The Phase Shift Analysis on Alpha-Alpha Scattering in the Energy Range from 22.9 to 28.9 Mev

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The scattering of alpha particles from Helium has been experimentally studied wi thincreased accuracy at five energies between 22.9 and 28.9 Mev. The phase shift analysis up to $L=6$ has been performed for each energy. The result shows that the S-, D- and G-wave phase shifts are necessary to fit the data and the I-wave phase shift is small in this energy range.

The level parameters for the G-state have been extracted using the dispersion theory and they are compatible with the alpha-alpha model of Be^8.

INTRODUCTION

The scattering of two alpha particles has been studied for many years. Although accuracies of the early experiments[1] were limited by the necessity of using natural alpha particles, these results are able to show the verification of the scattering theory of identical particles of Mott and the exsistence of nuclear forces other than the Coulomb force. Wheeler[2], in 1941, analyzed the early scattering data in terms of excited states in the compound nucleus Be^8,

Recently, the precise alpha-alpha scattering experiments have been performed by Heydenburg and Temmer[3] at the Carnegie Institution in the region from 0.15 to 3 Mev; by Russell, Phillips, and Reich[4] at the Rice Institute in the region from 2.5 to 5.5 Mev; by Jones, Phillips, and Miller[5] at the same Institute in the region from 5 to 9 Mev; by Berk, Steigert, and Salinger[6] at Yale University at the energy of 7.56 Mev; by Nilson, Jentschke. Briggs, Kerman, and Snyder[7,8], at the University of Illinois in the region from 12.3 to 22.9 Mev; by Bredin, Burcham, Evans, Gibson, McKee, Prowse, Rotblat, and Snyder[9] at the University of Birmingham in the region from 23.1 to 38.4 Mev; and by Conzett, Igo, Shaw, and Slobodrian[10] at Berkeley in the region from 36.8 to 47.3 Mev. All of these results have been analyzed in terms of partial wave phase shifts, and it is found that only nonzero phase shifts needed to fit the data to a laboratory energy of 47 Mev are those of $L=0$, 2, 4, 6, and 8. The behavior of these phase shifts has given evidence for several virtual excited states in Be^8 in this energy range; a narrow 0^+ ground state at 190 Kev bombarding energy, a broad 2^+ state at 3.1 Mev in Be^8, and an even broader 4^+ state at approximately 12 Mev in Be^8. The differential cross sections of alpha-alpha scattering are strongly energy-dependent, and consequently data at several neighboring energies are needed to explore resonance effects in the phase shifts. Therefore, sufficient justification was felt in repeating the

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measurement with increased accuracy at more closely spaced energy intervals above 23 Mev.

An attempt has been made to measure the differential cross section of alpha-alpha scattering at five energies between 23 and 29 Mev[11]. This paper reports the phase shifts analysis of these results.

DIFFERENTIAL CROSS SECTION MEASUREMENT

Alpha particles accelerated to an energy of 30 Mev by the Kyoto University 105*cm* cycrotron were used. The alpha particle beam was led into a scattering chamber in which a cylindrical Helium gas target was placed. After passing through the chamber, the beam was collected in a Faraday cup and integrated. A thin CsI crystal was used to detect elastically scattered alpha particles. A collimation system consisting of two rectangular slits was placed in front of the CsI crystal to determine the scattering volume and the solid angle subtended by the detector. The angular position of the detector was reproducible to $\pm 0.05°$ in the laboratory system.

The incident alpha particle beam was degraded to the desirable energies by placing Aluminum absorbers in front of the scattering chamber, then the mean energy of the beam at the scattering chamber was determined from its range in Aluminum with the accuracy of about 1%.

Absolute differential cross setions were evaluated from the number of detector counts, the integrated charge and the Helium pressure as described in Reference 11. The uncertainty in the absolute values of differential cross sections were separated from those in the relative values. The absolute uncertainty amounts to

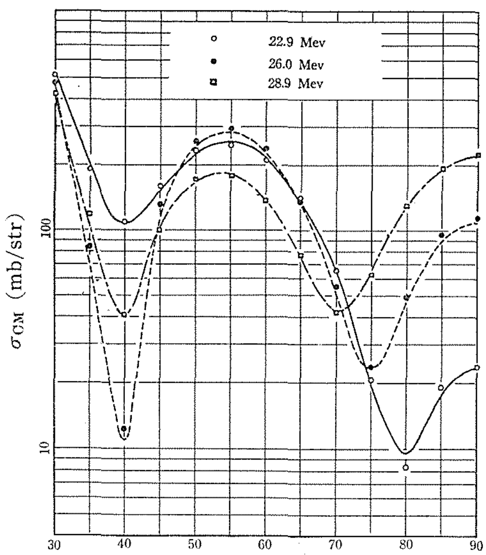

Fig. 1. Computed and observed differential cross sections of alpha-alpha scattering at 22.9, 26.0 and 28.9 Mev, curves computed with the best sets of the phase shifts obtained from the present work.

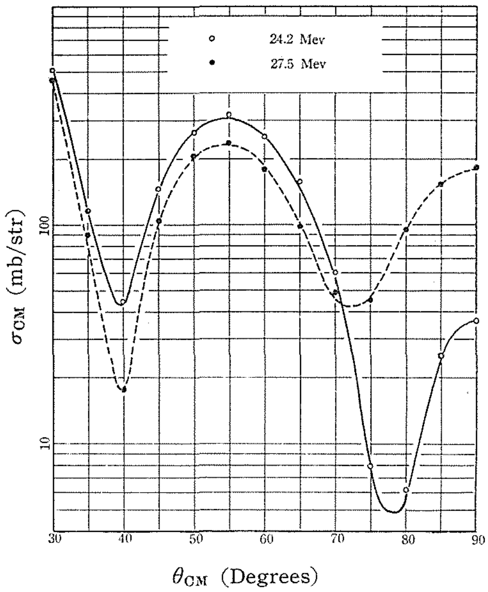

Fig. 2. Computed and observed differential cross sections of alpha-alpha scattering at 24.2 and 27.5 Mev, curves computed with the best sets of the phase shifts obtained from the present work.

3%, while the relative uncertainty varies from 1 to 10%. The measured cross sections at 22.9±0,3, 24.2±0.3, 26.0±0.3, 27.5±0.3, and 28.9±0.3 Mev are plotted in Figures 1 and 2.

PHASE SHIFT ANALYSIS

The center of mass differential cross section σ_{CM} of alpha-alpha scattering can be expressed in terms of the nuclear phase shifts α_L as follows[12],

$$\sigma_{CM}(\theta)=\frac{4^2e^4}{M^2V^2}\Big|\operatorname{cosec}^2(\theta/2)\exp\left[i\eta ln\operatorname{cosec}^2(\theta/2)\right]$$
$$+\sec^2(\theta/2)\exp\left[i\eta ln\sec^2(\theta/2)\right]$$
$$+\frac{2i}{\eta}\sum_{L=0,2,4}^{\infty}(2L+1)P_L(\cos\theta)\exp(2i\xi_L)\left[\exp(2i\alpha_L)-1\right]\Big|^2 \qquad (1)$$

with

$$\eta=\frac{4e^2}{\hbar V}, \qquad \xi_L=\sum_{S=1}^{L}\tan^{-1}(\eta/s), \qquad \xi_0=0,$$

where M denotes the mass of alpha particles, V is the relative velocity between alpha particles, θ is the center of mass scattering angle and $P_L(cos\,\theta)$ is the Legendre polynomial of order L.

The best set of phase shifts was extracted from the measured cross sections in the following manner which has been successfully applied to the phase shift analysis of pion-proton scattering[13]. Cramer[14] gives the method of the maximum likelihood to determine magnitudes from an indirect measurement. Suppose that we have a system,

$$
\begin{aligned}
f_1 &= a_{11}m_1 + a_{12}m_2 + \cdots\cdots + a_{1k}m_k, \\
f_2 &= a_{21}m_1 + a_{22}m_2 + \cdots\cdots + a_{2k}m_k, \\
&\cdots\cdots \\
f_n &= a_{n1}m_1 + a_{n2}m_2 + \qquad + a_{nk}m_k,
\end{aligned}
\tag{2}
$$

where the coefficients a_{ij}'are assumed to be known constants, and m_k are the unknown true values of parameters wich have to be determined form a measurement on the quantities f_i. According to the maximum likelihood method, the best estimates m_j^* of the parameters m_j and their errors are given as follows,

$$
m_s^* = \sum_{r=1}^{k} E_{rs}C_r \pm \sqrt{\frac{\chi^2 E_{ss}}{n-k}}, \tag{3}
$$

with $\quad C_r = \sum_{i=1}^{n} p_i a_{ir} x_i, \quad E_{rs} = B_{rs}/B,$

$B_{rs}=$Cofactor of b_{rs}; $B=\det|b_{rs}|$,

$b_{rs} = \sum_{i=1}^{n} p_i a_{ir} a_{is}$, $p_i=$Weight of x_i,

$\chi^2 = \sum_{i=1}^{n} p_i (x_i - f_i^*)^2,$

$f_i^*=$Calculated value from the m_i^* using Eq. (2)

$x_i=$Measured value of the quantity f_i.

However, in our case, the cross section is not a linear function of phase shifts, so that the following substitution has to be made to form it suitable for Cramer's treatment.

$$
f_i \equiv \sigma_i(\alpha_k + \delta\alpha_k) - \sigma_i(\alpha_k) = \sum_{j=1}^{k} \frac{\partial \sigma_i}{\partial \alpha_j}\delta\alpha_j,
$$

$x_i \equiv \alpha_i^m - \sigma_i^{cal}$, $\sigma_i^n=$Measured cross section,

$m_j \equiv \delta\alpha_j, \quad a_{ij} = \partial\sigma_i/\partial\alpha_j.$

Using these substitutions, we have to start with a certain set of phase shifts α_k^s from which we calculate the values of σ_i^{cal} and $\partial\sigma_i/\partial\alpha_j$, then a "best" set of corrections $\delta\alpha_k$ could be determined. This procedure would give us the best solution $\alpha^s{}_k + \delta\alpha_k$, only if the values of $\partial\sigma_i/\partial\alpha_j$ were independent of α_k. However, $\partial\sigma_i/\partial\alpha_j$ are depend on α_j, $\alpha_k^s + \delta\alpha_k$ are a set of phase shifts much closer to the true solution than the starting phase shifts α_k^s. It is necessary to repeat the analysis using $\alpha_k^s + \delta\alpha_k$ as the starting phase shifts. Normally about several iterations are sufficient.

The phase shift analysis has been performed with the assistance of an electric computer FACOM-128, using this mathematical procedure. The goodness of fit is expressed by the following quantity,

$$
\chi^2 = \sum_{i=1}^{n} p_i (\sigma^m{}_i - \sigma_i^{cal})^2,
$$

with $\quad p_i = 1/\varDelta_i^2,$

$\varDelta_i=$Standard deviation of the measured cross section.

The value of χ^2 for each iteration was calculated, and it decreased monotonously to a certain minimum value as the calculation carried on. The theory of the least squares implies the minimum value of χ^2 to be $n-k$, if we know the true standard deviation of the measurement. Therefore, a more sensitive measure of the goodness of fit is obtained from the root-mean-squares error ε, (*rms* error), defined as follows,

$$\varepsilon=(\chi^2/n-k)^{1/2}.$$

Our results show that the *rms* error varies from 1 to 1.6.

The best sets of phase shifts at all energies thus found are listed in Table 1 with the corresponding *rms* errors. The analysis was limitted up to the G-wave phase shift, since the results at Birmingham[9] and Berkeley[15] show that the K-wave phase shift ($L=8$) is essentially zero in the energy range from 30 to 44 Mev.

The errors in these phase shifts are assigned in the following manner. The values of σ_l^m and the corresponding relative errors were fed into the program as input data, then the computer found the best set of phase shifts and their errors defined by Eq. (3). This solution and error were accepted as the standard phase shift solution and the standard error. However, this error is only due to the relative error. Therefore, in order to evaluate the error arising from the absolute uncertainty, the values of σ_l^m were shifted by 3%, and the phase shift solution which corresponds to these σ_l^m was found. The difference between this phase shift solution and the standard solution is considered as the error arising from the absolute uncertainly. These errors, then, are combined with the standard errors in the usual manner, and these combined errors are listed in Table 1.

Table 1. The results of the phase shift analysis.

E_{lab}(Mev)	α_0(deg)	α_2(deg)	α_4(deg)	α_6(deg)	ε
22.9±0.3	−11.6±1.7	94.2±1.4	51.7±2.0	0.7±0.6	1.41
24.2±0.3	−12.6±1.9	93.0±2.3	72.5±2.5	2.1±1.0	1.61
26.0±0.3	−16.5±4.2	88.0±6.0	89.1±6.0	2.4±2.0	1.42
27.5±0.3	−28.8±4.2	81.1±3.3	97.7±4.5	0.4±1.0	1.15
28.9±0.3	−28.2±1.4	81.1±1.4	111.5±3.0	0.4±0.6	1.49

DISCUSSION

The phase shifts listed in Table 1 are plotted in conjunction with those from the results at Illinois and Birmingham in Figures 3 and 4.

The values of the S- and D-wave phase shifts seem to be consistent with each other and show no resonance behavior in this energy range. The I-wave phase shift is not accurately determined and shows no resonance behavior.

The values of the G-wave phase shift of the present analysis are systematically smaller than those derived at Birmingham. This discrepancy could be explained by the rather large uncertainty in the measured cross sections at Birmingham, since the absolute uncertainty is a serious source of errors in the phase shifrs, as is pointed out in Reference 9.

The variation of the G-wave phase shift can be fitted by the single-level dispersion theory of Wigner-Eisenbud[16] with a hard-sphere radious of $3.5\times10^{-13}cm$,

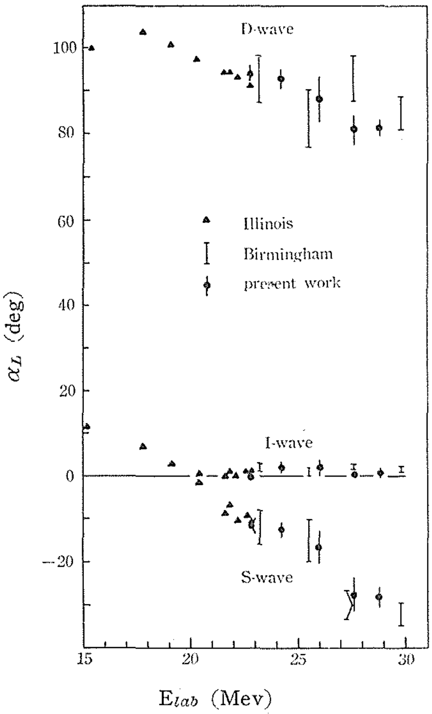

Fig. 3 Experimental alpha-alpha S-, D- and I-wave phase shifts.

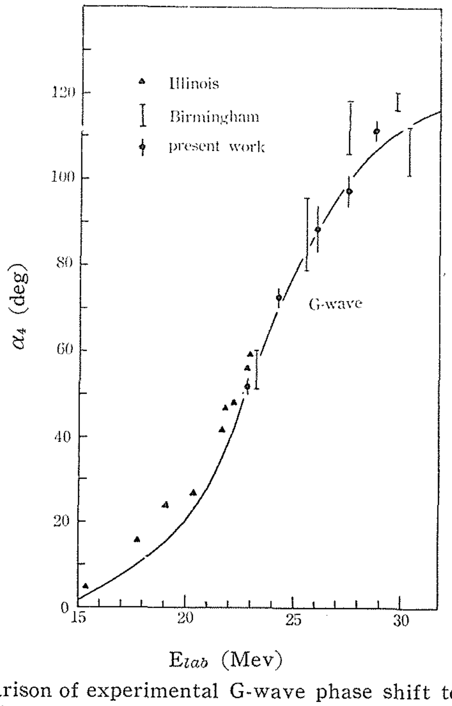

Fig. 4. Comparison of experimental G-wave phase shift to that obtained from the single-level dispersion theory with a hard-sphere radius of $3.5 \times 10^{-13} cm$, a reduced width of 3.5 Mev and a resonance energy of 12.5 Mev.

a reduced width of 3.5 Mev (1.3 Wigner limit) and a resonance energy of 12.5 Mev. Considering the reduced width of this state in comparison with the Wigner limit, $3h^2/2\mu a^2$ (=2.65 Mev), it is indicated that the G-state in Be^8 is almost wholly an alpha particle state. The lower states, the 0^+ ground state and the 2^+ excited state at 3.1 Mev in Be^8, are also confirmed as alpha particle states form the previous studies by Heydenburg *et al*[3]., Russell *et al*[4]., Jones *et al*[5]. and Nilson *et al*[8]. These three states ($0^+-2^+-4^+$), are predicted by both the shell model[17] and the alpha particle model[16], however, these experimental results lend credence to the latter model.

An alpha particle potential which is able to explain the behavior of these three wave phase shifts is most desirable. Among various types of alpha particle potentials, the Haefner potential[18] has been studied in detail by many authors[3,4,8]. Quite good fits were obtained by this potential, however in order to fit to all the phase shifts, it is necessary to use different values of the well depth for different values of *L*. None of potential of this form which would fit all the phase shifts without changing parameters has found.

As an attempt to solve this problem, Russell *et al*[4]. and Jones *et al*[5]. discussed the qualitative feature of the potential of alpha-alpha interaction which could explain the behavior of the S- and D-wave phase shifts. Russell *et al*., from their analysis of the low energy data, have presented a qualitative picture of the alpha-

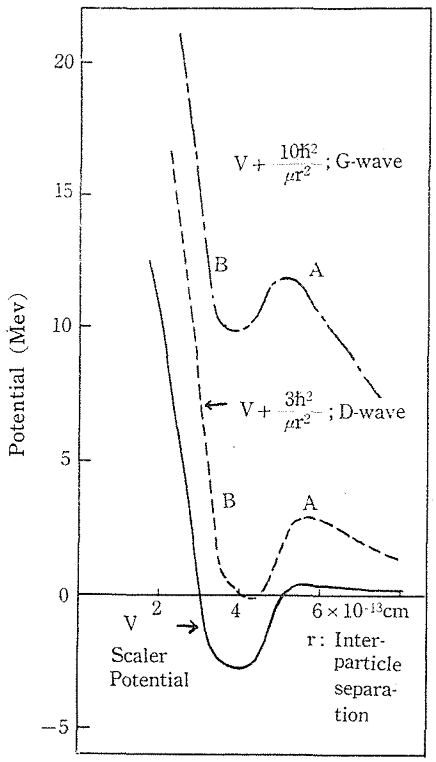

Fig. 5. Qualitative features of the alpha-alpha potential as presented by Russell *et al.* and Jones *et al.*

alpha potential as an attractive potential trough, of the order of $2\times10^{-13}cm$ wide and a few Mev deep, located at a radius of about $5\times10^{-13}cm$ and a potential core which must be more repulsive than the Coulomb potential. Jones et al. added a centrifugal potential, $L(L+1)\hbar^2/2\mu r^2$, to this scaler potential to explain the behavior of the D-wave phase shift. The qualitative shape of this potential is shown in Figure 5.

According to their analysis, the D-wave phase shift is fitted by the dispersion theory with a hard-sphere radius of $3.5\times10^{-13}cm$ up to 20 Mev in the laboratory system, while the lower energy D-wave phase shift (with $E_{lab}<6$ Mev) requires a larger hard-sphere radius. A possible explanation for this discrepancy is that at higher energies some of scattering does not occur at point A but rather at point B.

For the G-wave phase shift, this argument is also supported by the fact that the low energy G-wave phase shift (with $E_{lab}<23$ Mev) is fitted with a radius of $4.4\times10^{-13}cm$ as is described in Reference 8, while the present results are consistent with a radius of $3.5\times10^{-13}cm$. Therefore, it is hoped that these qualitative informations about the potential of alpha-alpha interaction would shed some light on finding a more sophisticated potential which can fit all these phase shifts.

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