Heat Transfer on a Flat Plate at One End of a Series-Connected Rectangular Duct in Pressurized He II

Hideki Tatsumoto, Koichi Hata, Katsuhiko Hama, Yasuyuki Shirai, and Masahiro Shiotsu

***Abstract*—Steady-state and transient heat transfer on a flat plate at one end of a two-series-connected duct with a sudden change of the cross-section were performed in subcooled He II at bath temperature of 1.8K in order to simulate the effect of the heat flow expansion in the cooling channel. The results of the steady-state critical heat flux (CHF) can be described within 15% by the correlation of CHF derived here. For the stepwise heat flux larger than the CHF, quasisteady-state exists for a certain duration called lifetime on the extension of Kapitza conductance curve. With increase in the stepwise heat flux, the lifetime becomes shorter and approaches that for one-dimensional duct. The experimental results were analyzed by the two-dimensional computer code named SUPER-2D developed by the authors based on the two fluid model and the theory of the mutual friction. The solutions of CHF and lifetime agreed well with the experimental data. The heat flow mechanism in the duct was clarified by the analysis.**

***Index Terms*—Critical heat flux, lambda transition, superfluid helium.**

I. Introduction

PRESSURIZED He II is expected as a coolant for a large-scale superconducting magnet because of its excellent cooling properties. The knowledge of the heat transfer in a He II channel with expansion is important in order to estimate the instability of the magnets caused by large pulsewise heat inputs. The heat transfer in He II has been mainly investigated experimentally and numerically for the one dimensional heat flow system. Pfotenhauer [1] treated the heat flow in the path as a series or parallel connection of the thermal resistance due to each Gorter–Mellink ducts. Tatsumoto *et al.* [2] measured CHFs on flat plates on one ends of the rectangular duct with two-dimensional extent of cross-section and derived the correlation of CHF that can describe the effect of the heat flow expansion in the duct.

There have been very small numbers of studies on the numerical analysis for the two-dimensional transient heat transfer in He II by using the two fluid model with some assumption. Whether the numerical code is adequate or not has not been clarified. Tatsumoto *et al.* [3] developed the two-dimensional numerical code (SUPER-2D) based on the two fluid model and the theory of the mutual friction. The adequacy of the code has been confirmed by comparison with the experimental data [2].

In this paper, steady-state and transient heat transfer in a series-connected duct with two-dimensional extent filled with pressurized He II was investigated experimentally and numerically at bath temperature, T_B, of 1.8 K.

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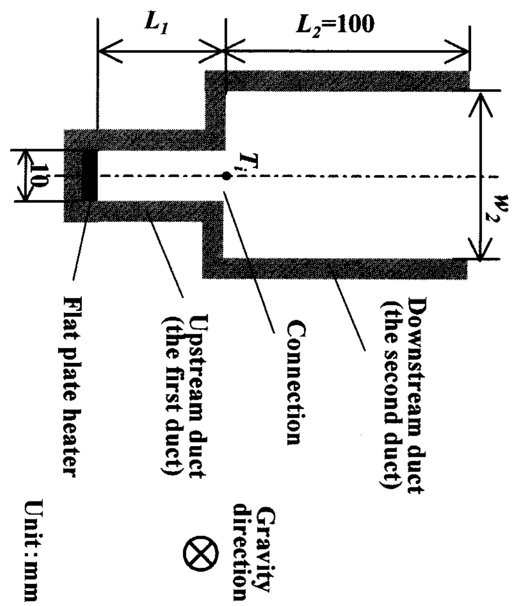

Fig. 1. Schematic of a series-connected duct.

II. Experiments

The schematic of a series-connected rectangular duct was shown in Fig. 1. All flat plate heaters made of Manganin were 10 mm in width, 40 mm in length and 0.1 mm in thickness. They were located at one ends of the first duct, which was the Gorter–Mellink duct with the length of L_1 and the cross-sectional area A_1 (40 mm $\times$ 10 mm). The other end of the second ducts with the length of L_2 (100 mm) and the cross-sectional area A_2 (40 mm $\times$ w_2), which is larger than A_1, is opened to a He II bath. Three of them has the inner width of second duct, w_2, of 16.5, 27.0 and 31.5 mm and the same L_1 of 20 mm. Only the ratios, A_1/A_2, were varied from 1.65 to 3.15. The others were 0, 10 and 30 mm in L_1 and the value of A_2/A_1 was fixed to be 2.7. The rectangular ducts were mounted horizontally in a pressurized He II bath at 1.8 K and 101.3 kPa.

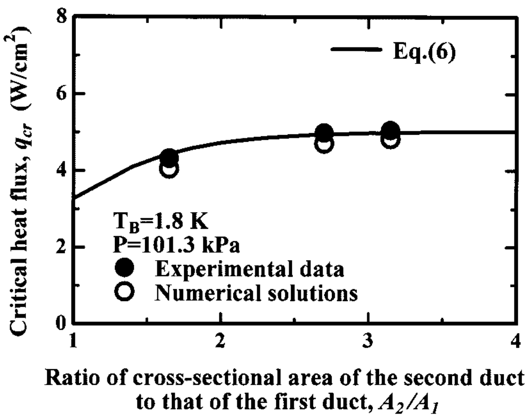

Fig. 2. Critical heat flux versus ratio of cross-sectional area of the second duct to that first duct.

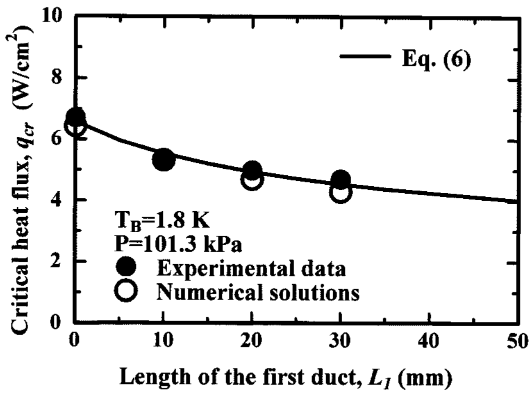

Fig. 3. Critical heat flux versus length of the first duct.

III. Experimental Results

A. Steady-State Critical Heat Flux

The experimental data of steady-state critical heat fluxes (CHFs), q_{cr}, are shown in Figs. 2 and 3. With the increment of A_2/A_1, the values of q_{cr} increase and approach a constant value. The increase of q_{cr} seems to disappear for $A_2/A_1 > 2.7$. As the value of L_1 increases to 10 mm, the values of the CHF rapidly decrease. With further increase of L_1, the decreasing rate of the CHF gradually becomes smaller.

B. A Correlation of CHF

Tatsumoto *et al.* [2] have derived the following correlation of CHF on a flat plate with the width of w and the length of l at one end of a rectangular duct with two-dimensional extent.

$$q_{cr} = \left(k \int_{T_B}^{T_\lambda} f(T)^{-1}\, dT \right)^{1/3} \quad (1)$$

where

$$k^{-1} = \{(A_h^3 L/A_d^3)^{1.5} + (w/0.78)^{1.5} \cdot \tanh\{5(A_d/A_h - 1)\}\}^{2/3} \quad (2)$$

where $f(T)^{-1}$ is effective thermal conductivity function, A_h is heated surface area, A_d is cross sectional area of the duct, and L is the length of the duct.

A correlation of CHF for a series-connected duct was derived as follows. Suppose that, when the liquid temperature adjacent to the heated surface reaches T_λ, the temperature averaged over the cross-section at the connection would be uniform at T_i and that at the duct end would be equal to bath temperature, T_B. As the individual duct carries the same heat, Q, under steady-state condition, the heat flux in the first duct is expressed by using Gorter–Mellink equation as follows

$$q_1 = Q/A_1 = \left((1/L_1) \int_{T_i}^{T_\lambda} f(T)^{-1}\, dT \right)^{1/3}. \quad (3)$$

On the other hand, the effect of the heat flow expansion to the second duct would be described by considering (1)

$$q_2 = Q/A_1 = \left(k_2 \int_{T_B}^{T_i} f(T)^{-1}\, dT \right)^{1/3} \quad (4)$$

where

$$k_2^{-1} = \{(A_1^3 L_2/A_2^3)^{1.5} + (w_1/0.78) \cdot \tanh\{5(A_2/A_1 - 1)\}\}^{2/3} \quad (5)$$

where w_1 is the inner width of the first duct.

The correlation of CHF can be obtained by combining (3) and (4) in terms of $Q = q_{cr} A_1$

$$q_{cr} = A_1^{-1} \left\{(L_1/A_1^3) + (1/k_2)/A_1^3\right\}^{-1/3} \cdot \left(\int_{T_B}^{T_\lambda} f(T)^{-1}\, dT \right)^{1/3}. \quad (6)$$

As shown in Figs. 2 and 3, all data of CHF exist near the curve given by (6). It was confirmed that the correlation can describe the data of CHF within 13% errors.

C. Transient Heat Transfer Caused by a Stepwise Heat Input

Typical behavior of transient heat transfer for the stepwise heat input with the height larger than q_{cr} is as follows. Initially, the heat input rapidly increases and then it takes a constant value, Q_s, after $t = t_A$. The rising time up to the Q_s is around 300 μs. The surface temperature difference and the heat flux remain constant at ΔT_s and q_s respectively, for a certain duration $(t_B - t_A)$, then they begin to increase and decrease, respectively, at a time $t = t_B$. The duration $t_L = t_B - t_A$ is defined as the lifetime of the quasisteady-state heat flux q_s. The transient heat flux increases along the Kapitza conductance curve and the curve's extrapolation, and then reaches a quasisteady-state on the extrapolated curve on $\log(q_s)$ vs. $\log(\Delta T)$ graph.

The data of lifetime, t_L, for various stepwise heat flux, q_s are displayed in Fig. 4. Shiotsu *et al.* [4] have presented the following correlation of lifetime for one-dimensional heat flow

$$t_L = a^{-4}\overline{\rho C_p}\,\overline{f(T)^{-1}}(T_\lambda - T_B)^2 q_s^{-4} \quad \text{for } t_L \geq 1.2 \text{ ms} \quad (7)$$

$$t_L = \overline{\rho C_p}\,\overline{B(T)^{-1}}(T_\lambda - T_B)^2 q_s^{-2} \quad \text{for } t_L < 1.2 \text{ ms} \quad (8)$$

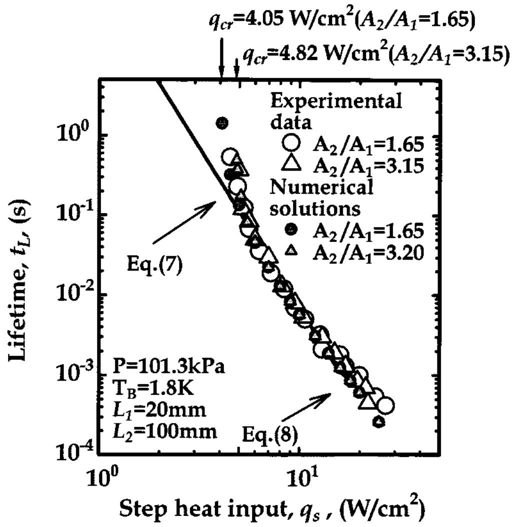

Fig. 4. Lifetime versus step heat flux.

where $a = 1.16$, $B(T)^{-1} = s^2T/A*$, $A* = 8000$ m^3/(kg s), ρ is the density of He II, and C_p is the specific heat.

The lifetime is infinite for $q_s < q_{cr}$. With increase in q_s from that corresponding to q_{cr}, the lifetime rapidly decreases and approaches the straight line with the gradient of -4 given by (7). It reaches the line at the lifetime of around 50 ms independent of A_2/A_1. For the q_s with $t_L < 1.2$ ms, the lifetime decreases along the line with the gradient of -2 given by (8).

IV. Numerical Analysis

A. Numerical Model

The experimental results of steady-state and transient heat transfer were analyzed by using a numerical code named SUPER-2D developed by Tatsumoto *et al.* [3] based on the two fluid model and the theory of the mutual friction. Two fluid equations used in the analysis is expressed as follows. Details of the numerical code has been described elsewhere [3].

Continuity Equation

$$\partial\rho/\partial t + \nabla\cdot(\rho_n v_n + \rho_s v_s) = 0 \tag{9}$$

Momentum Equation for the Total Fluid

$$\partial(\rho_n v_n + \rho_s v_s)/\partial t = -\nabla\cdot(\rho_n v_n v_n + \rho_s v_s v_s) - \nabla P + \eta\left\{\nabla^2 v_n + (1/3)\nabla(\nabla\cdot v_n)\right\} + \rho g \tag{10}$$

Momentum Equation for the Superfluid Component

$$\partial v_s/\partial t = -(v_s\cdot\nabla)v_s + S\nabla T - (1/\rho)\nabla P + (\rho_n/2\rho)\nabla|v_n - v_s|^2 + A\rho_n|v_n - v_s|^2(v_n - v_s) + g \tag{11}$$

Entropy Equation

$$\partial(\rho S)/\partial t = -\nabla\cdot(\rho S v_n) + A\rho_n\rho_s|v_n - v_s|^4/T \tag{12}$$

where v is the velocity, T is the temperature, P is the pressure, t is the time, η is the viscosity, S is the specific entropy, and A is the Gorter–Mellink mutual friction parameter. The subscripts, s and n, denote the superfluid and the normal fluid components, respectively.

As shown in Fig. 1, since the system can be regarded as a symmetric problem, the two fluid equations were solved on only half of the system by finite different method with the staggered grid with the length of 0.25 mm. Time integration was performed explicitly with a time step of 4.0×10^{-7}. The symmetric condition is applied at the center of the channel. The adiabatic condition is applied to the adiabatic duct wall except the heated section. The temperature at the channel end is kept constant at bath temperature, T_B. The liquid temperature on the heated surface is calculated by solving the form for a one-dimensional [5] with iteration

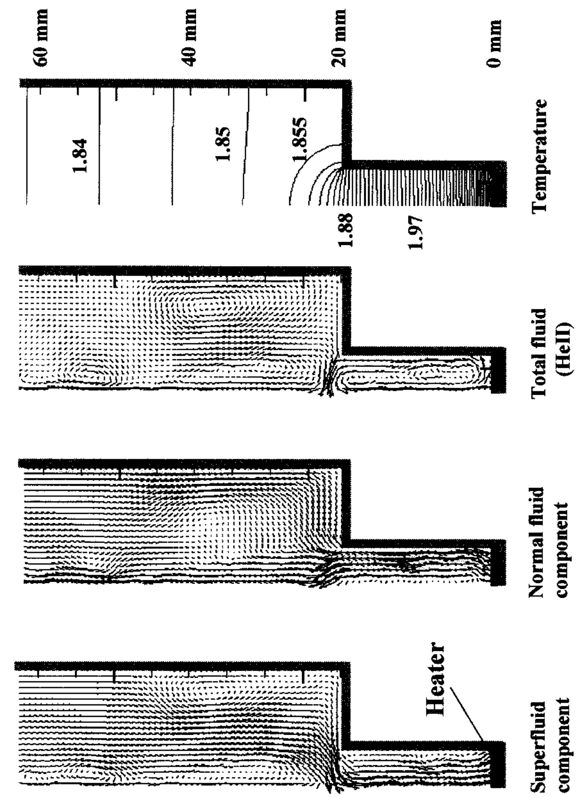

Fig. 5. Velocity for each component and temperature distribution in the duct filled with He II.

$$\rho C_P(\partial T/\partial t) = \partial\left(f(T)^{-1}(\partial T/\partial y)\right)^{1/3}/\partial y. \tag{13}$$

The full slip condition is applied to velocities of the superfluid component in the direction tangential to the wall, and velocities of the normal fluid component on the wall are set to be zero. In case of steady-state heat transfer analysis, when the $\partial T/\partial t$ at the heated surface converged, the temperature distribution was regarded to be steady-state. The heat flux applied to the heater was set to be 0.04 W/cm^2 higher than the previous value and the calculation was repeated. When the temperature reached T_λ, the heat flux was defined as the steady-state critical heat flux, q_{cr}. In case of transient heat transfer analysis, the heat flux was kept constant until the λ transition occurs.

V. Numerical Results

A. Steady-State Critical Heat Flux

The solutions of CHF were shown in Figs. 2 and 3 in terms of open symbols. The solutions of CHF given by SUPER-2D

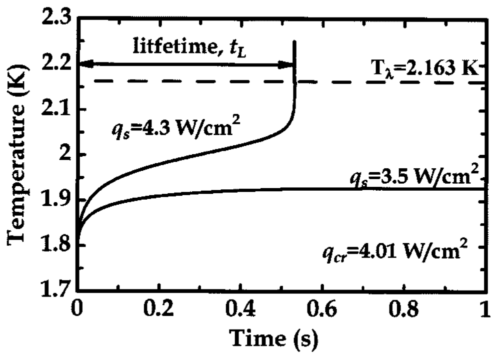

Fig. 6. Typical time trace of liquid temperature adjacent to the heated surface, T_h, for various step heat inputs, q_s.

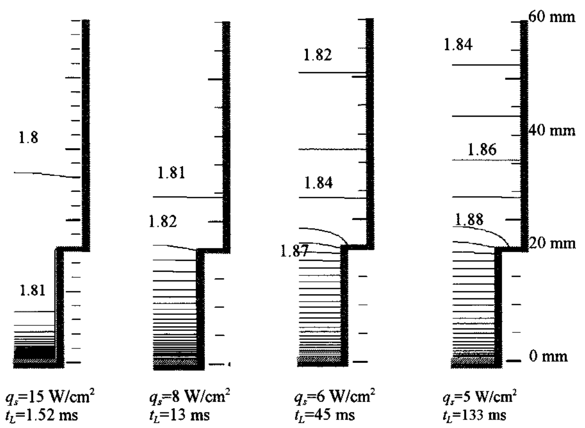

Fig. 7. Temperature distribution in a duct with $A_2/A_1 = 1.65$ for various step heat fluxes.

are in agreement with the experimental data within 8.0% difference. The velocity for each component and the temperature distribution for $A_2/A_1 = 3.15$ under steady-state critical heat flux condition are displayed in Fig. 5.

The vortices are formed because of the influence of the extent of the cross-section in superfluid component. The normal fluid that carries the entropy cannot expand to a uniform distribution rapidly at the connection as shown in Fig. 5. The distribution of the normal fluid almost remains unchanged for $A_2/A_1 > 2.7$ near the connection. It is, therefore, considered that the increasing rate of CHF is almost saturated for $A_2/A_1 > 2.7$.

B. Transient Heat Transfer Caused by a Step Heat Input

The typical time trace of calculated liquid temperature adjacent to the heated surface, T_h, for various step heat inputs, q_s is shown in Fig. 6. For the step heat flux larger than q_{cr}, the time delay from the onset of step heat input to the occurrence of λ transition exists. The time delay corresponds to the lifetime of the quasisteady-state mentioned above. For the q_s smaller than q_{cr}, the value of T_h remains at a constant value smaller than T_λ.

The solutions of lifetime are shown in Fig. 4 in terms of solid symbols. The solutions also reach the line given by (7) at a certain heat flux with t_L of around 50 ms independent of A_2/A_1 as well as the experimental data. For the q_s with $t_L \leq 1.2$ ms, however, they do not decrease along the line of (8) but along the extrapolated line by (7). The solutions are in good agreement with the experimental data except for $t_L < 1.2$ ms.

Fig. 7 show the temperature distribution in the duct with $A_2/A_1 = 1.65$ for various step heat inputs. For $q_s > 6$ W/cm^2, the temperature rise at the connection is slight It is, therefore, that the fast transient heat transfer is determined within the upstream duct with little influence of the heat flow expansion. The heat transfer would be regarded as one-dimensional. For $q_s < 6$ W/cm^2, the lifetime depends on A_2/A_1, because of the effect of the heat flow expansion.

VI. Conclusions

Heat transfer in a series-connected rectangular duct filled with pressurized He II was investigated experimentally and numerically. It is found that the steady-state CHFs depend strongly on the ratio of the cross-sectional area, A_2/A_1, and the length of the duct, L_1. The correlation of CHF for the duct with a sudden change of cross-sectional area can describe the experimental data within 13%. For the q_s smaller than the CHF, the lifetime is infinite. With the increase of q_s, it reaches the curve by (7) at the t_L of around 50 ms independently of A_2/A_1. After that, it agrees with the line by (7) and (8).

The solutions of CHF and lifetime are in good agreement with the experimental data. It is clarified from the analysis that the heat flow cannot expand immediately to the full cross-section. Therefore, the increasing rate of CHF becomes saturated for $A_2/A_1 > 2.7$. For $t_L \leq 50$ ms, the heat transfer can be regarded as one-dimensional since the transient heat transfer phenomena is almost determined within the upstream duct. For $t_L > 50$ ms, the t_L depends on A_2/A_1 because of the influence of the two-dimensional heat flow expansion.

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