RIMS Kôkyûroku Bessatsu
B92 (2023), 97–116

Descriptions of Excavations (*khāta*) in the *Triśatībhāṣya*: Surveying Method and Two Terms, *laṃba* and *kaḍī*

By

Taro Tokutake*

Abstract

The *Triśatībhāṣya* is an anonymous commentary on the Sanskrit arithmetic text *Triśatī* by Śrīdhara (ca. 800 CE). "Excavations" (*khāta*) is one of the topics in the *Triśatī*, and Śrīdhara offers rules and examples for calculating the dimensions of a pit with irregular sides (i.e., arithmetical means) and the capacity of a pit with uniform sides. The present paper attempts to restore the surveying method of excavations described in the *Triśatībhāṣya*, and to clarify the meanings of two terms, *laṃba* and *kaḍī*, employed in the commentary.

§ 1. Introduction

The *Triśatībhāṣya* (hereafter TrBh) is an anonymous commentary on the Sanskrit arithmetic text *Triśatī* (or *Triśatikā*, hereafter Tr) by Śrīdhara (ca. 800 CE). The TrBh is available only in a single complete manuscript (LD Institute, Ahmedabad, 1559: hereafter A_1) and is not contained in the edition published in Kāśī (hereafer K_{ED}).[1] In my recent study, I investigated the date and the place of the author of the TrBh through an analysis of the linguistic features, and concluded that he flourished in Western India some time between the 12th and 15th centuries CE.[2] The Tr presents arithmetic rules and examples briefly. On the other hand, the TrBh explains the computational procedures in detail.

Received November 29, 2022. Revised January 23, 2023.
2020 Mathematics Subject Classification(s): 01A32
Key Words: *Triśatībhāṣya*, excavations (*khāta*), surveying method, *laṃba*, *kaḍī*.
This work was supported by JSPS KAKENHI Grant Number JP21J23080, and by the Research Institute for Mathematical Sciences, an International Joint Usage/Research Center located in Kyoto University.
*Kyoto University, 606-8501 Japan.
email: tokutake.taro.36w@st.kyoto-u.ac.jp
[1] Kāśī is a historical name of the present-day Varanasi. In this paper, the verse numbers of the Tr follow K_{ED}.
[2] See Tokutake [2022b].

The Tr consists of "definitions [of number words and weights and measures]" (*paribhāṣā*), "fundamental operations" (*parikarman*) and "procedures" (*vyavahāra*). The procedures are further subdivided into eight topics:

1) Mixture (*miśraka*),
2) Series (*śreḍhī*),
3) Plane figures (*kṣetra*),
4) Excavations (*khāta*),
5) Piling (*citi*),
6) Sawing (*krakaca*),
7) Mounds of grain (*rāśi*), and
8) Shadows (*chāyā*).[3]

In the topic of excavations, Śrīdhara gives rules, Tr 52–53, for calculating the dimensions of a pit with irregular sides (i.e., arithmetical means) and the capacity of a pit with uniform sides. The author of the TrBh explains the rules in connection with surveying method, and presents a geometric figure in his commentary on Tr E88. The aim of this paper is twofold: to restore the surveying method described in the TrBh, and to clarify the meanings of two terms, *laṃba* and *kaḍī*, embedded in the figure.

For that purpose, first, I will recall the rules for excavations, Tr 52–53. Then, I will examine the example, Tr E88. Finally, I will investigate the meanings of the two terms, *laṃba* and *kaḍī*, through an analysis of actual usages of them.

§ 2. Rules for excavations

In this section, I will recall the rules for excavations in the Tr and try to restore the surveying method mentioned in the TrBh. For a better understanding of the rules, I will consult the description of them in Gaṇeśa's *Buddhivilāsinī* (1545 CE, hereafter BV) on Bhāskara II's *Līlāvatī* (1150 CE, hereafter L).

In the following, the text edited here of the Tr are based on both of K_{ED} and A_1, and that of the TrBh only on A_1. The word(s) cited in the TrBh from the Tr is printed in bold. I remove or add *daṇḍa*-s (/) in A_1 for the ease of reading. Phonological irregularities have been left in this edition just as they are in the manuscript.

Notation:

^{ac}	*ante correcturam*, i.e., the reading before the correction by the scribe
em.	emendation (I do not distinguish it from "correction" and "conjecture")

[3] For further details of the contents of the Tr, see Tokutake [2022a: 180–185].

^{pc}	*post correcturam*, i.e., the reading after the correction by the scribe
<A>	A is supplied by the editor.
°	truncation (of letters) in long Sanskrit words

§ 2.1. *Triśatī*

Śrīdhara offers a rule for the dimensions of a pit with irregular sides (i.e., arithmetical means) as follows.

Tr 52:
mukhatalamadhye pṛthutādairghye vā cet prajāyate viṣamam/[4]
vedhe vā viṣamayutiṃ sāmyāya bhajeta viṣamapadaiḥ//[5]

If, at the top, bottom and middle [of a pit], unequal [measures] of breadths, lengths or depths are produced, for the sake of the mean [measure], one should divide the sum of the unequal [measures] by [the number of] unequal places [where the unequal measures were taken].[6]

Let a_i, b_i, h_i be the lengths measured at n_1, n_2, n_3 places of the breadth, the length, and the depth, respectively. Their respective mean lengths are obtained by

$$\overline{a} = \frac{\sum_{i=1}^{n_1} a_i}{n_1}, \qquad \overline{b} = \frac{\sum_{i=1}^{n_2} b_i}{n_2}, \qquad \overline{h} = \frac{\sum_{i=1}^{n_3} h_i}{n_3}.$$

Then, the commentator explains the rule in connection with the surveying method.

TrBh on Tr 52 (A_1 fol. 22a):
khātaṃ ***mukhatalamadhye viṣamaṃ/ dairghye viṣamaṃ vā*** *bhavati/*[7]
tadā tatra viṣamasthāneṣu sūtraṃ pātayitvā ekīkṛtya tair ***viṣamapadair*** *bhāgaṃ apahṛtya* ***sāmyaṃ*** *khātasya āneyam iti//*[8]

An excavation [has] **unequal [measures] at the top, bottom and middle**. **Or, unequal [measures] of lengths** are [there]. Then, one should stretch (*pātayitvā*: lit. having stretched) cord[s] at the unequal places; put [them] into one (*ekīkṛtya*: lit. having put [them] into one); divide (*bhāgaṃ apahṛtya*: lit. having divided) [the single one] **by [the number of] unequal places [where the unequal measures were taken]**; and [thus] calculate **the mean [measures]** of the excavation.

[4] pṛthu°] K_{ED}, pūthu° A_1; °dairghye] K_{ED}, °dairghā A_1
[5] vedhe vā] K_{ED}, vedho bā A_1; bhajeta] K_{ED}, bhājat A_1; °padaiḥ] K_{ED}, °padai A_1
[6] Here and hereafter, a brief explanation of a word in translation is marked with parentheses ( ), and additions to the translation with square brackets [ ].
[7] dairghye viṣamaṃ] em., daurghye viṣa5maṃ A_1
[8] tair] em., ter A_1; °sthāneṣu] em., °sthānau A_1; bhāgaṃ] em., sāgaṃ A_1

The commentator intends, for instance, the following pit.

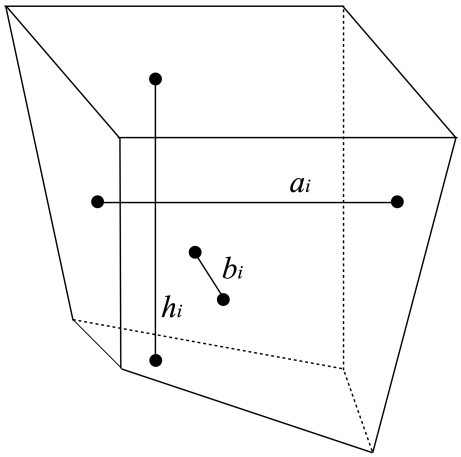


Figure 1.

The cords (*sūtra*-s) are "stretched" between the black dots in the above figure, and their lengths represented by a_i, b_i and h_i are measured. The expression "put [them] into one" means to reduce the cords stretched at the unequal places into one. What is intended by "divide" might be to fold the single cord, rather than to divide it arithmetically. The surveying method of the TrBh can be illustrated in the following way:

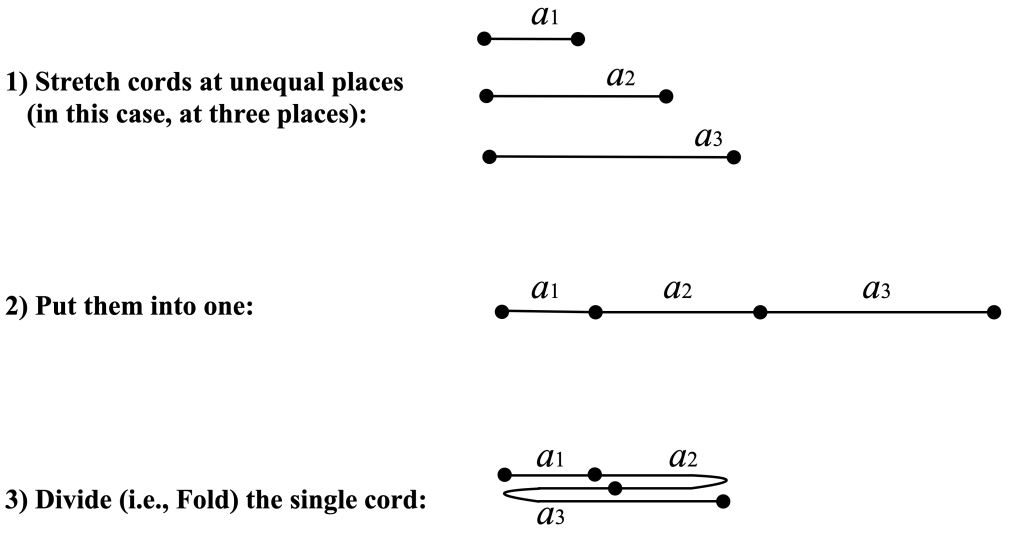


Figure 2.

The compound *sama-rajju* in the sense of the "mean cord" is found in *Brāhma-sphuṭasiddhānta* 12.44d.[9] It is possible that the above method of measurement is in-

[9]See Colebrooke [2005: 312–313].

tended there also. However, as Sarasvati [2007: 200] says, *Brāhmasphuṭasiddhānta* 12.44cd is elliptical and defies interpretation.

Then, a rule for obtaining the capacities of pits follows.

Tr 53:
samavistaradairghavadhe vedhena samāhate phalaṃ bhavati/[10]
khāte samabhujavedhe bāhughano jāyate gaṇitam//

When the product of the mean breadth and length is multiplied by the depth, the fruit (i.e., capacity) is produced. When an excavation has the equal arms and depth, the cube of the arm occurs as the calculated (i.e., capacity).

Capacities (V) of a rectangular pit with uniform sides (a, b, h) and of a cubic pit.

$$V = (ab)h, \qquad V = a^3.$$

There is no need to go into the details about the TrBh on Tr 53, because the commentator merely paraphrases the expressions in the Tr without providing new information.

§ 2.2. *Līlāvatī*

Bhāskara II gives the following rules for excavations.

L 214:
gaṇayitvā vistāraṃ bahuṣu sthāneṣu tadyutir bhājyā/
sthānakamityā samamitir evaṃ dairghye ca vedhe ca//
kṣetraphalaṃ vedhaguṇaṃ khāte ghanahastasaṃkhyā syāt//

Having measured the breadth in many places, their sum is to be divided by the number of the [unequal] places [where the unequal measures were taken]. The mean measure [is produced]. For the length and the depth, [their mean measures are also obtained] in this manner. The plane fruit (i.e., area) is multiplied by the depth. The number of cubic *hasta*-s of an excavation should be produced.[11]

The rule of L 214abcd is the same as Tr 52. With regard to the rule of L 214ef, when A and h denote the surface area and the depth, respectively, of a pit, its capacity (V) is obtained by

$$V = Ah.$$

[10] °dairghavadhe] A_1, °hatadairghye K_{ED}
[11] Cf. Colebrooke [2005: 97].

Gaṇeśa's commentary on the above rules is divided into two parts: explanation of the words and phrases in L 214, and a "proof" (*upapatti*).[12] As for the first part, it is sufficient to mention here only the following passage.

> BV on L 214 (Ānandāśrama edition p. 221):
> ⋯ *yathā yathā* ***bahuṣu sthāneṣu vistārā****dikaṃ gaṇyate tathā tathā* ***samamitiḥ*** *sūkṣmasūkṣmatarā* ***syād*** *iti spaṣṭam/* ⋯
>
> ⋯ The more **places the breadth**, etc. are measured, the more accurate **the mean measure should be**—this is clear. ⋯

The greater the numbers n_1, n_2 and n_3 are, the more accurate the values of $\overline{a}$, $\overline{b}$ and $\overline{h}$ will be. The second part of the BV is as follows.

> BV on L 214 (Ānandāśrama edition p. 221):
> *atropapattiḥ sugamā/* ***khāte vistārā****dikaṃ sarvasminn api pradeśe na hi* ***samamitiṃ*** *kṛtvā* ***bahuṣu sthāneṣu gaṇayitvā tadyutiḥ*** *sthānasaṃkhyayā hṛtā madhyasthasya* ***vistārā****dikasya mitiḥ* ***syāt****/*[13] *ekasmin padeśe tasmān nyūnam anyasminn adhikam ato madhyastha iti/ rūpatulyavedhe* ***kṣetraphala****tulyā eva* ***ghanahastāḥ*** *syuḥ/ ato* ***vedhaguṇaṃ kṣetraphalaṃ ghanahastasaṃkhyā syād*** *iti//*
>
> A "proof" for this (i.e., L 214) is easy to understand. In **an excavation**, when one observes that **the breadth**, etc. do not have **the same measure** at every spot, **having measured** [the breadth, etc.] **in many places, their sum** is divided by the number of the [unequal] places. The [mean] measure of **the breadth**, etc. standing at the middle **should be produced**. In one spot there is a shortage from it, and in another there is a surplus. Therefore, [the mean measure] stands at the middle.
> When the depth is equal to unity, [the number of] **the cubic *hasta*-s** should be equal to **the plane fruit** (i.e., the number of unit squares). Hence, **the plane fruit (i.e., area) is multiplied by the depth. The number of cubic *hasta*-s should be produced**.

The above "proof" can be divided into two parts: the "proof" for L 214abcd and that for L 214ef. In the former part, Gaṇeśa explains how to obtain the "mean measure" and what it stands for. As Figure 3 illustrates, he supposes at least three different measures: short (m_1), mean (m_m) and surplus (m_2) measures. The relationship between them, $m_1 < m_m < m_2$, is expressed as "[the mean measure] stands at the middle."

[12] For further details of "proof" (*upapatti*), see Sarma et al. [2008: vol. 1, 267–310].
[13] pradeśe na] em., pradeśena

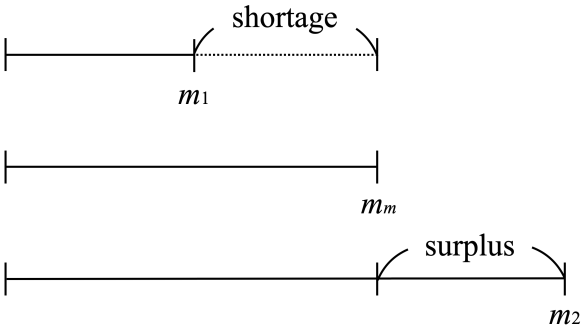


Figure 3.

In the latter part of the "proof," the commentator states: When $h = 1$, $V = A$. This "proof" of $V = Ah$ is based on the classical interpretation of area and volume:

area = number of unit squares,

volume = number of unit cubes.

As Figure 4 shows,

volume of an excavation
= number of unit cubes in it
= (number of unit cubes in one layer)×(number of the layers)
= (number of unit squares contained in the base)×(number of units contained in the depth)
$= A \times h$.

Gaṇeśa intends these steps, although he actually only refers to the following relationship: number of unit cubes in one layer = number of unit squares contained in the base.

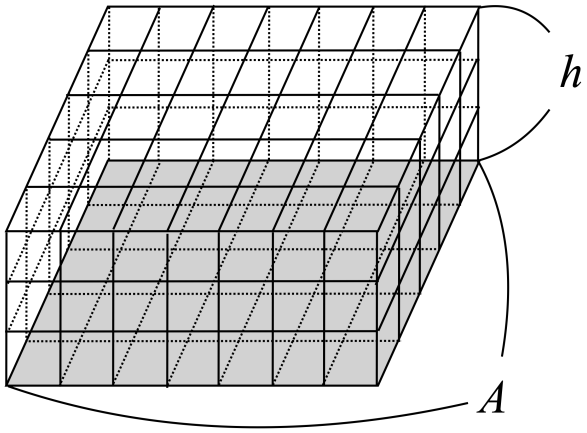


Figure 4.

§ 3. Example for an excavation

Śrīdhara gives the following example for an excavation.

Tr E88:
tricatuḥpañcakapṛthutā puṣkariṇī gaṇaka yatra vikhyātā/[14]
aṣṭau hastā vedhe dvādaśa dairghye ca kathaya phalam//[15]

O calculator, there is a famous lotus pond—where the depth is eight *hasta*-s and the length is twelve [*hasta*-s]—possessing three, four and five [*hasta*-s] as the breadth. Tell [me] the fruit (i.e., capacity).

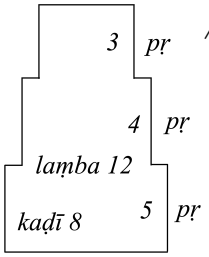
That is,

$$a_1 = 3, a_2 = 4, a_3 = 5 \text{ [}hasta\text{-s]};$$

$$b = 12 \text{ [}hasta\text{-s]};$$

$$h = 8 \text{ } hasta\text{-s}.$$

The commentator presents a "setting-down" (*nyāsa*)[16] and a solution for the problem.

TrBh on Tr E88 (A_1 fol. 22b):
nyāsaḥ 

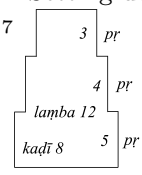
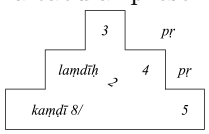
/[17] *pṛthuviṣamayutiḥ 3/*[18] *4/ 5/*[19] *yuti 12/* ***viṣamapada***
3/ bhāge datte labdhaṃ 4/ sarvatra samaṃ/ pṛthu sarvatra 4/ 8guṇite labdhaṃ 32/[20] *punar laṃbena 12 guṇite jātaṃ 384//*[21]

---

[14] °ñcakapṛthutā] em., °ñcakahastāḥ pṛthutā K_{ED}, °ñcakarāḥ pṛthutā A_1; puṣkariṇī] em., viṣamāttu K_{ED}, puṣkaraṇī A_1; gaṇaka yatra vikhyātā] A_1, yasya khātasya K_{ED}

[15] vedhe] A_1, vedho K_{ED}; dairghye] K_{ED}, dīrghye A_1; ca] em., om. $K_{ED}A_1$

[16] "Setting-down" is a tabular presentation of the numerical information given in the example.

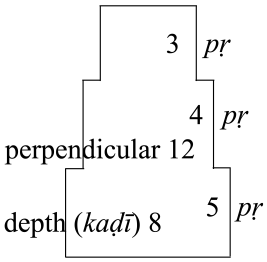
[17] ] em.,  A_1

[18] pṛthu°] em., ṣṛthu° A_1

[19] 4/ 5] em., 45 A_1

[20] pṛthu sarvatra 4/ 8°] em., vedha sarvatra 48 A_1

[21] punar laṃbe°] em., puna labe° A_1

Setting-down: 

.[22] The sum of unequal [values] for breadth is: 3, 4, 5. The sum is 12. **[The number of] unequal places [where the unequal measures were taken]** is 3. When [12 is] divided (*bhāge datte*: lit. when the part is given)[23] [by 3], what is obtained is 4. [This is] the same everywhere. [That is,] the breadth is [regarded as] 4 everywhere. Multiplied by 8, what is obtained is 32. Further multiplied by the perpendicular, i.e., 12, the result is 384.

That is,

$$\overline{a} = \frac{3+4+5}{3} = \frac{12}{3} = 4;$$

$$V = 4 \cdot 8 \cdot 12 = 32 \cdot 12 = 384 \text{ [}hasta\text{-s]}.$$

By the figure of the setting-down, the commentator intends the plane figure (i.e., top view)[24] of the following lotus pond:

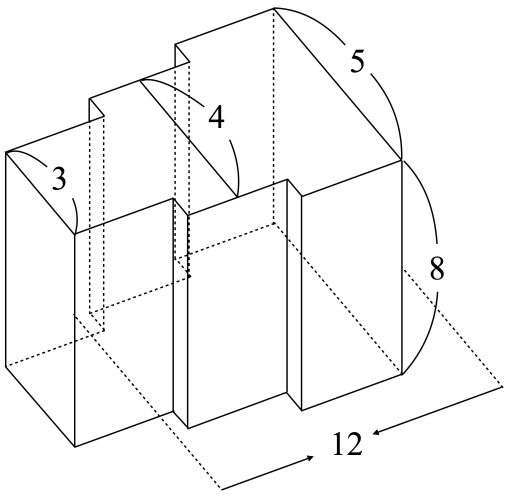


Figure 5.

The two words, *laṃba* and *kaḍī*, used here might correspond respectively to the *dairghya* "length" and *vedha* "depth" in Tr E88. However, this usage of *laṃba* is uncommon,

[22] *pṛ* is an abbreviation for *pṛthu* or *pṛthutā*.

[23] The same expression is found in Siṃhatilaka's commentary on the *Gaṇitatilaka* (see Hayashi [2019: 35, 127, etc.]).

[24] All the figures given in A_1 are illustrated as top views. For the functions of geometric figures, see Keller [2009].

and we rarely encounter the term *kaḍī* (or *kaṃḍī* in A_1) in Indian mathematical texts. Therefore, I will investigate the usage of the two terms in the following two sections.

§ 4. *laṃba*

In this section, I will discuss three different meanings of *laṃba* found in the TrBh: "perpendicular," "length" and "depth or height." It may be worth mentioning that Āryabhaṭa I employs, in his *Āryabhaṭīya* (499 CE, hereafter ĀBh), the term *lambaka* in the sense of "plumb-line," although it differs from the *laṃba* in question.

§ 4.1. *laṃba* in the sense of "perpendicular"

Figure 6 is a geometric figure illustrated in TrBh on Tr E77 (A_1 fol. 20a), and Figure 7 is an English translation of it:

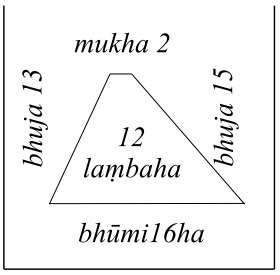


Figure 6. Text

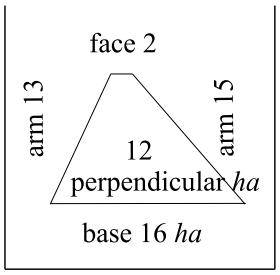


Figure 7. Translation

In Figure 6, *ha* in the compound *laṃba-ha* is an abbreviation for *hasta*. In this case, *laṃba* clearly means the perpendicular or the height of the above trapezium. The same usage of the term is found in TrBh on Tr E78–79, etc., and it is the most common usage in Indian mathematical texts.

§ 4.2. *laṃba* in the sense of "length"

The following stanza provides a rule for calculating the volume of a rectangular stone.

Tr 55:
dairghyāṅgulāni vistarapiṇḍāṅgulatāḍitāni bhajet/[25]
dvikṛticaturekaṣaḍbhir bhavanti pāṣāṇaphalahastāḥ//[26]

One should divide the length in *aṅgula*-s that is multiplied with the breadth

[25]dairghyāṅgu°] K_{ED}, dairghāṃgu° A_1; vistara°] A_1, vistṛti° K_{ED}; bhajet] A_1, vibhajet K_{ED}
[26]°catu°] K_{ED}, °thatu° A_1; pāṣāṇa°] K_{ED}, ṣoṇa° A_1

and the thickness in *aṅgula*-s, by [the number composed of] the square of two, four, one and six (i.e., 6144). The fruit (i.e., volume) of the stone [slab] in *hasta*-s (*pāṣāṇa-phala-hasta*) is produced.

When a, b and c respectively denote the length, the breadth and the thickness, all measured in *aṅgula*-s, of a rectangular stone, their product is divided by the constant 6144:

$$V_p = \frac{abc}{6144},$$

where V_p denotes "the fruit (i.e., volume) of the stone [slab] in *hasta*-s." This constant 6144 is obtained by $24^3 \cdot \frac{4}{9}$, where 24 *aṅgula*-s = 1 *hasta*. This conversion might be to take into account the weight of stone, but the details are not known.[27] An example for the above rule is as follows.

Tr E92:
sārdhatrikaravyāsā karārdhapiṇḍā śilā sakhe tasyāḥ/[28]
āyāmaḥ pañca karās tribhāgayuktāḥ phalaṃ kiṃ syāt//[29]

There is a stone [slab] whose breadth is three and a half *hasta*-s, whose thickness is half of a *hasta*, and whose length is five *hasta*-s increased by one-third. O friend, what would be the fruit (i.e., volume) [of the stone slab]?

That is,

$$a = 5\frac{1}{3} \text{ } hasta\text{-s}, \quad b = 3\frac{1}{2} \text{ } hasta\text{-s}, \quad c = \frac{1}{2} \text{ } hasta.$$

The following setting-down is given in the TrBh.

TrBh on Tr E92 (A_1 fol. 22b):
hastair aṃgulīkṛtair nyāsaḥ

128	*laṃba*
pṛ	*84*
12	*vedha*

[30]

Setting-down with the *hasta*-s converted into *aṃgula*-s:

128	perpendicular
pṛ	84
12	depth

[27] See SaKHYa [2009: 148–151] and Hayashi [2019: 350].
[28] °vyāsā] $K_{ED}A_1^{pc}$, °vyāsya sā A_1^{ac}; tasyāḥ] K_{ED}, tasyā A_1
[29] āyāmaḥ] K_{ED}, āyāma/ A_1; karās tribhāgayu°] K_{ED}, karā yu° A_1
[30] °līkṛ°] em., °lai kṛ° A_1; | 128 | laṃba | / pṛ | 84 | / 12 | vedha |] em., | 128 | lāṃ/ba | / pṛ 84 | 12 ve 5 | A_1

That is,

$$a = 5\frac{1}{3}\ \textit{hasta}\text{-s} = 128\ \textit{aṃgula}\text{-s},$$
$$b = 3\frac{1}{2}\ \textit{hasta}\text{-s} = 84\ \textit{aṃgula-s},$$
$$c = \frac{1}{2}\ \textit{hasta}\text{-s} = 12\ \textit{aṃgula}\text{-s}.$$

The length a (=128 *aṃgula*-s) is denoted by *laṃba*, although it is the emendation of *lāṃ/ba* in A_1. The same usage of the word is found in *Bālabodhāṅkavṛtti* (1428/29 CE) on *Pañcaviṃśatikā* 20.[31] Therefore, these usages attest that *dairghya* in Tr E88 corresponds with *laṃba* in TrBh on Tr E88.

§ 4.3. *laṃba* in the sense of "depth or height"

Śrīdhara offers the following example for the rules of Tr 52–53.

Tr E87:
dvitricatuṣkaravedhā puṣkariṇī pañcahastavistārā/[32]
ṣoḍaśahastāyāmā khātaphalaṃ kathyatām asyāḥ//[33]

There is a lotus pond whose depth is two, three and four *hasta*-s, whose breadth is five *hasta*-s, and whose length is sixteen *hasta*-s. Please tell [me] the fruit of excavation (i.e., capacity) of this [lotus pond].

That is,

$$a = 5\ \textit{hasta}\text{-s};$$
$$b = 16\ \textit{hasta}\text{-s};$$
$$h_1 = 2, \quad h_2 = 3, \quad h_3 = 4\ \textit{hasta}\text{-s}.$$

Then, the commentator provides a setting-down and a solution.

TrBh on Tr E87 (A_1 fols. 22ab):
āyāmaḥ 16 laṃbeṣu <

ve	*ve*	*ve*
2	*3*	*4*

vi 5>/[34]

The length is 16. For the perpendiculars (*laṃbeṣu*),

ve	*ve*	*ve*
2	3	4

, *vi* 5.[35]

[31] See Hayashi [2017: 40–44, 98–102].
[32] dvitri°] K_{ED}, dditṛ° A_1; °catuṣka°] $K_{ED}A_1{}^{pc}$, °caturthaṣka° $A_1{}^{ac}$; °ṣkariṇī] K_{ED}, °ṣkaraṇī A_1; °stārā] K_{ED}, °starā A_1
[33] °stāyāmā khāta°] K_{ED}, °staghame ṣāta° A_1; °masyāḥ] em., °māśu K_{ED}, °masyā A_1
[34] āyāmaḥ 16 laṃbeṣu] em., vyāmeḥ

pṛthuḥ/
kaḍīha 8

lābaṣa 16 A_1
[35] *ve* and *vi* are abbreviations for *vedha* and *vistāra* respectively.

In the above setting-down, *laṃbeṣu* probably conveys the meaning "for the depths." However, this case does not provide a strong evidence for the use of *laṃba* in the sense of "depth or height," because *laṃbeṣu* is an emendation of *lābaṣa* of A_1. The text in this paragraph is broken. Figure 8 shows the arrangement of the three boxes in this part of A_1.

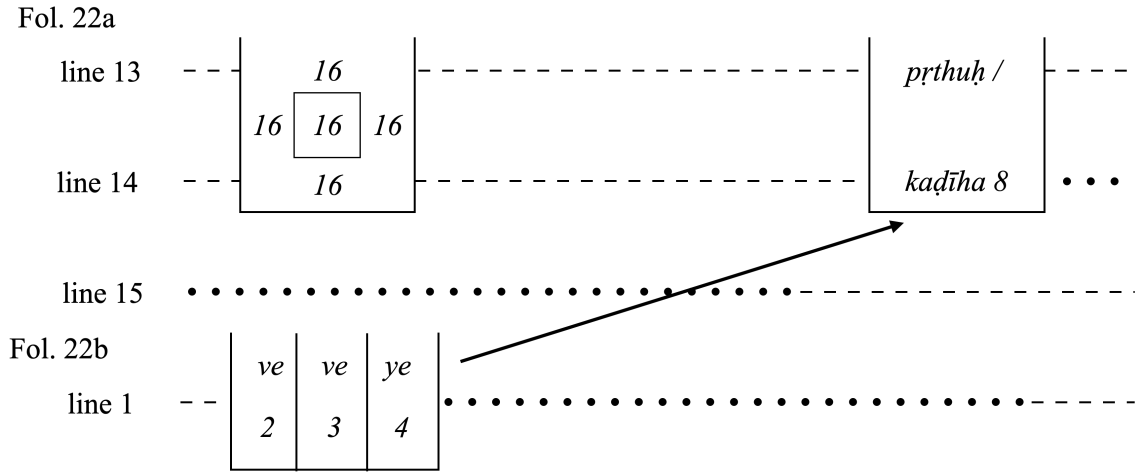


Figure 8.

The dashed lines - - - indicate the relevant sentences in A_1, the dotted lines ⋯ dittographies, and the arrow → the place where the third box, i.e., the box with three compartments in fol. 22b, line 1, should be located. The first box, i.e., the box at the left side in fol. 22a, lines 13–14, is for another example, Tr E90. The second box, i.e., the box at the right side, must be a part of the setting-down for Tr E88 (see §3 and §5) and should be replaced by the third box.

§ 4.4. *lambaka* in the sense of "plumb-line"

ĀBh 2.13d is a rule for correct production of a vertical line.

ĀBh 2.13d:
adha ūrdhvaṃ lambakenaiva//

Verticality (lit. top and bottom) [is to be brought about] with just a plumb-line.[36]

Bhāskara I glosses the above rule in his commentary (629 CE).

Bhāskara I's comm. on ĀBh 2.13d:
adha ūrdhvaṃ lambakenaiva/ *adha upalakṣitasya ya ūrdhvapradeśaḥ so 'va-*

[36] Cf. Keller [2006: vol. 1, 67].

lambakenaiva sādhyate/ ūrdhvapradeśasya vā yo 'dhaḥpradeśaḥ asāv apy avalambakenaiva/ avalambakaś ca gurudravyaikāgrāsaktaṃ sūtram iti//

<As for:> "**verticality with just a plumb-line**". The point above a <point> marked below is brought about with a plumb-line (*avalambaka*). Or, the point below a point above, also, is <brought about> with a plumb-line. And a plumb-line is a thread with a heavy object at one tip.[37]

From the above commentary, it is clear that "a thread with a heavy object at one tip" is denoted by the word *lambaka* or *avalambaka* which shares the same root *lamb* with *lamba*.

§ 5. *kaḍī*

We find three forms—*kaḍī*, *kaṃḍī* and *kā*—in A_1.

Fol. 22a, lines 13–14 (second box of Figure 8):

pṛthuḥ/
kaḍīha 8

;

Fol. 22b, line 3: *kaṃḍī 8/*;

Fol. 22b, line 9:

16	*ṣṛ*
kā	*12*

.

As I have mentioned in §4.3, the second box of Figure 8 is a part of the setting-down for Tr E88 and has been inserted into that for Tr E87 by mistake. It is clear that *kaḍīha* is composed of *kaḍī* and *ha*, and means "*hasta*-s of *kaḍī*."

As we have seen in §3, *kaṃḍī* in fol. 22b, line 3 is embedded into the geometric figure presented in TrBh on Tr E88.

According to the context of Tr E91, the box in fol. 22b, line 9 is to be emended to

16	*pṛ*
ka	*12*
4	*tala*

, where *pṛ* and *ka* are abbreviations for *pṛthu* and *kaḍī* respectively.

Tr E91:
kūpasya mukhavyāsaḥ ṣoḍaśa hastās tale tu catvāraḥ/[38]
vedho dvādaśa vidvan tat khātaphalaṃ samācakṣva//[39]

[37]Translation by Keller [2006: vol. 1, 70].
[38]kūpa°] K_{ED}, kapa° A_1; hastās tale] K_{ED}, hastā mūle A_1; tu] A_1, ca K_{ED}
[39]khāta°] K_{ED}, khāt° A_1; samācakṣva] K_{ED}, semācakṣaḥ A_1

The face (i.e., top) diameter of a circular well is sixteen *hasta*-s, [the diameter] at the bottom is four [*hasta*-s], and the depth is twelve [*hasta*-s]. Tell [me], O learned man, the fruit (i.e., capacity) of the excavation.

This is an example for capacity of a circular well. Śrīdhara gives the top diameter $d_1 = 16$ *hasta*-s, the bottom diameter $d_2 = 4$ [*hasta*-s], and the depth $h = 12$ [*hasta*-s]. The rule of Tr 54,[40]

$$V = \frac{\sqrt{\{d_1^2 + d_2^2 + (d_1 + d_2)^2\}^2 \cdot 10} \cdot h}{20 + 4},$$

is applied here in order to obtain the capacity V of a well shaped like an inverted truncated cone. The abbreviation *ka* for *kaḍī* in fol. 22b, line 9 clearly denotes the depth, $h = 12$.

In the following sections, I will discuss the meanings of *kaḍī* and *kaṃḍī* in detail.

§ 5.1. Meaning of *kaḍī*

Turner [1999: 131] lists four kinds of *kaṭa*-s in his dictionary, *A Comparative Dictionary of the Indo-Aryan Languages* (hereafter CDIAL). In the following, the symbol >, used as A > B, signifies the historical derivation of B from A, and m. and f. are abbreviations for masculine and feminine respectively.

kaṭa-[1] m. “twist of straw, mat”;
kaṭa-[2] m. “grass, Saccharum sara”;
kaṭa-[3] m. “thin, piece of wood, plank”; and
kaṭa-[4]/*kaṭi*- “hip.”

According to the CDIAL, the form *kaḍī*- can be derived from *kaṭa*-[1], *kaṭa*-[3] or *kaṭa*-[4]:

kaṭa-[1] > *kaḍī*- f. “metal ring” in Marāṭhī;
kaṭa-[3] > *kaḍī*- f. “small squared piece of timber” in Marāṭhī; or
kaṭa-[4]/*kaṭi*- > *kaḍī*- f. “hip, waist” in Prakrit.

Since I have no information on the usage of *kaṭa*-[1] > *kaḍī*- either in mathematical texts or in engineering texts, it seems not to be related with *kaḍī* in the TrBh.

[40] Tr 54: *mukhatalatadyogānāṃ vargaikyakṛteḥ padaṃ daśaguṇāyāḥ/ vedhaguṇaṃ caturanvitaviṃśatibhaktaṃ phalaṃ kūpe//* “The square of the sum of the squares of the face (i.e., top) and bottom [diameters], and of their sum, is multiplied by ten. The [square] root [taken] from [the product], multiplied by the depth and divided by twenty increased with four, [gives] the fruit (i.e., capacity) of a circular well.”

In another dictionaries of New Indo-Aryan such as *Oxford Hindi-English Dictionary*, etc., we can find *kaṭa*-[3] > *kaḍī*- in the meanings of "a beam, a squared rafter." This is an architectural term and seems to be related with *kaḍī* in the TrBh.

Acharya [1995: 106] lists possible meanings of *kaṭi*, "the hip-part of a building, the hip of an image, a flight of steps," and cites the following verse from Varāhamihira's *Bṛhatsaṃhitā* (6th century CE, hereafter BS).

> BS 56.11:
> *yo vistāro bhaved yasya dviguṇā tatsamunnatiḥ/*
> *ucchrāyād yas tṛtīyo 'ṃśas tena tulyā kaṭir bhavet//*
>
> The height of any temple must be twice its own width, and the flight of steps equal to a third part of the whole height (of the edifice).[41]
>
> The height of the temple shall be twice its breadth and its *kati* shall be a third of its height.[42]
>
> The height of a building should be twice its width and its kaṭi (*lit.* hip) should be (equal to) $\frac{1}{3}$ of its height.[43]
>
> The height of a temple should be double its width, and the height of the foundation above the ground consisting of steps (over which the edifice is built) equal to a third of this height.[44]

I have cited here four different English translations by Kern [1873], Iyer [1987], Acharya [1995] and Bhat [1995], respectively, in order of publication.[45] Kern [1873: 318] translates *kaṭi* as "the flight of steps." Iyer [1987: 282] does not translate the word into English. Acharya [1995: 106] objects to Kern's translation of *kaṭi* as follows:

> Dr. Kern translates 'kaṭi' by 'the flight of steps' (*J.R.A.S.*, N. S., Vol. VI, p. 318); but in this sense the word never occurs in dictionaries or literature; nor does this rendering suit the context here, first, because the description concerns a single-storeyed building, where the flight of steps, if there be any at the entrance, would not be usually one-third of the height of the whole building;

[41] Translation by Kern [1873: 318]. He refers to "temple" because this chapter, BS 56, deals with the structure of temples.
[42] Translation by Iyer [1987: 282].
[43] Translation by Acharya [1995: 106].
[44] Translation by Bhat [1995: part 1, 539]. He cites the text published at Varanasi (see [4] in References: Primary sources) which reads cd *pāda*-s as: *ucchrāyād yas tṛtīyāṃśas tena tulyā kaṭiḥ smṛtā*.
[45] Iyer's translation was first published in 1884–1885, Acharya's in 1934, and Bhat's in 1981.

secondly, the measures of the flight of steps mostly in buildings of more than one storey are never considered in any architectural treatises as being dependent on the height of the building or the storey.

Bhat [1995: part 1, 539] renders the term as "the height of the foundation above the ground consisting of steps (over which the edifice is built)," and glosses it as "hip or flight of steps in the foundation" in his note. His interpretation of *kaṭi* is probably based on the following commentary by Bhaṭṭotpala (fl. 966/969).

Bhaṭṭotpala's comm. on BS 56.11:[46]
sopānopari yato devagṛhasya prārambhaḥ sā kaṭir ucyate//

Above the steps, that from which [the main part of] a temple (*deva-gṛha*: lit. house of the god) begins is called *kaṭi*.

The steps are usually attached to the front of a temple or in the middle of the four sides, just like in Japanese temples, and we enter by the steps. The word *deva-gṛha* seems to indicate the main part of a temple where the statue of the deity is housed.

Given these evidences, I am unable to decide which of these views is preferable. It is noteworthy that Meister et al. [1989: vol. 2, pt. 1, 402–403] explain, in their glossary, *kaṭi* as

"waist"; wall (early synonym for jaṅghā)

and *jaṅghā* as

wall, wall frieze; elevation between vedībandha and śikhara.[47]

It follows from what has been said above that the *kaḍī* in the TrBh seems to have been derived from *kaṭa*-[3] or *kaṭa*-[4]/*kaṭi*- in architecture. It is likely that the commentator borrowed the term *kaḍī* from engineering text(s) or tradition in order to indicate "depth" in TrBh on Tr E88.

§ 5.2. Meaning of *kaṃḍī*

Three kinds of *kaṇṭa*-s are listed in the CDIAL:[48]

kaṇṭa-[1] m. "thorn";
kaṇṭa-[2] m. "boundary of a village"; and
kaṇṭa-[3] m. "backbone, podex, penis."

[46] The edition published at Varanasi adopts the different chapter number, BS 55.11. See [4] in References: Primary sources.
[47] *vedībandha* and *śikhara* are "basal wall-mouldings" and "tower, spire (in North India)" respectively. See Meister et al. [1989: vol. 2, pt. 1, 411, 414].
[48] See Turner [1999: 133–134].

According to the CDIAL, the form *kaṇḍī-* (or *kaṃḍī-*) can be derived from *kaṇṭa-*[3]:

kaṇṭa-[3] > *kaṇḍī-* f. "back" in Awāṇkārī dialect of Lahndā.

I have no information on the usage of *kaṇḍī* (or *kaṃḍī*) either in mathematical texts or in engineering texts. It is probable that *kaḍī* was originally intended by the author of the TrBh, and the scribe of A_1 or someone else incorrectly copied it as *kaṃḍī*.

§ 6. Concluding remarks

The author of the TrBh explains the rules for excavations, Tr 52–53, in connection with surveying method that consists of three steps: i) stretch cords at unequal places, ii) put them into one, and iii) divide (i.e., fold) the single cord. Since the compound *sama-rajju* is employed in *Brāhmasphuṭasiddhānta* 12.44d probably in the sense of the "mean cord," there is a small possibility that the above method is intended there also.

In BV on L 214, Gaṇeśa mentions the way to obtain more and more accurate values of mean lengths (i.e., $\overline{a}$, $\overline{b}$ and $\overline{h}$) and offers a "proof." In the former part of the "proof," he supposes three different measures—short (m_1), mean (m_m) and surplus (m_2) measures—and expresses their relationship, $m_1 < m_m < m_2$, as "[the mean measure] stands at the middle." In the latter part, the commentator provides a "proof" of $V = Ah$ which is based on the classical interpretation of area and volume.

There are three different meanings of *laṃba* in the TrBh: "perpendicular," "length" and "depth or height." The usage in the second meaning among them testifies that *dairghya* in Tr E88 corresponds with *laṃba* in TrBh on Tr E88. In addition, Āryabhaṭa I employs the term *lambaka* in the sense of "plumb-line," and Bhāskara I glosses it as "a thread with a heavy object at one tip."

We find three forms—*kaḍī*, *kaṃḍī* and *kā*—in A_1. Among them, *kaṃḍī* and *kā* should be emended to *kaḍī* and *ka* (i.e., an abbreviation for *kaḍī*) respectively. The form *kaḍī-* is derived from *kaṭa-*[3] and means "a beam, a squared rafter" in New Indo-Aryan. Otherwise, it is derived from *kaṭa-*[4] and denotes "hip, waist," "flight of steps in the foundation," or "wall." It is likely that the author of the TrBh borrowed the term *kaḍī* from engineering text(s) or tradition in order to indicate "depth" in TrBh on Tr E88.

Acknowledgments

I wish to express my sincere gratitude to Professor Takao Hayashi (Doshisha University) for his helpful comments and suggestions. I would also like to thank Professor Yuko Yokochi (Kyoto University) and Professor Somdev Vasudeva (Kyoto University) for their valuable comments on architectural issues.

Abbreviations

A_1	:	LD Institute, Ahmedabad, Manuscript 1559.
ĀBh	:	*Āryabhaṭīya* of Āryabhaṭa I.
BS	:	*Bṛhatsaṃhitā* of Varāhamihira.
BV	:	Gaṇeśa's *Buddhivilāsinī* on the *Līlāvatī*.
CDIAL	:	*A Comparative Dictionary of the Indo-Aryan Languages*.
L	:	*Līlāvatī* of Bhāskara II.
K_{ED}	:	Kāśī edition of the *Triśatī*.
Tr	:	*Triśatī* (alias *Triśatikā* and *Gaṇitasāra*) of Śrīdhara.
TrBh	:	*Triśatībhāṣya* (anonymous comm.) on the Tr.

References: Primary sources

[1] *Āryabhaṭīya of Āryabhaṭa, with the Commentary of Bhāskara I and Someśvara*, edited with Introduction and Appendices by Kripa Shankar Shukla. Āryabhaṭīya Critical Edition Series, Part 2. New Delhi: Indian National Science Academy, 1976.

[2] The *Bālabodhāṅkavṛtti* of Śambhudāsa. See [4] in Secondary sources.

[3] *The Bṛhat Sañhitá of Varáha-Mihira*, edited by H. Kern. Bibliotheca Indica; collection of Oriental works; New series; Nos. 51, 54, 59, 63, 68, 72 and 73. Calcutta: Baptist Mission Press, 1865.

[4] *Bṛhatsaṃhitā by Varāhamihirācārya with the Commentary of Bhaṭṭotpala*, edited by Avadhavihārī Tripāṭhī. 2 vols. Varanasi: Varanaseya Sanskrit Vishvavidyalaya, 1968.

[5] *Brāhmasphuṭasiddhānta of Brahmagupta*, edited with the editor's own commentary in Sanskrit by Sudhākara Dvivedī. Benares: Medical Hall Press, 1902.

[6] *Līlāvatī* of Bhāskara, edited with Ganeśa's *Buddhivilāsinī* and Mahīdhara's *Līlāvatīvivaraṇa* by Dattātreya Āpaṭe et al. Ānandāśrama Sanskrit Series 107, 2 vols. Poona: Ānandāśrama Press, 1937.

[7] *Triśatiká by Śrídharácárya*, edited by Sudhákara Dvivedí. Káśí: Pandit Jagannátha Śarmá Mehtá, 1899.

[8] Manuscript of the Tr with an anonymous commentary, the *Triśatībhāṣya*: LD Institute, Ahmedabad, 1559.

References: Secondary sources

[1] Acharya [1995]: Acharya, Prasanna Kumar. *A Dictionary of Hindu Architecture: Treating of Sanskrit Architectural Terms, with Illustrative Quotations from Silpāsastras, General Literature, and Archaeological Records*. London: Oxford University Press, 1934. Reprinted, New Delhi: Munshiram Manoharlal Publishers, 1995.

[2] Bhat [1995]: Bhat, M. Ramakrishna . *Varāhamihira's Bṛhat Saṁhitā, with English Translation, Exhaustive Notes and Literary Comments*. Part 1. Delhi: Motial Banarsidass, 1981. Reprinted, 1995.

[3] Colebrooke [2005]: Colebrooke, Henry Thomas. *Algebra with Arithmetic and Mensuration from the Sanskrit of Brahmagupta and Bhāskara*. London: John Murray, 1817. Reprinted under the title, *Classics of Indian Mathematics*, with a foreword by S. R. Sarma. Delhi: Sharada Publishing House, 2005.

[4] Hayashi [2017]: Hayashi, Takao (ed.). "The *Bālabodhāṅkavṛtti*: Śambhudāsa's Old-Gujarātī Commentary on the Anonymous Sanskrit Arithmetical Work *Pañcaviṃśatikā*," *SCIAMVS*, 18, 1–132.

[5] Hayashi [2019]: Hayashi, Takao. *Indo Sanjutsu Kenkyuu Gaṇitatilaka + Siṃhatilaka-chuu Zenyaku to Chuu* (A Study of Arithmetic in India: Annotated Japanese Translations of Śrīpati's Gaṇitatilaka and Siṃhatilaka's Commentary with an Introduction and Appendices), Tokyo: Kouseisha Kouseikaku, 2019.

[6] Iyer [1987]: Iyer, N. C. *The Bṛhat Saṁhitā of Varāha Mihira: Translated into English with Notes*. Part I. Madura: South Indian Press, 1884. Part II. Madras: Foster Press, 1885. Second revised edition in one volume, Delhi: Sri Satguru Publications, 1987.

[7] Keller [2006]: Keller, Agathe. *Expounding the Mathematical Seed: A Translation of Bhāskara I on the Mathematical Chapter of the Āryabhaṭīya*. 2 vols. Basel: Birkhäuser, 2006.

[8] Keller [2009]: Keller, Agathe. "Bhāskara I's Geometrical Diagrams in the *Āryabhaṭīyabhāṣya*," *Mathematics and Medicine in Sanskrit*. Edited by Dominik Wujastyk. Delhi: Motilal Banarsidass Publishers, 2009.

[9] Kern [1873]: Kern, H. "Art. XII.—The Bṛhat-Saṅhitâ; or, Complete System of Natural Astrology of Varâha-mihira," *Journal of the Royal Asiatic Society* 6, 1873, 279–338.

[10] Meister et al. [1989]: Meister, M. W., M. A. Dhaky, and K. Deva. *Encyclopaedia of Indian Temple Architecture: North India*, vol. 1, part 1 in 2 vols. Delhi: American Institute of Indian Studies, 1989.

[11] McGregor [1993]: McGregor, R. S. *Oxford Hindi-English Dictionary*. Oxford: Oxford University Press, 1993.

[12] SaKHYa [2009]: SaKHYa. *Gaṇitasārakaumudī: The Moonlight of the Essence of Mathematics by Ṭhakkura Pherū*, edited with Introduction, Translation, and Mathematical Commentary. New Delhi: Manohar, 2009.

[13] Sarasvati [2007]: Sarasvati Amma, T. A. *Geometry in Ancient and Medieval India*. First edition, Delhi: Motilal Banarsidass, 1979. Second revised edition, 1999. Reprinted, 2007.

[14] Sarma et al. [2008]: Sarma, K. V., K. Ramasubramanian, M. D. Srinivas and M. S. Sriram. *Gaṇita-Yukti-Bhāṣā (Rationales in Mathematical Astronomy) of Jyeṣṭhadeva*, Malayalam text critically edited with English translation, with explanatory notes in English. 2 vols. New Delhi: Hindustan Book Agency.

[15] Tokutake [2021]: Tokutake, Taro. "Indo Sanjutsusho *Triśatī*: Choshamisho no Chūshaku *Triśatībhāṣya* to tomoni" (Indian Arithmetic Book *Triśatī*: With the Anonymous Commentary *Triśatībhāṣya*), Master's thesis, Kyoto University, 2021.

[16] Tokutake [2022a]: Tokutake, Taro. "Study on an Example in the *Triśatī*, an Indian Arithmetic Book," *Historia Scientiarum* 31(2), 2022, 172–185.

[17] Tokutake [2022b]: Tokutake, Taro. "*Triśatībhāṣya* no Chosha no Zaisho to Nendai ni kansuru Ichi Kosatsu: Sanjutsu Hyogen to Gengoteki Tokucho no Bunseki wo tsūjite" (Place and Date of the Author of the *Triśatībhāṣya*: An Analysis of the Arithmetic Expressions and Linguistic Features), *Journal of Indian and Buddhist Studies* 70(2), 2022, 5–8.

[18] Turner [1999]: Turner, R. N. *A Comparative Dictionary of the Indo-Aryan Languages*. Delhi: Motial Banarsidass, 1999.