

**Mutual Commitment,
Norm Formation and
Indirect Regulation of Agent Society**

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Abstract

Norm formation is one of the most basic processes of social interaction. To clarify this process, we introduce mutual commitment and meta commitment among agents. R. Axelrod has analyzed the commitment structure of norm formation and the collapsing process of the norm as his norm and meta norm games [1]. He analyzes the games with genetic algorithm.

In this paper we generalize the games as mutual commitment processes of agents [6]. We also introduce the commitment by central authorities such as a government that enforces legal policies. We deal with the model from theoretical point of view. For the purpose of describing a social learning dynamics, we introduce replicator dynamics (RD). RD is usually used in evolutionary game theory. We induce RD from stochastic learning process.

At first we reformulate the norm game as a coupling model between the replicator dynamics of alternatives {C,D} and the dynamics of normative attitude. We also extend this model and introduce several types of centralized and decentralized commitment mechanisms on agent societies.

Key Words

Norm Game, Multi Agent System, Replicator Dynamics, Indirect Control, Indirect Regulation, Poly Agent System

1. Introduction

In this paper we focus on the concept of norm as a decentralized indirect regulation on a society. For this purpose, we analyze the mutual commitment and the meta commitment processes among agents as shown in figure 1.

The concept of norm is analyzed in multidisciplinary area of social science. It is difficult to give it a suitable definition but R. Axelrod has introduced the essential definition of it [1]. He defines norm and meta norm games and introduces mutual and hierarchical commitment structures on these games. He explains how norms are collapsed and maintained on the games.

We reformulate his norm and meta norm games using extended replicator dynamics for social learning. Replicator dynamics (RD) is usually derived from random matching no cooperative game that represents evolutionary process [3]. In this paper we extend RD and use it to describe social learning process.

Axelrod introduces his norm game as an extension of n-person Prisoner's Dilemma. He adds a new feature of normative attitude to the dilemma game, which is to punish defectors under the mutual monitoring and commitment processes.

In the norm game, the strategies of players consist of two dimensions. He introduces two set of alternatives, {Boldness, No- Boldness } and {Vengefulness, No-Vengefulness }. The former one represents if a player is enough bold to defect or not. In other words, it is an attitude to follow, or not to follow, the norm. The later one represents if a player is enough vengeful to punish the player who does not follow the norm. It is an indirect regulation of the norm through mutual commitment to other person's attitude.

He analyzes the norm game with genetic algorithm and shows how norms collapse because of the retrogression of normative commitment.

PROMOTING NORMS

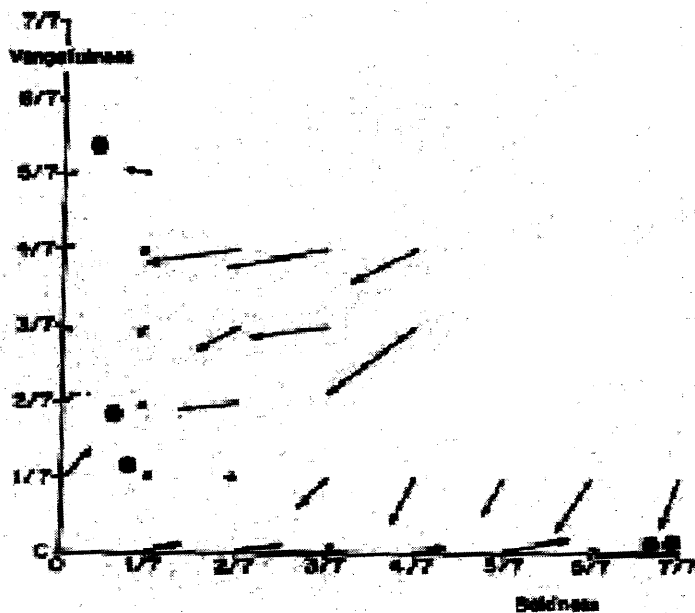


Figure 1-2. Norm Game Dynamics

Figure 1 Norm Game Dynamics: Norm Collapsing Process (Axcelrod The complexity of Cooperation Ch.3)

He also shows how to prevent the retrogression of normative commitment and how to maintain the norm. He introduces the concept of meta norm as mutual meta commitment for normative commitment. It means that a player is enough meta vengeful to punish the player who does not execute his normative commitment. If the meta norm mechanism is introduced in a society then normative commitment does not retrogress and norm dose not collapse.

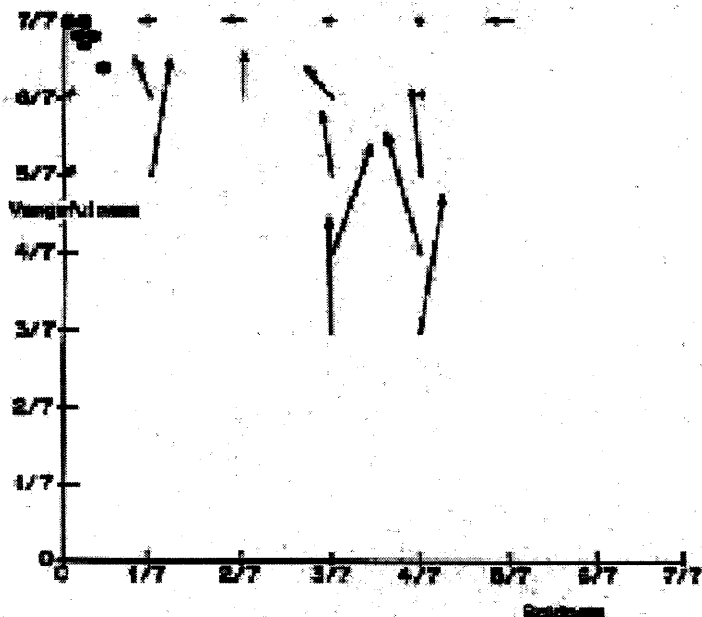


Figure 2 Meta Norm Game Dynamics: Norm Maintained (Axcelrod The complexity of Cooperation Ch.3)

We reformulate his norm game and meta norm game with extended replicator dynamics of social learning. For this purpose, we introduce the concept of decentralized indirect regulation or indirect control of agent society. We also introduce the social learning interpretation of RD, which is called social learning dynamics.

2. Social Learning Dynamics

RD is usually derived from random matching no cooperative game that represents

evolutionary process. This is what we call evolutionary interpretation of RD. In the context of social science, the evolutionary interpretation is not satisfied in many cases. Instead, we introduce the replicator dynamics under the social learning interpretation. The interpreted dynamics is called social learning dynamics. For this purpose we introduce the following Markov process on the alternatives { C, D}. In here "C" means "No- Boldness" and "D" means "Boldness". A player changes his decision under the following transition probability.

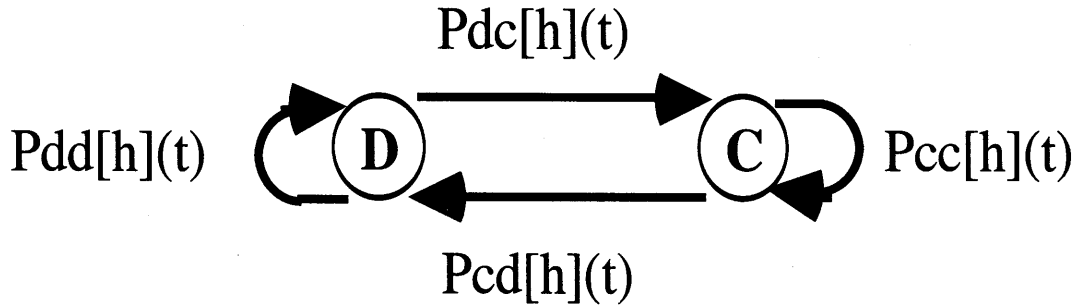


Figure 3 Markov Learning Process of Alternatives

Proposition 2.1

Let $P_{cd}[h](t) = h * P_{cd}[1](t) = h * P_d(t) * E_d(t) / W(t)$,
 $P_{dc}[h](t) = h * P_{dc}[1](t) = h * P_c(t) * E_c(t) / W(t)$,
 $P_{cc}[h](t) = 1 - h * P_d(t) * E_d(t) / W(t)$,
 $P_{dd}[h](t) = 1 - h * P_c(t) * E_c(t) / W(t)$.

Then we can introduce the dynamics of decision probability as follows. It is also interpreted to the population ratio of the choice of alternatives.

Proof:

$$\begin{aligned}
 P_c(t+h) &= P_c(t) * P_{cc}[h](t) + P_d(t) * P_{dc}[h](t) \\
 &= P_c(t) + h * P_c(t) * P_d(t) * \{E_c(t) - E_d(t)\} / W(t) \\
 \Delta [P_c[h](t)] &= P_c(t+h) - P_c(t) = h * P_c(t) * P_d(t) * \{E_c(t) - E_d(t)\} / W(t) \\
 dP_c(t)/dt &= \lim_{h \rightarrow 0} \Delta P_c[h](t) / h = P_c(t) * P_d(t) * \{E_c(t) - E_d(t)\} / W(t) \\
 &= P_c(t) * \{E_c(t) - W(t)\} / W(t) \quad \text{Q.E.D.}
 \end{aligned}$$

In the model a player changes his attitude slowly by himself under the influence of average payoff of alternatives. In other words we derive the dynamics not from direct interaction among agents and its payoff, but from average payoff of alternatives. Average payoff might be obtained by the random matching game in some cases or obtained from other social interactions as we mention in the following section, where we pay attention to the commitment processes among agents.

3. Social Learning Dynamics of Norm Game

3.1 Norm Game

In the norm game, if a player dose defect then he gets payoff of T=3. Pd and Pc denote the normalized population ratio of the selection of Boldness and No-Boldness respectively. Pc=1-Pd holds from definition. In this paper we used "C" for "NO-Boldness" and "D" for "Boldness" respectively. PV and Pnv denote the normalized population ratio of the selection of Vengefulness and No-Vengefulness respectively. Pnv=1- PV also holds.

As pointed earlier, "Vengefulness" represents the normative attitude under the mutual commitment process that is illustrated as follows.

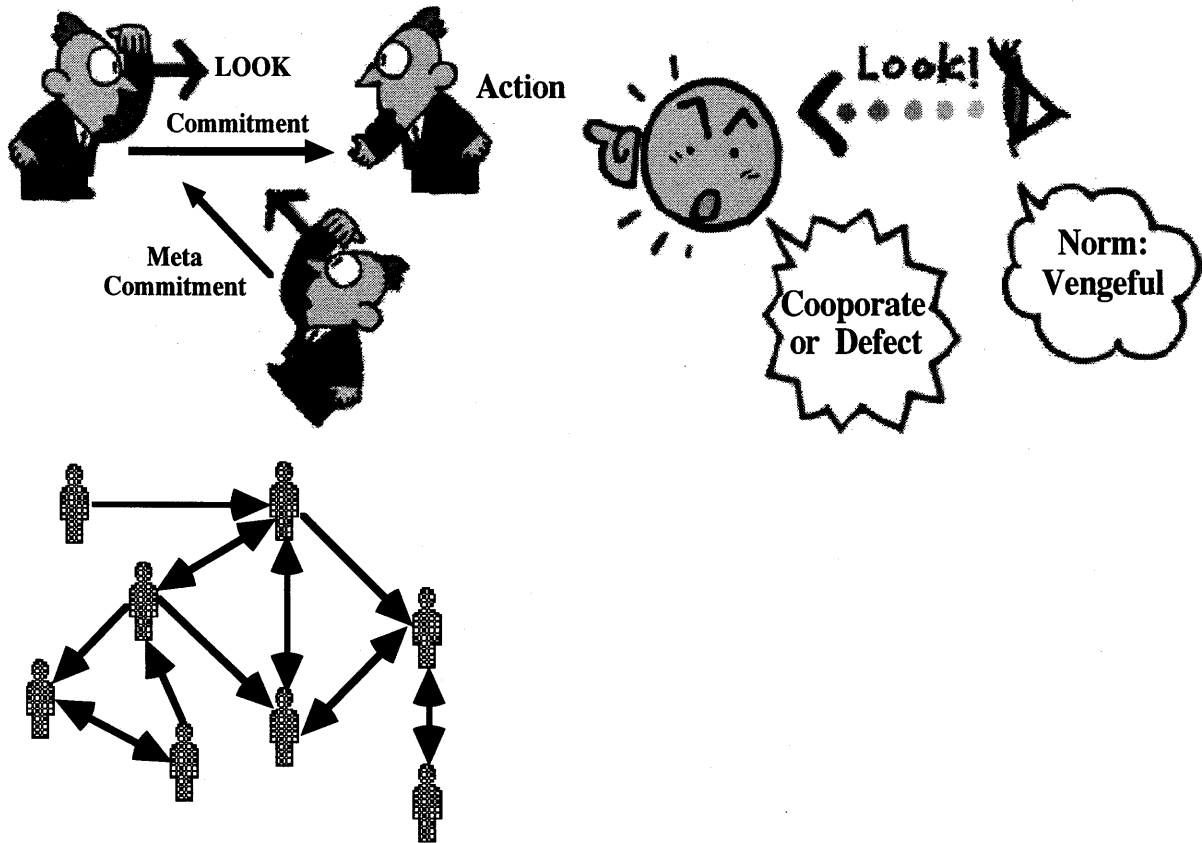


Figure 4 Mutual Reference and Commitment

For formulating replicator dynamics of the norm game we introduce average payoff structure as follows according to Axelrod's game structure.

Table 1. Average Payoff of Underlying and Commitment of Alternatives

Average Payoff	Interaction Factor	Commitment Factor
E[C]	$-a*Pd$	0
E[D]	$b*Pc$	$-c*PV$
E[V]	0	$-d*Pd$
E[NV]	0	0

Then we can introduce RD for norm game as follows.

Where we assume $a < d < b < c$. Let $a=1, b=3, c=9, d=2$ in the following simulation.

$$dPc/dt = Pc*(1-Pc)*(EC-ED) \quad (3.1.1)$$

$$dPV/dt = PV*(1-PV)*(EV-Env) \quad (3.1.2)$$

$$EC = -a*(1-Pc)$$

$$ED = b*Pc - c*PV$$

$$Env = 0$$

$$EV = -d*(1-Pc)$$

$$pd = 1-Pc$$

$$Pnv = 1-PV$$

Proposition 3.1

$Pc=1$ is a stable steady state of (3.1.1) if and only if $PV > \{(b-a)Pc + a\}/c$

Proof:

$Pc=1$ is a steady state of (3.1.1). The stability depends on the sign of $EC-ED$.

$$dPc/dt > 0 \Leftrightarrow EC - ED > 0 \Leftrightarrow -a*(1-Pc) > b*Pc - c*PV \Leftrightarrow PV > \{(b-a)Pc+a\}/c \quad \text{Q.E.D.}$$

We add the perturbation factor to this model. The result shows the collapsing process of the norm and it corresponds to the Axelrod's simulation by using genetic algorithm (GA) under the Complex Adaptive System (CAS) paradigm.

Proposition 3.2

- (1) Let $PV > \{(b-a)Pc+a\}/c$. Then $|dPV/dt| \leq d*\epsilon/4$, where $\epsilon = 1-Pc$ is small perturbation.
 (2) Let $PV < \{(b-a)Pc+a\}/c$. Then $|dPV/dt| \leq d*\alpha/4$, where $\alpha = 1-Pc$ become 1 as a result.

Proof:

(1) $PV > \{(b-a)Pc+a\}/c$ means $dPc/dt > 0$ from proposition 3.1. Then Pc become 1 with small perturbation ϵ . $0 \leq PV*(1-PV) \leq 1/4$, $EV = -d*(1-Pc) = -d\epsilon$ and $Env = 0$ hold. Then $|dPV/dt| = |PV*(1-PV)*(EV-Env)| \leq d*\epsilon/4$.

(2) $PV < \{(b-a)Pc+a\}/c$ means $dPc/dt < 0$ from proposition 3.1. Then Pc become 0. Then $|dPV/dt| = |PV*(1-PV)*(EV-Env)| \leq d*\alpha/4$ Q.E.D.

The proposition 3.2 (1) asserts that the decreasing rate of PV is small and in proportion to ϵ . The proposition 3.2 (2) asserts that the decreasing rate of PV becomes large in proportion to α .

Next we show the role of small group as the salt of the earth who prevent the collapse of norm by increasing the average payoff when $PV > \{(b-a)Pc+a\}/c$ holds.

Proposition 3.3

Let $EV = -d*\epsilon + \Delta$. Then the collapse of norm is prevented if $\Delta > d*\epsilon$ hold.

Proof:

$EV = -d*(1-Pc) + \Delta = -d\epsilon + \Delta$ and $Env = 0$ hold.

Then $EV - Env > 0 \Leftrightarrow -d\epsilon + \Delta > 0 \Leftrightarrow \Delta > d\epsilon$ hold. Q.E.D.

The proposition 3.3 asserts that the decreasing rate of PV is prevented by small compensation which is made by central authority or the effort of the salt of the earth.

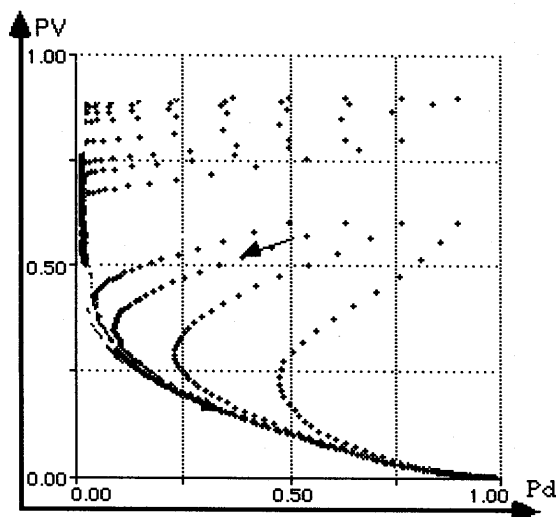


Figure 5 Norm Collapse

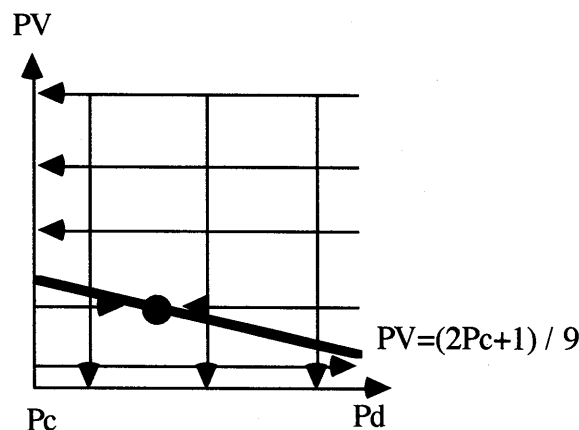


Figure 6 Bifurcation of Pc Dynamics

The norm game provides a new conceptual model of the norm. The GA simulation provides experimental discovery such as collapsing process of the norm. GA is a complex and complicated algorithm. It is difficult to extract systemic properties from the simulation.

It is also difficult to extend the model to more general centralized and decentralized regulation processes of an agent society under the mutual commitments.

We reformulate the mechanism with the population dynamics called RD. Once we give the mathematical formulation, it is easy to analyze and extend the mechanism.

The model consists of two 1 dimensional RD and its weak coupling. Population ratio of Vengefulness (V) becomes a bifurcation parameter for the dynamics of underlying alternatives {C,D}. If V is large enough then Pc increases. If V becomes small then the system bifurcates and Pd increases. The bifurcation structure is shown in the figure 6.

3.2 Meta Norm

Axelrod also introduced the concept of meta norm that is included in the concept of meta commitment among agents. It is illustrated in figure 7. In this figure, the thick arrows indicate normative commitments and the thin arrows indicate meta normative commitments.

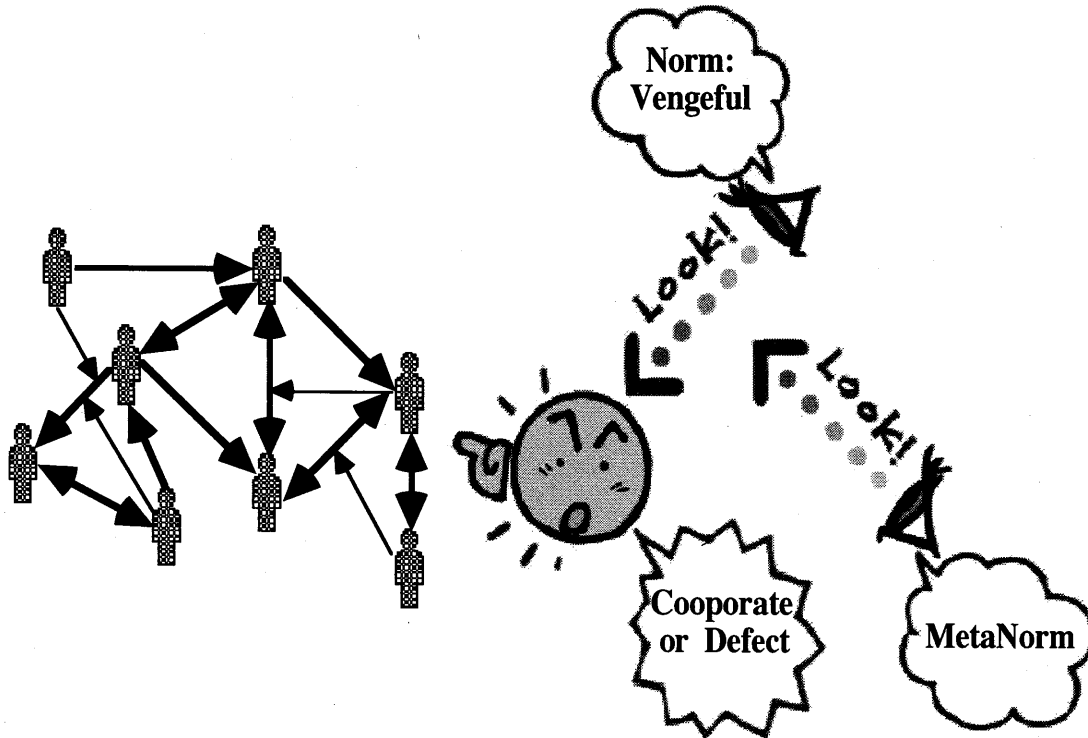


Figure 7. Commitment and Meta Commitment

We introduce the average payoff table of commitment and meta commitment of alternatives as follows.

Table 2. Average Payoff of Commitment and Meta Commitment of Alternatives

Average Payoff	Interaction Factor	Coupling Factor
E[V]	0	-d*Pd
E[NV]	0	-e*PMV
E[MV]	0	-d*Pnv
E[MNV]	0	0

Then we can introduce RD for meta norm game as follows.

Where we assume $a < d < b < c = e$. Let $a=1, b=3, c=9, d=2, e=9$ in the following simulation. We also introduce fluctuation in the simulation.

$$dPc/dt = Pc*(1-Pc)*(EC-ED) \quad (3.2.1)$$

$$dPV/dt = PV*(1-PV)*(EV-Env) \quad (3.2.2)$$

$$dPMV/dt = PMV*(1-PMV)*(EMV-EMnv) \quad (3.2.3)$$

$$EC = -a*(1-Pc), \quad ED = b*Pc - c*PV$$

$$EMnv = 0, \quad EMV = -d*(1-PV)$$

$$Env = -e*PMV, \quad EV = -d*(1-Pc)$$

$$Pd = 1-Pc, \quad Pnv = 1-PV$$

$$PMnv = 1-PMV$$

Figure 8 shows the result of the simulation of the meta norm.

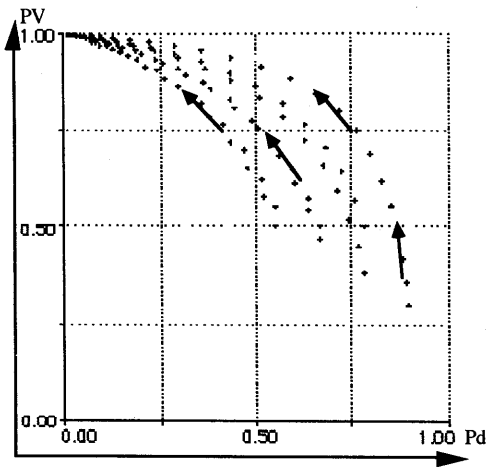


Figure 8 RD for Meta Norm

We have the same question that meta norm also collapse or not. The following proposition answer the question.

Proposition 3.4

Let ϵ and ϵ' be positive perturbation around $P_c=1$ and positive perturbation around $PMV=0$ respectively. If ϵ and ϵ' are same scale of positive perturbation and $c>d$ then $PV=1$ is stable.

Proof:

We show that $dPV/dt > 0$. $dPV/dt = PV*(1-PV)*(EV-Env)$.
 Then $dPV/dt > 0 \Leftrightarrow EV > Env \Leftrightarrow -d*(1-P_c) > -c*PMV$ holds. From the assumptions $-d*(1-P_c) > -c*PMV \Leftrightarrow -d\epsilon > -c*\epsilon'$ holds and ϵ and ϵ' are same scale of perturbation. Thus we can assume that $\epsilon = \epsilon'$. Then $c>d$ is the condition for $dPV/dt > 0$. Q.E.D.
 In the previous example $c=9, d=2$ satisfy the condition.

4. Centralized Commitment and Support

4.1 Type of Commitments

Once we formulate the mathematical model of the norm and analyze its bifurcation mechanism, we can extend the model to more general processes of mutual reference and commitments. We add centralized commitment structure in the society. We also introduce the positive sanction mechanism as comment process such as supporting attitude.

In the previous chapters we have analyzed the mutual reference and commitment processes. We call the mechanism as decentralized commitment, which effects as indirect regulation of agent societies. The norm and meta norm effect on the underlying alternatives as a decentralized indirect regulation. Now we try to design another indirect regulation mechanism of agent societies.

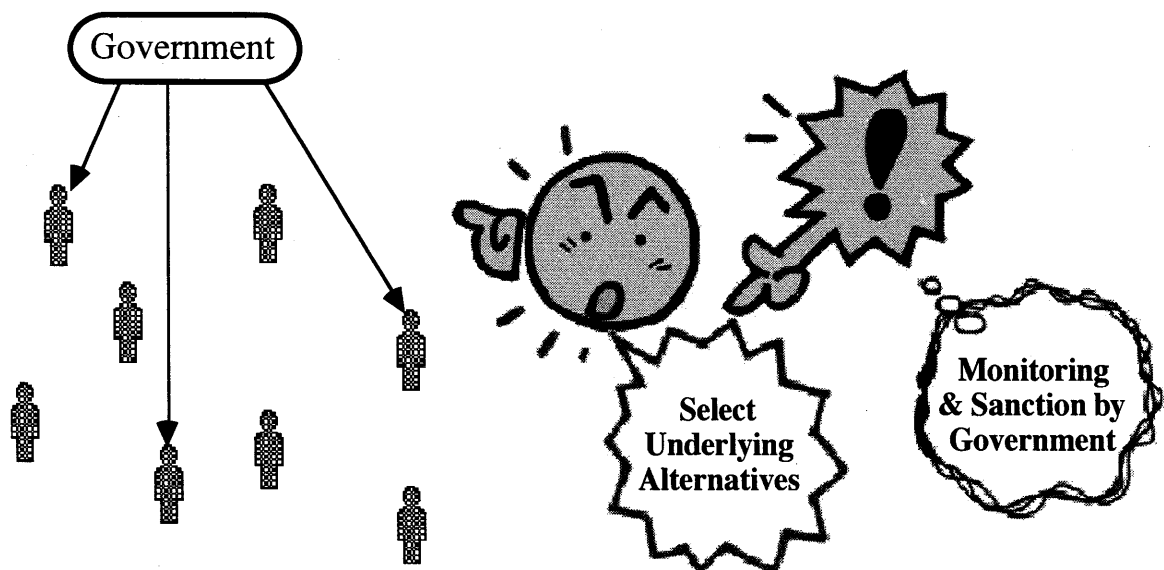


Figure 9 Centralized Monitoring and Sanction for Underlying Actions

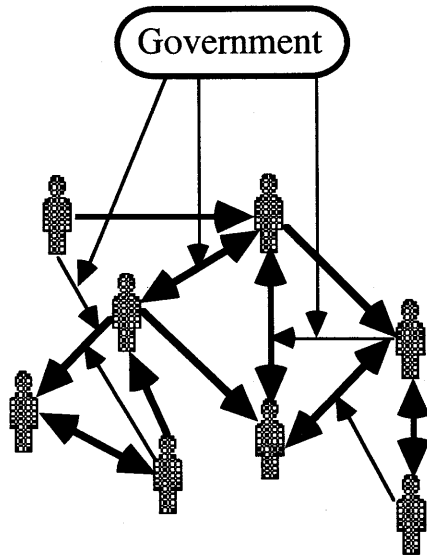


Figure 10 Centralized Monitoring and Sanction for Mutual Commitments

In this section we assume that there is an central authority such as government in the society and introduce centralized commitment processes. For example monitoring and a fine for a parking violation is a typical centralized commitment for underlying actions, which is shown in figure 9. We also pay attention to the centralized commitment for mutual commitment processes and meta commitment processes, which is shown in figure 10.

We also introduce positive sanction as a commitment to the model. In the norm and meta norm games, the sanction is negative penalty. Instead, we can use positive sanction such as supporting commitment instead of negative one.

4.2 Educational Effect and Supporting Commitment

To clarify our new concepts, we construct the model of educational effect and its support by government as follows. We introduce the underlying alternatives "A" and "B". In this case we assume that "ASup" and "BSup" are alternatives that mean supporting the underlying alternatives respectively. Alternatives "AKnow" and "Bknow" mean the belief that "A" is correct and "B" is correct respectively.

In the model we assume that "A" is a desirable attitude in the long run for the society and "B" is conventional attitude. We assume the dilemma situation shown in the table 3. We also assume that there is a political support for the "AKnow".

The initial conditions for the simulation assume that old belief such as "PbKnow" and "PBSup" are dominant. But the educational support changes the situation drastically. As the right knowledge spread, agents who support right manner increase. Then population of "A" spreads.

When we consider basic interaction layer, we get the following interaction table of payoff.

Table 3 Payoff of Interaction

	A	B
A	(a, a)	(-b, 0)
B	(0, -b)	(-a, -a)

Furthermore, we consider the following positive sanction as commitment shown by "ASup" and "BSup". We also assume meta commitment from correct knowledge shown by "AKnow" and "Bknow".

Table 4 Average Payoff for Alternatives of Supporting Commitments

Average Payoff	Interaction with Different type	Interaction with Same type	Coupling and Political Support
E[A]	-b*Pb	a*Pa	c*PASup

E[B]	0	-a*Pb	0
E[ASup]	0	d*Pa	e*PaKnow
E[BSup]	0	d*Pb	0
E[AKnow]	0	0	polsupA=1
E[BKnow]	0	0	0

Where "a", "b", "d", and "e" mean basic payoff of interaction, negative sanction from conservatives, bandwagon effects, and positive support by correct knowledge respectively.

At the first stage we are bound by convention. How can we do away with conventionalities? We are too conventional at the first stage because of the sanction from conservatives and bandwagon effects.

We assume $a < b$. Let $a=2$, $b=5$, $c=3$, $d=1$, and $e=2$ in the following simulation.

Then we can introduce RD for meta commitment game as follows. We also introduce fluctuation in the simulation.

$$dPa/dt = Pa*(1-Pa)*(EA-EB)$$

$$dPaKnow/dt = PaKnow*(1-PaKnow)*(EAKnow-EBKnow)$$

$$dPASup/dt = PASup*(1-PASup)*(EASup-EBSup)$$

$$EA = -b(1-Pa)+aPa+c*PASup,$$

$$EB = 0*Pa-a(1-Pa)+0*(1-PASup)$$

$$EASup = Pa+e*PaKnow,$$

$$EBSup = (1-Pa)+0*(1-PaKnow)$$

$$EAKnow = polsupA,$$

$$EBKnow = 0,$$

$$polsupA = 1$$

$$pb = 1-Pa,$$

$$PBSup = 1-PASup,$$

$$PbKnow = 1-PaKnow$$

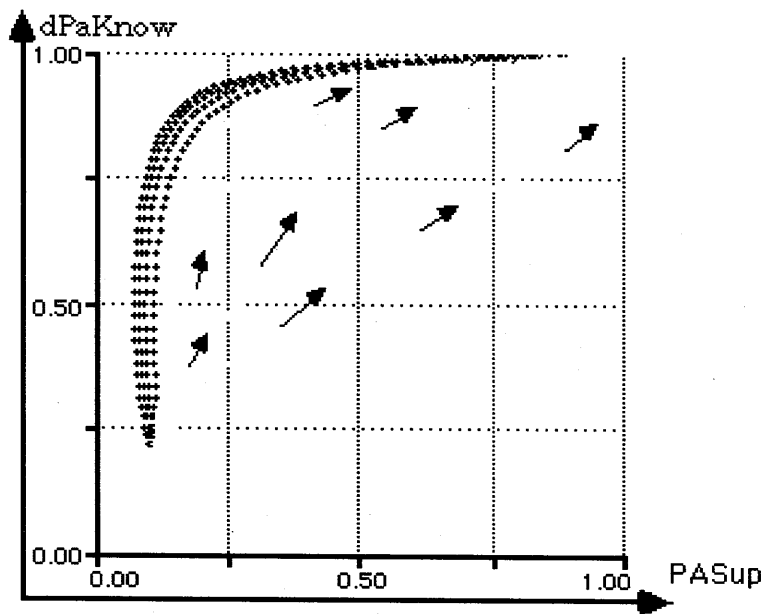


Figure 11 The spread of correct knowledge

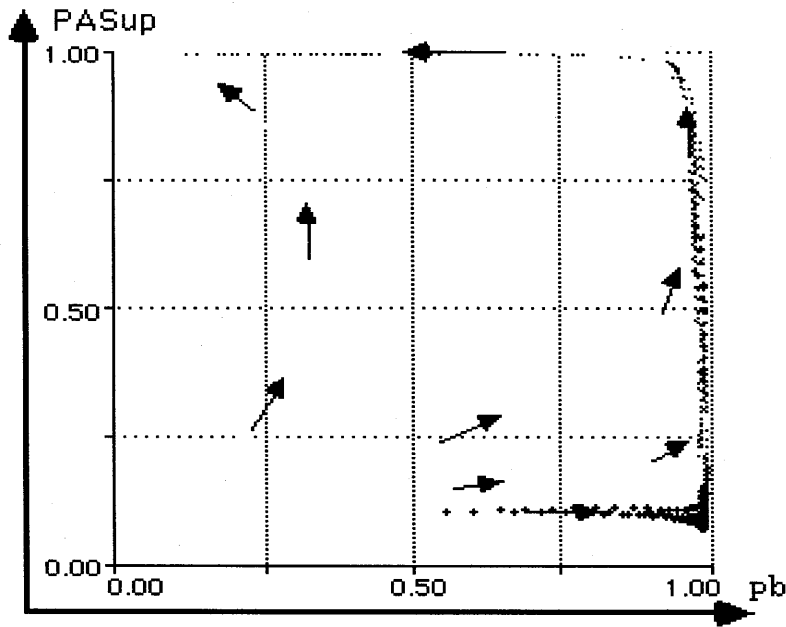


Figure 12 The spread of right attitude

Proposition 4.1

Let $PASup=0$. This means that there is no supportive commitment for attitude "A".

- (1) If $a < b$ then $Pb=1$ is stable, i.e., an existing conventional attitude is not overthrown.
- (2) If $a > 0$ then $Pa=1$ is stable, i.e., an existing desirable attitude is maintained.

Proof:

$$\begin{aligned}
 (1) \text{ We show that } dPa/dt < 0 &\Leftrightarrow EA-EB < 0 \Leftrightarrow -b*(1-Pa)+a*Pa+c*PASup+a*(1-Pa) < 0 \\
 &\Leftrightarrow a-b+c*PASup+b*Pa < 0 \\
 &\Leftrightarrow a+c*PASup < b-b*Pa
 \end{aligned}$$

If $PASup=0$, $Pb=1$, and $a < b$ then $dPa/dt < 0$ holds.

$$(2) dPa/dt > 0 \Leftrightarrow a+c*PASup > b-b*Pa.$$

Let $Pa=1$ and $a > 0$ then $dPa/dt > 0$ holds even if $PASup=0$. Q.E.D.

We are interested in a overthrowing process of existing conventional attitude. The proof shows that if $PASup$ is enough large and $a+c*PASup > b$ then $dPa/dt > 0$ holds for any $Pa > 0$.

Proposition 4.2

Let $PaKnow=1$ then $e > d$ is the condition of spreading supportive commitment of attitude "A" for any Pa .

Proof:

We show that $dPASup/dt > 0$.

$$dPASup/dt = PASup(1-PASup)(EASup-EBSup)$$

$$dPASup/dt > 0 \Leftrightarrow EASup-EBSup > 0 \Leftrightarrow e*PaKnow+d*Pa-d*(1-Pa) > 0$$

$$\Leftrightarrow e*PaKnow+d*Pa-d+d*Pa > 0 \Leftrightarrow e*PaKnow > d-2d*Pa$$

We assume that $PaKnow=1$ then $e*PaKnow > d-2d*Pa \Leftrightarrow e > d-2d*Pa$.

Then $e > d$ is the condition of $dPASup/dt > 0$ for any Pa .

We can show the typical interpretation of this model. A great deal of effort has been put into the project of birth control by the United Nations, but the centralized direct sanction policy has been failed. People do not want to change their belief. Now they say that the spread of right knowledge and its support for birth control are only the way.

5. Historical Change of Social Commitment Structure

Once we recognize hierarchical mutual commitment among agents, we can easily explain norm formation and other social order and its change. Furthermore, we can characterize historical change of the commitment structure in our society. In pre-modern society, decentralized commitment such as meta norm is a typical way of mutual commitments where the sanction is negative one such as punishment. Figure 13 shows the process.

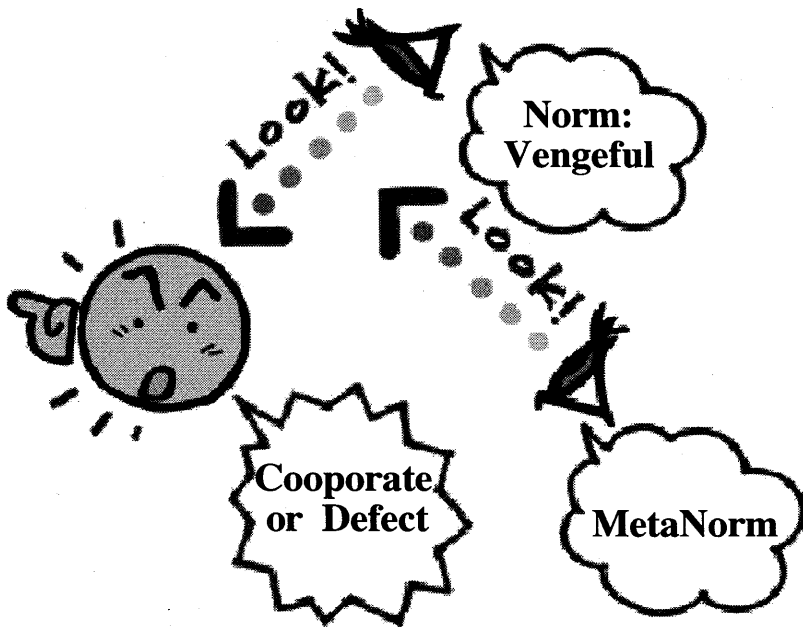


Figure 13 Indirect Control on Pre-Modern Society 1

Central authority might support the meta norm process, which is shown in figure 14.

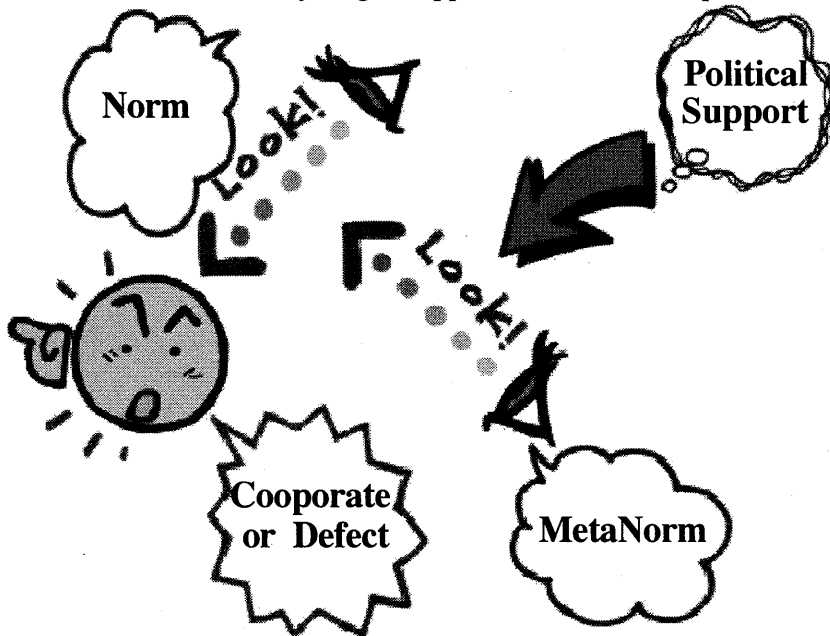


Figure 14 Indirect Control on Pre-Modern Society 2

In pre-modern society, it is difficult to manage the change of social order quickly. Modern society is highly fluid and varied. Modern society requires dynamic change for its economical development. In the society, new way of the formation of social norm is required. Under the nation state, the old way of mutual sanction became out-of-date. Two types of the way of commitments introduced by the nation state. One is the rule of law and another is education by mass media and school.

If someone breaks the law then he would be judged and punished by law under the social monitoring, which is shown in the following figure.

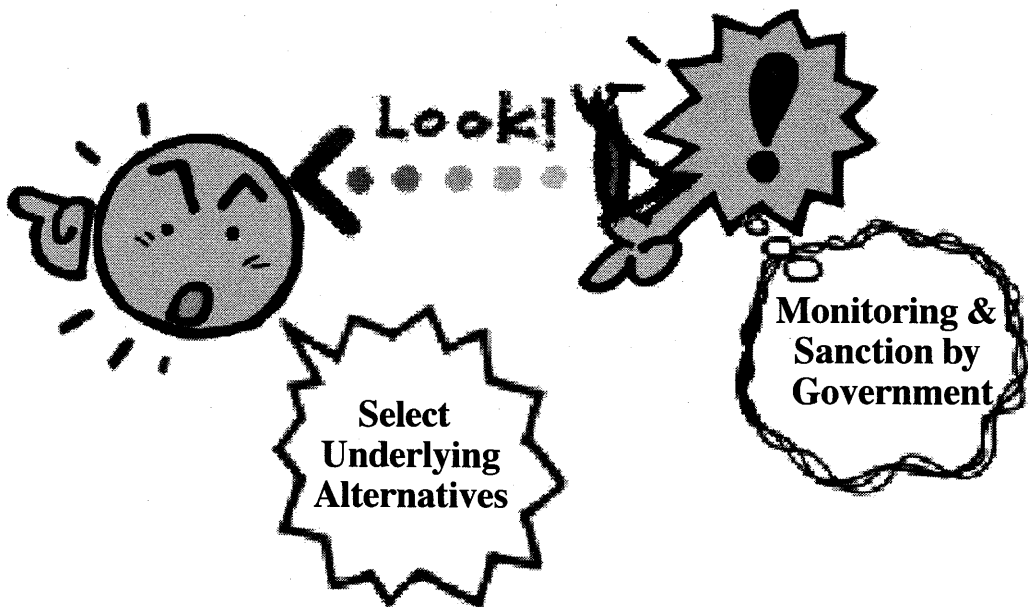


Figure 15 Monitoring and Punishment by Law

Another way of social commitment is developed in modern society. It consists of positive commitment such as support for underlying attitude, meta commitment depending on knowledge & information and educational support of knowledge. In modern society knowledge formation are supported or affected by education or mass media. The process is shown in figure 16.

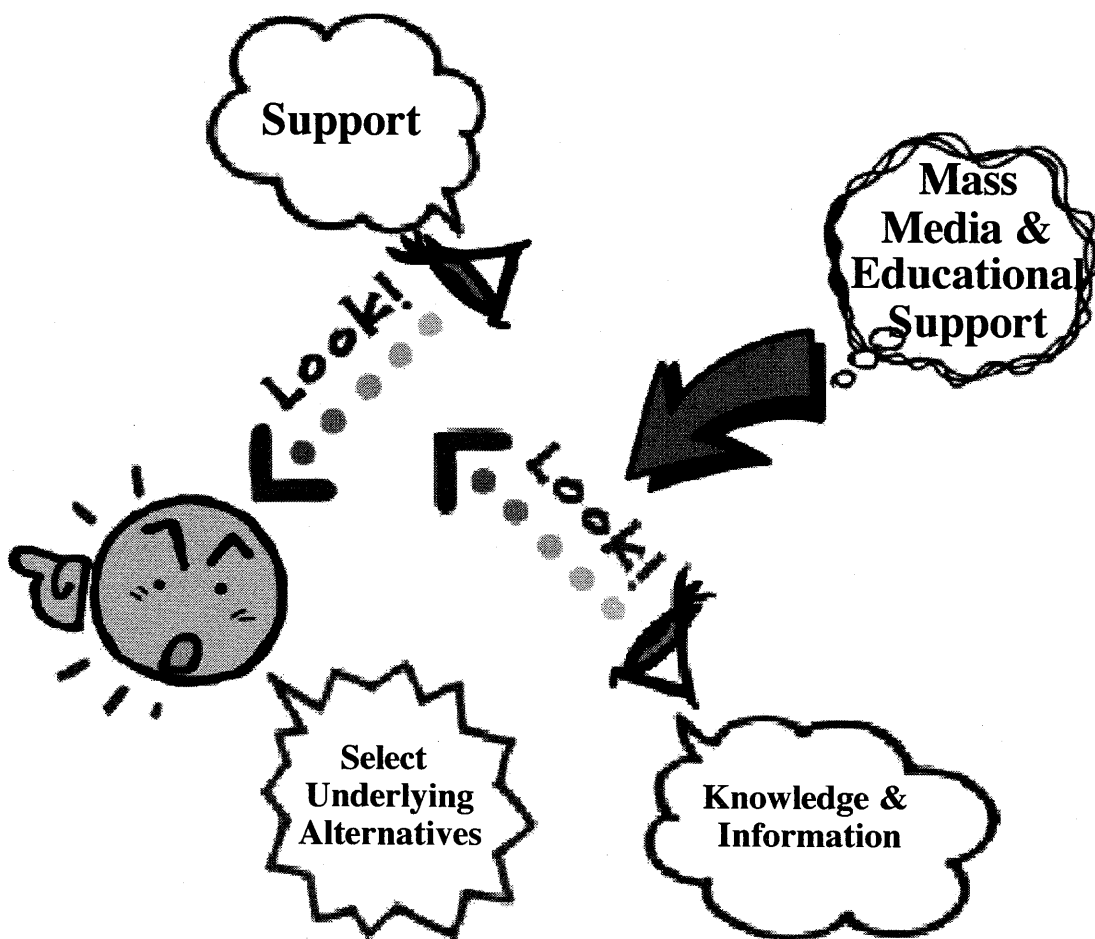


Figure 16 Meta Commitment by Knowledge & Information and Its Support

Of course this classification is a representative one in the period. There are different types of commitment structures in any period. In modern society, imposed evaluation of knowledge by education & mass media is dominant. New type of mutual commitment

structure is developing under the Internet & IT mediated world rapidly.

For evolving the evaluation landscape of underlying activities dynamically, we have to learn the knowledge about the situation. Knowledge creates our evaluation landscape, which dominates our supporting attitude.

In modern society, knowledge are supported by rather centralized systems. In post modern society it will be supported by some IT mediated social networks. Knowledge is created , shared and evaluated on the social networks. The networks compete with each other. Network reputation become an important principle of competition. It is shown in the next figure.

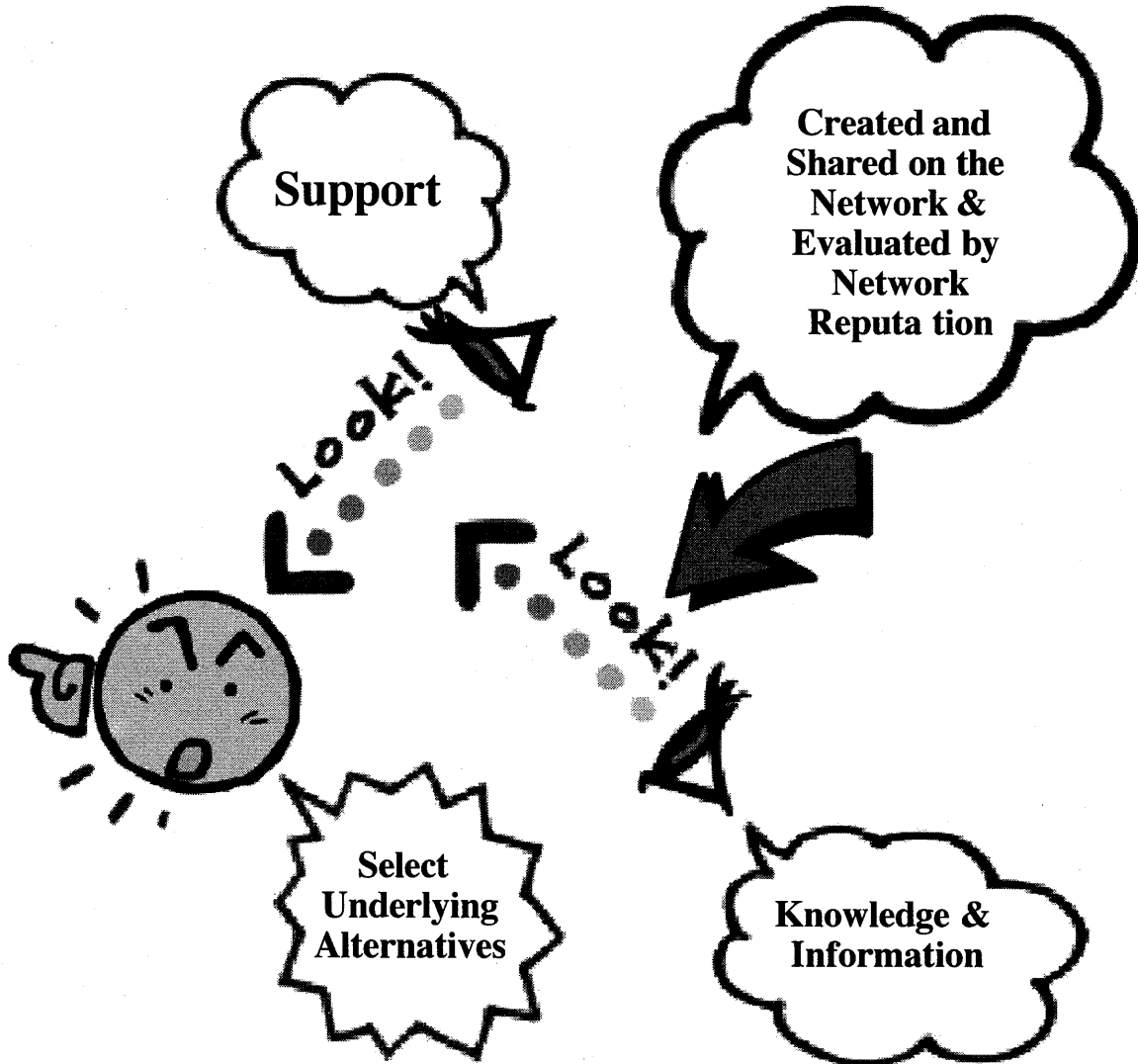


Figure 17 Meta Commitment by Knowledge & Information with Network Reputation

6. Conclusion

We formalized Axelrod's norm and meta norm games with weak coupled replicator dynamics. In this approach, we assume that RD represents social learning processes. It is different from evolutionary interpretation of RD. To justify this assumption, we derived RD from stochastic processes of social learning.

By using the social learning dynamics we have analyzed mutual and hierarchal commitment processes among agents. In usual rational decision making theory, we do not mention about these types of social learning and commitment processes. In other words we have introduced two types of extensions of the concept of rationality.

The one is learning rationality , which means that social learning processes of agents are rational in a certain sense. In our social learning we do not assume perfect nor complete information. Agents do not interact such as n person game. An agent decides by himself step by step depending on average payoff information of each alternative at the step. Macro social learning consists of the step by step dynamics of statistical decisions of agents.

The other is commitment rationality which means that agents have mutual and hierarchal commitments such as "norm" or "meta norm". Why are commitments rational in a

given situation? This question is a generalization of the one that why norm collapses or dose not collapses.

Robert Axelrod has introduced the concept of meta norm [3; ch. 3]. But it cause next question that why meta norm collapses or dose not collapses. R. Gaylord and D'Andria mentioned about his question in their book [8; ch. 5]. Jhon Maynerd Smith also points out that social norm is not always stable in his famous book, which is an origin of RD [9; ch. 13].

Our answer is simple. A commitment is rational if the alternative for the commitment is stable in the sense of social learning dynamics under the given social and political situations. The rationality of commitment comes from the stability of social learning dynamics.

We also introduced the higher order centralized and decentralized commitment processes that effect as indirect regulation or control mechanism of an agent society. The theory of hierarchical commitments for an agent society is a new frontier for agent based approaches. We have to develop the theory for designing our post modern society.

For developing this area, both of agents based heuristic simulation and theoretical analysis are important.

Reference

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