitt-Grohé (2017).

shock, we rely instead on Christiano et al. (2011) who indicate that the standard deviation of the monetary shock is an order of magnitude smaller than that of the technology shock. Given the relative magnitude, we calibrate the absolute size of those shocks so that the standard deviation of aggregate output matches that of Swedish data.

1.4 Results

In this section, we show how fluctuations in credit standards emerging from the deep habits in the banking section amplify the propagation mechanism of various shocks to the economy. For this purpose, we compare the results from the baseline model described in previous sections to the model in which the deep habits mechanism is shut off, i.e., we set $h^l = 0$ and $\rho_l = 0$.[29]

1.4.1 Dynamic Properties of the Models

We first consider the dynamic properties of the two models and compare them to the empirical moments. Table 1.2 illustrates the properties of our two models: (1) a model featuring the deep habits mechanism in the financial sector; and (2) a model without deep habits.

In general, the empirical moments are closely matched by the two models. First, both models allow us to generate the fact that while investment is more volatile than output, consumption is less so. Interestingly, output and investment become more volatile, but the standard deviation of consumption relative to output drops once we incorporate credit standards into the banking sector. These findings are in line with Aliaga-Díaz and Olivero (2010). Second, the two models accurately reproduce the correlation with output of consumption, investment, export, and import. Finally, regarding the persistence of macroeconomic variables, the autocorrelation coefficients of output, consumption, investment, and export are well captured by the two models.

Furthermore, the simulated results demonstrate that the introduction of credit standards in the financial sector better replicates the dynamic properties of the Swedish business cycles. Specifically, the deep habit model provides better fits for both the correlation with output of consumption and investment and the autocorrelation coefficients of output, consumption, and investment. More importantly, it generates the countercyclicality of the bank spread, which is in line with the data moment.

[29]The value of collateral retrieved in liquidation τ is re-calibrated so that the steady state of the LTV ratio remains unchanged at 0.75.

Table 1.2: Dynamic properties of the models

	Data	Deep habit model	No deep habit model
Standard deviation			
Output	2.38	2.38	2.16
Relative standard deviation to output			
Consumption	0.57	0.53	0.55
Investment	2.33	3.77	3.13
Export	2.59	1.12	1.13
Import	2.54	0.79	0.81
Correlation with output			
Consumption	0.59	0.89	0.91
Investment	0.89	0.90	0.93
Export	0.81	0.99	0.99
Import	0.77	0.90	0.89
Spread	-0.28	-0.67	–
Autocorrelation coefficients			
Output	0.92	0.83	0.82
Consumption	0.85	0.78	0.77
Investment	0.88	0.85	0.84
Export	0.86	0.85	0.84
Import	0.89	0.75	0.75
Spread	0.69	0.98	0.66

Notes: The data moments in the second column are calculated using the Swedish data for 1993Q1–2018Q3. All data series are collected from the International Financial Statistics (IFS). The empirical moments are computed after employing quadratic detrending to eliminate the trend component from the data. The last two columns present the simulated moments (100,000 periods) of the deep habits model and the model without deep habits.

1.4.2 Impulse Response Analysis

Next we consider the impulse response functions. To facilitate the comparisons, we display the impulse response functions (IRFs) for two models: the baseline model featuring deep habits in the banking sector and the model without the deep habits mechanism. Figure 1.1 presents the IRFs for a number of key variables to monetary policy shock. After a contractionary monetary policy shock, output, consumption, and investment decrease. Furthermore, the bank spread between the lending rate and the policy rate in the baseline model goes up due to the presence of deep habits. The mechanism is the following: When the shock arrives, output and demand for loans are both lower than usual. As a result, the future market share motive is dominated by the current profit one and banks try to exploit the lending relationship by increasing the bank spread to raise their current profit. The cost of lending increases, and thus investment decreases more in the baseline model than that in the model without the deep habits mechanism. In turn,

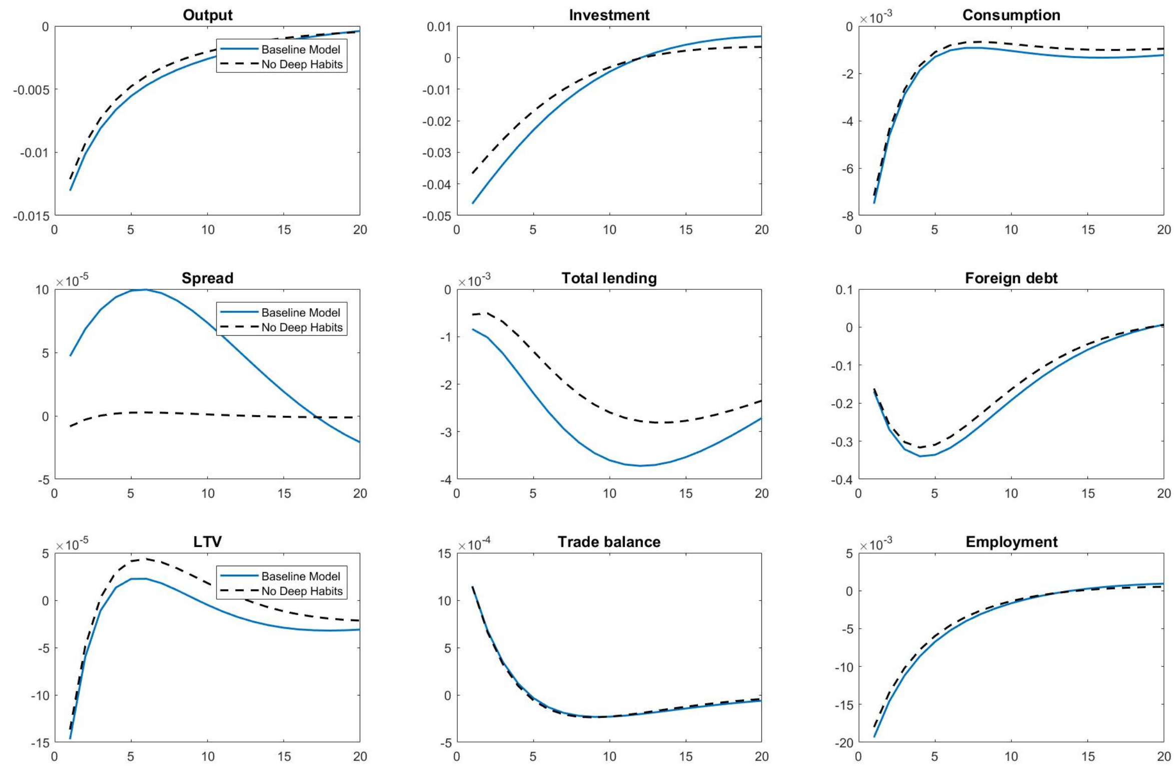

Figure 1.1: Impulse responses to the monetary policy shock, $\varepsilon_{r,t}$, of size one standard deviation in two different models: deep habits model (baseline) and no deep habits model

output, consumption, and employment all drop by more under the deep habits model. The model thus generates the countercyclical movement of the bank spread, which is in line with previous literature.

In addition, banks also tighten up the other credit standards, the collateral pledge. Specifically, the monetary policy shock causes a drop in the LTV ratio, which expresses an increase in the collateral requirement. The demand for loans, and thus investment, output, and consumption fall even further in the deep habits model. The countercyclical fluctuations in both interest rate spreads and collateral requirements amplify the propagation of the monetary policy shock to the economy.

As for the responses of openness-related variables, the monetary policy shock triggers contractions in domestic demand for foreign output, generating an increase in trade balance and a decrease in foreign debt in both models. This is supported by both empirical evidence (see, e.g., Lindé, 2003) and theoretical models (see, e.g., Christiano et al., 2011, Kollmann, 2001). It is also worth noting that under the baseline model, foreign debt decreases more than in the no

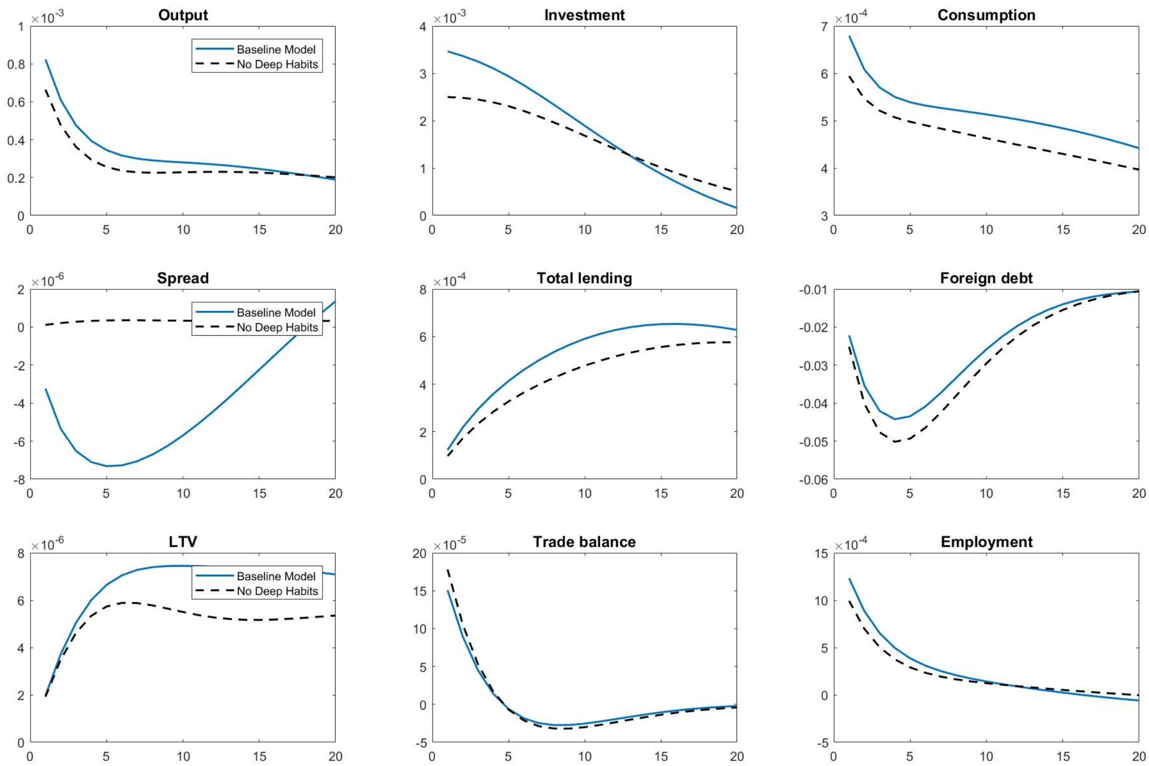

Figure 1.2: Impulse responses to the foreign demand shock, $\varepsilon_{y,t}$, of size one standard deviation in two different models: deep habits model (baseline) and no deep habits model

deep habits model. The explanation is as follows: Foreign debt is one source of funding that banks can utilize to offer loans to entrepreneurs and under the deep habits mechanism, total lending drops by more, the demand for foreign borrowing thus drops even further.

The dynamic effect of the foreign demand shock is presented in Figure 1.2. A positive foreign demand shock pushes up the foreign demand for domestic goods, and thus the exports. This drives up the production level of entrepreneurs, making them raise their demand for loans from banks, and thus also their demand for capital stock and labor. Because of the shock's persistence, output and demand for loans are expected to be high in the periods to come. Under the baseline model featuring the lending relationship, banks lower the interest rate spread and relax the credit constraint since current profit is not a priority at present. Therefore, investment, output, employment, and total lending increase by more in the presence of deep habits. Furthermore, it may seem surprising that foreign debt negatively correlates with output and the trade balance exhibits pro-cyclical behavior. The reason is that the foreign demand shock induces an increase in foreign assets, thus generating a decrease in foreign debt and an increase in trade

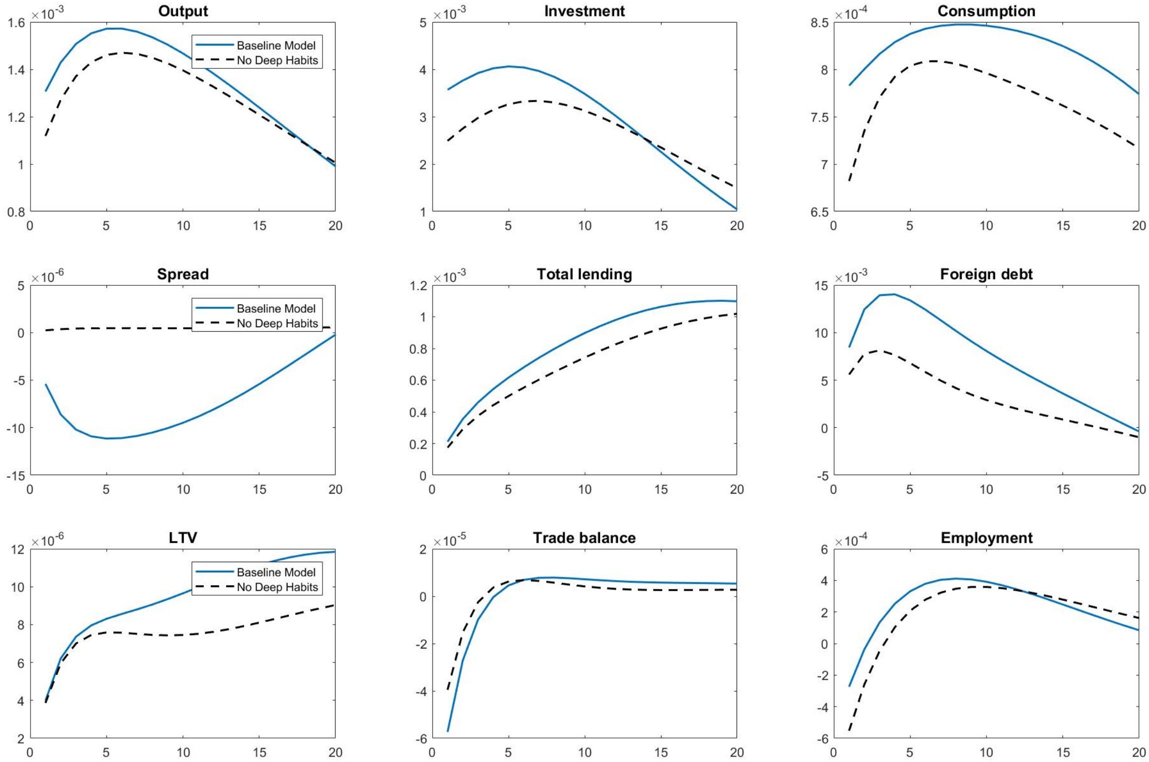

Figure 1.3: Impulse responses to the technology shock, $\varepsilon_{a,t}$, of size one standard deviation in two different models: deep habits model (baseline) and no deep habits model

balance (see, e.g., Lim and McNelis, 2008). Also, note that foreign debt falls by less in the baseline model in order to support a larger increase in total lending.

The IRFs in Figure 1.3 for the technology shock can be characterized as follows: A positive technology shock leads to an increase in consumption, investment, and output. Under the deep habits model, banks lower the lending margin and raise the LTV ratio. The explanation is that after the positive shock, output and demand for loans will be higher for periods to come due to the persistence of the shock, and thus the future profits of banks are expected to be higher. Consequently, banks are willing to sacrifice current profit for future market share by relaxing both the bank spread and the LTV ratio. Under the baseline model, the cost of lending falls, and thus investment increases by more than in the model without the deep habits mechanism. Since the increase in investment positively affects the capital stock, output is raised by more in the baseline model. In addition, the positive technology shock induces an increase in domestic demand for foreign output, triggering a rise in imports. This leads to a fall in the trade balance and a rise in foreign debt. Under the presence of deep habits, the foreign debt increases by

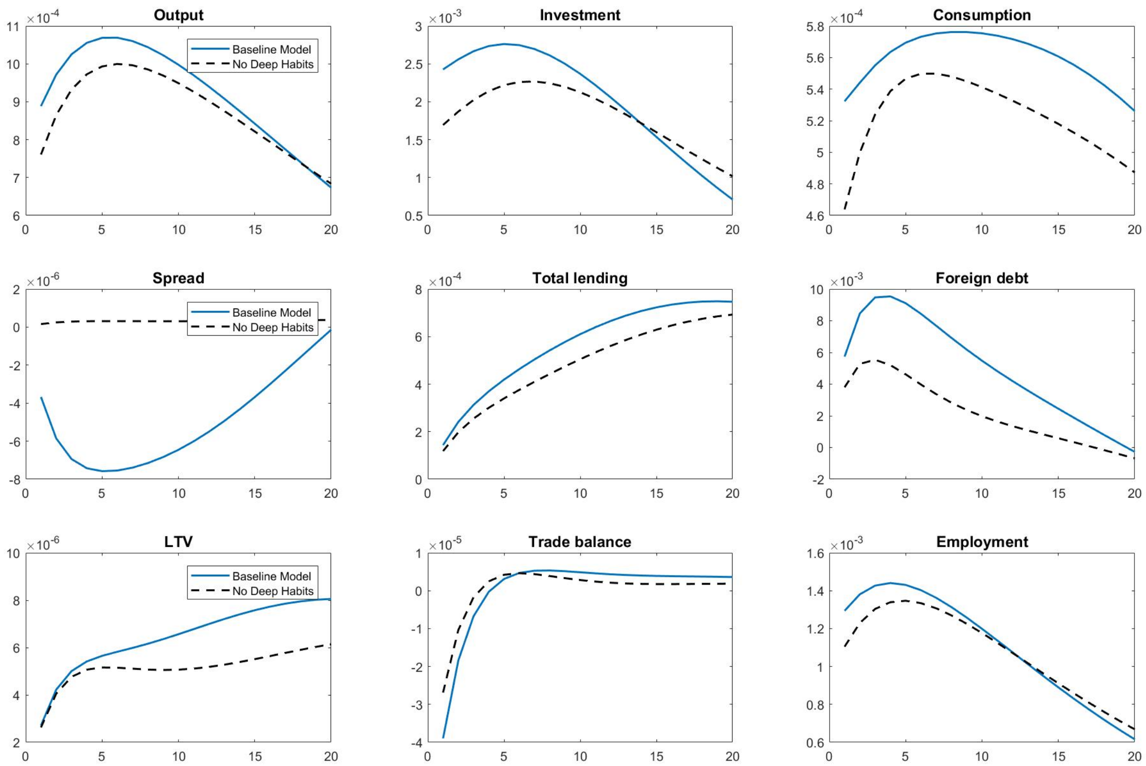

Figure 1.4: Impulse responses to the labor supply shock, $\varepsilon_{z,t}$, of size one standard deviation in two different models: deep habits model (baseline) and no deep habits model

more in the baseline model.

Similar results are obtained for a positive labor supply shock and the impact is illustrated in Figure 1.4. A positive labor supply shock drives up both the entrepreneurs' stock of capital and their demand for labor in current as well as future periods due to the persistence of the shock. The increase in the production level results in a higher demand for loans. Under the deep habits model, a fall in lending margin combined with an increase in the LTV ratio raises the entrepreneurs' demand for loans further. This is again the result of the deep habits mechanism: the future market share motive dominates the current profit motive since output is expected to be higher in future periods. Banks thus find it optimal to relax credit requirements and lower the bank spread. Analogous to the technology shock, the positive labor supply shock generates domestic demand expansion, resulting in a pro-cyclical response of foreign debt and a countercyclical response of trade balance. In addition, due to the existence of deep habits, the foreign debt increases by more in the baseline model as in the case of technology shock.

In conclusion, it is shown that for each shock in our model, the endogenized lending rela-

Table 1.3: Deep habits and aggregate fluctuations

Variance ratio	
Output	1.21
Total investment	1.76
Total consumption	1.16
Lending	1.32
Employment	1.20
Foreign debt	1.22
Trade balance	1.03

Notes: We use the theoretical variance of selected variables to compare two models. The variance ratios are then computed by dividing the variance of each variable in the baseline by that of their counterpart in the no deep habits model.

tionship works as a financial accelerator of macroeconomic shocks.

1.4.3 Deep Habits and Aggregate Fluctuations

Following Ravn (2016), we now compare the theoretical variance of macroeconomic variables in two models: the baseline model and the model without the deep habits mechanism. To facilitate the comparisons, we compute the variance ratios for macroeconomic variables by dividing the variance of each variable in the baseline by that of its counterpart in the model without deep habits. The results are shown in Table 1.3. It is clearly indicated that endogenous credit standards generate a significant increase in macroeconomic volatility. Specifically, the variance of consumption rises by 16 % when the deep habits mechanism is incorporated, while that of output increases by somewhat more (21 %). The variance of employment increases by 20 % due to the positive effect of output on the demand for labor. Moreover, the numbers show that the main source driving up the volatility of output is the investment volatility which goes up by 76 % under the deep habits model. This is the direct effect of the endogenous fluctuation in credit standards: during a boom the credit constraints are inclined to be reduced, which enables entrepreneurs to borrow more from the banks. The entrepreneurs, facing collateral constraints, employ the additional funding to push up the capital stock because the capital investment in turn can be used as collateral pledge to increase the access to credit further. This demonstrates how endogenous credit standards work as financial accelerators of macroeconomic innovation in our open economy setting.

Table 1.4: Robustness check

Variance ratio	Output	Investment	Consumption	Lending	Employment	Debt	Trade balance
Baseline	**1.21**	**1.76**	**1.16**	**1.32**	**1.20**	**1.22**	**1.03**
$\Theta = -1$	1.21	1.76	1.16	1.31	1.20	1.22	1.03
$\Theta = -2$	1.22	1.76	1.17	1.32	1.20	1.22	1.03
$\rho_l = 0.3$	1.01	1.14	1.01	1.03	1.01	1.00	0.95
$h^l = 0.3$	1.00	1.06	1.00	1.02	1.00	0.99	0.96
$\eta_l = 190$	1.28	1.98	1.22	1.40	1.26	1.29	1.06
$\eta_l = 300$	1.15	1.54	1.11	1.23	1.14	1.15	1.00
$\eta = 1.5$	1.20	1.74	1.13	1.32	1.19	1.18	0.94
$\eta = 2.5$	1.22	1.77	1.18	1.31	1.21	1.27	1.09
$v = 0.3$	1.22	1.76	1.17	1.30	1.20	1.23	1.04
$v = 0.4$	1.21	1.76	1.16	1.32	1.20	1.22	1.02

Notes: We use the theoretical variance of selected variables to compare two models. The variance ratios are then computed by dividing the variance of each variable in the baseline by that of their counterpart in the no deep habits model.

1.4.4 Robustness Check

In this section, we check the robustness of our model results presented in the previous section when the values of key parameters are adjusted. In addition, we compare our findings to those shown in the related literature.[30]

We first examine the sensitivity of our findings to changes in the elasticity of credit risk with respect to the difference between the LTV ratio and its steady state, Θ. In particular, we let this parameter vary from the baseline value of -1.5 to a (numerically) smaller value of -1, as well as to a (numerically) larger value of -2. The resulting variance ratios for this experiment are displayed in Table 1.4. It is clearly seen that our results are not sensitive to this elasticity since only minor changes are recorded as we change the value of Θ from -1 to -2.[31]

We next consider the robustness of our results to changes in the persistence of deep habits in banking, as captured by the parameter ρ_l. As the number illustrates, changing ρ_l results in significant changes in the variance ratios of macroeconomic variables. In addition, a low persistence of deep habits of 0.3 is sufficient to eliminate the additional volatility emerging from the lending relationship. Therefore, a certain degree of persistence is needed to observe any amplification. Similarly, changing the strength of deep habits h^l leads to major changes

[30] In each exercise, the value of collateral retrieved in liquidation τ is re-calibrated if necessary to guarantee that the regulatory LTV remains unchanged at 0.75.

[31] In fact, there are very small increases as we (numerically) increase the value of this elasticity from -1 to -2. In other words, deep habits induce larger effects as this elasticity is (numerically) higher. This seems confusing because given the relaxation of credit standards, it is more costly for banks to give loans in light of repayment probability when the elasticity is (numerically) higher, making banks less appealing. However, in this case, we have to increase the value of τ to keep the LTV ratio unchanged at 0.75, which offsets the negative effect of the (numerically) larger elasticity on the cost of a marginal increase in LTV ratio.

in our results, and a reduction of this parameter to 0.3 is enough to remove the amplification arising from deep habits in banking. The next two columns display the results for different elasticity of substitution between banks' loans η_l. It is clear that changing the value of η_l does not alter our conclusions from the baseline model. Specifically, a reduction of this value to 190 as in Aliaga-Díaz and Olivero (2010) leads to even stronger amplification, while with a higher value of 300, the model still displays substantial amplification.

As shown in the Table 1.4, we also report the robustness check for openness-related parameters. Given the uncertainty in the literature about the value of trade elasticity of substitution η, we allow this parameter vary from its baseline value of 2 to 1.5 or 2.5. It is clearly shown that variance ratios of macroeconomic variables displays only minor changes as we change the value of trade elasticity from 1.5 to 2.5. A similar result is obtained when we let the degree of openness v vary from 0.3 to 0.4. Changing v only leads to a very small deviation from our baseline results.

1.4.5 Policy

In the previous section, we have demonstrated that the fluctuation in credit standards serves as a financial accelerator of the business cycle. This suggests that while constructing policies to protect the economy from unfavorable financial market spillovers, policymakers should take this mechanism into consideration. In this section, we examine three alternative monetary policies that may be employed to alleviate the effect of endogenous credit standards.[32]

$$\log\left(\frac{R_t^d}{R^d}\right) = \rho_r \log\left(\frac{R_{t-1}^d}{R^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_l \log\left(\frac{l_t \Pi_t}{l_{t-1}\Pi}\right)\right] + \varepsilon_{r,t}, \quad (1.47)$$

$$\log\left(\frac{R_t^d}{R^d}\right) = \rho_r \log\left(\frac{R_{t-1}^d}{R^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_r \log\left(\frac{R_t^f}{R^f}\right)\right] + \varepsilon_{r,t}, \quad (1.48)$$

[32]The monetary rule has recently been modified to include several aspects of the open economy, such as the exchange rate and foreign interest rate. Lim and McNelis (2008) demonstrate that the domestic interest rate moves along with the foreign interest rate due to the activity of arbitrage and the assumption of a small open economy. Agyapong (2021), introducing the real exchange rate in the policy rule, investigates the effectiveness of the Taylor rule in predicting exchange rates.

Table 1.5: The impacts of different monetary policies

Credit growth augmented monetary policy

Variance ratio	$\phi_l = 0.01$	$\phi_l = 0.05$	$\phi_l = 0.07$	$\phi_l = 0.1$
Output	1.18	1.06	1.01	0.94
Total investment	1.71	1.52	1.44	1.33
Total consumption	1.14	1.03	0.99	0.93
Lending	1.28	1.16	1.11	1.04
Employment	1.16	1.04	0.98	0.91
Foreign debt	1.18	1.06	1.01	0.93
Trade balance	1.00	0.90	0.86	0.80

Foreign interest rate augmented monetary policy

Variance ratio	$\phi_r = 0.01$	$\phi_r = 0.05$	$\phi_r = 0.07$	$\phi_r = 0.1$
Output	1.19	1.11	1.07	1.01
Total investment	1.72	1.59	1.53	1.44
Total consumption	1.15	1.08	1.05	1.00
Lending	1.29	1.19	1.14	1.08
Employment	1.18	1.09	1.05	0.99
Foreign debt	1.19	1.10	1.06	1.00
Trade balance	1.01	0.94	0.90	0.86

Exchange rate augmented monetary policy

Variance ratio	$\phi_e = 0.01$	$\phi_e = 0.05$	$\phi_e = 0.07$	$\phi_e = 0.1$
Output	1.19	1.10	1.06	1.00
Total investment	1.72	1.59	1.52	1.43
Total consumption	1.14	1.07	1.04	0.99
Lending	1.30	1.24	1.21	1.16
Employment	1.17	1.08	1.03	0.97
Foreign debt	1.19	1.09	1.04	0.97
Trade balance	1.00	0.91	0.87	0.81

Notes: We use the theoretical variance of selected variables to compare two models. The variance ratios are then computed by dividing the variance of each variable in the credit growth (foreign interest, and exchange rate) augmented monetary policy by that of their counterpart in the no deep habits model.

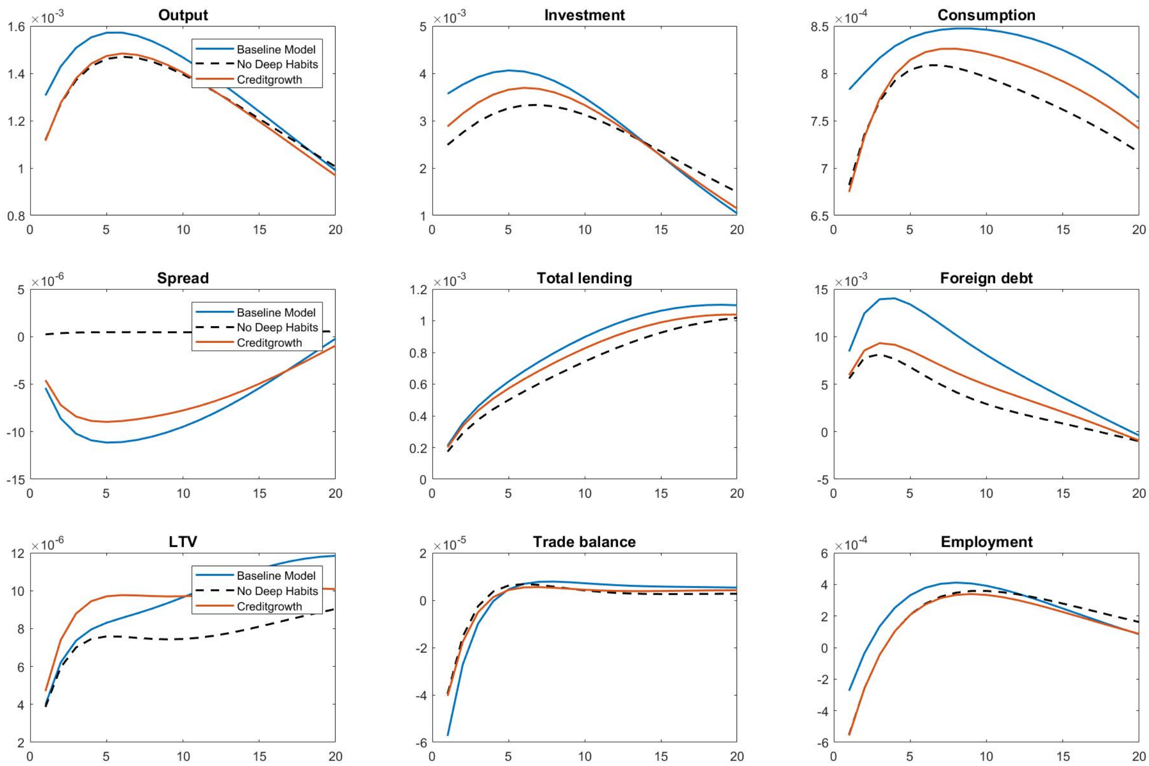

Figure 1.5: Impulse responses to the technology shock, $\varepsilon_{a,t}$, of size one standard deviation with and without the credit growth augmented policy

$$\log\left(\frac{R_t^d}{R^d}\right) = \rho_r \log\left(\frac{R_{t-1}^d}{R^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right)\right] + \varepsilon_{r,t}, \qquad (1.49)$$

where ϕ_l, ϕ_r, and ϕ_e denote the responses of the policy rate to the nominal credit growth, the deviation of the foreign interest rate from its steady state, and the deviation of the exchange rate from its steady state, respectively. Equation (1.47) is a credit growth augmented monetary policy; Equation (1.48) is a foreign interest rate augmented monetary policy; and Equation (1.49) is an exchange rate augmented monetary policy.

Aggregate fluctuations

We first illustrate how different monetary policies might be used to mitigate the influence of the deep habits mechanism on aggregate fluctuations. To this end, we compare the theoretical variances of selected macroeconomic variables in the model without the deep habits mechanism with those in: (1) the deep habits model with credit growth augmented monetary policy; (2) the deep habits model with foreign interest rate augmented monetary policy; and (3) the deep habits model with exchange rate augmented monetary policy, respectively. The results are presented in Table 1.5. Some interesting findings are obtained from this exercise. On the whole, these three policies do reduce the impact of deep habits on aggregate fluctuations. Second, the effectiveness of these policies increases when we increase the policy parameters (ϕ_l, ϕ_r, ϕ_e). For example, let us consider the results of credit growth augmented monetary policy. The variance ratios of output for $\phi_l = 0.01$ and $\phi_l = 0.05$ are 1.18 and 1.06, respectively, indicating the credit growth policy is more effective in reducing the effect of the deep habits mechanism when we increase ϕ_l. Similar conclusions are drawn for the other two policies. Last, all three policies, if well-constructed, can eliminate the majority of the additional fluctuations deriving from deep habits in the banking sector.

Impulse response analysis

We now consider the impulse response functions. To facilitate the comparisons, we display the impulse response functions for three models: (1) the baseline model; (2) the model without the deep habits mechanism; and (3) the deep habits model with credit growth augmented monetary policy. In Figure 1.5, we demonstrate the impacts of nominal credit growth augmented monetary policy in our model. The figure displays the impulse responses of selected variables to a positive technology shock in the baseline model and the model without deep habits, as well as the impulse responses generated when the baseline model is incorporated with the policy (1.47) with a value of $\phi_l = 0.1$. It is seen that the introduction of credit growth augmented monetary rule actually reduces the effect of endogenous credit standards on macroeconomic fluctuation significantly.[33] The explanation is that in the presence of interest rate responsiveness to nominal credit growth, the central bank can partially counteract the fluctuations of lending. Therefore, compared to the baseline model in which the simple monetary policy rule is applied, less credit is injected into the economy during the boom.

1.5 Conclusion

In the present study, we have augmented a small open economy model with three financial frictions: monopolistic competition, borrowing constraints and lending relationships. Following Aliaga-Díaz and Olivero (2010), we assume that entrepreneurs form deep habits in their

[33]Similar results are obtained for the two remaining policies. The explanation is as follows: By allowing the policy rate to react to changes in the foreign interest rate (or changes in the exchange rate), the central bank can indirectly counteract the fluctuations of lending through controlling foreign debt. As a consequence, less credit is poured into the economy during the boom, damping the impact of credit standards on aggregate fluctuations.

demand for banks' loans to incorporate the lending relationship in our framework. In this way, fluctuations in credit standards are endogenized as in Ravn (2016). In order to extend the current setting to the small open economy framework, we make use of the model of Galí and Monacelli (2016) with some modifications. First, we allow for a Calvo-type price setting of imported goods to incorporate the incomplete pass-through into the model as in Monacelli (2005). Second, we relax the assumption of a complete international asset market and use the debt elastic interest rate to close the model and induce stationary. We have then employed this framework to analyze how endogenous credit standards amplify the propagation mechanism of macroeconomic shocks to the economy.

Our analysis has demonstrated that countercyclical movements in credit standards indeed work as an amplifier of shocks to the economy. In particular, the existence of endogenous credit standards increases output volatility by approximately 21%. Furthermore, we have suggested three alternative tools for policymakers to mitigate the impact of endogenous credit standards on macroeconomic volatility. First, we have shown that credit growth augmented monetary rule succeeds in counteracting the fluctuation of lending, and thus decreasing the additional volatility considerably. Second, the exchange rate augmented monetary policy, if well-constructed, is considered an efficient tool to eliminate most of the additional fluctuations caused by deep habits in the banking sector. Finally, the introduction of the foreign interest augmented monetary rule also proves successful in dampening the effect of endogenous movements in lending standards.

For future research, we plan to incorporate housing into household consumption and the production function of entrepreneurs. Liu et al. (2013) find that the movements in land prices and the quantity of land are crucial factors in explaining the business cycle and that a large portion of the investment fluctuation can be attributed to a shock to land prices. Ravn (2016) investigates the impacts of commercial land on aggregate fluctuations and demonstrates that excluding land from entrepreneurs' production function reduces additional volatility emerging from the deep habits mechanism. Therefore, we anticipate that adding housing to our model can further amplify the effects of endogenous credit standards on the economy.

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Chapter 2

Modeling Inflation Dynamics: A Bayesian Comparison between GARCH and Stochastic Volatility

2.1 Introduction

Inflation and its volatility have received increasing attention in the economic literature due to their potential adverse impacts on the real economy. Theoretical studies demonstrate that high volatile inflation will cause an inefficient allocation of resources and thus may decrease economic growth and raise the unemployment rate (see, e.g., Friedman, 1977; Lucas, 2000). Given the cost of high volatile inflation, understanding the interaction between inflation and inflation uncertainty plays a key role in implementing an effective monetary policy.[1]

To document this relationship, empirical studies have to model inflation uncertainty. Two popular methods have been used extensively in the literature. In a conventional approach, this uncertainty can be modeled by a class of generalised autoregressive conditional heteroscedastic (GARCH) models, where the inflation volatility is a deterministic function of past data and the model parameters (see, e.g., Daal et al., 2005; Grier and Perry, 1998; Keskek and Orhan, 2010; Kontonikas, 2004). Alternatively, recent studies have employed stochastic volatility models, under which inflation uncertainty is treated as a latent variable that follows an autoregressive model of order one, AR(1), process (see, e.g., Berument et al., 2012; Chan, 2017; Stock and Watson, 2007). Unfortunately, these two types of volatility models are non-nested and their implied inflation volatilities demonstrate very different characteristics. Therefore, classical

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[1] Since no consensus on the terminologies between inflation uncertainty and inflation volatility has been reached in the literature, these two terms will be used interchangeably in this article.

econometric methods cannot be used to compare these two models. Since modeling inflation volatility plays a crucial part in documenting the nexus between inflation and its uncertainty, it is of great importance to straightly evaluate the model fit of these two types of volatility models by carrying out a formal model comparison. Yet, such a comparison is rarely performed in the literature.

The present paper fills the gap by comparing the model fit of commonly used GARCH models with that of their stochastic volatility counterparts in modeling the dynamics of inflation rates. We also penalise the complex models to avoid over-fitting. To this end, we employ a commonly used Bayesian model comparison approach, namely the Bayes factor, to investigate the evidence in support of the GARCH models against their stochastic volatility counterparts given the observed data. The Bayes factor is computed as the ratio of a likelihood of one particular model to that of another, and it can be used to assess the strength of evidence in favor of one model among two competing two models. Therefore, we need to calculate a marginal likelihood for each model first, and then use them to compute the Bayes factor. The marginal likelihood can be referred to as the data density, which indicates how likely it is that the observed data occurs given the model.

More specifically, for the Bayesian comparison exercise, we consider three GARCH specifications that are commonly used for modeling inflation volatility in empirical studies: (1) the standard GARCH, (2) GARCH with an asymmetric (or leverage) effect, and (3) GARCH in mean. We then select three stochastic volatility models which are closely parallel to GARCH models: (1) standard stochastic volatility, (2) stochastic volatility with a leverage effect, and (3) stochastic volatility in mean. First, by using pairwise comparison between GARCH models and their stochastic volatility counterparts (standard GARCH versus standard stochastic volatility, GARCH with a leverage effect versus stochastic volatility with a leverage effect, and GARCH in mean versus stochastic volatility in mean), we can evaluate which model (GARCH or stochastic volatility) is more strongly supported by the observed data. Second, we investigate which model features play a crucial role in modeling the inflation dynamics by directly comparing the more complex GARCH specifications with the standard one (and also the more complex stochastic volatility variants with the standard one). Finally, we examine the impact of inflation uncertainty on inflation.

The main findings, using the CPI data from 18 advanced economies, are obtained as follows. First, the stochastic volatility specifications generally outperform their GARCH counterparts, which demonstrates that inflation uncertainty is better documented as a latent variable under stochastic volatility models than as a deterministic conditional variance under GARCH models. This finding is consistent with the results in both the energy economic literature (see, e.g., Chan and Grant, 2016a) and finance literature (see, e.g., Kim et al., 1998) that favor the stochastic volatility models. Second, for all series considered, the inflation uncertainty feedback under both the stochastic volatility in mean and GARCH in mean is empirically important for modeling the dynamics of inflation rates. The relevance of the leverage effect, on the other hand, is found to be ambiguous under both classes of time-varying volatility models. Third, we find that inflation uncertainty has a positive impact on inflation, which confirms a hypothesis proposed by Cukierman et al. (1986).[2] Fourth, stochastic volatility in mean is the best model for all 18 series, followed by the GARCH in mean, which again confirms the importance of inflation volatility feedback. Finally, the forecast-based comparison results using the log predictive score for both the expanding and rolling samples confirm these findings.

There have been other studies that investigate the relationship between inflation and inflation uncertainty. However, to the best of our knowledge, this is the first study to compare the performance of GARCH models with that of their stochastic volatility counterparts in modeling inflation dynamics. Grier and Perry (1998) investigate the linkage between inflation and inflation uncertainty for G7 countries using the GACH models. They show that inflation Granger-causes inflation uncertainty for all G7 countries. However, mixed evidence on the impact of inflation volatility on inflation is found. Daal et al. (2005) employ the asymmetric power GARCH (PGARCH) model to explore the link between inflation and inflation volatility for 22 countries. They find that positive shocks to inflation have stronger effects on inflation volatility for Latin American countries. Berument et al. (2012), using the stochastic volatility in mean model to examine the interaction between inflation and inflation uncertainty for the United States, demonstrate that an innovation in inflation volatility results in an increase in inflation rates. Using data from Germany, the United States, and the United Kingdom, Chan (2017) introduces the time-varying parameter stochastic volatility in mean specification to model the inflation rates. He demonstrates that inflation volatility has a positive effect on the inflation rate for all three countries considered. Furthermore, the results clearly show that the

[2]They demonstrate that due to a lack of commitment, the monetary authorities are highly likely to generate inflation surprises by carrying out an expansionary monetary policy to stimulate the economy when facing a high inflation uncertainty environment. Thus, an increase in inflation uncertainty raises inflation. In contrast, Holland (1995) argues that in such a high inflation uncertainty environment, the state bank, owing to its stabilization incentive, should implement a tightening monetary policy to diminish the welfare cost of high volatile inflation, and thus lower the inflation rate.

volatility-related coefficients exhibit significant time-variation.

The remainder of this study is organised as follows. In Section 2, we outline two kinds of volatility models in modeling inflation dynamics, which are stochastic volatility and GARCH models. Section 3 gives a brief introduction of model comparison using the Bayes factor and introduces an importance sampling algorithm to compute marginal likelihoods with a view of evaluating these two classes of models. In Section 4, we provide the empirical findings which include the descriptive statistics, unit root tests, the Bayesian model comparison, and the estimation results of the two classes of time-varying volatility models. Section 5 presents the forecast-based comparison results using the log predictive score for both expanding samples and rolling samples. Finally, Section 6 concludes.

2.2 Model

2.2.1 GARCH Models

In this section, we introduce three common generalised autoregressive conditional heteroscedasticity (GARCH) models that are employed to model inflation uncertainty.[3] First, we consider a standard one, namely the GARCH(1,1) model (referred to as GARCH hereinafter):

$$\pi_t = \alpha + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_t^2), \tag{2.1}$$

$$\sigma_t^2 = \beta + \gamma\sigma_{t-1}^2 + \delta\epsilon_{t-1}^2, \tag{2.2}$$

where π_t is the inflation rate, σ_0^2 is a constant, and $\epsilon_0 = 0$. To make sure the variance process is always stationary, we impose the restriction $\gamma+\delta < 1$. It can be clearly seen that the conditional variance σ_t^2 representing a proxy for the inflation volatility is determined by past data and the model parameters.

Another common GARCH model that is widely used in modeling inflation uncertainty is the GARCH-GJR model developed by Glosten et al. (1993). The GARCH-GJR model accounts for asymmetric (leverage) effects of positive and negative disturbances on the conditional variance.

[3]The GARCH model is proposed by Bollerslev (1986) to generalise the earlier study on autoregressive conditional heteroscedasticity model by Engle (1982).

To be more specific, the conditional variance equation is defined as follows:

$$\sigma_t^2 = \beta + \gamma\sigma_{t-1}^2 + [\delta + \theta\mathbb{1}(\epsilon_{t-1} < 0)]\epsilon_{t-1}^2, \tag{2.3}$$

where $\mathbb{1}(\cdot)$ denotes an indicator function. The parameter θ captures the asymmetric effect: if $\theta > 0$, a negative shock would have a greater impact on inflation uncertainty; if $\theta < 0$, a negative shock would lower inflation uncertainty; and if $\theta = 0$, there is no asymmetric effect documented, and thus this specification becomes the standard GARCH model.

The last one we consider is the GARCH in mean model (referred to as GARCH-M) that accounts for potential volatility feedback on the inflation rates:

$$\pi_t = \alpha + \lambda\sigma_t^2 + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_t^2), \tag{2.4}$$

$$\sigma_t^2 = \beta + \gamma\sigma_{t-1}^2 + \delta(\pi_{t-1} - \alpha - \lambda\sigma_{t-1}^2)^2. \tag{2.5}$$

The effect of inflation volatility on inflation itself is captured by the parameter λ: when $\lambda > 0$, inflation uncertainty has a positive impact on the inflation rate; when $\lambda < 0$, inflation uncertainty has a negative impact on the inflation rate; and when $\lambda = 0$, inflation uncertainty has no impact on the inflation rate, and thus this specification reduces to the standard GARCH model.

2.2.2 Stochastic Volatility Models

In this section, we consider three stochastic volatility variants which are fairly close parallels to the three GARCH specifications just mentioned. In contrast to the GARCH specifications, the inflation uncertainty under the stochastic models is a latent variable following a stochastic process. The first model we consider is the standard stochastic volatility model, which is referred to as SV:

$$\pi_t = \alpha + \epsilon_t^\pi, \qquad \epsilon_t^\pi \sim N(0, e^{h_t}), \tag{2.6}$$

$$h_t = \alpha_h + \rho_h(h_{t-1} - \alpha_h) + \epsilon_t^h, \qquad \epsilon_t^h \sim N(0, \sigma_h^2). \tag{2.7}$$

Here, the inflation uncertainty is specified in a logarithmic form h_t that follows an AR(1) process. To make sure this process is always stationary, we impose the restriction $-1 < \rho_h < 1$. Note that the parameter σ_h^2 captures the uncertainty of future inflation volatility and that the two innovations ϵ_t^π and ϵ_t^h are assumed to be uncorrelated under the standard stochastic volatility model.

Next, we consider the counterpart of the GARCH-GJR specification, which is the stochastic volatility model with a leverage effect (see, e.g., Omori et al., 2007). Specifically, we accommodate a potential correlation between the two disturbances ϵ_t^π and ϵ_t^h as follows:

$$\pi_t = \alpha + \epsilon_t^\pi,$$

$$h_t = \alpha_h + \rho_h(h_{t-1} - \alpha_h) + \epsilon_t^h,$$
$$\begin{bmatrix} \epsilon_t^\pi \\ \epsilon_t^h \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} e^{h_t} & \rho e^{\frac{1}{2}h_t}\sigma_h \\ \rho e^{\frac{1}{2}h_t}\sigma_h & \sigma_h^2 \end{bmatrix}\right).$$

To model the potential correlation, we assume that ϵ_t^π and ϵ_t^h jointly follow a bivariate normal distribution. The correlation parameter ρ captures the leverage effect: if $\rho > 0$, a negative shock to inflation rate at time $t-1$ tends to decrease the inflation uncertainty at time t; if $\rho < 0$, a negative shock at time $t-1$ tends to increase the inflation uncertainty at time t; and if $\rho = 0$, there is no leverage effect documented, and this variant becomes the standard SV. We refer to this specification as SV-L.

Similar to the GARCH-M, the stochastic volatility in mean model proposed by Koopman and Hol Uspensky (2002) accommodates the possibility of volatility feedback:

$$\pi_t = \alpha + \lambda e^{h_t} + \epsilon_t^\pi, \qquad \epsilon_t^\pi \sim N(0, e^{h_t}), \tag{2.8}$$

$$h_t = \alpha_h + \rho_h(h_{t-1} - \alpha_h) + \epsilon_t^h, \qquad \epsilon_t^h \sim N(0, \sigma_h^2). \tag{2.9}$$

The parameter λ here captures the impact of inflation volatility on the inflation rate: when $\lambda > 0$, inflation volatility has a positive effect on the inflation rate; and when $\lambda = 0$, there is no volatility feedback documented.

2.3 Model Comparison

In this section, we provide a brief introduction of model comparison employing a prominent Bayesian criterion named the Bayes factor. In addition, we outline an adaptive importance sampling algorithm introduced in Chan and Eisenstat (2015) to compute the Bayes factor.

2.3.1 Bayes Factor

Suppose we have a set of L models $\{M_1, ..., M_L\}$ and need to compare them. Let $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_T)'$ be actual observed data, where T is the number of observations. Then, each model M_l, with $l \in (1, L)$ is constituted by two components: (1) a prior density $p(\boldsymbol{\Theta}_l \mid M_l)$, and (2) a likelihood function $p(\boldsymbol{\pi} \mid M_l, \boldsymbol{\Theta}_l)$ which relies on the parameter vector $\boldsymbol{\Theta}_l$. To perform a model comparison exercise, we employ a prominent Bayesian criterion, namely the Bayes factor, that is given by

$$BF_{ij} = \frac{p(\boldsymbol{\pi} \mid M_i)}{p(\boldsymbol{\pi} \mid M_j)}, \tag{2.10}$$

where $p(\boldsymbol{\pi} \mid M_l)$ is the marginal likelihood under the model $M_l, l = i, j$ and is computed as

$$p(\boldsymbol{\pi} \mid M_l) = \int p(\boldsymbol{\pi} \mid M_l, \boldsymbol{\Theta}_l) p(\boldsymbol{\Theta}_l \mid M_l) d\boldsymbol{\Theta}_l. \tag{2.11}$$

From this definition, we can simply interpret the marginal likelihood as a density of the data given the model M_l evaluated with the actual data $\boldsymbol{\pi}$. Thus, if the data is highly likely under the model M_l, the implied log marginal likelihood would be relatively small in absolute value and vice versa. In other words, if the Bayes factor $BF_{ij} > 1$, the model M_i is more favored by the observed data $\boldsymbol{\pi}$ than the model M_j.[4] Jeffreys (1998) provides a scale for a more concrete interpretation of the Bayes factor BF_{ij}: a Bayes factor in the interval (3, 10) indicates moderate evidence to support the model M_i; a Bayes factor in the interval (10, 30) provides strong evidence; a Bayes factor in the range (30, 100) provides very strong evidence; and if a Bayes factor is greater than 100, we have extreme evidence in favor of model M_i.

To calculate the Bayes factor, we need to compute the marginal likelihoods. In what follows, we outline an efficient method to compute the marginal likelihoods for both the GARCH-type and SV-type models.

[4]Note that another widely used model selection, namely the Bayesian information criterion (BIC) introduced by Schwarz (1978) is shown to be asymptotically convergent to the the logarithm of the Bayes factor (see, e.g., Kass and Raftery, 1995). In other words, both the Bayes factor and BIC asymptotically choose the same candidate model. More specifically, it can be easily checked that

$$\frac{(BIC_i - BIC_j) - \log BF_{ij}}{\log BF_{ij}}$$

converges to zero as T goes to infinity. Here T is the number of observations, and the BIC under the model M_l with $l \in i, j$ is computed as

$$BIC_l = \log f(\boldsymbol{\pi} \mid M_l, \boldsymbol{\Theta}_l) - \frac{n_l}{2} \log T,$$

where n_l is the number of estimated parameters, and $\boldsymbol{\Theta}_l$ is the maximum likelihood estimate value.

2.3.2 Importance Sampling for Marginal Likelihoods

One main challenge for calculating the marginal likelihood is to evaluate the integral in Equation (2.11) since it is often non-standard and of high dimension and thus cannot have an analytic solution. Following Chan and Eisenstat (2015), we compute the marginal likelihoods for both the stochastic volatility and GARCH models using an adaptive importance sampling algorithm. To this end, let $g(\boldsymbol{\Theta})$ be the proposal density. The marginal likelihood can then be rewritten as follows:[5]

$$p(\boldsymbol{\pi}) = \int \frac{p(\boldsymbol{\pi} \mid \boldsymbol{\Theta})p(\boldsymbol{\Theta})}{g(\boldsymbol{\Theta})} g(\boldsymbol{\Theta}) d\boldsymbol{\Theta}.$$

Let $\boldsymbol{\Theta}^{(i)}$ for all $i \in (1, N)$ be an independent draw obtained from the proposal density $g(\boldsymbol{\Theta})$, then the estimated marginal likelihood is computed as

$$\widehat{p(\boldsymbol{\pi})} = \frac{1}{N} \sum_{i=1}^{N} \frac{p(\boldsymbol{\pi} \mid \boldsymbol{\Theta}^{(i)})p(\boldsymbol{\Theta}^{(i)})}{g(\boldsymbol{\Theta}^{(i)})} \tag{2.12}$$

and is shown to be unbiased and simulation consistent. It is clear that the performance of this estimator depends heavily on the choice of the proposal density $g(\boldsymbol{\Theta})$. Chan and Eisenstat (2015) provide a way of obtaining an optimal proposal density by minimizing the Kullback-Leibler divergence (or cross-entropy distance) to the zero-variance density.[6] Once the proposal density $g(\boldsymbol{\Theta})$ is obtained, we can quickly construct the importance sampling estimator for the GARCH models as the corresponding likelihoods $p(\boldsymbol{\pi} \mid \boldsymbol{\Theta})$ are available analytically, and thus can be evaluated easily. As an example, the log-conditional likelihood $p(\boldsymbol{\pi} \mid \alpha, \beta, \gamma, \delta)$ under the standard GARCH model is given as follows:

$$\log p(\boldsymbol{\pi} \mid \alpha, \beta, \gamma, \delta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T} \log \sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T} \frac{(\pi_t - \alpha)^2}{\sigma_t^2}.$$

Unfortunately, we do not have an analytical form for the likelihoods $p(\boldsymbol{\pi} \mid \boldsymbol{\Theta})$ under the stochastic volatility models. Thus, we need to evaluate them by employing an importance sampling algorithm. More specifically, recall that the integrated (or observed-data) likelihood

[5] For simplicity, we drop out the notation M for the model in the expression.

[6] Chan and Eisenstat (2015) show that the posterior density $p(\boldsymbol{\Theta} \mid \pi)$ is the zero-variance density for estimating the marginal likelihood. Unfortunately, we can not use this density as a proposal due to its unknown normalizing constant.

under the stochastic volatility models is given as follows:

$$p(\boldsymbol{\pi} \,|\, \boldsymbol{\Theta}) = \int p(\boldsymbol{\pi}, \boldsymbol{h} \,|\, \boldsymbol{\Theta}) d\boldsymbol{h} = \int p(\boldsymbol{\pi} \,|\, \boldsymbol{h}, \boldsymbol{\Theta}) p(\boldsymbol{h} \,|\, \boldsymbol{\Theta}) d\boldsymbol{h},$$

where $p(\boldsymbol{\pi}, \boldsymbol{h} \,|\, \boldsymbol{\Theta})$ is the joint density of $\boldsymbol{\pi}$ and $\boldsymbol{h}$, $p(\boldsymbol{h} \,|\, \boldsymbol{\Theta})$ is the prior density of the log-inflation volatilities $\boldsymbol{h} = (h_1, h_2, .., h_T)$, and $p(\boldsymbol{\pi} \,|\, \boldsymbol{h}, \boldsymbol{\Theta})$ is the conditional likelihood. Let $g(\boldsymbol{h})$ be a proposal density; the integrated likelihood can then be rewritten as

$$p(\boldsymbol{\pi} \,|\, \boldsymbol{\Theta}) = \int \frac{p(\boldsymbol{\pi} \,|\, \boldsymbol{h}, \boldsymbol{\Theta}) p(\boldsymbol{h} \,|\, \boldsymbol{\Theta})}{g(\boldsymbol{h})} g(\boldsymbol{h}) d\boldsymbol{h}. \tag{2.13}$$

Suppose $\boldsymbol{h}^{(1)}$, $\boldsymbol{h}^{(2)}$,..., $\boldsymbol{h}^{(N)}$ are N independent draws from the proposal density $g(\boldsymbol{h})$, then the integrated likelihood $p(\boldsymbol{\pi} \,|\, \boldsymbol{\Theta})$ can be approximated by

$$\widehat{p(\boldsymbol{\pi} \,|\, \boldsymbol{\Theta})} = \frac{1}{N} \sum_{i=1}^{N} \frac{p(\boldsymbol{\pi} \,|\, \boldsymbol{h}^{(i)}, \boldsymbol{\Theta}) p(\boldsymbol{h}^{(i)} \,|\, \boldsymbol{\Theta})}{g(\boldsymbol{h}^{(i)})}. \tag{2.14}$$

2.4 Empirical Findings

2.4.1 Data

In this paper, we use quarterly CPI data for advanced economies obtained from the Federal Reserve Economic Data.[7] All data series are seasonally adjusted by the X-13-ARIMA SEAT (autoregressive integrated moving average, seasonal extraction in ARIMA time series) method developed by the U.S. Census Bureau. Inflation is then computed as the first difference of the log of CPI: $\pi_t = 400 * (\log CPI_t - \log CPI_{t-1})$. Table 2.1 displays the summary statistics and unit root tests for 18 advanced countries. From the table, we find that (for all data series): (1) the inflation distribution exhibits positive (right) skewness; (2) the inflation distribution tends to be leptokurtic owing to high excess kurtosis; (3) the Jarque-Bera (JB) test confirms these results: the test rejects the null hypothesis that the inflation rate follows a normal distribution; (4) both the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests reject the null hypothesis,

[7]The advanced economies, as defined in this study, are ones with GDP per capita over 40,000 US dollars under the IMF's list of countries by nominal GDP per capita. In addition, we only include countries with at least 50 consecutive years of data for CPI. The resulting data contains 18 economies: the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States), Australia, Austria, Belgium, Denmark, Finland, Luxembourg, the Netherlands, New Zealand, Norway, Sweden, and Switzerland. This is almost the same list considered by Uribe and Schmitt-Grohé (2017). In their study, they characterise the rich economies as all of those with purchasing power parity (PPP)-converted GDP per capita in 2005 U.S. dollars above 25,000.

Table 2.1: Summary statistics and unit root tests

Country	Mean	Median	Std.Dev	Skew	Ex.Kur	JB	ADF	PP
Canada	3.55	2.61	3.50	1.00	1.05	50.55***	-2.80***	-5.92***
France	4.05	2.67	3.72	1.15	0.57	55.22***	-1.81*	-2.88**
Germany	2.61	2.21	1.90	0.80	0.30	26.18***	-2.08**	-4.96***
Italy	5.67	3.97	5.36	1.47	1.76	115.86***	-1.69*	-2.70*
Japan	2.92	1.69	4.57	2.58	12.83	1878.9***	-3.66***	-6.44***
United Kingdom	5.06	3.26	4.97	2.17	5.89	525.56***	-2.35**	-4.35***
United States	3.70	3.19	3.03	0.81	2.92	110.65***	-2.60***	-5.23***
Australia	4.65	3.70	4.37	1.25	2.25	111.34***	-2.71***	-7.69***
Austria	3.23	2.75	2.34	1.09	1.48	68.37***	-2.74***	-6.96***
Belgium	3.48	2.90	3.01	1.28	2.73	137.94***	-2.47**	-5.78***
Denmark	4.31	2.71	4.17	1.35	1.98	98.93***	-3.20***	-6.39***
Finland	4.51	3.31	4.37	1.16	0.93	61.17***	-2.21**	-3.88***
Luxembourg	3.29	2.74	2.83	0.99	0.84	45.40***	-2.46**	-5.43***
Netherlands	3.31	2.55	2.87	0.94	1.04	45.61***	-2.82***	-7.13***
New Zealand	5.39	3.47	5.43	1.42	2.73	153.05***	-2.64***	-4.96***
Norway	4.46	3.61	3.79	1.05	2.97	130.89***	-2.87***	-8.02***
Sweden	4.26	2.96	4.17	1.11	1.21	62.94***	-2.89***	-6.82***
Switzerland	2.45	1.77	2.62	1.18	2.99	143.19***	-2.75***	-4.71***

Notes: *, **, and *** indicate the significance level of 10%, 5%, and 1%, respectively. Due to space constraints, we only report: (1) an ADF test in the absence of drift and trend; (2) a PP test with intercept. The period spans from 1961Q1 to 2018Q4 for all countries except for Denmark (1967Q1 to 2018Q4). Note also that here we display an "Excess Kurtosis" (Ex.Kur), which is simply a "Kurtosis-3".

implying that all the data series are stationary.[8]

2.4.2 Model Comparison Findings

In this exercise, we perform the model comparisons between the three commonly used GARCH variants and their SV counterparts using the algorithm presented in section 3. The results are shown in Table 2.2.[9] Some broad overviews are obtained from this exercise. On the whole, the best model for all 18 countries is the SV-M model, which is followed by the GARCH-M model. Second, with only a few exceptions of GARCH-vs-SV and GARCH-GJR-vs-SV-L pairs, the GARCH variants are outperformed by their SV counterparts. For example, let us consider the results for Canada. The log marginal likelihoods under the GARCH and SV specifications are -569.8 and -562.5, respectively, indicating a Bayes factor of 1480.30 in support of the SV model against its GARCH competitor. According to Jeffreys (1998), this

[8]We also perform: (1) an ADF test with drift; (2) an ADF test with both drift and trend; and (3) a PP test with trend. The findings also suggest that the inflation rates are stationary.

[9]We report the marginal likelihoods of the models because the Bayes factor is computed based on the marginal likelihoods. This is also a common method used by researchers.

demonstrates decisive evidence for choosing the former model. The Bayes factors for the two remaining pairs are even larger, which again indicates decisive evidence in favor of the SV-type models. This finding is consistent with the result in the energy economic literature that the stochastic volatility variants generally outperform their GARCH counterparts in modeling energy price dynamics (see, e.g., Chan and Grant, 2016a). Furthermore, stochastic volatility specifications have been shown to perform better in modeling financial returns (see, e.g., Kim et al., 1998; Yu, 2002).

Exceptions to this overall trend are the two pairs GARCH-vs-SV and GARCH-GJR-vs-SV-L for four countries. Interestingly, these two pairs follow the same pattern: whenever the SV model outperforms its GARCH counterpart, the SV-L model dominates the GARCH-GJR one and vice versa. However, the SV-M models outperform their GARCH-M counterparts for all the countries. For instance, let us consider the results for the United States. The log marginal likelihood for the GARCH and SV models are -513.8 and -520.0, respectively. This demonstrates a Bayes factor of 492.75 in support of the GARCH model against its SV competitor, showing decisive evidence for the former model. A similar conclusion is drawn for the GARCH-GJR-vs-SV-L pair with a smaller, but still relatively large Bayes factor of 221.41 in favor of GARCH-GJR. In contrast, the GARCH-M model is overwhelmed by the SV-M one with a Bayes factor of 1.94×10^{14} to support the latter model. Similar findings are found when this is applied to three other countries (Germany, Belgium, and Switzerland).

Now, we turn to examine which model characteristics play a crucial role in explaining the dynamics of inflation rates. First, we investigate the importance of the leverage effect by juxtaposing GARCH with GARCH-GJR and SV with SV-L. For both the GARCH-type and SV-type models, the results are rather mixed. In essence, accounting for the asymmetric effect increases the marginal likelihood for countries like Canada, Australia, and the United States, whereas the Netherlands, France, and the United Kingdom experience a decrease in their marginal likelihoods. These findings may seem surprising; since the GARCH-GJR nests the GARCH (the SV-L nests the SV), the former would be assumed to provide a better fit. However, remember that the marginal likelihood is in fact a density evaluation, and thus it suffers a penalty for the complexity of model. Therefore, these findings demonstrate that the cost of model complexity could exceed its benefit when referring to the leverage effect.

Finally, we compare the GARCH with GARCH-M and the SV with SV-M to explore the relevance of volatility feedback for modeling the inflation rates. It is clear that the volatility

Table 2.2: Log marginal likelihood of two classes of volatility models for 18 rich OECD countries' inflation

Type	GARCH	SV	GARCH-GJR	SV-L	GARCH-M	SV-M
Canada	-569.8	-562.5	-563.9	-556.4	-555.8	-516.8
	(0.09)	(0.01)	(0.15)	(0.02)	(0.02)	(0.03)
France	-543.8	-534.3	-551.5	-535.4	-479.0	-418.8
	(0.24)	(0.01)	(0.07)	(0.02)	(0.09)	(0.04)
Germany	-440.6	-447.1	-440.2	-444.4	-431.0	-354.6
	(0.29)	(0.01)	(0.13)	(0.01)	(0.08)	(0.03)
Italy	-599.2	-587.6	-606.8	-588.4	-543.9	-450.7
	(0.31)	(0.01)	(0.15)	(0.02)	(0.06)	(0.03)
Japan	-613.7	-597.9	-611.7	-599.2	-585.0	-524.4
	(0.11)	(0.04)	(0.18)	(0.04)	(0.06)	(0.02)
United Kingdom	-583.0	-565.9	-597.1	-567.0	-535.3	-476.9
	(0.36)	(0.01)	(0.36)	(0.03)	(0.07)	(0.04)
United States	-513.8	-520.0	-511.7	-517.1	-515.0	-482.1
	(0.11)	(0.03)	(0.32)	(0.02)	(0.05)	(0.03)
Australia	-578.3	-564.4	-573.5	-563.3	-551.7	-508.4
	(0.10)	(0.01)	(0.15)	(0.02)	(0.03)	(0.03)
Austria	-496.7	-486.0	-495.8	-486.6	-474.1	-435.8
	(0.21)	(0.01)	(0.25)	(0.02)	(0.05)	(0.03)
Belgium	-527.1	-536.4	-527.9	-533.6	-522.4	-483.9
	(0.13)	(0.02)	(0.10)	(0.01)	(0.05)	(0.04)
Denmark	-524.7	-483.5	-530.2	-485.0	-461.8	-435.2
	(0.29)	(0.01)	(0.21)	(0.01)	(0.04)	(0.02)
Finland	-610.8	-605.1	-608.1	-604.2	-551.4	-490.8
	(0.14)	(0.01)	(0.17)	(0.03)	(0.08)	(0.03)
Luxembourg	-523.0	-521.5	-523.2	-520.6	-517.8	-465.1
	(0.07)	(0.03)	(0.16)	(0.02)	(0.03)	(0.05)
Netherlands	-531.2	-518.7	-534.6	-519.9	-511.7	-485.3
	(0.12)	(0.01)	(0.10)	(0.01)	(0.05)	(0.03)
New Zealand	-658.7	-645.2	-653.1	-642.5	-637.4	-562.0
	(0.44)	(0.01)	(0.31)	(0.02)	(0.04)	(0.03)
Norway	-630.9	-598.9	-628.4	-599.1	-599.3	-555.7
	(0.05)	(0.02)	(0.10)	(0.02)	(0.07)	(0.02)
Sweden	-641.3	-624.2	-639.0	-619.7	-610.0	-533.9
	(0.18)	(0.01)	(0.17)	(0.02)	(0.04)	(0.01)
Switzerland	-502.3	-513.6	-501.6	-511.4	-490.2	-413.6
	(0.38)	(0.02)	(0.26)	(0.02)	(0.07)	(0.03)

Notes: The numbers in parentheses are numerical standard errors.

feedback plays a key role in explaining the dynamics of the inflation rates. More specifically, the Bayes factors for all the countries' data in support of the GARCH-M against GARCH are extremely large (for instance, 2.68×10^5 and 1636 for Canada and France, respectively), which implies decisive evidence in favor of the former. Similar findings are achieved for the SV models.

2.4.3 Bayesian Estimation Results

This section provides estimated results of model-specific parameters for both GARCH-type and SV-type specifications.[10] Because of space limits, we only report the posterior estimates for the G7 countries, which largely represent the findings for the remaining countries.

Bayesian Estimation for GARCH Models

The estimated results for the GARCH models are presented in Table 2.3. It can be easily checked that most of the parameter estimates across the three models for all countries are statistically different from zero. For instance, let us consider the results for Canada. In the GARCH model, the parameter α is estimated at 2.78, and its 95% credible interval is estimated to be $(2.74, 2.82)$, which excludes zero, indicating the estimate is statistically different from zero. A similar result is obtained by the GARCH-GJR model while the GARCH-M model experiences a relatively smaller estimate of α. This is due to the effect of the volatility feedback on the mean equation. The parameters describing the persistence of the inflation volatility equation (δ and γ) have quite similar estimated results across the three models and are statistically different from zero. More specifically, the inflation volatility equation is highly persistent for all three variants with the sum of the two parameters δ and γ ranging from 0.88 to 0.96, which is consistent with previous literature (see, e.g., Grier and Perry, 1998). Similar findings are found when this is applied for the remaining six countries.

We then further explore the dynamics of the inflation rates through model features. First, we consider the leverage effect. For Canada, the posterior estimate of θ is -0.39 and is statistically different from zero due to its credible interval excluding zero, implying that a negative shock at time $t-1$ would lower the inflation volatility at time t. A similar result is found for the United States with the leverage effect θ being estimated at -0.43.[11] However, the posterior

[10] The estimation method is presented in the Appendix.

[11] The negative estimate of the asymmetric effect is also found in the previous literature (see, e.g., Abbas Rizvi et al., 2014).

Table 2.3: Bayesian estimation for the GARCH models

Countries	**Parameters** **Models**	α	β	δ	γ	θ	λ
Canada	GARCH	2.78	1.35	0.42	0.45		
		(0.02)	(0.46)	(0.09)	(0.09)		
	GARCH-GJR	2.71	1.91	0.54	0.37	-0.39	
		(0.05)	(0.49)	(0.10)	(0.09)	(0.11)	
	GARCH-M	1.54	0.41	0.31	0.65		0.09
		(0.17)	(0.19)	(0.06)	(0.06)		(0.02)
France	GARCH	3.14	0.51	0.38	0.54		
		(0.06)	(0.26)	(0.09)	(0.10)		
	GARCH-GJR	3.16	0.56	0.40	0.53	-0.04	
		(0.01)	(0.27)	(0.09)	(0.09)	(0.12)	
	GARCH-M	0.60	0.06	0.13	0.86		0.46
		(0.17)	(0.03)	(0.02)	(0.02)		(0.03)
Germany	GARCH	2.18	0.40	0.35	0.53		
		(0.06)	(0.11)	(0.07)	(0.06)		
	GARCH-GJR	2.11	0.44	0.41	0.50	-0.17	
		(0.06)	(0.12)	(0.07)	(0.07)	(0.09)	
	GARCH-M	0.64	0.11	0.12	0.84		0.57
		(0.31)	(0.04)	(0.04)	(0.04)		(0.15)
Italy	GARCH	4.38	1.27	0.56	0.35		
		(0.12)	(0.44)	(0.11)	(0.12)		
	GARCH-GJR	4.49	1.37	0.59	0.33	-0.05	
		(0.06)	(0.39)	(0.11)	(0.11)	(0.12)	
	GARCH-M	1.80	0.04	0.16	0.83		0.22
		(0.20)	(0.02)	(0.04)	(0.04)		(0.04)
Japan	GARCH	1.87	0.70	0.21	0.73		
		(0.05)	(0.33)	(0.05)	(0.05)		
	GARCH-GJR	1.80	1.52	0.29	0.64	-0.22	
		(0.06)	(0.52)	(0.07)	(0.07)	(0.10)	
	GARCH-M	-0.36	0.19	0.07	0.91		0.20
		(0.23)	(0.08)	(0.01)	(0.02)		(0.03)
United Kingdom	GARCH	3.61	0.59	0.39	0.57		
		(0.08)	(0.25)	(0.07)	(0.07)		
	GARCH-GJR	3.98	1.34	0.48	0.49	-0.23	
		(0.07)	(0.42)	(0.07)	(0.07)	(0.10)	
	GARCH-M	1.75	0.04	0.17	0.82		0.18
		(0.11)	(0.02)	(0.02)	(0.02)		(0.01)
United States	GARCH	2.97	1.10	0.64	0.24		
		(0.05)	(0.30)	(0.10)	(0.09)		
	GARCH-GJR	2.98	1.30	0.66	0.25	-0.43	
		(0.05)	(0.35)	(0.10)	(0.10)	(0.13)	
	GARCH-M	2.36	0.72	0.49	0.42		0.06
		(0.18)	(0.24)	(0.10)	(0.10)		(0.02)

Notes: In this table, we report the estimated posterior means of the parameters. The numbers in parentheses are posterior standard errors.

estimate of θ are insignificant for countries like France, Germany, and Italy, showing no asymmetric effect. These findings support the ranking of marginal likelihood shown in the previous section.[12] In other words, the leverage effect is found to be mixed. This finding is in line with previous literature. For example, Daal et al. (2005) employ the PGARCH model to capture the asymmetric effect of inflation volatility for 22 countries and show that there are mixed results regarding the relevance of the leverage effect.

Finally, we investigate the impact of inflation volatility on the inflation rate. It is clearly shown that the volatility feedback plays a crucial role in modeling the inflation rates. As an example, let us consider the results for France. The volatility parameter λ is estimated at 0.46 and is statistically significant, implying that inflation volatility has a positive effect on the inflation rate. The same conclusions are drawn for the remaining countries. The finding is consistent with the ranking of marginal likelihood that favors GARCH-M over GARCH.

Bayesian Estimation for Stochastic Volatility Models

Table 2.4 provides results for the three stochastic volatility variants. Similar to the findings from the GARCH models, most of the posterior estimates across the three variants are statistically significant. Also, all the models imply high persistence of the inflation volatility equation with the posterior estimate of ρ_h ranging from 0.92 to 0.98, which is in line with previous literature. For instance, Chan (2017) proposes the time-varying parameter stochastic volatility in mean (TVP-SVM) variant to model inflation dynamics and shows high persistence of 0.963 for the transition of inflation volatility.

Next, we consider the importance of the leverage effect for modeling inflation rates. Similar to the GARCH-GJR results, the posterior estimate of the leverage parameter ρ under SV-L is consistent with the findings from the marginal likelihoods. More specifically, recall that Canada experiences an increase in the marginal likelihood when adding the leverage effect, and thus we would expect that little mass around zero is observed in the posterior distribution of ρ. This is indeed the case since the 95% credible interval of ρ excludes zero. In addition, a positive correlation ($\rho = 0.67$) indicates that a negative shock at time $t-1$ decreases the volatility at time t, which is in line with the GARCH-GJR findings. Similarly, the posterior estimate of ρ for the United States is 0.37 and is statistically significant from zero. In the instance of France, the posterior estimate of ρ under SV-L is 0.15, but is insignificant, indicating that no leverage effect

[12] Similar conclusions are drawn for the remaining countries. This means the posterior estimates of an asymmetric effect parameter θ is in line with the ranking of the marginal likelihoods. The results can be found in the Appendix.

Table 2.4: Bayesian estimation for the stochastic volatility models

Countries	**Parameters** **Models**	α	α_h	ρ_h	σ_h^2	ρ	λ
Canada	SV	2.01	1.52	0.97	0.09		
		(0.16)	(0.88)	(0.02)	(0.04)		
	SV-L	2.01	1.20	0.92	0.08	0.67	
		(0.14)	(0.30)	(0.02)	(0.03)	(0.16)	
	SV-M	-0.12	1.12	0.97	0.03		0.80
		(0.86)	(0.62)	(0.02)	(0.01)		(0.28)
France	SV	1.97	1.34	0.98	0.09		
		(0.12)	(1.11)	(0.01)	(0.03)		
	SV-L	1.97	1.11	0.97	0.10	0.15	
		(0.12)	(0.86)	(0.02)	(0.04)	(0.13)	
	SV-M	-1.84	0.14	0.97	0.02		4.38
		(0.65)	(0.53)	(0.02)	(0.00)		(1.13)
Germany	SV	1.82	0.65	0.95	0.15		
		(0.10)	(0.64)	(0.03)	(0.06)		
	SV-L	1.81	0.42	0.89	0.13	0.44	
		(0.10)	(0.29)	(0.03)	(0.06)	(0.15)	
	SV-M	-2.34	-0.63	0.93	0.02		9.22
		(0.57)	(0.29)	(0.03)	(0.00)		(2.62)
Italy	SV	2.42	1.57	0.99	0.11		
		(0.11)	(1.23)	(0.01)	(0.03)		
	SV-L	2.43	1.15	0.97	0.12	0.17	
		(0.11)	(0.95)	(0.02)	(0.04)	(0.13)	
	SV-M	-2.92	-0.15	0.97	0.02		8.26
		(0.66)	(0.49)	(0.02)	(0.00)		(2.26)
Japan	SV	0.65	1.94	0.98	0.07		
		(0.20)	(1.08)	(0.01)	(0.03)		
	SV-L	0.61	1.77	0.96	0.09	0.17	
		(0.20)	(0.87)	(0.03)	(0.05)	(0.21)	
	SV-M	-1.41	1.26	0.97	0.05		0.94
		(0.34)	(0.62)	(0.02)	(0.02)		(0.19)
United Kingdom	SV	2.27	1.17	0.99	0.12		
		(0.12)	(1.25)	(0.01)	(0.04)		
	SV-L	2.29	0.95	0.97	0.13	0.14	
		(0.12)	(1.01)	(0.02)	(0.04)	(0.14)	
	SV-M	0.66	0.64	0.97	0.06		1.46
		(0.31)	(0.72)	(0.02)	(0.01)		(0.34)
United States	SV	2.75	1.21	0.96	0.15		
		(0.13)	(0.71)	(0.02)	(0.06)		
	SV-L	2.72	1.03	0.92	0.15	0.37	
		(0.14)	(0.39)	(0.03)	(0.06)	(0.12)	
	SV-M	0.77	0.66	0.98	0.04		0.84
		(0.34)	(0.71)	(0.02)	(0.01)		(0.17)

Notes: In this table, we report the estimated posterior means of the parameters. The numbers in parentheses are posterior standard errors.

is found. Same conclusions are drawn for Italy, Japan and the United Kingdom. Contrary to the finding from the GARCH-GJR model, the asymmetric effect for Germany under the SV-L is estimated to be 0.44 and its 95% credible interval excludes zero. This is, however, consistent with the finding from the model comparison using the log marginal likelihood.

Finally, we investigate the relevance of the inflation volatility feedback in explaining the dynamics of the inflation rates. Similar to the GARCH-M findings, the volatility feedback parameter λ under SV-M is estimated to be positive and is significantly different from zero for all countries, implying that the volatility feedback is relevant in modeling the inflation rates. This finding is in line with the empirical work of Berument et al. (2012), who employ the SV-M model and demonstrate that an innovation in inflation volatility generates an increase in inflation. Furthermore, the estimate of the volatility parameter λ under SV-M is considerably larger than that under the GARCH-M, which shows that a relatively stronger volatility feedback is found under the SV-M. These findings can be generalised to all the remaining countries considered.

2.5 Forecast-Based Comparison

In this exercise, we perform the forecast-based comparisons between the GARCH specifications and their SV counterparts. More specifically, we compare these models by employing the log predictive score for both expanding samples and rolling samples.[13] A greater value of the log predictive score demonstrates better prediction performance, and vice versa.

2.5.1 Expanding Samples

First, we calculate the one-step-ahead density forecast $p(\pi_{t+1} \,|\, \mathbf{\Pi}_{1:t})$ under a certain model. Clearly, it is the predictive density for π_{t+1} computed at time t by employing the data from periods 1, 2,..., t. This predictive density is then evaluated at the actual observed data π^o_{t+1} by computing the log predictive likelihood $\log p(\pi_{t+1} = \pi^o_{t+1} \,|\, \mathbf{\Pi}_{1:t})$. It is apparent that this log likelihood will be large when the actual value π^o_{t+1} is highly likely under the predictive density, and vice versa. Second, we repeat the above exercise using the data up to time $t+1$, $t+2$, and so forth. Lastly, the log predictive score for the expanding samples is computed as the sum of

[13]Interested readers are referred to Geweke and Amisano (2011) for a more in-depth consideration of the log predictive score using the expanding samples and rolling samples.

Table 2.5: Log predictive score of two classes of volatility models for both the expanding samples and rolling samples (Canada)

	GARCH	SV	GARCH-GJR	SV-L	GARCH-M	SV-M
Expanding	-84.15	-75.16	-82.29	-73.38	-70.96	-68.16
Rolling	-86.15	-75.47	-84.38	-73.27	-70.94	-69.89

the log predictive likelihoods:

$$\sum_{t=t_0}^{T-1} \log p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Pi}_{1:t}).$$

Here, $t_0 + 1$,..., T are evaluation periods. The one-step-ahead predictive likelihood $p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Pi}_{1:t})$ can be computed as follows:

$$p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Pi}_{1:t}) = \int p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Theta}, \mathbf{\Pi}_{1:t}) p(\mathbf{\Theta} \,|\, \mathbf{\Pi}_{1:t}) d\mathbf{\Theta}.$$

Suppose $\mathbf{\Theta}^{(1)}$, $\mathbf{\Theta}^{(2)}$,..., $\mathbf{\Theta}^{(N)}$ are N draws from the posterior distributions of the parameters, then the predictive likelihood can be approximated by

$$p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Pi}_{1:t}) = \frac{1}{N} \sum_{i=1}^{N} p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Theta}^{(i)}, \mathbf{\Pi}_{1:t}).$$

2.5.2 Rolling Samples

While the expanding samples make use of the entire sample, the rolling samples employ the most recent data of CPI, and the log predictive score is computed as:

$$\sum_{t=t_0+1}^{T-1} \log p(\pi_{t+1} = \pi_{t+1}^o \,|\, \pi_t, \pi_{t-1}, ..., \pi_{t-t_0}).$$

Similarly, the one-step-ahead predictive likelihood for the rolling samples $p(\pi_{t+1} = \pi_{t+1}^o \,|\, \pi_t,$ $\pi_{t-1}, ..., \pi_{t-t_0})$ can be approximated using draws $\mathbf{\Theta}^{(i)}$ $(i = 1, 2, ..., N)$ from the posterior distributions of the parameters:

$$p(\pi_{t+1} = \pi_{t+1}^o \,|\, \pi_t, \pi_{t-1}, ..., \pi_{t-t_0}) = \frac{1}{N} \sum_{i=1}^{N} p(\pi_{t+1} = \pi_{t+1}^o \,|\, \mathbf{\Theta}^{(i)}, \pi_t, \pi_{t-1}, ..., \pi_{t-t_0}).$$

2.5.3 Forecasting Results

This section provides the forecasting results of the two exercises for Canada.[14] The evaluation period for both the expanding samples and rolling samples is from 2009Q1 to 2018Q4. The results are presented in Table 2.5. The forecast-based comparison findings are widely similar to the model comparison findings using the Bayes factor. More specifically, the SV specifications produce better forecast performance than their GARCH competitors. In addition, the volatility feedback improves the predictive density significantly for both the GARCH and SV variants. Yet, the importance of the leverage effect is found to be rather ambiguous. For instance, let us consider the forecasting results of the expanding sample. The log predictive score for the GARCH and SV models are -84.15 and -75.16, respectively. This demonstrates better density predictions of the SV model against its GARCH counterpart. A similar conclusion is drawn for the GARCH-GJR-vs-SV-L and the GARCH-M-vs-SV-M pairs. Moreover, we find that the volatility feedback is important for modeling the inflation dynamics by comparing the GARCH with GARCH-M and the SV with SV-M. Next, we investigate the relevance of the leverage effect by comparing the forecast performance of the GARCH and SV specifications with that of the GARCH-GJR and SV-L models. The numbers show that the GARCH-GJR and SV-L specifications give better density forecasts than the GARCH and SV models, respectively, which demonstrates the importance of the leverage effect for Canada.[15] These findings are confirmed when we employ the rolling samples.

2.6 Conclusion

In this paper, we have performed a Bayesian estimation to evaluate three widely used GARCH specifications and their stochastic volatility counterparts in modeling inflation rates for 18 advanced countries. By employing a formal Bayesian comparison criterion−the Bayes factor−to compare a variety of models, we find that the GARCH variants are generally surpassed by their stochastic volatility counterparts in modeling inflation dynamics. In addition, the stochastic volatility in mean model is shown to be the best one for all 18 countries considered. The forecast-based comparison results using the log predictive score for both the

[14]The results for the other countries can be found in the Appendix.

[15]We found no improvement in the density prediction of the GARCH-GJR (SV-L) over the GARCH (SV) for countries like France, Germany, Italy. This finding is in line with the result from the model comparison using the log marginal likelihood. To summarise, the relevance of the asymmetric effect is shown to be mixed.

expanding samples and rolling samples confirm these findings.

The study also investigates which model characteristics are important in modeling inflation rates. We show that the inflation volatility feedback is a crucial feature that we should take into consideration when modeling the inflation rates. Moreover, inflation volatility has a positive impact on the inflation rate, which confirms a hypothesis introduced by Cukierman et al. (1986). However, we find mixed results when taking the leverage effect into consideration.

For future research, it would be of considerable interest to allow for time-varying parameters in both GARCH and stochastic volatility specifications and evaluate the effectiveness of these models in modeling inflation dynamics. In addition, the interaction between CPI inflation and macroeconomic variables has been a topic of great interest. As a result, incorporating macroeconomic variables into present models and extending them to multivariate GARCH and stochastic volatility variants would also be desirable.

2.A Appendix

2.A.1 Hyper-Parameters Setting

GARCH-Type Models

We assume independent priors for α and the group of parameters $\Delta = (\beta, \gamma, \delta)'$ as follows:

$$\alpha \sim N(\alpha_0, V_\alpha), \qquad \log \Delta \sim N(\Delta_0, V_\Delta)\mathbb{1}(\gamma + \delta < 1).$$

We impose the inequality $\gamma + \delta < 1$ to induce stationarity. For the GARCH-GJR, the coefficient of leverage ρ is assumed to have a uniform prior conditional on Δ. As for the volatility feedback parameter λ under the GARCH-M, we assume a normal distribution: $\lambda \sim N(\lambda_0, V_\lambda)$. Following Chan and Grant (2016a), we consider the noninformative priors as follows: $\alpha_0 = 0$, $\Delta_0 = (1, \log 0.1, \log 0.8)$, $\lambda_0 = 0$, $V_\alpha = 5$, $V_\Delta = diag(5, 1, 1)$, and $V_\lambda = 100$.

SV-Type Models

Similar to the GARCH-type models, we use independent priors for α, α_h, ρ_h, and σ_h^2 as follows:

$$\alpha \sim N(\alpha_0, V_\alpha), \quad \alpha_h \sim N(\alpha_{h0}, V_{\alpha_h}), \quad \rho_h \sim N(\rho_{h0}, V_{\rho_h})\mathbb{1}(|\rho_h| < 1), \quad \sigma_h^2 \sim IG(\sigma_{h0}, V_{\sigma_h}).$$

Here $IG(\cdot)$ is the inverse-gamma distribution. We also impose a restriction $|\rho_h| < 1$ to induce stationarity. As for the SV-L, we assume a uniform prior for the leverage parameter ρ. For the SV-M, we assume that the volatility feedback parameter λ follows a normal distribution $\lambda \sim N(\lambda_0, V_\lambda)$. To obtain similar dynamics for the inflation volatility as in the GARCH variants, we also set noninformative priors for the parameters of the SV models: $\alpha_{h0} = 1$, $\rho_{h0} = 0.97$, $\sigma_{h0} = 5$, $V_{\alpha_h} = 5$, $V_{\rho_h} = 0.1^2$, and $V_{\sigma_h} = 0.16$.

2.A.2 Bayesian Estimation

In this section, we provide a brief discussion of Bayesian estimation for our models. Two classes of time-varying models are estimated by the Markov chain Monte Carlo (MCMC) methods. More specifically, we generate Markov samplers to draw from the posterior distributions and use these independent draws to calculate moments of interest such as the log marginal likelihoods, the posterior means, and standard deviations.

GARCH Models

For the GARCH models, following Chan and Grant (2016a), we draw from the posterior distributions using the Metropolis-Hastings (MH) algorithm. We group parameters into blocks and sequentially sample from conditional distributions. As an example, let us consider the standard GARCH model. We first divide the parameters into two blocks: (1) α and (2) $\kappa = (\beta, \gamma, \delta)$. We then sequentially sample from the two conditional distributions $p(\alpha|\boldsymbol{\pi}, \kappa)$ and $p(\kappa|\boldsymbol{\pi}, \alpha)$. To this end, we resort to the Metropolis-Hasting algorithm for sampling as these conditional distributions are not standard,. More specifically, we use a Gaussian distribution centered at the sample mean $\bar{\pi}$ with the variance s^2/T, where s^2 is the sample variance, to draw α. For a block κ, we use a Gaussian distribution with the mean and the covariance matrix being set to be the mode of $p(\kappa|\boldsymbol{\pi}, \alpha)$ and the outer product of the scores, respectively. For the two remaining GARCH variants, the algorithm remains unchanged, but additional blocks are required to deal with additional parameters.

Stochastic Volatility Models

For stochastic volatility variants, a main challenge is to jointly draw the log-inflation volatilities $\boldsymbol{h} = (h_1, h_2, ..., h_T)$ conditional on the observed data and model parameters. To this end, we employ the acceptance-rejection Metropolis-Hastings algorithm proposed by Chan (2017) to draw $\boldsymbol{h}$. A main feature of this approach is the use of fast band and sparse matrix routines which take advantage of the specialty of the problem, namely, that the Hessian of the log-conditional density of $\boldsymbol{h}$ contains only a few non-zero elements along the diagonal band. In general, this approach has proved to be more efficient than the conventional Kalman filter. [16]

Standard Stochastic Volatility Model First, we discuss the algorithm for the standard stochastic volatility specification. For convenience, we refer to this algorithm as the baseline one. Let $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_T)$ be the observed data. The posterior draws can then be attained by sequentially sampling from

1. $p(\boldsymbol{h} \mid \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2)$;
2. $p(\alpha \mid \boldsymbol{\pi}, \boldsymbol{h}, \alpha_h, \rho_h, \sigma_h^2) = p(\alpha \mid \boldsymbol{\pi}, \boldsymbol{h})$;
3. $p(\alpha_h \mid \boldsymbol{\pi}, \boldsymbol{h}, \alpha, \rho_h, \sigma_h^2) = p(\alpha_h \mid \boldsymbol{h}, \rho_h, \sigma_h^2)$;
4. $p(\sigma_h^2 \mid \boldsymbol{\pi}, \boldsymbol{h}, \alpha, \alpha_h, \rho_h) = p(\sigma_h^2 \mid \boldsymbol{h}, \alpha_h, \rho_h)$;
5. $p(\rho_h \mid \boldsymbol{\pi}, \boldsymbol{h}, \alpha, \alpha_h, \sigma_h^2) = p(\rho_h \mid \boldsymbol{h}, \alpha_h, \sigma_h^2)$.

In the first step, we need to jointly draw log-inflation volatilities $\boldsymbol{h}$, which is a key ingredient to implement the acceptance-rejection Metropolis-Hastings algorithm. The fundamental idea is to approximate the target $p(\boldsymbol{h} \mid \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2)$ using a Gaussian density. Note that from Bayes' Theorem, we have:

$$p(\boldsymbol{h} \mid \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2) \propto p(\boldsymbol{\pi} \mid \boldsymbol{h}, \alpha) p(\boldsymbol{h} \mid \alpha_h, \rho_h, \sigma_h^2).$$

Hereinafter, we derive the explicit expressions for the two conditional densities, $p(\boldsymbol{\pi} \mid \boldsymbol{h}, \alpha)$ and $p(\boldsymbol{h} \mid \alpha_h, \rho_h, \sigma_h^2)$. The former density $p(\boldsymbol{\pi} \mid \boldsymbol{h}, \alpha)$ can be approximated by a Gaussian distribution in $\boldsymbol{h}$. To this end, we approximate the conditional density $\log p(\boldsymbol{\pi} \mid \boldsymbol{h}, \alpha) = \sum_{t=1}^{T} \log p(\pi_t \mid h_t, \alpha)$ around a point $\bar{\boldsymbol{h}} = (\bar{h}_1, \bar{h}_2, ..., \bar{h}_T)'$ which is chosen to be the mode of $p(\boldsymbol{h} \mid \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2)$ by using a second-order Taylor expansion to obtain[17]

$$\begin{aligned} \log p(\boldsymbol{\pi} \mid \boldsymbol{h}, \alpha) &\approx \log p(\boldsymbol{\pi} \mid \bar{\boldsymbol{h}}, \alpha) + (\boldsymbol{h} - \bar{\boldsymbol{h}})' \boldsymbol{F} - \frac{1}{2} (\boldsymbol{h} - \bar{\boldsymbol{h}})' G (\boldsymbol{h} - \bar{\boldsymbol{h}}) \\ &= -\frac{1}{2} (\boldsymbol{h}' \boldsymbol{G} \boldsymbol{h} - 2\boldsymbol{h}'(\boldsymbol{F} + \boldsymbol{G}\bar{\boldsymbol{h}})) + a_1, \end{aligned} \tag{2.15}$$

[16] This approach has been used recently, for example, by Chan and Jeliazkov (2009) and McCausland et al. (2011) for linear state space models, and by McCausland (2012) and Djegnéné and McCausland (2014) for non-linear state space models.

[17] We choose the mode of conditional density $p(\boldsymbol{h} \mid \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2)$ as a point to expand since it can be quickly computed by the Newton-Raphson method. Interested readers are referred to Kroese et al. (2013) for a more detailed explanation.

where a_1 is a constant, and $\boldsymbol{F} = (F_1, F_2, ..., F_T)'$ and $\boldsymbol{G} = diag(G_1, G_2, ..., G_T)$ are the gradient and the negative Hessian of the log-conditional density of π_t evaluated at $\bar{\boldsymbol{h}}$, respectively. F_t and G_t for all $t \in (1, T)$ are computed as follows:[18]

$$F_t = \frac{\partial}{\partial h_t} \log p(\pi_t \,|\, h_t, \alpha)|_{h_t = \bar{h_t}} = -\frac{1}{2} + \frac{1}{2} e^{-h_t}(\pi_t - \alpha)^2,$$

$$G_t = -\frac{\partial^2}{\partial h_t^2} \log p(\pi_t \,|\, h_t, \alpha)|_{h_t = \bar{h_t}} = \frac{1}{2} e^{-h_t}(\pi_t - \alpha)^2.$$

Next, we consider the conditional density $p(\boldsymbol{h} \,|\, \alpha_h, \rho_h, \sigma_h^2)$. It is proved that this density is Gaussian (see, e.g., Chan and Grant, 2016b). Let $\boldsymbol{H}_{\rho_h}$ be the following matrix:

$$\boldsymbol{H}_{\rho_h} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -\rho_h & 1 & 0 & \dots & 0 \\ 0 & -\rho_h & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & -\rho_h & 1 \end{bmatrix}.$$

Then, we can rewrite the volatility equation of the standard stochastic volatility model in (2.7) as follows:

$$\boldsymbol{H}_{\rho_h} \boldsymbol{h} = \boldsymbol{\Upsilon} + \boldsymbol{\epsilon}^h, \quad \boldsymbol{\epsilon}^h \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_h), \tag{2.16}$$

where $\boldsymbol{\Upsilon} = (\alpha_h, (1 - \rho_h)\alpha_h, ..., (1 - \rho_h)\alpha_h)'$, $\boldsymbol{\epsilon}^h = (\epsilon_1^h, \epsilon_2^h, ..., \epsilon_T^h)'$, and $\boldsymbol{\Sigma}_h = diag(\sigma_h^2/(1 - \rho_h^2), \sigma_h^2, ..., \sigma_h^2)$. Since the determinant of $\boldsymbol{H}_{\rho_h}$ is one, the matrix is invertible regardless of the value of ρ_h. Therefore, from Equation (2.16), we obtain:

$$(\boldsymbol{h} \,|\, \alpha_h, \rho_h, \sigma_h^2) \sim N(\boldsymbol{\Upsilon}_h, (\boldsymbol{H}_{\rho_h}' \boldsymbol{\Sigma}_h^{-1} \boldsymbol{H}_{\rho_h})^{-1}),$$

where $\boldsymbol{\Upsilon}_h = \boldsymbol{H}_{\rho_h}^{-1} \boldsymbol{\Upsilon}$. Hence, the log-conditional density can be written as follows:

$$\log p(\boldsymbol{h} \,|\, \alpha_h, \rho_h, \sigma_h^2) = -\frac{1}{2}(\boldsymbol{h}' \boldsymbol{H}_{\rho_h}' \boldsymbol{\Sigma}_h^{-1} \boldsymbol{H}_{\rho_h} \boldsymbol{h} - 2\boldsymbol{h}' \boldsymbol{H}_{\rho_h}' \boldsymbol{\Sigma}_h^{-1} \boldsymbol{H}_{\rho_h} \boldsymbol{\Upsilon}_h) + a_2, \tag{2.17}$$

where a_2 is a constant parameter independent of $\boldsymbol{h}$. Finally, combining Equations (2.15) and

[18] From Equation (2.6), we can derive the log-conditional density of π given the parameter α and the volatility h_t as follows:

$$\log p(\pi_t \,|\, h_t, \alpha) = -\frac{1}{2} h_t - \frac{1}{2} \log(2\pi) - \frac{1}{2} e^{-h_t}(\pi_t - \alpha)^2.$$

(2.17), we obtain the following result:

$$\log p(\boldsymbol{h} \,|\, \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2) = \log p(\boldsymbol{\pi} \,|\, \boldsymbol{h}, \alpha) + \log p(\boldsymbol{h} \,|\, \alpha_h, \rho_h, \sigma_h^2) + a_3 \\ \approx -\frac{1}{2}(\boldsymbol{h}'\boldsymbol{K}_h\boldsymbol{h} - 2\boldsymbol{h}'\boldsymbol{k}_h) + a_4, \tag{2.18}$$

where a_3 and a_4 are constant parameters independent of $\boldsymbol{h}$, $\boldsymbol{K}_h = \boldsymbol{H}'_{\rho_h}\boldsymbol{\Sigma}_h^{-1}\boldsymbol{H}_{\rho_h} + \boldsymbol{G}$ and $\boldsymbol{k}_h = \boldsymbol{F} + \boldsymbol{G}\bar{\boldsymbol{h}} + \boldsymbol{H}'_{\rho_h}\boldsymbol{\Sigma}_h^{-1}\boldsymbol{H}_{\rho_h}\boldsymbol{\Upsilon}_h$. The expression in (2.18) can be shown as the log-kernel of $N(\tilde{\boldsymbol{h}}, \boldsymbol{K}_h^{-1})$ with $\tilde{\boldsymbol{h}} = \boldsymbol{K}_h^{-1}\boldsymbol{k}_h$.[19] In other words, we can approximate the joint conditional density $p(\boldsymbol{h} \,|\, \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2)$ by the Gaussian density with the mean vector $\tilde{\boldsymbol{h}}$ and variance vector $\boldsymbol{K}_h^{-1}$. It is easy to check that $\boldsymbol{K}_h$ is a tridiagonal matrix, and hence, we can quickly obtain $\tilde{\boldsymbol{h}}$ by solving the linear system $\boldsymbol{K}_h\boldsymbol{x} = \boldsymbol{k}_h$ for $\boldsymbol{x}$ without computing the inverse matrix $\boldsymbol{K}_h^{-1}$. Moreover, it is quite fast to sample from the density $N(\tilde{\boldsymbol{h}}, \boldsymbol{K}_h^{-1})$ by using the precision sampler introduced in Chan and Jeliazkov (2009).

The posterior draws for α, α_h, and σ_h^2 can be easily obtained as their corresponding conditional distributions are Gaussian:

$$1. \quad (\alpha \,|\, \boldsymbol{\pi}, \boldsymbol{h}) \sim N(\tilde{\alpha}, K_\alpha),$$

where $\tilde{\alpha} = K_\alpha(\alpha_0 V_\alpha^{-1} + \sum_{t=1}^T \pi_t e^{-h_t})$ and $K_\alpha^{-1} = V_\alpha^{-1} + \sum_{t=1}^T e^{-h_t}$.

$$2. \quad (\alpha_h \,|\, \boldsymbol{h}, \rho_h, \sigma_h^2) \sim N(\tilde{\alpha_h}, K_{\alpha_h}),$$

where $\tilde{\alpha_h} = K_{\alpha_h}(\alpha_{h0}V_{\alpha_h}^{-1} + \boldsymbol{X}'_{\alpha_h}\boldsymbol{\Sigma}_h^{-1}\boldsymbol{y}_{\alpha_h})$ and $K_{\alpha_h}^{-1} = V_{\alpha_h}^{-1} + \boldsymbol{X}'_{\alpha_h}\boldsymbol{\Sigma}_h^{-1}\boldsymbol{X}_{\alpha_h}$. Here, $\boldsymbol{X}_{\alpha_h}$ and $\boldsymbol{y}_{\alpha_h}$ are defined as $\boldsymbol{X}_{\alpha_h} = (1, 1-\rho_h, ..., 1-\rho_h)'$ and $\boldsymbol{y}_{\alpha_h} = (h_1, h_2 - h_1\rho_h, ..., h_T - h_{T-1}\rho_h)$.

$$3. \quad (\sigma_h^2 \,|\, \boldsymbol{h}, \alpha_h, \rho_h) \sim IG(\sigma_{h0} + T/2, \tilde{V}_{\sigma_h}),$$

where $\tilde{V}_{\sigma_h} = V_{\sigma_h} + [(1-\rho_h^2)(h_1 - \alpha_h)^2 + \sum_{t=2}^T (h_t - \alpha_h - \rho_h(h_{t-1} - \alpha_h))^2]/2$.

Finally, we can draw from $p(\rho_h \,|\, \boldsymbol{h}, \alpha_h, \sigma_h^2)$ by employing an independence-chain Metropolis-Hastings algorithm with the proposal density $N(\tilde{\rho_h}, K_{\rho_h})\mathbb{1}(|\rho_h| < 1)$, where $\tilde{\rho_h} = K_{\rho_h}(V_{\rho_h}^{-1}\rho_{h0} + \boldsymbol{X}'_{\rho_h}\boldsymbol{Z}_{\rho_h}/\sigma_h^2)$ and $K_{\rho_h}^{-1} = V_{\rho_h}^{-1} + \boldsymbol{X}'_{\rho_h}\boldsymbol{X}_{\rho_h}/\sigma_h^2$, with $\boldsymbol{X}_{\rho_h} = (h_1 - \alpha_h, h_2 - \alpha_h, ..., h_{T-1} - \alpha_h)'$ and $\boldsymbol{Z}_{\rho_h} = (h_2 - \alpha_h, h_3 - \alpha_h, ..., h_T - \alpha_h)'$.

[19] Interested readers are referred to Kroese and Chan (2014) for a detailed proof.

Stochastic Volatility in Mean Model Some modifications of the baseline algorithm are required in order to estimate the stochastic volatility in mean model. First, the log-conditional density of π_t given parameters (α, λ) and the volatility h_t now become

$$\log p(\pi_t \,|\, h_t, \alpha, \lambda) = -\frac{1}{2}h_t - \frac{1}{2}\log(2\pi) - \frac{1}{2}e^{-h_t}(\pi_t - \alpha)^2 - \frac{1}{2}\lambda^2 e^{h_t} + (\pi - \alpha)\lambda.$$

Hence, the first and second derivatives of this log-conditional density with respect to h_t are as follows:

$$\frac{\partial}{\partial h_t}\log p(\pi_t \,|\, h_t, \alpha, \lambda) = -\frac{1}{2} - \frac{1}{2}\lambda^2 e^{h_t} + \frac{1}{2}e^{-h_t}(\pi_t - \alpha)^2,$$
$$\frac{\partial^2}{\partial h_t^2}\log p(\pi_t \,|\, h_t, \alpha, \lambda) = -\frac{1}{2}\lambda^2 e^{h_t} - \frac{1}{2}e^{-h_t}(\pi_t - \alpha)^2.$$

We can then sample $\boldsymbol{h}$ from the joint distribution as in Step 1 of SV model. A second adjustment is that in Step 2 of the baseline algorithm, we need to jointly draw (α, λ) from $p(\alpha, \lambda \,|\, \boldsymbol{\pi}, \boldsymbol{h})$. This can be done easily since the joint conditional distribution is Gaussian. Specifically, we define $\boldsymbol{\Psi}$ as $\boldsymbol{\Psi} = (\alpha, \lambda)$. The conditional distribution is shown as follows:

$$1. \quad (\boldsymbol{\Psi} \,|\, \boldsymbol{\pi}, \boldsymbol{h}) \sim N(\tilde{\boldsymbol{\Psi}}, \boldsymbol{K}_{\boldsymbol{\Psi}}),$$

where $\tilde{\Psi} = \boldsymbol{K}_{\boldsymbol{\Psi}}(\boldsymbol{V}_{\boldsymbol{\Psi}}^{-1}\Psi_0 + \boldsymbol{X}_{\boldsymbol{\Psi}}'\boldsymbol{\Sigma}_{\boldsymbol{\pi}}^{-1}\boldsymbol{\pi})$ and $\boldsymbol{K}_{\boldsymbol{\Psi}}^{-1} = \boldsymbol{X}_{\boldsymbol{\Psi}}'\boldsymbol{\Sigma}_{\boldsymbol{\pi}}^{-1}\boldsymbol{X}_{\boldsymbol{\Psi}} + \boldsymbol{V}_{\boldsymbol{\Psi}}^{-1}$. Here, $\boldsymbol{V}_{\boldsymbol{\Psi}}$, $\boldsymbol{\Psi}_0$, and $\boldsymbol{\Sigma}_{\boldsymbol{\pi}}$ are defined as

$$\boldsymbol{V}_{\boldsymbol{\Psi}} = diag(V_\alpha, V_\lambda), \quad \boldsymbol{\Psi}_0 = (\alpha_0, \lambda_0), \quad \boldsymbol{\Sigma}_{\boldsymbol{\pi}} = diag(e^{h_1}, ..., e^{h_T}), \quad \boldsymbol{X}_{\boldsymbol{\Psi}} = \begin{bmatrix} 1 & e^{h_1} \\ \vdots & \vdots \\ 1 & e^{h_T} \end{bmatrix}.$$

Stochastic Volatility with Leverage To fit this stochastic volatility specification, a few modifications are required. First, we need to draw from $p(\boldsymbol{h} \,|\, \boldsymbol{\pi}, \alpha, \alpha_h, \rho_h, \sigma_h^2, \rho)$, where $\boldsymbol{h}$ now is defined as $\boldsymbol{h} = (h_1, h_2, ..., h_{T+1})$, which means $\boldsymbol{h}$ is of length $T + 1$. The conditional density of π_t given parameters and h_t, h_{t+1} is as follows:

$$(\pi_t \,|\, h_t, h_{t+1}, \alpha, \alpha_h, \rho_h, \sigma_h^2, \rho) \sim N\left(\alpha + \frac{\rho}{\sigma_h}e^{\frac{1}{2}h_t}(h_{t+1} - \rho_h h_t - \alpha_h(1 - \rho_h)), e^{h_t}(1 - \rho^2)\right).$$

Hence, the log-conditional density is as follows:

$$\log(\pi_t \mid h_t, h_{t+1}, \alpha, \alpha_h, \rho_h, \sigma_h^2, \rho) = -\frac{1}{2}\log(2\pi(1-\rho^2)) - \frac{1}{2}h_t$$
$$- \frac{1}{2(1-\rho^2)}e^{-h_t}\left(\pi_t - \alpha - \frac{\rho}{\sigma_h}e^{\frac{1}{2}h_t}(h_{t+1} - \rho_h h_t - \alpha_h(1-\rho_h))\right)^2.$$

To sample $\boldsymbol{h}$, we go through a similar procedure as in the baseline algorithm with only slight changes. Second, we need one extra step to sample ρ from the conditional distribution $p(\rho \mid \boldsymbol{\pi}, \boldsymbol{h}, \alpha, \alpha_h, \rho_h, \sigma_h^2)$. It can be checked that the log-conditional distribution of ρ is as follows:

$$\log p(\rho \mid \boldsymbol{\pi}, \boldsymbol{h}, \alpha, \alpha_h, \rho_h, \sigma_h^2) \propto \log p(\rho) - \frac{T}{2}log(1-\rho^2) - \frac{1}{2(1-\rho^2)}\left(m_1 - \frac{2\rho m_2}{\sigma_h} + \frac{\rho^2 m_3}{\sigma_h^2}\right).$$

Here, $p(\rho)$ is a prior distribution; m_1, m_2, and m_3 are defined as $m_1 = \sum_{t=1}^T e^{-h_t}(\pi_t - \alpha)^2$, $m_2 = \sum_{t=1}^T e^{-h_t/2}(\pi_t - \alpha)\epsilon_t^h$, and $m_3 = \sum_{t=1}^T (\epsilon_t^h)^2$. The remaining parameters can be drawn similarly as the standard stochastic volatility model.

2.A.3 Estimation Results for All 18 Countries

In this section, we present the estimated results for the all 18 countries. Because of space limit, we only report the posterior estimates for the leverage effect and volatility feedback. The results are presented in Table 2.6. It is clear that the posterior estimates of volatility feedback λ for both the GARCH-M and SV-M are all positive and statistically significant, which demonstrates that inflation volatility has a positive impact on the inflation rate. However, we find mixed results when taking the leverage effect into account. These findings are in line with the results of the model comparison using the marginal likelihood.

2.A.4 Forecasting Comparison Results for All 18 Countries

This section provides the forecasting comparison results for all 18 countries. The results are presented in Table 2.7 for the expanding samples and Table 2.8 for the rolling samples. It is clear that the SV specifications surpass their GARCH counterparts in performing the density forecast of the inflation rate for both the expanding samples and rolling samples. This result is consistent with the finding of the model comparison using the log marginal likelihood.

Table 2.6: The posterior estimates of the leverage effect and volatility feedback for all 18 countries

Country	GARCH-GJR (θ)	GARCH-M (λ)	SV-L(ρ)	SV-M(λ)
Canada	-0.39	0.09	0.67	0.80
	(0.11)	(0.02)	(0.16)	(0.28)
France	-0.04	0.46	0.15	4.38
	(0.12)	(0.03)	(0.13)	(1.13)
Germany	-0.17	0.57	0.44	9.22
	(0.09)	(0.15)	(0.15)	(2.62)
Italy	-0.05	0.22	0.17	8.26
	(0.12)	(0.04)	(0.13)	(2.26)
Japan	-0.22	0.20	0.17	0.94
	(0.10)	(0.03)	(0.21)	(0.19)
United Kingdom	-0.23	0.18	0.14	1.46
	(0.10)	(0.01)	(0.14)	(0.34)
United States	-0.43	0.06	0.37	0.84
	(0.13)	(0.02)	(0.12)	(0.17)
Australia	-0.24	0.32	0.37	0.74
	(0.08)	(0.06)	(0.15)	(0.19)
Austria	-0.13	0.61	0.21	1.90
	(0.07)	(0.08)	(0.15)	(0.63)
Belgium	-0.27	0.25	0.43	2.36
	(0.12)	(0.08)	(0.13)	(0.63)
Denmark	0.00	0.34	-0.05	0.92
	(0.05)	(0.05)	(0.13)	(0.17)
Finland	-0.17	0.33	0.52	3.66
	(0.17)	(0.03)	(0.26)	(1.02)
Luxembourg	-0.17	0.23	0.33	3.40
	(0.08)	(0.05)	(0.15)	(0.95)
Netherlands	-0.01	0.37	0.12	1.34
	(0.09)	(0.07)	(0.14)	(0.38)
New Zealand	-0.66	0.09	0.48	1.61
	(0.13)	(0.01)	(0.16)	(0.55)
Norway	-0.39	0.64	0.24	0.61
	(0.12)	(0.02)	(0.15)	(0.16)
Sweden	-0.25	0.17	0.66	1.26
	(0.09)	(0.02)	(0.18)	(0.24)
Switzerland	-0.25	0.25	0.34	4.74
	(0.15)	(0.05)	(0.11)	(1.40)

Notes: The period spans from 1961Q1 to 2018Q4 for all countries except for Denmark (1967Q1 to 2018Q4). The numbers in parentheses are numerical standard errors.

Table 2.7: Log predictive score of two classes of volatility models for all 18 countries (Expanding samples)

Type	**GARCH**	**SV**	**GARCH-GJR**	**SV-L**	**GARCH-M**	**SV-M**
Canada	-84.15	-75.16	-82.29	-73.38	-70.96	-68.16
France	-93.59	-75.08	-93.96	-74.93	-74.01	-63.83
Germany	-65.20	-59.74	-64.95	-59.13	-56.88	-50.68
Italy	-109.48	-82.66	-109.55	-83.77	-97.38	-71.87
Japan	-97.23	-86.42	-97.12	-86.05	-90.10	-85.63
United Kingdom	-89.77	-64.22	-89.77	-64.93	-65.93	-65.61
United States	-85.22	-84.54	-84.10	-83.08	-82.10	-80.06
Australia	-90.56	-77.24	-89.52	-77.01	-77.89	-76.82
Austria	-71.43	-66.05	-71.17	-65.91	-59.88	-58.74
Belgium	-81.36	-80.85	-81.33	-80.34	-79.98	-75.99
Denmark	-93.60	-73.83	-94.69	-73.69	-72.68	-71.95
Finland	-98.52	-80.44	-99.15	-79.32	-71.16	-70.33
Luxembourg	-79.66	-76.65	-78.95	-76.99	-75.51	-67.04
Netherlands	-83.03	-76.39	-83.35	-76.19	-73.84	-73.72
New Zealand	-108.95	-90.56	-107.45	-90.96	-90.35	-83.45
Norway	-95.93	-79.49	-93.69	-78.52	-80.02	-77.58
Sweden	-101.04	-85.86	-99.10	-82.08	-78.68	-71.19
Switzerland	-88.08	-80.33	-87.20	-82.45	-84.05	-70.52

Notes: The period spans from 1961Q1 to 2018Q4 for all countries except for Denmark (1967Q1 to 2018Q4). The evaluation period is from 2009Q1 to 2018Q4.

Table 2.8: Log predictive score of two classes of volatility models for all 18 countries (Rolling samples)

Type	**GARCH**	**SV**	**GARCH-GJR**	**SV-L**	**GARCH-M**	**SV-M**
Canada	-86.15	-75.47	-84.38	-73.27	-70.94	-69.89
France	-92.86	-73.30	-93.74	-73.46	-71.45	-63.44
Germany	-63.83	-57.64	-63.54	-57.73	-57.48	-49.64
Italy	-109.75	-81.90	-110.05	-82.77	-93.22	-75.42
Japan	-96.48	-86.20	-94.96	-85.68	-89.97	-85.61
United Kingdom	-90.59	-63.38	-90.24	-64.15	-65.62	-63.73
United States	-87.21	-86.35	-87.11	-86.09	-82.56	-80.69
Australia	-93.07	-77.98	-92.67	-77.73	-75.93	-73.95
Austria	-69.75	-64.83	-69.58	-64.58	-60.11	-56.85
Belgium	-81.55	-80.42	-81.44	-79.92	-77.50	-75.31
Denmark	-88.95	-74.76	-89.21	-72.99	-70.65	-69.15
Finland	-96.81	-77.79	-97.04	-76.31	-75.11	-69.01
Luxembourg	-79.85	-76.44	-79.28	-76.68	-73.58	-67.08
Netherlands	-82.36	-75.89	-81.56	-75.77	-72.90	-73.21
New Zealand	-110.51	-89.74	-108.90	-90.41	-88.62	-84.02
Norway	-94.08	-78.50	-92.14	-77.52	-81.89	-76.92
Sweden	-100.61	-82.27	-98.15	-80.01	-76.91	-71.73
Switzerland	-84.55	-76.37	-82.83	-78.70	-81.51	-68.79

Notes: The period spans from 1961Q1 to 2018Q4 for all countries except for Denmark (1967Q1 to 2018Q4). The evaluation period is from 2009Q1 to 2018Q4.

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Chapter 3

Monetary Policy in Practice: Do Central Banks Respond to Movements in Exchange Rate and Credit Growth?

with Nguyen Minh Phuong[a]

3.1 Introduction

Crisis after crisis, some factors that are the sources of the crises have been gaining a lot of attention. For example, in the aftermath of the Asian financial crisis in 1997, a hot debated topic was whether monetary policy should support exchange rate stability or let it be determined freely by the market. Some countries, such as Thailand, floated their exchange rates and did not manipulate their foreign exchange markets as a part of monetary policy anymore. For some others, such as Vietnam, the exchange rate is still in the set of policies controlled by central banks. Another example is that since the recent global financial crisis in 2008, the role of central banks in protecting financial markets from periods of excessively high credit growth has been gaining popularity. This leads to a renewed interest in the leaning against the wind policy, which emphasizes the importance of credit conditions. It is agreed that financial markets have an important influence on business cycle fluctuations and that credit growth should be controlled by macroprudential policies (leaning against the wind policy) such that it does not create too much risk in the economy (see, e.g., Melina and Villa, 2018). The remaining debate is whether the central bank or another independent entity should control these policies.

A question arising from these crises is whether central banks should control credit and/or exchange rates, and in which cases they would create the higher welfare for the economy. It

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is also worth noting that compared with the popular inflation-targeting framework, it might be the case that it is better for the economy if central banks also care about credit and/or exchange rates. To address this question, this paper constructs a dynamic stochastic general equilibrium (DSGE) model, which has become the workhorse in macroeconomics, and extends it to include credit and small open economy features. We estimate this model using the full-information likelihood-based multivariate approach and data from Thailand.

Our small open economy framework follows closely the setting of Galí and Monacelli (2016) with a few modifications. First, instead of assuming that the international financial market is complete to close the open economy model, we relax this strict assumption and allow for an incomplete financial market. To close our open model and induce stationarity, we employ the debt elastic interest rate as in Schmitt-Grohé and Uribe (2003). Second, we follow Gambacorta and Signoretti (2014) and incorporate financial frictions in our model in order to investigate the relevance of credit growth in implementing monetary policy. Lastly, we introduce the incomplete exchange rate pass-through in our model given the crucial roles of this feature in the small open economy framework (see, e.g., Monacelli, 2005; Dong, 2013).

The dynamic general equilibrium framework takes into consideration the fact that prices, credit growth, and exchange rates are simultaneously determined. As a result, this approach enables us to overcome the endogenity problems that often emerge from a single-equation estimation. Furthermore, the full-information likelihood-based multivariate approach allows us to utilize the cross-equation restrictions that associate the decision rules of economic agents to the parameters of the policy reaction function (see, e.g., Lubik and Schorfheide, 2007). Accordingly, we choose the prior densities for policy function variants and the remaining structural parameters and estimate the model using the Bayesian method. The posterior distributions and log marginal likelihoods are employed to evaluate the appropriateness of different monetary policy specifications. Although this approach has been used extensively to address several economic questions, to the best of our knowledge, it has not been used to investigate the importance of credit growth and exchange rate movements in designing the monetary policy rule. Our study is the first to do so. We apply our estimation strategy to investigate how the Bank of Thailand has constructed its policy rules. More specifically, we examine if the policy rate reacts to fluctuations in exchange rates and credit growth.[1]

[1]Thailand floated their exchange rate after the 1997 Asian financial crisis and adopted a flexible inflation targeting regime in 2000. Under this regime, the Bank of Thailand formulates their policies not only to ensure price stability but also to preserve economic growth and financial stability. Furthermore, as a typical developing country, the trade openness of Thailand is relatively large; thus, the movements in the international relative prices

Our analyses indicate that the Bank of Thailand has adjusted policy interest rates in response to exchange rate movements. The introduction of credit growth in the policy rule, however, is not important for the monetary authorities of Thailand. The findings are robust to various specifications of the monetary policy rules. Furthermore, we demonstrate that domestic shocks contribute remarkably to the business cycles. Terms of trade disturbance, despite having a minor role in most macroeconomic variables, explains the largest proportion of exchange rate movement, followed by country risk premium shocks.

Our study is related to several strands of literature. First, the relevance of exchange rates in designing monetary policy has been explored in previous literature. Clarida et al. (1998), using a univariate framework to estimate the policy functions for developed countries, demonstrate that policymakers in Japan and some developed countries react to exchange rate misalignments. Similar findings for emerging economies are obtained in the study of Calvo and Reinhart (2002), who show that monetary authorities employ policy rates to smooth the movements in exchange rates. Recent studies investigating the importance of exchange rates, however, employ a multivariate approach to address the possible endogenous relationships between the exchange rate and the policy interest rate. As an example, Bjørnland and Halvorsen (2014), employing a structural vector autoregressive (VAR) model that is identified by a combination of zero and sign restrictions, investigate how policy rates react to exchange rate fluctuations in six open economies. They demonstrate that there is an instantaneous reaction in the policy rate following an exchange rate shock in four countries. Lubik and Schorfheide (2007) and Dong (2013), using a dynamic stochastic general equilibrium (DSGE) framework, examine the relevance of exchange rates in constructing policy reaction functions.

Our study is also related to the literature that examines the importance of credit growth in the implementation of monetary policy. Gambacorta and Signoretti (2014) address this issue using the DSGE framework. They discover that the leaning against the wind policy is favorable in the case of supply side shocks, whereas the standard Taylor rule and inflation targeting are less effective. Along the same line, Melina and Villa (2018) employ the DSGE model with banking to investigate the role of credit growth in constructing the monetary policy for the United States. They indicate that during the Great Moderation, policy rates respond to the fluctuations in credit growth, and this finding remains unchanged regardless of various alternative specifications of policy rules employed.

might have significant impacts on the domestic business cycle. Accordingly, the Bank of Thailand may explicitly respond to exchange rate fluctuations to lessen the effects of international price changes.

The remainder of this study is organized as follows. Section 2 presents the structural small open economy model. The estimated strategy is reported in Section 3. In Section 4, we present the estimated results. Finally, Section 5 concludes.

3.2 Model

This section presents the small open economy DSGE model. The economy is inhabited by a representative household; a wholesale good producer; domestic retailers; importers; and a representative bank. The representative household, populated by a continuum of members, chooses consumption, labor supply, and deposits to maximize its utility subject to the budget constraint. The wholesale good producer borrows from banks to finance both wage bills and investment and produces a homogeneous output. The domestic retailers then differentiate the output at no cost and resell it in a monopolistically competitive market. The importers also operate in a monopolistic competition, importing differentiated goods from the world economy and selling them in the home economy. The representative bank consists of two units: a wholesale branch and a retail branch. The wholesale branch collects deposits from the household, borrows foreign debt, and issues wholesale loans to the retail branch. The retailer branch buys wholesale loans from the wholesale branch and lends them to the domestic wholesale goods producer.

3.2.1 Households

The utility function of a representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\psi} N_t^{1+\psi} \right) Z_t,$$

where $\beta \in (0,1)$ is the discount factor; σ and ψ denote the inverse of the intertemporal elasticity of consumption and the inverse of the Frisch elasticity of labor supply N_t, respectively; and Z_t is the preference shock and is assumed to follow an AR(1) process. Consumption C_t is a composite index and has the constant elasticity of substitution form as follows:

$$C_t = \left[(1-v)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + v^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{3.1}$$

Here, parameter $\eta > 0$ measures the intratemporal elasticity of substitution between domestic and imported goods. $v \in (0, 1)$ denotes the share of imported goods in the consumption basket of the home country, representing the degree of trade openness of the economy. $C_{H,t}$ is the domestic final good consumption index, provided by domestic retailers. $C_{F,t}$ is the imported good consumption index, imported by the importers. $C_{H,t}$ and $C_{F,t}$ have the following forms, respectively:[2]

$$C_{H,t} = \left[\int_0^1 (C_{Hj,t})^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad C_{F,t} = \left[\int_0^1 (C_{Fj,t})^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$ denotes the elasticity of substitution across final goods within each category of domestic or foreign goods.

In each period t, a representative household faces two optimization problems: an optimal allocation of goods and a utility maximization. First, the optimal allocation of expenditures between domestic and imported goods implies

$$C_{H,t} = (1-v) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad C_{F,t} = v \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \tag{3.2}$$

where $P_t = [(1-v)P_{H,t}^{1-\eta} + vP_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ denotes the consumer price index or CPI. $P_{H,t} = (\int_0^1 P_{Hj,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$ and $P_{F,t} = (\int_0^1 P_{Fj,t}^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$, in turn, are the price indexes of domestic and imported final goods, respectively, both expressed in the home currency.

Second, the household, taking interest rates, wage, consumer price index, and profits as given, chooses its consumption C_t, labor supply N_t, and deposit B_t to maximize its utility subject to the budget constraint. The optimization problem can be characterized as follows:

$$\max_{C_t, N_t, B_t} E_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{1}{1+\psi} N_{t+s}^{1+\psi} \right] Z_{t+s}, \tag{3.3}$$

subject to

$$P_t C_t + B_t \leq W_t N_t + (1 + r_{t-1}^d) B_{t-1} + \Pi_t. \tag{3.4}$$

Here, we assume that the household invests its savings as bank deposit B_t in nominal terms that pay the nominal interest rate r_t^d, which equalizes to the central bank's policy rate; earns the nominal wage W_t from supplying labor; and receives nominal profit income Π_t from ownership of wholesale good producer, retailers, and importers. Equation (3.3) represents the utility

[2] Here, we assume that all goods are tradable, as in Galí and Monacelli (2016).

maximization and Equation (3.4) is the budget constraint. The first-order conditions yield

$$C_t^{-\sigma} = E_t \left[\beta C_{t+1}^{-\sigma} (1 + r_t^d) \frac{P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t} \right], \tag{3.5}$$

$$N_t^{\psi} = \frac{W_t}{P_t} C_t^{-\sigma}. \tag{3.6}$$

Equation (3.5) is the Euler equation, which states that the marginal utility of consuming one unit of consumption today must be equal to the discounted marginal utility of saving today and consuming one unit of consumption tomorrow adjusted for inflation. Equation (3.6) is the labor supply equation, which equalizes the marginal rate of substitution between leisure and consumption with real wage.

3.2.2 Wholesale Good Producer

A representative wholesale good producer is assumed to operate in a perfectly competitive, borrowing from the bank, employing capital K_t, labor N_t, and exogenous stochastic aggregate productivity shock A_t to produce a homogeneous wholesale output. The production function is assumed to take the standard Cobb-Douglas form as follows:

$$Y_t^w = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{3.7}$$

where α denotes the capital share of income and Y_t^w is the wholesale output. The wholesale producer chooses its capital K_{t+1}, investment I_t, employment N_t, and bank's loan L_t to maximize the present value of future expected nominal net cash flows, taking the wage W_t, nominal borrowing rate r_{t-1}^l, and wholesale price P_t^w as given. The problem of the wholesale good firm can be summarized as follows:

$$\max_{K_{t+1}, I_t, N_t, L_t} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} [P_{t+s}^w Y_{t+s}^w - (1 + r_{t-1+s}^l) L_{t-1+s}], \tag{3.8}$$

subject to

$$K_{t+1} = (1 - \delta) K_t + I_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right], \tag{3.9}$$

$$L_t = W_t N_t + P_{H,t} I_t, \tag{3.10}$$

where r^l_{t-1} is the nominal borrowing rate. Equation (3.8) represents the sum of discounted nominal net cash flows. Since the producer is owned by households, the discount factor is given by $\Lambda_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+s}} \frac{Z_{t+1}}{Z_t}\right)$. Equation (3.9) is the standard evolution of capital, with δ being the depreciation rate. Following Smets and Wouters (2007), the investment adjustment cost is introduced to dampen the volatility of investment over the business cycle.[3] More specifically, the investment adjustment cost is assumed to take the quadratic form: $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, where $\gamma > 0$ captures the elasticity of the investment adjustment cost to movements in investment. The financial constraint (3.10) demonstrates that the firm needs to borrow external funds from banks to finance both its wage bill and investment expenditures. In the absence of this constraint, it is always optimal for firms to finance their needs through internal resources. The first-order conditions for the above problem lead to the following results:

$$(1-\alpha)P^w_t A_t K^\alpha_t N^{-\alpha}_t = W_t E_t[\Lambda_{t,t+1}(1+r^l_t)], \tag{3.11}$$

$$Q_t = E_t\Lambda_{t,t+1}[\alpha P^w_{t+1} A_{t+1} K^{\alpha-1}_{t+1} N^{1-\alpha}_{t+1} + Q_{t+1}(1-\delta)], \tag{3.12}$$

$$\begin{aligned} P_{H,t}E_t[\Lambda_{t,t+1}(1+r^l_t)] = Q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)\right] \\ + E_t\Lambda_{t,t+1}\left[Q_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]. \end{aligned} \tag{3.13}$$

Q_t is the Lagrange multiplier associated with the capital law of motion and can be interpreted as the shadow price of installed capital or Tobin's margin Q. The shadow price of capital includes the contribution from the adjustment costs due to new capital stock and the effect of depreciation. Equation (3.11) is the labor input function, equalizing the nominal marginal product of labor with the nominal marginal cost of labor. The latter, in turn, depends on the nominal wage and the borrowing rate. Equation (3.12) equates the marginal cost of acquiring one extra unit of installed capital with the expected sum of marginal product of capital and marginal savings arising from (1-δ) fraction of capital not having to borrow. Finally, equation (3.13) shows that the marginal cost of investment is equal to its marginal benefit.

[3] Analogue to consumption, we assume that investment I_t is a composite index of differentiated goods, i.e $I_t = \left(\int_0^1 I_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$.

3.2.3 Domestic Retailers

To introduce price stickiness, we allow for monopolistic competition to occur at the retail level, as in Bernanke et al. (1999). A continuum of firms, indexed by $j \in [0, 1]$, buys the homogeneous wholesale goods from the domestic producer at the wholesale price P_t^w in a competitive market, differentiates these goods at no cost, and sells them in a monopolistically competitive market at the price $P_{Hj,t}$. The total domestic final good is therefore a composite of individual retail goods:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $Y_{j,t}$ denotes the output of retailer j and Y_t indicates the total final goods. Given the above composite that integrates individual retail goods into final goods, the demand for retailer j's goods is as follows:

$$Y_{j,t} = \left(\frac{P_{Hj,t}}{P_{H,t}} \right)^{-\epsilon} Y_t.$$

For simplicity, the export price of the domestic final good $P^*_{Hj,t}$ is assumed to be flexible and determined by the law of one price. Thus, the corresponding price index is given by

$$P_{H,t} = \left(\int_0^1 P_{Hj,t}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

To introduce the sticky price, we use a Calvo pricing setting with θ_H being the degree of price stickiness. Thus, the probability that the price set at time t will still hold at time $t + s$ is θ_H^s. The retailer then chooses the selling price $P_{Hj,t}$, taking as given the demand curve and the price of the wholesale goods P_t^w, to maximize the expected discounted profits. The problem of retailers can be summarized as follows:

$$\max_{P_{Hj,t}} \sum_{s=0}^{\infty} E_t[\theta_H^s \Lambda_{t,t+s}(\bar{P}_{H,t} - P_{t+s}^w)Y_{j,t+s}], \tag{3.14}$$

subject to

$$Y_{j,t} = \left(\frac{\bar{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} Y_t, \tag{3.15}$$

where $\Lambda_{t,t+s}$ denotes a relevant stochastic discount factor for retailers to discount their future profits and defined as above. Equation (3.14) represents the sum of expected discounted profits of retailers, while equation (3.15) is the demand for goods of retailer j. The first order condition with respect to $\bar{P}_{H,t}$ yields the following:

$$\bar{P}_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (\frac{C_{t+s}}{C_t})^{-\sigma} (\frac{P_t}{P_{t+s}}) (\frac{Z_{t+s}}{Z_t}) (P_{H,t+s})^{\epsilon+1} Y_{t+s} mc_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (\frac{C_{t+s}}{C_t})^{-\sigma} (\frac{P_t}{P_{t+s}}) (\frac{Z_{t+s}}{Z_t}) (P_{H,t+s})^{\epsilon} Y_{t+s}}, \tag{3.16}$$

where $mc_{t+s} = \frac{P^w_{t+s}}{P_{H,t+s}}$ denotes the real marginal cost of domestic retailer j in terms of final goods price at period $t + s$; $\frac{\epsilon}{\epsilon-1} > 1$ is the markup earned by retailers. Equation (3.16) demonstrates that the retailer's re-optimized price is set so that, in expectation, the discounted sum of marginal revenue is equal to the discounted sum of nominal cost, given the probability that the price set at time t is fixed at time $t + s$ is θ_H^s. Furthermore, due to price stickiness, the optimal price is purely forward-looking, which is the weighted average of future nominal marginal costs. It is well-known that with this kind of Calvo price setting, all domestic retailers who can reset their prices at time t choose the same price. Therefore, the aggregate price index of final domestic goods evolves according to

$$P_{H,t} = [\theta_H P_{H,t-1}^{1-\epsilon} + (1 - \theta_H)(\bar{P}_{H,t})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}. \tag{3.17}$$

3.2.4 Importers and Incomplete Exchange Rate Pass-through

The setup here, which closely follows Monacelli (2005), features an incomplete exchange rate pass-through and allows the deviations from the law of one price to be gradual and persistent. The incomplete exchange rate pass-through is induced by the price setting of the importers according to the Calvo (1983) pricing rule. The price stickiness in the import sector creates the difference between the world price and the domestic price of imported foreign goods. As a result, there is a deviation from the law of one price in the short run.

The domestic market is populated by monopolistically competitive local retailers, indexed by $j \in (0, 1)$, who import differentiated goods from the rest of the world at a cost $\mathcal{E}_t P^*_{Fj,t}$, where $\mathcal{E}_t$ and $P^*_{Fj,t}$ denote the exchange rate and the foreign-currency price of the imported good j, respectively. The optimization problem of importers can be written as follows:

$$\max_{P_{Fj,t}} \sum_{s=0}^{\infty} \Lambda_{t,t+s} \theta_F^s [\bar{P}_{F,t} - \mathcal{E}_{t+s} P^*_{Fj,t+s}] C_{Fj,t+s}, \tag{3.18}$$

subject to

$$C_{Fj,t+s} = \left(\frac{\bar{P}_{F,t}}{P_{F,t+s}} \right)^{-\epsilon} C_{F,t+s}, \tag{3.19}$$

where θ_F denotes the degree of price stickiness; $\Lambda_{t,t+s}$, defined as above, denotes a relevant stochastic discount factor for importers to discount their future profits. Equation (3.18) represents the sum of expected discounted profits of importers, while equation (3.19) is the demand function. The first order condition with respect to $\bar{P}_{F,t}$ yields the following:

$$\bar{P}_{F,t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\theta_F)^s (\frac{C_{t+s}}{C_t})^{-\sigma} (\frac{P_t}{P_{t+s}}) (\frac{Z_{t+s}}{Z_t}) (P_{F,t+s})^{\epsilon+1} C_{F,t+s} Lg_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\theta_F)^s (\frac{C_{t+s}}{C_t})^{-\sigma} (\frac{P_t}{P_{t+s}}) (\frac{Z_{t+s}}{Z_t}) (P_{F,t+s})^{\epsilon} C_{F,t+s}}, \tag{3.20}$$

where $Lg_{t+s} = \frac{\mathcal{E}_{t+s} P^*_{F,t+s}}{P_{F,t+s}}$ denotes the real marginal cost of importers in terms of the final imported good price. Note that this measure does not depend on j, so all importers have the same marginal cost. Therefore, the aggregate import price index is

$$P_{F,t} = [\theta_F P_{F,t-1}^{1-\epsilon} + (1 - \theta_F)(\bar{P}_{F,t})^{1-\epsilon}]^{\frac{1}{1-\epsilon}}. \tag{3.21}$$

Let us define the terms of trade tot_t as $tot_t = \frac{P_{F,t}}{P_{H,t}}$. Following Uribe and Schmitt-Grohé (2017), we let the log-linear form of the terms of trade follow an AR(1) process:

$$\widehat{tot_t} = \rho_{tot} \widehat{tot_{t-1}} + \sigma_{tot} \varepsilon_{tot,t}, \tag{3.22}$$

where $\widehat{tot_t}$ denotes the log-linear form of the terms of trade; ρ_{tot} captures a persistence of the shock; σ_{tot} denotes a standard deviation; and $\varepsilon_{tot,t}$ is an innovation to terms of trade. Equation (3.22) demonstrates the evolution of the terms of trade shock. Note also that this specification allows us to investigate the impacts of the terms of trade shock on the business cycle of Thailand. This interesting question has received increasing attention in recent studies on the small open economies (see, e.g., Schmitt-Grohé and Uribe, 2018). In addition, the specification of terms of trade shock helps overcome the issue of the singularity in the model.

3.2.5 Banks

The representative bank, which is composed of a wholesale branch and a retail branch, is included in this model to analyze the role of banks in transmitting the impact of credit policy by the central bank on the economy.

3.2.5.1 Wholesale Branch

The wholesale branch operates under perfect competition. It collects deposits B_t from the household at the nominal interest rate r_t^d set by the central bank, borrows foreign debt, and issues wholesale loans L_t to the retail branch at the wholesale loan rate r_t^w. Moreover, the bank has its own capital K_t^b, which is accumulated through the retained earnings according to

$$K_t^b = (1 - \delta_b)K_{t-1}^b + \Omega J_{t-1}^b, \tag{3.23}$$

where J_t^b is the aggregate profit of the bank, including both the wholesale and retail branches; δ_b captures the costs involved in managing bank capital; and Ω denotes the share of profits used to increase bank capital.[4] This δ_b fraction is set at a value to ensure that the bank meets its target capital-to-loan ratio ν_b in the steady state. The target capital-to-loan ratio is exogenously given, and the bank has to pay a quadratic cost of the form $\frac{\kappa}{2}(\frac{K_t^b}{L_t} - \nu_b)^2 K_t^b$ if it adjusts its capital-to-loan ratio from this target value (see, e.g., Gerali et al., 2010). The cost is proportional to K_t^b and parameterized by κ.

Different from the assumption of complete international asset markets as in Gali and Monacelli (2005), we employ the debt elastic interest rate to close our open economy model and induce stationarity. Accordingly, we assume that the wholesale branch can borrow nominal external debt, denominated in foreign currency and denoted by D_t, from the rest of the world at the interest rate r_t^f. Specifically, r_t^f is increasing in the level of debt and is given by

$$r_t^f = r_t^* + p(D_t), \tag{3.24}$$

where r_t^* denotes the world interest rate and is assumed to follow an AR(1) process and $p(.)$ a country-specific interest rate premium. The function $p(.)$ is supposed to be strictly increasing and convex, i.e., the interest rate at which the wholesale branch borrows from abroad is an increasing and convex function of the level of its external debt. We use the following function form for the country-specific interest rate premium as in Uribe and Schmitt-Grohé (2017):

$$p(D_t) = \Psi \left(e^{\frac{D_t}{P_t^*} - \frac{\bar{D}}{P^*}} - 1 \right) + e^{\Theta_t} - 1,$$

where P_t^* is the price level of the world economy, $\bar{D}$ and P^* are the steady state values of D_t

[4]For simplicity, we assume that the bank reinvests all its profits into new bank capital, i.e., $\Omega = 1$.

and P_t^*, respectively. The parameter Ψ determines the elasticity of the domestic interest rate to changes in the level of foreign debt. Θ_t denotes the country spread or country risk premium and is assumed to follow an AR(1) process.

In each period t, the bank chooses the amounts of deposit B_t, foreign debt D_t, and loan L_t to maximize its expected discounted profits subject to a balance-sheet constraint, taking the net interest rates and exchange rate as given. The optimization problem of the wholesale branch can be written as follows:

$$\max_{L_t,B_t,D_t} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[(1+r^w_{t-1+s})L_{t-1+s} - (1+r^d_{t-1+s})B_{t-1+s} \right. \tag{3.25}$$

$$\left. -(1+r^f_{t-1+s})D_{t-1+s}\mathcal{E}_{t+s} - \frac{\kappa}{2}\left(\frac{K^b_{t+s}}{L_{t+s}} - \nu_b\right)^2 K^b_{t+s} \right],$$

subject to

$$L_t = B_t + D_t\mathcal{E}_t + K^b_{t+s}, \tag{3.26}$$

where $\Lambda_{t,t+s}$ denotes the stochastic discount factor for the wholesale branch and is defined as above. Equation (3.25) represents the sum of expected discounted profits of the wholesale branch, while equation (3.26) is the balance sheet of the branch. The first-order condition for the above problem yields

$$1 + r^d_t = \left(1 + r^f_t + \frac{D_t}{P^*_t}\Psi e^{\frac{D_t}{P^*_t} - \frac{\bar{D}}{P^*}}\right) E_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right), \tag{3.27}$$

$$\Lambda_{t,t+1}[r^d_t - r^w_t] = \kappa\left(\frac{K^b_t}{L_t} - \nu_b\right)\left(\frac{K^b_t}{L_t}\right)^2. \tag{3.28}$$

Equation (3.27) is the uncovered interest rate parity condition, which shows the relationship among domestic interest rate, foreign interest rate, and expected change in exchange rates. Equation (3.28) shows the relationship between the wholesale loan interest rate and the deposit rate. In the steady state, the wholesale loan rate is equal to the deposit rate.

3.2.5.2 Retail Branch

The retail branch operates in a monopolistically competitive environment. It purchases wholesale loans from the wholesale branch, differentiates them at no cost, and lends them to the domestic wholesale good producer. The retail branch can fix the loan rate r^l_t by applying a constant markup μ_b to the wholesale loan rate, i.e., $r^l_t = r^w_t + \mu_b$.

3.2.6 Monetary Policy Rule

The central bank sets its nominal interest rates according to a Taylor-type reaction function, depending on inflation and output gap:

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right)\right] + \varepsilon_{r,t}, \tag{3.29}$$

where r^d and Y are the steady state values of the nominal policy interest rate and output, respectively. ρ_r, ϕ_π, and ϕ_y are policy parameters referring to interest rate smoothing and the responsiveness of the nominal interest rate to the inflation rate and the output gap, respectively. π_t, defined as $\pi_t = \frac{P_t - P_{t-1}}{P_t}$, is the inflation rate, and π is the target rate of inflation. $\varepsilon_{r,t}$ is the monetary shock, which can be interpreted as any error or contingent event underlying the central bank's control over the policy instrument.

In order to investigate the importance of the exchange rate and credit growth in the policy reaction function, we check the hypothesis of the standard Taylor rule, in which monetary policy reacts to fluctuations in neither the exchange rate nor credit growth, against the following three alternative policies:

Exchange rate augmented Taylor rule

$$\begin{aligned}\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\Bigg[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) \\ +\phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right)\Bigg] + \varepsilon_{r,t}.\end{aligned} \tag{3.30}$$

Credit growth augmented Taylor rule

$$\begin{aligned}\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\Bigg[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) \\ +\phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_l \log\left(\frac{l_t}{l_{t-1}}\frac{\pi_t}{\pi}\right)\Bigg] + \varepsilon_{r,t}.\end{aligned} \tag{3.31}$$

Generalized Taylor rule

$$\begin{aligned}\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\Bigg[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) \\ +\phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right) + \phi_l \log\left(\frac{l_t}{l_{t-1}}\frac{\pi_t}{\pi}\right)\Bigg] + \varepsilon_{r,t}.\end{aligned} \tag{3.32}$$

First, in the exchange rate augmented Taylor rule, in addition to inflation and output, we let the policy rate also respond to the exchange rate gap to smooth the impact of exchange rate volatility. It is worth noting that ex_t, defined as $ex_t = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$, measures changes in the exchange rate while ex denotes its corresponding steady state. Therefore, the parameter ϕ_e captures the reaction of the policy rate to exchange rate fluctuations. Second, the credit growth augmented Taylor rule allows us to test the relevance of credit growth when designing monetary policy. The variable l_t, defined as $l_t = \frac{L_t}{P_t}$, is the real lending. Accordingly, the parameter ϕ_l denotes the responsiveness of the nominal interest rate to movements in credit growth. Finally, we consider the generalized Taylor rule, in which we let the policy rate react to inflation, output gap, exchange rate movements, and credit growth fluctuations.

3.2.7 Good Market Clearing

Domestic goods are used for domestic consumption, investment, and exports. Therefore, the good market clearing is as follows:

$$Y_t = C_{H,t} + I_t + C^*_{H,t},$$

where $C^*_{H,t}$ is the foreign demand for domestically produced goods and is defined as follows:

$$C^*_{H,t} = v\left(\frac{P^*_{H,t}}{P^*_t}\right) C^*_t.$$

Since the size of the open economy is small enough compared to the world economy, we can neglect its impact on the world economy. Therefore, the price level of the world economy equalizes the price of foreign products, i.e., $P^*_t = P^*_{F,t}$. The foreign demand can thus be rewritten as follows:

$$C^*_{H,t} = v\left(\frac{P_{F,t}}{P_{H,t}} Lg_t\right)^{\eta} C^*_t.$$

Employing the world economy's market clearing condition $C^*_t = Y^*_t$, we have

$$Y_t = C_{H,t} + I_t + v\left(\frac{P_{F,t}}{P_{H,t}} Lg_t\right)^{\eta} Y^*_t,$$

where Y^*_t denotes the foreign output and is assumed to follow an AR(1) process.

3.2.8 Exogenous Shocks

There are seven structural shocks in our model: preference, technology, terms of trade, foreign interest rate, foreign demand, country risk premium, and monetary shocks. The first six shocks follow AR(1) processes as follows:

$$\log Z_t = \rho_z \log Z_{t-1} + \sigma_z \varepsilon_{z,t}, \tag{3.33}$$

$$\log A_t = \rho_a \log A_{t-1} + \sigma_a \varepsilon_{a,t}, \tag{3.34}$$

$$\widehat{tot_t} = \rho_{tot} \widehat{tot_{t-1}} + \sigma_{tot} \varepsilon_{tot,t} \tag{3.35}$$

$$r_t^* = r^* + \rho_{r^*}(r_{t-1}^* - r^*) + \sigma_{r^*} \varepsilon_{r^*,t}, \tag{3.36}$$

$$\log Y_t^* = \rho_y \log Y_{t-1}^* + \sigma_y \varepsilon_{y,t}, \tag{3.37}$$

$$\log \Theta_t = \rho_{risk} \log \Theta_{t-1} + \sigma_{risk} \varepsilon_{risk,t}. \tag{3.38}$$

The parameters ρ_x with $x \in \{z, a, tot, r^*, y, risk\}$ capture the persistence of the shocks, while the parameters $\sigma_{x,t}$ denote the standard deviations. The remaining monetary shock $\varepsilon_{r,t}$ is assumed to follow a white noise process. Note also that it is conventional to model the monetary policy shock as a white noise process (see, e.g., Smets and Wouters, 2003; Adolfson et al., 2007; Ireland, 2011).

3.3 Calibration and Estimation Strategy

We first report the calibration strategy for some structural parameters and later estimate our model using the data from Thailand.[5]

3.3.1 Data

We employ quarterly data from 2000Q3 to 2019Q4 on the following seven macroeconomic series for the estimation: GDP, CPI inflation rate, short-term nominal interest rate, commodity terms of trade, nominal exchange rate, U.S. interest rate, and U.S. GDP. All data series are first seasonally adjusted by the X-13-ARIMA SEAT (autoregressive integrated moving average, seasonal extraction in ARIMA time series) method developed by the U.S. Census Bureau and

[5]Note that our model is first linearized and then estimated using DYNARE (Adjemian et al., 2011).

Table 3.1: Calibrated parameters

Parameter	Value	Description
β	0.99	Discount factor
ϵ	6	Elasticity of substitution across final goods
α	0.32	Capital share in production
δ	0.025	Depreciation rate of capital
δ_b	Implied value	Cost involving in managing bank capital
ν_b	0.09	Target capital-to-loan ratio in steady state
μ_b	0.0067	Bank markup
r^*	0.005	World interest rate
Ψ	0.001	Debt elasticity interest rate
π	0.005	Steady state gross inflation target

then detrended using the one-sided HP filter. Data are obtained from the International Financial Statistics of the International Monetary Fund.

3.3.2 Calibration

In this section we show the calibration strategy. Table 3.1 presents the calibrated values.

The time unit in the model corresponds to one quarter. Based on parameter values commonly used in much of the related business-cycle literature, the discount factor β is set equal to 0.99, and the capital share α in the production function equal to 0.32. In addition, the depreciation rate δ equalizes to 0.025, which amounts to an annual depreciation of 10 percent. Following standard literature, the parameter ϵ capturing the elasticity of substitution across different varieties of goods is set at 6 in order to target a steady state gross mark-up of 1.2.

Regarding the banking block of the model, the constant bank markup μ_b that the retail branch charges upon the wholesale lending rate is calibrated to target the net interest margin of 2.68% (average over the periods 2000-2020). Accordingly, μ_b is set at 0.0067. Following Gerali et al. (2010), we set the target capital-to-loan ratio ν_b equal to 0.09. And finally, the steady state conditions of the banking sector imply $\delta_b = \frac{\mu_b + r^d \nu_b}{\nu_b} - \pi$. Thus, the cost of managing bank capital δ_b is set at 0.0908 initially.

The steady state of the world interest rate r^* is calibrated to match the average U.S. interest rate. Accordingly, r^* is set at 0.005. Following Schmitt-Grohé and Uribe (2003), we assign a relatively small value of 0.001 for the debt elastic interest rate parameter Ψ. The steady state of inflation rate is calibrated to match the inflation targeting of 2% in Thailand. Thus, π is set at 0.005.

3.3.3 Choice of Prior

The structural model is estimated using Bayesian methods. The prior distributions for the structural parameters of the model are chosen mainly based on past literature. We use three types of prior distributions. They are the gamma, inverse-gamma, and beta distributions. Accordingly, the gamma distributions are used for the intertemporal elasticity of substitution, capital adjustment cost, bank capital adjustment cost, and policy parameters in the Taylor rule. Meanwhile, the inverse-gamma distributions are employed for the intratemporal elasticity of substitution and the standard deviations of seven structural innovations. Finally, the beta distributions are applied to the curvature of disutility from labor, the degree of openness, Calvo price indexes, interest rate smoothing, and the persistence of shocks. An overview of the prior densities is provided in the third column of Table 3.2. Further details are discussed below.

We employ the gamma distribution with a mean of 1.5 and a standard deviation of 0.4 for the intertemporal elasticity of substitution following the studies of Havranek et al. (2015). The prior for the degree of openness follows the beta distribution with a mean of 0.50 and a standard deviation of 0.05. This assumption is quite reasonable because Thailand's economy is relatively open. Following Christiano et al. (2011), we use the gamma distribution for the capital adjustment cost. Similarly, a gamma distribution is applied to the bank's capital adjustment cost. However, we set a larger prior mean of 11 for this adjustment cost based on the studies of Gambacorta and Signoretti (2014).

Standard literature on the real business cycle often demonstrates that prices change every four months. Accordingly, we employ the beta distribution with a mean of 0.75 and a standard deviation of 0.075 as the prior density for both Calvo indexes θ_H and θ_F. This prior setting is based on Christiano et al. (2011). Since there is a high proportion of people under 40 years old in Thailand, we believe that the inverse of the Frisch elasticity of labor supply ψ would be quite small. Therefore, we use the beta distribution with a mean of 0.5 and a deviation of 0.1 as the prior density for the inverse of the Frisch elasticity of labor supply. Note also that a small magnitude of this parameter indicates that there are strong responses of labor supply to wage and macroeconomic data.

We use relatively loose priors for the parameters of the Taylor rule. Specifically, the prior for the interest rate smoothing ρ_r is set at 0.50 with a standard deviation of 0.10. We let ϕ_π and ϕ_y center at 1.5 and 0.5, respectively, two values widely associated with the Taylor rule. Similarly, we apply the gamma distribution with a mean of 0.5 for both ϕ_e and ϕ_l.

Regarding the stochastic processes, the prior for the persistence of shocks follows the beta distribution with a prior mean of 0.8 and a standard deviation of 0.075. For the magnitude of shock innovation, we use the inverse gamma density following standard literature.

3.4 Estimated Results

In this section, we start by reporting the estimated structural parameters for the small open economy model using Thai data. We then present results for the model comparison using the marginal likelihoods. Next, we examine the sources of business cycles in Thailand using two commonly used techniques: forecast error variance decomposition and impulse response functions. Finally, we report the robustness checks.

3.4.1 Estimated DSGE Model Parameters

Table 3.2 presents the posterior estimates for the structural parameters of the exchange rate augmented Taylor rule.[6] In addition to posterior means, we report the 90% credible intervals using the Metropolis Hastings algorithm. Specifically, we generate 400,000 draws, and the first 200,000 draws are discarded. Overall, the data appear really informative for most structural parameters as the posterior densities substantially differ from their prior distributions. In addition, the estimated results are largely consistent with the previous literature.

Regarding the posterior estimates of the Taylor rule, we find that the interest rate is highly persistent, with the interest rate smoothing being estimated at 0.94. The estimated value of the coefficient on the inflation term is 1.50, implying that the Bank of Thailand follows an adequately anti-inflationary policy and that the Taylor rule is satisfied (ϕ_π is greater than 1). Furthermore, the results indicate the Bank of Thailand's concern for fluctuations in both output ($\phi_y = 0.14$) and exchange rate ($\phi_e = 0.26$). It is also worth noting that the posterior estimate of ϕ_e falls within reasonable ranges. Lubik and Schorfheide (2007) show the posterior estimates of this parameter to be 0.07, 0.14, 0.04, and 0.13 for Australia, Canada, New Zealand, and the UK, respectively. Justiniano and Preston (2010) find slightly higher estimates at 0.29, 0.29, and 0.07 for Australia, Canada, and New Zealand, respectively.

The Calvo index for domestic price θ_H is estimated at 0.73, which demonstrates that domestic retailers, on average, reset their prices optimally every four quarters. The import price,

[6]Because of space constraints, we only display the estimated results for the exchange rate augmented Taylor rule. The results for the three remaining monetary policies are distributed in the Appendix.

Table 3.2: Prior and posterior distribution of estimated parameters, the exchange rate augmented Taylor rule

Parameter	Parameter description	Prior Dist	Mean	SD	Posterior Mean	90% Interval
σ	Intertemporal elasticity of substitution	Gamma	1.50	0.40	0.61	[0.30, 0.90]
ψ	Curvature of disutility from labor	Beta	0.50	0.10	0.53	[0.36, 0.69]
η	Intratemporal elasticity of substitution	IG	1.50	Inf	1.09	[0.74, 1.42]
v	Degree of openness	Beta	0.50	0.05	0.44	[0.38, 0.51]
γ	Capital adjustment cost	Gamma	2.58	0.50	3.18	[2.31, 4.03]
θ_H	Calvo index for domestic price	Beta	0.75	0.075	0.73	[0.68, 0.80]
θ_F	Calvo index for imported price	Beta	0.75	0.075	0.66	[0.59, 0.74]
κ	Bank capital adjustment cost	Gamma	11.00	5.00	11.61	[3.20, 19.59]
ρ_r	Interest rate smoothing	Beta	0.50	0.10	0.94	[0.93, 0.95]
ϕ_π	Taylor rule, inflation	Gamma	1.50	0.25	1.50	[1.17, 1.80]
ϕ_y	Taylor rule, output	Gamma	0.50	0.25	0.14	[0.06, 0.22]
ϕ_e	Taylor rule, exchange rate	Gamma	0.50	0.25	0.26	[0.13, 0.38]
ρ_y	Persistence, foreign demand shock	Beta	0.80	0.075	0.91	[0.87, 0.96]
ρ_a	Persistence, technology shock	Beta	0.80	0.075	0.54	[0.42, 0.65]
ρ_z	Persistence, preference shock	Beta	0.80	0.075	0.62	[0.54, 0.71]
ρ_{tot}	Persistence, terms of trade shock	Beta	0.80	0.075	0.57	[0.51, 0.62]
ρ_{r^*}	Persistence, world interest rate shock	Beta	0.80	0.075	0.88	[0.84, 0.92]
ρ_{risk}	Persistence, country risk premium	Beta	0.80	0.075	0.87	[0.80, 0.95]
$100\sigma_y$	SD, foreign demand shock	IG	0.15	Inf	0.51	[0.44, 0.57]
$100\sigma_a$	SD, technology shock	IG	0.15	Inf	3.67	[2.16, 5.26]
$100\sigma_z$	SD, preference shock	IG	0.15	Inf	3.54	[1.92, 5.04]
$100\sigma_r$	SD, monetary shock	IG	0.15	Inf	0.08	[0.06, 0.09]
$100\sigma_{tot}$	SD, terms of trade shock	IG	0.15	Inf	0.81	[0.66, 0.96]
$100\sigma_{r^*}$	SD, world interest rate shock	IG	0.15	Inf	0.07	[0.06, 0.08]
$100\sigma_{risk}$	SD, country risk premium	IG	0.15	Inf	0.19	[0.10, 0.26]

Notes: In this table, we report the posterior estimates for the exchange rate augmented Taylor rule. IG stands for the inverse-gamma distribution, SD is a standard deviation, and Dist is a distribution. The 90% credible intervals are displayed in square brackets.

on the other hand, is more flexible. Specifically, the posterior estimate of imported price stickiness θ_F is 0.66, demonstrating that importers reoptimize their prices every three quarters. This finding is in line with the standard literature. For instance, Christiano et al. (2011) find that the degree of price stickiness is substantially lower for importers than for domestic firms. Similar results can be found in Adolfson et al. (2007), Justiniano and Preston (2010).

The posterior estimates of the remaining structural parameters are largely in line with the previous literature. The parameter γ capturing the investment adjustment cost is estimated at 3.18, which falls within feasible ranges. Melina and Villa (2018), using the closed economy model, estimate this parameter to be 2.57 for the United States. Similarly, the posterior estimate of 2.58 for the Swedish economy is shown in the study of Christiano et al. (2011). Meanwhile,

Table 3.3: Log marginal likelihoods for four different monetary rules

Laplace approximation	Baseline $\phi_e = \phi_l = 0$	Generalized $\phi_e \& \phi_l$	Exchange rate ϕ_e	Credit growth ϕ_l
Log marginal likelihood	2077	2068	2080	2064
Bayes factor		1.23×10^{-4}	20.09	2.26×10^{-6}
Kass-Raftery statistics		-18	6	-26
Modified harmonic mean	**Baseline $\phi_e = \phi_l = 0$**	**Generalized $\phi_e \& \phi_l$**	**Exchange rate ϕ_e**	**Credit growth ϕ_l**
Log marginal likelihood	2078	2068	2081	2064
Bayes factor		4.54×10^{-5}	20.09	8.32×10^{-7}
Kass-Raftery statistics		-20	6	-28

we find a relatively large value of 11.61 for the bank capital adjustment cost, which is consistent with standard literature. Furthermore, the estimated results demonstrate a significant degree of persistence in the stochastic processes. More specifically, of seven shocks in our model, the foreign demand shock has the highest degree of autocorrelation ($\rho_y = 0.91$). Previous studies also document a substantial degree of persistence in the foreign demand shock (see, e.g., Gali and Monacelli, 2005; Lubik and Schorfheide, 2007; Le, 2021).

3.4.2 Model Comparison

Now, we address the main question whether the Bank of Thailand reacts to fluctuations in either the exchange rate or credit growth.[7] The log marginal likelihood, the Bayes factor, and the Kass-Raftery statistics are presented in Table 3.3. Here, we consider two approaches in order to compute the log marginal likelihood: (1) the Laplace approximation and (2) the modified harmonic mean estimator.

The relevance of the exchange rate

Regarding the Laplace approximation, the results demonstrate that the exchange rate augmented monetary policy outperforms the other three policies, which indicates the importance of exchange rate movements when setting the monetary policy. Specifically, the log marginal likelihood of the standard Taylor rule and the exchange rate augmented Taylor rule are, respectively, 2077 and 2080, implying a Bayes factor of 20.09. This indicates strong evidence in support of the exchange rate augmented monetary rule against the standard one. The Kass-Raftery statistic, calculated as twice the log of the Bayes factor, confirms this finding. With a

[7]We estimate the model using the same data for four alternative policy rules. Because of space constraints, we do not display the estimated results. The posterior estimates, impulse response functions, and variance decomposition are broadly similar to those presented in this paper.

Kass-Rattery statistic of 6, we also find strong evidence in favor of the exchange rate augmented rule. Similar results are obtained using the modified harmonic mean estimator approach.

The relevance of credit growth

The introduction of credit growth into the Taylor rule, however, leads to a worse model in terms of log marginal likelihood. As an example, let us consider the Laplace approximation. The log marginal likelihoods of the standard Taylor rule and the credit growth augmented Taylor rule are 2077 and 2064, respectively. This translates to a Bayes factor of 2.26×10^{-6} in favor of the credit growth augmented rule against the standard one, demonstrating overwhelming evidence for the latter model. This finding is also consistent with the Kass-Raftery statistic. The statistic of -26 provides decisive evidence in favor of the standard rule against the credit growth augmented rule. The abovementioned results are again confirmed when we employ the harmonic mean estimator approach. Thus, the Bank of Thailand has not adjusted policy rates in response to the fluctuations in credit growth.

The generalized Taylor rule

We further investigate the introduction of both credit growth and the exchange rate into the standard Taylor rule. Interestingly, we find that while the generalized Taylor rule outperforms the credit growth augmented rule, it is outperformed by the standard Taylor rule in terms of log marginal likelihood. For instance, let us consider the Laplace approximation. The Bayes factor of 1.23×10^{-4} in favor of the generalized monetary rule against the standard one provides an overwhelming decision in support of the latter. The Kass-Raftery statistic also confirms this finding. A reasonable explanation is that the inclusion of credit growth in the monetary rule results in a substantial decrease in the log likelihood, while there is a relatively small increase in the likelihood when the exchange rate movement is incorporated into the policy rule. Note also that we draw similar conclusions when the modified harmonic mean estimator is employed.

3.4.3 Forecast Error Variance Decomposition

To investigate the relevance of structural shocks to the aggregate fluctuations in the Thai economy, we calculate the forecast error variance decomposition. More specifically, we compute both unconditional and conditional variance decompositions. The results are displayed in Tables 3.4 and 3.5. Some broad overviews can be drawn from this exercise. Overall, the fluctuations in most macroeconomic variables are mainly driven by domestic shocks, namely, the preference shock and technology shock, which is consistent with standard literature. Second,

Table 3.4: Conditional variance decomposition, the exchange rate augmented Taylor rule

Quarter 1	**Output**	**Inflation**	**Policy rate**	**Lending rate**	**Exchange rate**
Preference	85.29	27.07	35.40	49.08	1.98
Technology	6.78	53.23	25.37	40.16	2.17
Foreign output	0.26	0.02	0.09	0.00	0.54
Policy	6.43	10.14	21.24	3.84	9.75
Foreign interest	0.11	0.61	2.09	0.81	4.71
Terms of trade	0.39	4.86	1.85	0.66	49.37
Country risk premium	0.75	4.06	13.97	5.45	31.48
Quarter 4	**Output**	**Inflation**	**Policy rate**	**Lending rate**	**Exchange rate**
Preference	62.31	25.39	46.97	50.85	2.04
Technology	29.17	50.15	24.94	33.10	2.27
Foreign output	0.34	0.06	0.09	0.03	0.54
Policy	6.87	13.52	8.16	2.60	9.68
Foreign interest	0.13	0.74	2.52	1.71	4.77
Terms of trade	0.28	5.22	0.60	0.34	48.74
Country risk premium	0.90	4.92	16.72	11.37	31.96
Quarter 8	**Output**	**Inflation**	**Policy rate**	**Lending rate**	**Exchange rate**
Preference	58.43	25.66	48.20	49.38	2.05
Technology	33.63	49.89	21.45	29.30	2.30
Foreign output	0.33	0.07	0.14	0.10	0.54
Policy	6.28	13.58	6.00	2.24	9.63
Foreign interest	0.13	0.74	3.13	2.46	4.81
Terms of trade	0.29	5.17	0.44	0.30	48.48
Country risk premium	0.91	4.88	20.64	16.23	32.19
Quarter 16	**Output**	**Inflation**	**Policy rate**	**Lending rate**	**Exchange rate**
Preference	58.21	25.69	48.39	48.96	2.05
Technology	33.98	49.85	19.42	27.04	2.30
Foreign output	0.32	0.08	0.24	0.20	0.54
Policy	6.14	13.56	5.23	2.02	9.61
Foreign interest	0.14	0.74	3.49	2.84	4.82
Terms of trade	0.30	5.18	0.42	0.29	48.38
Country risk premium	0.92	4.90	22.82	18.63	32.30
Quarter 32	**Output**	**Inflation**	**Policy rate**	**Lending rate**	**Exchange rate**
Preference	58.17	25.73	49.34	49.81	2.05
Technology	34.03	49.83	18.81	26.19	2.30
Foreign output	0.32	0.08	0.32	0.27	0.54
Policy	6.12	13.55	4.95	1.92	9.61
Foreign interest	0.14	0.74	3.47	2.85	4.82
Terms of trade	0.31	5.17	0.43	0.31	48.36
Country risk premium	0.92	4.90	22.68	18.63	32.31

Note: The values presented here are measured in percent. The table displays the conditional variance decomposition at different periods.

Table 3.5: Unconditional variance decomposition, the exchange rate augmented Taylor rule

	Output	**Inflation**	**Policy rate**	**Lending rate**	**Exchange rate**
Preference	58.16	25.73	49.58	49.99	2.05
Technology	34.04	49.83	18.73	26.07	2.31
Foreign output	0.33	0.08	0.34	0.29	0.54
Policy	6.12	13.55	4.91	1.91	9.61
Foreign interest	0.14	0.74	3.45	2.84	4.82
Terms of trade	0.31	5.17	0.43	0.32	48.36
Country risk premium	0.92	4.90	22.57	18.57	32.31

Note: The values presented here are measured in percent.

the contributions of foreign output shock and foreign interest shock on Thai business cycles are relatively small. Interestingly, exchange rate fluctuations are largely explained by the terms of trade shock and, to a lesser degree, by the country risk premium shock. Although both unconditional and conditional variance decompositions indicate a minor role of the terms of trade shock in most macroeconomic variables, the fact that approximately 48.50% of exchange rate movements are attributed to terms of trade shock would provide clear evidence that the Bank of Thailand reacts to fluctuations in exchange rates to smooth the effect of international price fluctuations.

Output

The results from both conditional and unconditional variance decompositions indicate that the fluctuations in output in Thailand are largely driven by preference shock σ_z. Specifically, approximately 85.50% of fluctuations are attributed to this shock in the first quarter. Although there is a slight decrease in the contribution of preference shock in the long run, it still accounts for around 58% of output fluctuations. The dominance of the demand-side disturbance in explaining the business cycles is well documented in the model with nominal rigidities (see, e.g., Galí, 2002, Nguyen, 2020). Meanwhile, technology shock is another important source driving up the volatility of output. Accordingly, it accounts for more than one third of the output movements in Thailand.

The contributions of the remaining shocks are relatively small. Monetary policy innovation contributes less than 10%, which is in line with previous studies (see, e.g., Lubik and Schorfheide, 2007). Furthermore, we find negligible impacts of foreign shocks on the volatility of Thai GDP.

Inflation

The variance decomposition technique demonstrates that technology shock is the primary source driving up the volatility of domestic CPI inflation. Accordingly, nearly half of the fluctuations in inflation are contributed by technology shocks. Furthermore, we find that there is a sizable impact of preference shock on CPI inflation in Thailand. Specifically, preference innovation explains approximately one-fourth of the volatility of CPI inflation. Thus, technology shock and preference shock together are significantly responsible for approximately 75% of the fluctuations in CPI inflation.

Similar to the case of Thai GDP, monetary policy shocks contribute around 13.50% of the volatility of CPI inflation. In addition, results indicate that foreign shocks play a minor role in domestic CPI inflation in Thailand.

Exchange rate

Interestingly, we find that external-sector associated shocks play a substantial role in explaining the exchange rate movements. Specifically, the terms of trade shock accounts for nearly half of the fluctuations in the exchange rate, followed by the country risk premium shock with approximately one third. Note also that the exchange rate captures the relative ratio of the international goods price denominated in terms of the domestic currency to the domestic goods price. Therefore, it is appropriate that external-sector associated innovations are substantially responsible for the large proportion of the fluctuations in the exchange rate. Furthermore, the fact that terms of trade shocks account for nearly half of the exchange rate movement shows that the Bank of Thailand smoothies the effects of fluctuations in international relative prices by allowing policy rates to react to changes in the exchange rate.

Domestic shocks, on the other hand, account for a small proportion of the variations in the exchange rate in Thailand. Specifically, while the monetary policy shock contributes nearly 10%, technology and preference shocks together account for less than 5%.

3.4.4 Impulse Response Functions

In this section, we further investigate the model dynamics using the impulse response functions. Specifically, we display the impulse responses of six selected variables to various structural shocks. The solid line denotes the posterior mean, while the dashed lines indicate the point-wise 90% posterior probability band. A few broad overviews can be drawn from this exercise. Overall, all six key macroeconomic variables respond remarkably to structural shocks, and the impacts of structural shocks are statistically significant in most cases. Second, the responses of macroeconomic variables to structural shocks exhibit expected patterns. For instance, the supply-side innovations, such as technology shock, generate countercyclical movements between CPI inflation and domestic output. The demand-side shocks like preference disturbance, however, enable CPI inflation and domestic output to move in the same direction. Finally, all structural shocks demonstrate temporary impacts on the Thai economy. Specifically, key variables quickly return to their steady state level after approximately eight quarters.

We present the dynamic responses of macroeconomic variables to structural shocks below. Due to space constraints, only the impulse response functions of technology shock and preference shock are depicted.

Preference shock

Figure 3.1 shows the impulse response functions for six key variables to a preference shock. After a positive preference shock, both output and output growth increase by 2 percent. Furthermore, as a domestic demand-side shock, it triggers an increase of 0.35 percentage points in CPI inflation. The increases in both output and CPI inflation lead to a rise in the policy rate through the mechanism of policy reaction functions. Specifically, the policy rate increases by 0.05 percentage points on impact, and it reaches a peak of 0.06 percentage points shortly after two quarters. This, in turn, translates into a 0.1 percentage point rise in the lending rate. It is also worth noting that all of these effects of the preference shock on key variables are statistically significant because the 90% posterior probability interval excludes zero.

Technology shock

The dynamic responses of key variables to technology shock are presented in Figure 3.2. When a positive shock hits the economy, output increases on impact by 0.5 percent and peaks at approximately 0.8 percent in the second quarter. CPI inflation, on the other hand, decreases by 0.5 percentage points. The countercyclical movements between CPI inflation and output when facing a supply-side disturbance are well documented in previous literature. In addition, a drop in domestic CPI inflation triggers a decline of roughly 0.04 percentage points in the policy rate. This in turn leads to a fall of around 0.1 percentage points in the lending rate. And lastly, the nominal exchange rate appreciates as a result of lower interest rates and CPI inflation.

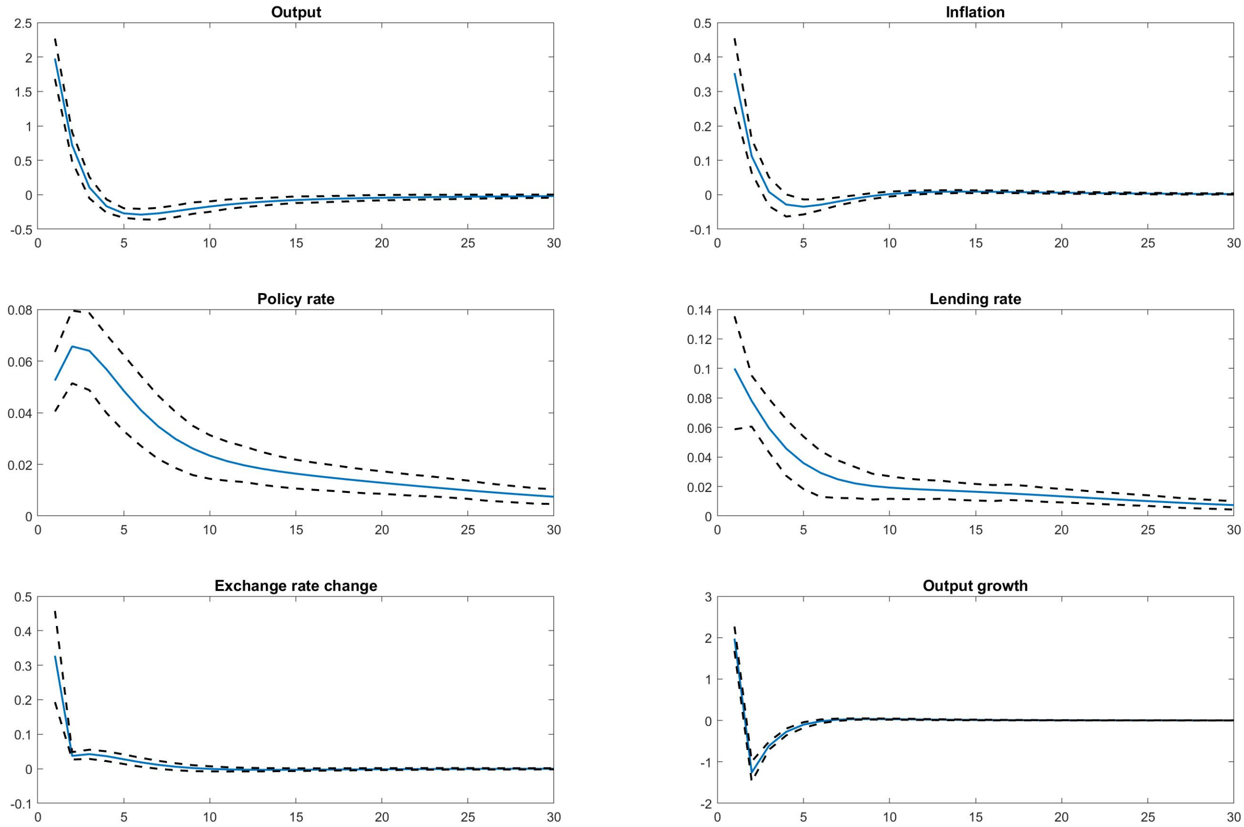

Figure 3.1: Impulse responses to preference shock, the exchange rate augmented policy rule

Notes: The figure displays posterior means and 90% credible intervals for output, inflation, policy rate, lending rate, exchange rate change, and output growth to one standard deviation of shock.

Table 3.6: Log marginal likelihoods for the models taking measurement errors into consideration

	Baseline	Generalized	Exchange rate	Credit growth
	$\phi_e = \phi_l = 0$	$\phi_e \& \phi_l$	ϕ_e	ϕ_l
Log marginal likelihood	2112	2112	2115	2093
Bayes factor		1	20.09	5.60×10^{-9}
Kass-Raftery statistics		0	6	-38

Notes: The log marginal likelihood is computed using the Laplace approximation.

3.4.5 Robustness Checks

Since the structural estimation of small open economy monetary rules is the main focus of this study, we report the sensitivity of previous findings by considering various specifications of the policy functions. First, we investigate the robustness of our findings by incorporating

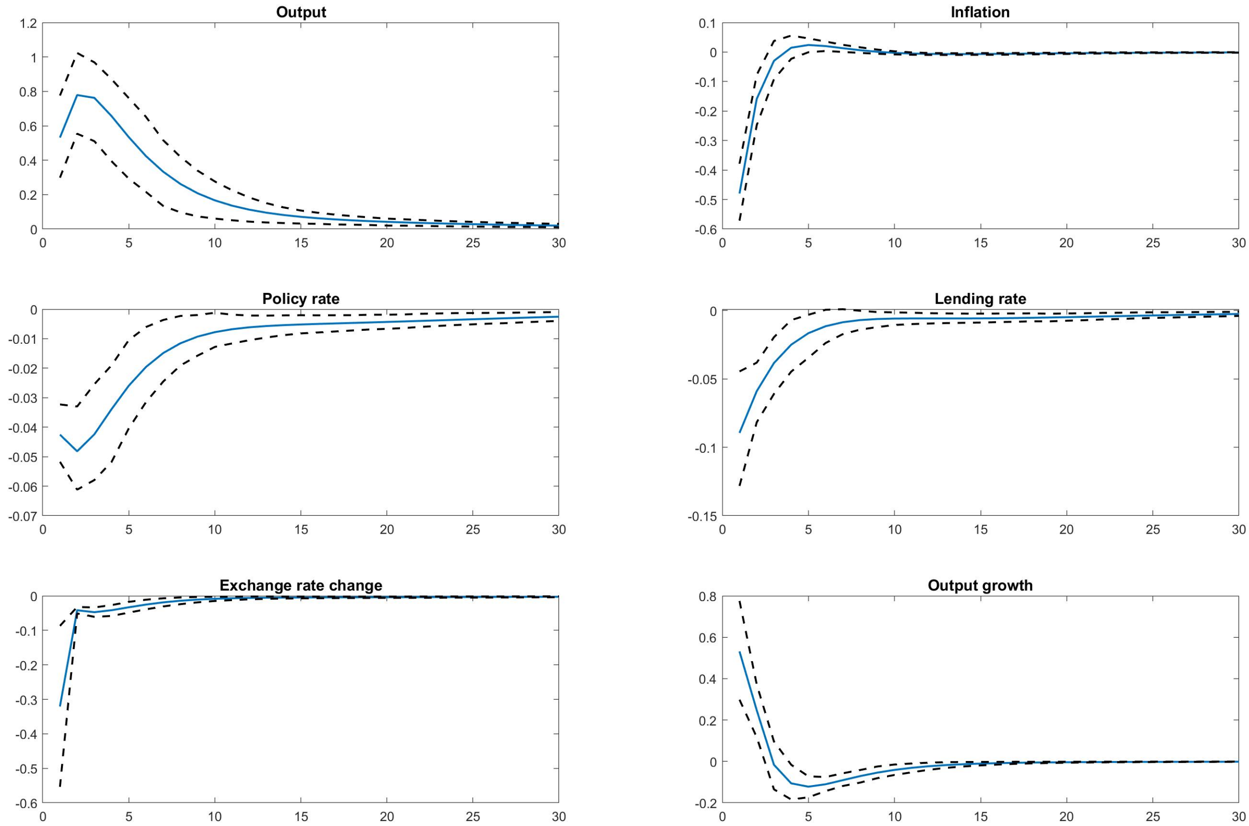

Figure 3.2: Impulse responses to technology shock, the exchange rate augmented policy rule

Notes: The figure displays posterior means and 90% credible intervals for output, inflation, policy rate, lending rate, exchange rate change, and output growth to one standard deviation of shock.

measurement errors into the observable equations. In particular, we introduce measurement errors for domestic variables except for the nominal interest rate. This is reasonable given that macroeconomic data are frequently measured with significant noise (see, e.g., Adolfson et al., 2008; Christiano et al., 2011). The log data densities, Bayes factors, and Kass-Raftery statistics are displayed in Table 3.6. Clearly, under this specification, the marginal data densities for all four different monetary rules improve remarkably. As an example, let us consider the case of the standard Taylor rule. The marginal likelihoods for the standard Taylor rule with and without measurement errors being introduced in the observable equations are 2077 and 2112, respectively, implying a Bayes factor of 1.59×10^{15} in favor of the model with measurement errors. This results in decisive evidence to support the standard Taylor rule with measurement errors. Similar findings are applied to the remaining three monetary rules. As for the importance of exchange rate movement in the policy reaction function, similar to our previous findings, the

Table 3.7: Log marginal likelihoods for different monetary rules

Output growth inclusion	Baseline $\phi_e = \phi_l = 0$	Generalized $\phi_e \& \phi_l$	Exchange rate ϕ_e	Credit growth ϕ_l
Log marginal likelihood	2074	2066	2078	2061
Bayes factor		3.35×10^{-4}	54.60	2.26×10^{-6}
Kass-Raftery statistics		-16	8	-26
Expected inflation	**Baseline $\phi_e = \phi_l = 0$**	**Generalized $\phi_e \& \phi_l$**	**Exchange rate ϕ_e**	**Credit growth ϕ_l**
Log marginal likelihood	2080	2064	2079	2063
Bayes factor		1.27×10^{-14}	0.14	1.71×10^{-15}
Kass-Raftery statistics		-32	-2	-34

Notes: The log marginal likelihood is computed using the Laplace approximation.

introduction of exchange rate movement in the Taylor rule leads to a better model fit, as measured by the log marginal likelihood, than the standard Taylor rule. A Bayes factor of 20.09 in favor of the exchange rate augmented Taylor rule against the standard one demonstrates strong evidence that the Bank of Thailand reacts to changes in exchange rates. On the contrary, under the credit growth augmented policy rule, the marginal likelihood deteriorates remarkably. The likelihood of this policy is 19 smaller on a log-scale than that of the standard Taylor rule, which translates into a Kass-Raftery statistic of -38 and a Bayes factor of almost zero. This provides overwhelming evidence to support the view that the Bank of Thailand does not respond to fluctuations in credit growth.

Next, we incorporate the output growth into the policy reaction functions of four alternative monetary rules and check the results.

Baseline rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right] + \varepsilon_{r,t}, \tag{3.39}$$

Exchange rate augmented Taylor rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right)\right] + \varepsilon_{r,t}, \tag{3.40}$$

Credit growth augmented Taylor rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_l \log\left(\frac{l_t}{l_{t-1}}\frac{\pi_t}{\pi}\right)\right] + \varepsilon_{r,t}, \quad (3.41)$$

Generalized Taylor rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_t}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right) + \phi_l \log\left(\frac{l_t}{l_{t-1}}\frac{\pi_t}{\pi}\right)\right] + \varepsilon_{r,t}, \quad (3.42)$$

where the parameter ϕ_{dy} captures the responsiveness of the policy rate to changes in output growth. Table 3.7 presents the log marginal likelihood, Bayes factor, and Kass-Raftery statistics for four alternative models associated with monetary rules. Overall, similar results are drawn for model comparison. Specifically, the inclusion of the exchange rate in the policy function increases the log marginal likelihood, indicating that the Bank of Thailand responds to exchange rate movements in order to reduce the effect of fluctuations in international relative prices. The credit growth augmented Taylor rule, however, decreases the log marginal likelihood significantly. With the Bayes factor of 2.26×10^{-6}, we find overwhelming evidence in support of the baseline rule over the credit growth augmented policy rule. Furthermore, the numbers show that there is a slight decrease in the marginal likelihood when we incorporate output growth in the reaction function. For example, let us consider the baseline Taylor rules. The log marginal likelihood drops from 2077 to 2074 when output growth is introduced, implying a Kass-Raftery statistic of -6 in favor of output growth augmented Taylor rule over a baseline one. This provides strong evidence for the latter. Similar conclusions are drawn for the three remaining pairs.[8]

A third robustness check concerns the re-estimation of the model under expected inflation monetary rules, in which the policymakers adjust the policy rate based on the gap between expected inflation and the inflation target. This specification is relevant because Thailand has formally adopted the inflation targeting regime as the policy framework since May 2000. An advantage of the inflation forecast-based rules is their consistency with the real practices of the

[8]Note that we only report the results of log marginal likelihoods using the Laplace approximation. Similar to the previous section, we obtain virtually identical results when using the modified harmonic mean estimator.

central bank. Thus, in addition to previous specifications, we investigate the following expected inflation monetary rules:

Baseline rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_{t+1}}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right] + \varepsilon_{r,t}, \tag{3.43}$$

Exchange rate augmented Taylor rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_{t+1}}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right)\right] + \varepsilon_{r,t}, \tag{3.44}$$

Credit growth augmented Taylor rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_{t+1}}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_l \log\left(\frac{l_t}{l_{t-1}}\frac{\pi_t}{\pi}\right)\right] + \varepsilon_{r,t}, \tag{3.45}$$

Generalized Taylor rule

$$\log\left(\frac{1+r_t^d}{1+r^d}\right) = \rho_r \log\left(\frac{1+r_{t-1}^d}{1+r^d}\right) + (1-\rho_r)\left[\phi_\pi \log\left(\frac{1+\pi_{t+1}}{1+\pi}\right) + \phi_y \log\left(\frac{Y_t}{Y}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \phi_e \log\left(\frac{ex_t}{ex}\right) + \phi_l \log\left(\frac{l_t}{l_{t-1}}\frac{\pi_t}{\pi}\right)\right] + \varepsilon_{r,t}. \tag{3.46}$$

The log marginal likelihood, Bayes factor, and Kass-Raftery statistics for expected inflation policy rules are reported in Table 3.7. The findings from model comparisons are virtually identical to the above-mentioned ones. Specifically, the inclusion of credit growth in the baseline rule leads to a significant decrease in the marginal likelihood, indicating that the Bank of Thailand does not react to fluctuations in credit growth. Interestingly, we find that while there are slight decreases in the marginal likelihoods of three policies (generalized, exchange rate, and credit growth) once the expected inflation is introduced, the log marginal likelihood of the baseline rule increases slightly from 2077 to 2080. This provides slight evidence in support that the Bank of Thailand employs the inflation targeting as monetary framework in practice.

Finally, we check the sensitivity of our main results by relaxing the priors on the parameters

Table 3.8: Log marginal likelihoods for the models with alternative priors for policy parameters

	Baseline $\phi_e = \phi_l = 0$	Generalized $\phi_e \& \phi_l$	Exchange rate ϕ_e	Credit growth ϕ_l
Log marginal likelihood	2077	2070	2079	2067
Bayes factor		9.12×10^{-4}	7.39	4.54×10^{-5}
Kass-Raftery statistics		-14	4	-20

Notes: The log marginal likelihood is computed using the Laplace approximation.

of the policy rule. Specifically, we increase both the prior means and standard deviations of the response coefficients. More specifically, we apply the gamma density with a mean of 0.75 and a standard deviation of 0.50 for ϕ_y (standard Taylor rule), ϕ_e (exchange rate augmented Taylor rule), and ϕ_l (credit growth augmented Taylor rule). In addition, the standard deviation of interest rate smoothing ρ_r increases to 0.50. Interestingly, the posterior estimates for all structural parameters are virtually unchanged. This indicates that our estimated results are robust to changes in the prior setting of policy parameters. This finding is in line with the study of Lubik and Schorfheide (2007), who demonstrate fairly weak restrictions between the structural equations and the monetary rule. Meanwhile, we find that the data are clearly informative since the posterior densities are significantly pulled away from the prior. The marginal data density reported in Table 3.8 remains unchanged for both the standard Taylor rule and the exchange rate augmented policy rule, while we find slight improvements in the marginal likelihoods of the credit growth and the generalized Taylor rules. However, it is interesting that the rankings of monetary policy rules based on marginal likelihoods are clearly the same: the Bank of Thailand reacts to exchange rate fluctuations, while there is no response of the policy rate to movements in credit growth.

3.5 Conclusion

In the present study, we incorporate financial frictions, incomplete exchange rate pass-through, and sticky prices in a small open economy setting and estimate the model using the Bayesian estimation technique to answer the question of whether the central bank reacts to exchange rate and credit growth. Our analyses demonstrate that the introduction of the exchange rate in the monetary policy rules is crucial for the monetary authorities of Thailand. However, we do not find evidence that the Bank of Thailand adjusts their policy rate in response to credit growth movements. These findings are robust to various specifications of the monetary policy rules. Furthermore, we indicate that domestic shocks contribute remarkably to the business cycles. Interestingly, terms of trade disturbance, despite playing a minor role in most macroeconomic variables, explains the largest proportion of exchange rate movement, followed by the country risk premium shock.

3.A Appendix

3.A.1 Estimated Results

Table 3.9 presents the estimated results for the three remaining monetary rules. In addition, we also present the estimated results for the exchange rate augmented Taylor rule when alternative priors of policy parameters are employed. Overall, the posterior estimates of model parameters are robust to various monetary policy rules.

Table 3.9: Prior and posterior distribution of estimated parameters for different policy rules

Parameter	Standard		Credit growth		Generalization		Alternative prior	
	Mean	90% Interval	Mean	90% Interval	Mean	90% Interval	Mean	90% Interval
σ	0.71	[0.29, 1.15]	0.51	[0.24, 0.77]	0.48	[0.24, 0.69]	0.59	[0.29, 0.88]
ψ	0.50	[0.34, 0.67]	0.50	[0.34, 0.66]	0.51	[0.35, 0.67]	0.53	[0.36, 0.68]
η	1.12	[0.73, 1.50]	1.25	[0.80, 1.66]	1.12	[0.76, 1.48]	1.11	[0.74, 1.45]
v	0.44	[0.37, 0.51]	0.44	[0.38, 0.51]	0.45	[0.38, 0.51]	0.44	[0.38, 0.51]
γ	3.18	[2.29, 4.01]	3.13	[2.30, 3.99]	3.16	[2.30, 4.03]	3.16	[2.33, 4.00]
θ_H	0.76	[0.69, 0.83]	0.71	[0.65, 0.78]	0.70	[0.64, 0.76]	0.73	[0.67, 0.79]
θ_F	0.66	[0.58, 0.74]	0.65	[0.57, 0.73]	0.66	[0.59, 0.73]	0.66	[0.58, 0.73]
κ	11.78	[3.52, 19.68]	11.43	[3.42, 19.08]	11.38	[3.04, 19.03]	11.66	[3.54, 19.62]
ρ_r	0.94	[0.93, 0.95]	0.94	[0.93, 0.95]	0.94	[0.93, 0.95]	0.94	[0.92, 0.95]
ϕ_π	1.45	[1.14, 1.73]	1.54	[1.21, 1.85]	1.54	[1.24, 1.85]	1.46	[1.04, 1.86]
ϕ_y	0.15	[0.06, 0.24]	0.14	[0.05, 0.23]	0.14	[0.05, 0.22]	0.12	[0.03, 0.20]
ϕ_e					0.31	[0.16, 0.46]	0.24	[0.10, 0.37]
ϕ_l			0.02	[0.00, 0.03]	0.03	[0.00, 0.04]		
ρ_y	0.91	[0.86, 0.96]	0.91	[0.86, 0.95]	0.91	[0.87, 0.96]	0.91	[0.87, 0.96]
ρ_a	0.52	[0.41, 0.64]	0.54	[0.43, 0.65]	0.55	[0.43, 0.67]	0.55	[0.43, 0.66]
ρ_z	0.62	[0.54, 0.71]	0.63	[0.54, 0.72]	0.63	[0.55, 0.71]	0.63	[0.55, 0.71]
ρ_{tot}	0.56	[0.50, 0.61]	0.56	[0.50, 0.61]	0.57	[0.51, 0.63]	0.56	[0.51, 0.62]
ρ_{r^*}	0.88	[0.84, 0.92]	0.88	[0.85, 0.92]	0.88	[0.84, 0.92]	0.88	[0.84, 0.91]
ρ_{risk}	0.87	[0.80, 0.95]	0.87	[0.79, 0.94]	0.87	[0.80, 0.94]	0.87	[0.80, 0.94]
$100\sigma_y$	0.50	[0.44, 0.57]	0.51	[0.44, 0.57]	0.50	[0.43, 0.57]	0.51	[0.44, 0.57]
$100\sigma_a$	4.58	[2.00, 7.40]	3.12	[1.78, 4.46]	2.86	[1.81, 3.90]	3.57	[2.07, 5.11]
$100\sigma_z$	3.95	[1.83, 6.31]	2.98	[1.59, 4.34]	2.90	[1.71, 4.07]	3.40	[1.86, 4.86]
$100\sigma_r$	0.07	[0.06, 0.08]	0.07	[0.06, 0.08]	0.08	[0.07, 0.10]	0.08	[0.06, 0.09]
$100\sigma_{tot}$	0.80	[0.64, 0.95]	0.82	[0.67, 0.97]	0.81	[0.66, 0.95]	0.81	[0.66, 0.96]
$100\sigma_{r^*}$	0.07	[0.06, 0.08]	0.07	[0.06, 0.08]	0.07	[0.06, 0.08]	0.07	[0.06, 0.08]
$100\sigma_{risk}$	0.21	[0.11, 0.30]	0.23	[0.12, 0.33]	0.19	[0.11, 0.28]	0.18	[0.10, 0.26]

Notes: In this table, we report the posterior estimates for three alternative monetary rules: (1) the standard Taylor rule; (2) the credit growth augmented Taylor rule; and (3) the generalized Taylor rule. In addition, we also present the estimated results for the exchange rate augmented Taylor rule when alternative priors of policy parameters are employed.

3.A.2 Prior and Posterior Densities

Figures 3.3, 3.4, and 3.5 display the prior and posterior densities for our model parameters. It is clearly seen that the posterior densities significantly differ from their prior densities for most structural parameters. This demonstrates that our data are quite informative for most parameters.

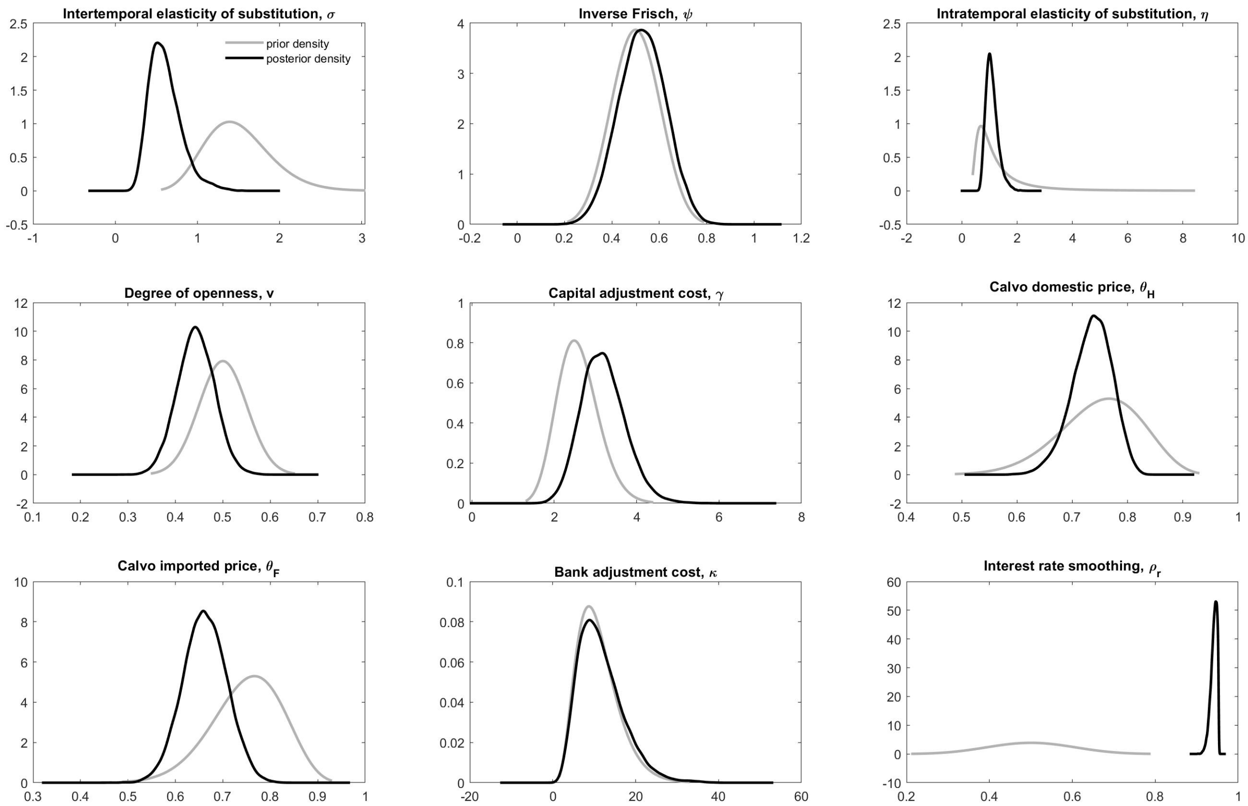

Figure 3.3: Prior and posterior densities

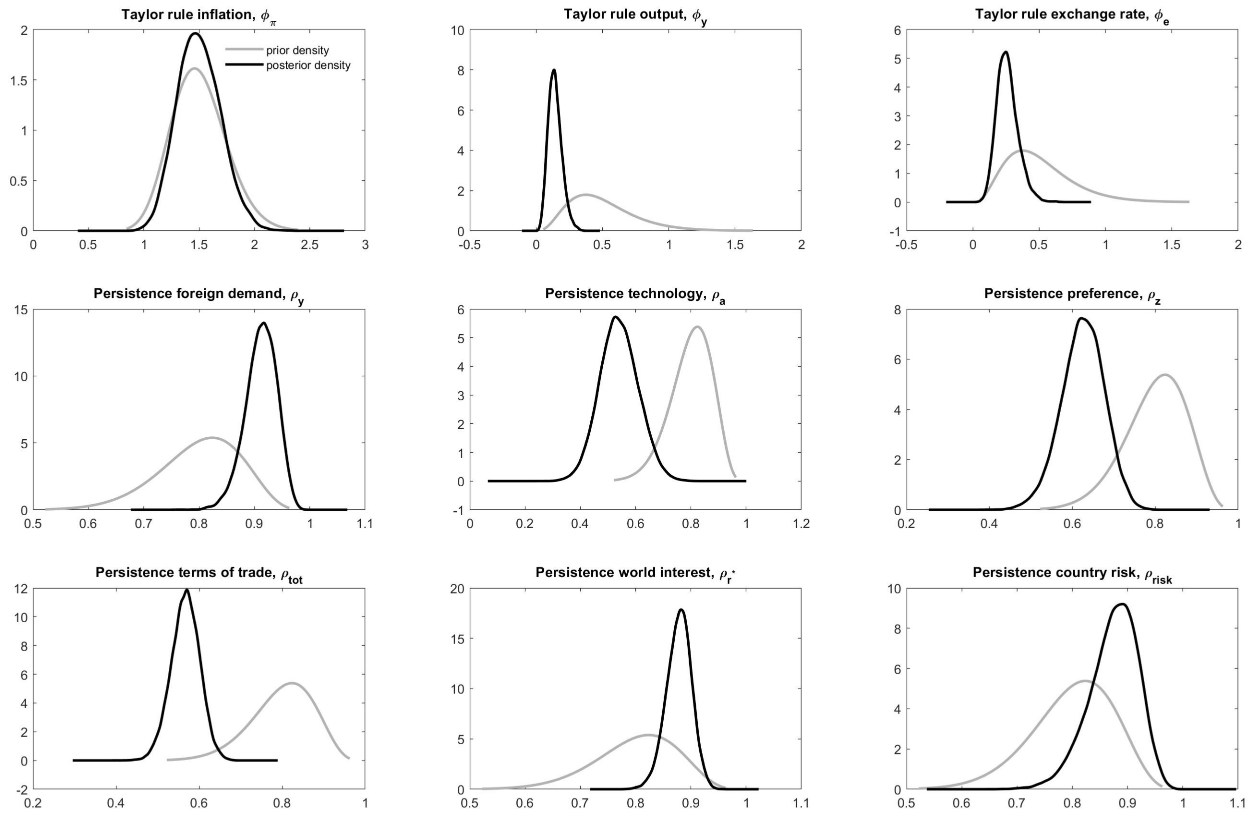

Figure 3.4: Prior and posterior densities

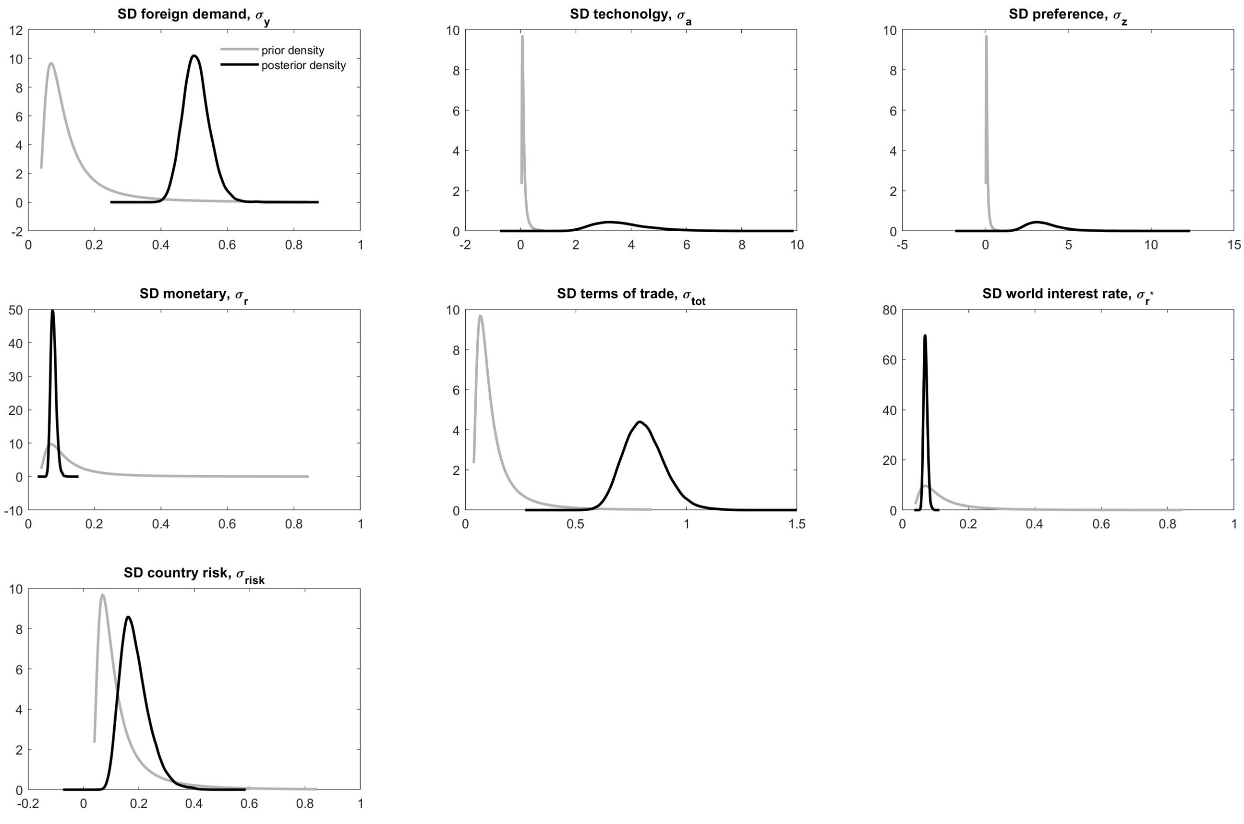

Figure 3.5: Prior and posterior densities

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