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Equilibrium Analysis for a Migration Model

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Abstract

This paper gives a theoretical equilibrium analysis for a deterministic migration model among n regions in the case of zero natural growth. First, this paper shows that a migration equilibrium always exists if residents' utility functions are continuous. Second, this paper gives some conditions for the stability of a migration equilibrium. Specifically, extending the necessary condition of Tabuchi (1986), this paper provides conditions which are sufficient to ensure a stable migration equilibrium. Although the model is basic and simple, this paper provides a complete theoretical analysis and derives concise results, which have very intuitive explanations.

Keywords: Migration; Equilibrium; Stability

1 Introduction

Economics theories of migration begin with the assumption that the migration decision is based on a comparison of economic and social conditions in the origin and destination regions. This paper assumes that residents are homogeneous and individual decisions to migrate depend on the utility discrepancy of regions. Although a resident's utility depends on many factors, for the sake of simplicity, this paper supposes that the utility of a resident is a function $u_i(P_i)$, where P_i is the population size of region i. Let the total population be $\overline{P}$. Although the number of population in a region should be an integer, we suppose that $\overline{P}$ is larger enough so that we can treat all P_i as continuous variables.

We call a population state of n regions $\mathbf{P}^* = \langle P_1^*, \ldots, P_n^* \rangle$ a (*migration*) *equilibrium* if no resident wants to migrate. In the words of utility, it should hold at equilibrium $\mathbf{P}^*$ that

$$\begin{cases} u_i(P_i^*) = u^* & \text{if } i \text{ is a region with } P_i^* > 0, \\ u_i(0) \le u^* & \text{if } i \text{ is a region with } P_i^* = 0, \end{cases} \tag{1}$$

where $u_i(0) = \lim_{\epsilon>0,\epsilon\to 0} u_i(\epsilon)$, $i = 1, \ldots, n$. If all $P_i^* > 0$, we call $\mathbf{P}^*$ an *interior equilibrium*. Otherwise, we call $\mathbf{P}^*$ a *corner equilibrium*.

To the author' knowledge, none gives any condition to ensure the existence of a migration equilibrium. Therefore Section 2 provides an existence result, which says that an equilibrium (interior or corner) always exists if utility function $u_i(\cdot)$ is continuous for all i.

An equilibrium may collapse if some residents migrate by accident. Therefore it is necessary to consider the stability of an equilibrium. To do so, we have to derive a differential equation as a dynamic model. Migration equilibrium $\mathbf{P}^* = \langle P_1^*, \ldots, P_n^* \rangle$ is called *stable* if the stationary solution $P_i(t) = P_i^*$ of the dynamics is (*locally*) *asymptotically stable*. In otherwards, even if the population of the regions happens to run off $\mathbf{P}^*$ a little, the population will return to

$\mathbf{P}^*$. In mathematical terms, $\mathbf{P}^*$ is stable if for any positive number ϵ and initial time t_0, there exists a neighborhood $N(\mathbf{P}^*)$ of $\mathbf{P}^*$ such that for any $\mathbf{P}^0 \in N(\mathbf{P}^*)$, every solution $\mathbf{P}(t) = \langle P_1(t), \ldots, P_n(t)\rangle$ of the differential equation with initial value $\mathbf{P}(t_0) = \mathbf{P}^0$ satisfies $||\mathbf{P}(t) - \mathbf{P}^*|| (\triangleq \max_{i=1,\ldots,n} |P_i(t) - P_i^*|) < \epsilon$ for all $t \geq t_0$ and $\lim_{t\to\infty} \mathbf{P}(t) = \mathbf{P}^*$.

It is important to find some convenient conditions to ensure the stability of an equilibrium. For example, in the recent "economic geography" literature (Krugman, 1991; Fujita, Krugman and Veneables, 1999), researchers are interested in the process of regions' agglomeration, by examining the change of a stable equilibrium when transportation cost converges to zero. However, since there is no useful theoretical conclusion to ensure equilibrium stability in the case of multiple regions, researchers either restrict their study to the case of two regions, or only concern some special equilibria while setting the stability conditions aside. The purpose of this paper is to fill the theory gap of stability research.

Recently, some developed countries have a trend toward a zero natural growth rate. Therefore we suppose that $\overline{P}$ is a constant number. To investigate the stability of a migration equilibrium, Section 3 derives the following migration dynamics, by assuming that the migratory population size is proportional to the utility discrepancy.

$$\frac{dP_i(t)}{dt} = \sum_{j=1}^{n} [u_i(P_i(t)) - u_j(P_j(t))], \quad i = 1, 2, \ldots, n, \tag{2}$$

where t denotes time.

The above model is not new. When $u_i = u_j$ for all regions i and j, the above dynamics is used in Okabe (1980) and Tabuchi (1986). This paper adopts this more general model because each region i may have its uncontrollable region specific factors including some natural amenities (see the Appendix of Berglas (1984)). Besides, Boadway and Flatters (1982) consider a similar two-region migration model including public sector goods and mobile capital. Recently, Nakajima (1995) provides a stability condition for a two-region model including both mobile capital and labor.

A similar dynamics, called "replicator dynamics", which is routinely used in evolutionary game theory (Weibull, 1995), was proposed in Chapter 5 of Fujita, Krugman and Veneables (1999) as follows:

$$\frac{dP_i(t)}{dt} = \kappa\Big(u_i(P_i) - \sum_{j=1}^{n} \frac{P_i(t)}{\overline{P}} u_i(P_i(t))\Big) P_i(t),$$

where κ is the speed of adjustment (see Section 3 later). Two dynamics are distinguished as follows. First, our residents do not reproduce themselves. The only reason for the population increase in a region is that some residents of other regions move in. Therefore the right side of (2) (the increase speed) is not proportional to the present population $P_i(t)$, which happens in the replicator dynamics model. Second, the average utility is weighted by the population distribution in the replicator dynamics, while the average in our dynamics is a simple one without weight. Third, this paper derives a sufficient and necessary condition to ensure the equilibrium stability of our dynamics but no similar result is known for the replicator dynamics. Finally, although the dynamics are different, any migration equilibrium corresponds to a stationary solution of both dynamics.

Although it is important to find some convenient conditions for evaluating the stability of equilibrium $\mathbf{P}^* = \langle P_1^*, \ldots, P_n^*\rangle$, such kind of research in a general n-region case seems to be quite mathematically complex. Therefore Okabe (1980), Boadway and Flatters (1982), Nakajima (1995) only consider the case of two regions. An essential development in this field was done by Tabuchi (1986), who gives a necessary condition in the general case of n regions. It will be clear that Tabuchi's condition is very important to form a sufficient one. To illustrate his condition, we now assume that $u_i(\cdot)$ is differentiable for all $i = 1, \ldots, n$, and consider an interior equilibrium $\mathbf{P}^*$. By renaming the regions if necessary, we let $u_1'(P_1^*) \geq u_2'(P_2^*) \geq \ldots \geq u_n'(P_n^*)$. Then Tabuchi's necessary condition is

$$(n-1)u_1'(P_1^*) + u_i'(P_i^*) \leq 0, \quad \forall i = 2, \ldots, n. \tag{3}$$

To form a sufficient one, this paper strengthens condition (3) by adding

$$\text{The inequality of (3) holds strictly for at least one } i = 2, \ldots, n; \tag{4}$$

$$\text{If } u_1'(P_1^*) = 0, \text{ then } u_2'(P_2^*) < 0. \tag{5}$$

When $n = 2$, (3) – (5) degenerate to expression $u_1'(P_1^*) + u_2'(\overline{P} - P_1^*) < 0$, which appears in Okabe (1980) ((10) of Lemma 1, pp. 357) and Boadway and Flatters (1982) (expression (6), pp.619). Our conditions have a very intuitive explanation. Following Boadway and Flatters, we call region i with P_i^* *underpopulated* if $u_i'(P_i^*) > 0$ and *overpopulated* if $u_i'(P_i^*) < 0$. Tabuchi (1986) says that if there are two or more underpopulated regions, then the equilibrium is not stable. On the other hand, if all regions are overpopulated, then each resident's utility decreases if he/she migrates hence the equilibrium is stable. If region 1 is underpopulated and (3)—(5) hold, then $u_i'(P_i^*) < 0$ for $i = 2, \ldots, n$ by (3), and the residents of regions $2, \ldots, n$ may prefer migration to region 1. In the case that each region of $2, \ldots, n$ has one resident migrating to region 1, the utility of a new comer of region 1 increases by approximately $(n-1)u_1'(P_1^*)$. However, the utility of a remained resident in region i increases approximately by $-u_i'(P_i^*) \geq (n-1)u_1'(P_1^*)$, where the inequality is implied by (3). If the inequality holds strictly for i then no resident of region i prefers moving and the equilibrium becomes stable. Finally, (5) excludes the case that $u_1'(P_1^*) = u_2'(P_2^*) = 0$, in which residents of regions 1 and 2 may migrate free to each other without changing utilities. What we will do in Section 3 is to theoretically prove that (3) – (5) actually form a set of conditions which is sufficient to ensure the stability of an interior equilibrium $\mathbf{P}^*$, and further generalize the result to the case of corner equilibrium.

2 The existence of an equilibrium

The existence of an equilibrium is extensively discussed in the field of local public good economies. For example, Nechyba (1997), Konishi (1996) and Bewley (1981). Since public good and tax are considered in their models, a resident's utility function depends on at least (i) the region i where the resident lives; (ii) the public goods distribution and (iii) the amount of private good consumption. To ensure the existence, some standard conditions for the utility functions are required, for example, the continuity, the monotony and quasi-convexity in the public goods and private goods.

Our migration model (1) is similar but different, because we assume that a resident's utility is only related to the region (where the resident lives) and the population in the region. Our model seems to be simpler, hence we can expect a conclusion with fewer conditions. In fact, this section shows, to ensure the existence of a migration equilibrium, we only need the continuity of functions $u_i(\cdot)$ for all $i = 1, \ldots, n$.

Theorem 1 *If utility function $u_i(\cdot)$ is continuous for any region i, then there exists at least one equilibrium $\mathbf{P}^* = \langle P_1^*, \ldots, P_n^* \rangle$.*

It is important to note that our conclusion does not affirm the existence of an interior equilibrium. In fact, it may happen that there are only some corner equilibria. A corner equilibrium is important in the study of core-periphery structure of regions' agglomeration (Fujita, Krugman and Venables, 1999).

Also, the theorem says nothing about the uniqueness of an equilibrium. For example, in the case of two regions shown in Figure 1, there are three equilibria, one interior and two corners.

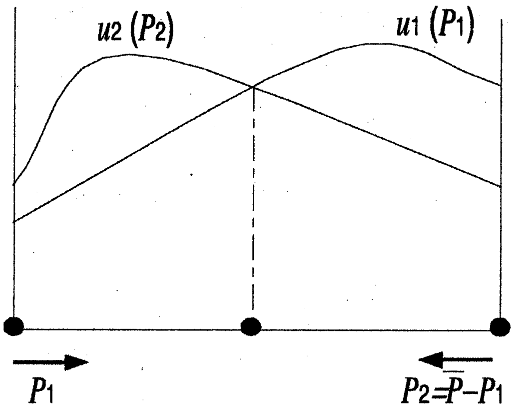

Figure 1: Three equilibria

3 Stability Analysis

This section provides a set of convenient conditions to ensure the stability of a migration equilibrium, which may be interior or corner. Specifically, we show that an interior migration equilibrium is stable if (3)—(5) holds at the equilibrium, and a similar result holds for a corner equilibrium. To the author's knowledge, this is the first result concerning a sufficient condition for a stable migration equilibrium in a general case of n regions, although the model is basic and simple.

Supposes that $u_i(\cdot)$ is differentiable and hence continuous. Therefore there always exists an equilibrium $\mathbf{P}^* = \langle P_1^*, \ldots, P_n^* \rangle$ by Theorem 1. Furthermore we suppose that $u_i(\cdot)$ satisfies the so-called Lipschitz condition (P. 306 of Takayama (1985)).

We first consider the case of interior equilibrium. Since $P_i^* > 0$ holds for any region i, we can limit our concern to a neighbor of $\mathbf{P}^*$ such that $P_i(t) > 0$, where t is the considered time. Let Δt be a small time period such that $P_i(t + \Delta) > 0$ for all region i, and denote $P_{ji}(t, t + \Delta t)$ as the population moving from region $j \neq i$ to region i during the time period from t to $t + \Delta t$. A positive value of $P_{ji}(t, t + \Delta t)$ means that some residents move from j to i, and a negative value means that some residents move from i to j, therefore $P_{ji}(t, t + \Delta t) = -P_{ij}(t, t + \Delta t)$. Since residents move from a low-utility region to a high-utility region, we suppose that the migration population is proportional to the utility discrepancy. Then for sufficiently small Δt, it holds that

$$P_{ji}(t, t + \Delta t) = \kappa_{ji} \Delta t [u_i(P_i(t)) - u_j(P_j(t))], \qquad \text{for } i, j = 1, \ldots, n, i \neq j, \tag{6}$$

where κ_{ji} is the so-called speed of adjustment (Metzler, 1945), which measures the speed with which residents migrate between regions i and j corresponding to a given utility discrepancy between regions i and j. Since $P_{ji}(t, t+\Delta t) = -P_{ij}(t, t+\Delta t)$, it holds that $\kappa_{ji} = \kappa_{ij}$. Since all the features of a region is included in its utility function and all residents are homogeneous, residents' decisions to migrate only depend on the utility discrepancy. Therefore, independent of the names of regions, residents in region i respond to the utility discrepancy of any other region with the same speed of adjustment. That is $\kappa_{ij_1} = \kappa_{ij_2}$ for all $j_1, j_2 \neq i$. Therefore, $\kappa_{ij} = \kappa$ for all i and $j \neq i$. We can simply normalize residents' utility function so that $\kappa = 1$. So in the following arguments, we always let $\kappa_{ij} = 1$.

For convenience, define $P_{ii}(t, t+\Delta t) = 0$ for any i, t and Δt. Then (6) holds for all i and j. Hence

$$P_i(t+\Delta t) = P_i(t) + \sum_{j=1}^{n} P_{ji}(t, t+\Delta t) = P_i(t) + \Delta t \sum_{j=1}^{n} [u_i(P_i(t)) - u_j(P_j(t))],$$

and

$$\frac{dP_i(t)}{dt} = \lim_{\Delta t \to 0} \frac{P_i(t+\Delta t) - P_i(t)}{\Delta t} = \sum_{j=1}^{n} [u_i(P_i(t)) - u_j(P_j(t))],$$

which leads to dynamics (2). Since we have supposed that $u_i(\cdot)$ is differentiable and satisfies the Lipschitz condition, by extending the domain of definition of $u_i(\cdot)$ from $[0, \overline{P}]$ to $(-\infty, \infty)$ suitably, we know that there is a unique and continuous solution of (2) with any initial value around $\mathbf{P}^*$ (Theorem 3.B.1 and its Remarks of Takayama (1985)).

Summing up all the equations of (2), we find $\sum_{i=1}^{n} dP_i(t)/dt = 0$, which is consistent with the fact that $\sum_{i=1}^{n} P_i(t) = \overline{P}$ is a constant. Therefore we can revise (2) as follows.

$$\frac{dP_i}{dt} = (n-1)u_i(P_i) - \sum_{\substack{j=1 \\ j \neq i}}^{n-1} u_j(P_j) - u_n(\overline{P} - \sum_{j=1}^{n-1} P_j), \quad i = 1, \ldots, n-1, \tag{7}$$

Following Tabuchi (1986), we denote $\mu_i = u_i'(P_i^*)$ and denote the characteristic polynomial of matrix A as $\Psi_A(\lambda) = \det[A(\lambda)]$, where $A = [a_{ij}]_{(n-1)\times(n-1)}$, $A(\lambda) = [a(\lambda)_{ij}]_{(n-1)\times(n-1)}$, and

$$a_{ij} = \begin{cases} (n-1)\mu_i + \mu_n, & \text{for } i = j \\ -\mu_j + \mu_n, & \text{for } i \neq j \end{cases}, \tag{8}$$

$$a(\lambda)_{ij} = \begin{cases} (n-1)\mu_i + \mu_n - \lambda, & \text{for } i = j \\ -\mu_j + \mu_n, & \text{for } i \neq j \end{cases}.$$

After renaming the regions if necessary, we suppose that $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_n$. There are $n-1$ roots (possibly multiple) of the characteristic equation $\Psi_A(\lambda) = 0$, which are called eigenvalues. It is known that equilibrium $\mathbf{P}^*$ of (7) is stable if all the real parts of the eigenvalues of A are negative (Gantmacher, 1960), and is unstable if there is at least one eigenvalue is with positive real part. We can show that all the eigenvalues of A are real numbers, which are all negative if and only if (3)—(5) hold. Furthermore, from Tabuchi (1986), we know that if (3) is violated then there is at least one positive eigenvalue and the dynamics is unstable. Therefore, we affirm

Theorem 2 *An interior equilibrium is stable if* (3)–(5) *hold at the equilibrium; if* (3) *is violated, then the equilibrium is unstable.*

Remark We can further show that if (3) holds but (4) or (5) is violated, then 0 is an eigenvalue of A. Since the differential equation theory does not completely disclose the critical case with a zero eigenvalue, we are not sure whether the dynamics is stable or unstable in this case. However, almost all economics researches (Okabe, 1980; Nakajima, 1995) omit the discussion of this case and think that having a zero eigenvalue implies that the dynamics is unstable. In this sense, (3) – (5) become necessary and sufficient conditions for the stability.

Next we turn to the case of corner equilibrium. In migration study, it is reasonable to suppose that the inequality in (1) holds strictly[1] . That is,

$$u^* > u_j(0) \qquad \text{for all } j \text{ such that } P_j^* = 0. \tag{9}$$

By (1) and (9), we can rename the regions so that

$$\begin{cases} P_i^* \neq 0 \ \text{ and } \ u_i(P_i^*) = u^*, & \text{for } i = 1, \dots, n_1; \\ P_j^* = 0 \ \text{ and } \ u_j(0) < u^*, & \text{for } j = n_1 + 1, \dots, n; \\ u_1'(P_1^*) \geq \cdots \geq u_i'(P_i^*) \geq \cdots \geq u_{n_1}'(P_{n_1}^*). \end{cases} \tag{10}$$

Consider an initial population distribution $\mathbf{P}(t) = \langle P_1(t), \dots, P_n(t)\rangle$. If $P_j(t) = 0$, then none moves from region j to other regions but some residents may migrate into region j. Therefore (6) should be revised as follows

$$P_{ji}(t, t+\Delta t) = \begin{cases} \Delta t[u_i(P_i(t)) - u_j(P_j(t))], & \text{if } P_i(t) > 0, P_j(t) > 0, \\ \Delta t \min\{0, u_i(P_i(t)) - u_j(P_j(t))\}, & \text{if } P_i(t) > 0, P_j(t) = 0, \\ \Delta t \max\{0, u_i(P_i(t)) - u_j(P_j(t))\}, & \text{if } P_i(t) = 0, P_j(t) > 0, \\ 0, & \text{if } P_i(t) = P_j(t) = 0. \end{cases}$$

Hence the dynamics for a corner equilibrium takes the following form: for i=1, ..., n,

$$\frac{dP_i(t)}{dt} = \begin{cases} \displaystyle\sum_{j=1,\dots,n|P_j(t)>0} [u_i(P_i(t)) - u_j(P_j(t))] & \text{if } P_i(t) > 0, \\ \qquad + \displaystyle\sum_{j=1,\dots,n|P_j(t)=0} \min\{0, u_i(P_i(t)) - u_j(P_j(t))\}, & \\ \displaystyle\sum_{j=1,\dots,n|P_j(t)>0} \max\{0, u_i(P_i(t)) - u_j(P_j(t))\}, & \text{if } P_i(t) = 0. \end{cases} \tag{11}$$

Note that the above equations imply that $\sum_{i=1}^n dP_i(t)/dt = 0$ for all t, which is consistent with the assumption of constant population $\overline{P}$.

Remember that $u_i(\cdot)$ satisfies the Lipschitz condition, by extending the domain of definition of $u_i(\cdot)$ from $[0, \overline{P}]$ to $(-\infty, \infty)$ suitably, we know that there is a unique and continuous solution of (11) with any initial condition around $\mathbf{P}^*$. Our stability conclusion for a corner equilibrium is stated in the following theorem. The result is also very intuitive. Starting from initial distribution around $\mathbf{P}^*$, the residents in regions $n_1 + 1, \dots, n$ will migrate to regions $1, \dots, n_1$ because the utilities in regions $n_1 + 1, \dots, n$ are lower. Conditions (12) – (14) are similar to (3) – (5), which ensure that the population distribution $\langle P_1^*, \dots, P_{n_1}^*\rangle$ will be stable if there are only n_1 regions totally.

[1] An equilibrium may be either stable or unstable if $u^* = u_j(0)$ instead of (9) holds for a region j. The mathematical analysis for this case is very troublesome and this case seems to be unlikely in a migration equilibrium of real life. Therefore almost all economics researchers (for example, Fujita, Krugman and Veneables, 1999) omit the discussion of this case, even when $n = 2$.

Theorem 3 *If $n_1 = 1$, then equilibrium* $\mathbf{P}^* = \langle P_1^*, \ldots, P_n^* \rangle$ *satisfying* (10) *is always stable. If $n_1 \geq 2$, then equilibrium* $\mathbf{P}^*$ *satisfying* (10) *is stable if*

$$(n_1 - 1)u_1'(P_1^*) + u_i'(P_i^*) \leq 0, \quad \forall i = 2, \ldots, n_1, \tag{12}$$

$$\textit{the inequality of } (12) \textit{ holds strictly for at least one } i = 2, \ldots, n_1, \tag{13}$$

$$\textit{if } u_1'(P_1^*) = 0, \ \textit{then } u_2'(P_2^*) < 0. \tag{14}$$

If (12) *is violated, then* $\mathbf{P}^*$ *is unstable.*

Similar to the remark after Theorem 2, if we treat the critical case of zero eigenvalue as unstable, then (12) – (14) become necessary and sufficient conditions.

From Theorems 2 and 3, we know that in the case of Figure 1, two corner equilibria are stable and the interior one is not. Although Theorem 1 ensures that at least one equilibrium, it does not ensure the existence of a stable equilibrium. In the case that $n = 2$, $u_1(P_1) = u_2(\overline{P} - P_1)$, each residential distribution forms an equilibrium but none is (asymptotically) stable.

4 Conclusions

This paper investigates a deterministic migration model among n regions in the case of zero natural growth. First, it is shown that the continuity of residents' utility functions ensures the existence of a migration equilibrium. Then sufficient conditions are given for the stability of such an equilibrium.

Although the model used in this paper is basic and simple, but the discussion here is rigorous. To the author's knowledge, no other paper shows the existence of a migration equilibrium explicitly and none has given any sufficient condition for the stability of a migration equilibrium in the case of n regions before.

It is known that the recent economic geography literature is strongly related to stability analysis of a migration equilibrium, the results of this paper is expected to be applicable in the research of economic geography.

Finally, the results of this paper are valuable to be extended to some more realistic and more complex models. First, our results depend strongly on the assumption that the utility function $u_i(P_i)$ is only related to the population size of region i. A general function may be related to the population sizes of other regions. The stability conditions derived in this paper do not hold in this general model. Second, our model is deterministic but there are many good stochastic models (Tabuchi, 1986; Weidlich and Haag, 1988; O'Connell, 1997). The stability research on some stochastic models may reveal more important facts.

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