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A Unified Approximation of Optimal Shutdown Schedules Based on a Brownian Motion Process

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1 Introduction

Since *the Energy Star Project* by U.S. government, the power management for computer systems has received considerable attention all over the world. As a computer consists of a number of electric components and devices, the problems of power management to reduce the energy consumption have to be discussed in terms of each component unit such as IC chip [1], microprocessor [2], CPU, disk drive, display and so on. Also, since the measurement technique to estimate the electrical power consumed in each component has been developed recently [3, 4], some interesting attempts have been made to reduce the electrical power in the real computer operation [5, 6]. In general, the power management should be carried out at each level of the hierarchical computer design process; circuit level, layout level, logic level, behavioral level, architectural level, *etc.* In particular, *the system level* power management techniques have emerged as one of the most applicable design methodologies in practice, because they do not assume the development of new low-power devices. For the details on the system level power management techniques, see [7, 8].

The dynamic power management, as it is generically known, can provide a control scheme that dynamically reconfigures an electric system to provide the requested services and performance levels with a minimum number of active components or a minimum load on such components [7, 8]. The design methods will be useful for the operating system and the control system of peripheral devices. Especially, the dynamic power management plays an important role to achieve energy efficiency in the operating system, since the application software programs are monitored and controlled by it. It is, however, known that typical operating systems like UNIX, WindowsOS and MacOS were not designed originally with energy efficiency in mind.

The most simple way to establish the power reduction in the current operating systems is to add the ability to selectively shutdown the peripherals which are not currently being used. In fact, this method called the *shutdown approach* or the *shutdown policy* has been applied to the power saving in the hard disk [9] as well as VLSI circuits system [7, 10, 11]. The typical example for the dynamic power management is the mobile computing with limited capacity of battery [12, 13, 14]. Unlike a desktop personal computer, the electrical power in a mobile computer must be carefully rationed among all of the components and peripherals. For such systems, the shutdown approach will be useful to reduce the electrical power consumed in the operating period.

In this paper, we present a stochastic model for computer systems which employ the shutdown approach. We consider two models proposed in Okamura *et al.* [15, 16] and develop a unified approach to integrate these models theoretically. Based on the general arrival assumption on tasks, we formulate the expected electrical power consumption per unit time in steady state and propose a unified approximate method by applying the so-called diffusion approximation.

2 Dynamic Power Management

Consider a stochastic model for the dynamic power management [17]. In the typical dynamic power management, internal states of the underlying computer system are classified into three states (see Fig. 1).

Busy: The *busy state* means that the system is active, *i.e.*, the system is processing tasks requested. From the viewpoints of operating system, the busy state can be regarded as the state where the operating system serves the tasks requested by users.

Idle: In the *idle state*, the system waits for receiving an additional request. In the operating system, it means that the system is processing the light tasks on memory.

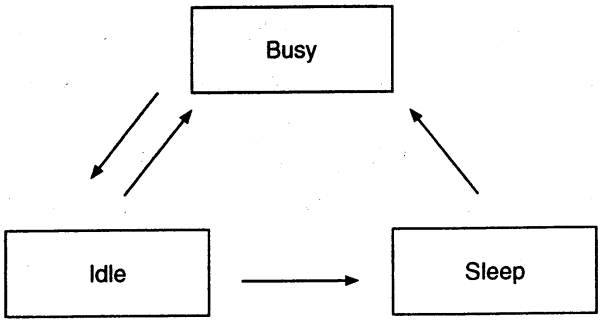

Figure 1: Configuration of the dynamic power management.

Table 1: An example of delayed times at a CPU device.

from		to	delayed time
busy	$\rightarrow$	idle	~ 10 (μs)
idle	$\rightarrow$	busy	~ 10 (μs)
idle	$\rightarrow$	sleep	~ 90 (μs)
sleep	$\rightarrow$	busy	~ 160 (ms)

Inactive (Sleep): The *inactive state* is usually called the *sleep state*. In personal computers, the inactive state is referred as *sleep state* or *hibernation state*

The electrical power consumption per unit time in each state; busy, idle or sleep state, depends on the processing performance. On the other hand, it is usually reported that the delayed time occurs at the transition of state. Table 1 presents an example of delayed times at a CPU device [17]. From the physical principles of electricity, it can be observed that the system may waste higher electrical power instantaneously at the transition time from low-power states to high-power states. This instantaneous electrical power is generally called the *wake up power*. The nature of higher wake up power makes the optimal power-saving design difficult.

Based on these characterizations for power consumptions, we construct a stochastic dynamic power management model for computer systems. To simplify the mathematical treatments, we make three assumptions in the stochastic shutdown model.

Assumption A: The electrical power consumption per unit time in both the busy state and the idle state is equivalent.

Assumption B: If the system transfers from a high-power state to a low-power state, it does not take delayed times, *i.e.*, the state transition can be completed in a moment.

Assumption C: The wake up power is not wasted when the system transfers from the idle state to the busy state. The wake up power is wasted uniformly during the delayed time while the system transfers from the sleep state to the busy state.

Assumption A indicates that the following relationship has to hold in terms of power consumption;

$$(\text{Busy}) = (\text{Idle}) > (\text{Sleep}).$$

One of the most important factors in the design for shutdown schedules is the trade-off between the amount of electrical power saved by shutdown and wasted by waking up from the sleep state. Since Assumption A is related with the electrical power consumptions in both busy state and idle state, it does not affect the trade-off relationship as well as the design of shutdown schedule. Assumption B is related with the delayed time when the system transfers from high-power states to low-power states. In Benini and De Micheli [17], it is pointed out that the delayed times to transfer from high-power states to low-power states are much smaller than the other delayed times. Furthermore, unlike the wake up power, the instantaneous electrical power consumption caused by transitions from high-power states to low-power states can be negligible. Under this fact, the effect of Assumption B for the shutdown scheduling seems to be sufficiently small. Assumption C concerns the wake up power. It is known that the behavior of wake up power in practice has the bursty. Also, as the delayed time is shorter, the more often and higher wake up power will be needed, namely, the wake up power is inversely proportional to the delayed time. Hence, it can be seen that

$$\text{(Wake up power wasted per unit time)} \times \text{(Delayed time)}$$

is approximately constant, that is to say, the amount of the wake up power wasted during the delayed time can be estimated by the mean wake up power wasted per unit time during the delayed time, regardless of the electrical characteristics and the behavior during the delayed time.

For the stochastic model above, the following shutdown schedule is performed to save the electrical power consumption.

Shutdown policy: If the system has spent a certain constant time period in waiting for a request, the system can transfer from the idle state to the sleep state automatically. When the system is in the sleep state, the system wakes up and goes to the busy state if an additional request occurs. The sojourn time length in the idle state is said the *shutdown timing* or the *shutdown schedule.*

The problem is to derive the optimal shutdown schedule which maximizes the effect of electrical power saving.

Okamura *et al.* [15, 16] have considered the stochastic shutdown models under the additional assumptions;

Assumption D [15]: If other requests arrive at the system in the busy period, they can be canceled immediately.

Assumption D' [16]: If other requests arrive at the system in the busy, they can be stored in the buffer and can be processed under the First-Come First-Service (FCFS) discipline.

Assumption E: Requests arrive as a sequence of independently and identically distributed random variables.

If the system has a finite buffer whose capacity is K (≥ 1), the assumptions D and D' can be regarded as special cases such as $K = 1$ and $K \to \infty$. In other words, the results in Okamura *et al.* [15, 16] can be integrated by considering the stochastic shutdown model with a finite buffer. The purpose of this paper is to establish a common approximation for two shutdown models in [15, 16].

3 Model Description

In this paper, the following notation is used:

$\{X(t);\ t \geq 0\}$: cumulative number of arrival requests at time t (renewal process)

S_k: processing time for the k-th task (random variable)

τ (> 0): delayed time to transfer from the idle state to the busy state

$s + \tau$ (> 0): delayed time to transfer from the sleep state to the busy state

t_0: shutdown schedule (decision variable; $0 \leq t_0 < \infty$)

K (> 1): capacity of buffer

P_1 (> 0): electrical power consumption per unit time in the idle and busy states,

P_2 (> 0): wake up power per unit time during the delayed time period ($P_2 > P_1$).

Suppose that the occurrence of requests follows a renewal process with an inter-arrival time distribution $F(t)$, which has mean; $1/\lambda$ (> 0) and variance; σ_a^2 (> 0). Let S_k denote the processing time for the k-th task required, and S_k for $k = 1, 2, \cdots$ are the non-negative i.i.d. (independently and identically distributed) random variables having an absolutely continuous probability distribution function $G(t)$ with finite mean $1/\mu$ (> 0) and variance σ_s^2 (> 0). When a request occurs in the sleep state, the system wakes up and goes to the busy state after elapsing the delayed time $s + \tau$. If the buffer is not full, other requests are stored in the buffer and are processed under the FCFS discipline. Otherwise, *i.e.*, if the

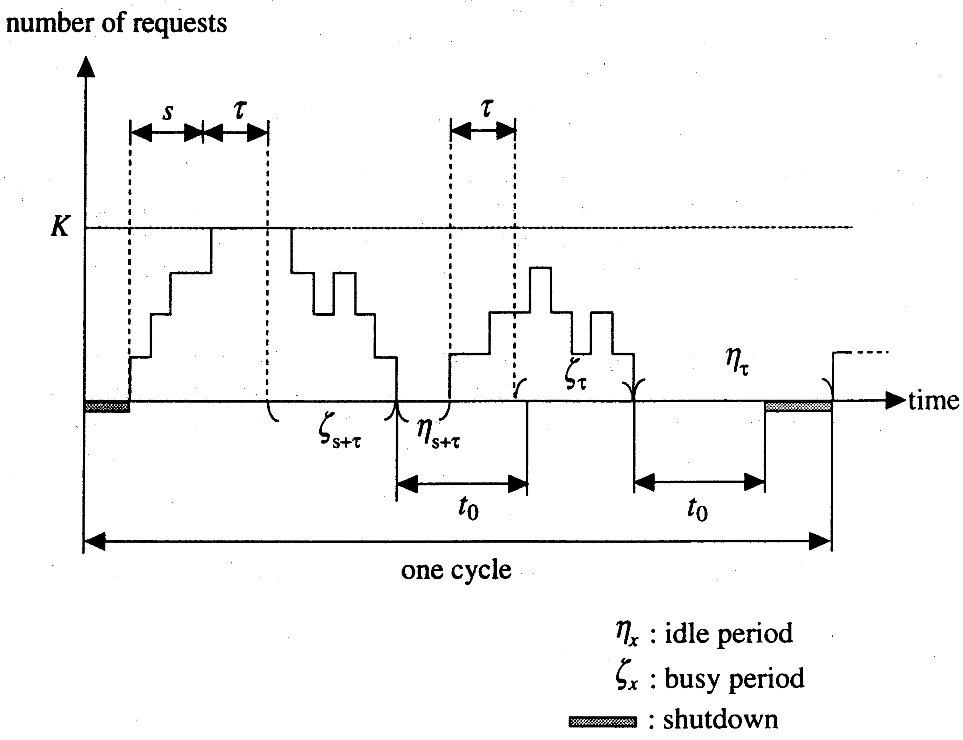

Figure 2: Possible realization of the shutdown model with a finite buffer.

buffer is full, other requests are canceled. When the tasks stored in the buffer have been completed, the system transfers to the idle state. If a new request is received before the amount of successive sojourn time in the idle state becomes t_0, then the system has to start processing the task after elapsing the delayed time τ. Otherwise, *i.e.*, if a new request does not arrive before the amount of successive sojourn time in the idle state becomes t_0, the system goes to the sleep state.

Figure 2 illustrates the possible realization of the shutdown model with a finite buffer. Based on Assumptions A and C, it is assumed that the electrical power consumption per unit time is P_1 in the busy and idle states, and that the electrical power consumption per unit time during the delayed time period is P_2. To simplify the analysis, the electrical power consumption in the sleep state is assumed to be zero.

4 Optimal Shutdown Schedule

4.1 Formulation

Consider the expected electrical power consumption per unit time in the steady state as the power-saving measure. The formal definition of the expected electrical power consumption per unit time in the steady state is given by

$$V(t_0) = \lim_{t \to \infty} \frac{\text{E[amount of electrical power consumed in } [0,t)]}{t}. \tag{1}$$

Define the time period from the end of sleep state to the next one as one cycle. Let $\zeta_v^{(K)}$ and $\eta_v^{(K)}$ denote a processing period (a busy period) and an idle period during one cycle, respectively, provided that the capacity of buffer is K and that the delayed time is v. The probability distribution function of an idle period is $I^{(K)}(\cdot|v)$. It can be seen that the probabilities that the system executes shutdown in the first idle period and in the second or later period become $I^{(K)}(t_0|s+\tau)$ and $I^{(K)}(t_0|\tau)$, respectively. Thus, the expected number of transitions from the idle state to the busy state is thus given by

$$\mathrm{E}[L_{s,\tau}^{(K)}(t_0)] = I^{(K)}(t_0|s+\tau)/\overline{I}^{(K)}(t_0|\tau), \tag{2}$$

where $\overline{I}^{(K)}(\cdot|t) = 1 - I^{(K)}(\cdot|t)$. Furthermore, the following relationship between the busy and idle periods holds;

$$\rho_v^{(K)} = \frac{\mathrm{E}[\zeta_v^{(K)}]}{\mathrm{E}[\eta_v^{(K)}] + v + \mathrm{E}[\zeta_v^{(K)}]}, \tag{3}$$

where $\rho_v^{(K)}$ is the traffic intensity in the $GI/GI/1/K$ queueing system with a delayed time v. Using the loss probability $q_v^{(K)}(K)$, the traffic intensity is given by

$$\rho_v^{(K)} = \{1 - q_v^{(K)}(K)\}\rho, \tag{4}$$

where $\rho = \lambda/\mu$. On the other hand, from the well-known Miyazawa's intensity conservation law [18, 19], we have the following relationships;

$$1 - p_v^{(K)}(0) = \rho\{1 - q_v^{(K)}(K)\} + \lambda v q_v^{(K)}(0) \tag{5}$$

and

$$p_v^{(K)}(0) = \lambda \mathrm{E}[\eta_v^{(K)}] q_v^{(K)}(0), \tag{6}$$

where $p_v^{(K)}(0)$ and $q_v^{(K)}(0)$ represent the steady-state probabilities that the buffer is empty at arbitrary time and at arrival points, respectively. From Equations (3)–(6), we obtain the expected time length of one cycle as

$$\begin{aligned} T^{(K)}(t_0) &= \frac{1}{1-\rho_{s+\tau}^{(K)}}\left\{s + \tau + \mathrm{E}[\eta_{s+\tau}^{(K)}]\right\} + \frac{1}{1-\rho_\tau^{(K)}}\left\{\tau + \mathrm{E}[\eta_\tau^{(K)}]\right\}\mathrm{E}[L_{s,\tau}^{(K)}(t_0)] \\ &= \frac{1}{\lambda q_{s+\tau}^{(K)}(0)} + \frac{1}{\lambda q_\tau^{(K)}(0)}\mathrm{E}[L_{s,\tau}^{(K)}(t_0)]. \end{aligned} \tag{7}$$

Similarly, the expected power consumed during one cycle is given by

$$\begin{aligned} C^{(K)}(t_0) =& (P_2 - P_1)s + P_1 T^{(K)}(t_0) + P_1\Big\{\mathrm{E}[\eta_{s+\tau}^{(K)} \wedge t_0] - \mathrm{E}[\eta_{s+\tau}^{(K)}] \\ &+ (\mathrm{E}[\eta_\tau^{(K)} \wedge t_0] - \mathrm{E}[\eta_\tau^{(K)}])\mathrm{E}[L_{s,\tau}^{(K)}(t_0)]\Big\}, \end{aligned} \tag{8}$$

where, in general, $a \wedge b = \min(a, b)$. We therefore derive the expected power consumption per unit time in the steady state;

$$V^{(K)}(t_0) = C^{(K)}(t_0)/T^{(K)}(t_0), \tag{9}$$

so that the problem is to find the optimal shutdown timing t_0^* minimizing $V(t_0)$.

Remark 1: If $K = 1$, then this model is reduced to the renewal model in Okamura *et al.* [15]. On the other hand, if $K \to \infty$, then this model is consistent with the queueing model in the literature [16].

4.2 Poisson Arrival Case

Let $W^{(K)}(\cdot|v)$ denote a probability distribution function of the time length until the buffer becomes empty at an arrival point in the steady state provided that the delayed time is v. It can be seen that the distribution of an idle period is given by

$$\overline{I}^{(K)}(x|v) = \frac{\int_0^\infty \overline{F}(x+u)dW^{(K)}(u|v)}{\int_0^\infty \overline{F}(u)dW^{(K)}(u|v)}, \tag{10}$$

where $\overline{F}(\cdot) = 1 - F(\cdot)$. Since $\overline{F}(x+y) = \overline{F}(x)\overline{F}(y)$ in the case of Poisson arrival stream, we can derive the following result for the optimal shutdown schedule.

Theorem 1: Suppose that $\rho_v^{(K)} < 1$. If $P_1 - \lambda(P_2 - P_1)s \geq 0$, then the optimal shutdown schedule is $t_0^* = 0$. Otherwise, *i.e.* if $P_1 - \lambda(P_2 - P_1)s < 0$, then $t_0^* \to \infty$.

Figure 3 summarizes Theorem 1. From this figure, it is found that the simple on-off switching policy is optimal, *i.e.* it is optimal to shutdown the system at the beginning of idle state or not to do at all. It is interesting that the capacity of buffer does not affect the shutdown decision in the Poisson arrival case.

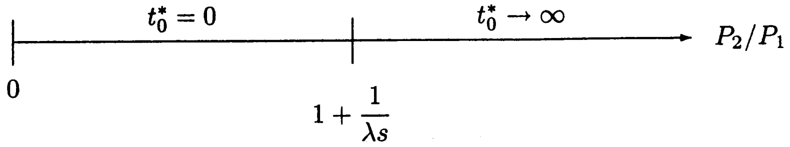

Figure 3: Optimal shutdown schedule.

5 A Unified Approximate Method

In the general arrival case, it is not easy to express the closed form of the stationary distribution $W^{(K)}(x|v)$. We thus propose an approximate formulation for $W^{(K)}(x|v)$ based on the diffusion approximation.

Let $\{Y_v^{(K)}(t); t \geq 0\}$ denote the time until the buffer becomes empty at arbitrary time t. More specifically, we suppose that drift parameter; μ_w and diffusion parameter; σ_w, where

$$\mu_w = \lambda(1 - q_0^{(K)}) - \mu, \quad (< 0) \tag{11}$$

and

$$\sigma_w^2 = \sigma_a^2(1 - q_0^{(K)}) + \sigma_s^2, \quad (> 0). \tag{12}$$

If we treat an ordinary $GI/G/1/K$ queueing system, *i.e.*, a queueing system without a delayed time, then it is well known that the queueing process is approximated by a reflected Brownian motion with drift and diffusion parameters; μ_w and σ_w. However, it is clean that the approximation based on the reflected Brownian motion can not be applied since the queueing process with a delayed time has jumps under a certain condition. We therefore propose an alternative approximation based on a diffusion process having the following properties;

- The diffusion process can take negative values.
- A Poisson arrival occurs with the rate λ.
- The diffusion process has a jump to the delayed time v with the rate λ if the process takes a negative value (see fig. 4).

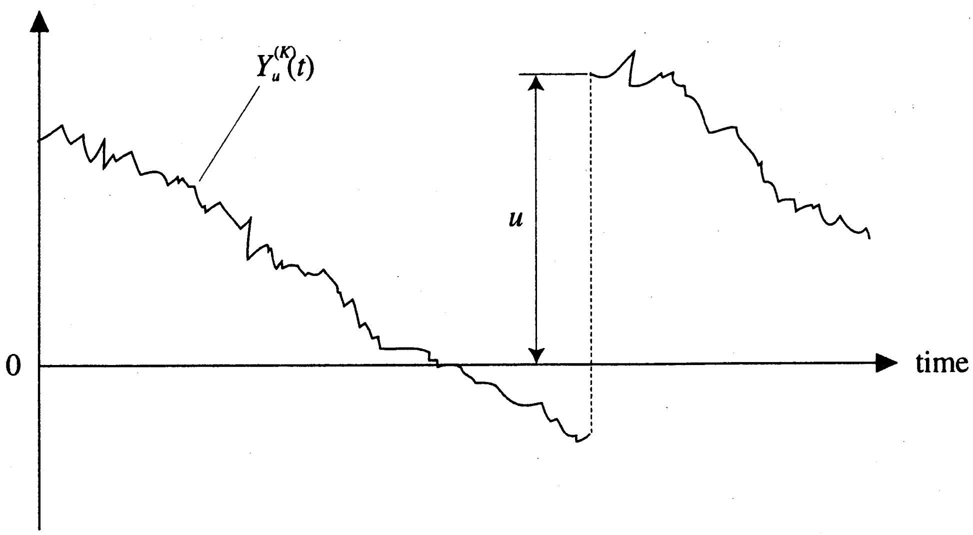

Figure 4: Configuration of the diffusion process with jumps.

Define the time period from the occurrence time of a jump to the next one as one cycle. Let N denote the number of arrivals during one cycle, and N_x the number of arrivals while the process is above the level x during one cycle. Further we define the difference between the processes at the n-th arrival and at the $(n+1)$-st arrival during one cycle as $\tilde{Y}_v^{(K)}$. The probability density function of $\tilde{Y}_v^{(K)}$ can be derived by

$$f(x) = \frac{1}{\xi}\exp\left\{\frac{\mu_w}{\sigma_w^2}x\right\}\exp\left\{-\frac{\xi}{\sigma_w^2}|x|\right\}, \tag{13}$$

where $\xi = \sqrt{\mu_w^2 + 2\lambda\sigma_w^2}$. Using the probability density function $f(x)$, the expected number of arrivals during one cycle, provided that the delayed time is v, is given by

$$\mathrm{E}[N|v] = 1 + \frac{\lambda v}{|\mu_w|} + \frac{(\lambda/|\mu_w|)\int_0^\infty uf(u)du + \int_0^\infty f(u)du}{1 - \int_0^\infty f(u)du}. \tag{14}$$

Let $g_x(v)$ denote the probability that the diffusion process is x at the first passage time to the level x or the level 0. The probability $g_x(v)$ can be derived from the property of the Brownian motion as follows.

$$g_x(v) = \begin{cases} \frac{1-\phi(v)}{1-\phi(x)} & \text{for } 0 \le v \le x, \\ 1 & \text{for } x < v, \end{cases} \tag{15}$$

where $\phi(v) = \exp\{-(2\mu_w v)/\sigma_w^2\}$. By using $g_x(v)$, the expected number of arrivals while the process is above the level x during one cycle is also given by

$$\begin{aligned}\mathrm{E}[N_x|v] =& \frac{\lambda(v-x)}{|\mu_w|}U(v-x) + g_x(v)\left\{\int_0^\infty (1+\mathrm{E}[N_x|x+y])f(y)dy + \int_{-x}^0 \mathrm{E}[N_x|x+y]f(y)dy\right\} \\ &+ (1-g_x(v))\left\{\int_x^\infty (1+\mathrm{E}[N_x|y])f(y)dy + \int_0^x \mathrm{E}[N_x|y]f(y)dy\right\},\end{aligned} \tag{16}$$

where $U(\cdot)$ is a step function, that is,

$$U(x) = \begin{cases} 0 & \text{for } x < 0, \\ 1 & \text{for } x \ge 0. \end{cases} \tag{17}$$

Let $\psi^{(K)}(x|v)$ denote the stationary distribution of the diffusion process with jumps at arrival points. The approximate forms of $q_v^{(K)}(0)$ and $W^{(K)}(x|v)$ are

$$q_v^{(K)}(0) \approx \psi^{(K)}(0|v) = \frac{1}{\mathrm{E}[N|v]} \tag{18}$$

and

$$\overline{W}^{(K)}(x|v) \approx \frac{\overline{\psi}^{(K)}(x|v)}{\overline{\psi}^{(K)}(0|v)} = \frac{\mathrm{E}[N_x|v]}{\mathrm{E}[N|v]-1}, \tag{19}$$

respectively. Therefore, if $K = 1$, then the approximate formulations can be obtained by Equations (18), (19) and $q_v^{(1)}(1) = 1 - q_v^{(1)}(0)$. Similarly, if $K \to \infty$, then the approximate formulations can be obtained by Equations (18), (19) and $q_v^{(\infty)}(\infty) = 0$.

6 Concluding Remarks

In this paper, we have considered the stochastic shutdown model with a finite buffer. Taking a finite buffer into consideration, we have integrated two models in Okamura *et al.* [15, 16]. Furthermore, based on the stochastic shutdown model with a finite buffer, we have proposed a unified approximation method. In the future research, we will investigate the accuracy of approximation method proposed in this paper.

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