1. New Methods of Dielectric Measurement in the Centimeter Wave Region*

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INTRODUCTION

In the previous paper,[1] we have proposed new methods of dielectric measurement using wave guide in the centimeter wave region. We have started from Maxwell's equations and obtained the exact explicit expressions of $\varepsilon^* = \varepsilon' - j\varepsilon''$ under the conditions that the tangential components of the electric and the magnetic vectors are continuous at two boundary surfaces of the sample and the air columns, that is, the field impedances which are the ratios of these vectors are continuous.

Roberts and von Hippel[2] utilized the graphical solutions of a transcendental function which contains ε^* implicitly, but our theory includes as a special case the method of Surber and Crouch[3] which was obtained circuit-theoretically, and possesses the merits that both the sample deformation and the frequency variation which are often used in approximate measurements are unnecessary.

In our theory, the end of the sample is not necessarily open or short, so several methods are produced by adequate combinations of two positions of the terminating plate. In the present paper, concerning especially three methods among those which are in principle proposed in the previous paper the explicit expressions of ε^* are transformed to the more convenient forms which are expressed in terms of measured quantities alone, and the experiments performed on cetyl alcohol ($C_{15}H_{31}CH_2OH$) which have shown the coincidence with the theory are described.

These methods can in principle be applied to measure any ε^*, no matter whether the loss be large or small, but in case of the sample of too small loss, the effect of the losses of the guide wall, the terminating plate and others must be considered. Accordingly in the present paper, the fundamental equation including these losses is derived, and by use of this equation, the effect of the loss of the terminating plate on the sample length and the electrical position of the terminating plate is discussed.

1. THE FUNDAMENTAL EQUATION INCLUDING THE LOSSES OF THE GUIDE WALL, THE TERMINATING PLATE AND OTHERS

As shown in Fig. 1 when there are two air columns 1 and 3 on both sides of the sample column and the end of the column 3 is short-circuited, the electric and

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the magnetic fields in each column are expressed with the superposition of the incident wave and the multiply-reflected waves, and concerning one mode, they are expressed by the real part of $Ae^{j\omega t+\gamma x}$ (A is the complex amplitude[4]).

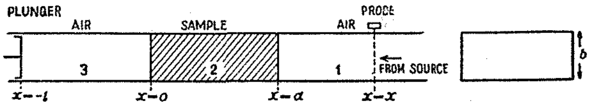

Fig. 1

The transversal components of the field of H_{01} mode which we now use are expressed as follows:

$$\left.\begin{aligned} E_i(x)_t &= A_{ti}e^{\gamma_i x}(1+r_i)e^{j\omega t} \\ H_i(x)_t &= \frac{A_{ti}}{Z_i}e^{\gamma_i x}(1-r_i)e^{j\omega t} \end{aligned}\right\} , \tag{1}$$

where
$$\left.\begin{aligned} Z_i &= \frac{j\omega\mu_i}{\gamma_i} \\ A_{ti} &\propto \frac{\pi}{b}\sin\frac{\pi v}{b} \end{aligned}\right\} \tag{2}$$

$$r_i(x) = R_i e^{-2\gamma_i x} \tag{3}$$

and $i=1,2,3$ shows the field in each corresponding column, Z_i is the field impedance in the x-direction, r_i the complex reflection coefficient (R_i is the reflection coefficient at $x=0$) and γ_i the propagation constant. As $\gamma_1 \fallingdotseq \gamma_3$, and $Z_1 \fallingdotseq Z_3$, we let those be represented γ_g and Z_g respectively, and those in the column 2 by γ_a and Z_a respectively. In the previous paper, γ_g, γ_a, Z_g and Z_a are not used, but γ_1, γ_2, Z_1 and Z_2 are preserved. The suffix "t" showing the transversal component is omitted in the following.

Now, if we consider the loss of the terminating plate, at $x=-l$ the field impedance, $E_3(-l)/H_3(-l)$ ($\equiv X$) is not zero, and from (1) we obtain:

$$X = \frac{1+r_3(-l)}{1-r_3(-l)} = \frac{1+R_3e^{2\gamma_g l}}{1-R_3e^{2\gamma_g l}} \quad . \tag{4}$$

Combining this with (1), at $x=0$ the following equations are derived:

$$\frac{E_3(0)}{H_3(0)} = Z_g\frac{1+R_3}{1-R_3} = Z_gX'$$

$$\frac{E_2(0)}{H_2(0)} = Z_a\frac{1+R_2}{1-R_2}$$

where
$$X'(l) \equiv \frac{1+R_3(l)}{1-R_3(l)} = \frac{X+\tanh\gamma_g l}{X\tanh\gamma_g l+1} \quad . \tag{5}$$

From (2) and the condition of continuity at $x=0$, we obtain

$$R_2 = \frac{\frac{\gamma_a}{\gamma_g}X'-1}{\frac{\gamma_a}{\gamma_g}X'+1} \quad .$$

Substituting this R_2 into the following formula of T defined in the previous paper:

$$T=\frac{\gamma_g}{\gamma_a}\,\frac{e^{2\gamma_a d}+R_2}{e^{2\gamma_a d}-R_2}\quad, \tag{6}$$

we can derive the following equation:

$$\frac{\gamma_d}{\gamma_g}T=\frac{e^{2\gamma_a d}+\dfrac{\dfrac{\gamma_a}{\gamma_g}X'-1}{\dfrac{\gamma_a}{\gamma_g}X'+1}}{e^{2\gamma_a d}-\dfrac{\dfrac{\gamma_a}{\gamma_g}X'-1}{\dfrac{\gamma_a}{\gamma_g}X'+1}} \tag{7}$$

or

$$e^{2\gamma_a d}\left(\frac{\dfrac{\gamma_a}{\gamma_g}T-1}{\dfrac{\gamma_a}{\gamma_g}T+1}\right)=\frac{\dfrac{\gamma_a}{\gamma_g}X'-1}{\dfrac{\gamma_a}{\gamma_g}X'+1}$$

If we let T_1 and T_2 be the values of T corresponding to the positions of the plunger (the terminating plate) at $l=l_1$ and $l=l_2$, and also X_1' and X_2' be the corresponding values of X, we have

$$\frac{\dfrac{\gamma_a T_1-\gamma_g}{\gamma_a T_1+\gamma_g}}{\dfrac{\gamma_a T_2-\gamma_g}{\gamma_a T_2+\gamma_g}}=\frac{\dfrac{\gamma_a X_1'-\gamma_g}{\gamma_a X_1'+\gamma_g}}{\dfrac{\gamma_a X_2'-\gamma_g}{\gamma_a X_2'+\gamma_g}}$$

$$\frac{X_2'+T_1}{\gamma_a^2 T_1 X_2'+X}=\frac{X_1'+T_2}{\gamma_a^2 T_2 X_1'+\gamma_g^2}\,.$$

and then we obtain the general expression:

$$\left(\frac{\gamma_a}{\gamma_g}\right)^2=\frac{(T_2-T_1)+(X_1'-X_2')}{X_1'X_2'(T_2-T_1)+T_1T_2(X_1'-X_2')} \tag{8}$$

where $\gamma_g=\alpha_g+j\beta_g$, $\alpha_g=\alpha_{cg}+\alpha_{sg}+\alpha_{ag}+\alpha_j$, and α_{cg}, α_{sg}, α_{ag} and α_j denote the attenuations by the wall loss, the slot, the air loss and the junction loss respectively.

Now, if there is no loss in the terminating plate, and the waves are perfectly reflected by the plate, $X=0$ (i.e. $R_3=-e^{-2\gamma_g l}$)in (5), and so

$$X_1'=\tanh\,\gamma_g l_1\ \ (\equiv K_1)$$

$$X_2'=\tanh\,\gamma_g l_2\ \ (\equiv K_2)$$

Accordingly (8) is reduced to

$$\left(\frac{\gamma_a}{\gamma_g}\right)^2=\frac{(T_2-T_1)+(K_1-K_2)}{K_1K_2(T_2-T_1)+T_1T_2(K_1-K_2)} \tag{9}$$

Furthermore, when α_g is neglected, and X_1' and X_2' are replaced by K_1 and K_2, we obtain:

$$K_1 = j\tan\ \beta_g l_1$$
$$K_2 = j\tan\ \beta_g l_2$$

and these agree with (12) shown in the previous paper.

The expression in which α_g is considered, corresponding to (28) in the previous paper is as follows:

$$\left(\frac{\gamma_d}{\gamma_g}\right)^2 = \frac{\varepsilon' - \left(\frac{\lambda}{\lambda_c}\right)^2 - j\varepsilon''}{\varepsilon_g' - \left(\frac{\lambda}{\lambda_c}\right)^2 - j\varepsilon_g''}$$

where

$$\left.\begin{aligned} \varepsilon_g' &\equiv \left(\frac{\lambda}{2\pi}\right)^2\left[\alpha_g^2 - \left(\frac{2\pi}{\lambda_g}\right)^2\right] + \left(\frac{\lambda}{\lambda c}\right)^2 \doteqdot 1 \\ \varepsilon_g'' &\equiv \frac{\alpha_g}{\pi\lambda_g}\lambda^2 \quad . \end{aligned}\right\}.$$

Combining (8) with the above equation, we obtain the explicit expression of ε^*:

$$\frac{\varepsilon' - \left(\frac{\lambda}{\lambda_c}\right)^2 - j\varepsilon''}{\varepsilon_g' - \left(\frac{\lambda}{\lambda_c}\right)^2 - j\varepsilon_g''} = \frac{(T_2 - T_1) + (X_1' - X_2')}{X_1'X_2'(T_2 - T_1) + T_1T_2(X_1' - X_2')}\ . \tag{10}$$

This is the fundamental equation of measurement in which the losses of the guide wall, the terminating plate and others are considered. The value of X' in (10) can be calculated from (5) and T is given independently of α_g as follows:

$$T = \frac{\Gamma + j\cot\frac{2\pi}{\lambda_g}x_0}{1 + j\Gamma\cot\frac{2\pi}{\lambda_g}x_0} \tag{11}$$

and this is (23) in the previous paper. In (11), x_0 is the position of the minimum of the electric field intensity and Γ the VSWR. For practice of measurement, we choose two adequate lengths of the column 3, $l = l_1$ and $l = l_2$ by adjusting the plunger and for respective lengths we measure Γ and x_0 in the column 1. Then ε' and ε'' of the sample can be determined by (10).

2. FORMULAE OF ε^* IN TERMS OF MEASURED QUANTITIES WHEN THE LOSSES OF THE GUIDE WALL, THE TERMINATING PLATE AND OTHERS CAN BE NEGLECTED

In the following, we present only the measurement methods referring to the position x_0 of minimum E_1 and those referring to the position x_0' of maximum E_1 are to be reported in the next paper. Then, when the losses of the guide wall, the terminating plate and others are neglected, we put $\alpha_g=0$ and $X=0$, then the expressions of ε' and ε'' are reduced from (10) to

$$\left.\begin{aligned}\varepsilon' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{PR+QS}{R^2+S^2}+\left(\frac{\lambda}{\lambda_c}\right)^2\\ \varepsilon'' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{PS-QR}{R^2+S^2}\end{aligned}\right\} \tag{12}$$

with

$$\left.\begin{aligned}P &\equiv (\Gamma_2-\Gamma_1)(\theta_1\theta_2+1)-(k_1-k_2)(\Gamma_2\theta_1+\Gamma_1\theta_2)\\ Q &\equiv (\theta_2-\theta_1)(\Gamma_1\Gamma_2-1)+(k_1-k_2)(\theta_1\theta_2-\Gamma_1\Gamma_2)\\ R &\equiv k_1k_2(\Gamma_1-\Gamma_2)(\theta_1\theta_2+1)-(k_1-k_2)(\Gamma_1\theta_1+\Gamma_2\theta_2)\\ S &\equiv k_1k_2(\theta_1-\theta_2)(\Gamma_1\Gamma_2-1)+(k_1-k_2)(\Gamma_1\Gamma_2\theta_1\theta_2-1)\end{aligned}\right\}, \tag{13}$$

where

$$\theta_i \equiv \tan\frac{2\pi}{\lambda_g}x_{oi}$$

$$k_i \equiv \tan\frac{2\pi}{\lambda_g}l_i$$

and suffix $i=$ 1, 2 show numbers of order of the measurement.

Calculation with (12) and (13) for arbitrary lengths l_i of the column 3 is generally very complicated, but it is comparatively simplified by the following values of l:

i) $l=(2n+1)\frac{\lambda_g}{4}$

ii) $l=n\frac{\lambda_g}{2}$

iii) l such that $x_o=(2n+1)\frac{\lambda_g}{4}$

iv) l such that $x_o=n\frac{\lambda_g}{2}$,

where $n=1,2,3,\cdots\cdots\cdots$.

By an adequate combination of two pairs of l and x_0 (of which one pair or two may be selected from the above mentioned four pairs) the convenient method is produced.

In this paper, we select two pairs from the above four. Thus we obtain convenient methods as follows:

1) method of combining i) with iii) (letting $i=1,2$ be in this order)
2) method of combining i) with iv) (〃)
3) method of combining i) with ii) (〃)
4) method of combining iii) with ii) (〃)
5) method of combining iv) with ii) (〃)
6) method of combining iii) with iv) (〃).

Among the above methods, method 3 when $n=0$ is the so called open-short method by W. H. Surber and G. E. Crouch and the explicit expressions of ε' and ε'' in methods 2, 4 and 6 which we have used in our experiment reported in this paper, are reduced respectively to:

in the case of Method 2,

$$\left.\begin{aligned}
\varepsilon' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{\dfrac{\Gamma_1}{\Gamma_2}(1+\theta_1^2)+k_2\theta_1(1-\Gamma_1^2)}{\left[\dfrac{\Gamma_1}{\Gamma_2}(\theta_1-k_2)+k_2\right]^2+\left[\dfrac{1}{\Gamma_2}+k_2\theta_1\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)\right]^2}+\left(\frac{\lambda}{\lambda_c}\right)^2 \\
\varepsilon'' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{\theta_1\left[\dfrac{1}{\Gamma_2}+k_2\theta_1\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)\right]-\Gamma_1\left[\dfrac{\Gamma_1}{\Gamma_2}(\theta_1-k_2)+k_2\right]}{\left[\dfrac{\Gamma_1}{\Gamma_2}(\theta_1-k_2)+k_2\right]^2+\left[\dfrac{1}{\Gamma_2}+k_2\theta_1\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)\right]^2}
\end{aligned}\right\} \quad (14)$$

in the case of Method 4,

$$\left.\begin{aligned}
\varepsilon' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{k_1\dfrac{\Gamma_2}{\Gamma_1}(1+\theta_2^2)+\theta_2(1-\Gamma_2^2)}{k_1[1+(\theta_2\Gamma_2)^2]}+\left(\frac{\lambda}{\lambda_c}\right)^2 \\
\varepsilon'' &= \left[1-\left(\frac{\lambda}{\lambda c}\right)^2\right]\frac{\dfrac{\Gamma_2}{\Gamma_1}\left[\theta_2^2(\Gamma_2-\Gamma_1)-k_1\theta_2\Gamma_2\right]+\left[\dfrac{1}{\Gamma_1}(k_1\theta_2+1)-\Gamma_2\right]}{k_1[1+(\theta_2\Gamma_2)^2]}
\end{aligned}\right\} \quad (15)$$

and in the case of Method 6,

$$\left.\begin{aligned}
\varepsilon' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{(k_1-k_2)^2\dfrac{\Gamma_1}{\Gamma_2}-\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)^2k_1k_2}{\left[\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)k_1k_2\right]^2+\left[(k_1-k_2)\dfrac{\Gamma_1}{\Gamma_2}\right]^2}+\left(\frac{\lambda}{\lambda_c}\right)^2 \\
\varepsilon'' &= \left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]\frac{(k_1-k_2)\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)\left(k_1k_2+\dfrac{\Gamma_1}{\Gamma_2}\right)}{\left[\left(\dfrac{1}{\Gamma_2}-\Gamma_1\right)k_1k_2\right]^2+\left[(k_1-k_2)\dfrac{\Gamma_1}{\Gamma_2}\right]^2}
\end{aligned}\right\} \quad (16)$$

For the lengths of the sample used in our experiments by methods 1 and 5, the differences of measured values of x_0 have had major influences on the calculations, so the required mechanical accuracy of the apparatus is large.

Accordingly, we omit the reports of methods 1 and 5 here, but we are planning to examine methods 1 and 5 with adequate lengths of sample and to report about the results in the near future. In method 3 the situation is similar to that stated above concerning methods 1 and 5.

In fact, for $n=0$ *i.e.*, by Surber's method, in our experiments the measurements have been often accompanied with errors, but by adjusting the length l, we have been able to attain the practical measurement by method 3, as shown later in the present paper.

We have experienced that method 6 is particularly practical and simple both in operation and in calculation.

3. EXPERIMENT

The block diagram of the apparatus used in our measurements is shown in Fig. 2.

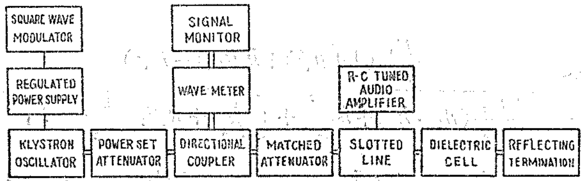

Fig. 2

Shimadzu Seisakusho has kindly made this apparatus by our request. The wave guide is of inside dimensions: 22.86 ± 0.04 mm × 10.16 ± 0.04 mm and of outside dimensions: 25.4 mm × 12.7 mm and made of brass. Its inner wall is gold-plated and the standing wave detector has a slotted line of width 2.5 mm. The scale of slotted line and the length of dielectric cell have been determined by means of a travelling microscope.

The frequency used is 9450 Mc/sec ($\lambda=3.172$ cm). The wave length within the guide, λ_g is 4.412 cm which was obtained as an average value from measurements off maximum and minimum points of the standing wave in the air column 1. The cut-off wave length, λ_c is 4.572 cm which has been determined by the geometry of the guide and agrees in 4 significant figures with the value calculated by formula, $1/\lambda_c^2 = 1/\lambda^2 - 1/\lambda_g^2$, using the above measured values of λ and λ_g.

The sample used for the examination of these methods is commercial cetyl alcohol ($C_{15}H_{31}CH_2OH$) which has been broken to pieces and melted and then gradually solid-

Table 1.

Sample A $d=3.295$ cm, cross section: 2.22×1.00 cm^2				
	Method 2	Method 4	Method 6	Method 3
Γ_1	3.29	5.48	2.23	5.48
Γ_2	5.75	2.23	5.75	3.29
θ_1	-1.03540			($\theta_1'=-0.19765$)
θ_2		-0.19765		($\theta_2'=1.03540$)
k_1		-1.09938	-1.09938	
k_2	0.19026		0.19026	
ε'	2.317	2.319	2.310	2.321
ε''	0.503	0.411	0.565	0.356
$\tan\delta$	22×10^{-2}	18×10^{-2}	24×10^{-2}	15×10^{-2}

ified into the shape of the guide. The signs A and B in the following tables distinguish the samples which correspond to different manufactures and different heat-treatments.

The results of experiments performed by different methods and with different sample lengths are shown in Tables 1, 2 and 3. It must be noticed that in Table 3 the calculatlons have not been so precise as in Tables 1 and 2.

The values of θ_1' and θ_2' are calculated by $\theta_i' = \tan \dfrac{d+n\frac{\lambda_g}{2}}{\lambda_g}$ which is Surber's expression.

One can see directly the coincidence especially among the values of ε'

Table 2.

Sample B d=4.351 cm, cross section: 2.22×1.00 cm²						
Γ_1	Γ_2	k_1	k_2	ε'	ε''	tan δ
3.21	10.82	−0.38106	0.73722	2.315	0.324	14×10^{-2}

by method 6.

Table 3.

Sample A				
	Method 3		Method 6	
	d=3.18 cm	d=3.23 cm	d=3.18 cm	d=3.23 cm
Γ_1	4.62	5.10	2.26	2.25
Γ_2	5.50	4.30	5.98	5.90
θ_1'	-0.4214	-0.3091		
θ_2'	0.4955	0.6878		
k_1			-0.5327	-0.6977
k_2			0.4623	0.3225
ε'	2.56	2.44	2.33	2.39
ε''	0.259	0.175	0.350	0.464
tan δ	10×10^{-2}	7×10^{-2}	15×10^{-2}	19×10^{-2}

In performing these experiments we have payed attention to the following points.

1) Stabilization of the power source and the frequency.
2) Characteristics of the detector, the amplifier and the multiplier.
3) Sensitivity of the probe.
4) To minimize the clearance crossing the electric field direction between the sample

and the wall.

5) To make the length of the column 3 sufficiently long, that is, in stead of the lengths, $l=\lambda_g/4$ and $l=0$ which Surber has used in his open-short method, we have let $l=(2n+1)\lambda_g/4$ and $l=n\lambda_g/2(n\gg 1)$ in method 3 in order to avoid the inconvenient effect of the gap between the sample and the plunger in Surber's method. The reason is discussed in § 4.

6) To choose the adequate sample lengths to avoid the errors which attend the measurement when k_i and θ_i are large in absolute values.

4. DISCUSSION

1) Effects of the Loss of the Terminating Plate on the Sample Length

We have stated that method 3 when $n=0$ agrees with the open-short method, which W. H. Surber and G. E. Crouch have proposed as the measurement method of the medium and high loss materials.

Accordingly they have not considered the effects of the loss of the guide wall, but considered those of the terminating impedances behind the sample R_{sc} and R_{oc} which are effectively not 0 and ∞ because the wall loss may be neglected against the sample loss, but R_{sc} and R_{oc} may affect the measurement with certain lengths of the sample even if the deviations of R_{sc} and R_{oc} from 0 and ∞ are very small.

Concerning method 3, we can of course derive the correction factor corresponding to the above consideration from (7). This factor agrees with that derived circuit-theoretically by Surber. Namely, we transform (7) to the following form:

$$\frac{\gamma_a}{\gamma_g}T=\frac{\left(\frac{\gamma_a}{\gamma_g}X'+1\right)+\left(\frac{\gamma_a}{\gamma_g}X'-1\right)e^{-2\gamma_a d}}{\left(\frac{\gamma_a}{\gamma_g}X'+1\right)-\left(\frac{\gamma_a}{\gamma_g}X'-1\right)e^{-2\gamma_a d}} \tag{7$'$}$$

Distinguishing T_s' and X_s' when the termination is short $(l=n\lambda_g/2)$ and open $(l=(2m+1)\lambda_g/4)$ with the suffixes s and o, we can derive the following equations:

$$T_s=Z_a'\frac{\tanh\gamma_a d+\left(\frac{X_s'}{Z_a'}\right)}{1+\left(\frac{X_s'}{Z_a'}\right)\tanh\gamma_a d} \tag{17}$$

$$T_o=Z_a'\frac{\coth\gamma_a d+\left(\frac{Z_a'}{X_o'}\right)}{1+\left(\frac{Z_a'}{X_o'}\right)\coth\gamma_a d}, \tag{18}$$

where $Z_a'=\dfrac{Z_a}{Z_g}=\dfrac{\gamma_g}{\gamma_a}$ (normalized impedance), [5]

$$X_s' = \frac{X+\tanh\frac{n\alpha_g}{2}\lambda_g}{X\tanh\frac{n\alpha_g}{2}\lambda_g+1} \doteqdot X+\frac{n\alpha_g}{2}\lambda_g \tag{17$'$}$$

$$X_o' = \frac{X+\coth\frac{(2m+1)\alpha_g}{4}\lambda_g}{X\coth\frac{(2m+1)\alpha_g}{4}\lambda_g+1} \doteqdot \frac{1}{X+\frac{(2m+1)\alpha_g}{4}\lambda_g}, \tag{18$'$}$$

which are obtained from (5), and when $n=0$ and $m=$any integer, from (17)$'$ and (18)$'$ the relation:

$$\frac{1}{X_o'} \cong X_s + \frac{(2m+1)}{4}\alpha_g\lambda_g \tag{19}$$

holds. As X is sufficiently small, we have from (17),(18),(17)$'$ and (18)$'$

$$T_sT_o \cong Z_d'^2\left[1+\frac{2}{\sinh 2\gamma_a d}\left(\frac{X_s'}{Z_d'}-\frac{Z_d'}{X_o'}\right)\right]. \tag{20}$$

Eq. (36) in the previous paper is written here with the present notations:

$$T_sT_o = \left(\frac{\gamma_g}{\gamma_a}\right)^2.$$

Therefore the second term of (20) gives the correction factor concerning method 3 which becomes very large at minima of $\sinh 2\gamma_a d$, which occurs for $d=n\lambda_a/4$, when the present notations Z_d', T_s, T_o, X_s and X_o are replaced by Surber's notations Z_d, Z_{sc}, Z_{oc}, R_{sc} and R_{oc} respectively. We choose the sample lengths, as shown in the tables, with respect of the effects mentioned above, but as $2\gamma_a d$ is not small in the medium loss, for instance cetyl alcohol and other high loss materials, the correction factor is still small.

2) Effects of the Loss of the Terminating Plate on its Position

The expressions of ε^* we have presented here, have been derived under the condition that the terminating impedance X is zero, but when X is not zero, how small it may be, it affects not only the sample length but the electrical position of the termination and becomes a cause of error when $X\neq0$; the error Δ caused by putting $X=0$ is considered, which is from (5) written as follows:

$$\Delta = \left|\frac{X+\tanh\gamma_g l}{X\tanh\gamma_g l+1} - \tanh\gamma_g l,\right|, \tag{21}$$

which is easily reduced to:

$$\begin{aligned}\Delta &= \frac{1}{\left|\frac{1}{2}\sinh 2\gamma_g l+\frac{\cosh^2\gamma_g l}{X}\right|}\\ &\cong \frac{|X|}{\cosh^2\left(\alpha_g l+\frac{2\pi}{\lambda_g}l\right)}\end{aligned} \tag{22}$$

From this expression it is found that the large error evidently attends the measurement with $l \fallingdotseq (2m+1)\lambda_g/4$, and this is one of disadvantages of the measurement using open termination.

It is also obvious that the larger m is, the smaller becomes Δ, at $l=(2m+1)\lambda_g/4$ and this is the reason for us to use the open termination with large m.

CONCLUSION

In the present paper we have proposed several new methods to determine ε^* by measuring the position of E_{min} in front of the sample column and VSWR and reported about the experiments performed on cetyl alcohol, in which the values of the dielectric constant ε' shown in Table 1 and 2 agree in 3 significant figures: $\varepsilon'=2.31$.

On the other hand, according to the results[6] reported about the measurements in the meter wave region, for instance 5480 c/s, the dielectric constant and the conductivity of cetyl alcohol change largely in the neighbourhoods of the melting point (about 50°C) and of the transition point (about 44°C) and are greatly affected by the heat-treatment, and the values of $\tan\delta$ exist within the extent of about $0.02\sim0.8$.

From our experimental results of cetyl alcohol obtained with various sample lengths as shown in the tables, it may be concluded that $\varepsilon'=2.3$ and $\tan\delta=(10\sim25)\times10^{-2}$.

Also, we have experienced that the curve of the standing wave in the air column in front of the sample becomes sharper in the neighbourhood of the position E_{max} than in that of the position E_{min}, for the properly selected positions of the plunger. So the method based on an appropriate combination of the measurements of the position E_{min} and the position E_{max} seems to be better than that reported in the present paper and will be discussed in the next report of this work.

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