General Dynamic Pricing Algorithms Based on Universal Exponential Booking Curves

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Contents

Chapter 1

Introduction

Dynamic pricing is a sales strategy that dynamically adjusts prices according to supply and demand conditions. Especially in Japan, dynamic pricing has been spreading to various industries in recent years: the hotel and airline ticket industries, which require making effective use of vacant assets, as well as railways, expressways, theme park tickets, food delivery services, beauty salons, and taxis (Nikkei (2022a); Nikkei (2022c); Nikkei (2021b); Nikkei (2022b); Nikkei (2021a); Nikkei (2022d)). Some of these projects, such as railroads and highways, are large-scale, with the government designing the system, and will involve significant changes in social structure.

There are two primary factors behind the spread of dynamic pricing; the first is the development of technologies, and the second is the social situation.

The development of technologies is in the IT field, including EC, sensors, digital price tags, high-speed online communications, and machine learning with big data. These innovations have shortened the time interval from observing the demand-supply situation to the pricing process, reduced various human costs, and provided the IT infrastructure to support the spread of dynamic pricing.

The most significant change in the social situation is the COVID-19 pandemic. In particular, as a strategy for economic growth in response to the decline in offline consumption, a coexistence of preventing overcrowding and stimulating demand is urgently required. Dynamic pricing is expected to be used in the railroads, expressways, and theme parks to overcome the issue (Nikkei (2022c); Nikkei (2021b)).

Meanwhile, in food delivery services and beauty salons, the system accomplished by dynamic pricing in which employees receive compensation commensurate with the value they provide can improve people's working environment (Nikkei (2021a); Nikkei (2022b)). In addition, dynamic pricing of food products and electricity are contributing to the SDGs, such as the issue of food loss and stable energy supply (Nikkei (2022b); Nikkei (2022d)).

According to the background above in Japan, dynamic pricing is expected to play various roles, including measures to address environmental issues, improve the working environment for individuals, and empower economic development. Therefore, spreading dynamic pricing is an irreversible trend in society.

1.1 Research fields on dynamic pricing

In general in the world, the application fields have been diverse in various industries, including energy (Faruqui & Sergici (2010); Hu et al. (2015); Joskow & Wolfram (2012); Ito et al. (2018)), retail (Garbarino & Lee (2003)), sharing economy (Garbarino & Lee (2003); Gibbs et al. (2018); Chen & Sheldon (2016); Altinay & Taheri (2018)), perishable products (Akçay et al. (2010); Subramanian et al. (1999); Akçay et al. (2010); Levin et al. (2010)), and perishable assets (Relihan III (1989); Yoon et al. (2017); Kimes (1989); Anderson & Xie (2010); Gallego & Van Ryzin (1994); Gallego & Hu (2014); Den Boer (2015a); Guo et al. (2013); Abrate et al. (2019); Abrate et al. (2012)). In each industry, the modeling styles for optimization are different because of the different factors that vary demand or supply (Elmaghraby & Keskinocak (2003a); Den Boer (2015a)).

In this study, we focus specifically on the perishable asset industry. In particular, dynamic pricing in the perishable asset industry —i.e., the products are required to heighten occupancy with finite inventories and sales periods— is a crucial measure in revenue management (RM), whose objective is to maximize revenue over a finite sales horizon. RM began in the airline ticket industry in the late 1980s (Relihan III (1989); Kimes (1989); Talluri et al. (2004); Anderson & Xie (2010)). In addition to the significant development of RM in the airline and hotel industries, it has been diverse in the leisure industry, including car rentals (Geraghty & Johnson (1997); Li & Pang (2017); Yang et al. (2021)), golf (Pekgün et al. (2014); Wang et al. (2015); Enz & Canina (2017); Guo et al. (2012); cruises (Sun et al. (2011); Biehn (2006); Kimes & McGuire (2001)), train tickets (Armstrong & Meissner (2010)), and theme parks (Heo & Lee (2009)).

We first surveyed the substance of the current state of dynamic pricing, which is spreading in various perishable assets industries as above, using both of previous research and fieldwork.

1.2 Terminology

- Dynamic pricing
  Dynamic pricing is one of the sales strategies to maximize profit or adjust the number of sales by dynamically changing prices in response to changes in demand and supply conditions. Although the above definition can be applied to any industry since demand and supply conditions cannot remain constant for any commodity, an item with the following characteristics is more proper to apply a dynamic pricing strategy: perishable, rarity, and low controllability for the supply. Commodities often the subject of academic research include electricity, food, trendy clothing, hotels, and airline tickets. Dynamic pricing in this study is defined in perishable assets industries.

- Perishable assets industry
  The perishable assets industry is characterized by a finite inventory and a finite sales period (sales horizon), and in addition, it has a reservation sales style. In other words, the asset products have a pile-up nature, where all the booked quantity of products do not consume until a deadline. The typical product in perishable asset industries includes airline tickets, hotels, car rentals, live events tickets, movie theater, and cruises. A similar product

type is perishable items, such as food and trendy clothing, which differ from perishable assets because they do not have a pile-up nature.

- Revenue management
  Revenue management is a collective name for profit maximization efforts in the perishable assets industry. Dynamic pricing is one of the measures of revenue management.

- Booking curve
  A booking curve, defined in perishable industries, represents a time series expressed as a cumulative number of bookings forward to the sales deadline. Generally, we have two definitions for booking curves: a "net" demand-based booking curve, which only includes bookings that have been consumed, and a "gross" demand-based booking curve, which provides for bookings that existed at the time of the observation date. A detailed explanation of them is in Chapter 2. In this study, we mainly use the net demand-based booking curve.

1.3 Dynamic pricing in perishable assets industry

Research on dynamic pricing in the perishable assets industry is divided into two categories: theoretical-based and empirical. Theoretical-based researches constitute the algorithms to maximize profits and simulation. The main objective of these studies is to discuss the effect of dynamic pricing on profits and sales by operating according to some realistic assumptions and strategies. In other words, giving reasonable assumptions with formulation for factors such as demand, supply, prices, competition, and inventories, they derive the expected results through the optimal strategy based on the formulation. The assumptions include the following examples in the hotel and airline industry; the demand function following a parametric function system (Subramanian et al. (1999); Guo et al. (2013); Ling et al. (2015)), divided demand stages of booking horizon (Elmaghraby & Keskinocak (2003b); Den Boer (2015b)), cancellation rate depending on product grade or time of booking (Subramanian et al. (1999); Yoon et al. (2017)), and price competition with competitors (Gallego & Hu (2014)). In addition, in the car rentals industry, assumptions include the issue of fleet and time-segmentation in days which is a specific property of the service (Li & Pang (2017)). Thus, many researchers have proposed various sophisticated models by formulating industry-specific strategies for implementing RM and have proved the necessity of introducing dynamic pricing.

Meanwhile, empirical-based studies are based on actual data. However, they are much less common than theoretically-based studies. Some of them also have the objective of discussing whether or not introducing a dynamic pricing strategy will increase revenue (Piga & Bachis (2006); Abrate et al. (2012); Abrate et al. (2019)). In particular, they construct a model based on externally obtained data on actual dynamic prices reflecting RM professional's measure and explain the effects of actual dynamic pricing.

These theoretical and empirical studies have given the necessity of a dynamic pricing strategy for maximizing revenue and the evidence of introducing it by some industries. Thus, in addition to the already-mentioned industries, they have constructed the basis of introducing dynamic pricing in various perishable assets industries such as express buses, activities, movie theaters, beauty salons, conference rooms, and function spaces.

The empirical-based models introduced above, however, are not based on a strategy for maximizing their property revenue but on actual practitioners' pricing data. Only a few studies construct an RM strategy model to maximize revenue by using actual sales data and providing techniques that RM professionals can use (Aziz et al. (2011)). In other words, one of the reasons for this is the difficulty of getting actual sales data. In the past, RM research was difficult to expand because actual sales data could not be obtained due to confidentiality issues (Sa (1987)), however the commonly available data for analysis are increasing (Antonio et al. (2019)). Given that studies based on available sales datasets are a few cases, we believe that developing theories and techniques based on actual data can be essential to accelerate the introduction of dynamic pricing.

Our fieldwork with several firms confirmed that the technology has not yet been implemented in the RM field. Although we surveyed RM instances in some industries; hotels, express buses, car rentals, leisure activities, and beauty salons, RM practitioners were engaged in trial and error in each facility by monitoring various kinds of data.

In particular, even in the hotel industry, where dynamic pricing is a standard strategy, the RM professionals assigned to each facility had considered various factors, and there was no uniformity in operation in the industry. The diversity was not only due to differences in business hotel and resort hotel types but also to differences in the management approaches in the facilities and the experience and intuition of the RM professionals. For example, the RM professional's judgment was based on a complex assessment of the following factors; past sales prices, estimation of how much should be sold and when based on the booking curve, competitors' prices, trend-based product development, and nearby events. RM professionals made comprehensive decisions on products, prices, and sales channels. Changes in market and sales conditions in the actual business field are not fixed enough to be formulated across facilities, much less across industries.

However, in investigating specific RM operational procedures, we found some elements that could be technologized. Specifically, the judgment of current market conditions based on the booking curve is common to all sectors. The booking curve is a time series expressed as a cumulative number of bookings. Although many forecasting and dynamic pricing algorithms based on booking curves have been developed in the RM research field (Brumelle & Walczak (2003); Levin et al. (2010); Weatherford & Bodily (1992); Lee (1990); Rajopadhye et al. (2001); Morales & Wang (2010); Haensel & Koole (2011); Weatherford & Kimes (2003a); Tse & Poon (2015); Lee (2018); Webb et al. (2020); Fiori & Foroni (2020); Sun et al. (2011); Reyes (2006); Yang et al. (2021); Pekgün et al. (2014); Enz & Canina (2017); Weatherford et al. (2001); Ma et al. (2014); Lee et al. (2005); Armstrong & Meissner (2010); Queenan et al. (2007); Kimes & McGuire (2001); Webb et al. (2021)), we confirm through our fieldwork that no actual data-driven theory and technology by which RM professionals generally can use have been established

Table 1.1: A part of the actual sales data set in a hotel A. The column set of use date and booking date in each record enables us to aggregate booking curves. The column use date represents stay date in hotels.

use date	id	price	rooms	room type code	booking date	is canceled	cancel date	booking lead time days
2019-10-01	180707	48008	2	W1	2019-09-02	true	2019-09-07	29
2019-10-01	180803	32184	1	KID2	2019-09-02	false		29
2019-10-01	181410	20401	1	Y2	2019-09-07	false		24
2019-10-01	181411	21311	1	WY1	2019-09-07	false		24
2019-10-01	181351	21602	1	W1	2019-09-07	true	2019-09-16	24
2019-10-01	181853	24002	1	W1	2019-09-11	false		20
2019-10-01	182041	26730	1	KID1	2019-09-12	false		19

yet.

Based on these backgrounds, this study constructs a theory of the booking curve based on actual data. It derives an implementable dynamic pricing technology. A comprehensive proposal with these theories and technology is expected to provide the basis for developing a fundamental technology in perishable asset revenue management. The actual sales data we use is obtained in the hotel and car rental industries for two years. In the next section, we introduce the contents of the sales data the cooperating companies provided.

1.4 The data analyzed

In this study, we analyze actual sales data collected over two years (from the beginning of 2019 to the end of 2020) for a total of six properties in the hotel and car rental industries. Tab. 1.1 shows a part of the sales data recorded in a hotel (hotel "A" in this thesis, see Sec. 2.4 in detail). The data constitution also almost holds for other properties; Tab. 1.2 shows a part of the sales data recorded in a car rental property (car rental "D" in this thesis). We have "whole" sales data, that is, it has all of the booking data coming in each of the six properties. Sales data were recorded by the "bookings" unit, that is, its originals contained who made a reservation, at what time, which product was chosen, how many were booked, at

Table 1.2: A part of actual sales data set in a car rental property that we analyzed. The column of car class code corresponds to room type code in hotel's data. The overnight use of a rental car makes two records of use date.

use date	id	price	qty	car class code	booking date	is canceled	cancel date	booking lead time days
2019-10-01	297741	3067	1	S	2019-09-12	true	2019-09-28	19
2019-10-01	297665	2300	1	S	2019-09-12	true	2019-09-30	19
2019-10-01	297653	2258	1	S	2019-09-12	false		19
2019-10-01	297703	2455	1	S	2019-09-13	false		18
2019-10-01	297827	8135	1	W8	2019-09-14	false		17

what price, and through which sales channel.

The data recorded by booking unit enables us to aggregate booking curves. Table 1.3 shows a part of booking curves that we analyzed in this study. Note that, we focus "net" demand based booking curve, which is defined as a cumulative number of bookings based on only the actual demand consumed (see Sec. 2.3 in detail).

In addition, the price change history data was provided by hotel A. The data acquisition period is from the beginning of 2019 to September 2020. Table 1.4 shows a part of it.

The outline of this study is to justify a new general law for booking curves using these data and to propose several applications based on the derived the law.

1.4.1 Data Management

As for data acquisition and management, since partners' systems rarely output data for third-party automatically or effortlessly, we often extract manually from the core system. In the study of data science, information about the actual conditions of data collection through companies should be shared, including the methods of acquisition and the challenging issues. We show some hard things we experienced and how to handle them as follows.

- Hard things

Table 1.3: A part of data shows net demand based booking curve in hotel A. The column of cumulative qty shows a booking curve.

stay date	days for deadline	cumulative qty
2019-10-01	0	80
2019-10-01	1	75
2019-10-01	2	71
2019-10-01	3	67
2019-10-01	4	64
2019-10-01	5	60
2019-10-01	6	59
2019-10-01	7	59
2019-10-01	8	59
⋮	⋮	⋮
2019-10-01	29	47
2019-10-01	30	46
⋮	⋮	⋮
2019-10-01	60	38
⋮	⋮	⋮
2019-10-01	201	0

- Only one year of data. Sales data older than one year has already been lost.
- The table structure of the downloaded data is hard to understand.
- It is extremely time-consuming to download data.
- Money is required to output data.
- Always only "yesterday's" actual data can be output (daily retrieval work is required, even on weekends).
- They reject due to the confidential issue (impossible to handle).

• How to handle them

- Developing a data submission system (AWS) for each customer system

Table 1.4: A part of data of changes in price in hotel A. Prices in the columns of price (before) and price (after) represent prices of a standard room per night with two people use with breakfast, which is one of the most standard use in hotel A.

use date	price change date	days for deadline	price (before)	price (after)	price action
2019-10-17	2019-10-15	2	11220	10010	down
2019-10-17	2019-10-07	10	12100	11220	down
2019-10-17	2019-08-30	48	13200	12100	down
2019-10-18	2019-10-15	3	15400	10010	down
2019-10-18	2019-10-06	12	14300	15400	up
2019-10-18	2019-08-30	49	13200	14300	up
2019-10-19	2019-10-15	4	18700	15400	down
2019-10-19	2019-10-06	13	20350	18700	down
2019-10-19	2019-09-23	26	17050	20350	up
2019-10-20	2019-10-06	14	14300	15400	up
2019-10-21	2019-10-06	15	14300	15400	up
2019-10-22	2019-10-15	7	15400	14300	down

– Developing an RPA (robotics process automation) system on the partner's PC at the partner's office.

Since we continuously performed from 2018 to 2021, we could get two-year sales data for 2019 and 2020.

1.5 Outline of the thesis

We first explain a new statistical law for booking curve time series based on actual sales data from theoretical and empirical justification. Second, we develop a time series forecasting method related to the new law. Third, we propose a practical dynamic pricing algorithm that is statistically evidence-based and human behavioral-based established.

Chapter 1 reports a survey of dynamic pricing in the perishable assets industry based on theoretical modeling, empirical investigation, and implementable technology. We introduce the importance of developing an evidence-based pricing technology considering that few dynamic pricing algorithms have been implemented despite spreading the strategy in various fields. In addition, we show the outline of actual sales data we analyzed in this study.

In Chapter 2, we explain a new statistical law, average booking curves draw exponential functions (ABCDEF law), based on actual data, modeling in the statistical physics framework, and empirical justification for the causality of the model. The law derives a proposed dynamic pricing algorithm in this study.

Chapter 3 describes the analytical time series forecasting method. We propose a new forecasting method for exponential decay time series. We evaluate its performance by forecasting random variables series and actual time series data; that is, booking curves that draw exponential decay functions.

Chapters 4 proposes practical dynamic pricing algorithms based on universal exponential booking curves. We show a learning process to give suggested prices, which leads to the target quantity demanded, based on the idea of the relation between changes in price and response in the slopes of booking curves. An addition, we show the effectiveness of proposed algorithms considering the consumers' behavioral complexity required in revenue management. The algorithm which provides behavioral-oriented and evidence-based information to RM professionals is expected to be a basis of practical dynamic pricing technology.

Chapter 5 concludes.

Chapter 2

Cross-Industry Statistical Law: Average Booking Curves Draw Exponential Functions

2.1 Introduction

This study explains a new universal statistical law, recently discovered, regarding the booking curves time series.

In the perishable assets industry, which includes hotels and airline ticketing, revenue management (RM), whose objective is to maximize profits by managing inventory over a finite sales horizon, has been used for more than 30 years. (Relihan III (1989); Kimes (1989); Lee (1990); Reyes (2006); Anderson & Xie (2010)). The application fields have been diverse such as rental cars, golf, cruise lines, railway tickets, theme parks (Guo et al. (2012); Geraghty & Johnson (1997); Yang et al. (2021); Sun et al. (2011); Biehn (2006); Heo & Lee (2009); Pekgün et al. (2014); Wang et al. (2015); Enz & Canina (2017); Armstrong & Meissner (2010)). A booking curve is represented by the cumulative time series of bookings. It has traditionally been a primary subject in RM efforts, as it directly relates to occupancy forecasting, which is a critical issue in RM. (Kimes (1989); Anderson & Xie (2010); Guo et al. (2012)). In the case of airline tickets, a 20%

improvement in forecasting accuracy has been reported to increase sales by 0.5-3% (Lee (1990)). Therefore, the use of booking curves for forecasting in RM has been widely studied by many researchers and professionals in the perishable assets industries. This has led to the development of several advanced and highly accurate forecasting models, including exponential smoothing, stochastic process models, generalized linear mixed models, deep learning, and methods utilizing neural nets that incorporate seasonal and day-of-week trends into their parameters. (Lee (1990); Rajopadhye et al. (2001); Morales & Wang (2010); Haensel & Koole (2011); Weatherford & Kimes (2003a); Tse & Poon (2015); Lee (2018); Webb et al. (2020); Fiori & Foroni (2020); Sun et al. (2011); Reyes (2006); Yang et al. (2021); Pekgün et al. (2014); Enz & Canina (2017); Weatherford et al. (2001); Ma et al. (2014); Lee et al. (2005); Armstrong & Meissner (2010); Queenan et al. (2007); Webb et al. (2021); Shintani & Umeno (2022b)).

However, mistiming the launch of RM measures due to people's booking window shift is a challenge RM professionals face. Since the fundamental idea of RM is to sell the right product at the right price and at the right time, RM professionals need to understand how the timing of people's bookings changes with the shifts in the demand-supply environment to ensure the effective use of advanced forecasting. Conjoint analysis for managers indicates that the factor most likely to affect performance in pricing, which is the most typical measure in RM, is appropriate timing (that is, the booking window) (Lee (2016)). Therefore, it is important to measure changes in booking curves quantitatively, that is, the booking window shift, to ensure the performance of RM.

In general, the causes of changes in booking curves can be divided into micro factors and macro factors (Tse & Poon (2015); Webb et al. (2020)). Micro-environmental factors have been studied and incorporated into daily forecasting models, including seasonality, trends in a day of the week, pricing, and promotions. Macro-environmental factors have posed a challenge for RM professionals when considering the appropriate timing for taking RM measures and include factors, e.g., the development of technologies, economic conditions, and people's preferences.

Hotel and airline ticketing industries have reported that a complex (that is, "grow" and "shrink" (Webb et al. (2020))) booking window shift has occurred due to the spread of IT technologies, and the booking window has shrunk due to extreme delays in decision-making during the COVID-19 pandemic (Uğur & Akbıyık (2020); Garrow & Lurkin (2021)). To find appropriate opportunities to sell products at the right price and timing, RM professionals have been required to analyze these complex booking window shifts, which can occur both gradually and rapidly due to macro-environmental factors. In other words, it is essential to have information to understand macroscopic changes in booking curves of each industry and property due to the changing times to take full advantage of sophisticated forecasting algorithms. In this study, therefore, while we focus on booking curves, which use "days" units in conventional forecasting modeling, we choose "months or a year" as a unit of time to evaluate the booking window shift. By doing so, we provide an explanation that summarizes the effect of changes in the demand-supply environment on people's booking patterns.

Based on this background, we analyzed actual sales data provided by six properties —three each in the hotel and car rental industries—, whose data acquisition period is for two years, including the periods before and after the COVID-19 pandemic. We studied the average booking curves, a new statistical series defined as an average time series of booking curves over a period. We explain a new universal statistical law characterized by the exponential function, which we called the ABCDEF (average booking curves draw exponential functions) law.

2.2 Summary

This section presents an overview of the ABCDEF law. A booking curve time series exists in capacity-constrained assets (perishable assets) industries such as hotels, airline tickets, car rentals, golf, cruise lines, rental cars, and theater seats. It is defined, in this study, as a cumulative time series of bookings toward a deadline over a finite sales horizon.

Strictly speaking, there are two types of definitions of booking curves. In this study, we use a "net" demand-based booking curve that reflects only the actual demand consumed (Rajopadhye et al. (2001); Morales & Wang (2010); Reyes (2006); Haensel & Koole (2011); Queenan et al. (2007)). The other booking curve is represented as a "gross" demand-based booking curve (on-hand-based booking curve generally). The latter is often used in studies for RM (Lee (1990); Weatherford & Kimes (2003a); Tse & Poon (2015); Lee (2018); Webb et al. (2020); Sun et al. (2011); Enz & Canina (2017); Weatherford et al. (2001); Ma et al. (2014)).Each of these definitions is divided according to whether or not canceled bookings are included in the time series; the former does not include them (that is, aggregate only consumed bookings), and the latter includes them ("gross" shows the original data at the time). When $q(t)$ is the number of bookings that are received at t days before the deadline, the (net demand-based) booking curve is expressed as a cumulative time series of $q(t)$. In other words, the booking curve $X(t)$ at t days before the deadline is defined as follows:

$$X(t) \stackrel{\text{def}}{=} \int_t^\infty q(s)ds, \tag{2.2.1}$$

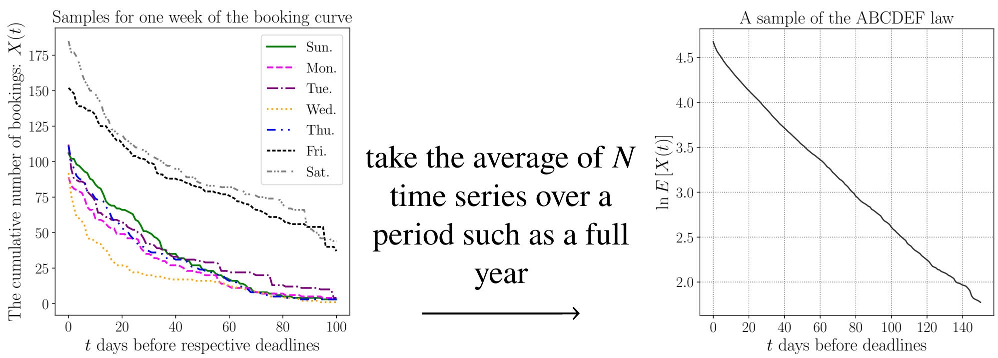
We study the characteristics of booking curves $X(t)$ with two years of actual sales data for six properties. Here, we introduce a statistical series based on booking curves, that is, the average booking curve, which is defined as the average statistical time series of N bookings curves over the observation period such as months or a year, and is defined as follows:

$$\mathbb{E}[X(t;n)] \stackrel{\text{def}}{=} \frac{1}{N}\sum_{n=1}^{N} X(t;n). \tag{2.2.2}$$

The ABCDEF law derived in this study shows that an average booking curve (2.2.2) is universally characterized by the exponential function as follows:

$$\mathbb{E}[X(t;n)] \simeq A\exp(-\beta t). \tag{2.2.3}$$

where A is a parameter that depends on quantity demanded and β is a parameter that



(a) (b)

Figure 2.1: An outline of the ABCDEF law. Figure (a) shows the (net demand-based) booking curves for a hotel for one week in 2019. Since demand varies depending on the day of the week and other factors, the transitions of booking curves have different trends. Figure (b) shows an average booking curve time series with a logarithmic scale derived from data for the year 2019 in the same property. The straight line with a logarithmic scale represents the exponential law of the average booking curve.

indicates the demand-supply environment surrounding the property. Figure 2.1 shows an example of the law provided from actual data. As for the implications of the two parameters, A is mainly determined by the initial inventory of the product and the magnitude of demand. While the parameter β indicates the demand-supply environment surrounding the property, it also illustrates patterns in people's booking horizons, which can be observed based on the shape of the function. These characteristics in β are two sides of the same coin. For example, the value of β becomes larger when the ratio of advance booking increases, and vice-versa with the ratio of last-minute bookings. In other words, a larger value is created by high demanded conditions such as vacations or discounts, where consumers tend to book in advance. In contrast, a smaller value is caused due to last-minute bookings, such as on-the-day bookings by strategic consumers for last-minute discounts.

Note that, while the values of these two parameters, A and β, differ for each property as expected, they also differ in the same property according to the time shift. In this study, therefore, our argument of "universality" in the ABCDEF law is based on the uniqueness of each industry and each demand-supply environment.

We justify the universality of the ABCDEF law from the following three perspectives: data confirmation, modeling in the statistical physics framework, and empirical justification for the causality of the model.

The data confirmation section shows the two-year actual sales data and fitting based on Eq. (2.2.3) for each property. We investigate by considering the difference in economic conditions, in addition to the difference in properties: we divided the data acquisition periods into groups according to before, middle, and after the COVID-19 epidemic. In the modeling section, we explain that an average booking curve follows an exponential function under a specific requirement for the demand-supply environment surrounding the property. We call the requirement a homogeneous demand-supply bath assumption (HDSBA: 2.5.2 in details). In the empirical justification section, to provide a basis for the causality of the model, we discuss whether the exponential function is derived when the HDSBA is satisfied. Specifically, we introduce a quantitative definition of a homogeneous "degree" of the environment, which characterizes the HDSBA. We investigate a correlation between the homogeneous "degree" and an exponential property, described as fitting deviation by exponential functions and average booking curves.

The ABCDEF law, justified from the above three perspectives, provides booking curves with its usefulness besides daily forecasting in RM. New statistics based on the averaged property of booking curves can provide more information about people's booking timing patterns, which change depending on various factors, e.g., the economic environment, preferences, pricing, and promotions. Thus, the statistics facilitate RM professionals to take measures at appropriate timing.

2.3 Literature Review

Our literature review mainly focuses on booking curves and windows, which are closely tied to each other because a booking curve only represents the cumulative time series of the number of bookings over a booking window. Thus when booking windows shift, the time-series patterns in booking curves also change.

2.3.1 History

A booking curve has been used as one of the most important elements in forecasting for RM whose objective is to maximize revenue over a finite sales horizon in the perishable assets industries. As for the history of RM, it originates from yield management (control an allocation for finite inventory) in the airline industry in the United States due to the deregulation of pricing in the 1970s (Relihan III (1989); Kimes (1989)).Subsequently, yield management derives RM with the fundamental idea of the right product, at the right price, and at the right time. RM has become part of public research themes from the late 1980s, and many models and business instance reports have been illustrated over the past 30 years in various perishable assets industries. (Lee (1990); Reyes (2006); Anderson & Xie (2010); Jerath et al. (2010); Guo et al. (2012); Geraghty & Johnson (1997); Yang et al. (2021); Sun et al. (2011); Biehn (2006); Heo & Lee (2009); Pekgün et al. (2014); Wang et al. (2015); Enz & Canina (2017); Armstrong & Meissner (2010)).A study on airline tickets among them showed that a 20% improvement in the accuracy of quantity demand forecasting led to a 0.5-3% increase in revenue. Based on such arguments, booking curves, which are directly related to occupancy forecasting, play an important role in the management (Lee (1990)). In the past, RM research was difficult to expand because actual sales data could not be obtained due to confidentiality issues (Sa (1987)), however, the commonly available data for analysis are increasing (Antonio et al. (2019)).

Booking curves represent a cumulative time series of bookings, and has two types defined in previous studies; "gross" (in general on-hand) demand-based booking curves and "net" demand-based booking curves. They are divided by types to compile cancelation data. The former shows actual observation numbers at the time, in other words, it includes lost bookings after the observation day due to cancellations. The latter does not include canceled bookings; all booking data to be recorded are finally presented. The former is generally used in RM studies because it has more realistic situations for forecasting. In contrast, the latter is often used to estimate imaginary demand that might have been achieved without the constraints of limited inventory in case of selling out. The commonalities of both booking curves are used for predicting the (actual or imaginary) number of bookings at the deadline based on historical or current dynamic time-series data.

As for forecasting models, various ones have been proposed using booking curves or booking processes. The most traditional and representative forecasting model is the exponential smoothing (including its expansions) (Harvey (1993); Chatfield (1978); Chatfield (1980); Rajopadhye et al. (2001); Morales & Wang (2010); Weatherford & Kimes (2003a); Sun et al. (2011); Weatherford et al. (2001); Queenan et al. (2007)). In recent studies, various approaches, such as stochastic process models (including Poisson processes, negative binomial processes, generalized linear mixed models,), neural nets, pick up algorithms, and advance booking models have been proposed (Reyes (2006); Lee (1990); Weatherford & Kimes (2003a); Tse & Poon (2015); Lee (2018); Webb et al. (2020); Sun et al. (2011); Enz & Canina (2017); Ma et al. (2014)).Since some of them are versatile, they spread in various perishable assets industries with some vertical advances such as dealing with the peculiar parameter in each field. While various sophisticated forecasting models using the booking curve have been developed, it has become clear that the utility of booking curves has mainly been biased towards "forecasting" and has few other purposes in any industry.

2.3.2 What affects booking curves

A macroscopic change in booking curves and people's booking window shifts are two sides of the same coin. In general, what influences on booking window shifts are divided into macro factors, such as technology developments and changes in economic conditions, and micro factors, such as seasonality, day of the week trends, pricing, sales channel allocations, and promotions. (Tse & Poon (2015); Webb et al. (2020)). In particular, IT technologies, one of the macro factors, e.g., adoption of online agencies, mobile APPs, and RM operation systems, made it possible for consumers to make bookings anytime and anywhere and enabled strategies such as pricing for RM professionals. Thereby, IT technologies resulted in booking window shifts with complex characteristics like "grow" and "shrink" (Webb et al. (2020)) through the evaluation of changes in an average lead day of bookings, which is used to measure booking window shifts. For example, it is reported that high average lead days were observed for vacations, and low average lead days were caused by delayed bookings by strategic consumers for last-minute discounts that caused unfulfilled inventories. (Webb et al. (2020)). The COVID-19 pandemic also caused booking window shifts. Various industries, including airline ticketing reported that people's booking windows were delayed due to uncertainties such as anomalous infection risks (Uğur & Akbıyık (2020); Garrow & Lurkin (2021)).

The establishment of models to provide enough information regarding people's booking pattern shifts, which result in macroscopic changes in booking curves, is expected to maintain revenue performance.

2.4 The Data Analyzed

We used a variety-rich dataset on booking curves provided by several firms to which we are grateful. The data was collected for a two-year period, including before and after the COVID-19 pandemic, from six properties in two industries; three each in the hotel and car rental industries. This data is sufficient to allow us to investigate across not only multiple fields, but also multiple economic environments.

This section presents a description of the data analyzed in terms of the following aspects: property information, actual sales data, and periods divided according to economic environment shifts.

2.4.1 Industries, properties, and sales data

Table 2.1 shows information on the properties from which the acquisition data was provided. Each property is located in Japan and is dispersed into areas with different economic characteristics, such as city, regional city, and resort.

Table 2.1: Profiles for six properties in two industries which provide the actual sales performance data.

Property	Location	Region	Characteristics of services
hotel (property A)	Tochigi	regional city	resort hotel
hotel (B)	Osaka	city	business hotel
hotel (C)	Kyoto	city	business hotel
car rental (D)	island Ishigaki	resort	passenger car
car rental (E)	Naha	resort	passenger car
car rental (F)	island Miyako	resort	passenger car

Table 2.2: A part of the actual sales data set in hotel A. The column set of use date and booking date in each record enables us to aggregate booking curves. This table is the same as Table 1.1.

use date	price	rooms	room type code	booking date	is canceled	cancel date	booking lead days
2019-10-01	48008	2	W1	2019-09-02	true	2019-09-07	29
2019-10-01	32184	1	KID2	2019-09-02	false		29
2019-10-01	21602	1	W1	2019-09-07	true	2019-09-16	24
2019-10-01	26730	1	KID1	2019-09-12	false		19

Following this, Tab. 2.2 shows a part of the sales data recorded in hotel A. The data constitution also holds for other properties. We have "whole" sales data, that is, it has all of the booking data coming in each property. Sales data were recorded by the "bookings" unit, that is, its originals contained who made a reservation, at what time, which product was chosen, how many were booked, at what price, and through which sales channel. We can access the data that describes when, how many, at what price while considering confidentiality, which allows us to summarize data for booking curves. In addition, in property A, D, E, and F, we can also access cancelation data, which have information regarding the canceled day in the same row of the booking records. Although cancelation data are not necessary to summarize the net demand-based booking curves, which is the subject

Table 2.3: The information of periods divided in terms of economic environments. Our validation in this study is based on the comparison across these four periods.

Period	Time range	Remark for market conditions
2019 ('19)	Jan. 1 - Dec. 31 (2019)	"regular" demand
2020-A (20-A)	Jan. 1 - Mar. 31 (2020)	the first COVID-19 epidemic
2020-B (20-B)	Apr. 1 - Jul. 21 (2020)	a state of emergency
2020-C (20-C)	Jul. 22 - Dec. 31 (2020)	tourism campaign

of this study's analysis, it can improve the quality of the data for future studies.

Note that, especially in hotel A, we did not aggregate booking curves from chartered days, which are defined as having more than 50% group travel customers, to exclude people's extraordinary booking patterns. We can confirm whether a day is chartered or not in the repository.

2.4.2 Economic environments

We divide the data acquisition period into groups because of some events which largely influenced economic environments for every property. Since COVID-19 crisis occurred all over the world, including Japan, every property was economically affected. Expressing the period 2019 as "regular" demand, we regard the period 2020 as "irregular" demand. Then, we divide 2020 into three groups according to events which significantly changed the market condition or people's mobility in the whole country. Table 2.3 shows definitions and characteristics of the period in 2019 and three periods in 2020. There were three major events in 2020 according to the economic shifts due to the COVID-19 epidemic. From mid-January to February, 2020, the first infection case was observed in Japan, subsequently the number of cases increased, and restrictions on immigration and gatherings were imposed (Karako et al. (2021)). We define the first period as from January to March.

In April, the Japanese government declared a state of emergency, which continued until late May. The state of emergency resulted in a strong restriction on people's mobility, with decreases up to 30% in urban areas (Morita et al. (2020);

Table 2.4: Changes year-on-year in quantity demanded and the average lead days of bookings which represent the outline of the demand environment.

	Relative ratio: Quantity demanded				Relative ratio: Average lead days (actual days)			
	'19	20-A	20-B	20-C	'19	20-A	20-B	20-C
hotel A	-	0.74	0.30	0.76	-(46)	0.95(35)	0.24(13)	0.63(28)
hotel B	-	0.58	0.08	0.23	-(45)	1.58(51)	0.09(5)	0.15(7)
hotel C	-	0.42	0.06	0.55	-(50)	1.05(43)	0.19(9)	0.27(15)
car rental D	-	0.99	0.27	0.79	-(50)	0.92(38)	0.46(22)	0.57(31)
car rental E	-	0.69	0.16	0.60	-(45)	1.02(41)	0.44(20)	0.58(27)
car rental F	-	0.70	0.22	0.92	-(51)	1.02(43)	0.53(28)	0.60(32)

Nagata et al. (2021)). On July 22, as the wave of the number of the infected largely decreased, the Japanese government launched a tourism campaign called "Go-To Travel," which assisted people in domestic travel by subsidizing up to 50% to encourage economic activities. This tourism campaign ended on December 28 due to the re-spread of COVID-19 infections. (Morita et al. (2020); Nagata et al. (2021)).Thus, we define the second and third periods as April 1 to July 21, and July 22 to December 31, respectively.

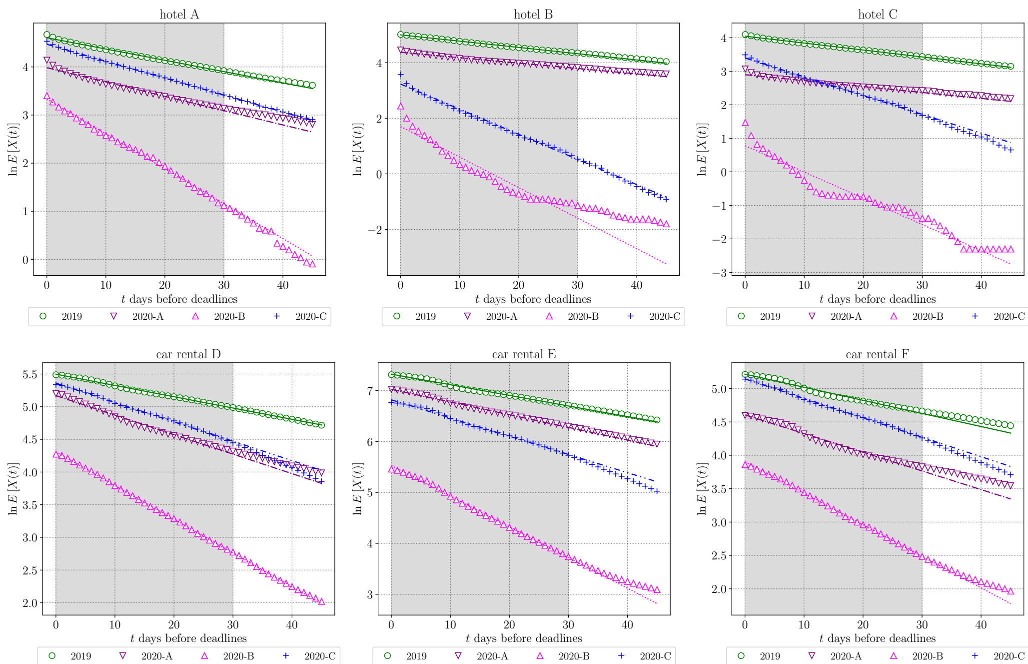
Here, we check the states of economic environment in each property over the divided periods. Table 2.4 shows the relationship between the quantity demanded and the average lead day of bookings, which is a statistic to measure changes in the booking window shifts (Webb et al. (2020); Uğur & Akbıyık (2020); Garrow & Lurkin (2021)), year on year in the same period. Note that an average lead days is defined as $\underset{n}{\mathbb{E}}\left[\left(\sum_n q(t;n)\right)^{-1}\sum_n tq(t;n)\right]$. The changes in quantity demanded had the following general trends in all properties; 20-A shows a slight or up to 50% decrease, 20-B shows a significant decrease of up to 70%-90%, and 20-C represents a general recovery with differences among properties, though they don't reach the previous year's level. This seems to reflect the effect on mobility in each characteristic of the period, the COVID-19 first pandemic, a state of emergency, and the tourism campaign.

Besides, the average lead days of bookings also had mostly common trends across all properties; 20-A represents the same level as the previous year, 20-B shows large decreasing fluctuations up to 50%-90% reduction (shrink), 20-C shows slightly or decent recovery (grow) along with quantity demanded recovery, especially up to 60% in four properties A, D, E, and F. The significant decreases in 20-B, can be attributed to the tendency to delay decision-making due to heightened uncertainty (Uğur & Akbıyık (2020); Garrow & Lurkin (2021)). Considering the changes in these two economic representative values, we confirm that the four periods, including 2019, are considerably different regarding the demand environment and people's booking patterns. The following sections investigate the booking curves across properties and economic environments. Then, we use the four periods above as economically different periods.

2.5 A Universal Statistical Law: Average Booking Curves Draw Exponential Functions

2.5.1 The data confirmation

We compiled average booking curves (2.2.2) for each property listed in Tab. 2.1 and for each period listed in Tab. 2.3. Figure 2.2 illustrates a total of 24 series of average booking curves, which was constituted by a combination of the six properties and the four periods, and fitting lines based on Eq. (2.2.3). The table in Fig. 2.2 shows the fitting parameters and the number of aggregated days. Note that, in the table, we adopt $\tau(= 1/\beta)$, which is generally called a time constant in exponential decay functions, and analytically represents the day at which $1/e \times 100 \simeq 36.8$ percentile cumulative bookings are fulfilled when under the exponential law.



	A: magnitude of demand				τ(= 1/β): environment				N: aggregated days			
	’19	20-A	20-B	20-C	’19	20-A	20-B	20-C	’19	20-A	20-B	20-C
A	100.2	53.8	28.0	87.5	42.6	33.6	13.8	28.4	290	68	55	114
B	148.5	79.4	5.5	25.3	44.3	51.4	9.0	11.0	365	91	97	163
C	57.2	18.1	2.2	29.8	48.4	56.6	12.8	17.8	365	84	40	119
D	224.3	175.8	74.4	212.8	57.9	33.5	19.4	33.7	365	91	112	163
E	1515.3	1124.8	251.7	915.5	48.0	40.1	16.6	28.0	365	91	112	163
F	184.2	100.1	49.6	171.5	51.0	35.7	21.2	34.3	365	91	112	163

Figure 2.2: Visualization of the average booking curves (marker plots) and fitted lines based on Eq. (2.2.3) for each property and economic environment. The parameters A and $\tau(= 1/\beta)$ show the fitting parameter, and N represents aggregated days for booking curves. The fitting interval is set as $0 \leq t \leq 30$ and represented by the shaded part.

Although the parameters A and β represent the magnitude of demand and the demand-supply environment surrounding the property, respectively, they show unique values for different periods, even at the same property. Confirming how the average booking curves draw exponential functions, in 2019, we see the coincidence in at least five of the six properties, excluding car rental F. For the three periods in 2020, relatively large deviations from the exponential function were observed in the hotels B and C in economic environment 20-B. In addition, the following environments can be divided into cases with slightly more significant deviations; hotel A × 20-A, hotel C × 20-A, and car rental F × 20-A.

Even if we evaluate these six patterns as not satisfying the exponential functions law, we can confirm the non-trivial coincidence in 75% (18/24) environments. In other words, the average booking curves are well characterized by exponential functions with unique parameters. These results imply a "universal" exponential law of average booking curves since it is a property observed not only across industries such as hotels and car rentals but also across economic environments such as before and after COVID-19. We attempt to model the dynamics which derive the exponential nature of average booking curves in the following subsection.

2.5.2 Modeling of exponential booking curves in the statistical physics framework

First, we model a booking curve by a function based on its time-series characteristics. Since a booking curve $X(t)$ is a net demand-based booking curve, it builds up according to the deadline. Therefore, we define a booking curve of a day n using the following monotonically non-decreasing function ϕ_n.

$$X(t;n) = A_n \phi_n(t;\beta_{n,t}), \tag{2.5.1}$$

where $A_n(> 0)$ is a reached occupancy constant, and $\beta_{n,t} > 0$ denotes a time-depending variable that reflects the build-up way of a time series. The function $\phi(t, \beta_{n,t})$ satisfies the following properties:

$$\begin{aligned} &\phi_n(0;\beta_{n,0}) = 1, \quad \lim_{t\to\infty} \phi_n(t;\beta_{n,t}) = 0, \\ &\phi_n(t;\beta_{n,t}) \geq \phi_n(t+1;\beta_{n,t+1}) \quad \text{for } \forall t \geq 0. \end{aligned} \tag{2.5.2}$$

The parameter $\beta_{n,t}$ varies time-dependent even when comparing days that reach the same sales quantity due to demand and supply environments such as seasonality

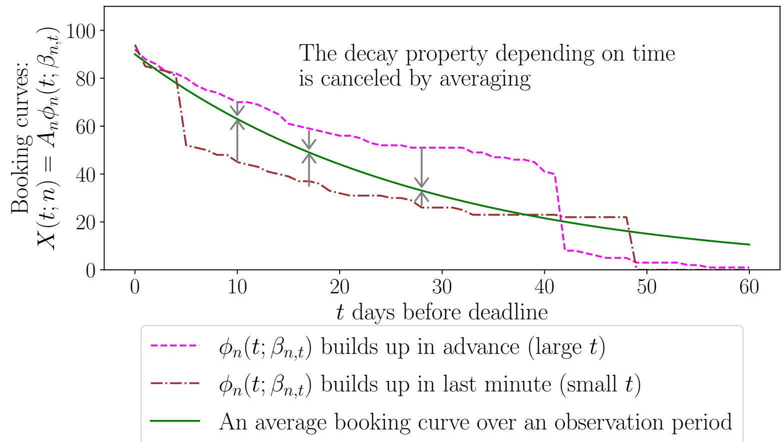


Figure 2.3: Visualization of actual booking curve samples with time-dependent $\beta_{n,t}$ and an average booking curve.

trends, pricing and promotions. However, the polarity of build-up patterns among observed days is canceled by averaging as Fig. 2.3 illustrates.

Here, we set the following assumptions. It is assumed that the property's demand-supply environment over an observation period is homogeneous enough to entirely level the time-dependent polarity of build-up patterns. In other words, a homogeneous demand-supply environment leads to a uniform rate of change by unit day in the average booking curve. This assumption describes that the average booking curve satisfies the following mathematical properties;

$$\begin{aligned} \mathbb{E}[X(t;n)] &= \frac{1}{N}\sum_n A_n\phi_n(t;\beta_{n,t}) \\ &\to A\phi(t;\beta), \end{aligned} \tag{2.5.3}$$

$$\frac{\phi(0)}{\phi(1)} = \frac{\phi(1)}{\phi(2)} = \cdots = \frac{\phi(t-1)}{\phi(t)} = \frac{\phi(t)}{\phi(t+1)} = \cdots \quad \text{for } \forall t \geq 0, \tag{2.5.4}$$

where $\phi(t;\beta)$ represents monotonically non-decreasing function and satisfies:

$$\begin{aligned} &\phi(0;\beta) = 1, \quad \lim_{t\to\infty}\phi(t;\beta) = 0, \\ &\phi(t;\beta) \geq \phi(t+1;\beta) \quad \text{for } \forall t \geq 0. \end{aligned} \tag{2.5.5}$$

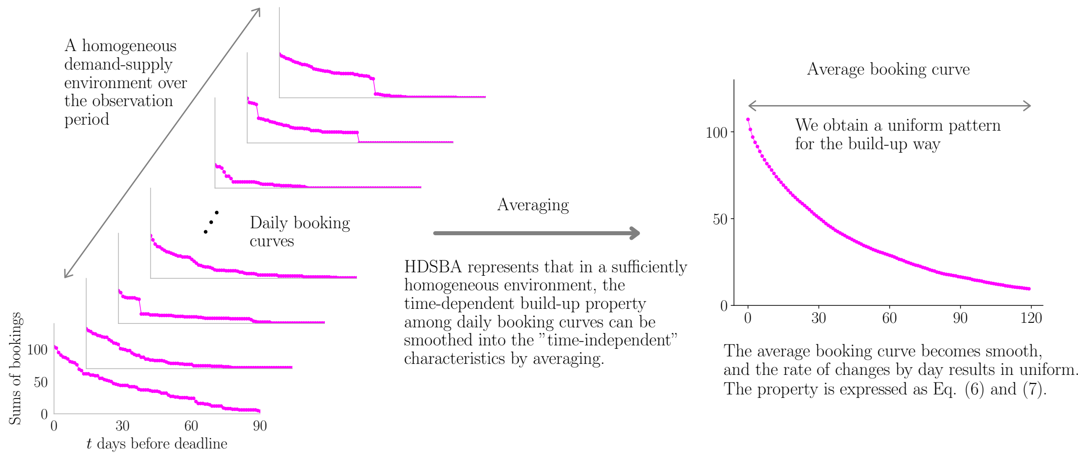


Figure 2.4: The visualization of the HDSBA.

Because a demand-supply environment surrounding the property is characterized by various factors such as economic backgrounds, people's preferences, competitors, and operations of RM measures, this assumption lies in a macroscopical mechanism. We call this assumption the homogeneous demand-supply bath assumption (HDSBA), which is similar to the concept of of the heat bath in statistical mechanics (Landau & Lifshitz (2013)). The concept of the HDSBA is visualized in Fig. 2.4.

Under this assumption and given properties, we discuss the characteristics of the function $\phi(t;\beta)$ in detail. From Eq. (2.5.4) and $\phi(0) = 1$, we obtain the following equation:

$$\begin{aligned} \phi(t) &= \phi(1)\phi(t-1) \\ &= \phi(1)\phi(1)\phi(t-2) = \cdots = \{\phi(1)\}^t \quad \text{for } \forall t \geq 0. \end{aligned} \tag{2.5.6}$$

Here, since the function $\phi(t)$ is leveled enough, it is reasonable to assume that $\phi(t)$ is smoothly differentiable except $t = 0$. Hence, Eq. (2.5.6) is transformed with the derivative $\phi'(t)$ to:

$$-\frac{1}{t^2}\ln\phi(t) + \frac{1}{t}\frac{\phi'(t)}{\phi(t)} = 0 \quad \text{for } \forall t > 0 \tag{2.5.7}$$
$$\text{via} \quad \frac{1}{t}\ln\phi(t) = \ln\phi(1),$$

From Eqs. (2.5.5), (2.5.6), and (2.5.7), the function ϕ is uniquely determined as the exponential decay function $\phi(t;\beta) = \exp(-\beta t)$ (see the supplemental document for an explanation), and we consequently obtain the formula about average booking curves: $\mathbb{E}[X(t)] = A\exp(-\beta t)$. Note that a differential equation $\frac{d\phi(t)}{dt} = -\beta\phi(t)$, which implies the property that an increase in the sums of bookings is proportional to the number of cumulative bookings, as is shown in the explanation document. That implication is similar to "guests volume invites guests," typified by the bandwagon effect and word-of-mouth (Xie et al. (2014); Zhou & Duan (2016)).

2.5.3 Empirical justification for causality

We empirically justify the causality of the above model. The key point of the proposed model is the causal relationship that the HDSBA results in the exponential law of average booking curves. To verify this causality in-depth, we define a degree of the cause and a degree of the result. In other words, we quantitatively provide a homogeneous "degree" of the demand-supply environment and a degree of similarity between fitted exponential functions and average booking curves. By doing so, we attribute the model's causality to the degrees' correlations. Note that that is strictly a correlation and not causality, but it significantly supports the validity of the consequence. Here, while we also use the two years data from 2019 to 2020 in this section, we compare annual data, not the periodized data in Tab. 2.3. That is because, as discussed in Sec. 2.4, the economic environments in 2019 and 2020 differ as regular vs. irregular. The objective is to find the emergence of a remarkable homogeneous "degree" of demand over the year by comparing annual data.

We focus on the quantity demanded, which reflects the changes in economic conditions, one of the compositions of the environment surrounding the property. We define the homogeneous "degree" of an environment by the magnitude of variation of the quantity demanded throughout the year, in other words, by the coefficient of variation. Thus, the small coefficient of variation denotes the small fluctuation in quantity demanded and the high degree of the homogeneous environment throughout the year. The mathematical definition of the coefficient of variation is as:

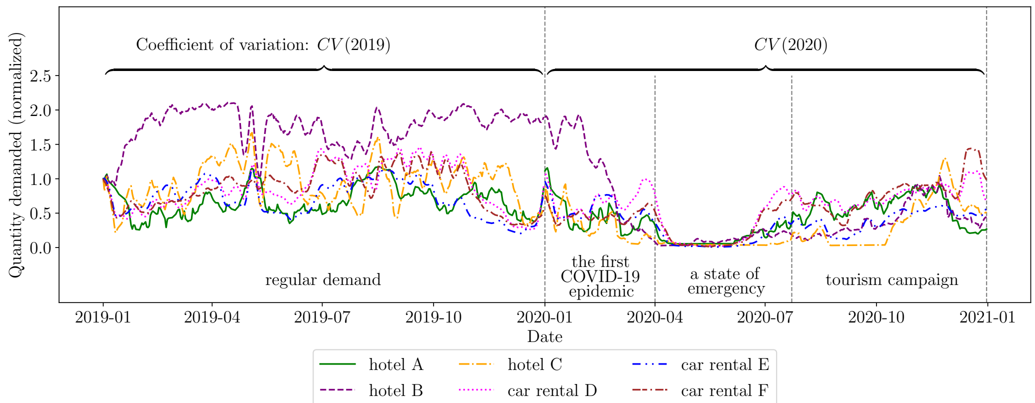
$$CV = \frac{\sigma_{\mathrm{q}}}{\mu_{\mathrm{q}}} \times 100\ (\%), \tag{2.5.8}$$

where μ_{q} and σ_{q} are the mean and the standard deviation of seven days moving average time series of the quantity demanded for each year. We derive CV from the annual quantity demanded data for each year and each property.

Meanwhile, the degree of similarity between fitted exponential functions and average booking curves is defined by the mean squared error MSE as follows:

$$MSE(N; t^*) = \frac{1}{t^*} \int_0^{t^*} |\ln(\mathbb{E}[X(s; n)]) - (\ln A - s/\tau)|^2 \, ds, \tag{2.5.9}$$

where N is the number of days in each period, that is, the number of booking curves series $X(t)$ to be aggregated, and t^* is a constant that gives the integral interval. Figure 2.5 illustrates the relationship between CV and MSE, which denotes the degree of causes and the degree of results in the proposed model, respectively.



	2019		2020	
	MSE	CV %	MSE	CV %
hotel A	3.61×10^{-4}	33.31	5.96×10^{-4}	54.52
hotel B	1.48×10^{-4}	15.31	6.02×10^{-3}	100.89
hotel C	2.17×10^{-4}	33.54	2.28×10^{-3}	78.37
car rental D	1.00×10^{-4}	36.80	1.49×10^{-4}	51.88
car rental E	4.02×10^{-4}	35.57	4.22×10^{-4}	62.95
car rental F	6.99×10^{-4}	33.52	2.26×10^{-4}	61.64

$\downarrow$

The correlation coefficient between CV and MSE is 0.79.

Figure 2.5: Verify the proposed model's causality by comparing the values CV and MSE, which imply one of the degrees of causes and results, respectively. The graph shows the quantity demanded time series, representing the seven-day moving average with normalized values on January 1, 2019, like 1 for each property. The value CV is calculated from the series in the graph based on Eq. (2.5.8).

As for CV in the table in Fig. 2.5, comparing 2019 and 2020, they were 1.5 to 6.5 times larger in 2020 through all properties. Looking more closely, the rate of increase from the previous year at properties B and C was more significant than at the other properties. This result reflects the fact seen in Tab. 2.4. As for the exponential degree, MSE in properties B and C in 2020—the environments with the highest CV— were one digit order larger than the other environments. Although we see a case with inverse correlation, such as the property F, we confirm that changes in CV were positively linked to changes in MSE. Finally, the correlation coefficient between CV and MSE calculated from the results in the table was 0.79.

This result of the correlation between the "degree" of a homogeneous environment and the exponential property of average booking curves yields sufficient evidence for the causality of the model that the HDSBA leads to the exponential law. Note that we visualize actual average booking curves for each year of 2019 and 2020 for each property in the same way as Fig. 2.2 in the supplemental document.

Furthermore, as mentioned in Sec. 2.4, we did not aggregate booking curves for chartered days to exclude group travel customers, having different booking patterns from usual customers, whose objectives consist of escorted tours or school excursions. Thus, we discuss the case with chartered days in the supplemental document and explain one of the limitations of the ABCDEF law; we show that the high ratio of group customer results does not satisfy the ABCDEF law according to mixed (non-homogeneous) booking patterns. That implies that "customer attributes" can become one of the elements defining a homogeneous demand-supply environment, in addition to the economic background.

As described above, Sec. 2.5 has presented the evidence for the ABCDEF law from three perspectives; data confirmation, modeling in the statistical physics framework under the HDSBA, and empirical justification for causality of the model. It is highly expected that the average booking curves, a time series inherent in perishable asset industries, can universally follow the exponential law regardless of economic environment or industry with the comprehensive justification.

2.6 Discussion

This section discusses how the ABCDEF law can contribute to the industry's future development. One of the advantages of the application is that it is industrially unlimited. Some applications for forecasting or dynamic pricing that can be applied in various perishable asset industries have already been proposed (Shintani & Umeno (2022b); Shintani & Umeno (2022a)).

However, the ABCDEF law provides a booking curve, mainly used as a daily means of forecasting, with another usefulness. In the following section, we explain quantitative and informative statistics indicating people's booking patterns based on the ABCDEF law.

2.6.1 A new statistic for measuring booking window shifts

As seen in Sec. 2.3, a conjoint analysis for hotel managers reported that timing is the most important factor in pricing. In addition, complex shifts in the booking window have provided challenges for RM professionals to adapt continuously. In other words, recent changes have let us pay more attention to the booking window shifts to maintain or improve the performance of RM.

Here, we have the following proposal. While the average lead day of bookings has been used as a statistic to measure booking window shifts, we are not sure whether this is a helpful statistic. That is because the average lead day of bookings is less informative; in other words, it only has limited information about the entire booking horizon to determine the appropriate timing to take RM measures. Besides, an average lead day cannot reflect the bookings at the deadline day ($t = 0$) in which the most number of bookings emerge according to the ABCDEF law.

Based on this statement, we believe that a "booking pace baseline statistic"—a new statistic that quantifies how fast bookings build up under the current demand-supply environment—can give RM practitioners more information about people's booking patterns, that is, booking window shifts, than a conventional average lead day of bookings.

In addition, we propose that β, one of the parameters inherent in the ABCDEF law, can serve as a sample of "booking pace baseline statistics." While the parameter β reflects the environment surrounding the property in the ABCDEF law, it also illustrates patterns in people's booking horizons, which can be recognized from the shape of the function. Therefore, it has more information about the build-up pace over whole booking horizons. Table 2.5 compares the characteristics of average lead days and "booking pace baseline statistics" based on β. The booking pace baseline statistics characterized by β is considered superior to the conventional statistics in terms of the amount of information about the entire time series. The unique booking pattern for whole booking horizons periodically derived is expected to be a practical measure for RM professionals in making strategies,

Table 2.5: Comparison of the two statistics for evaluating booking window shifts; average lead days and booking pace baseline statistics given by β.

	Statistics for measuring booking window shift	
	Average lead days	β:booking pace baseline statistics
Versatility	○	○
Information about characteristics of time series	less: just one point of time	more: whole booking horizons

including pricing, in which the timing is the crucial factor.

2.7 Conclusion

This study is based on actual sales data for the two years of 2019 and 2020 from six properties in multiple industries, including the hotels and car rental fields. We investigated macroscopic aspects of booking curves, considering the difference in the economic environment characterized before, middle, and after the COVID-19 epidemic. We justified the universality of the ABCDEF law from the following three perspectives: data confirmation, modeling in the statistical physics framework, and empirical justification for the causality of the model. In particular, in the modeling section, we introduced an assumption for the environment surrounding the property (a homogeneous demand-supply bath assumption: HDSBA) and explained the mechanism that average booking curves follow exponential functions. In the empirical justification for the causality section, we reinforced the basis by investigating the correlation of two "degrees" defining a homogeneous environment and exponential property, respectively.

The ABCDEF law provides a booking curve with its usefulness besides daily forecasting in RM. The new measure for booking window shift, "booking pace baseline statistics," characterized by the parameter in the ABCDEF law, provides RM practitioners with practical information about people's booking patterns in the current demand-supply environment. It is expected to be helpful in RM in various perishable asset industries, based on selling products at the right price and timing since people's booking windows have changed due to environmental shifts.

2.8 Supplementary material

2.8.1 Proof of leading $\phi(t;\beta)$ into exponential decay functions

We show that the function ϕ, which satisfies the following Eqs. (2.8.1) and (2.8.3) in the main section, is derived uniquely into an exponential decay function.

$$\begin{aligned} &\phi(0) = 1, \quad \lim_{t\to\infty} \phi(t) = 0, \\ &\phi(t) \geq \phi(t+1) \quad \text{for } \forall t \geq 0. \end{aligned} \tag{2.8.1}$$

$$\phi(t) = \{\phi(1)\}^t \quad \text{for } \forall t \geq 0, \tag{2.8.2}$$

$$-\frac{1}{t^2}\ln\phi(t) + \frac{1}{t}\frac{\phi'(t)}{\phi(t)} = 0 \quad \text{for } \forall t > 0 \tag{2.8.3}$$

When we show $\phi'(t)$ as $\frac{d\phi}{dt}$, (2.8.3) is transformed to the following separable differentiate equation:

$$\frac{1}{\phi\ln\phi}d\phi = \frac{1}{t}dt, \tag{2.8.4}$$

$$\text{via} \quad \frac{d\phi}{dt} = \frac{1}{t}\phi\ln\phi,$$

where $\phi\ln\phi(t) \neq 0$ because of (2.8.1) and $t > 0$.

We solve the differential equation (2.8.4). Taking the antiderivative for both sides of (2.8.4) leads to

$$\int \frac{1}{\phi\ln\phi}d\phi = \int \frac{1}{t}dt.$$

We calculate respectively:

$$
\begin{aligned}
\text{[The left side]} &= \int \frac{1}{\phi \ln \phi} d\phi \\
&= \int \frac{1}{z} dz \ (\because \ \text{let} \ \ z = \ln \phi, \ \text{then} \ d\phi = \exp(z) dz) \\
&= \ln |z| + \text{const} \\
&= \ln |\ln \phi| + \text{const}, \\
\text{[The right side]} &= \ln t + \text{const}.
\end{aligned}
$$

Here, considering $\ln|\ln \phi(t)| = \ln t + \text{const}$ and $\ln \phi(t) < 0$ ($\because$ (2.8.1) and $t > 0$), we obtain the function $\phi(t)$ to follow with a certain positive constant C_0:

$$
\begin{gathered}
\phi(t) = \exp(-C_0 t), \\
\text{via } -\ln \phi(t) = \exp(\ln t + \text{const}) = C_0 t.
\end{gathered}
$$

Considering (2.8.1), the function $\phi(t;\beta)$ is uniquely determined as the exponential decay function with a certain positive constant $\beta (= C_0) > 0$:

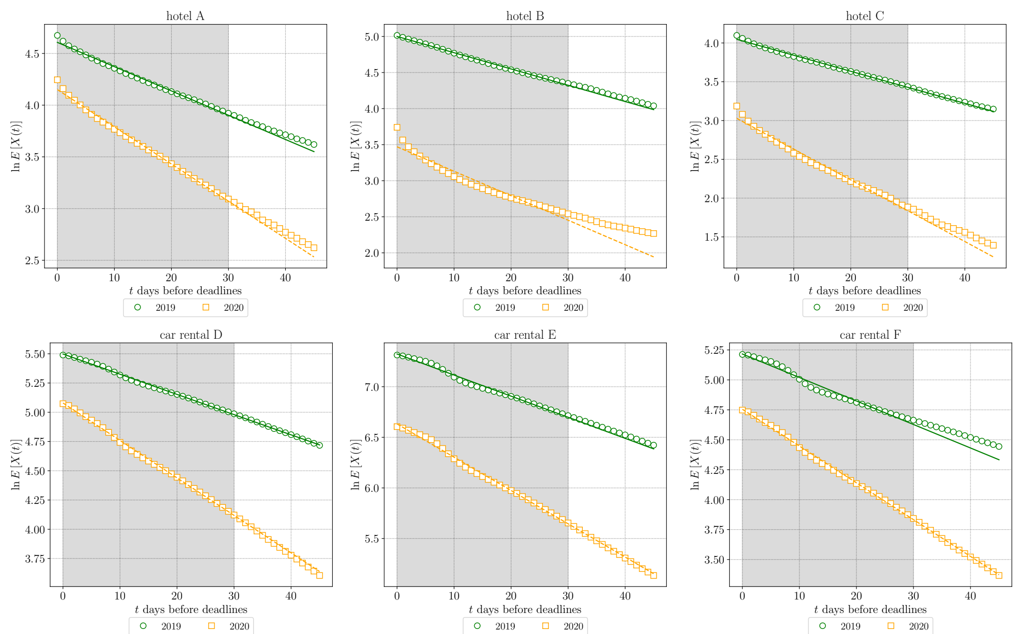
$$
\phi(t) = \exp(-\beta t).
$$

Finally, the function $\phi(t;\beta)$ satisfies (2.8.2).

Note that the function ϕ satisfies a differential equation $\frac{d\phi(t)}{dt} = -\beta\phi(t)$, which implies the property that the increase in the sums of bookings is proportional to the number of cumulative bookings.

2.8.2 Data confirmation for 2019 and 2020 with MSE and CV

In the main section, to verify the causality of the proposed model, we investigate the correlation of the following two variables: the coefficient of variation CV, which represents the quantitative degree of a homogeneous demand-supply environment, and the mean squared error MSE, which is defined as fitting deviation between exponential functions and average booking curves. Then we show the positive correlation (the coefficient of correlation is 0.79) in CV and MSE. In this supplementary document, we visualize the average booking curves for each year of 2019 and 2020 in Figure 2.6. The average booking curves deviate from exponential functions according to large MSE, especially in hotels B and C in 2020. Their environments had more significant low demand especially in the state of emergency period, which caused exceedingly last-minute As for their environments, the most significant low demand occurred mainly during the state of emergency period, which caused a considerable bias in last-minute bookings. Due to the unstable demand condition, we confirm how the average booking curves deviate from exponential functions.



	2019		2020	
	MSE	CV %	MSE	CV %
hotel A	3.61×10^{-4}	33.31	5.96×10^{-4}	54.52
hotel B	1.48×10^{-4}	15.31	6.02×10^{-3}	100.89
hotel C	2.17×10^{-4}	33.54	2.28×10^{-3}	78.37
car rental D	1.00×10^{-4}	36.80	1.49×10^{-4}	51.88
car rental E	4.02×10^{-4}	35.57	4.22×10^{-4}	62.95
car rental F	6.99×10^{-4}	33.52	2.26×10^{-4}	61.64

Figure 2.6: Figures show average booking curves and exponential functions based on annual data of 2019 and 2020. Table is the same as in Figure 5 in the main paper. The fitting interval, represented by the shaded part ($0 \leq t \leq 30$) is the same as in the main paper.

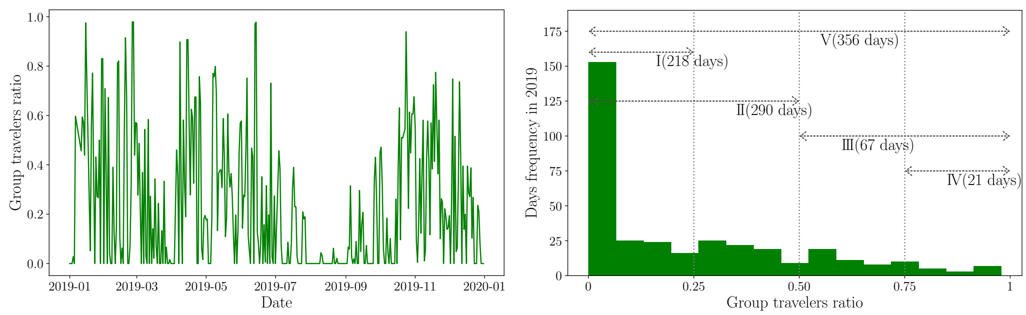


Figure 2.7: The information about group travelers ratio in hotel A in 2019. The left figure shows the ratio of group travelers in terms of used rooms unit in 2019. The right figure illustrates the histogram of the ratio shown in the left figure. The labels I, II, III, IV, and V, represent the aggregating targets for the identical and group travelers, and type II corresponds to the subject in the main paper.

2.8.3 Investigation of booking curves, including group travelers in hotel A

This supplementary document visualizes the limitation of the ABCDEF law when including group travelers with unusual booking patterns in hotel A.

In the hotel industry, group travelers whose objective includes escorted tours or school excursions have a different booking pattern from individual travelers; hotel A has mainly individual travelers and sometimes receives group travelers. Figure 2.7 illustrates the information about the group ratio in hotel A in 2019 and defines five types of aggregation patterns. Figure 2.8 shows that average booking curves with a high rate of group guests result in not drawing exponential functions. Note that type II is the same subject as shown in the main paper, and type I shows high exponential property as with type II.

These results imply that one of the limitations of the ABCDEF law results from the mixture of booking behaviors. In other words, customer attributes, which generate differences in booking behavior patterns, can become one of the elements defining a homogeneous demand-supply environment.

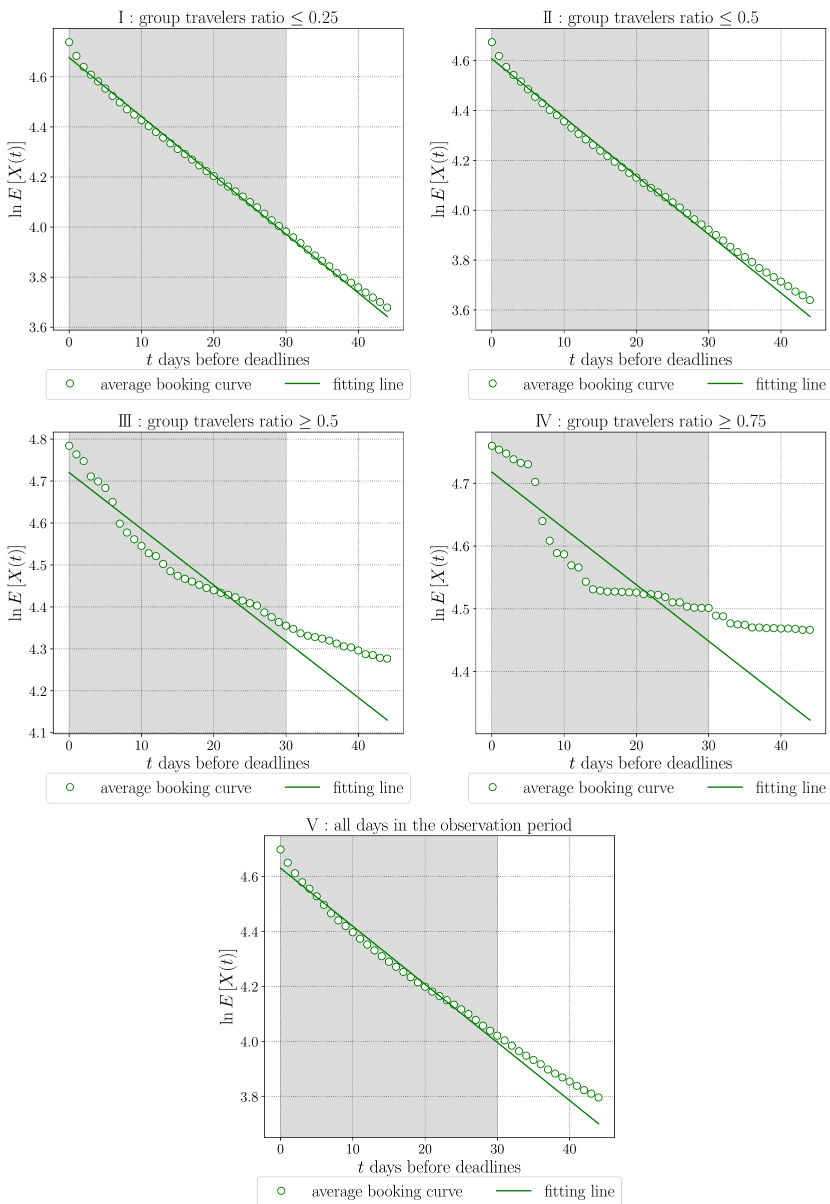


Figure 2.8: This figure illustrates the average booking curves in each aggregate pattern of I, II, III, IV, and V, defined in Fig. 2.7.

2.8.4 A segmentation-detailed analysis framework

Finally, we discuss a practical application based on the ABCDEF law. A "segmentation-detailed" analysis for booking curves provides opportunities for RM professionals to find improvement potential in terms of timing. In other words, we provide a framework to investigate factors impacting demand quantity and booking pace parameters. Thus, the analysis is based not only on "booking pace baseline statistics" $\beta(= 1/\tau)$ but also on the magnitude of demand A.

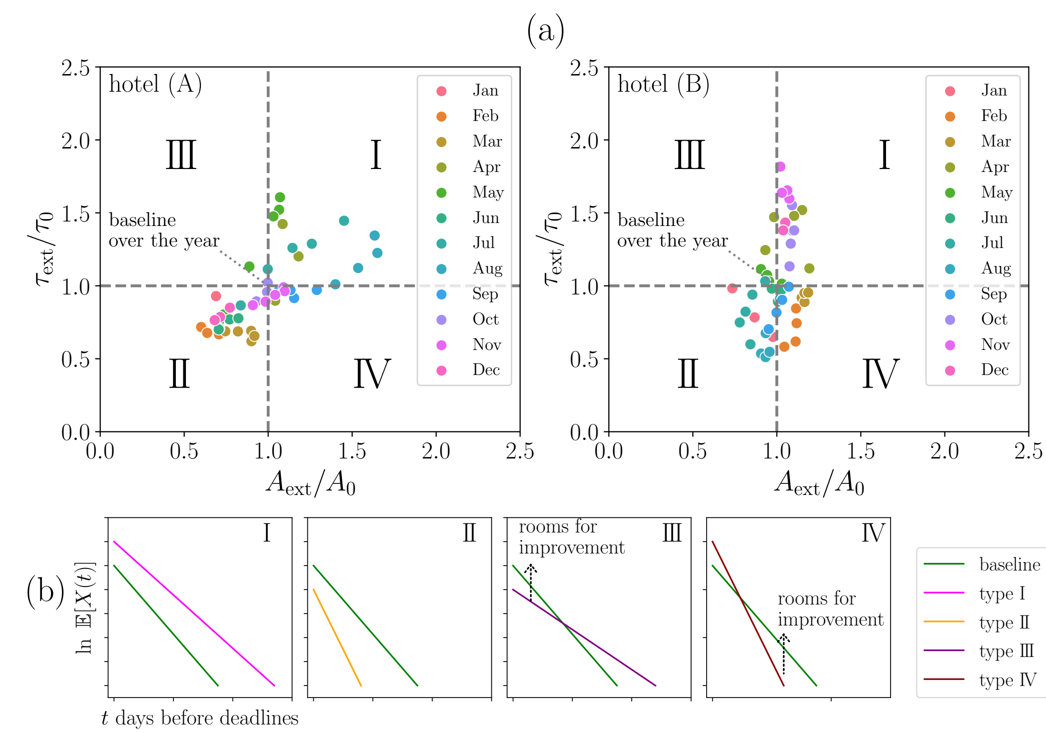
We divide the processes of proposing the analysis framework into three steps. First, we identify unique microscopic differences for each property in terms of a specific segmentation, contrary to the fact that the properties macroscopically follow the common ABCDEF law. In particular, we choose seasonality as a segment and visualize how and what factors lead to changes in parameters for each property. Second, we categorize the changes in parameters in booking curves into four types and show the improvement potential of each kind with visualizations. Third, we offer to select segments based on a variety of categories, such as products and users, in addition to seasonality. Furthermore, we have a notable segment based on booking horizon, which is derived from the "booking pace baseline statistics." According to these steps, The segmentation-detailed analysis in each property provides discovering original improvement potential and is expected to be a framework for constructing unique RM strategies.

First, while the average booking curve in the ABCDEF law is uniquely characterized by an exponential function with parameters regarding the magnitude of demand and booking pace, the daily booking curves differ for both due to some factors, including seasonality or day of week trends, as seen in Fig. 2.1. In addition, causal elements and degrees of changes in parameters also differ depending on properties or economic environments. We confirm them with an example by studying the data for the year 2019, which is a part of the historical data and generally represented "regular" demand. By applying the seasonality segment in the data, we can grasp the annual seasonal demand and booking pattern without a case of a rapid social change such as the COVID-19 epidemic.

Figure 2.9 is an example of materials for "segmentation-detailed" analysis that visualize how the parameters A and $\beta(= 1/\tau)$ change. We choose seasonality (from month to month in the figure) as a segment here. As seen in Fig. 2.9, different properties have different directions, magnitudes of scattering, and different seasonality factors that generate changes. As for seasonality, for example, the highest demand trend (Type I) was observed in July-September for the resort hotel A in the regional area and in October-December for the business hotel B in the city area. This reflects that the major purpose of use varies from one property to another; vacation at hotel A or tourism due to a particular seasonal tradition at hotel B, for instance. As for scattering direction, while an increase in A (demand grow up) is generally correlated with an increase in τ (booking window expansion), that does not appear in hotel B. This resulted from the effort for high occupancy throughout the year with RM measures, including largish dynamic pricing. As mentioned above, a comparative analysis for average booking curves based on a segment (e.g., monthly in the figure) provides microscopic trends on property-specific changes in demand and booking pace.

Second, Fig. 2.9.b and the table illustrate typical time-series shapes of average booking curves corresponding to types I - IV, and present rooms for improvement at each type, respectively. With the implication that RM professionals get an opportunity to discuss the effective measures in advance in each season resulted in types III or IV because the subsequent seasons is likely to generate the same changes as in latest year in demand and booking paces.

Third, while we adopt the segmentation unit as months, that is, seasonality, in Fig. 2.9.a, we can give a variety of other classification segments, which lead to deeper analysis. In other words, we discover segments that visualize the factor results in types III and IV by analysis based on a variety of details, such as product type (e.g., room for hotels, car type for car rentals, etc.), by sales channel (e.g., online travel agency or company website for travel), by day of the week, by the user (number of users, age, distance from facility to residence, etc.), or by segments created by any combination thereof. This supports RM professionals in finding improvement potential and considering a variety of effective measures. In addition, we have another detail about the booking horizon. Although a booking horizon is generally divided at equal time intervals (e.g., every week), it is possible to analytically divide the horizon into intervals which provide equal volume of bookings by taking advantage of the exponential law Shintani & Umeno (2022b). For example, when given τ = 51 and 5 as the number of divisions for a booking horizon, periods which emerge with equal volume of bookings are analytically determined as, from day 0 to day 10, from 11 to 25, from 26 to 45, from 46 to 81, before day 82, regardless A. All of these periods account for 20% of all the bookings on average when under the ABCDEF law. By doing so, we can adopt the division of the booking horizon as one of the segmentations for discovering what period tends to emerge more or less than the baseline (20% in the above example) of bookings. As seen in Fig. 2.2, since the values τ differ for each property or economic condition, this booking horizon segmentation gives the unique "time scale" to the environment. The only reason this notable approach is available is because the ABCDEF law describes the shape of average booking curves over the whole booking horizon, that is, the advantage of "booking pace baseline statistics". This segmentation-detailed analysis based on the ABCDEF law in each property gives discovering original improvement potential primarily in terms of timing, and is expected to be a framework for constructing unique RM strategies.



	I	II	III	IV
quantity	up	down	down	up
pace	spread	shrink	spread	shrink
status example	high demand	low demand	lack of last minute appeal	excessive last minute discount
improvement potential in RM	less	less	in last minute	in advance

Figure 2.9: An example of materials to find improvement potential in RM strategy in terms of timing. Figure (a) shows how seasonality causes changes to the demand parameter A and the booking pace parameter τ. Note that, baselines, that is, the values of A_0 and τ_0, are a result of the ABCDEF law using annual data in 2019, and parameters A_{ext} and τ_{ext} are derived from each extracted data (30 business days with every 7 days interval). Figure (b) and the table outlines the shape and characteristics for each classification based on bias in parameters, respectively.

Chapter 3

Time-Rescaling Regression Method for Exponential Decay Time-Series Predictions

3.1 Introduction

In previous studies, many methods for the analysis and prediction of stationary and non-stationary time series have been proposed. Typical examples include ARIMA, state space models, and exponential smoothing methods (Yaffee & McGee (2000); Montgomery et al. (2015); Brockwell et al. (2016); Kalpakis et al. (2001); Hyndman et al. (2008)). In this study, we especially consider the prediction of a time series that follows an exponential decay function, which is a non-stationary time series. Our prediction method can be used for analyzing the time series with the exponential law that has been discovered in various systems around the world (Evans & Evans (1955); Landau & Lifshitz (2013); Vollmer (2009); Casas & Quintanilla (2005); Karagiannis et al. (2010); Shepard (1987); Shintani & Umeno (2021)).

3.2 Methods

We introduce the exponential decay function f as:

$$f(t; A, \tau) \stackrel{\text{def}}{=} A \exp\left(-\frac{t}{\tau}\right), \tag{3.2.1}$$

where $A > 0$ and $\tau > 0$ are constants and represent the magnitude and exponential time constant, respectively. A time series where the time variable t counts down from a certain duration was prepared in Eq. (3.2.1); that is, the time t decreases with the passage of time in the time series. We study methods for predicting $f(t = 0)$ from the progress data in the interval $t^* \leq t \leq T$ for a time series that is approximately according to Eq. (3.2.1), where T and t^* are the start and current time of observation. Here, we can safely assume that the exponential time constant τ is estimated empirically because various types of data such as booking curves actually follow exponential decay functions (Chapter 2) We show two existing methods and one proposed method for time series predictions.

Table 3.1 outlines the three methods. First, EM. 1 was defined as a linear

Table 3.1: The definition of the regression target time series and regression equations for the two existing methods (EM. 1 and EM. 2) and proposed method (PM). All the methods are analytical and based on linear regression. Given a certain $t^* > 0$, we predict the value of $f(0)$ using the data corresponding to $t \geq t^*$.

Method	regression targets	regression equation	estimation for $f(0)$
EM.1	$\{[t, \ln f(t)] \mid t^* \leq t \leq T\}$	$\ln f(t) = w_1 t + w_0$	$\exp(w_0)$
EM.2	$\{[t, \ln f(t)] \mid t^* \leq t \leq T\}$	$\ln f(t) = -\frac{1}{\tau} t + w_0$	$\exp(w_0)$
PM	$\{[i, g(i) \left(\equiv f(t^i_{\text{scaled}})\right)] \mid i \text{ satisfy } t^* \leq t^i_{\text{scaled}}\}$	$g(i) = -\frac{w_0}{N} i + w_0$	$w_0 (= g(0))$

regression of the time series $\{[t, \ln f(t)] \mid t^* \leq t \leq T\}$. EM. 1 is the general method for analyzing exponential time series and has two unestimated parameters A and τ. Second, we utilized the time constant τ in EM. 2. Although EM. 2 was also defined as a linear regression for the time series $\{[t, \ln f(t)] \mid t^* \leq t \leq T\}$, and the value of the slope $-\frac{1}{\tau}$ has already been determined. In EM. 1 and EM. 2, using the characteristics of the exponential function and replacing the vertical axis with the logarithmic scale, we analytically studied exponential time series using linear regressions. Third, we propose a method that results from linear regression with the analytical rescaling of the time axis. We used a division constant N and linearly divided the value of the vertical axis $f(t)$ into N sections. For the division point i, when we defined t^i_{scaled} as follows:

$$t^i_{\text{scaled}} \stackrel{\text{def}}{=} \tau \ln \frac{N}{N-i}. \tag{3.2.2}$$

Then, $f(t^i_{\text{scaled}})$ is obtained as follows:

$$f(t^i_{\text{scaled}}) = -\frac{A}{N} i + A. \tag{3.2.3}$$

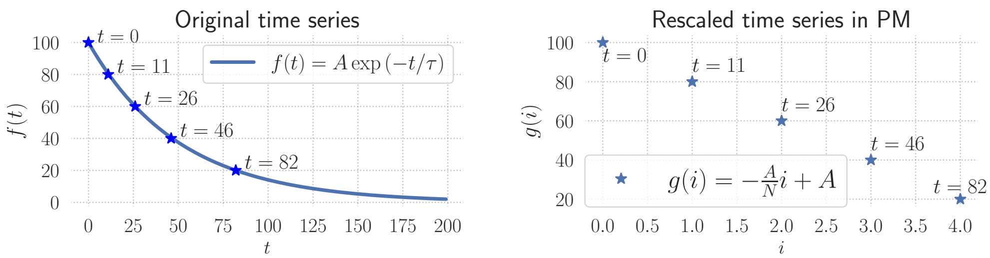
When we consider the expression (3.2.3) as a function of i and define it as $g(i)$, the time series $\{[i, g(i)]\}$ is analytically modeled using the linear regression, where $g(0)$ corresponds to the value of $f(0)$. We call our proposed method the time-rescaling regression method because we derive the regression equation from rescaling the time axis. Figure 3.1 shows an example time series in PM.

3.3 Performance evaluation

We investigate the estimation accuracy for the exponential decay time series, which is approximately expressed by Eq. (3.2.1).

3.3.1 Dataset

We generated a time series $X(t)$ based on Eq. (3.2.1) using a random-numbered time series with the following non-stationary Poisson process: We defined $q(t)$ as the number of occurrences of a certain event at time t, which is the time to a certain end time $t = 0$. Then, we defined $X(t)$ as the cumulative number of occurrences of events up to time t. Here, we assumed that $q(t)$ follows a Poisson process with $\lambda(t)$ as an average. In addition, $\lambda(t)$ is a function of time and is defined by the



i	0	1	2	3	4	$i = 0, 1, \cdots, N-1$
t^i_{scaled}	0	11	26	46	82	$= \tau \ln \frac{N}{N-i}$
$f(t^i_{\text{scaled}})$	A	$0.8A$	$0.6A$	$0.4A$	$0.2A$	$= -\frac{A}{N}i + A \equiv g(i)$

Figure 3.1: An example of the regression target time series in PM. The table shows an example of the regression target time series in the PM, where the division constant is $N = 5$, and the time constant $\tau = 51$. In addition, the figure shows a visualization of the regression target time series in the EM. 1, EM. 2, and PM, when $A = 100$.

following function:

$$\lambda(t) \overset{\text{def}}{=} \frac{A}{\tau} \exp\left(-\frac{t}{\tau}\right), \tag{3.3.1}$$

where $\tau > 0$ is a time constant, and $A > 0$ is a constant parameter that controls the number of event occurrences. Then, the probability density function of $q(t)$ is given by the following Poisson distribution:

$$P(q(t) = k; t) = \frac{\lambda(t)^k}{k!} \exp(-\lambda(t)). \tag{3.3.2}$$

With Eqs. (3.3.1) and (3.3.2), the cumulative number of events $X(t)$ is expressed as:

$$X(t) \overset{\text{def}}{=} \int_t^\infty q(s)ds. \tag{3.3.3}$$

Note that, considering the average number of events that occur at a certain $t = s$ is represented by $\lambda(s)$, when we assign $\hat{X}(t)$ as the maximum likelihood time series of $X(t)$ with the above Poisson process, $\hat{X}(t)$ is obtained as in Eq. (3.2.1):

$$\begin{aligned}\hat{X}(t) &= \int_t^{\infty} \lambda(s) ds \\ &= \int_t^{\infty} \frac{A}{\tau} \exp\left(-\frac{s}{\tau}\right) ds \;=\; A \exp\left(-\frac{t}{\tau}\right).\end{aligned}$$

Here, we generated a time series that appropriately follows the exponential decay function (3.2.1) according to the above non-stationary Poisson process with a fixed constant τ. Next, we compared the prediction accuracy of the three methods and investigated the parameters that affect the accuracy. We gave survey target parameters for the start time T of the observation and the magnitude of the number of occurrences A; thus, we prepared multiple time series with different values of T and A. Figure 3.2 shows some example time series of $q(t)$ from the above non-stationary Poisson process. The time-series data set to be predicted is obtained by accumulating a random-numbered time series $q(t)$ using equation (3.3.3). Then, the integration interval is given by $t^* \leq t \leq T$ using the current time t^* and start time T of the observation.

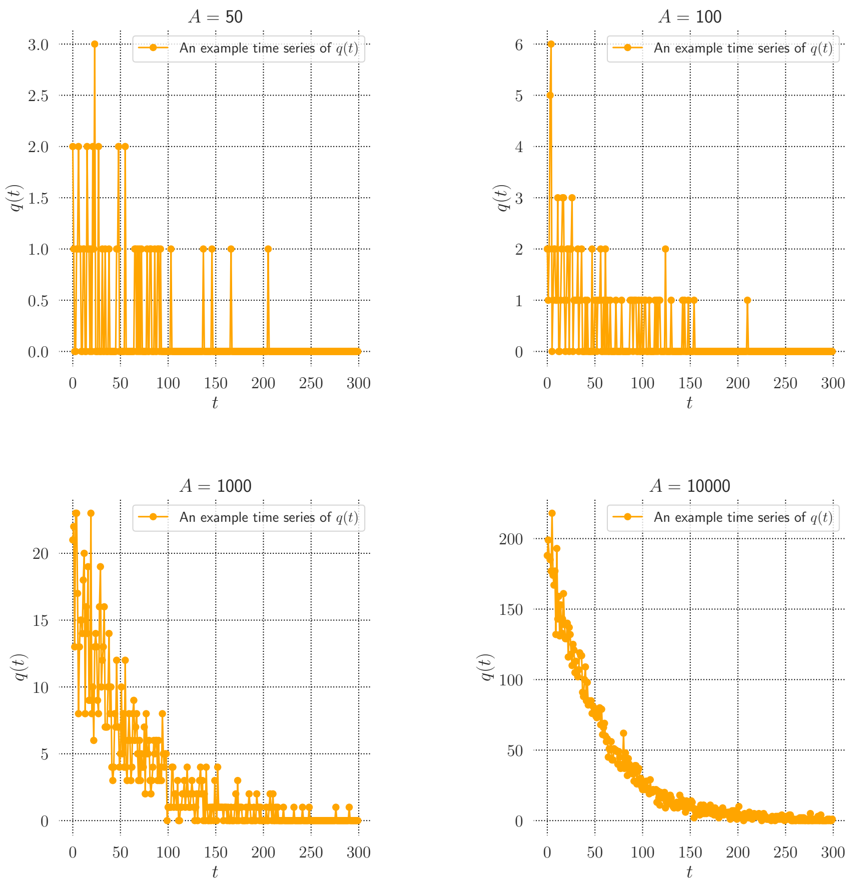


Figure 3.2: Examples of the time series $q(t)$ obtained using the non-stationary Poisson process model in Eqs. (3.3.1), (3.3.2) and (3.3.3) with the parameters $\tau = 51$, $T = 300$ and $A = 50, 100, 1000, 10000$.

3.3.2 Verification

We investigated several time series derived from the non-stationary Poisson process with different values of A and T. To evaluate prediction accuracy, we used the mean absolute percentage error ($MAPE = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right|$) and the error rate: ($\frac{y_i - \hat{y}_i}{y_i}$) for n samples of the true value y_i and predicted value $\hat{y}_i$ in order to normalize differences that depend on the value of A. Figure 3.3 shows the relationship between $MAPE$ and A and T. Figure 3.4 shows some concrete examples of the error rate histograms with a fixed $T = 300$. The length of the time series to be regressed is proportional to the size of the starting point T, especially in EM.1 and EM. 2. As expected (from the central limit theorem), it was confirmed that the prediction accuracy of EM. 1 and EM. 2 was inversely proportional to the size of T. On the other hand, the prediction accuracy of the PM was less than half for small T, regardless of the value of A. Note that the superiority of the PM is remarkable for small T, which indicates less data.

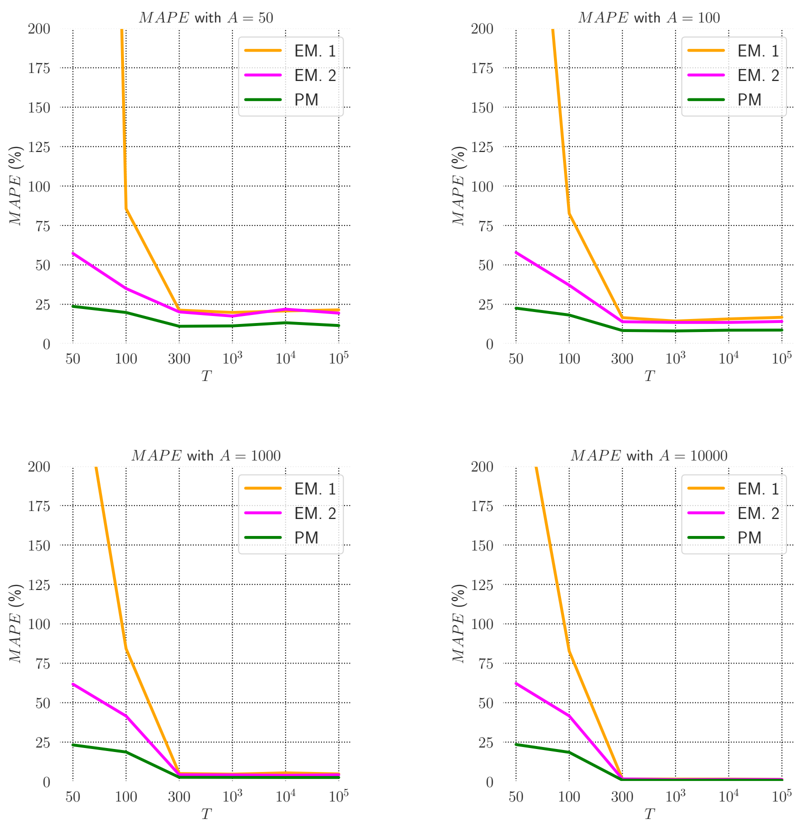


Figure 3.3: The relations between $MAPE$ and some parameters of A and T. This figure illustrates $MAPE$ calculated from the three methods for the time series generated based on the non-stationary Poisson process with some values of A and T, where $\tau = 51, t^* = 31, N = 11$ and the number of time series to be averaged was 200. Then, t^i_{scaled} were calculated as rounded values, and we adopted $\ln(x+1)$ to make x logarithmic to avoid calculation errors in EM. 1 and EM. 2. It was confirmed that $MAPE$ in EM. 1 and EM. 2 decreases as T increased, because T represents the length of the time series used for regression. On the other hand, the prediction accuracy of PM was less than half for small T, regardless of the value A, which indicates that PM is remarkably superior to existing methods for small T, which indicates less data.

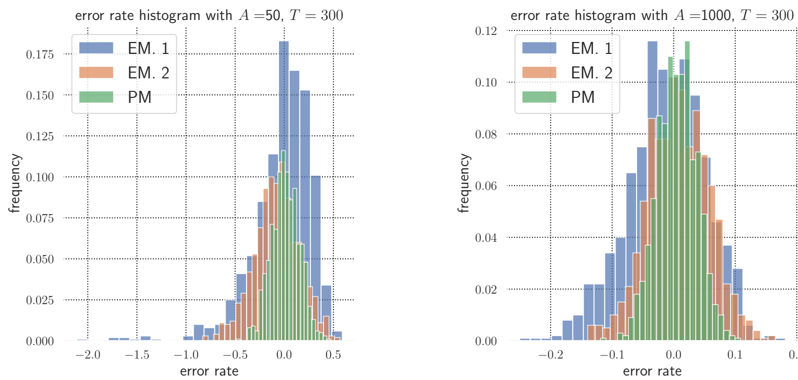


Figure 3.4: Examples of the error rate histograms of three methods for predicting time series with parameters $\tau = 51, N = 11$ and $t^* = 31$, where the number of time series is 1000. It was confirmed that the PM has a high prediction accuracy regardless of the value of A.

3.4 Application for real data

Here, we apply the three methods to actual data and compare their accuracies. We introduced the following real-data time series according to the exponential decay function (3.2.1) in a previous study.

In the service industry, it is found that the booking curve universally follows an exponential decay function (Shintani & Umeno (2021)). Here, the booking curve represents the cumulative sum of the number of reservations toward the usage day, which is used in many industries, including hotels and rental cars. Note that when we assign $q(t)$ as the number of reservations t days before the usage day, the booking curve has the same structure as the non-stationary Poisson process model of Eqs. (3.3.1), (3.3.2) and (3.3.3).

Here, according to the previous research, although the booking curves follow the exponential decay functions when they are averaged over a period, such as some usage days; in this study, we assumed that the booking curve of each usage day generally follows an exponential function; therefore, we used the three methods to forecast the quantity demanded for each usage day by using the data of the booking curve observed up to t^* days before. We used the actual sales data in hotels and the car rental industry.

The target properties were three hotels and three car rentals, and the acquisition period of data was all of 2019 (excluding chartered days and non-business days). Figure 3.5 and Table 3.2 show the accuracy of the demand forecast obtained using the three methods, where $t^* = 30$. The figure shows the prediction error rate histograms for each method, and the table shows the results for $MAPE$. The prediction accuracy with PM is approximately 10% to 30%, which confirms that it is sufficiently superior to other methods.

In addition, these accuracies were almost as accurate as those obtained using other, multi-variable, nonparametric machine learning forecasting models (GBDT algorithms represented by Catboost and Xgboost). The results show that the model can forecast with high accuracy using parametric and a small number of variables, taking advantage of the universal law of booking curves. Hence, this model is expected to be used as a baseline for forecasting, and more advanced forecasting models can be constructed by considering various external information.

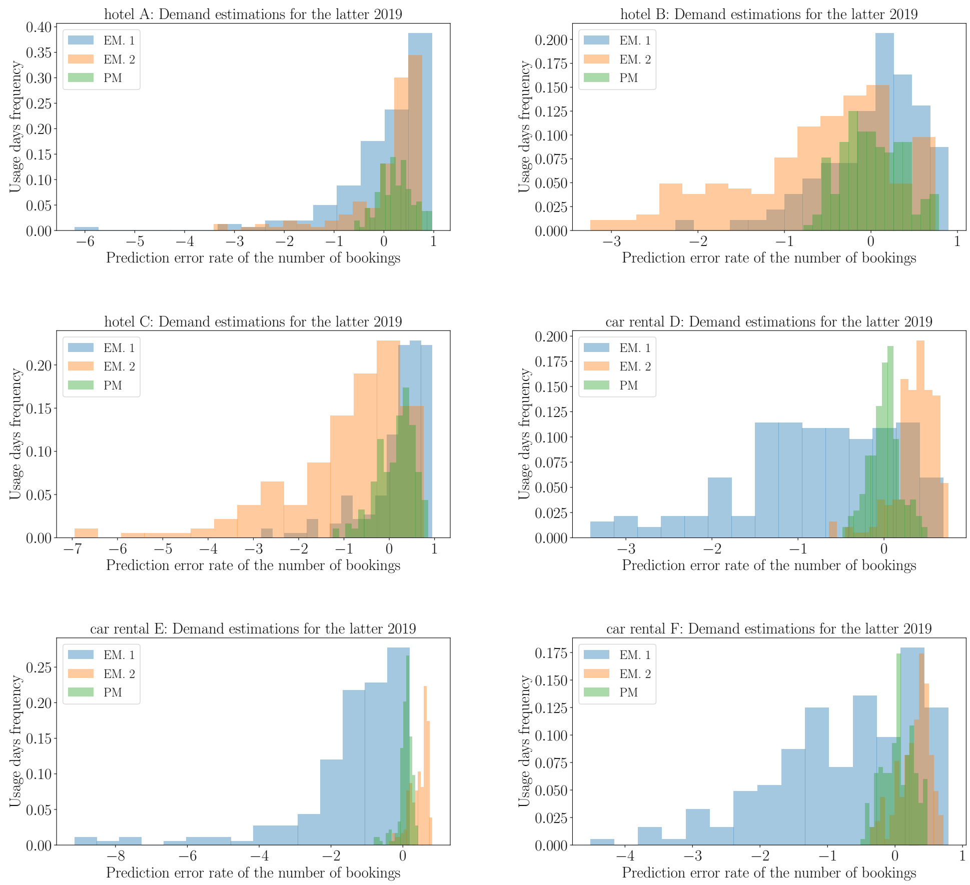


Figure 3.5: The results of the predictions with the three methods for actual booking data time series. We predicted the reservation demand forecast on the usage days of each property, where the data can be regressed in the interval $t^* \leq t$. Then, we used $t^* = 30$ and the division constant $N = 13$. We prepared a full-year reservation data set in 2019 based on the previous study (Shintani & Umeno (2021)) and divided it into two periods. One is the first half year, that is, from January 1 to June 30, and the second is the latter half year, that is, from July 1 to December 31, 2019. Note that because the time constant τ must be obtained empirically in the EM. Two and PM, we calculated τ by fitting the averaged data for the first half year into the linear regression with interval the $0 \leq t \leq 60$; the calculated parameters τ and A are listed in the table. We predicted the reservation demand forecast for the latter half of 2019 with the above estimated τ. It is confirmed that PM has the smallest error in all six properties.

Table 3.2: This table shows the $MAPE$ in applying the three prediction methods to the actual sales dataset. As the same to the results in Fig. 3.5, the proposed method shows high prediction accuracy in all the properties.

Target		$MAPE$ (%)		
Property	Area	EM.1	EM.2	PM
hotel A	Tochigi	64.88	57.79	33.08
hotel B	Osaka	43.92	82.32	28.78
hotel C	Kyoto	57.87	110.74	36.50
car rental D	Naha	92.20	39.55	13.99
car rental E	Island Ishigaki	143.65	46.99	16.52
car rental F	Island Miyako	96.00	32.81	19.51

3.5 Discussions

3.5.1 Mechanism of the proposed method

Here, we explain why the prediction accuracy is improved by regression using time-scaled data, such as the proposed method. In general, to improve the accuracy of time-series predictions, increasing the magnitude of influence as the time gets closer to the prediction target time has been proposed, as represented by the exponential smoothing method. Note that, the linear regression approach using logarithmical scale for exponential decay time series as in EM. 1 and EM. 2 results in the same weights of the coefficients in the time series regardless of the time; therefore, the existing methods reduce the magnitude of influence for time closer to the prediction target time and result in reduced prediction accuracy.

On the other hand, although the weights of the coefficients are uniform, the data points used for regression can be analytically designed to be densely distributed in a small range of t. As a result, the magnitude of influence at a time close to the prediction target time would be relatively large, which is similar to the exponential smoothing method. This is a reason why PM results in better prediction accuracy than the existing methods.

3.5.2 Attempt for practical use with clients

The application of the model proposed in this study, especially forecasting combined with the ABCDEF law as shown in Sec. 3.4, was experimentally tested to verify the usefulness of predicting the number of sales for RM with the cooperation of several clients. That is because, as mentioned above (Chapters 1 and 2), forecasting is the essential element of RM operations. We believe that our forecasting method can be proposed as a business service to support RM operations.

The verification method was to collect daily reservation and cancellation data on the previous day, perform forecasting using the process described in this chapter, and provide the information via the web-based information system. During the test period, RM practitioners daily checked the forecasted data. They used the information to help their RM operations in properties, and we received feedback periodically, such as every two weeks. Table 3.3 shows the test duration and participating clients. The customer responses gave several good points and requested improvements, listed below.

Table 3.3: Information about the test on forecasting using the proposed methodology.

Target properties	Period start	Period end	Learning period for determining τ
hotel A	2020.9	2020.12	2020.7 - 2020.11
car rental D	2020.12	2021.2	2020.8 - 2020.11
car rental E	2020.12	2021.2	2020.8 - 2020.11
car rental F	2020.12	2021.2	2020.8 - 2020.11

- Good point
  - The logic used for forecasting is not a black box, but rather, it is based on past trends in the booking curve, making it easy to understand.
  - Linearly visualized predictions lead to improved reliability.
- Requested improvements

- The accuracy of the forecast was worse than expected before the experiment.
- Even if more accurate predictions were given, they are too macroscopic information to be useful as necessary information for RM operations.

The first request, deterioration in forecast accuracy, particularly concerning requested improvements, was due to extreme changes in economic conditions. The test period included the end of the tourism campaign, shown as "Go to travel" in chapter 2 due to the increase in COVID-19 infections. Hence people's traveling and booking patterns significantly changed during the test period, and the difference in the trend of the booking curve between the learning periods and the test periods was significant.

Regarding the second point that the RM measures are too macroscopic, the elements which RM practitioners focus on are a wide variety, such as products, channels, and consumer preferences. In addition, practitioners investigate the cause of changes in each factor, considering competitors' prices and events in the area. Therefore, practitioners told that the forecasting of the entire property is incomplete and difficult to use for RM, which needs a comprehensive understanding of the property's environment.

These requests were valuable opinions in creating a helpful service. We want to refer to them as we consider the value of offerings in the future.

3.6 Conclusion

We analytically introduced two existing methods and one proposed method for time-series prediction according to the exponential decay function. All the methods were linear regression models from rescaling one of the axes of the time series, it was the times axis that was rescaled in the proposed method. From using a non-stationary Poisson process to generate a random-numbered time series following an exponential decay function and comparing the prediction accuracy of the three methods, we showed that the proposed method provides the highest prediction accuracy even when using a part of the time series. In addition, we verified the superiority of the proposed method in predicting the real sales data. Finally, we explained why the proposed method resulted in a higher prediction accuracy than the existing methods and confirmed the generality of the proposed method. Our proposed method can be applied as an analytical time-series prediction method for real data expressed by exponential functions, which are ubiquitous in the world.

In addition, we tested the usefulness of forecasting combined with the ABCDEF law in the hotel and car rental industry. As a result, although we recognize that the current forecasting does not reach a practical level for RM practitioners in their day-to-day work, we find some improvements with the cooperation of clients.

Chapter 4

General Dynamic Pricing Algorithms Based on Universal Exponential Booking Curves

4.1 Introduction

Dynamic pricing is a pricing strategy for optimizing profits, sales, and quantity demanded, and its algorithms have been proposed in previous studies (Den Boer (2015b); Qiang & Bayati (2016); Elmaghraby & Keskinocak (2003a); Umeno et al. (2019)).

In this study, we propose a new dynamic pricing algorithm that can be applied based on the universal exponential law of booking curves in services with reservations, which has been discovered in recent research Shintani & Umeno (2021). In a business environment with dynamic pricing, it is difficult to analyze the relationship between price and demand based on fundamental microeconomic principles. We confirm an example based on the real data of a hotel that has actually introduced dynamic pricing. Moreover, for the practical use of our proposed algorithm, we propose a parametric model for analyzing the relationship between price and demand with learning from historical data. A dynamic pricing algorithm using the proposed model that can continuously update the optimum parameters with learning is expected to be a new practical pricing strategy.

4.2 The fundamental principle and reality

As a fundamental principle in microeconomics, the law of demand states that prices are inversely proportional to quantity demanded under certain conditions (Mankiw (2014); Black et al. (2012)); Fig. 4.1(**a**) illustrates the statement of the law where the non-increasing function is called a demand function (Mankiw (2014); Black et al. (2012); Besbes & Zeevi (2009)). Fig. 4.1(**b**) shows the

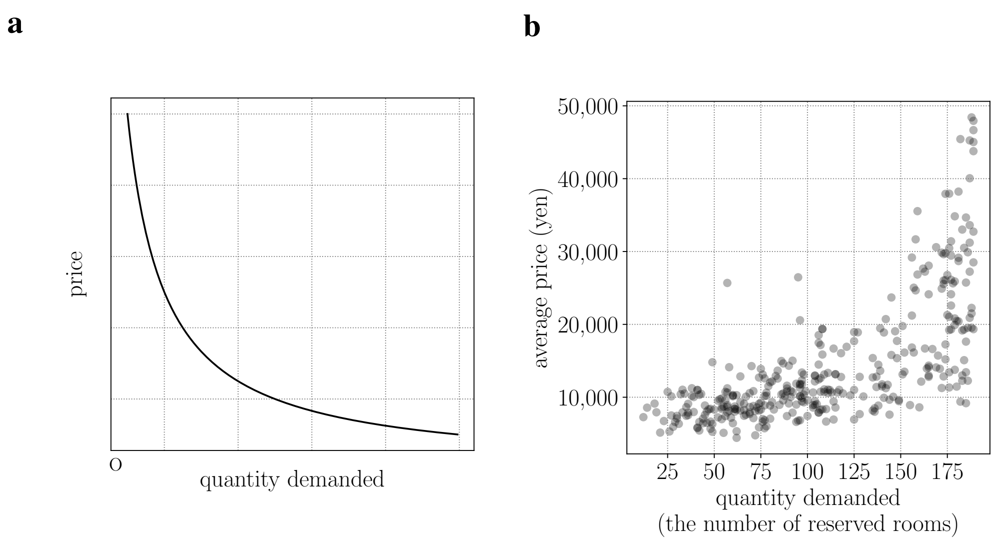


Figure 4.1: The relation between price and quantity demanded. Figure 4.1(**a**) shows the concept of the law of demand, known as the fundamental principle between demand and price. Figure 4.1(**b**) shows the relationship between the reserved number of rooms per day and the average price per room a day based on the real data obtained in 2019 in the hotel A.

relationship between the daily number of reserved rooms and average sales price per reserved room a day in a certain resort hotel (hotel A) in Japan in 2019, and it evidently contradicts the statement of the law of demand that the higher price, the lower quantity demanded. However, as expected, the reason is that the premise of the law of demand—the same for all other conditions—is not satisfied; the fluctuating hotel demand owing to weekends and seasonality causes the staff in the hotel to adjust prices to optimize the sales strategy considering the finite rooms in the facility. The efforts to optimize the sales strategy, including price adjustments according to the demand situation, are referred to as revenue management in the hotel industry (Ivanov & Zhechev (2012); Weatherford & Kimes (2003b)). The revenue manager in the hotel A that played a role in revenue management carried out all of the sales strategy decisions manually; therefore, we were not involved in the decision-making process vis-à-vis prices. Revenue managers are required to evaluate proper prices according to their purposes, such as optimizing profits, sales, and quantity demanded, based on historical sales data. To evaluate how the quantity demanded changes in response to a change in price, there is an approach in microeconomics that involves using the price elasticity of demand, which is the value derived from the slope of the demand function and indicates the quantity demanded response rate with a 1% change in price (Mankiw (2014); Black et al. (2012); Sharma (2020); Parkin et al. (2007)). Therefore, the difficulty in obtaining a proper demand function in a dynamic pricing environment as described above results in the difficulty in the continuity of carrying out dynamic pricing. Thus, we propose a dynamic pricing algorithm based on the universal law in the service industry with reservation, as we introduce below.

4.3 Exponential laws of booking curves

According to a previous study, it has been discovered that booking curves in service industries with reservations, such as hotels and rental cars, are universally represented by exponential functions (Shintani & Umeno (2021)). A booking curve $X(t)$, which is a time series of the cumulative number of reservations up to the usage day, is represented as follows:

$$X(t) = \int_t^\infty q(s)ds, \tag{4.3.1}$$

where $q(t)$ is the number of reservations t days before the usage day. In previous studies, the average booking curve for time series $\{X(t;n)|n = 1, 2, \cdots, N\}$ for N days, which is defined as $\mathbb{E}[X(t;n)] \overset{\text{def}}{=} \frac{1}{N}\sum_{n=1}^{N} X(t;n)$, is expressed as the following exponential function:

$$\mathbb{E}[X(t)] \simeq A \exp(-\beta t), \tag{4.3.2}$$

where, A is a parameter that depends on the magnitude of demand and the capacity of the facility, β is an environment variable that surrounds the property; the larger parameter β, the higher the demand because of the high rate of reservations beforehand. It is also shown that the more stable β is throughout the observation period, the closer the average booking curve is to the exponential function (Shintani & Umeno (2021)). Figure 4.2 illustrates examples of real booking curves obtained in the hotel A in 2019, revealing that parameter β is sufficiently stable in 2019, as shown in Fig. 4.2(**b**) (Shintani & Umeno (2021)).

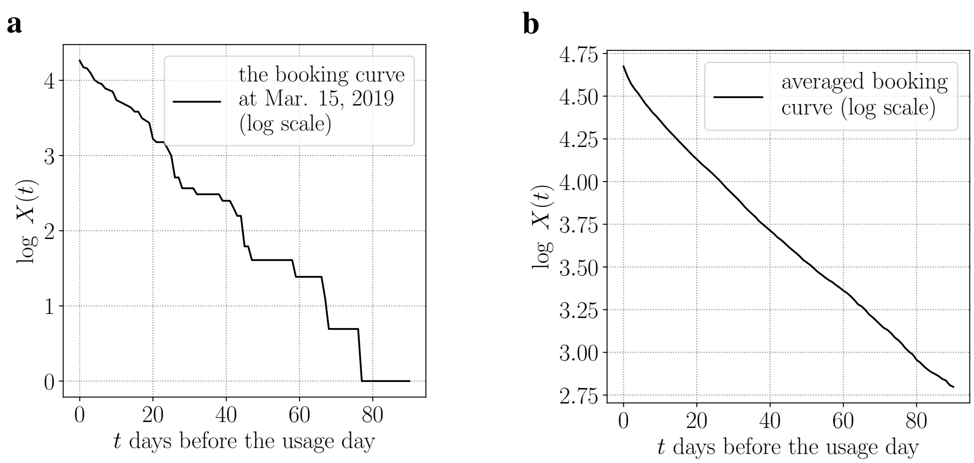


Figure 4.2: Examples of real booking curves. Figure 4.2(**a**) illustrates an example of a booking curve at March 15, 2019. Figure 4.2(**b**) illustrates the average booking curve in 2019, which shows the exponential laws as has been revealed in previous studies.

In addition, according to previous research, a time series prediction method for improving the accuracy of demand forecast using the exponential function law of booking curves has been proposed (Shintani & Umeno (2022b)). This approach is referred to as the time-rescaling regression method. According to this, when we model a booking curve of a certain usage day i as:

$$X(t;i) \simeq A^i \exp(-\beta t), \tag{4.3.3}$$

where A^i is the total quantity demanded, the method gives a high accuracy of predicted A^i at the time t^* (> 0) than general ones using booking curve data observed up to time t^* days before the usage day.

4.4 Concept of dynamic pricing based on the ABCDEF law

The universal booking curve exponential law leads to a concept of a dynamic pricing algorithm (Umeno et al. (2019)). The objective is to optimize the number of sales products with a finite inventory and sales horizon depending on the booking curve process. Given a finite inventory $Q_{\max}$, we consider a situation in which we have a booking curve with the sales price $\text{Price}_{\text{before}}$ at t^* days before the usage day. Based on Model (4.3.3) with the time series over $t \geq t^*$ and the time-rescaling regression method (Shintani & Umeno (2022b)), we obtain the predicted quantity demanded A^i_{pred}. Here, given the quantity targeted $Q^i_{\text{target}} \leq Q_{\max}$ on the usage date i, we have an opportunity to adjust the price by comparing A^i_{pred} and Q^i_{target} for each usage day. This is the proposed dynamic pricing algorithm based on the exponential law of booking curves, and the Fig. 4.3 shows the concept.

To practically utilize our algorithm, however, there are two issues that must be considered. First, we need to provide a way to simulate the magnitude of price changes in response to demand. Second, it is difficult to give an optimal time for changing prices. As expected, the smaller t^* of prediction time with changing price, the higher accuracy of the predicted quantity demanded A^i_{pred} and the larger changes in price to reach the quantity targeted Q^i_{target}; the larger t^* is the opposite. Regarding this issue, we need another objective function, such as maximizing the

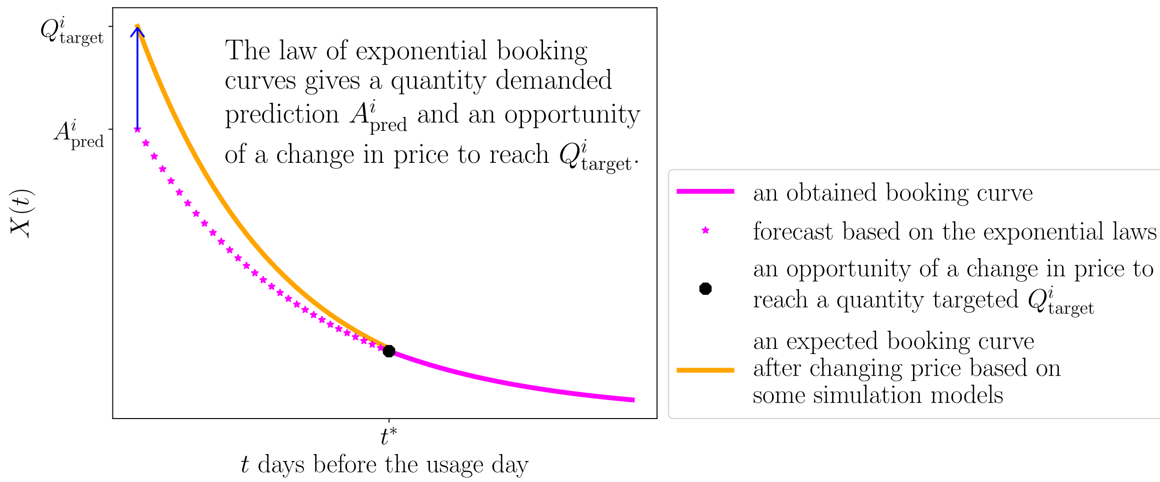


Figure 4.3: The concept of our proposed dynamic pricing algorithm. The exponential booking curve law provides a quantity predicted A^i_{pred} based on a booking curve observed up to time t^* and an opportunity to adjust the price to reach the quantity targeted Q^i_{target}.

expected revenue, to give the optimum price in the business environment. Thus, it is required to extend to an algorithm that can handle price changes more than once. Among these issues, the first one—how to evaluate the response in demand by changes in price—is introduced below with a learning system based on the historical data of changing prices.

4.5 Learning model based on historical data with changes in price

To practically utilize the proposed algorithm, it is necessary to simulate changes in demand due to price changes. Here, we propose a learning model that quantitatively evaluates the magnitude of changes in demand on changes in price based on the historical data .

We confirmed in the Fig. 4.1(**b**) that the revenue manager in the hotel A sold rooms at different prices each day during 2019. Furthermore, the revenue manager changed the price in the middle of the sales period depending on the progress of the sales situation. Figure 4.4 illustrates the historical data of price changes actually carried out by the revenue manager in the hotel A and a concrete example that shows how the price changes affect the booking curve.

As shown in the Fig. 4.4(**b**), it can be confirmed that the slope of the booking curve represented on the logarithmic scale, which corresponds to parameter β, changes before and after the price change. We evaluate the change in a booking curve caused by changes in price for all of the days represented in Fig. 4.4(**a**), which results in employing the learning model to simulate the magnitude of the response in demand on changes in price. Note that we assume in the proposed algorithm that the rate of changes in parameter β with respect to the rate of changes in price does not depend on the demand situation or the price at the time. This assumption makes it possible to evaluate the relationship between price and demand, although we encounter difficulties in analyzing a demand function correctly, similar to Fig. 4.1(**b**). Figure 4.5 shows how to evaluate changes in parameter β before and after changes in price.

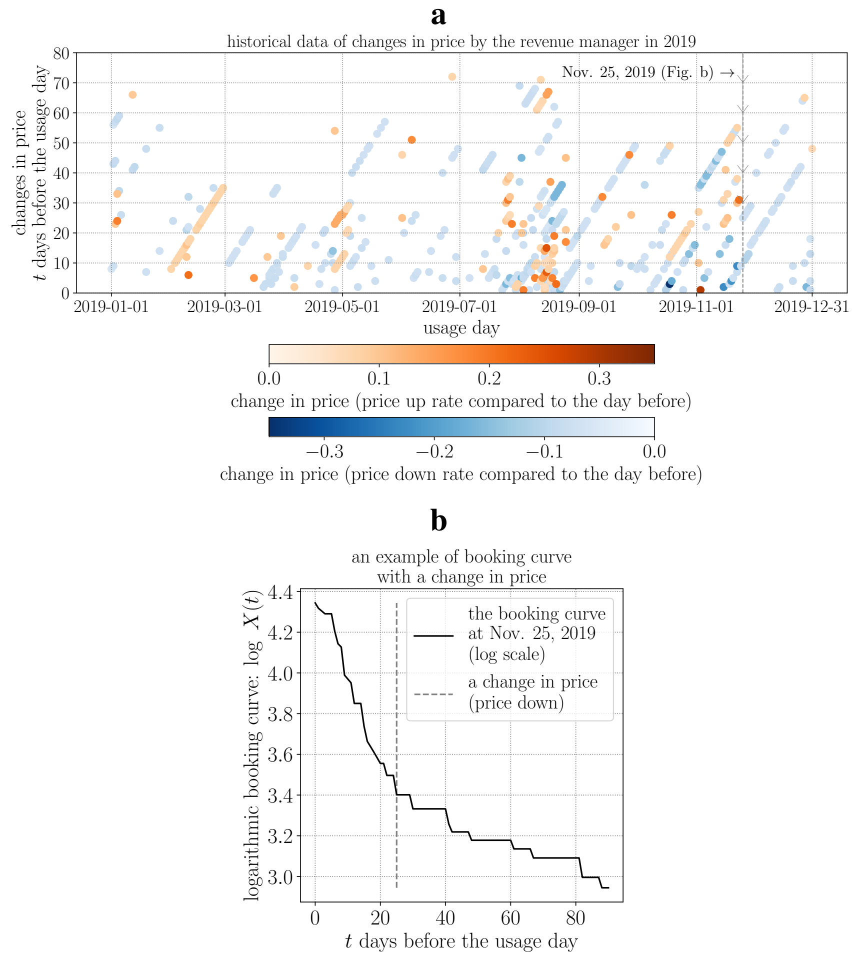


Figure 4.4: Actual historical data of changes in price by the revenue manager in the hotel A. Figure 4.4(**a**) shows the price change history for the standard room that the revenue manager in the hotel A carries out in 2019. All of these pricing phenomena were independent to us authors. Because the revenue manager changed the prices for several days or weeks at a time, we see the dots to line up in an upward direction in Fig. 4.4(**a**). Figure 4.4(**b**) shows an example of one day (Nov. 25, 2019) and the process of the booking curve. After lowering the price, the slope—corresponding to parameter β—in the booking curve expressed on the logarithmic scale has changed; therefore, it is confirmed that the price change affects the quantity demanded through changes in β.

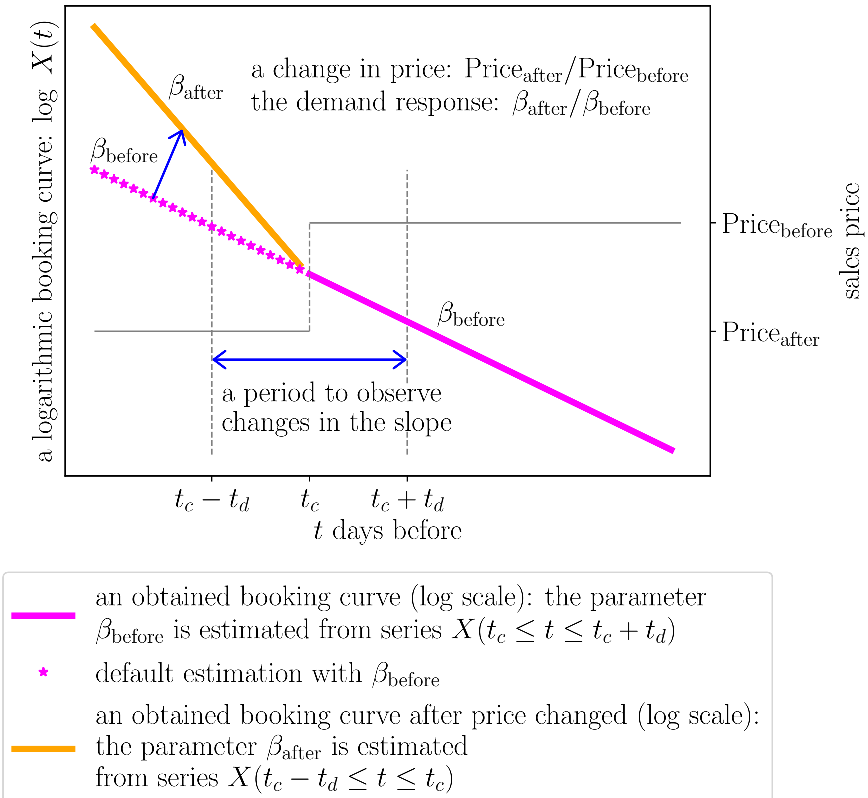


Figure 4.5: The definition of demand responses according to changes in price. The variables or parameters used before and after changing price provide the definition of the demand response according to changes in price. The parameters t_c and t_d give a day of changing price and an interval to calculate the changes in slopes, respectively. We analyze all of the changing price data represented in Fig. 4.4(**a**) using the definition.

Let $\mathrm{Price}_{\mathrm{before}}$ and $\mathrm{Price}_{\mathrm{after}}$ be prices before and after changes, respectively; then, we obtain the rate of changes in price as $\mathrm{Price}_{\mathrm{after}}/\mathrm{Price}_{\mathrm{before}}$. In addition, we define β_{before} and β_{after} as the parameters estimated from the partial booking curves, respectively, obtained during the period before and after changes in price, then we obtain the rate of changes in demand depending on β as $\beta_{\mathrm{after}}/\beta_{\mathrm{before}}$.

Here, we model the relationship between changes in price and the demand response as a function. Note that the term demand response here does not refer to demand response in economic terms but rather to a change in the number of sales, that is, responses on quantity demanded, based on an engineering perspective. Considering that no changes in price result in no response in demand, we define $f_\alpha(x)$ based on a rectangular hyperbola as one of demand curve models as follows:

$$f_\alpha(x) \stackrel{\text{def}}{=} 1/x^\alpha. \tag{4.5.1}$$

where, α is an *elasticity* parameter. We estimate $\hat{\alpha}$ by the regression using a set of points $(\text{Price}_{\text{after}}/\text{Price}_{\text{before}}, \beta_{\text{after}}/\beta_{\text{before}})$ derived from the historical data represented in Fig. 4.4(**a**); Figure 4.6 shows the result.

The estimated function $f_{\hat{\alpha}}(x)$ makes it possible to simulate the magnitude of the response in demand due to changes in price. When we obtain parameters A_{pred} and β_{before} based on Model Eq. (4.3.3) using the booking curve observed up to the time t^*, we can calculate the quantity updated $\overline{A_{\text{pred}}}$ by price changes as:

$$\overline{A_{\text{pred}}} = A_{\text{pred}} \exp\left((f_{\hat{\alpha}}(R_{\text{price}}) - 1)\beta_{\text{before}} t^*\right), \tag{4.5.2}$$

where $R_{\text{price}} \stackrel{\text{def}}{=} \text{Price}_{\text{after}}/\text{Price}_{\text{before}}$. This is derived from solving the following equation considering the updated booking curve through the point $(t^*, A_{\text{pred}} \exp(-\beta_{\text{before}} t^*))$:

$$\overline{A_{\text{pred}}} \exp(-f_{\hat{\alpha}}(R_{\text{price}})\beta_{\text{before}} t^*) = A_{\text{pred}} \exp(-\beta_{\text{before}} t^*).$$

This is the proposed model to evaluate and simulate the magnitude of the demand changes in response to changes in price using historical data. Even if the property has multiple types of products such as standard or luxury rooms in a hotel, this model can be applied using the booking curve for each product. In actual dynamic pricing, although there are various purposes, such as optimizing sales, profits and quantity demanded, this model is expected to provide useful simulations in any case. Actually, the proposed model requires some improvements, such as a better definition of the regression function. These constitute our future work.

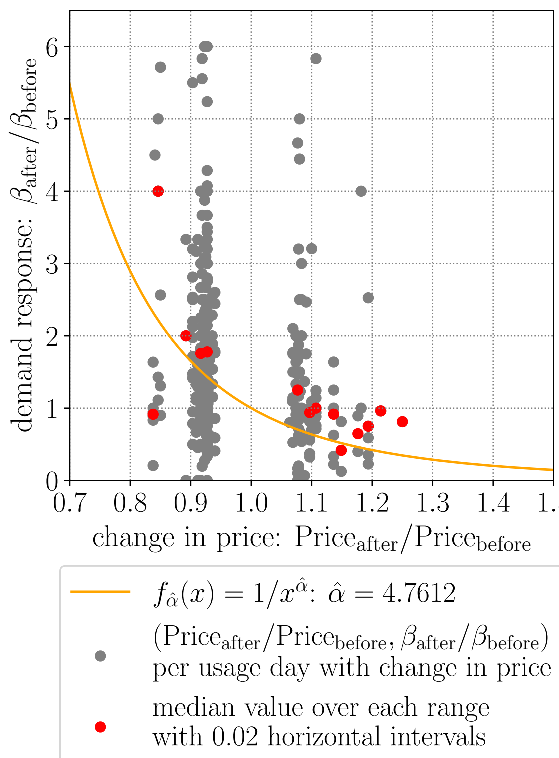


Figure 4.6: Parameter $\hat{\alpha}$ estimation of $f_{\hat{\alpha}}(x)$ for demand response according to change in price modeling. The gray dots represent the change in demand for the price change calculated based on the definition in Fig. 4.5 with respect to the daily data shown in Fig. 4.4(**a**). The reason for many changes in price data around 0.92 was because the revenue manager in the hotel had many opportunities to reduce prices by approximately 8 %. The red dots represent the median values within the range of 0.02 intervals in the horizontal axis. The curve line is the function $f_{\hat{\alpha}}(x)$ with estimated α according to the least regression for red dots. Here, gray dots represent the days that have changing prices between 10 and 90 days before the usage day. We set $t_d = 10$.

4.6 Discussion

4.6.1 Development for practical use

In the above section, we introduced a mathematical procedure for dynamic pricing, which is based on the ABCDEF laws. However, only statistical-based processing is not yet practical in a business environment and has issues to be overcome.

In addition to the experiments described in Sec. 3.5, the dynamic pricing algorithm and the method of evaluating price elasticity proposed in this chapter also had the opportunity to receive feedback on improvements toward the development of a solution to support RM operations, based on the cooperation of our clients.

In particular, the model for elasticity between price and number of sales was evaluated as seemingly challenging to give a proper and parametric model. Besides the accuracy of the values, several other suggestions for improvement were received; the model is unreliable based only on time-series patterns of historical booking curves. The demand condition is analyzed in detail by investigating various factors such as product sales, customer preferences, and competitors' prices. Therefore, using only time-series patterns as the basis for the forecast is still unreliable. These points are common to the requests regarding testing presented in Sec. 3.5, and it became clear that they are critical points.

Considering valuable requests obtained from our partners, we need two critical challenges in proposing a practical dynamic pricing algorithm in a business environment. First, a suggestion derived from just statistical laws has less familiar logic for RM practitioners. Second, a single requested price based on a property-specific booking pace is too macroscopic to be useful as necessary information for RM operations. That is because the bookings in the property consist of various factors (e.g., multiple consumer attributes, multiple types of products), which RM always comprehensively consider. In other words, these issues call for more familiar logic and detailed segmentation to describe the sales environment properly. Then, the proposed algorithm can respond to these critical issues as follows.

Although the ABCDEF law is justified in a whole property under a condition of a homogeneous demand-supply environment in Chapter 2, the law can be applied to a segmented group of customers who have a homogeneous booking pattern based on its universality. In other words, we can provide a unique marketing measure and price for each segmented group of customers with each booking pace parameter $\tau(= 1/\beta)$ in the ABCDEF law.

We show two existing approaches and a new one for segmentation. For examples of existing ones, the segmentation strategy has been introduced in the airline and hotel industries. The airline industry has already adopted a popular marketing strategy based on a difference in the booking lead time days among customers (Relihan III (1989); Sa (1987); Kimes (1989); Subramanian et al. (1999)). For airline RM professionals, business travelers and pleasure travelers can be easily split into separate groups because they have different booking lead-time days; business travelers tend to book at the last minute, and pleasure travelers in advance. That results in the different values of τ for each customer group, and RM professionals have different marketing or pricing plans for the different types of customers. The same structure, in which the overall bookings are composed of consumers with different τ due to the mixture of advance-oriented bookings and middle-or-last-minute-oriented bookings, has been observed in the hotel industry between group travelers and individual travelers (see Chapter 2).

In addition to these dividing methods based on the "use case," we introduce a new approach based on "customers' historical purchasing pattern" by analyzing actual sales data in a beauty salon. We found that average booking curves draw exponential functions with the different parameter τ according to the number of experienced visit times, which is one indicator of loyalty (see the supplementary document in this chapter).

The result implies that it is expected to implement unique marketing, pricing, and other measures for each divided group by customer attributes such as loyalty using customers' historical booking data. In other words, dividing customers into some groups based on customer attributes or customers' behavioral pattern makes it possible to apply the proposed dynamic pricing algorithm for each segmented group parallelly.

Therefore, the segmentation approach, primarily based on customers ' historical data, is expected to provide practitioners with familiar logic and practical contributions since the historical data reflects actual experiences. The statistical-evidence-based and human-behavioral-based dynamic pricing algorithm is expected to become a technological foundation used in various perishable asset industries.

4.7 Conclusion

We propose a new general dynamic pricing algorithm with a finite sales horizon based on the cross-industry exponential law of booking curves discovered in recent research (Shintani & Umeno (2021)). In the general business environment with dynamic pricing, it is often difficult to correctly analyze the relationship between price and quantity demanded; we confirm these using actual data in a hotel. We also propose a learning model to evaluate the magnitude of the response in demand due to changes in price based on historical data, which makes our algorithm practical in terms of the availability of simulation by changing price; we introduce these using real data in a hotel. Our proposed dynamic pricing algorithm and learning model are expected to be practical pricing strategies that are not limited to the hotel industry because they are based on the universal laws of exponential booking curves.

In addition, receiving valuable requests for practical use from our partners, we propose a more helpful algorithm that is established by analyzing customers' historical booking data. That is given from the fact that the ABCDEF law, which is the basis of the dynamic pricing algorithms, is individually satisfied in the segmented group by customer loyalty characterized by experienced times of use by analyzing the actual data in a beauty salon. The more personalized algorithm has a familiar logic for practitioners because the historical data reflects actual sales experiences. Therefore, the statistical-evidence-based and human-behavioral-based algorithm is expected to become a technological foundation used in various perishable asset industries.

4.8 Supplementary Material

4.8.1 Analysis based on customer's loyalty in a beauty salon

We analyzed actual sales data in a beauty salon. We found that there are different booking patterns in parameter τ in the ABCDEF law among divided customers based on the number of times they have ever used the salon, which is one indicator for loyalty.

The data analyzed

We collected actual sales performance data from a beauty salon in Tokyo, whose capacity of services per day is around 40, from January 1, 2018, to December 31, 2018. Table 4.1 outlines the collected data and the data analyzed in this chapter. We analyze only data with the cutting or including cutting menu, the most popular menu item in the beauty salon, to investigate how difference emerges in customers' booking lead time days according to the number of experienced visits. Table 4.2 shows a part of the sales performance data. In addition to being able to identify when customer reserves, we can identify how many times the customer visited the salon using the customer id. Then, we divide all bookings based on the number of experienced visits and analyze each booking curve.

Table 4.1: The summary of actual sales data in a beauty salon property in 2018. Analyzed data only consists of menus, including cutting.

	Collected data	Analyzed data
The total users	6240	4143
The number of unique customers	1993	1334
Average lead time days	8.357372	7.1006517

Analysis based on the times of experienced visits

First, we study how booking lead time days differ depending on the number of times the customer has visited the salon; Figure 4.7 shows the results. We see that

Table 4.2: A part of the actual sales data set in a beauty salon. Note that the experienced visit times are compiled from 2016, prior to the observation period.

customer id	visit date	start time	reservation date	menu id	service menu	lead time days	Xth visits
1000265	2018-10-01	14:30	2018-09-30	1	cut	1	20
1000474	2018-10-01	12:30	2018-09-28	1	cut	3	45
1000550	2018-10-01	11:00	2018-09-09	1	cut	22	17
1000914	2018-10-01	15:00	2018-09-22	21	cut + color	9	24
1000979	2018-10-01	14:00	2018-09-29	21	cut + color	2	26
1001564	2018-10-01	18:00	2018-09-26	21	cut + color	5	39
1003263	2018-10-01	16:00	2018-10-01	1	cut	0	1
1003264	2018-10-01	17:30	2018-10-01	1	cut	0	1

the lead time days increase with the number of experienced visits; in other words, customers tend to reserve more in advance. These results imply that the booking pattern differs depending on the level of loyalty characterized by the number of visits (Matsumoto (2021)).

Second, we study the difference in the booking curve. We divide bookings into two groups first-time use and repeat use. Figure 4.8 shows the fitting of average booking curves into exponential functions for each group and parameters with fitting. While the exponential property of the repeat-use group is not high, the value of τ, the parameter that controls the pattern of bookings, is more than twice as large as that of the first-time visiters.

In addition, we divide bookings by whether until Xth visits or not and investigate booking curves for each group, as Figure 4.9 illustrates. We find that average booking curves are close to exponential functions for a low loyalty group composed of first-time customers and a high loyalty group consisting mainly of high-repeat customers. Given that customer loyalty strengthens according to the number of experienced visits, the exponential property of the average booking curve increases as the group is composed of highly pure loyal customers. Since customer loyalty represents one of the customer attributes, the result is consistent with the statement that customer attributes are one of the factors defining a homogeneous demand-supply environment, as seen in chapter 2.

In this section, we showed that average booking curves draw exponential

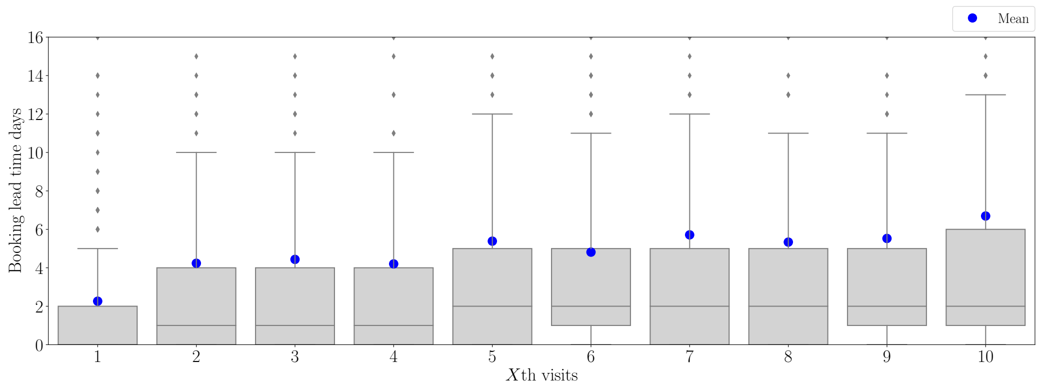
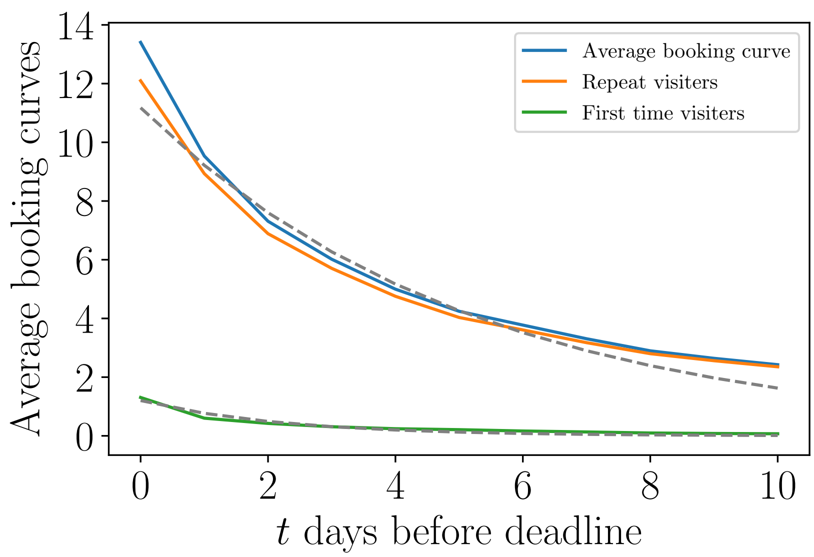


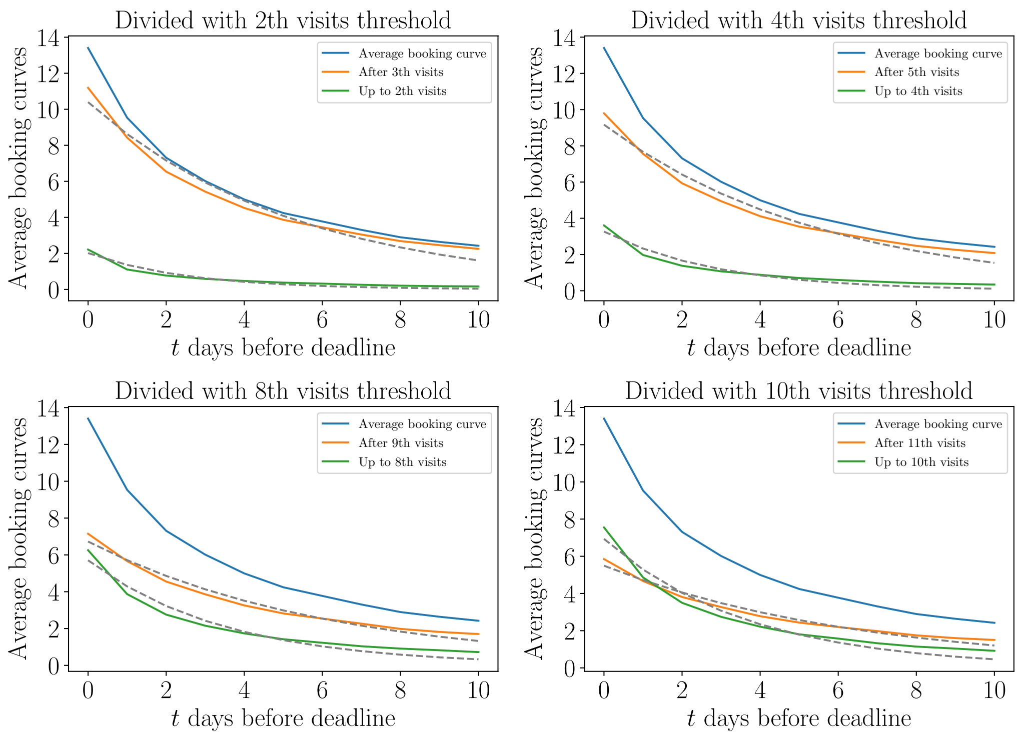
Figure 4.7: Relationship between the number of visits for the salon and booking lead time days. We confirm that booking lead time days tend to expand as the number of experienced visit times increases. Note that the box plot consists of two boxes, the upper line, the lower line, and outliers. Boxes represent the central 50% percentile volume. The upper line represents the maximum value smaller than the third quartile + 1.5 × interquartile range (IQR), and the lower represents the maximum value larger than the first quartile -1.5 × IQR.

functions with the different parameter τ according to the number of experienced visit times identified using customer information. By applying these results, it is expected to implement unique marketing, pricing, and other measures for each divided group by customer attributes such as loyalty using customers' historical booking data.



	First-time visiters	Repeat visiters
τ	2.226334	5.187725
MSE	0.007606	0.310669

Figure 4.8: How do the customers' booking patterns differ whether first-time-visit or not? The value MSE represents the mean squared error between average booking curves and fitting exponential functions with from 0 to 10 days of fitting interval.



	MSE	
X	Up to Xth visits customers	After $(X+1)$th visits customers
1	**0.007606**	0.310669
2	0.022033	0.239561
3	0.039599	0.193170
4	0.055354	0.159252
5	0.072356	0.132761
6	0.088067	0.110376
7	0.110419	0.087225
8	0.129942	0.068463
9	0.144238	0.056036
10	0.162424	**0.045012**

Figure 4.9: We divide customers based on whether up to Xth visits or not and compare average booking curves and fitting errors for each segment. Figures show divided booking curves; the sum of the orange line and green line is equal to the blue line in each figures. The table implies that the exponential property of average booking curves increases according to the homogeneity of customer loyalty.

Chapter 5

Conclusion

Chapter 1 is an introduction. Dynamic pricing has been introduced to various industries in Japan in recent years. We show the background of the spreading in terms of technology development and social situation shift. As for a social situation shift, dynamic pricing is expected to be one of the measures to solve various social issues, such as strategic economic recovery from the COVID-19 pandemic, improving people's working environment, and achieving the SDGs. Therefore, it is important to provide practical knowledge of dynamic pricing to support proper decision-making in its implementation.

In this research, we mainly focus the dynamic pricing in the perishable assets industry. We report a survey of dynamic pricing in the perishable assets industry based on theoretical modeling, empirical investigation, and implementable technology. We introduce the importance of developing an evidence-based pricing technology considering that few dynamic pricing algorithms have been implemented despite spreading the strategy in various fields. In addition, we show the outline of actual sales data we analyzed in this study.

In Chapter 2, our study investigated the macroscopic aspects of booking curves with actual sales data across six properties belonging to two industries for two years, studying aspects like large shifts in the economic environment. We propose a new cross-industry and cross-economic-environment universal statistical average booking curves draw exponential functions (ABCDEF) law, which shows that average booking curves draw exponential functions from three perspectives; data confirmation, modeling in the statistical physics framework, and empirical justification for modeling causality. The ABCDEF law and included parameters provide more information to describe the people's booking pattern over the whole booking horizon even when the demand-supply environment changes. Further, the fact that the time constant of the booking curve is linked to properties' characteristics and economic conditions means that the ABCDEF law can be applied to situation-specific demand forecasting and dynamic pricing algorithms.

In Chapter 3, we proposed a new analytical time series prediction method for exponential decay time series. The method is established based on time-rescaling for time series and needs only a part of the time series. The method improves forecasting accuracy not only for random variable series prediction but also for quantity-demanded forecasting of booking curves which draw exponential functions. We discuss the mechanism of achieving high performance in connection with typical time series prediction methods. Besides, we tested the usefulness of forecasting in the hotel and car rental industry and report the some improvements with the cooperation of clients.

In Chapter 4, we propose a new general dynamic pricing algorithm for perishable asset industries based on the ABCDEF law. We also propose a learning model to evaluate the magnitude of the response in quantity demanded due to changes in price based on historical data, which makes our algorithm practical in terms of the availability of simulation by changing price; we introduce these using actual data in a hotel.

However, according to partners' opinions, only macroscopically statistical-based processing is not yet practical in a business environment. We have issues to overcome; familiar logic for practitioners and detailed segmentation to adapt to the complexity of RM. Therefore, we propose a more practical algorithm that is established by analyzing customers' historical booking data in a beauty salon. That is given from the fact that the ABCDEF law, which is the basis of the dynamic pricing algorithms, is individually satisfied in the segmented group by customer loyalty characterized by experienced times of use. The time constant of booking curves individually determined reflects the booking pattern depending on customer attributes, that is, human behavior.

The analysis based on customers' historical purchasing data makes the proposed dynamic pricing algorithm adaptable in personalized segmentation. The more personalized algorithm has familiar logic for practitioners because the historical data reflects actual sales experiences. The statistical-evidence-based and human-behavioral-based algorithm is expected to become a technological foundation used in various perishable asset industries.

Finally, this work pushes dynamic pricing up to one of the major econophysics subjects, whose main field has been stocks and cryptocurrency. The expansion of fields dealing with dynamic pricing makes a multifaceted knowledge base. We look forward to more opportunities to support proper decision-making in dynamic pricing worldwide.

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List of author's papers and patents related to this thesis

1. Masaru Shintani and Ken Umeno, Cross-industry statistical law: average booking curves draw exponential functions, in preparation for submitting to *Physica A* in July 2022.

2. Masaru Shintani and Ken Umeno, Time-rescaling regression method for exponential decay time-series predictions, *JSIAM Letters*, **14**, pp.45–48 (2022).

3. Masaru Shintani and Ken Umeno, General dynamic pricing algorithms based on universal exponential booking curves, *JSIAM Letters*, **14**, pp.49–52 (2022).

4. Ken Umeno, Masaru Shintani, et al., 'Demand forecasting systems, pricing systems, information proceesing systems and computer programs', JP Patent 7109027 (2019) (In Japanese).

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- Chapter 2 is based on paper 1.
- Chapter 3 is based on paper 2.
- Chapter 4 is based on paper 3.

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