# Theories of Optimal Control and Transport with Entropy Regularization

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## Abstract

This dissertation investigates closely connected optimization problems, optimal control and optimal transport, in terms of entropy regularization. The goal of the optimal control problem is to control systems so that some performance index is optimized. On the other hand, the optimal transport problem seeks to find the most efficient way of transporting mass from a given distribution to another. There has recently been increasing interest in entropy-regularized versions of both problems called maximum entropy (MaxEnt) optimal control and entropic optimal transport, respectively, because of their computational advantages and so on. Entropic optimal transport is known to be equivalent to the so-called Schrödinger bridge problem whose origin lies in quantum mechanics. In addition, the Schrödinger bridge problem can be reformulated as a stochastic optimal control problem with density constraints at initial and final times.

In the light of the above relationship between stochastic optimal control and Schrödinger bridge (or equivalently entropic optimal transport) problems, one of the primary goals of this dissertation is to reveal the connection between MaxEnt optimal control and Schrödinger bridge problems. To this end, we first study Kullback-Leibler (KL) control, a special class of stochastic optimal control problems. The KL control problem has the KL divergence as a control cost while the Schrödinger bridge problem minimizes the KL divergence over probability measures on a path space. Despite this similarity, KL control requires the unrealistic assumption that the transition distribution is fully controllable, which hides the relationship between KL control and Schrödinger bridges. To resolve this issue, we reformulate the KL control problem so that it does not require unrealistic assumptions. Then we establish the equivalence between the reformulated KL control and MaxEnt optimal control, and a clear link between our KL control and Schrödinger bridges. Moreover, our reformulation broadens the applicability of the results of KL control such as an efficient algorithm for computing the optimal control. Next, we focus on linear systems and consider the MaxEnt optimal control with density constraints at initial and final times. Then, we derive the explicit form of the optimal solution. In addition, we reveal that the obtained solution gives the solution of the Schrödinger bridge problem associated with a linear system.

Lastly, we describe how entropy regularization is beneficial in control system design through the problem of stabilizing a collection of agents to a desired distribution shape with minimum cost. This can be formulated as an optimal transport problem over dynamical systems, which is challenging due to its high computational cost. To resolve this issue, by using entropy regularization, we propose a dynamical transport algorithm integrating model predictive control and the Sinkhorn algorithm, which is an iterative algorithm to efficiently solve entropic optimal transport problems. The notable feature of the proposed method is that it achieves cost-effective transport in real time by performing control and transport planning simultaneously. Moreover, we reveal that the entropy regularization enables us to ensure the convergence property of the proposed method.