Structural Dynamics Approaches for Mitigating Seismic Risk of Italian Existing Buildings (Project No. 2022L-02)

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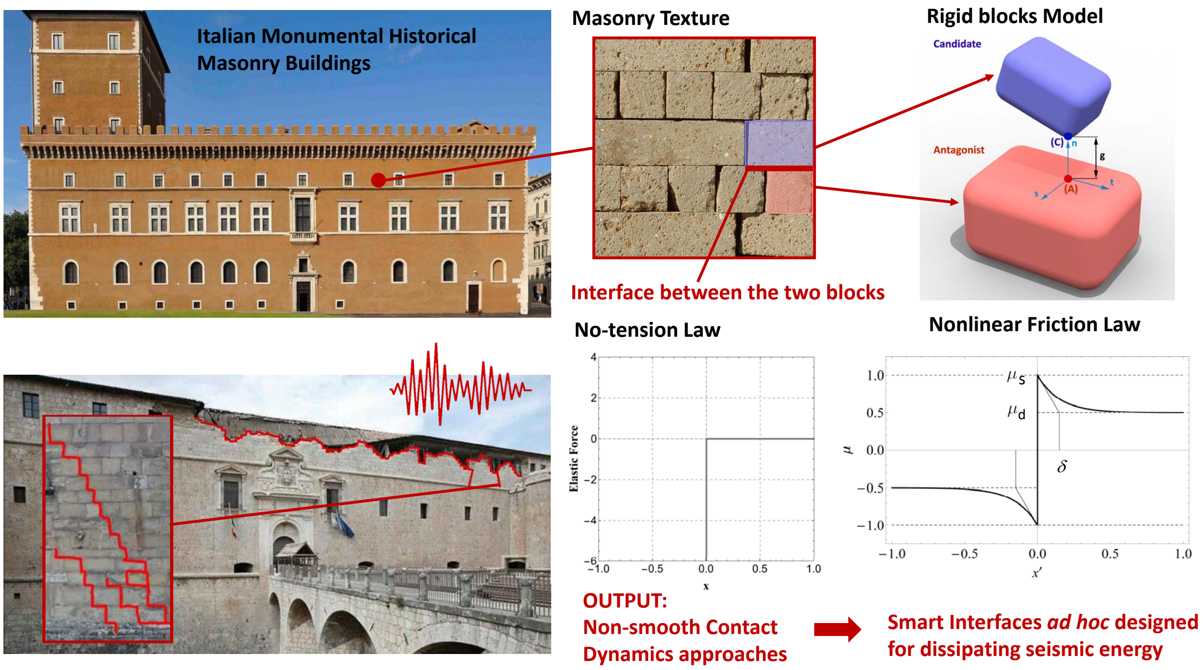
Graphical Abstract



Abstract of the research project

The research project aims to gain insights into the non-smooth contact dynamics of existing masonry structures, which represent the major part of the historical architectural heritage of Italian cities and are sensibly vulnerable with respect to moderate-large earthquakes, that relatively often occur in the Italian territory, characterised by high seismicity rate. Additionally, masonry constructions should not be demolished and reconstructed due to their architectural and historical values, according to ICOMOS and ISCARSAH guidelines [1].

In the last decades, several numerical methods have been proposed in the inherent literature in order to faithfully grasp the actual behaviour of these structures, and, more importantly, to define *ad hoc* seismic retrofit interventions. Despite the numerous different approaches suggested, the debate within the scientific community is still alive and does not converge on a unique structural model. With the aim of providing a novel understanding of the complex mechanics of masonry structures, we enhance some discrete element methods addressed by the inherent literature, also giving raising further issues into non-smooth contact dynamics of rigid

masonry blocks separated by mortar joints. In detail, more refined models are uploaded to account for some dissipative mechanism, e.g. the rocking phenomena and the contact friction between rigid blocks and several parametric analyses are implemented, in order to provide a comprehensive view on the competition of the above-mentioned effects on the strong nonlinear dynamics of masonry panels. These results can be helpful for defining novel smart engineering interfaces that replace the mortar joints at specific parts of the structures, thus improving the seismic behaviour of the whole complex. Therefore, it is believed that this research can respond to the need to adapt the existing architectural heritage in high seismicity areas like Italy with minimally invasive interventions that do not alter the building's structure, thus preserving its historical, artistic and architectural value.

Contents

1 Introduction and the state-of-the-art

1.1 Masonry Buildings

The existing Italian building stock is mainly made by masonry structures characterized by a large number of typologies in the whole national territory. Unfortunately, experience from recent earthquakes has highlighted that existing masonry buildings, both ordinary and of historic-monumental value, represent one of the classes of structures most vulnerable to earthquakes. The high vulnerability of these buildings, is related to several aspects including certainly the age of construction and the implementation over time of multiple interventions that have evidently modified their original structural behaviour.

Earthquakes certainly represent one of the natural events with the greatest destructive capacity and, therefore, seismic improvement/retrofit of existing structures nowadays becomes of vital importance, both for the protection of people's lives and for the preservation of buildings of historical-monumental interest and value.

Improvement/adaptation interventions should corroborate structural safety without changing the architectural value, shape, or weight of the elements. For this reason, the interventions carried out should comply with the most stringent rules of modern restoration culture, which require minimization of variation with respect to the authenticity, reversibility and compatibility of the work. In this field, traditional intervention techniques do not always meet these criteria, which should satisfy structural requirements and conservation needs. In these cases, the use of innovative techniques can be helpful in solving the problem.

These difficulties are even greater if the buildings being reinforced are subject to constraints. Indeed, in such cases, the choice of reinforcement materials and techniques is further reduced, as it is conditioned by specific requirements for compatibility with the existing construction.

In addition, urban built development should be guided according to a sustainable approach, simultaneously pursuing economic development, social welfare and environmental protection. This leads to research and development of technologies and materials that can offer specific performance, with low economic, environmental and social costs.

The improvement of structural performance can be pursued through the development of reinforcement systems based on innovative materials and techniques that are chemically, physically, and mechanically compatible with existing materials, as well as complying with guiding criteria (such as reversibility, durability, low impact on the existing geometric and structural configuration).

The development of research and technology in recent decades has made possible to use composite materials in several industrial sectors, thanks to the refinement of the chemical knowledge, capable of meeting the performance requirements. This innovation has also involved the construction sector, an area where composite materials represent a valid alternative to traditional reinforcement techniques and can be very effective from the point of view of the compatibility and durability of the intervention. However, although composite materials has been successfully employed for seismic retrofit interventions of existing masonry structures, research on novel and innovative techniques, also with the possibility of exploiting the advancements into the composite materials know-how, are here provided, with an unconventional key to understanding the structural behaviour of masonry structures.

It should be stressed that the major part of Italian existing masonry buildings are not sufficiently adequate to resist to seismic loads, however, since they should not be demolished and reconstructed due to their architectural and historical values, according to ICOMOS and ISCARSAH guidelines [1], innovative and non-invasive retrofit interventions should be defined.

The earthquakes of the last 30 years have led to a greater awareness of seismic risk as well as the issues closely related to the vulnerability of existing buildings, especially those ones of historic monumental interest. In this perspective, Italy's historic city centres are to be considered historical architectural heritage as they are evidence of past civilizations and a trace of their urban and architectural culture.

Therefore, the preservation of the typological characteristics of such structures becomes an aspect of fundamental importance, but at the same time, safeguarding them from collapse may conflict with this aspect. Consequently, combining the needs of preservation with those related to prefixed safety standards, means adopting analysis systems specifically developed for such types of artefacts, as well as, intervention solutions with innovative, reversible and compatible methods and techniques, which take into account that the historical architectural heritage has been tested for events of significant intensity.

As well-known, masonry buildings can exhibit local and/or global collapse. Most of the damage detected in masonry structures following medium-large earthquakes is due to the triggering of local-type kinematics while a residual part is due to global collapse. Moreover, the failure of the masonry structure understood as a global organism can generally occur in the case where none of the local mechanisms are triggered. Therefore, the study of the structural behavior of masonry buildings generally starts from the study of local mechanisms. The analysis of such mechanisms requires accurate knowledge of the masonry element or portion of the structure to be investigated. This is because behaviour, as well as criticality, is strongly influenced by the construction details of the elements, such as may be, the connections and masonry texture. The capacity of the building against seismic actions is highly dependent on the quality of the connections especially of the masonry intersections (e.g. the connection between two interior orthogonal panels or the buildings angle) and the horizontal load-bearing structures, both intermediate and roofing. In detail, the typology of the horizontal systems affects the seismic response: vaults, for instance, result in thrusts on the façade masonry if not adequately resisted; thrusting roofs also result in out-of-plane actions; old wooden floors are not rigid in their own plane, thus not distributing actions evenly over the whole structure. Therefore, the process of knowledge of the structure is not always immediate, since, as above-mentioned, the age (or historicity) of the building can heavily affect the structural layout. As a result, the first step of analysing an existing building is the knowledge of its history, thus deriving useful information for defining the load-bearing structure. The time of construction, the geographical area to which it belongs, and the development of its construction, which could take place in several stages with the succession of different techniques and materials, provide important indications for structural engineers.

Masonry is a composite material whose mechanical properties depend on the characteristics of the individual components (stone elements and mortar joints), but also on the masonry texture and the shape of the elements composing it. It is essential, therefore, to know the construction methods (texture, single or double facing, presence of a sack filled with inconsistent material, presence of connecting diatons between the wall) as well as the materials of which it is made (pebbles, hewed stones, squared stones, bricks, bedding mortar, etc...) and their degradation. In addition, the cracking pattern, with cracks that may be in some way expected due to the vertical loads or produced by the incipient initiation of kinematic mechanisms, is a very valuable source of knowledge of the current or past distribution of stresses in masonry. The study of collapse mechanisms involves the fundamental assumption of monolithic behavior of the portion of masonry under study. This condition is not always verified and sometimes, in the presence of low-quality masonry, the phenomenon of masonry disintegration can be triggered. This is a brittle collapse and dissipates a much smaller amount of energy than a first- or second-mode

mechanism. Therefore, it is advisable, even before approaching the study of the kinematic mechanism, to provide for appropriate interventions (restyling of joints, connection of different faces in multi-headed masonry by means of diatons, etc.) designed to avoid the occurrence of this phenomenon.
In the inherent literature [2–12] two types of collapse mechanism are conventionally analysed: the out-of-plane and the in-plane mechanism.

1.2 Out-of-plane mechanisms for masonry structures subjected to seismic loads

Generally, out-of-plane collapses mainly occur when the structure is not able to adequately respond with global behaviour to the earthquake and there is the triggering of kinematic mechanisms due to the forces that strike the masonry panel orthogonally to its plane. Such mechanisms are also referred as "first-mode mechanisms" since they require less energy for their activation and therefore are the first ones to be triggered, if proper restraining among orthogonal walls is not registered, thus not guaranteeing the box behaviour.
The most frequent mechanisms, without claiming to be exhaustive, are:

- vertical overturning of the wall
- overturning of the wall with two side wings
- corner failure
- vertical arch
- horizontal arch
- roof/floors collapse.

Moreover, these mechanisms may have variations depending on the involvement of the entire element (façade, gable, corner, etc.) or only part of it due to the presence of multi-lined masonry disconnected from each other.

The vertical overturning of the wall due to the seismic loads represents one of the most frequent and dangerous damage situations. The mechanism can be modelled as a rigid rotation of the whole panel or a portion of it around a horizontal cylindrical hinge placed at its base, activated by stresses orthogonal to the plane containing it. This phenomenon occurs when the wall invested by the seismic action orthogonal to it, is not effectively constrained to other elements such as the roof, floors or perpendicular walls. The experimental evidence confirm that with the same constraint conditions and masonry quality, the the kinematism is triggered first in walls normal to the seismic action. Such kinematic motion may affect the entire façade or only superior parts of it, as represented in Figure 1.

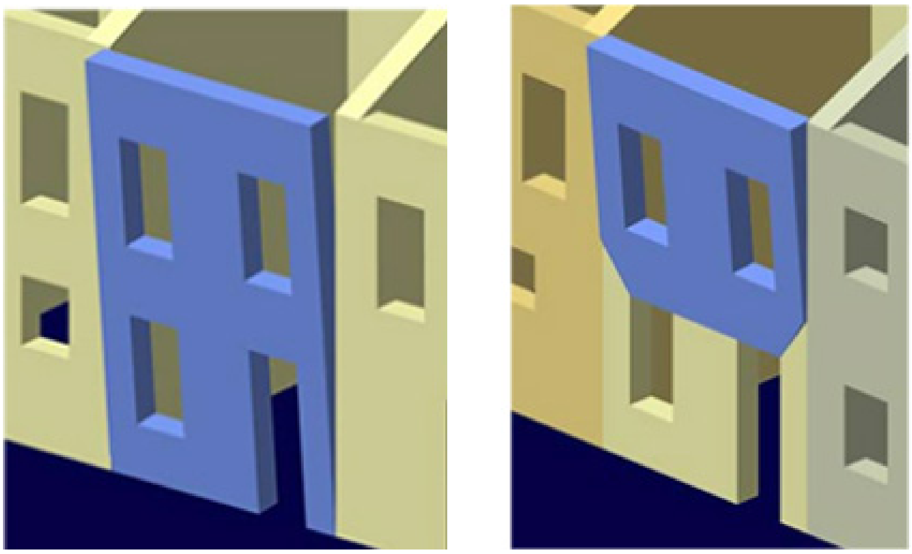


Figure 1: The vertical overturning of the wall mechanisms [13].

Signs of incipient activation of this mechanism can easily be detected by observing the intersection between the wall subject to the mechanism and the one orthogonal to it. Indeed, in such a case, it will not be difficult to detect one or more vertical lesions at the intersection of the two wall panels, with greater amplitude in correspondence of the top of the building (Figure 2). In addition, a careful interior inspection may show that the roof beams or floor slabs have slipped out.

Figure 2: The mapping of the cracks that anticipates the vertical overturning of the wall [13].

The overturning of the masonry wall with one or two sides wings occurs, as well as the previous mechanism, through a rigid rotation of entire façades or portions of walls with respect to a horizontal cylindrical hinge placed at the base of the panel and accompanied by the dragging of parts of the walls orthogonal to it (Figure 3). The size of the portion of the orthogonal wall that undergoes tilting along with the façade depends strongly on the masonry texture. The angle of inclination and the extension of the crack are more evident for regular masonry that irregular one. In addition, the presence of openings may in some cases condition the spread of the failure in the orthogonal wall.

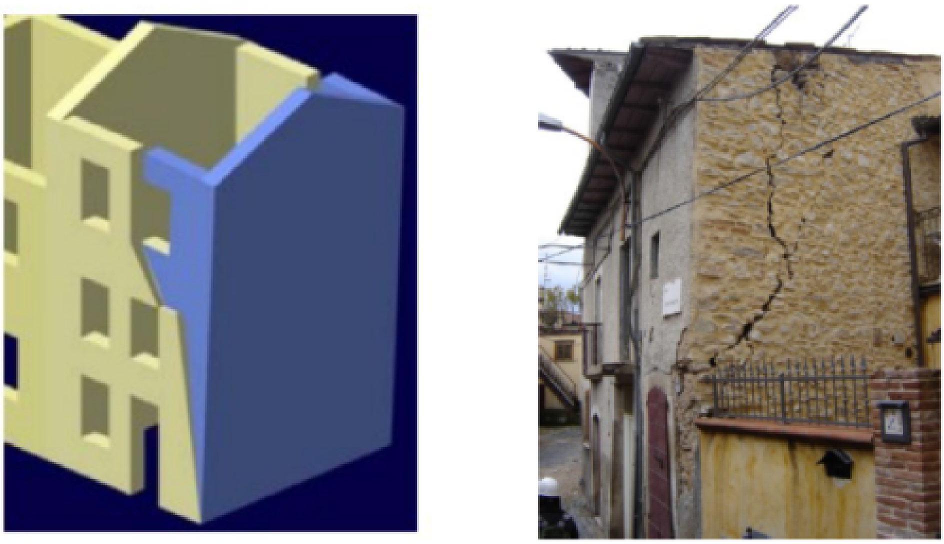


Figure 3: The overturning of the wall with two side wings attached to the orthogonal façade [13].

The corner failure is achieved through a rigid rotation of a detachment wedge, bounded by diagonal cracks in the competing walls in the free angles, with respect to a hinge located at its base. These kinematic mechanisms are common in buildings with concentrated thrusts at the head of corners transmitted by the struts of the roofs. It is assumed that the tilting of the corner occurs in the direction of the strut thrust and the rigid kinematic motion is defined by the rotation of the defined macro-element around an axis perpendicular to the vertical plane that forms an angle of about 45 degree with the walls converging into the angle, as represented in Figure 4.

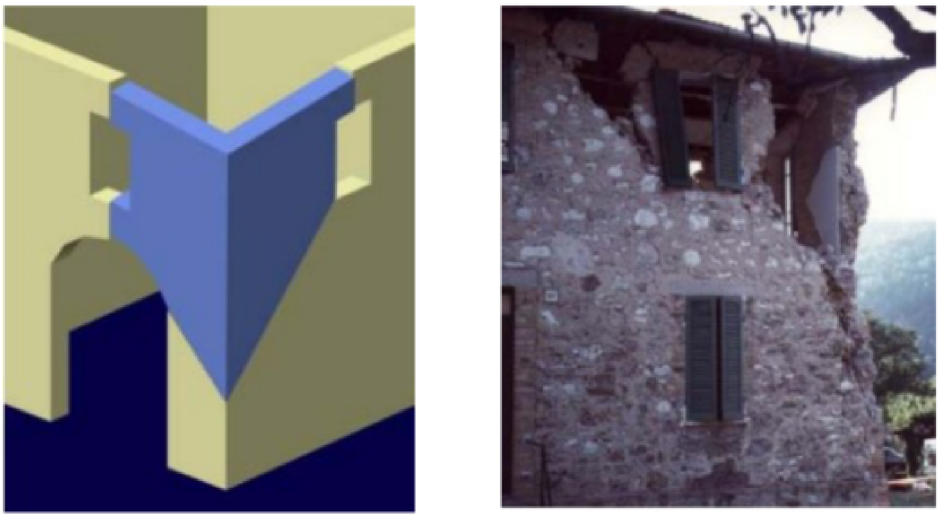


Figure 4: The corner failure [13].

The vertical arch consists of the vertical bending mechanism of a wall and it is the formation of a horizontal cylindrical hinge that divides the panel into two blocks. It is generally modelled as the mutual rotation of the blocks around this axis by out-of-plane actions (Figure 5.

Figure 5: The vertical arch mechanism in masonry façades [13].

The horizontal bending mechanism, i.e. the horizontal arch of a wall occurs with the ejection of material from the top zone of the wall and the detachment of wedge-shaped bodies accompanied by the formation of oblique and vertical cylindrical hinges for out-of-plane actions, as sketched in Figure 6.

Figure 6: The horizontal arch mechanism in masonry façades [13].

Finally, the roof/floor collapse, also named the gable collapse is manifested by the ejection of the superior part of the gable wall and the detachment of wedge-shaped bodies defined by oblique and vertical cracking sections by out-of-plane actions, as depicted in Figure 7.

Figure 7: The gable collapse mechanism [13].

The kinematic motion is generally caused by the cyclic hammering action of the roof ridge beam. In the seismic phase, the presence of large ridge beams causes the transfer of high thrust to the gable wall and can result in the detachment of wedge-shaped macro-elements and the establishment of buckling conditions that are manifested through their rotation around oblique hinges.

1.3 In-plane mechanisms for masonry structures subjected to seismic loads

If the masonry structure is constructed in order to avoid the occurrence of first mode mechanisms thanks to the box behaviour, the whole structural complex is involved in the seismic response, thus implying that the strengths of the elements in their own plane are activated. With respect to this second case, these mechanisms are defined second mode mechanisms and mainly involve the flexural and shear strengths of the elements of the masonry piers. Intuitively, one can easily note that the collapse of a wall element due to the damage in its own plane requires much more energy that the one needed for the activation of a kinematic mechanism outside the plane.

The behaviour of the walls in the plane strongly depends on the size of the masonry piers. The aspect ratio, i.e. the ratio between the panel height and the the panel base, together with the gravitational and shear loads involved, can result in either eccentric axial force crises, typically for slender panels, or shear failure, in the case of spandrel panels with limited gravitational loads, or tensile shear crises, as analytically shown in Figure 8.

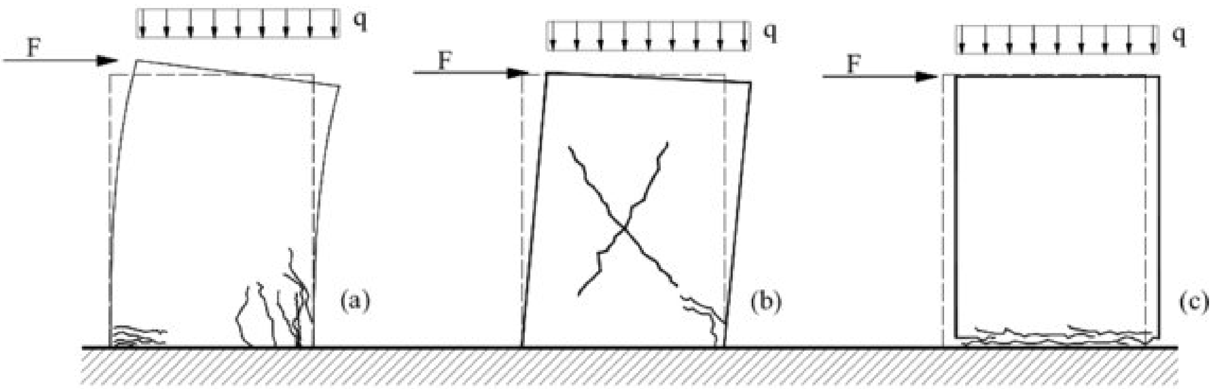


Figure 8: In-plane crises mechanisms: (a) flexural failure, (b) shear-diagonal failure and (c) shear-sliding failure [14].

It should be stressed that the in-plane failure mode is associated to the brittleness of the material and that manifests itself with detachment fractures, such as those ones reported in Figure 9. Such fractures consist in cracks that generally separate neatly two parts of seemingly intact material and are usually the "good" ones, that is those contributing to the accommodation and release of stress. From the figure, one can note that for masonry structures consisting of bricks and mortar joints, the cracks mainly occur along the mortar beds.

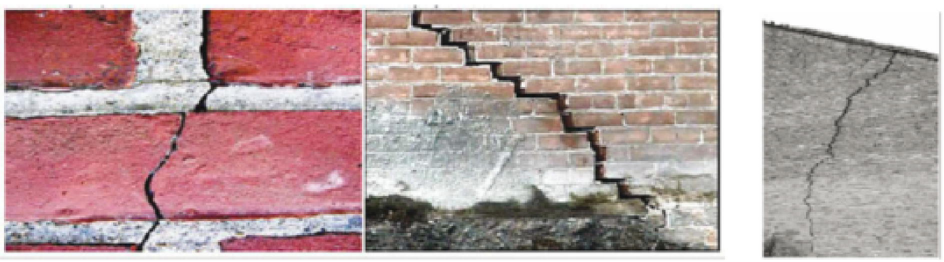


Figure 9: Fracture of detachment in brick walls at different scales [15].

Masonry structures have been remodelled and in some cases heavily modified over the centuries, and past interventions have often been found to be fallacious and contrary to their design idea only increased the vulnerabilities of masonry structures. For this reason, knowing the entire life history of the structure is the most important step in the analysis of masonry buildings, especially if they are of historical-monumental interest. Maintenance works with structural consequences, which have been most frequently adopted in the past, include the following: replacement of floor or roof slab; insertion of connection systems of poorly adhered masonry walls; opening and closing of compartments.

Figure 10 shows the effects produced by replacing a deformable wooden roof with a more rigid mixed steel and concrete slab, arranged with a different warping than the original one. The new arrangement involves a different resisting system than the one originally conceived. The change in warping certainly induces a different distribution of loads on the load-bearing vertical elements. Moreover, it may induce the occurrence of a crack pattern of the wall close to the opening, due to the rearrangement of vertical loads not initially contemplated. Therefore, the replacement of the roofing has caused a further disruption in the masonry pier.

Figure 10: The crack in the masonry wall as a result of floor replacement with change of warping [13].

On the one hand, the replacement of flexible floor slabs with rigid diaphragms can avert the collapse of the structure due to out-of-plane mechanisms, on the other hand, it changes the response of the resisting wall elements from flexural to shear behaviour. In such a case, it is necessary to verify the adequacy of the structures to the new stress system.

There are frequent cases where lightweight pushing roofs (e.g. vaults and wooden trusses) have been replaced by heavy reinforced concrete ones, often adopted in order to meet modern safety requirements. However, collapses and disruptions suffered in recent seismic events show how this mode of intervention, if not well designed and executed, has been inadequate especially with respect to safety, if the suitability of the structural system to the increase of inertial forces has not been assessed and an appropriate constraint between the new roof and the masonry has not been designed (Figure 11).

Figure 11: Damage due to roofing replacement [13].

Therefore, static failures of building fabrications arise from disruptive actions of different kinds. Old age is the inexorable enemy of buildings, undermining the material by making it vulnerable to thermal, hygrometric and weathering variations. Each disruptive cause induces alterations in the static equilibrium regime of the masonry system; exceeded certain limits, these alterations give rise to static failures that manifest themselves through cracks and deformations. From the diagnosis of the cracking pattern, it is possible to trace back to the failure that generated it and its cause, through a direct cause-and-effect relationship. It is clear, therefore, that continuous monitoring of the proper functioning of the structures and systems by the users of the asset themselves, as well as planned maintenance that makes use of the various technological innova-

tions in the field of structural engineering, represent key aspects in order to significantly reduce the vulnerability of existing buildings and pursue the dual purpose of conservation and seismic safety.
In this perspective, numerical methods for faithfully grasping the complex structural behaviour of this heterogeneous material have been provided in the last decades, in order to theoretically predict the stresses and cracking distributions of these typologies of structures.

1.4 Numerical methods for masonry structures

Numerical methods for masonry structures have become increasingly important in recent years due to the need for accurate and efficient analysis of these complex structures. As a result, various numerical methods have been developed to accurately analyse these structures while considering their unique properties and behaviour. However, the complexity of masonry structures, their heterogeneity, and their specific failure mechanisms make accurate numerical modelling particularly challenging [4, 7].
Numerical modelling of masonry structures can be roughly classified into three categories: continuum-based, discrete element methods and limit analysis-based model. Continuum-based approaches treat masonry as a homogeneous material, while discrete element methods simulate the behaviour of individual building elements and their interaction with each other. Both approaches have their advantages and disadvantages, and the choice of method will depend on the specific structural and material characteristics of the masonry structure being modelled.
Within the continuum-based methods, finite element analysis (FEA) is one of the most popular numerical methods used for masonry structures [16–24], since it is well-known FEA is a powerful tool that can model complex geometries and provide accurate predictions of the structural response. However, one of the challenges of using FEA for masonry structures is the need to accurately model the behaviour of masonry units and the mortar beds that binds them together. Generally, homogenization procedures and multi-scale approaches are applied and the material constitutive law is deduced from a homogenization procedure which associates the structural-scale model to a material-scale model (which assumes the main heterogeneities of masonry) of a representative volume element (RVE) of the structure. In detail, accurate modelling of the mechanical behaviour of masonry materials is crucial to obtain reliable results from finite element analysis of masonry structures. Several approaches have been developed to represent

the behaviour of masonry materials, including linear elastic models, nonlinear isotropic models, and nonlinear anisotropic models. More sophisticated models consider the interaction between mortar and masonry units, accounting for fracture effects, residual tensile stress, and progressive damage.

Moreover, the choice of finite element type is critical to accurately represent the structural behaviour of masonry. The main types of elements used for masonry analysis are brick elements, shell elements, and interface elements. Brick elements are suitable for three-dimensional modelling of masonry walls, while shell elements are preferred for modelling masonry planes or shells subjected to predominantly in-plane loads. Interface elements are used to model discontinuities between masonry layers, such as mortar joints. Also the definition of constitutive models assume a key role in the application of FEM (finite element method) on masonry structures. The most common constitutive models currently employed in the inherent literature include linear elastic, nonlinear elastic, damage-plasticity, and damage with progressive fracture models. These models consider stress-strain nonlinearity, damage, fracture, and anisotropic behaviour of masonry. Finite element analyses for masonry structures can be performed using various analysis techniques, such as linear static analysis, nonlinear static analysis, dynamic analysis, and time history analysis. Linear static analysis provides an initial understanding of the structural response, while nonlinear static analysis considers material nonlinearity and large deformations. Dynamic analysis is used to evaluate the dynamic response of masonry structures under seismic or impact loads. Time history analysis incorporates the time-dependent nature of the loading to capture transient and dynamic effects. However, verification and validation of finite element models are crucial to ensure the accuracy and reliability of results. Verification involves comparing the numerical results with analytical or experimental solutions for simplified problems. Validation involves comparing the numerical results with experimental data obtained from real masonry structures. These processes help assess the appropriateness of the selected modelling approaches and validate the numerical predictions.

Another numerical method frequently used for masonry structures is the discrete element method (DEM). DEM is used to model the behaviour of individual masonry units and their interactions. This method can simulate the behaviour of masonry during various loading scenarios, including compression, tension, and shear. The DEM method is particularly useful in analysing localized damage or cracking in masonry structures. As well-known, the discrete element method is a numerical approach that treats materials as a collection of discrete particles interacting with each other through contact forces. DEM has been extensively used for the analysis of granular materials, and its application to masonry structures has gained significant attention [25–33]. Masonry structures are composed of discrete units (bricks or blocks) held together by mortar, making DEM a suitable method for faithfully capturing their behaviour. In DEM, the behaviour of each discrete particle is determined by considering its motion, interactions with neighbouring particles, and contact forces. The particles can experience translational and rotational motion, and their interactions are governed by constitutive laws and contact algorithms.

Material modelling in DEM for masonry structures involves the characterization of both masonry units and mortar beds. The discrete particles representing masonry units can be assigned properties such as shape, size, and mechanical behaviour. The properties of the mortar can be defined based on its elastic properties, compressive and tensile strength, and bond behaviour. The material models can be calibrated using experimental data or analytical solutions.

It should be stressed that contact algorithms play a crucial role in DEM simulations of masonry structures as they determine the interactions between particles and the generation of contact forces. Various contact models, such as Hertzian contact, linear elastic contact, or bonded contact, can be employed depending on the desired level of realism. Contact detection algorithms, such as the spatial grid method or the neighbour list method, are used to efficiently identify and compute contact forces between particles.

Therefore, DEM offers several advantages for analysing masonry structures, such as its ability to capture the discrete nature and complex behaviour of masonry units, the ability to simulate progressive failure and cracking, and the consideration of non-linear material behaviour. However, it also has limitations, including computational requirements, difficulties in capturing the macro-scale response of structures, and challenges in accurately representing the bond behaviour between masonry units.

In the recent years, the application of DEM to masonry structures is an active area of research, and ongoing developments aim to overcome the limitations and enhance its capabilities. This includes the development of advanced constitutive models, improved contact algorithms, and the integration of DEM with other numerical methods for multi-scale analysis. Additionally, experimental validation and benchmarking against real masonry structures are crucial for the validation and wider acceptance of DEM in practical applications.

Finally, among models adopted in literature for the analysis of masonry structure, it is necessary to mention the approaches based on the limit analysis of rigid blocks. In detail, masonry structures are conceived as rigid bodies and the geometry of the construction and the loading condition represent the input needed by these numerical models. Generally, the limit analysis and its related theorems, i.e. the static or kinematic theorem, have been widely utilized for the analysis of the equilibrium and/or collapse of masonry structures. These methods provide an efficient and rigorous way to determine the load-carrying capacity and failure modes of masonry structures. Within this framework of limit analyses-based models, several novel solutions have been provided, typically following the Heyman's rigid no tension assumption [34]. In particular, in the context of masonry structures, limit analysis provides a valuable tool for understanding the failure behavior, load-carrying capacity, and collapse mechanisms [35–42]. This method takes into account the non-linear behaviour of masonry materials and provides insights into the safety and stability of these structures. The hypotheses of the plasticity theory are also assumed in the limit analysis approaches for masonry structures, although this can represent a strong assumption for masonry material. Thanks to the limit analysis, it is possible to describe the non-linear behavior of masonry materials and the formation of plastic hinges under loading. The yield criteria and flow rules are essential components of plasticity theories and define the onset of yielding and the subsequent plastic deformation. Various plasticity theories, such as the Drucker-Prager or the Mohr-Coulomb models, have been used for masonry structures. Moreover, understanding the failure mechanisms in masonry structures is a crucial point for this analyses. Failure modes in masonry can include diagonal tension, shear, flexural bending, and compression. Different failure mechanisms may govern different parts of the structure, such as walls, arches, or columns. The identification and analysis of failure mechanisms are necessary for accurate prediction of the collapse load and structural behaviour. In this perspective, limit analysis and its related methods have been successfully applied to various practical applications in masonry structures. These include the design of arches, vaults, and domes, stability analysis of retaining walls, assessment of existing structures, and optimization of structural configurations. Limit Analysis provides valuable insights into the load-carrying capacity and safety factors of masonry structures, aiding in their design, retrofitting, and preservation.

However, as above-mentioned limit analysis has some limitations for grasping the structural behaviour of masonry construction, including the assumption of perfect plasticity, neglecting the effect of cracking and damage, and difficulty in capturing the influence of time-dependent phenomena.

2 Aims of the research project

Within this framework, since as well-established the good restraining among orthogonal masonry walls and the box behaviour of masonry structures should be verified and guaranteed in advance in seismic areas for avoiding first mode failure mechanisms, this research project mainly focuses on the in-plane mechanisms of masonry structures. In particular, this study investigates the nonlinear dynamics of masonry panels consisting of interconnected rigid components (e.g. the bricks), with the aim of defining innovative smart interfaces, for conceiving and designing novel structures that can meet the increasingly high-performance requirements demanded especially in structural engineering applications. Therefore, this research project provides some upgrades for discrete element methods adopted for masonry structures, which to the scale of the masonry panel are generally preferred for their accuracy and reliability, in order to trace back the real structural behaviour of these heterogeneous structures. The masonry piers can be considered and modelled as multi-body systems, whose fundamental units are the bricks and the interfaces among blocks are the mortar beds. In general, a multi-body system is defined as a collection of subsystems, called bodies, components, or substructures. The motion of the subsystems is kinematically constrained by different types of joints, and each subsystem or component may experience large translational and rotational displacements. In this sense, basics of this research is the understanding of the motion of subsystems made by rigid blocks interconnected through joints that can be *ad hoc* designed for optimizing the system in reference to an assigned target function, i.e., for instance, the increase of the overall strength of the structure, the reduction of the absolute displacements or accelerations of each blocks with respect to a certain input, the increase of the energy dissipating by the several interfaces. As a consequence, the starting sample of this research is a panel made by rectangular blocks connected through mortar joints. The main idea is to re-design the joints located at selected portions of the structure as smart engineered interfaces that can dissipate energy by exploiting some mechanism, e.g. the rocking phenomena and the contact friction among the several blocks. To do this, the bodies are modelled as rigid blocks and non-smooth contact dynamics approaches are considered for faithfully grasping the strong nonlinear dynamics of the overall structure, by following some methods provided in the inherent literature [43–53]. More refined models are then uploaded to account for the competition of all the effects above-mentioned and numerical applications are implemented to validate the proposed approach.

Therefore, it is believed that this research can respond to the need to gain a deeper understanding of the complex structural behaviour of masonry structures, by developing a theoretical refined model that can allow to faithfully describe the nonlinear dynamics of systems made by rigid bricks interconnected through mortar joints. An in-depth understanding of this dynamics can certainly help structural engineers to conceive new interfaces that can improve the in-plane seismic behaviour of masonry panels, by means of non-invasive interventions.

3 Methods

3.1 Problem's formulation

Starting from the models provided by several scholars [11,47,48,54–58] discrete element methods are employed for accounting for the non-smooth contact dynamics phenomena of the complex made by rigid blocks and joints. In this context, the main lack in the inherent literature is based on the fact that the friction and the damping effects that arise in the interfaces among bricks cannot be faithfully and accurately grasped by means of the provided approaches. Generally, according to the present state-of-art, the Signorini's contact law [59, 60] and the Coulomb model [61–63] are utilized for describing the contact dynamics among the blocks, as summarised in Figure 12).

However, since the the strong nonlinear phenomena occurring among blocks are the most relevant for predicting the structural response, more refined models are required for grasping, just to highlight a few issues, the different yield strengths of the composite material with respect to tension or compression field, and also the influence of the variation of bricks' relative velocity on the friction forces. To in-depth investigate this contact dynamics, a two-dimensional panel made by regular rigid blocks with rectangular shape is analysed as first sample of this study and a discrete element model has been developed in the symbolic environment of Mathematica© [64]. A single horizontal degree of freedom is assumed for each rigid block, by considering the in-plane behaviour of the wall. For each block, modelled as a point mass at its centre of gravity, interactions with nearest neighbouring blocks are assumed and several aspect ratios of the panel can be reproduced, by arbitrary varying the number of lines and columns of

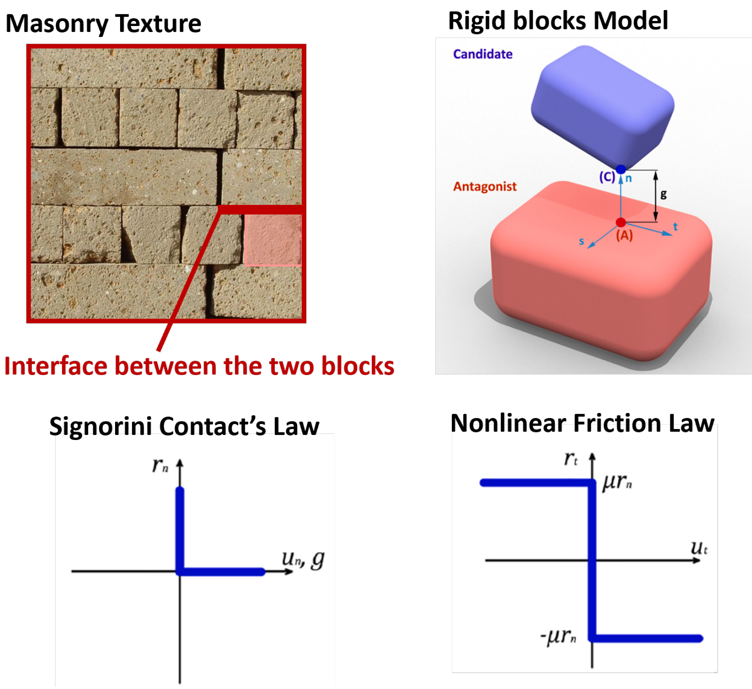


Figure 12: Two-dimensional panel made by several rigid blocks and Signorini's contact law and Coulomb friction model assumed in the inherent literature.

masonry bricks (Figure 13).

In addition, since old masonry panel often exhibit irregular pattern in terms of size and typology of bricks, it is necessary to introduce the possibility to vary –still in a fully parametric way– the dimensions of the rows and bricks is taken into account, leading to several configurations of masonry textures, as depicted in Figure 14.

A further update is then considered for exploiting the possibility of connecting in series two or more masonry panels, still accounting for the in-plane behaviour of the whole complex, as represented in Figure 15.

Masses' interactions rule the dynamics of the overall system and should be carefully defined in order to reproduce friction phenomena, dissipative effects and contact forces among the various blocks. Several models are then uploaded to accurately characterize each interaction.

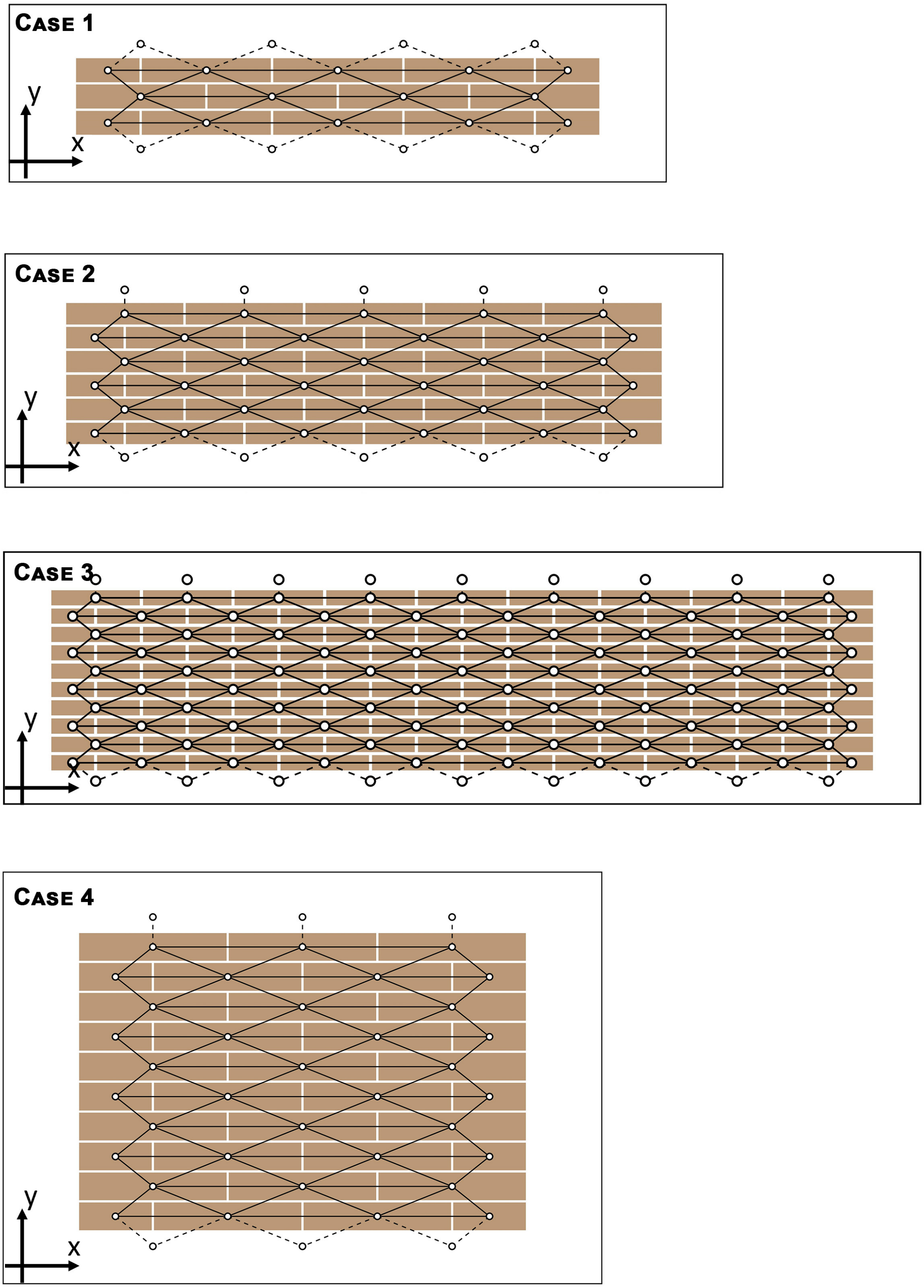


Figure 13: Graphical representation of the interactions among each block, by selecting several samples of masonry panel.

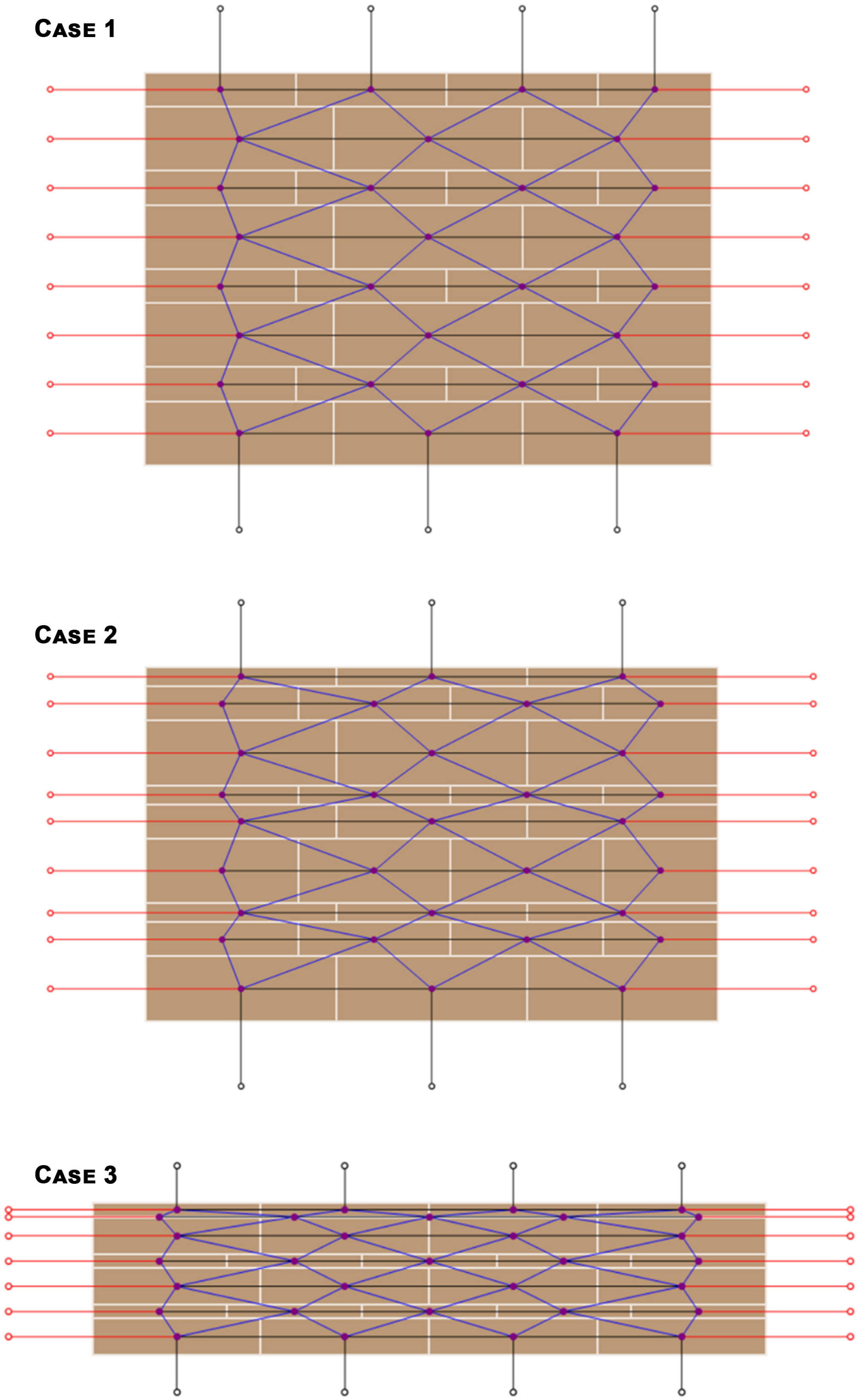


Figure 14: Graphical representation of some irregular patterns of masonry panel that can be modelled by the developed parametric code.

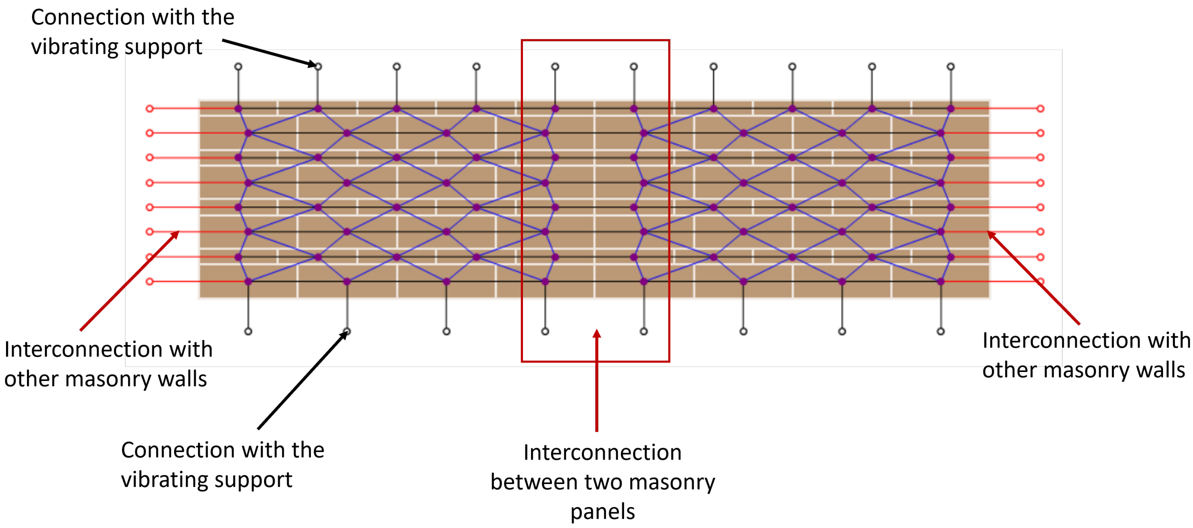


Figure 15: Graphical representation of two masonry panels connected.

3.2 Constitutive laws and dynamics equations

Let's imagine reproducing the experimental setup classically employed for studying the response of a panel made by rigid blocks subjected to horizontal accelerograms provided thanks to an actuator, as depicted in Figure 16.

In order to model the in-plane behaviour of the panel, interactions among bricks are defined, by uploading a nonlinear law to capture friction phenomena [65–68], while utilizing the classical Kelvin-Voigt model for taking into account the dissipative effects and the stiffness in the interfaces among the bodies. Additionally, the vertical loads and the self-weight of each mass are considered, since the gravity loads affect the friction forces that arise in the interface.
For instance, with reference to the Element i of Figure 16, the differential equation of motion can be made explicit as follows:

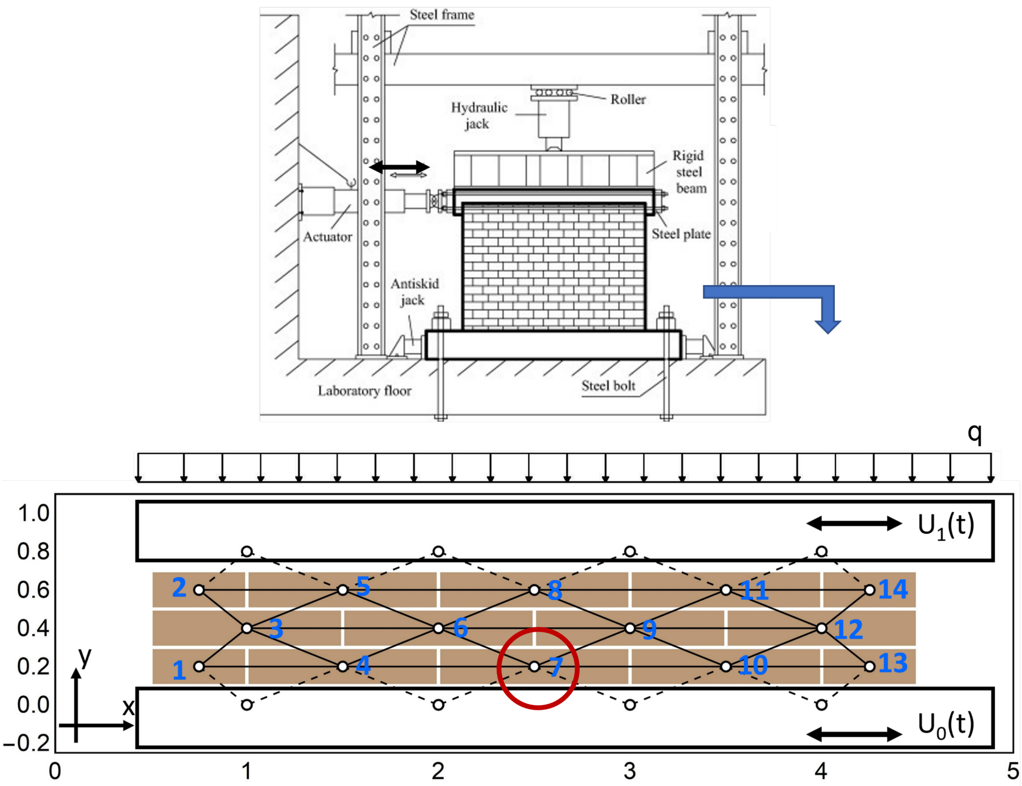


Figure 16: The experimental setup that can be grasped by the provided model with $U_0(t)$ and $U_1(t)$ referring to the displacement histories that can be assigned to the supports of the panel.

$$
\begin{aligned}
&2K[7,0](-U[0][t]+x[7][t])+K_{NL}[7,4](-x[4][t]+x[7][t])+\\
&+K_L[7,6](-x[6][t]+x[7][t])+K_L[7,9](x[7][t]-x[9][t])+K_{NL}[7,10](x[7][t]-x[10][t])+\\
&+2C[7,0]\,(-U'[0][t]+x'[7][t])+C[7,4]\,(-x'[4][t]+x'[7][t])+\\
&+C[7,6]\,(-x'[6][t]+x'[7][t])+2\,(g\,m[7]+P_0[7])\,\mu\,(-U'[0][t]+x'[7][t])+\\
&+(g\,m[7]+P_0[7])\,\mu\,(-x'[4][t]+x'[7][t])+(g\,m[7]+P_0[7])\,\mu\,(-x'[6][t]+x'[7][t])+\\
&+(g\,m[7]+P_0[7])\,\mu\,(x'[7][t]-x'[9][t])+C[7,9]\,(x'[7][t]-x'[9][t])+\\
&+(g\,m[7]+P_0[7])\,\mu\,(x'[7][t]-x'[10][t])+C[7,10]\,(x'[7][t]-x'[10][t])+\\
&+m[7]x''[7][t]=0,
\end{aligned}
\tag{3.1}
$$

where μ is the friction coefficient, $K_L[i]$ refer to the linear stiffness of mortar beds belonging to different lines, $K_{NL}[i]$ is the in-line stiffness of mortar joints (being the subscript NL referred to the nonlinear law which is addressed in the subsequent text), $C[i]$ is the damping parameter accounting for some dissipative effects (e.g. rocking phenomena), g represents the gravity acceleration, while $m[i]$ and $x[i]$, $x'[i]$ and $x''[i]$ respectively refer to the mass and displacement, velocity and acceleration of the *i*-th degree of freedom. Additionally, $P_0[i]$ indicates the vertical loads acting on the *i*-th block, which is the superposition of the gravity load of the upper blocks and the vertical load applied over the beam, say q ($P_0[i] = qL_{brick} + \gamma(y_{max} - y[i])$), by defining as γ and L_{brick} respectively the specific weight and the length of the masonry brick.

With respect to the in-line stiffness of mortar joints, appointed in Equation 3.1 as K_{NL}, a rigid no-tension constitutive law has been adopted, as suggested by the pioneering book by Heyman [34]. This nonlinear elastic law is generally used in masonry engineering to analyze and design structures with masonry components. This approach focuses on considering the mechanical behaviour of masonry materials without taking into account tensile stresses, as masonry has limited tensile strength compared to its compressive strength. In detail, in the rigid no-tension approach, masonry is treated as a rigid material that can only withstand compressive stresses. This simplification allows engineers to model masonry structures using constitutive laws that focus solely on compressive behaviour. By neglecting the tensile strength of masonry, the approach aims to prevent crack propagation and failure in tension. The constitutive law used in this approach typically involves assuming a linear-elastic or nonlinear behavior for masonry in compression. The material properties, such as compressive strength, Young's modulus, and

Poisson's ratio, are determined based on experimental tests and empirical data specific to the masonry type being considered. This model is on the safe-side: by adopting the constitutive law rigid no-tension approach, engineers can design masonry structures that rely on compressive strength and minimize the potential for tensile stress-induced failure. This approach is particularly suitable for applications where the loads predominantly generate compressive forces, such as gravity loads in most building structures. It is important to note that while the rigid no-tension approach simplifies the analysis and design of masonry structures, it does have limitations. It assumes that masonry behaves as a perfectly rigid material, neglecting the potential for small deformations and cracking that may occur under certain conditions. Therefore, it is crucial to consider other factors, such as the effects of settlement, thermal expansion, and external loads, to ensure the overall performance and stability of the masonry structure. In conclusion, by utilizing this model, the material is assumed rigid in compression and can elongate freely, a positive deformation of the element being interpreted as a measure of fracture into the material (either smeared or concentrated). However, it is essential to consider the limitations of this approach mainly due to the strong assumptions made: indeed, masonry structures have an even if weak resistance to tensile stresses and neglecting it can be not convenient for faithfully grasping some specific tensile failure phenomena.

As previously discussed, uploading the no-tension law for the in-line stiffness of vertical mortar joints is physically reasonable and on the other hand it represents a good choice also for avoiding the interpenetration among bricks, not compatible with the assumption of rigid bodies, as sketched in Figure 17.

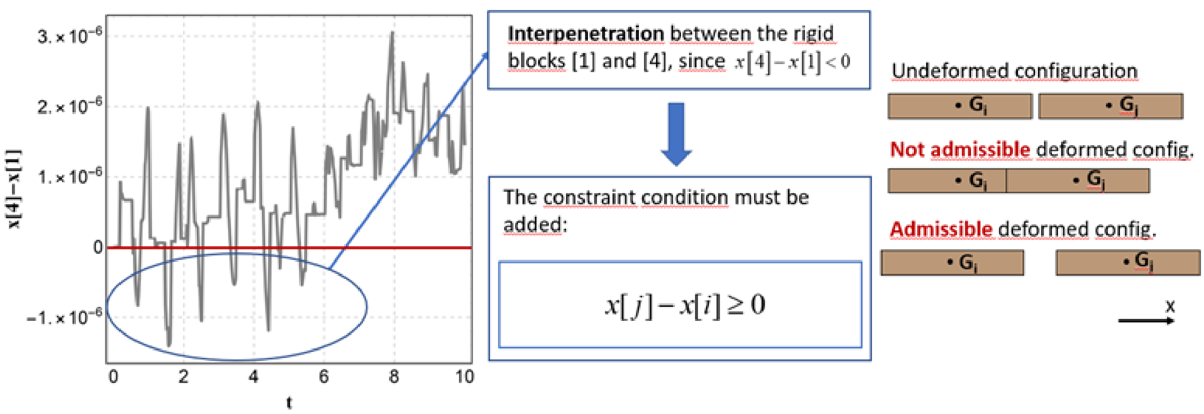


Figure 17: Admissible and non-admissible deformed configurations for two adjacent bricks.

To consider this issue, generally the model required to be revised, by adding a constraint condition that avoids the interpenetration among the blocks, i.e. inequalities in terms of displacements of each couple of bricks should be uploaded to the model. The interpenetration condition strongly affects the dynamics of the multibody system, as depicted in Figure 18, where, by selecting two sample blocks, a comparison between the response registered with and without the constraint is provided. If this approach would be adopted, further algebraic conditions should be considered in the ordinary-differential-equations (ODEs) system describing the motion of each degree-of-freedom (e.g. the element [7] 3.1), thus strongly increasing the nonlinearities of the system and consequently the computational time.

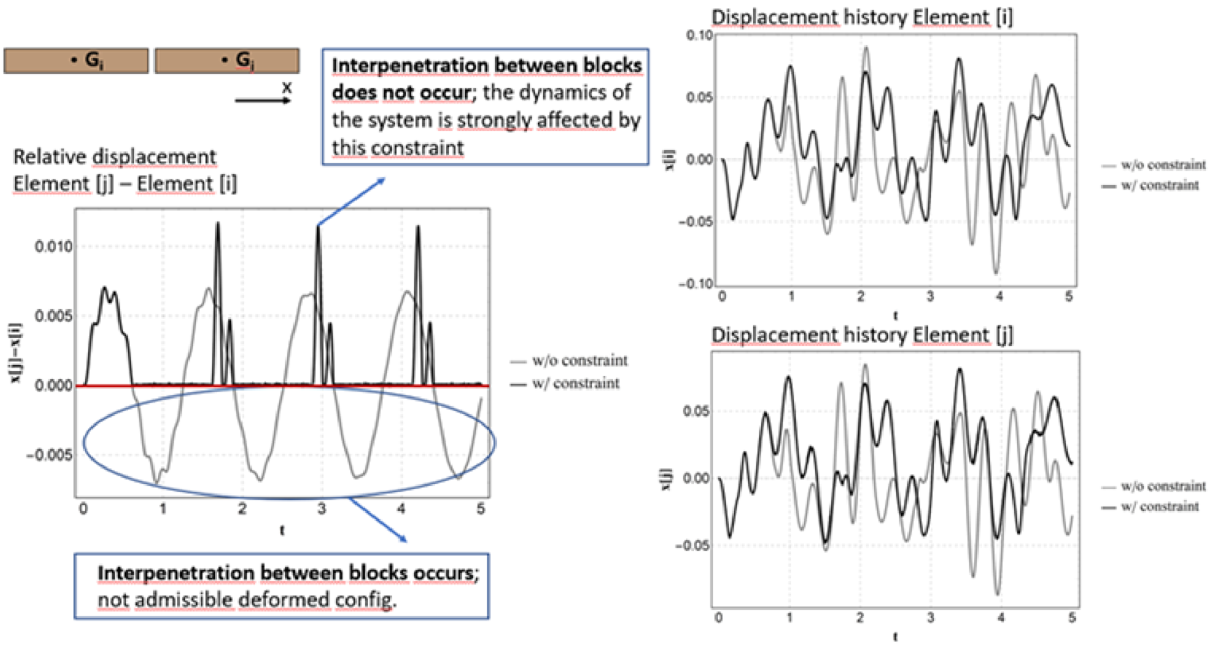


Figure 18: Displacement and velocity histories of the elements [j] and [i], by comparing constrained and free configurations.

However, considering a rigid no-tension law for the in-line stiffness of mortar links, automatically solve this issue without uploading further algebraic conditions, since make less convenient from an energetic point of view to the bricks to go in compression (Figure 19a). Moreover, for the mortar joints out-of-line herein modelled as elastic links, it is physically reasonable to assume an elastic linear behaviour, by considering constant for all degree-of-freedom the stiffness parameter K_L (Figure 19b).

With respect to the friction forces that arise in the interfaces, non linear dry friction models are assumed. In detail, as above mentioned, DEM applications of masonry structures provided in literature (e.g. [31, 47, 50]), for the sake of simplicity, consider Coulomb-like friction. As well-known, Coulomb-like friction, also appointed as dry or static friction, is a simple yet widely employed model to describe frictional forces. It follows the principles formulated by Charles-

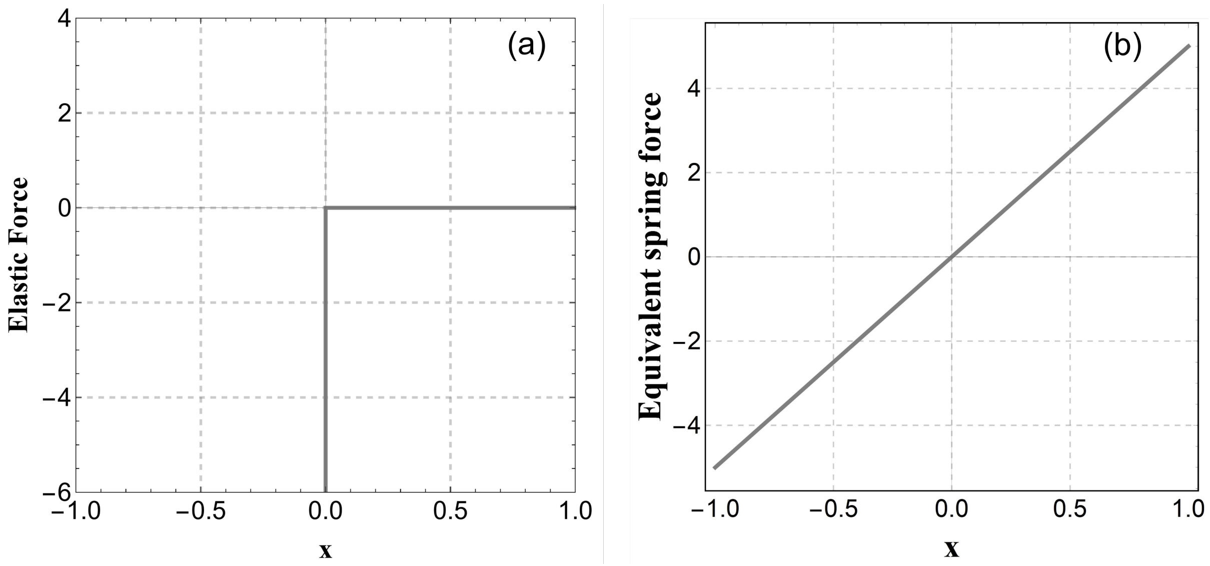


Figure 19: Elastic terms: (a) no-tension law for in-line links and (b) linear elastic law for out-of-line links.

Augustin de Coulomb in the late 18th century. According to this model, the frictional force between two surfaces is proportional to the normal force pressing them together and is independent of the contact area. The basic equation for Coulomb-like friction is $F = \mu N$, where F represents the frictional force, μ is the coefficient of friction, and N denotes the normal force. The coefficient of friction, μ, is a dimensionless quantity that characterizes the frictional properties of the materials in contact. It indicates the amount of force required to initiate or maintain relative motion between the surfaces. Therefore, Coulomb-like friction assumes that the frictional force remains constant until a threshold is reached, commonly known as the static friction limit. Once this threshold is surpassed, the surfaces start sliding, and the frictional force transitions to a different regime.

To overcome the limitation of constant kinematic coefficient of Coulomb-like laws, nonlinear dry friction models provide a more sophisticated approach to describe frictional behavior in various scenarios. Unlike Coulomb-like friction, nonlinear models [69–72] account for additional factors that influence the frictional force, such as relative velocity, slip rate, or contact pressure. One commonly used nonlinear friction model is the Stribeck curve. The Stribeck curve describes the relationship between friction and velocity in a system. It exhibits three distinct regions: static friction, mixed friction, and viscous friction. At low velocities, the static friction dominates, and the frictional force is high, making it difficult to set the surfaces in motion. As the velocity increases, the friction decreases and reaches a minimum at the mixed friction region. In this region, the surfaces are partially separated by a lubricating film, reducing the frictional forces. Finally, at high velocities, the viscous friction region takes over, where the frictional force increases due to fluid viscous effects. Therefore, nonlinear dry friction models offer a more comprehensive understanding of frictional phenomena in various engineering applications. They provide insights into the dynamic behavior of sliding or rolling contacts, allowing for more accurate predictions and optimizations. For these reasons, in order to take into account the importance of accurately predicting the friction behaviour in masonry constructions, a nonlinear dry friction model (Figure 20), where the friction force depends on the relative velocity of each couple of blocks, is uploaded to the system, with the following form:

$$\mu\left(x'\right)=\frac{\left(2\arctan\left[\frac{\beta x'[i]}{\delta}\right]\right)\left(\left(\mu_s-\mu_d\right)e^{-\left|\frac{|x[i]|}{\delta}\right|}\right)}{\pi}+\frac{\mu_d\left(2\arctan\left[\frac{\beta x'[i]}{\delta}\right]\right)}{\pi}+\alpha\left|x'[i]\right|x[i], \tag{3.2}$$

being β and α respectively the slope of the first branch in the transition between static and dynamic friction and the slope of the second branch in the slip phase, δ representing the velocity beyond which the slip phenomenon occurs, μ_s and μ_d referring to the static and dynamic friction coefficients.

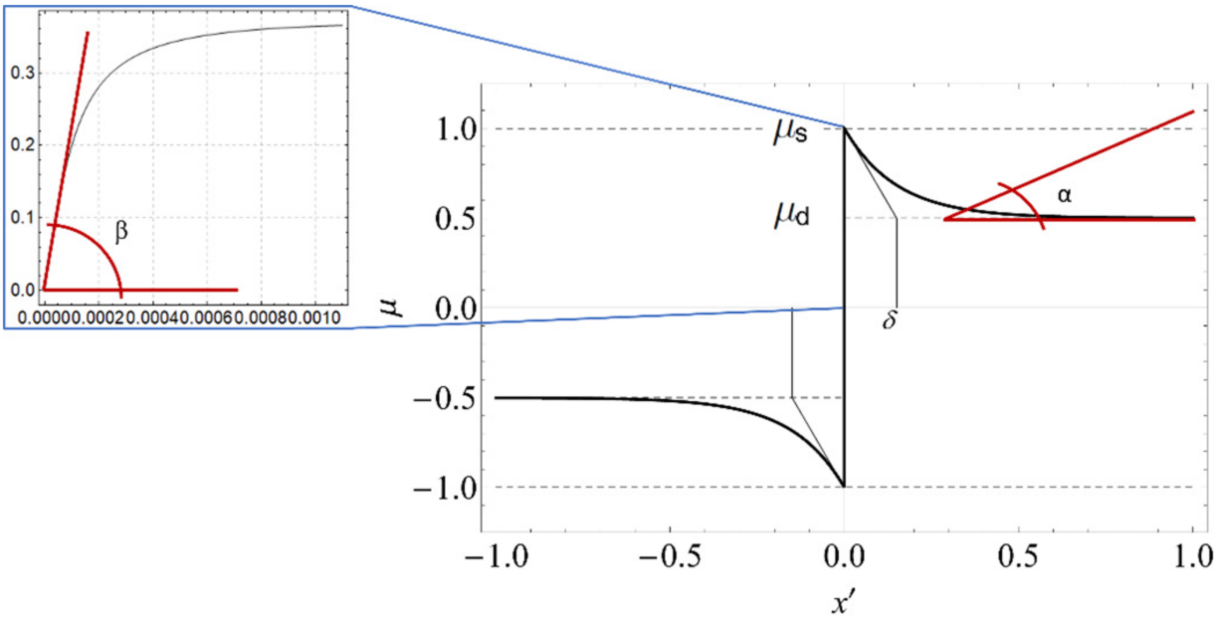


Figure 20: Nonlinear dry friction model assumed in the multi-blocks system, being α set equal to 0.

Finally, a linear equivalent viscous damping is assumed, with aim of condensing all further dissipative effects, such as rocking phenomena and/or micro-fracturing of mortar joints, in a unique constant parameter, i.e. the damping ratio of the whole system.

4 Numerical results

4.1 Calibration of parameters and dimensionless form of the system

Several research papers are precisely and carefully investigated for deriving the experimental values of the mechanical properties for masonry constructions. In detail, as expected, there is a large variation of parameters depending on the quality of masonry in terms of bricks size, masonry texture, nature of the joints and regular or irregular patterns [73–78]. For the sake of comparison, a table collecting the ranges of parameters of masonry panels reported in some references is provided in the followings (Table 1). It should be noted that, although the Poisson's ratios and the friction angle parameters vary approximately in the same range for all specimens considered in the several papers, the stiffness of the joints are more sensible to variation of mortar beds, being therefore significant the scatter between the vertical and the horizontal joints.

Table 1: Mechanical parameters of masonry panels given in papers p1 [76], p2 [77] p3 [78] and p4 [75]. The length, base and height of bricks are appointed as L, B and H, with γ, E and ν indicating respectively the specific weight, the Young modulus and the Poisson modulus. The brick's thickness is defined as t, being the friction angle (proportional to the static friction coefficient μ_s) appointed as ϕ. K_V and K_H respectively indicate the stiffness of the vertical and horizontal mortar joints.

	Bricks: sizes, weight and mechanical parameters						**Mortar Joint**	**Friction Angle**		**Stiffness of the joints**	
	$L[m]$	$B[m]$	$H[m]$	$\gamma[N/m^3]$	$E[MPa]$	$\nu[-]$	$t[m]$	ϕ_{min}	ϕ_{max}	$K_V[N/m]$	$K_H[N/m]$
p1	0.25	0.12	0.055	19620	6050	0.14	0.015	20°	40°	9.00 E+07 - 1.62 E+08	5.40 E+07 - 1.53 E+08
p2	0.25	0.12	0.055	14010	2.00E+04	0.2	0.015	15°	38°	1.80 E+07	2.16 E+07
p3	0.5	0.299	0.2	14715	1550-1400	0.2	0.015	/	/	6.69 E+06±2.14 E+06	6.69 E+07±2.14 E+06
p4	0.25	0.12	0.07	/	1.45E+04-2.6E+04	0.156-0.2	0.010-0.015	17.2°	62.2°	1.34 E+07±1.07 E+08	1.34 E+07±1.07 E+08

In order to validate the model, several parametric analyses based on the numerical set of parameters of interests (1) are carried out. However, before implementing the numerical analyses, with the aim to easy manage the strictly necessary number of parameters, the system's equations have been re-written in a dimensionless form, taking advantage from the application of the Buckingham π Theorem. For instance, making reference to Figure 21, for the sake of simplicity, let us consider the sole 3$^{\text{th}}$ degree-of-freedom, whose equation of motion particularises as:

$$\begin{aligned} &m_3\frac{d^2x_3}{dt^2} - C_{3,1}\frac{dx_1}{dt} - C_{3,2}\frac{dx_2}{dt} + \left(C_{3,1}+C_{3,2}+C_{3,4}+C_{3,5}\right)\frac{dx_3}{dt} + \\ &- C_{3,4}\frac{dx_4}{dt} - C_{3,5}\frac{dx_5}{dt} - K_{3,1}x_1 - K_{3,2}x_2 + \left(K_{3,1}+K_{3,2}+K_{3,4}+K_{3,5}\right)x_3 + \\ &- K_{3,4}x_4 - K_{3,5}x_5 = 0. \end{aligned} \tag{4.1}$$

With respect to equation 3.1, it should be highlighted that in 4.1, the Leibniz notation is used for differentiation instead of the Lagrangian one, just for stressing the mathematical steps for achieving the dimensionless form.

By setting $m_i/M = \mu_i$, $K_{i,j}/K = k_{i,j}$ and $C_{i,j}/C = \gamma_{i,j}$ the equation 4.1 is re-written as follows:

$$\begin{aligned} &\mu_3 M\frac{d^2x_3}{dt^2} + C\left(-\gamma_{3,1}\frac{dx_1}{dt} - \gamma_{3,2}\frac{dx_2}{dt} + \left(\gamma_{3,1}+\gamma_{3,2}+\gamma_{3,4}+\gamma_{3,5}\right)\frac{dx_3}{dt} - \gamma_{3,4}\frac{dx_4}{dt} - \gamma_{3,5}\frac{dx_5}{dt}\right) + \\ &+ K\left(-k_{3,1}x_1 - k_{3,2}x_2 + \left(k_{3,1}+k_{3,2}+k_{3,4}+k_{3,5}\right)x_3 - k_{3,4}x_4 - k_{3,5}x_5\right) = 0. \end{aligned} \tag{4.2}$$

By dividing both members of Equation 4.2 of the quantity M, the following expression is given:

$$\begin{aligned} &\mu_3\frac{d^2x_3}{dt^2} + \frac{\mathrm{C}}{M}\left(-\gamma_{3,1}\frac{dx_1}{dt} - \gamma_{3,2}\frac{dx_2}{dt} + \left(\gamma_{3,1}+\gamma_{3,2}+\gamma_{3,4}+\gamma_{3,5}\right)\frac{dx_3}{dt} - \gamma_{3,4}\frac{dx_4}{dt} - \gamma_{3,5}\frac{dx_5}{dt}\right) + \\ &+ \frac{\mathrm{K}}{M}\left(-k_{3,1}x_1 - k_{3,2}x_2 + \left(k_{3,1}+k_{3,2}+k_{3,4}+k_{3,5}\right)x_3 - k_{3,4}x_4 - k_{3,5}x_5\right) = 0. \end{aligned} \tag{4.3}$$

By setting $u_i = x_i/\mathit{\Lambda}$ and $t = \tau/\mathit{\Omega}$, one can derive the following quantities:

$$\begin{aligned} \frac{dx_{\rm i}}{dt} &= \frac{d\left(u_{\rm i}\varLambda\right)}{d\tau}\frac{d\tau}{dt} = \varLambda\varOmega \\ \frac{d^2x_{\rm i}}{dt^2} &= \frac{d}{dt}\frac{dx_{\rm i}}{dt} = \frac{d}{dt}\left(\varLambda\varOmega\dot{u}_i\right) = \varLambda\varOmega\ddot{u}_i\frac{d\tau}{dt} = \varLambda\varOmega^2\ddot{u}_i. \end{aligned} \tag{4.4}$$

Then, by recalling that the damping coefficient C is equal to $2\xi\varOmega M$ and by substituting expressions 4.4 in Equation 4.3, the subsequent equation can be obtained:

$$\begin{aligned} &\mu_3\varLambda\varOmega^2\ddot{u}_3 + 2\dot{\zeta}\varOmega\left(-\gamma_{3,1}\varLambda\varOmega\dot{u}_1 - \gamma_{3,2}\varLambda\varOmega\dot{u}_2 + \left(\dot{\gamma}_{3,1} + \gamma_{3,2} + \gamma_{3,4} + \gamma_{3,5}\right)\varLambda\varOmega\dot{u}_3 - \gamma_{3,4}\varLambda\varOmega\dot{u}_4 + \right. \\ &\left. - \gamma_{3,5}\varLambda\varOmega\dot{u}_5\right) + \varOmega^2\left(-k_{3,1}\varLambda u_1 - k_{3,2}\varLambda u_2 + \left(k_{3,1} + k_{3,2} + k_{3,4} + k_{3,5}\right)\varLambda u_3 - k_{3,4}\varLambda u_4 + \right. \\ &\left. - k_{3,5}\varLambda u_5\right) = 0 \end{aligned} \tag{4.5}$$

After some algebraic manipulations, the final equation of motion is derived, being all terms dimensionless parameters:

$$\begin{aligned} &\mu_3\ddot{u}_3 + 2\zeta\left(-\gamma_{3,1}\dot{u}_1 - \gamma_{3,2}\dot{u}_2 + \left(\gamma_{3,1} + \gamma_{3,2} + \gamma_{3,4} + \gamma_{3,5}\right)u_3 - \gamma_{3,4}\dot{u}_4 - \gamma_{3,5}\dot{u}_5\right) + \\ &+ \left(-k_{3,1}u_1 - k_{3,2}u_2 + \left(k_{3,1} + k_{3,2} + k_{3,4} + k_{3,5}\right)u_3 - k_{3,4}u_4 - k_{3,5}u_5\right) = 0 \end{aligned} \tag{4.6}$$

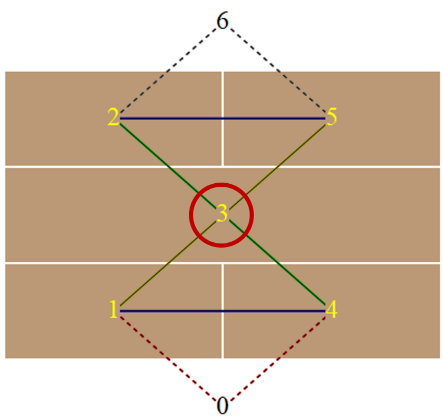


Figure 21: Sketch of part of masonry panel assumed as sample reference for making explicit the dimensionless form of dynamics equations.

It is worth to noticing that the stiffness terms that appears in Equation 4.1 are the sole constant ones (i.e. the K_L terms). The nonlinear behaviour of the in-line elastic links is obtained by assuming the no-tension law (Figure 20a) numerically reproduced, by setting vanishing stiff-

ness for the tensile branch, being instead a large number (greater of several orders of magnitude with respect to the K_L parameters) utilized for tracing back the *theoretically infinite* slope of the compressive part.
In this light, by extending and generalizing this procedure to each degree-of-freedom, the dimensionless form of the ODE system can be easily derived.

4.2 Parametric analyses and discussion

In order to validate the model, several parametric analyses based on the numerical set of parameters of interests, derived from the inherent literature are carried out. In particular, the numerical values of parameters used for the parametric analyses are obtained from the experimental tests summarised in Table 1 and are here reported: with regard to the bricks' characteristics, the following parameters are adopted, i.e. length, $L = 0.250$ [m], width, $B = 0.120$ [m], height, $H = 0.055$ [m], specific weight, $\gamma = 18000$ [N/m^3]; the width and the thickness of mortar beds are respectively equal to 0.120 [m] and 0.01 [m]; the parameters of the friction law are static and dynamic friction coefficients, $\mu_s = 0.40 - 0.80$, $\mu_d = 0.28 - 0.56$, transition velocity stick-slip phenomena, $\delta = 0.001$ [m/s], slopes of the friction model, $\alpha = 0$, $\beta = 100$; the mechanical parameters of the mortar beds are stiffness of the vertical joint, $K_V = 5.4$x10^7 [N/m], stiffness of the horizontal joint, $K_H = 5.4$x10^7 [N/m]; damping ratios for all interfaces (i.e. the vertical and horizontal joint, the interface beam-block on the top and on the ground) are set equal to 0.05.
Some relevant cases are here reported and commented. The first numerical test assumes all parameters nonzero and considers a regular pattern for the masonry panel, as depicted in Figure 22. By looking at the plots in the Figure, two observations can be deduced: *i)* from the phase portrait, it can be seen that by removing the noise induced by the transient phase, the jumps in the plot are mainly attributable to the stick-slip effect caused by friction: *ii)* from the plot (c), it can be observed that due to the no-tension law, the interpenetration between blocks never occurs, since all relative displacements for each pair of bricks are always positive. It should be highlighted that for all subsequent Figures reporting the results of numerical tests, the deformations among bricks are over-scaled just for the sake of visualization.

Analogously, the same case is reproduced by considering an irregular pattern of masonry pier,

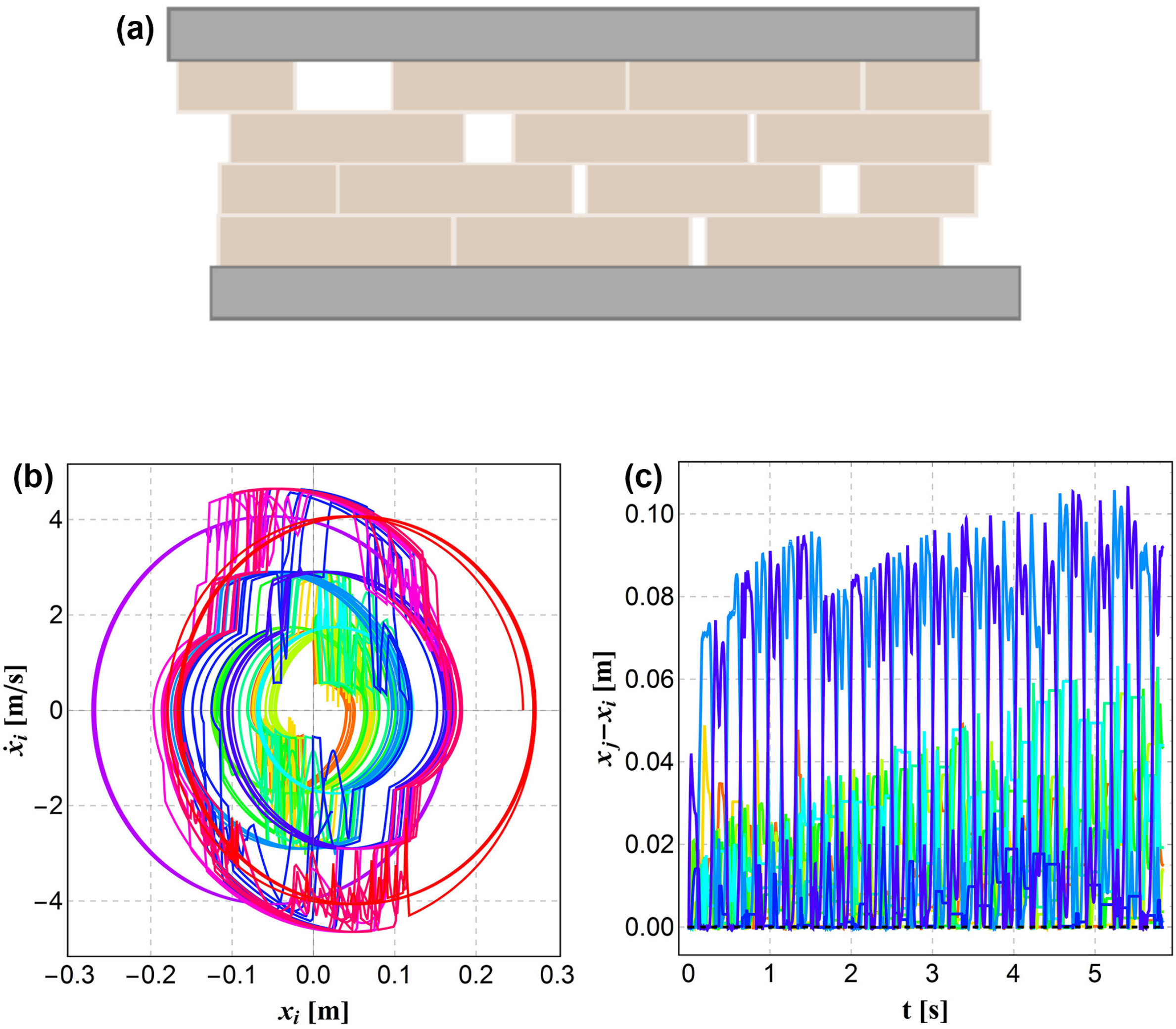


Figure 22: Results of the case 1, regular pattern, being all parameters nonzero: (a) graph reproducing the dynamics of the masonry panel at a selected time step $\tilde{t}$; (b) phase portrait; (c) time-history of the relative displacements among bricks.

being also adopted a different aspect ratio (Figure 23). It should be noted that the dynamics' response is qualitatively similar to the previous one.

A further numerical case is analysed, by assuming all parameters nonzero except for the damping ratio here set as 0 (Figure 24). From this configuration, it can be seen that the magnitude of displacements is lightly increased with respect to the correspondent damped case (Figure 22). This result –that can be in some way expected for a linear system– is not trivial in presence of nonsmooth friction uploaded to the ODE system.

In order to clarify the influence of the friction term on the overall dynamics, two further cases are implemented. In the first one, a frictionless case is reproduced, by imposing the friction

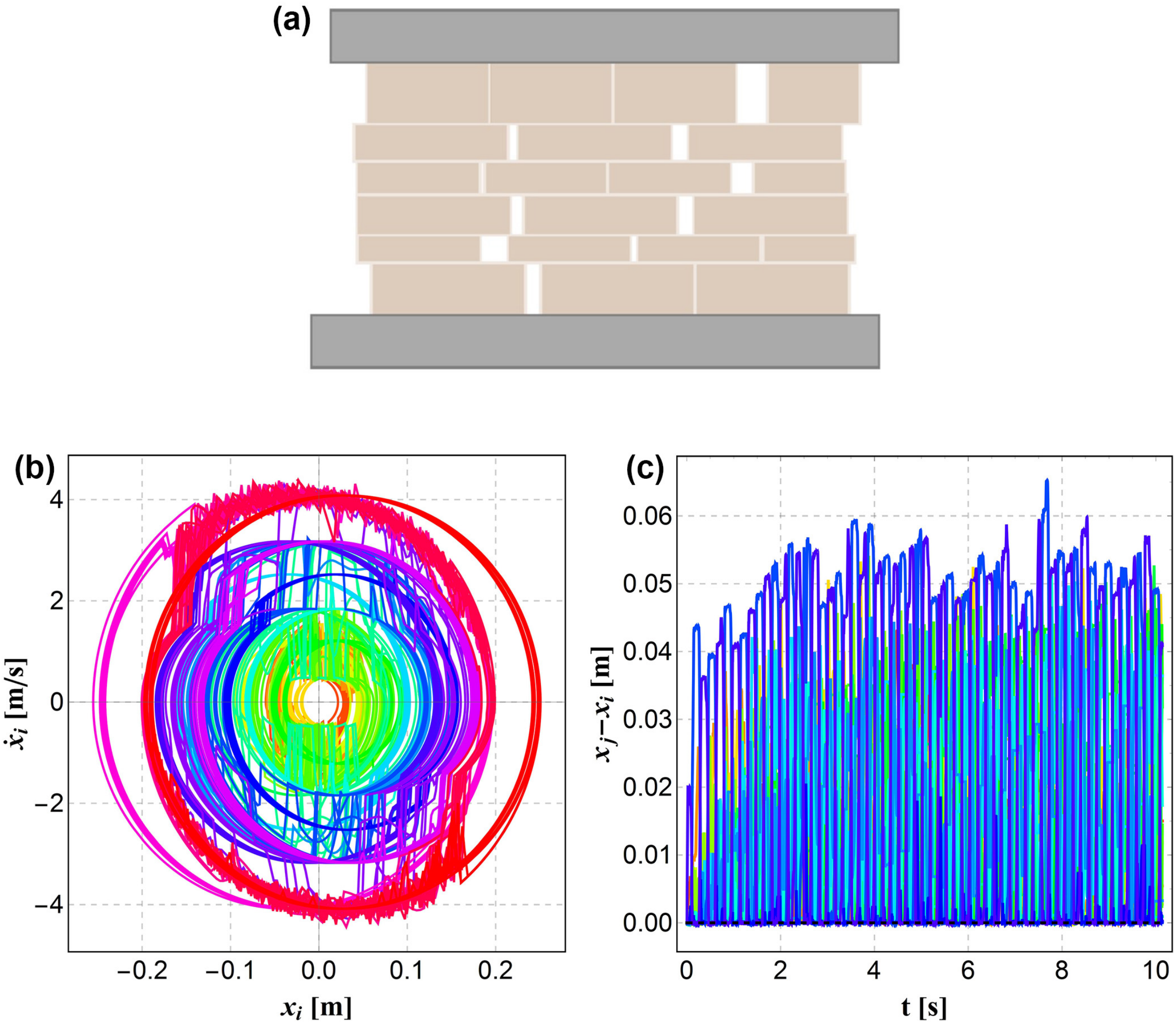


Figure 23: Results of the case 2, irregular pattern, being all parameters nonzero: (a) graph reproducing the dynamics of the masonry panel at a selected time step $\tilde{t}$; (b) phase portrait; (c) time-history of the relative displacements among bricks.

forces tend to 0. As one can note from Figure 25, representing this last frictionless configuration, the jumps in the phase portrait are not registered and trajectories typically observed in multiple degrees-of-freedom linear systems are reported. Additionally, with respect to the previous numerical tests, it is worth highlighting that the absence of friction remarkably reduce the magnitude of the relative displacements among bricks, as depicted in Figure 25 (c).

Finally, for the sake of completeness, the configuration characterised by vanishing elastic forces among blocks is taken into account and numerically simulated. The Figure 26 shows the results of the dynamic response of a masonry panel where the stiffness constants of the horizontal mortar beds are assumed zero. In detail, it can be seen that the absence of the elastic term that link bricks belonging to different lines strongly affects the displacements and velocities histories

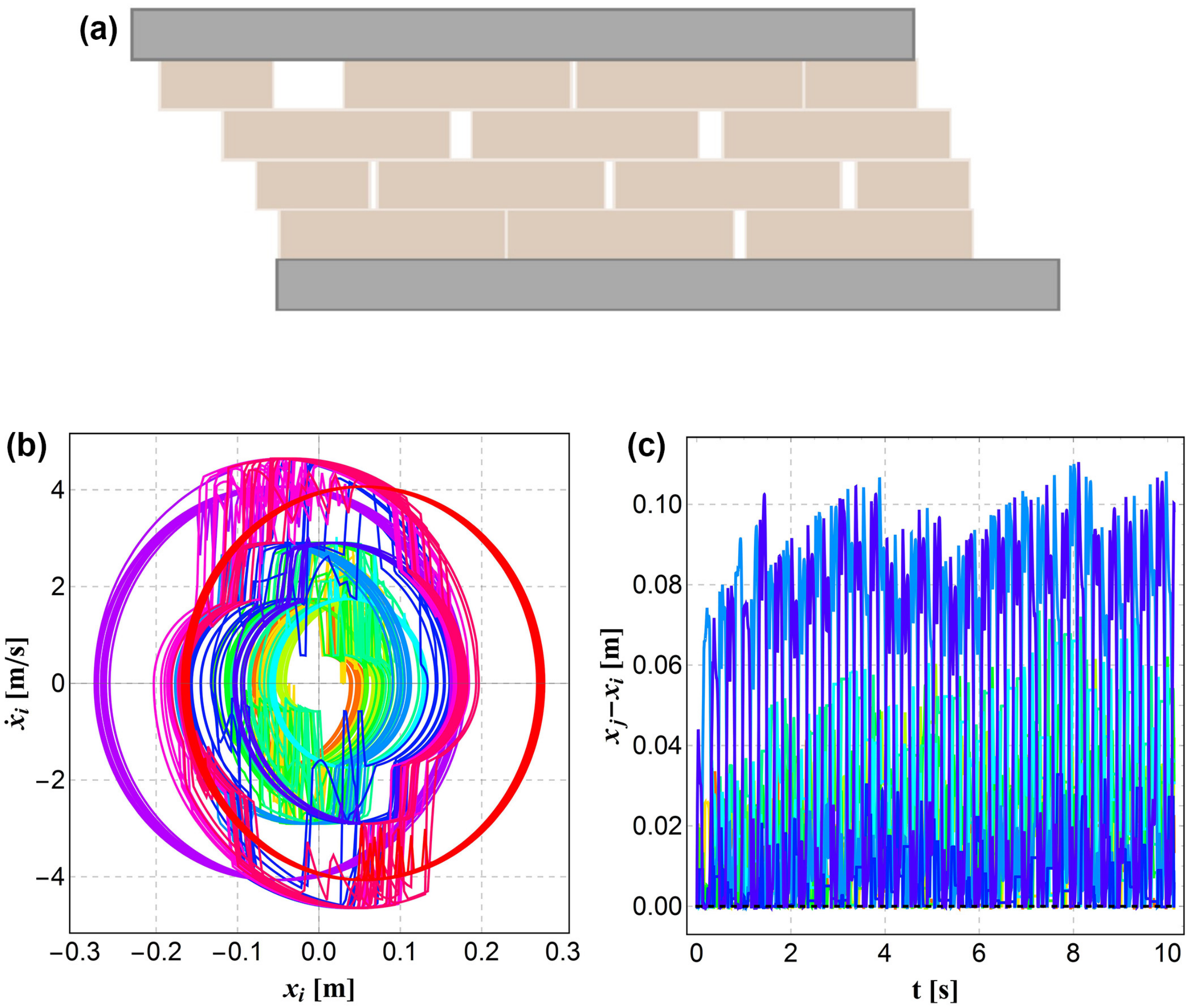


Figure 24: Results of the case 3, regular pattern, being all parameters nonzero with the exception of the damping parameters assumed vanishing: (a) graph reproducing the dynamics of the masonry panel at a selected time step $\tilde{t}$; (b) phase portrait; (c) time-history of the relative displacements among bricks.

and does not allow the activation of stick-slip effect.

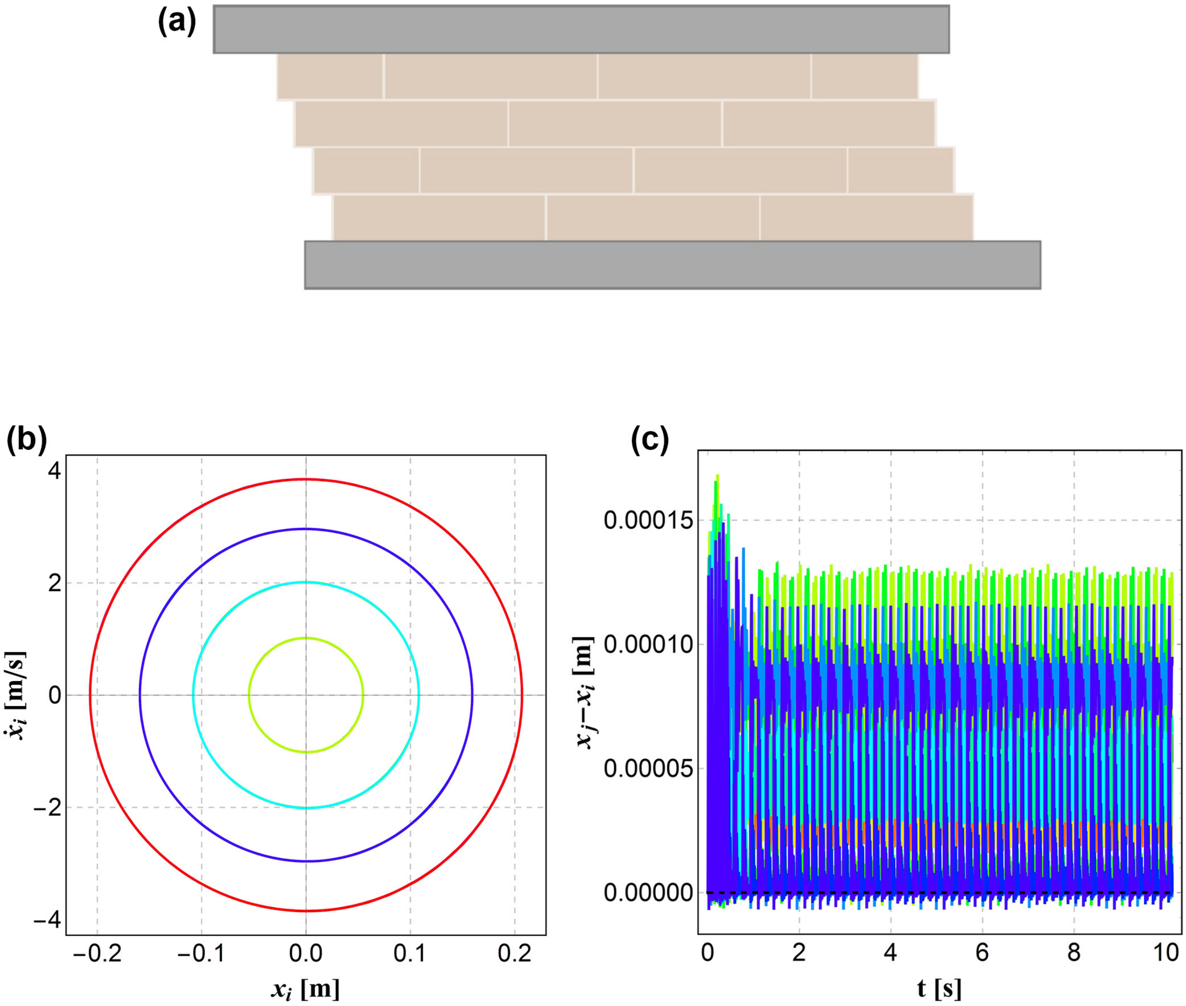


Figure 25: Results of the frictionless case 4, regular pattern: (a) graph reproducing the dynamics of the masonry panel at a selected time step $\tilde{t}$; (b) phase portrait; (c) time-history of the relative displacements among bricks.

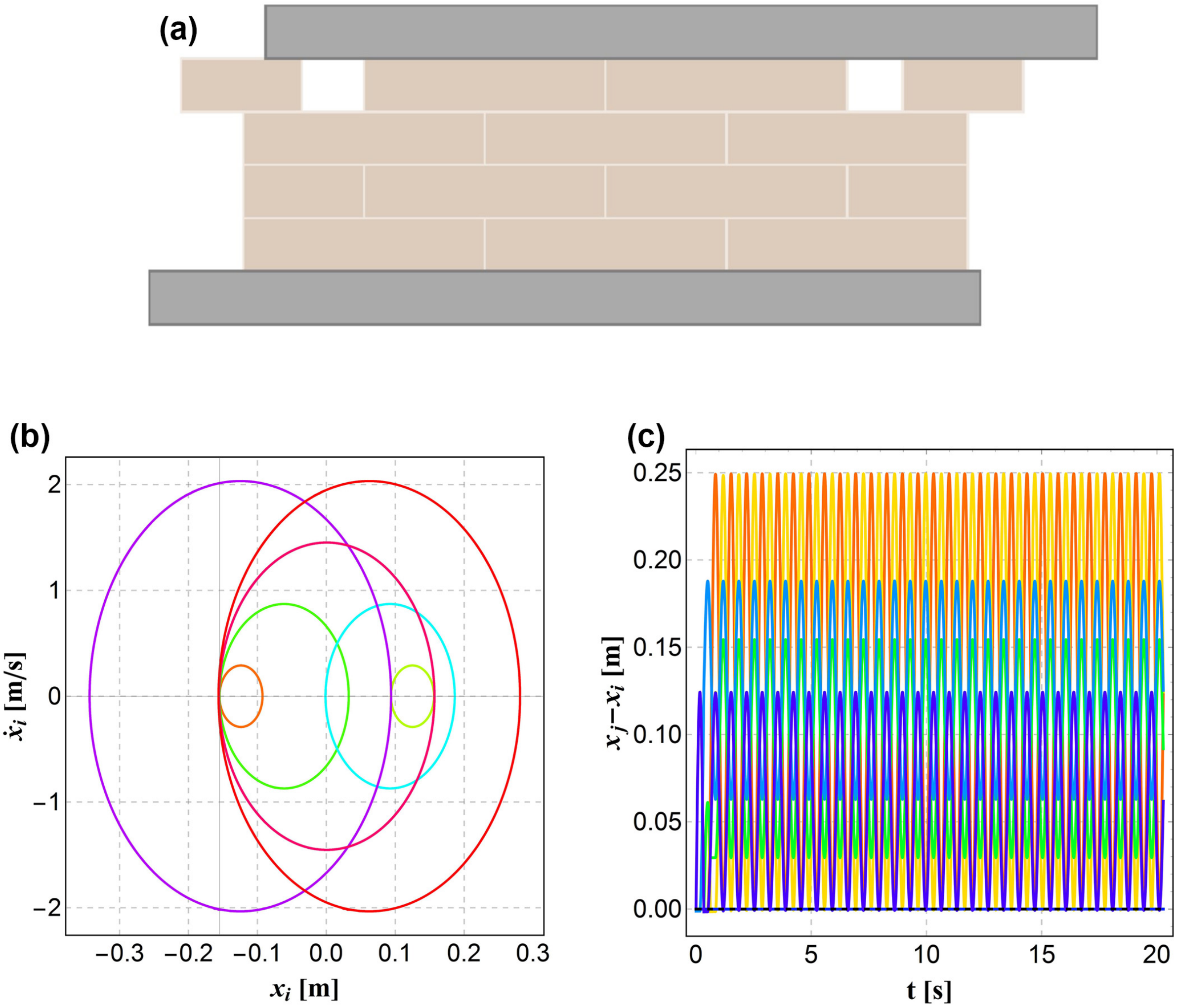


Figure 26: Results of the case 5, with vanishing stiffness constants of horizontal mortar beds, regular pattern: (a) graph reproducing the dynamics of the masonry panel at a selected time step $\tilde{t}$; (b) phase portrait; (c) time-history of the relative displacements among bricks.

5 Insights and new perspectives for retrofit applications

In order to understand the numerical results provided in the previous section, the relative accelerations among bricks are chosen as performance parameters, for preliminary assessing the structural behaviour of the panel subjected to ground motion. In this light, with reference to cases 1 and 2 (regular and irregular patterns) of Figures 22 and 23, the relative accelerations among blocks are plotted in Figure 27 at a selected time instant. From the Figure, one can be observe that, depending on the masonry texture and the size of mortar joints and bricks, some links located at certain points experience higher accelerations. At this stage, the reasons of this nonhomogeneous behaviour of the mortar beds is not clear and need further more refined and accurate analyses. However, with the aim of reducing the dynamic response of the masonry panel, after having faithfully grasping its behaviour thanks to DEM model here implemented, a possible solution can consist of replacing these critic mortar beds (characterized by high values of relative accelerations) with novel engineered smart interfaces that can dissipate energy, thus reducing the concentration of stresses. It would be worth exploring this solutions, which could allow to address the twofold need to provide a minimally invasive intervention that actually improves the seismic behaviour of the structure and at the same time can match the sustainable requirements of the last design trends.

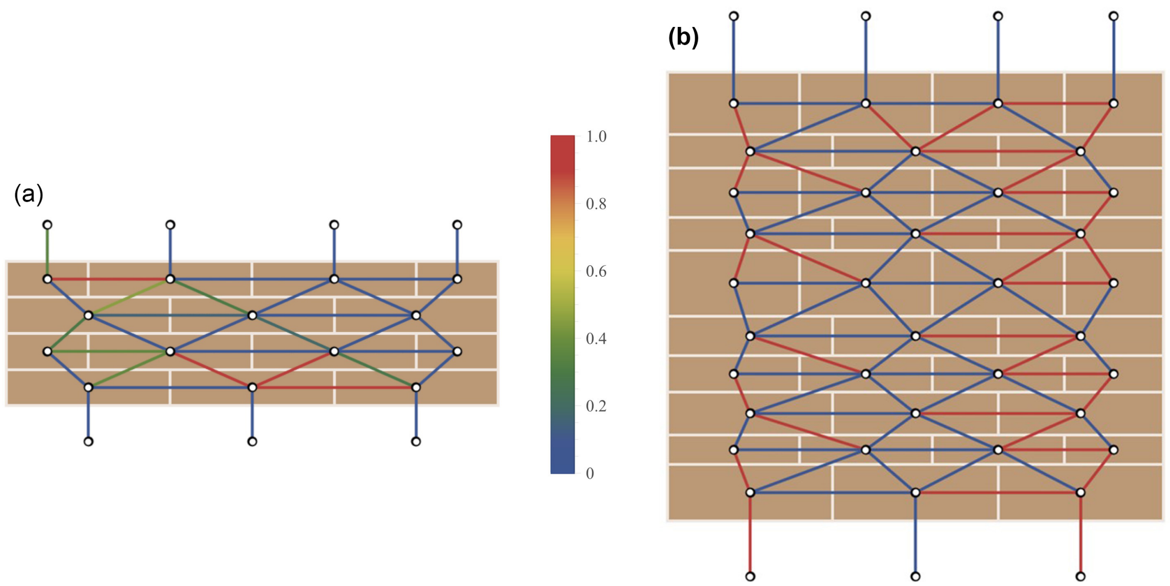


Figure 27: Graphics of the relative accelerations among bricks, being the chromatic scale from 0 to 1 defined as the gradient of the ratio between the relative acceleration of the block and the maximum one registered during the whole time history: (a) regular pattern, (b) irregular pattern.

6 Conclusions

Based on the findings of the parametric analyses implemented, some conclusions can be drawn:

- for the selected numerical set of parameters, the dynamics of the system seems to be affected by the stiffness and the friction parameters;

- the model confirms the experimental observation in literature: the stiffness of the horizontal and vertical joint interfaces play a key role in the dynamic response;

- when stiffness parameters are low (i.e. friction forces mainly rule the behaviour), the dynamics of the system is strongly changed;

- the interpenetration condition, here obtained through the no-tension rigid law, corresponds *de facto* to an event locator and increases nonlinarities in the system;

- when the friction forces tend to zero, the dynamics of the system is heavily modified, thus certifying that both stiffness and friction terms are relevant for describing the physical behaviour of the multibody panel.

Several other questions remain to be addressed and require accurate investigation, as an example, next stage of this research should be:

- calibrate and vary the order of magnitude of viscous, elastic and friction forces in order to further validate the numerical set of parameters;

- extend the range of the variation of the parameters of the system by considering physically reasonable intervals of engineering interests still derived by experimental tests;

- evaluate the patterns of masonry panels characterized by higher values of relative displacements between two contiguous blocks;

- model possible cracking lines in the panel, by stressing the system with several accelerograms;

- define the minimum horizontal acceleration needed for determining the first cracking.

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